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September 1964

APPROXIMATE METHOD FOR DETERMINING
STATIC STRESSES IN REGULAR SUPPORT BLOCK
(BASED ON PLATE ANALYSIS)

(Title Unclassified)

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ABSTRACT

During nuclear reactor operation, the support block is subjected to static, dynamic and thermal loadings. This report demonstrates a method for predicting the stresses in a regular support block subjected to static pressure loadings. A computer program has been developed, based upon this solution, from which numerical results have been obtained. A comparison is included in this report between the predicted theoretical failure load for the block and experimental values obtained during the A5 Test Series (Support Block Structural Integrity - Mechanical and Thermal Tests).

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INTRODUCTION

One of the primary components of the nuclear reactor engine is the graphite support block (Figure 5). During reactor operation, these blocks are subjected to static, dynamic and thermal loadings. The regular support block is a modified hexagonal perforated cylinder with a center cylindrical hole through which a tie rod passes. (The irregular support block, which will not be considered here, contains two or three tie rods.)

The purpose of this analysis is to demonstrate a method for predicting the stresses in a regular support block subjected to static pressure loadings. (The pressure loadings result from the reaction of the fuel elements on the top surface of the block and the reaction of the tie rod washer on the support block seat.) This analysis is based upon modifying thin plate theory to include the effects of shearing stresses and lateral pressures on plate deflections.

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SUMMARY

Appendix A contains a detailed derivation of the three-dimensional stress state in the support block. Comparisons between calculated failure loads for the block and experimental values indicate that the theoretical analysis is conservative. Theoretical failure loads ranging from 3,200 to 3850 lb. were determined while experimental values range from 3,800 to 4,200 lb.

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DISCUSSION

Appendix A contains a detailed stress analysis for the support block considering only static pressure loadings. This analysis was obtained considering the block to be circular and behave as a perforated thick plate. A detailed listing of the assumptions made in the analysis is included in the appendix.

A computer program (Ref. 1) has been established for the stress equations derived in the appendix. This program enables prediction of the three-dimensional stress state in the block as a function of the tie rod load. (The tie rod results in pressure loadings being transmitted to the top surface and seat of the support block.) Figs. 1, 2, 3 and 4 have been obtained from this program. These figures describe the stresses on the bottom surface ($Z = +h/2$) (Figure 7) of the block with respect to radial positions for an arbitrary tie rod load (P), and the stresses with respect to axial positions for the lower half of the block ($0 \leq Z \leq +h/2$) at particular radii. The following constants (Figure 7) were used to obtain these figures:

$$R_1 = 0.184 \text{ in.}$$

$$R_2 = 0.374 \text{ in.}$$

$$R_3 = 1.125 \text{ in.}$$

$$h = 1.6 \text{ in.}$$

$$\nu = 0.3 \text{ and } 0.4$$

(Poisson's Ratio (ν) can be corrected in the equations to account for block perforations. However, since the ligament efficiency of the support block is approximately 0.45, curves obtained in Ref. 5 for steel plates indicate the effective elastic constant (ν^*) being identical with ν for this value. Therefore, it was assumed for the numerical calculations presented here that ν and ν^* were identical.)

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During experimental testing of the support blocks, it was visually observed that failure initiated at the corner of the support block seat ($r = R_2$, $Z = h/2$). Failure loads varying between 3,800 to 4,200 lb. have been recorded. Using $\nu^* = 0.4$ and $h = 1.6$ in., (Actual height (h) of the block is 2.25 in. with a depth to the block seat of 1.5 in. Since the material extending below the block seat was assumed to have no influence during the analysis, by choosing $h = 1.6$ in. instead of 1.5 in., part of this material is taken into consideration) the following stress values, based upon the analysis presented in the Appendix, are calculated for $r = R_2$ and $Z = h/2$: (Figure 7)

$$\sigma_r = -2.92 P/\pi$$

$$\sigma_\theta = -3.25 P/\pi$$

$$\sigma_z = -9.43 P/\pi$$

In order to calculate a theoretical failure load (P), let us assume that the support block will fail in shear. Experimental shearing stress values for ATJ graphite range between 3,300 psi to 4,000 psi. Letting these values represent τ_p in Equation 72 of the Appendix (Mohr's Circle Analysis) and using the values for σ_r , σ_θ , and σ_z presented above, we calculate a theoretical failure load (P) for the corner of the support block seat of 3,200 lb. and 3,850 lb., respectively. Comparing these to experimental values (3,800 to 4,200 lb.), we find that the theoretical analysis is conservative.

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STRESS VERSUS RADIAL POSITION AT
BOTTOM SURFACE OF SIMULATED SUPPORT BLOCK

FIGURE 1

STRESS VERSUS RADIAL POSITION AT
BOTTOM SURFACE OF SIMULATED SUPPORT BLOCK

STRESS VERSUS AXIAL POSITION FOR
 BOTTOM HALF OF SIMULATED SUPPORT BLOCK

FIGURE 3

STRESS VERSUS BLOCK AXIAL POSITION FOR
 BOTTOM HALF OF SIMULATED SUPPORT BLOCK

NOTES:

- (a) Applicable for $R = 0.464$
- (b) $\nu^* = 0.4$; $h = 1.6$ inches
- (c) Curves shown as dotted lines since only three points used for plotting
- (d) Computer program - Ref. 1

FIGURE 4

FIGURE 5
SUPPORT BLOCK AND ASSOCIATED HARDWARE

APPENDIX A

SUPPORT BLOCK STRESS ANALYSIS

During reactor operation, the support block is subjected to static, dynamic and thermal loadings. The support block is a hexagonal perforated cylinder with a center cylindrical hole through which a tie rod passes. In a regular support block, one tie rod is utilized; one, two, or three tie rods are used with the irregular block.

The objective of this analysis is to demonstrate a method for predicting the stresses in the NRX-A regular support block. This solution was obtained from linear elasticity theory and eliminates the complexities of non-linear theory that would have to be used when analyzing the irregular block. Furthermore, only static loading will be considered; thermal and dynamic loading, although important, will be neglected.

Figure 6 shows a simulated regular support block under pressure loadings p_1 and p_2 -- p_1 represents the interaction of the fuel elements and the block, and p_2 the interaction between the tie rod washer and the block. (Figure 6 - not to scale)

REGULAR SUPPORT BLOCK (Simulated)

Fig. 6

The complex geometry of the support block eliminates any exact solution for the stress distribution. However, an approximate solution for the stresses in the block can be obtained by considering the block to behave as a thick plate under pressure loadings p_1

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and p_2 . (This system could have been analyzed by considering the block to be a finite length cylinder. However, until recently, finite length cylinder theory did not exist. Today, numerical solutions are obtainable for finite length cylinders wherein the stress gradients at geometric discontinuities remain finite. However, to avoid numerical analysis, thick plate theory was chosen as the first approach to solving the support block problem.) Direct solutions of thick plates under specific loadings can be obtained using Legendre polynomials (Ref. 2). However, an alternate method exists which modifies thin plate theory by considering the influence of shearing stresses and lateral pressures on plate deflections. (Ref. 3). The alternate approach will be chosen for this analysis.

Before proceeding with the analysis, several assumptions will be made:

- (a) The cross-section of the support block will be assumed circular. In the actual case, the modified hexagonal periphery of the block will result in large stress gradients near the corners of the hex. Experimental tests have shown that failure does not initiate at the outer periphery. Therefore, neglecting the stress gradients existing there should not appreciably effect this analysis.
- (b) Hooke's Law is assumed to be valid for the graphite support block. The non-linearity of the graphite stress-strain curve will be neglected.
- (c) The block will be considered to be elastic, homogeneous, and isotropic. Discontinuities arising from manufacturing processes tend to make graphite a highly heterogeneous material. The assumption of isotropy implies

$$K_{ijmn} = \text{Constant}$$
$$\sigma_{ij} = K_{ijmn} \epsilon_{mn}$$

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- (d) The initial part of the analysis will consider the support block to be a thin solid circular plate having a center circular hole. This solution will then be corrected for plate thickness and perforations.
- (e) Small deflection theory will be assumed valid. Graphite, being a brittle material, should not undergo large deflections prior to failure.
- (f) Mohr's failure theory for brittle materials will be assumed applicable to the graphite support block.
- (g) The interlocking feature of the support blocks that could cause shear loading around the outer periphery is neglected.
- (h) The stresses in the support block are assumed not significantly effected by the material extending below the top of the support block seat.

The above assumptions are considered primary for the completion of this analysis. Other assumptions that will apply to a particular section of the analysis will be made and noted.

Let us first consider a thin circular plate subjected to pressure loadings p_1 and p_2 (Fig. 7).

THIN CIRCULAR PLATE

Fig. 7

In order to analyze the loading conditions in Fig. 7, the solution will be divided into two parts:

- (1) A circular plate under uniform pressure (p_1) over its entire face and simply supported around the outer periphery, and
- (2) A circular plate under uniform pressure loading (p_2) over part of its face and simply supported around the outer periphery.

The superposition of these two solutions will thus yield the deflection of the plate loaded as shown in Fig. 7.

Figure 8 represents the loading condition that will be considered first.

Fig. 8

The general differential equation that must be satisfied is

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \right] (\phi r) = -\frac{Q}{D} \quad (1)$$

where Q is the shearing force per unit length of circumference. Q can be determined from Fig. 8 as

$$Q_1 = \frac{p_1 r}{2} - \frac{p_1 R_1^2}{2r} \quad (2)$$

Combining Equations 1 and 2 yields for the slope of the plate

$$\phi = -\frac{p_1 r^3}{16D} + \frac{p_1 R_1^2 r}{8D} (2 \ln r - 1) + \frac{C_1 r}{2} + \frac{C_2}{r} \quad (3)$$

The slope of the plate can also be defined as

$$\phi = \frac{dw}{dr} \quad (4)$$

from which by integrating the deflection of the plate is

$$w_1 = \frac{p_1 r^4}{64D} - \frac{p_1 R_1^2 r^2}{8D} (\ln r - 1) - \frac{C_1 r^2}{4} - C_2 \ln r + C_3 \quad (5)$$

where C_1 , C_2 and C_3 are constants of integration and can be determined from the boundary conditions.

The boundary conditions are:

$$(a) \quad M_r = 0, w_1 = 0, \text{ at } r = R_1$$

$$(b) \quad M_r = 0, \text{ at } r = R_3$$

where M_r is the bending moment in the radial direction and is defined as

$$M_r = D \left(\frac{d\phi}{dr} + \nu \frac{\phi}{r} \right) \quad (6)$$

Substituting the boundary conditions into Equations 5 and 6 yields for the deflection:

$$w_1 = \frac{p_1}{8D} \left[\frac{r^4}{8} - R_1^2 r^2 (\ln r - 1) - \frac{A r^2}{2} - B \ln r + C \right] \quad (7)$$

where

$$A = \frac{(R_1^2 + R_3^2)(1 + 3\nu)}{(1 + \nu)} + \frac{2R_1^2 \ln R_1 - 2R_3^2 \ln R_3}{(R_3^2 - R_1^2)} - 2R_3^2 \ln R_3 \quad (8)$$

$$B = R_1^2 R_3^2 \left[\frac{(1 + 3\nu)}{(1 - \nu)} + \frac{(1 + \nu)(2R_1^2 \ln R_1 - 2R_3^2 \ln R_3)}{(1 - \nu)(R_3^2 - R_1^2)} \right] \quad (9)$$

and

$$C = R_1^2 \left[R_1^2 (\ln R_1 - 9/8) + \frac{(1 + 3\nu)}{2(1 - \nu)} (R_1^2 + R_3^2) + \frac{(R_1^2 \ln R_1 - R_3^2 \ln R_3)}{(R_3^2 - R_1^2)} \left(1 + \frac{2(1 + \nu)}{(1 - \nu)} R_3^2 \ln R_1 \right) + R_3^2 \left\{ \frac{(1 + 3\nu)}{(1 - \nu)} \ln R_1 - \ln R_3 \right\} \right] \quad (10)$$

Now let us consider the deflection due to the pressure loading (p_2) in Fig. 9.

Fig. 9

Equation 1 must also be satisfied for this case. However, due to the plate only carrying pressure over part of its face, two independent shearing forces exist.

For the region $R_1 \leq r \leq R_2$

$$Q_2 = \frac{p_2 r}{2} - \frac{p_2 R_1^2}{2r} \quad (11)$$

And combining Equations 1 and 11 and integrating yields

$$\phi_2 = -\frac{p_2 r^3}{16D} + \frac{p_2 R_1^2 r}{8D} (2 \ln r - 1) + \frac{K_1 r}{2} + \frac{K_2}{r} \quad (12)$$

and from Equation 4,

$$w_2 = \frac{p_2 r^4}{64D} - \frac{p_2 R_1^2 r^2}{8D} (\ln r - 1) - \frac{K_1 r^2}{4} - K_2 \ln r + K_3 \quad (13)$$

For the region $R_2 \leq r \leq R_3$

$$Q_3 = \frac{p_2}{2r} \left(R_2^2 - R_1^2 \right) \quad (14)$$

and combining Equations 1 and 14 and integrating yields

$$\phi_3 = - \frac{p_2 r}{8D} \left(R_2^2 - R_1^2 \right) (2 \ln r - 1) + \frac{H_1 r}{2} + \frac{H_2}{r} \quad (15)$$

and from Equation 4,

$$w_3 = \frac{p_2 r^2}{8D} \left(R_2^2 - R_1^2 \right) (\ln r - 1) - \frac{H_1 r^2}{4} - H_2 \ln r + H_3 \quad (16)$$

where K_1, K_2, K_3, H_1, H_2 and H_3 are constants of integration and can be determined from the boundary conditions. The boundary conditions are:

- (a) $M_r = 0, w_2 = 0$ at $r = R_1$
- (b) $w_2 = w_3, \phi_2 = \phi_3, M_{r2} = M_{r3}$ at $r = R_2$
- (c) $M_r = 0$ at $r = R_3$

Substituting the above boundary conditions into Equations 12, 13, 15 and 16 yields the deflections for $R_1 \leq r \leq R_2$ as

$$w_2 = \frac{P_2}{8D} \left[\frac{r^4}{8} - R_1^2 r^2 (\ln r - 1) - \frac{Fr^2}{4} - \bar{G} \ln r + H \right] \quad (17)$$

and for $R_2 \leq r \leq R_3$ as

$$w_3 = \frac{P_2}{8D} \left[r^2 \left(R_2^2 - R_1^2 \right) (\ln r - 1) - \frac{Ir^2}{4} - J \ln r + K \right] \quad (18)$$

where

$$F = \left[R_1^2 \left\{ -R_3^2 \left[4 \ln R_3 + \frac{2(1-\nu)}{(1+\nu)} \right] - R_1^2 \left[\frac{1+3\nu}{1+\nu} - 4 \ln R_1 \right] \right\} + \right. \\ \left. R_2^2 \left\{ \frac{2R_2^2(1-\nu)}{(1+\nu)} \left[2 \ln R_2 - 3/2 \right] + R_3^2 \left[4 \ln R_3 + \frac{2(1-\nu)}{(1+\nu)} \right] - \right. \right. \\ \left. \left. R_2^2 \left[\frac{4(1-\nu)}{(1+\nu)} \ln R_2 - \frac{2(1-\nu)}{(1+\nu)} \right] - 2R_3^2 \left[2 \ln R_2 - 1 \right] \right\} \right] \quad (19)$$

$$\bar{G} = \frac{R_1^2(1+\nu)}{2(1-\nu)} F - \frac{R_1^4}{(1-\nu)} \left[\frac{(1+3\nu)}{2} - 2(1+\nu) \ln R_1 \right] \quad (20)$$

$$H = R_1^4 \left[\ln R_1 - 9/8 \right] + \frac{R_1^2 F}{4} + \bar{G} \ln R_1 \quad (21)$$

$$I = R_2^2 \left[4 \ln R_2 - 2 \right] + F \quad (22)$$

$$J = \frac{R_3^2 (1 + \nu)}{2(1 - \nu)} I - R_3^2 \left(R_2^2 - R_1^2 \right) \left[\frac{2(1 + \nu)}{(1 - \nu)} \ln R_3 + 1 \right] \quad (23)$$

and

$$K = \frac{R_1^2 R_2^2}{4} \left[4 \ln R_2 - 2 \right] + \left[J - \bar{G} \right] \ln R_2 - R_2^4 \left[\ln R_2 - 9/8 \right] \quad (24)$$

Equations 7, 17 and 18 define the deflection of the thin plate shown in Fig. 2.

The solutions will now be corrected to include the effects of transverse shearing stress and lateral pressure. First, consider the loading shown in Fig. 3. From the exact solution for thick plates (Ref. 2), it can be shown that the shearing stress (τ_{rz}) varies parabolically through the thickness of the plate in the same manner as in beams of narrow rectangular cross-section. From Equation 2, the transverse shearing force is

$$Q_1 = \frac{p_1 r}{2} - \frac{p_1 R_1^2}{2r}$$

or, using the same formula as for a rectangular beam (Ref. 4)

$$\tau_{rz} = \frac{Q_1}{N} \int_z^{h/2} z dA \quad (25)$$

and therefore,

$$\tau_{rz} = \frac{3p_1}{h^3} (h^2/4 - z^2) (r - R_1^2/r) \quad (26)$$

The shearing strain can be defined as

$$\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \quad (27)$$

where for the case being considered u is independent of z and

$$\frac{\partial u}{\partial z} = 0 \quad (28)$$

Combining Equations 26 and 27 with Hooke's Law and integrating, the additional deflection due to shearing stresses is

$$w_{1s} = \frac{3p_1}{Gh^3} \left[h^2/4 - z^2 \right] \left[R_1^2 \ln (r/R_1) - 1/2 (r^2 - R_1^2) \right] \quad (29)$$

The lateral pressure acting on the plate produces a concave curvature similar to that which occurs in a uniformly loaded beam supported near its edge. The pressure (p_1) per unit area produces a radial elongation on the top surface of the plate:

$$u_r = \frac{\nu p_1}{E} \quad (30)$$

and zero on the bottom surface. Assuming a straight line relationship to hole, an approximate radius of curvature can be found as

$$\frac{1}{2R} = \frac{\nu p_1}{2hE} \quad (31)$$

and the additional deflection of the plate is

$$w_{11} = -\frac{1}{2R} \left[r^2 + R_1^2 - 2R_1 r \right] \quad (32)$$

or,

$$w_{11} = -\frac{\nu p_1}{2hE} \left[r^2 + R_1^2 - 2R_1 r \right] \quad (33)$$

In deriving Equation 33, a necessary boundary condition assumption is $\phi = 0$ at $r = R_1$. This assumption is valid since a linear relationship was assumed to hold for u_r . Therefore, considering the plate in Fig. 3 to be thick, the total deflection is

$$\bar{w}_1 = \frac{p_1}{8D} \left[\frac{r^4}{8} - R_1^2 r^2 (\ln r - 1) - \frac{Ar^2}{2} - B \ln r + C \right] - \quad (34)$$

$$\frac{\nu p_1}{2hE} \left[r^2 + R_1^2 - 2R_1 r \right] + \frac{3p_1}{Gh^3} \left[h^2/4 - z^2 \right] \left[R_1^2 \ln(r/R_1) - 1/2(r^2 + R_1^2) \right]$$

where A, B and C are constants defined by Equations 8, 9 and 10.

Proceeding in the same manner as was used to derive Equation 29, the additional deflections due to shearing stress for the plate loaded as shown in Fig. 4 are:

for $R_1 \leq r \leq R_2$

$$w_{2s} = \frac{3p_2}{Gh^3} \left[h^2/4 - z^2 \right] \left[R_1^2 \ln(r/R_1) - 1/2(R^2 + R_1^2) \right] \quad (35)$$

and for $R_2 \leq r \leq R_3$

$$w_{3s} = \frac{3p_2}{Gh^3} \left[h^2/4 - z^2 \right] \left[R_2^2 \ln R_2 - R_1^2 \ln R_1 - (R_2^2 - R_1^2) (\ln r + 1/2) \right] \quad (36)$$

Note also, that at $r = R_2$

$$w_{2s} = w_{3s}; \quad \phi_{2s} = \phi_{3s}$$

Also, the additional deflections resulting from the lateral pressure (p_2) are

for $R_1 \leq r \leq R_2$

$$w_{2l} = -\frac{\nu p_2}{2hE} (r^2 - 2R_1 r + R_1^2) \quad (37)$$

and for $R_2 \leq r \leq R_3$

$$w_{3l} = \frac{p_2 R_2^2}{hE} + \frac{\nu p_2}{2hE} [R_2^2 - R_1^2] - \frac{p_2 R_2 r}{hE} \left[(1+\nu) - \frac{\nu R_1}{R_2} \right] \quad (38)$$

Therefore, the total deflection of the plate resulting from pressure loading, p_2 (Fig. 4) is:

for $R_1 \leq r \leq R_2$

$$\begin{aligned} \overline{w}_2 = & \frac{p_2}{8D} \left[\frac{r^4}{8} - R_1^2 r^2 (\ln r - 1) - \frac{Fr^2}{4} - \overline{G} \ln r + H \right] - \frac{\nu p_2}{2hE} [r^2 - 2R_1 r + R_1^2] + \\ & \frac{3p_2}{Gh^3} \left[h^2/4 - z^2 \right] \left[R_1^2 \ln(r/R_1) - 1/2(r^2 - R_1^2) \right] \end{aligned} \quad (39)$$

and for $R_2 \leq r \leq R_3$

$$\begin{aligned} \overline{w}_3 &= \frac{p_2}{8D} \left[r^2 (R_2^2 - R_1^2) (\ln r - 1) - \frac{r^2}{4} - J \ln r + K \right] + \\ &\frac{3p_2}{Gh^3} \left[h^2/4 - z^2 \right] \left[R_2^2 \ln R_2 - R_1^2 \ln R_1 - (R_2^2 - R_1^2) (\ln r + 1/2) \right] + \\ &\frac{p_2 R_2^2}{hE} + \frac{\nu p_2}{2hE} \left[R_2^2 - R_1^2 \right] - \frac{p_2 R_2 r}{hE} \left[(1 + \nu) - \frac{\nu R_1}{R_2} \right] \end{aligned} \quad (40)$$

where F, G, H, I, J and K are constants defined by Equations 19 through 24.

The deflection of the support block can be defined by superimposing Equations 34, 39 and 40. Therefore, for $R_1 \leq r \leq R_2$

$$w_\eta = \overline{w}_1 - \overline{w}_2 \quad (41)$$

and

$$\begin{aligned} w_\eta &= \frac{(p_1 - p_2)}{8D} \left[\frac{r^4}{8} - R_1^2 r^2 (\ln r - 1) \right] - \frac{r^2}{32D} \left[2Ap_1 - Fp_2 \right] - \\ &\frac{\ln r}{8D} \left[p_1 B - p_2 G \right] + \frac{1}{8D} \left[p_1 C - p_2 H \right] - \frac{\nu(p_1 - p_2)}{2hE} \left[r^2 + R_1^2 - 2R_1 r \right] - \\ &\frac{3(p_1 - p_2)}{Gh^3} \left[h^2/4 - z^2 \right] \left[1/2(r^2 - R_1^2) - R_1^2 \ln(r/R_1) \right] \end{aligned} \quad (42)$$

and for $R_2 \leq r \leq R_3$

$$w_\zeta = \overline{w}_1 - \overline{w}_3 \quad (43)$$

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which yields

$$\begin{aligned}
 w_z = & \frac{P_1}{8D} \left[\frac{r^4}{8} - R_1^2 r^2 (\ln r - 1) \right] - \frac{P_2}{8D} \left[r^2 (R_2^2 - R_1^2) (\ln r - 1) \right] - \\
 & \frac{r^2}{32D} \left[2A_1 - I_{P_1} \right] - \frac{\ln r}{8D} \left[B_{P_1} - J_{P_2} \right] + \frac{1}{8D} \left[C_{P_1} - K_{P_2} \right] - \\
 & \frac{3P_1}{Gh^3} \left[\frac{h^2}{4} - z^2 \right] \left[\frac{1}{2} (r^2 - R_1^2) - R_1^2 \ln (r/R_1) \right] - \frac{\nu P_1}{2hE} \left[r^2 + R_1^2 - 2R_1 r \right] + \\
 & \frac{P_2 R_2 r}{hE} \left[(1 + \nu) - \frac{\nu R_1}{R_2} \right] - \frac{\nu P_2}{2hE} \left[R_2^2 - R_1^2 \right] - \frac{P_2 R_2^2}{hE} \quad (44)
 \end{aligned}$$

where the constants were previously defined and the flexural modulus can be defined as

$$D = \frac{Eh^3}{12(1 - \nu^2)} \quad (45)$$

The stresses can now be calculated directly from Equations 42 and 44. In the general theory of thin plates, the shear and lateral deflection terms appearing in Equations 42 and 44 would be neglected when calculating the bending moments. However, in order to be conservative in the remainder of the analysis, these terms will be included in the bending moment equations. Rewriting Equation 6, the bending moment in terms of plate slope in the radial direction is

$$M_r = D \left[\frac{d\phi}{dr} + \frac{\nu\phi}{r} \right] \quad (46)$$

and in the tangential direction

$$M_\theta = D \left[\frac{\phi}{r} + \frac{\nu d\phi}{dr} \right] \quad (47)$$

and in terms of plate deflection:

$$M_r = -D \left[\frac{d^2 w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right] \quad (48)$$

and

$$M_\theta = -D \left[\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right] \quad (49)$$

The radial and tangential bending stresses can be determined from

$$\sigma_r = \frac{12 M_r z}{h^3} \quad (50)$$

and

$$\sigma_\theta = \frac{12 M_\theta z}{h^3} \quad (51)$$

or, upon substituting Equations 48 and 49 the stresses become

$$\sigma_r = -\frac{12Dz}{h^3} \left[\frac{d^2 w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right] \quad (52)$$

and

$$\sigma_\theta = -\frac{12Dz}{h^3} \left[\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right] \quad (53)$$

or, by using Equation 45,

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$$\sigma_r = - \frac{Ez}{(1-\nu^2)} \left[\frac{d^2 w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right] \quad (54)$$

and

$$\sigma_\theta = - \frac{Ez}{(1-\nu^2)} \left[\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right] \quad (55)$$

Therefore, by combining Equations 42, 44, 52 and 53, we obtain for $R_1 \leq r \leq R_2$

$$\begin{aligned} \sigma_r = & \frac{3(p_2 - p_1)z}{2h^3} \left[(3+\nu) \frac{r^2}{2} - 2R_1^2(1+\nu) \ln r - R_1^2(1-\nu) \right] + \\ & \frac{3(1+\nu)z}{4h^3} \left[2Ap_1 - Fp_2 \right] - \frac{3(1-\nu)z}{2h^3 r^2} \left[p_1 B - p_2 \bar{G} \right] + \\ & \frac{12\nu z(p_1 - p_2)D}{h^4 E} \left[(1+\nu) - \frac{\nu R_1}{r} \right] + \\ & \frac{36(p_1 - p_2)D}{Gh^6} \left[\frac{h^2 z}{4} - z^3 \right] \left[(1+\nu) + \frac{R_1^2}{r^2} (1-\nu) \right] \end{aligned} \quad (56)$$

and

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$$\begin{aligned} \sigma_{\theta} = & \frac{3(p_2 - p_1)z}{2h^3} \left[\frac{r^2(3+\nu)}{z} - 2R_1^2(1+\nu)\ln r + R_1^2(1-\nu) \right] + \\ & \frac{3(1+\nu)z}{4h^3} \left[2Ap_1 - Fp_2 \right] + \frac{3(1-\nu)z}{2h^3 r^2} \left[p_1 B - p_2 \bar{G} \right] + \\ & \frac{12\nu(p_1 - p_2)zD}{h^4 E} \left[(1+\nu) - \frac{R_1}{r} \right] + \\ & \frac{36D(p_1 - p_2)z}{Gh^6} \left[\frac{h^2}{4} - z^2 \right] \left[(1+\nu) - \frac{R_1^2}{r^2} (1-\nu) \right] \end{aligned} \quad (57)$$

and for $R_2 \leq r \leq R_3$

$$\begin{aligned} \sigma_r = & \frac{3p_1 z}{2h^3} \left[2R_1^2(1+\nu)\ln r + R_1^2(1-\nu) - \frac{r^2}{2}(3+\nu) \right] + \frac{3(1+\nu)z}{4h^3} \left[2Ap_1 - Ip_2 \right] + \\ & \frac{3p_2 z}{2h^3} \left[2(R_2^2 - R_1^2)(1+\nu)\ln r + (R_2^2 - R_1^2)(1-\nu) \right] - \frac{3(1-\nu)z}{2h^3 r^2} \left[Bp_1 - Jp_2 \right] + \\ & \frac{36Dzp_1}{Gh^6} \left[\frac{h^2}{4} - z^2 \right] \left[(1+\nu) + \frac{R_1^2}{r^2} (1-\nu) \right] - \frac{36(1-\nu)p_2 z}{Gh^3} \left[\frac{h^2}{4} - z^2 \right] \left[\frac{R_1^2 - R_2^2}{r^2} \right. \\ & \left. - \frac{12zD}{h^3} \left[-\frac{\nu p_1}{hE} \left\{ (1+\nu) - \frac{\nu R_1}{r} \right\} + \frac{\nu p_2}{hE} \left\{ (1+\nu) \frac{R_2}{r} - \frac{\nu R_1}{r} \right\} \right] \right] \end{aligned} \quad (58)$$

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and

$$\begin{aligned} \sigma_{\theta} = & \frac{3p_1 z}{2h^3} \left[2R_1^2 (1+\nu) \ln r - \frac{r^2}{3} (1+3\nu) - R_1^2 (1-\nu) \right] + \frac{3(1+\nu)z}{4h^3} \left[2Ap_1 - Ip_2 \right] + \\ & \frac{3p_2 z}{2h^3} \left[2(R_2^2 - R_1^2) (1+\nu) \ln r - (R_2^2 - R_1^2) (1-\nu) \right] + \frac{3(1-\nu)z}{2h^3 r^2} \left[Bl_1 - Jp_2 \right] + \\ & \frac{36p_1 Dz}{Gh^6} \left[\frac{h^2}{4} - z^2 \right] (H\nu) - (1-\nu) \frac{R_1^2}{r^2} + \\ & \frac{36p_2 (1-\nu) Dz}{Gh^6} \left[\frac{h^2}{4} - z^2 \right] \frac{R_1^2 - R_2^2}{r^2} - \frac{12Dz}{h^3} - \frac{\nu p_1}{hE} \left\{ (1+\nu) - \frac{R_1}{r} \right\} + \\ & \frac{p_2}{hE} \left\{ (1+\nu) \frac{R_2}{r} - \frac{\nu R_1}{r} \right\} \end{aligned} \quad (59)$$

where the constants were previously defined.

The support block loading is uniformly distributed about the central axial (z) of the block. The shearing stresses in the r - θ plane and the θ - z plane will vanish due to symmetrical deformation. Therefore

$$\tau_{r\theta} = \tau_{\theta r} = \tau_{\theta z} = \tau_{z\theta} = 0 \quad (60)$$

The shearing stress in the r - z plane can be determined from Equations 29, 35, and 36.

Therefore,

for $R_1 \leq r \leq R_2$

$$\bar{\tau}_{rz} = G \left[\frac{\partial w_{1s}}{\partial r} - \frac{\partial w_{2s}}{\partial r} \right] \quad (61)$$

or,

$$\bar{\tau}_{rz} = \frac{3(p_1 - p_2)}{rh^3} \left[\frac{h^2}{4} - z^2 \right] [R_1^2 - r^2] \quad (62)$$

and for $R_2 \leq r \leq R_3$

$$\bar{\tau}_{rz} = G \left[\frac{\partial w_{1s}}{\partial r} - \frac{\partial w_{3s}}{\partial r} \right] \quad (63)$$

and therefore,

$$\bar{\tau}_{rz} = \frac{3p_1}{h^3} \left[\frac{h^2}{4} - z^2 \right] \left[\frac{R_1^2}{r} - r \right] + \frac{3p_2}{h^3} \left[\frac{h^2}{4} - z^2 \right] \left[\frac{R_2^2 - R_1^2}{r} \right] \quad (64)$$

The compressive stress σ_z can be determined by considering the equation of equilibrium in the $r - z$ plane. This equation can be written as

$$\frac{\partial \sigma_z}{\partial z} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \quad (65)$$

and noting that $\tau_{\theta z}$ was found to be zero from Equation 56, therefore, Equation 61 becomes

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \quad (66)$$

For $R_1 \leq r \leq R_2$, the boundary conditions are

$$\sigma_z = -p_2 \text{ at } z = +\frac{h}{2}$$

and

(67)

$$\sigma_z = -p_1 \text{ at } z = -\frac{h}{2}$$

Therefore, using Equation 58, and the above boundary conditions,

$$\sigma_z = \frac{6(p_1 - p_2)}{h^3} \left[\frac{h^2 z^2}{4} - \frac{z^3}{3} \right] - \frac{(p_1 + p_2)}{2} \quad (68)$$

For $R_2 \leq r \leq R_3$, the boundary conditions are

$$\sigma_z = 0 \text{ at } z = +\frac{h}{2}$$

and

(69)

$$\sigma_z = -p_1 \text{ at } z = -\frac{h}{2}$$

Therefore, using Equation 60 and the above boundary conditions,

$$\sigma_z = \frac{6p_1}{h^3} \left[\frac{h^2 z^2}{4} - \frac{z^3}{3} \right] - \frac{p_1}{2} \quad (70)$$

Before preceding further with the analysis, let us list the equation numbers that are applicable for determining the stress distribution in the support block. For the region $R_1 \leq r \leq R_2$, Equations 42, 54, 55, 56, 57, 62 and 68 apply and for $R_2 \leq r \leq R_3$, Equations 44, 54, 55, 58, 59, 60, 64 and 70 apply.

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The stress state in the support block can be represented by Fig. 10.

STRESS STATE IN SUPPORT BLOCK

Fig. 10

The maximum stresses in the support block can be obtained from a three dimensional Mohr's Circle. (Ref. 4) The general expressions for maximum and minimum principal stresses are given below. The principal normal stresses are given by

$$\sigma_p = \frac{(\sigma_\alpha + \sigma_\beta)}{2} \pm \sqrt{\left(\frac{\sigma_\alpha - \sigma_\beta}{2}\right)^2 + \tau_{\alpha\beta}^2} \quad (71)$$

and the principal shearing stress is

$$\tau_p = \sqrt{\left(\frac{\sigma_\alpha - \sigma_\beta}{2}\right)^2 + \tau_{\alpha\beta}^2} \quad (72)$$

where α and β are the coordinate representations of the particular plane of the block being considered.

The foregoing equations have considered the support block to be solid. In order to consider the perforations existing in the block, the elastic constants have to be modified.(Ref.5)

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Exact modification of elastic constants can only be obtained from experimental data. (Ref. 5 - Values obtained for steel plates having a Poisson's Ratio of 0.3.) Once these are known, the nominal bending and membrane stresses and deflections of an equivalent solid plate having the effective modified perforated plate elastic constants E^* and ν^* and the same dimensions as the perforated plate can be calculated.

The deflection of the perforated plate is the same as that of the solid plate. Since the thickness to perforated hole pitch ratio for the support block is greater than two, the perforated plate can likewise be considered "thick." Therefore, the same set of effective elastic constants can be used for bending and in-plane loads. The maximum stresses in a perforated plate would occur within the ligament sections (the material between the cylindrical perforations).

However, due to stress concentrations existing in the support block seat at $r = R_2$, the maximum stress should be obtained there.

The average stresses through the thickness and across the ligament sections can, however, be determined (Fig. 11).

TOP VIEW OF LIGAMENT SECTION

Fig. 11

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The average stress normal to the ligament section in the $r - \theta$ plane at any depth (z) is

$$(\sigma_y)_{ave} = \frac{R}{S} \left[\sigma_r \cos^2 \psi + \sigma_\theta \sin^2 \psi \right] \quad (73)$$

and the average shear stress is

$$(\tau_{xy})_{ave} = \frac{R}{2S} (\sigma_r - \sigma_\theta) \sin 2\psi \quad (74)$$

The stress $(\sigma_x)_{ave}$ must be zero at the periphery of the ligament section. Moreover, this stress is usually small at the center of the section. Therefore, for our purposes, this stress will be considered negligible. The average stresses through the thickness of the block are:

$$(\sigma_z)_{ave} = \frac{R}{S} \sigma_z \quad (75)$$

and

$$(\tau_{xy})_{ave} = \frac{R}{S} \tau_{rz} \sin \psi \quad (76)$$

$$(\tau_{yz})_{ave} = \frac{R}{S} \tau_{rz} \cos \psi \quad (77)$$

The above equations together with Equations 71 and 72 determine the maximum principal stresses in the support block. The angle ψ defined in Fig.11, can be determined by maximizing either the equation for the maximum principal normal stress, or the maximum principal shear stress.

Before a failure theory can be developed for the support block, the structural properties of graphite must be examined. At room temperature, graphite in general, behaves as a brittle material. (Ref. 6) At temperatures between 3000°F to 4000°F, graphite goes through a transition resulting in ductile behavior at temperatures above 4000°F. Therefore, for ambient temperatures, a failure theory must be based upon brittle material behavior.

The compressive strength (8500 psi) of graphite is considerably higher than its shear or tensile strengths (3000-4000 psi). Therefore, a compression mode of failure (not considering bearing pressure at the support block seat) is unlikely. Since most brittle materials fail by the mechanism of shear, it is likely to expect graphite to also fail by shear. Based upon Mohr's Theory of Brittle Materials (Ref. 7), the shear yielding strength can be determined from:

$$\tau_{yp} = \frac{\sigma_{yp} \sigma'_{yp}}{\sigma_{yp} + \sigma'_{yp}} \quad (78)$$

where σ_{yp} and σ'_{yp} are the yield strengths of graphite in simple tension and compression, respectively.

The foregoing analysis has been based upon the assumption that the block acts as a thick plate. Perforations can be considered by using effective instead of actual elastic constants for graphite. The relationship between effective and actual material properties have been obtained experimentally using steel plates. (Ref. 5)

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APPENDIX B
NOMENCLATURE

r	=	Radius at any point
z	=	Thickness at any point
R_1	=	Radius of cylindrical hole
R_2	=	Radius of support block seat
R_3	=	Outer radius of support block
h	=	Thickness of support block
p_1	=	Pressure loading caused by fuel elements
p_2	=	Pressure loading caused by tie rod washer
ν	=	Poisson's Ratio
E	=	Young's Modulus
G	=	Shear modulus
D	=	Flexure modulus
$*$	=	Effective constants
ϕ	=	Slope of plate
w	=	Deflection of plate
u	=	Radial plate deflection
$1,2,3,$ s, l	=	Subscripts representing different cases being considered
—	=	Total
σ_r	=	Radial stress
σ_θ	=	Tangential stress
σ_z	=	Compressive stress
τ_{rz}	=	Shearing stress

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r, θ, z = Subscripts representing direction
 M = Bending moment
 Q = Shearing force per unit length
 w, η, ζ = Total support block deflection
 σ_p, τ_p = Principal stresses
 ψ = Defined in Fig. 11 in Appendix A

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