Final Report

Division of Nuclear Physics in the Department of Energy

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Title: Towards the Chiral Limit in QCD

Principal Investigator: S.Chandrasekharan

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Susan Lasley, Assistant Director,
Office of Research Support,
Box 90077, Duke University,
Durham NC 27708.

I. INTRODUCTION

Computing hadronic observables by solving QCD from first principles with realistic quark masses is an important challenge in fundamental nuclear and particle physics research. Although lattice QCD provides a rigorous framework for such calculations many difficulties arise. Firstly, there are no good algorithms to solve lattice QCD with realistically light quark masses. Secondly, due to critical slowing down, Monte Carlo algorithms are able to access only small lattice sizes on coarse lattices. Finally, due to sign problems it is almost impossible to study the physics of finite baryon density.

Lattice QCD contains roughly three mass scales: the cutoff (or inverse lattice spacing) a^{-1} , the confinement scale Λ_{QCD} , and the pion mass m_π . Most conventional Monte Carlo algorithms for QCD become inefficient in two regimes: when Λ_{QCD} becomes small compared to a^{-1} and when m_π becomes small compared to Λ_{QCD} . The former can be largely controlled by perturbation theory thanks to asymptotic freedom. The latter is more difficult since chiral extrapolations are typically non-analytic and can be unreliable if the calculations are not done at sufficiently small quark masses. For this reason it has been difficult to compute quantities close to the chiral limit. The essential goal behind this proposal was to develop a new approach towards understanding QCD and QCD-like theories with sufficiently light quarks.

The proposal was based on a novel cluster algorithm discovered in the strong coupling limit with staggered fermions [1]. This algorithm allowed us to explore the physics of exactly massless quarks and as well as light quarks. Thus, the hope was that this discovery would lead to the complete solution of at least a few strongly coupled QCD-like theories. The solution would be far better than those achievable through conventional methods and thus would be able to shed light on the chiral physics from a new direction. By the end of the funding period, the project led to 6 publications, one in *physical review letters*, three in *physical review* as rapid communications and two conference proceedings. A long and detailed publication on the phase diagram of two-color QCD was just submitted to hep-lat archive. All the publications are listed in the sections titled *Papers published or submitted* and *Published conference proceedings*. Based on the projects completed, it is clear that the goal of the proposal was indeed partially realized.

The funds from this grant was used to essentially fund the education of a graduate student, Fu-Jiun Jiang. Mr. Jiang is expected to graduate in the summer of 2006 and has been offered a postdoctoral position at Bern University in Switzerland. He will be working with Professor Uwe-Jens Wiese there. In addition the funds was used to partially support a postdoctoral associate Costas Strouthos. Some of the funds were used for travel by Chandrasekharan, Fu-Jiun Jiang and Costas Strouthos. Chandrasekharan was also funded by an umbrella grant, during the funding period of this grant. The work done under that grant was reported separately in the progress report of that grant submitted in January 2006 and does not include the work reported here.

II. COMPLETED RESEARCH

A. Universality of the Chiral Phase Transition

S. Chandrasekharan and F.-J. Jiang

Lattice QCD calculations in the past have tried to determine the the universality class of the finite temperature chiral phase transition in QCD. Unfortunately, today the answer remains unclear. This is particularly true with staggered fermions, where there is a remnant $U(1)$ chiral symmetry. Since the symmetry properties do not change with the gauge coupling, one expects that the phase transition belongs to the $O(2)$ universality class in the strong coupling limit also. Using our algorithm we have shown this with unprecedented precision. In this calculation we worked with $U(3)$ gauge fields instead of $SU(3)$ since it is algorithmically simpler and the difference is expected to be unimportant. Thanks to our new algorithm we could work directly in the chiral limit and perform calculations at spatial volumes as large as 192^3 . Our work has been published as a rapid communications [2].

B. Kosterlitz-Thouless Universality in Dimer Models

S. Chandrasekharan and Costas Strouthos

Since the relevant chiral symmetry in strong coupling QCD with staggered fermions is $U(1)$, theoretical arguments would suggest that in $2 + 1$ dimensions the chiral transition should belong to the Kosterlitz-Thouless universality class. We found that this is indeed true. Apart from building confidence in universality arguments, this extension to lower dimensions is of interest to the condensed matter physics community in the context of models of high T_c superconductivity. Our work was published in [3].

C. Failure of Mean Field Theory at Large N

S. Chandrasekharan and Costas Strouthos

It is usually believed that Strong Coupling Lattice QCD with a large number of colors is governed by mean field theory. In fact there are many calculations that are performed using this assumption. We have discovered that although this is true at zero temperatures, the finite temperature chiral phase transition in this theory is not governed by mean field theory. As discussed above, strong coupling lattice QCD with staggered fermions, undergoes a second order phase transition in the $O(2)$ universality class. We have found that the critical region, where the $O(2)$ critical exponents are seen, does not shrink with N , as one might expect from arguments of Kogut, Stephanov and Strouthos. Since the result was new and controvertial we were able to publish our findings in Physical Review Results [5].

D. Connecting lattice QCD with chiral perturbation theory

S. Chandrasekharan and Costas Strouthos

Most calculations of lattice QCD are performed at heavy and unrealistic quark masses. In order to extract physically meaningful quantities, the results from these calculations have to be extrapolated to small quark masses using Chiral Perturbation Theory (ChPT). One of the difficulties has been that the non-analytic singularities that ChPT predicts are not seen in the calculations. Thus, it is not clear if ChPT is valid in the range where extrapolations are being performed.

At strong couplings, lattice QCD with staggered fermions has a chirally broken phase where the low energy physics of the Goldstone bosons must be described by ChPT. Since, using the new algorithm we can compute quantities at small quark masses, we can learn about the difficulties associated with chiral extrapolations.

We have studied this question recently. We focused at ChPT at finite temperatures since the chiral singularities are power-like and it is possible to avoid lattice artifacts in the chiral expansion by working close to the second order chiral phase transition. We discovered that results are indeed consistent with the predictions of ChPT, however quark masses required were much smaller than those reachable with conventional algorithms. This study makes it clear that it is important to continue to develop better algorithms to approach the chiral limit of QCD. Our work was published as a rapid communications in [4].

E. Phase Diagram of Two Color Lattice QCD in the Chiral Limit

S. Chandrasekharan and F.-J. Jiang

Given the difficulties of studying the phase diagram of QCD, several groups have focused their attention on QCD-like theories which do not suffer from sign problems at $\mu \neq 0$. The sign problem is avoided due to special properties of the fermion action which makes the fermion determinant real and positive. As a price for this baryons turn out to be bosons. In spite of this QCD-like theories provide interesting toy models for QCD. In particular these theories have a rich phase diagram. In addition, the low energy physics is rich and is described by some form of chiral perturbation theory. Thus, these theories allow us to study some of qualitative physics of QCD in a simpler setting.

A famous example of a QCD-like theory is two-color QCD which been extensively studied over the years both theoretically Two-color lattice QCD with staggered fermions (2CLQCD) is especially interesting due to an enhanced $U(2)$ symmetry at zero quark mass and baryon chemical potential. The long distance physics of 2CLQCD in the $T - \mu$ plane is described by $O(N) \times O(2)$ ($N = 2, 3$) field theories. Due to this connection it may have applications to certain phenomena in condensed matter physics such as superfluid ${}^3\text{He}$. Although some progress has been made in uncovering important qualitative features of the phase diagram of 2CLQCD, many quantitative questions have remained: (1) What is the order of the finite temperature chiral transition at zero and non-zero chemical potentials? (2) Can the low

FIG. 1: The phase diagram of two-color lattice QCD with staggered fermions at strong couplings.

energy physics at small T and μ be captured by an appropriate chiral perturbation theory. (3) What is the order of the phase transition that occurs when the lattice gets saturated with baryons at $T = 0$? The reason for the lack of quantitative progress can be traced to the fact that the difficulty of calculations are similar to those in QCD at zero chemical potential. Hence, all previous studies have been limited to small lattice sizes and relatively large quark masses.

In our work we extend the directed path algorithm [1] to study 2CLQCD in the chiral limit. Using our new algorithmic techniques, we were able to uncover the phase diagram completely in the strong coupling limit. We found clear evidence that the model undergoes a weak first order phase transition at $\mu = 0$ which becomes second order at a finite μ . The universality class of these phase transitions are governed by a $O(N) \times O(2)$ field theory with collinear order, with $N = 3$ at $\mu = 0$ and $N = 2$ at $\mu \neq 0$. The universality class of the second order phase transition at $\mu \neq 0$ appears to be governed by the decoupled XY fixed point present in the $O(2) \times O(2)$ field theory. Finally we showed that the quantum ($T = 0$) phase transition as a function of μ is a second order mean field transition. The phase diagram is given in figure 1.

A preliminary version of this work appeared in a recent conference proceedings [6] and the a long and detailed paper was submitted to the hep-lat archive recently [7]. This work forms the core of Fu-Jiun Jiang's thesis.

GRADUATE STUDENT SUPPORTED ON THIS GRANT

Graduate Student	Entered Program	Joined Group	Mentor	Expected Graduation
Fu-Jiun Jiang	2000	2002	Chandrasekharan	2006

Fu-Jiun Jiang has been Chandrasekharan's student since the summer of 2002. He has worked on strongly coupled QCD and Two-Color QCD which will form his thesis. He is currently working on verifying the predictions of chiral perturbation theory in two color QCD. He is expected to graduate in the summer of 2006 and will join Bern University in the fall of 2006.

INVITED TALKS

1. "Lattice Gauge Theories: Challenges and Opportunities", S. Chandrasekharan, Aspen workshop on *Deconfined Criticality and Gauge Theories in Condensed Matter*. Jul 18-Aug 15, 2005.
2. "*Strong Coupling Lattice QCD in the Chiral Limit*", theoretical physics seminar presented at the university of Washington St. Louis, October 2004.
3. "*New Approaches in Strong Coupling QCD* ", invited talk presented at the 12th International Conference on Recent Progress in Many Body Theories, Santa Fe, NM, 23-27, August 2004.
4. "*Strong Coupling QCD at finite T and μ* ", invited talk presented at the workshop on dense nuclear matter, Institute for Nuclear Theory, May 2004.
5. "*Chiral Limit in Strong Coupling Lattice QCD*", theoretical physics seminar presented at Jefferson Laboratory, April 2004.

CONTRIBUTED TALKS

1. "Chiral Limit of Two-Color QCD at Strong Couplings", Fu-Jiun Jiang International Symposium on Lattice Field Theory, Dublin, July 25-30, 2005.

PAPERS PUBLISHED OR SUBMITTED

1. "Phase Diagram of 2-color QCD", S. Chandrasekharan and F. J. Jiang, [arXiv:hep-lat/0602??].
2. "Failure of mean field theory at large N ", S. Chandrasekharan and C. G. Strouthos, Phys. Rev. Lett. **94**, 061601 (2005). [arXiv:hep-lat/0410036].
3. "Connecting lattice QCD with chiral perturbation theory at strong coupling", S. Chandrasekharan and C. G. Strouthos, Phys. Rev. D **69**, 091502 (2004) [arXiv:hep-lat/0401002].

4. “Chiral limit of strongly coupled lattice QCD at finite temperatures”, S. Chandrasekharan and F. J. Jiang, Phys. Rev. D **68**, 091501 (2003), [arXiv:hep-lat/0309025].
5. “Kosterlitz-Thouless universality in dimer models”, S. Chandrasekharan and C. G. Strouthos, Phys. Rev. D **68**, 091502 (2003), [arXiv:hep-lat/0306034].

PUBLISHED PROCEEDINGS

1. “Chiral limit of 2-color QCD at strong couplings”, S. Chandrasekharan and F. J. Jiang, PoS **LAT2005**, 198 (2005), [arXiv:hep-lat/0509117].
2. “Chiral and critical behavior in strong coupling QCD”, S. Chandrasekharan, Nucl. Phys. Proc. Suppl. **129**, 578 (2004), [arXiv:hep-lat/0309098].

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 - [6] S. Chandrasekharan and F. J. Jiang, PoS **LAT2005**, 198 (2005) [arXiv:hep-lat/0509117].
 - [7] “Phase Diagram of Two-Color QCD”, S. Chandrasekharan and F.-J. Jiang, hep-lat/0602031.