DISCLAIMER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

DISTRIBUTION

This report has been distributed according to the category "Physics and Mathematics" as given in "Standard Distribution Lists for Unclassified Scientific and Technical Reports" TID-4500 (14th Ed.), October 1, 1958. A total of 650 copies was printed.

AI

CONTENTS

TABLE

FIGURES

Page

ABSTRACT

The prompt neutron lifetime of the SRE was measured by both the oscillation and random noise techniques. Measurement by use of the oscillation technique gave a prompt neutron lifetime of $(5.25 \pm 0.35) \times 10^{-4}$ sec for a calculated β of 7×10^{-3}. The measured noise response indicated a lifetime of $(5.25 \pm 0.7) \times 10^{-4}$ sec. Both measured values are in agreement with the calculated value of 5×10^{-4} sec.

Four experiments utilizing the noise analysis technique were performed to determine the prompt neutron lifetime of the KEWB. All four experiments gave results which agreed within 3%. For an estimated β of 8×10^{-3}, the measured value obtained was $(7.8 \pm 0.3) \times 10^{-5}$ sec. This is in reasonable agreement with both the energy independent calculated value of 6.6×10^{-5} sec and the value of 6.2×10^{-5} sec obtained from the experimental inhour equation.

The oscillation technique has been found to be better suited for lifetime determinations in reactors where the prompt neutron break frequency is less than 5 cps. Reactor noise analysis is more suitable for reactors which have prompt neutron lifetime break frequencies above 20 cps.

I. INTRODUCTION

A. EXPERIMENTAL APPROACH

The quantity, ℓ/β, which is the ratio of the prompt neutron lifetime to the effective fraction of delayed neutrons, β, is uniquely determined by the measured break frequency of the modulus of the reactor transfer function. The experimental measurement of ℓ/β serves to make possible the predictions of the high frequency or short period responses of a reactor and to corroborate or disprove theoretical calculations. Beta effective is represented by β throughout this report.

B. TWO METHODS

Several experimental methods of measuring that portion of the transfer function which determines ℓ/β have been used at various reactor installations. One method is to deliberately introduce a sinusoidal change of reactivity in the reactor and measure the amplitude and phase of the frequency response. This is called the oscillation[1] technique. A second method is to measure the power spectral density[2] of the inherent random fluctuations of the reactor neutron density. A third obtains the power spectral density from the Fourier transform of the auto-correlation function[2] of these same random fluctuations. The last two methods are referred to as random noise techniques.[2] The oscillation technique and direct power spectral density measurement are analyzed in this report. Both techniques were used successfully on the SRE. The noise analysis was used successfully on the KEWB and an oscillation experiment is scheduled.

C. SMALL SIGNAL THEORY

The concept of measuring the prompt neutron lifetime of a nuclear reactor by determining its frequency response is based upon the assumption that, for a specific reactor, the prompt neutron break frequency ω_p, which is equal to ℓ/β, is sufficiently greater than the break frequencies of the delayed neutrons that the effect of the delayed neutrons on the high frequency portion of the plotted frequency response curves of the reactor is negligible. The basic principle of applying small signal theory to derive the linearized transfer function of a nuclear reactor is well covered in the literature.[3] The resultant normalized transfer function as converted for frequency response is given by

$$\frac{\dfrac{\delta N_o(j\omega)}{N_o}}{\dfrac{\delta\rho(j\omega)}{\beta}} = \frac{1}{j\omega\dfrac{\ell}{\beta}\left[1 + \displaystyle\sum_{i=1}^{6}\frac{\beta_i/\beta}{\dfrac{\ell}{\beta}(j\omega + \lambda_i)}\right]} , \qquad \ldots(1)$$

where

N_o = average neutron flux or power level

δN_o = small change in flux level about the average

$\delta\rho$ = small change in reactivity input to the reactor

ℓ = prompt neutron lifetime for a finite reactor

$j = \sqrt{-1}$

ω = frequency in rad/sec

λ_i = decay constant for delayed neutrons in the i' th group

β_i = effective fraction of delayed neutrons in the i' th group.

Figure 1 is a plot of equation (1). If, in equation (1) the prompt neutron break frequency, ω_p, which is equal to β/ℓ, can be assumed large compared to all λ_i, then in the region of this frequency equation (1) becomes

$$\frac{\dfrac{\delta N_o(j\omega)}{N_o}}{\dfrac{\delta\rho(j\omega)}{\beta}} = \frac{1}{j\omega\dfrac{\ell}{\beta} + 1} , \qquad \ldots(2)$$

and it is seen that the break frequency is determined by ℓ/β.

In a large graphite reactor, the assumption that the prompt neutron break frequency, ω_p, is very much larger than any of the λ_i's which establish the break frequencies of the delayed neutron groups, may not be valid. If the measured value of ℓ/β for the SRE of 0.075 sec is used to obtain $\omega_p = \beta/\ell$ and this value

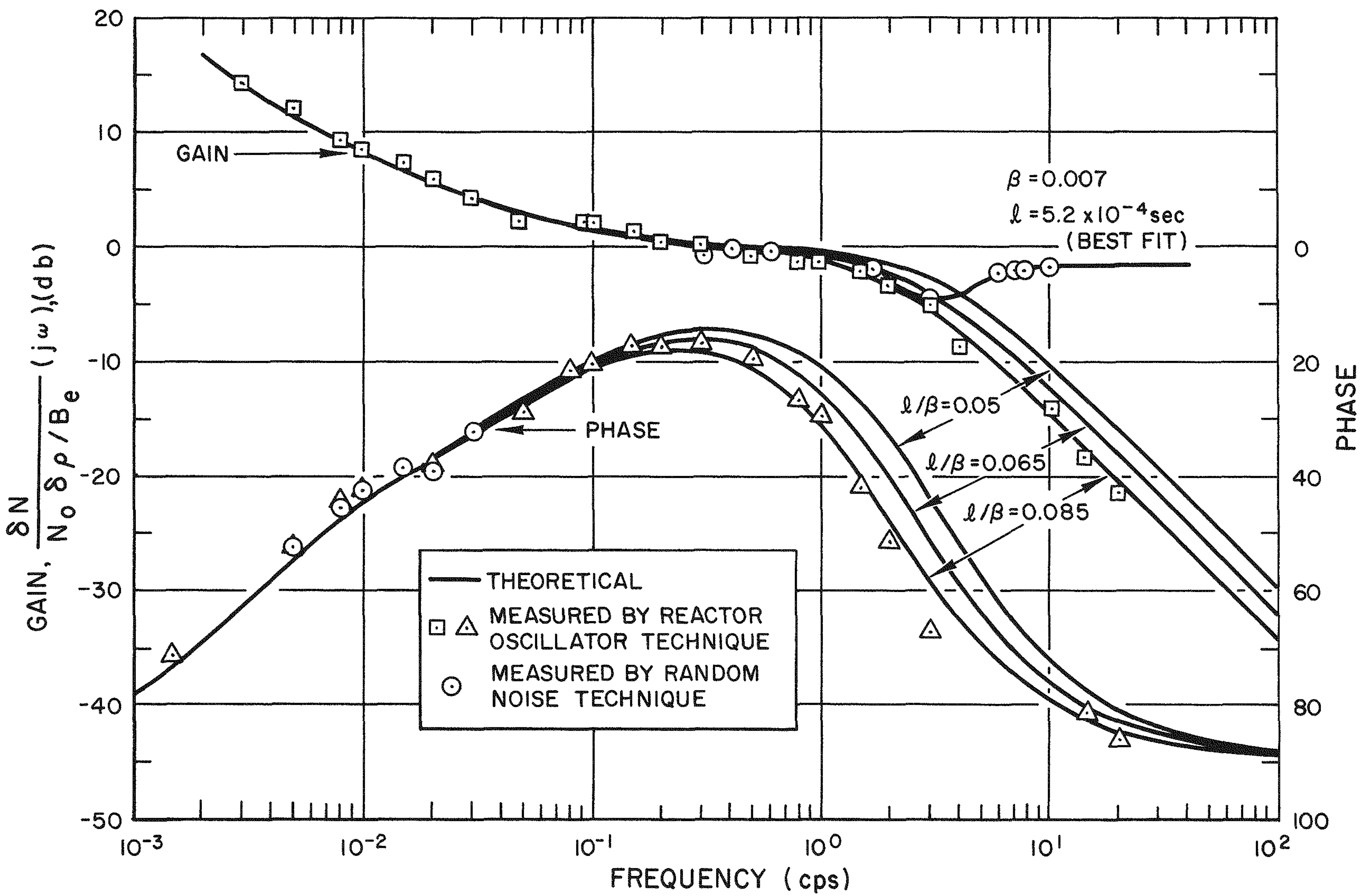


Figure 1. SRE Zero Power Transfer Function

inserted in equation (1) along with Keepin and Wimmett's[4] delayed neutron constants, the use of equation (2) results in a value for ℓ/β which would be approximately 2% too large. This error, however, is of the order of magnitude of the accuracy of the equipment used. For this reason, and since error contributions are made by other factors, such as the effects of an unknown quantity of photo neutrons assumed present due to the γ,n reaction of beryllium elements in the core, no attempt has been made to introduce correction factors. If, however, the measured value of 0.0097 sec for ℓ/β of the KEWB is used as before in equation (1), then the use of equation (2) results in no appreciable error.

II. OSCILLATION MEASUREMENTS OF THE SRE

A. THE OSCILLATOR

The best known method of obtaining the frequency response of a nuclear reactor is by use of the oscillation technique. In this method a device is used to cause a sinusoidal change in reactivity in the reactor. The amplitude and phase shift of response in power to this change is then measured.

The reactor oscillator rotor, measuring equipment, and procedure which were used on the SRE to cause the sinusoidal change in reactivity have been thoroughly covered in another report,[1] and are, therefore, only briefly described here. The SRE reactor oscillator is a rotary mechanism which consists of a rotor and stator each of which has squares of poison which shadow each other during rotation in such a manner as to create four cycles of sinusoidal reactivity change per revolution of the rotor. The disassembled oscillator is shown in Figure 2.

B. EXPERIMENTAL RESULTS

The response of the SRE to both oscillation and noise measurement is shown in Figure 1 along with calculated theoretical curves for several values of ℓ/β. The dependability of the measurement, as performed by oscillation, is indicated by the manner in which both the amplitude and phase curves tend to follow ℓ/β = 0.075 from 0.1 to 1.0 cps. The deviation in the phase curve, before any attenuation in amplitude is noticed, is a result of induced torsional strain in the long rotor at the higher frequencies.

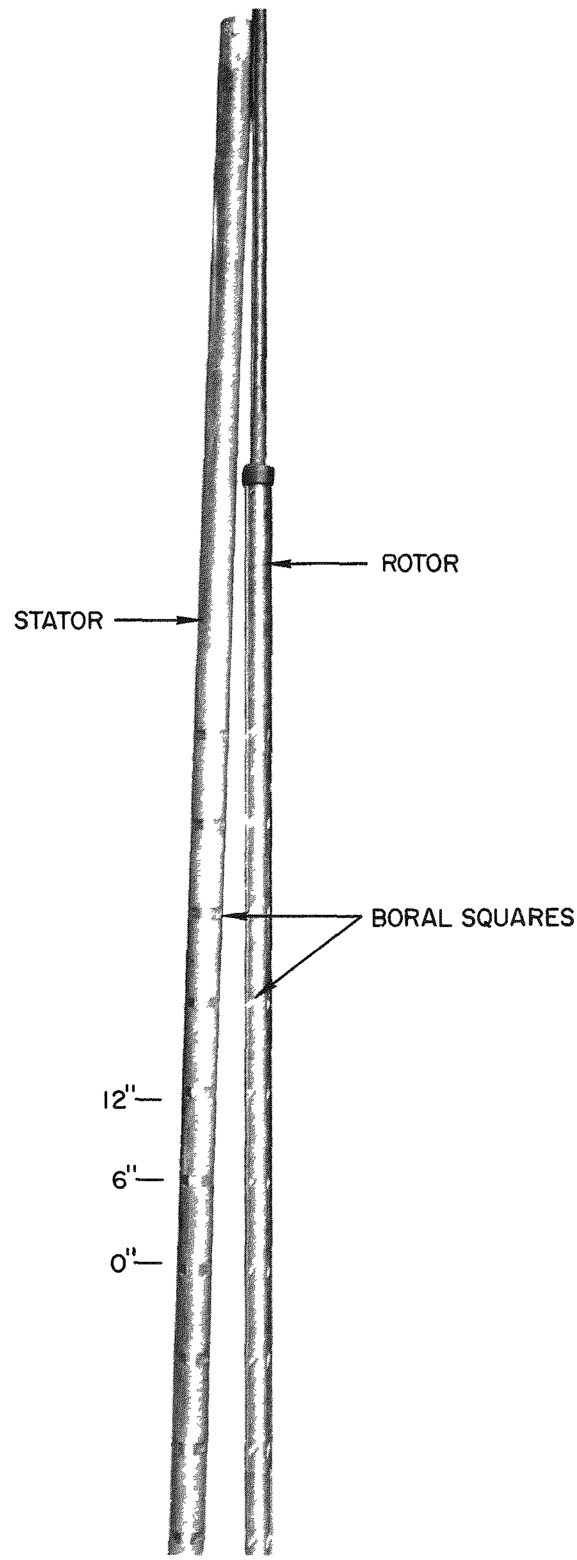


Figure 2. Disassembled View of the
Oscillator Rotor and Stator

III. REACTOR NOISE THEORY

A. RANDOM PROCESSES

Reactor noise analysis is a comparatively new field when compared to reactor oscillation or period measurements. In recent years, using random variables to control or describe physical systems or phenomena has become an exact science.

The random variable of interest may be an externally applied disturbance or an inherent internal fluctuation of the system. In a reactor it is the latter. The variable must be describable in the language of probability theory to be useful. A familiar use of a describable random function in nuclear reactors is the approximately "normal distribution" of neutron propagation through a medium. The fact that the distribution is approximately "normal" makes possible the "Monte Carlo" codes.

The spontaneous emission of electrons from a thermionic cathode surface, or the spontaneous decay of a radioactive substance is described as having "Poisson distribution." In a nuclear reactor, the assumption is made that the spontaneous creation of neutrons through fission has the same distribution as radioactive decay. Subsequent experimental results justify this assumption.

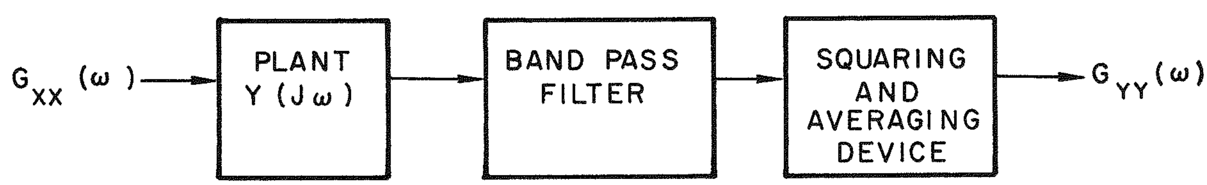
B. WHITE NOISE

The power spectral density of a random variable having a "Poisson distribution" is very unique in that, where all other power spectral densities are a function of frequency, this one is equal to a constant. This is to say that the power output in each unit band width of the spectrum would be equal to a constant. A random process having a constant power spectral density is referred to as a "white noise." A complete derivation of the concept may be found in section 3.7 of reference 2.

C. EXPERIMENTAL APPROACH

It is now readily apparent that, if a system or plant is excited by a white noise, either externally applied or inherent, it is only necessary to analyze the output of the plant over the range of frequencies of interest in order to describe the plant frequency response since the input is known to have constant amplitude for all frequencies. The basic experimental system for such an analysis is illustrated in Figure 3. In this system it is assumed that the band pass filter and squaring device introduce no extraneous noises and have perfect frequency response. Each frequency of input has constant power amplitude since the input power spectral density $G_{xx}(\omega)$ is equal to a constant. The amplitude is attenuated by the plant transfer function, $Y(j\omega)$, to give the output power spectral density, $G_{yy}(\omega)$. The band pass filter permits analysis of a particular frequency band, and the squaring device converts the signal to units of power. It has been proven[2] that when a white noise is passed through a linear system or plant, the output is still a white noise which is attenuated only by the system or plant characteristics. Thus, the output of the squaring device of Figure 3 is a white noise equal to a constant multiplied by the modulus of the reactor transfer function squared. The system equation is

$$G_{yy}(\omega) = G_{xx}(\omega) \left| Y(j\omega) \right|^2 = K \left| Y(j\omega) \right|^2 \quad . \qquad . . . (3)$$



$G_{XX}(\omega)$ = INPUT POWER SPECTRAL DENSITY = CONSTANT
$G_{XX}(\omega)$ = OUTPUT POWER SPECTRAL DENSITY = $G_{XX}(\omega) \left| Y(j\omega) \right|^2$
$Y(J\omega)$ = PLANT TRANSFER FUNCTION

Figure 3. Basic Power Spectral Density Measurement System

C. E. Cohn[5] and M. N. Moore[6] demonstrated the feasibility of using the inherent reactor noise to measure the high frequency portion of the power spectral density of a reactor. The creation of neutrons in a reactor produces "reactor noise" due to the random process of fission. This noise is assumed as nearly a true "white noise." Since the power spectral density of white noise is a constant, the measured power spectral density of the reactor response to the generated noise is simply the squared modulus of the reactor transfer function multiplied by a constant provided that the signal-to-noise ratio is large and that the frequency response of the measuring circuitry is perfect.

In determining the prompt neutron lifetime of the KEWB and SRE, a Krohn-hite band pass filter was used which had a frequency range from 0.02 to 2000 cps. Additional equipment included: an unshielded ionization chamber; an electrometer; three operational amplifiers for amplification, bias, and integration of the final signal; a vacuum thermocouple used as a squaring device; an oscilloscope; and a recording oscillograph. The schematic circuit is shown in Figure 4.

Physically, measuring equipment does not, as assumed in equation (3), have perfect frequency response. Therefore, the general equation must include both the transfer function of the plant (hereafter called reactor) and the measuring equipment. Equation (4) is the result of this inclusion.

$$G_{yy}(\omega)f = \left|Y_1(j\omega)\right|^2 \left|Y_2(j\omega)\right|^2 G_{xx}(\omega)f + \left|Y_2(j\omega)\right|^2 G_{nn}(\omega)f \quad , \qquad \ldots (4)$$

where

$Y_1(j\omega)$ = transfer function of the reactor

$Y_2(j\omega)$ = transfer function of the ionization chamber and instrumentation

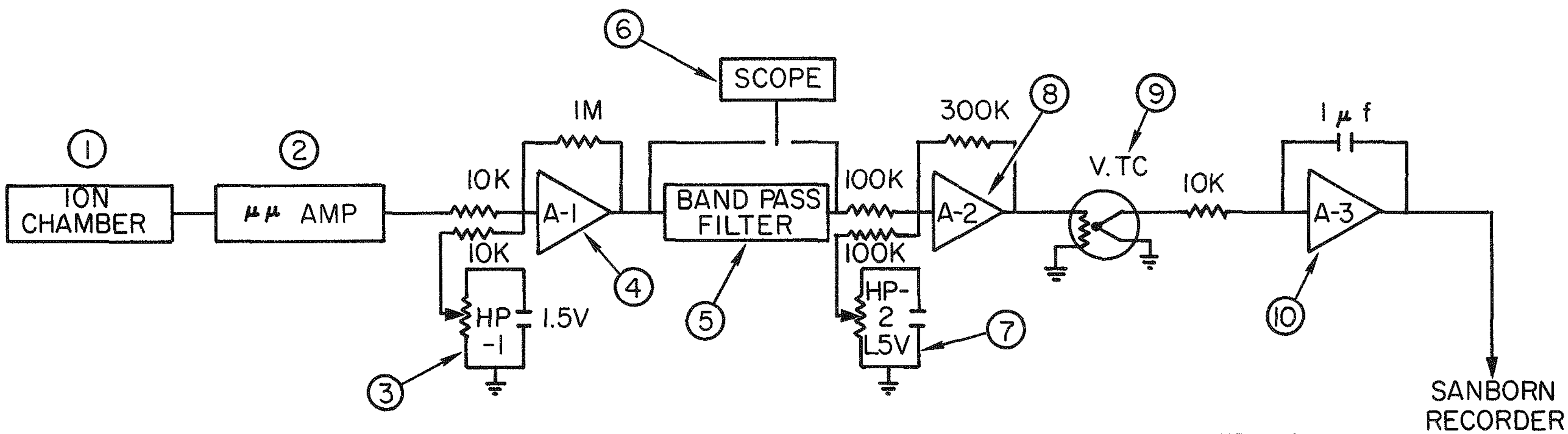
$G_{xx}(\omega)$ = white noise power spectral density of input signal (a constant); in a reactor it is the inherent fluctuations in neutron density.

$G_{nn}(\omega)$ = approximately white noise power spectral density of the ionization chamber and instrumentation noise (a constant)

$G_{yy}(\omega)$ = output power spectral density

f = the mean frequency of the band at which the measurement is being made; f is included in equation (4) because the commerical band pass filter used has an adjustable percent band pass width rather than a constant band pass width.

From the experimentally determined plot of equation (4), it is desired to obtain the reactor transfer function $Y_1(j\omega)$. The power spectral densities are constants determined by the plotted curves. The only unknown, other than $Y_1(j\omega)$, is $Y_2(j\omega)$ which is the transfer function of the entire circuit of Figure 4.



(1)	UNSHIELDED IONIZATION CHAMBER
(2)	KEITHLY MODEL 410 ELECTROMETER
(3)	1K HELIPOT
(4)	SUMMING AMPLIFIER
(5)	BAND-PASS FILTER (KRHON-HITE MODEL 330-A)
(6)	OSCILLOSCOPE
(7)	1K HELIPOT
(8)	SUMMING AMPLIFIER
(9)	VACUUM HERMOCOUPLE AMER. THERMO. ELECT. TYPE # 21
(10)	INTEGRATING AMPLIFIER

Figure 4. Schematic of Noise Measuring Circuit

$|Y_2(j\omega)|^2$ may be obtained by placing a gamma source next to the ionization chamber shown in Figure 4 and measuring the response of the system. Since a gamma source generates a true white noise, equation (3) is a true representation of the system, but must include the f of equation (4) if a constant percent band width is used. This calibration is not always required, for, if the minimum break frequency of the measuring equipment is known to be beyond the maximum frequency of interest, the system noise response may be assumed to be a linear function of frequency for a constant percent band pass. In actual measurements on the KEWB and SRE it was shown that gamma calibration was unnecessary.

When the total response of equation (4) is divided by the system response of equation (3) (with the frequency f included), the resultant is an equation of the form

$$G_{yy}(\omega) = A + B\left|Y_1(j\omega)\right|^2 \quad , \qquad \ldots (5)$$

where $Y_1(j\omega)$ is reduced to equation (2) which is the high frequency portion of the reactor transfer function. Thus, equation (5) becomes

$$G_{yy}(\omega) + A + \frac{B}{1 + (\omega \ell/\beta)^2} \quad . \qquad \ldots (6)$$

Equation (6) is sketched in Figure 5. In Figure 5, "A" represents the level of the ionization chamber and measuring equipment noise. The major portion of "A" is generated in the ionization chamber and is a function of the reactor power level. "B" represents the level of reactor noise and is proportional to the square root of the reactor power.

In some of Cohn's[5] early work only the portion to the right of the dotted vertical line in Figure 5 was obtained in experimental measurement. He therefore devised a computer code to determine the best fitting curve to equation (6) by use of the mean-squared error criteria.

If the flat portion (A + B) and the slope in the region of $\omega = \beta/\ell$ have been clearly determined, it is only necessary to plot the square root of $\left|Y_1(j\omega)\right|^2$ and compare this plotted curve to plots of calculated $Y_1(j\omega)$ for several values of ℓ/β.

When the horizontal portion (A + B) of the experimental curve is shifted vertically to coincide with the horizontal portion of the calculated curves, the best value of ℓ/β is obtained. This is the method that was used on both the SRE and the KEWB.

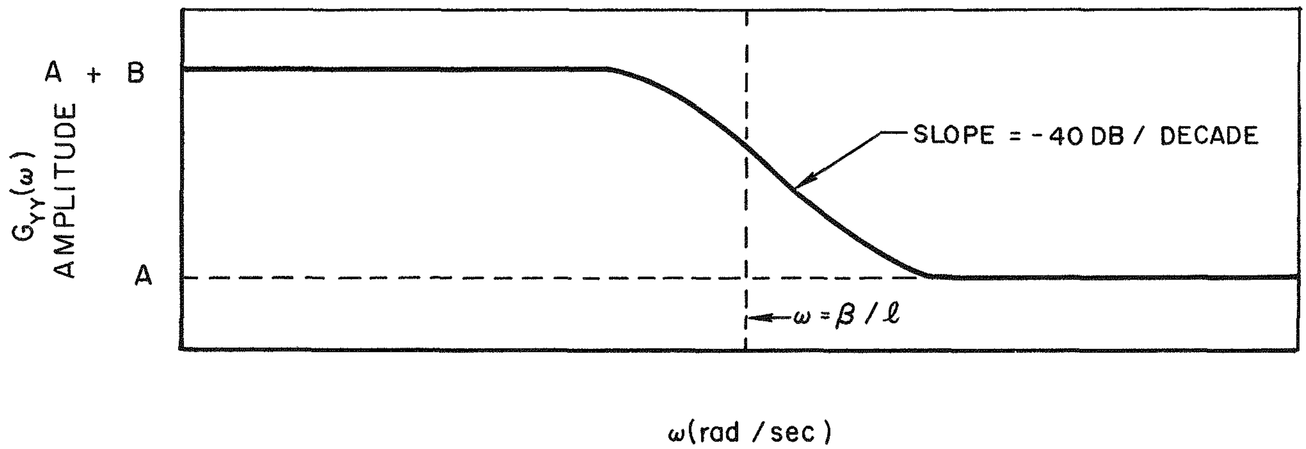


Figure 5. Typical Measured Reactor Power Spectral Density (corrected for system noise)

IV. KEWB NOISE EXPERIMENT

A. SPECIFIC EQUIPMENT

Actual measurements at the KEWB were conducted with several different types of equipment. Figure 6 is a plot of the response for three different conditions of measurement. These are plots of $G_{yy}(\omega)f$ of equation (4), which includes ionization chamber and system noise. Each of the three runs was conducted twice, first with the band pass filter set to pass a band width equal to 5% of the mean frequency setting, then repeated for a band pass filter setting of 10% band pass width. No difference was noted as between 5% and 10% band width measurements. The specific conditions for each experiment were as follows:

<u>Curve 1</u>

a) A Keithley electrometer was set on the 3×10^{-7} ampere range. (All capacitors were removed from the feedback circuits.)

b) A lead-shielded, uncompensated ionization chamber was located in the graphite reflector.

c) 10% band width on band pass filter.

d) Reactor power level was 4 watts.

<u>Curve 2</u>

a) An E-H electrometer was set on the 3×10^{-9} ampere range.

b) An unshielded, uncompensated ionization chamber was located next to the outer layer of the graphite reflector.

c) 10% band width.

d) Reactor power level was 6 watts.

<u>Curve 3</u>

a) A Keithley electrometer was set on the 3×10^{-8} ampere range. (All feedback capacitors removed).

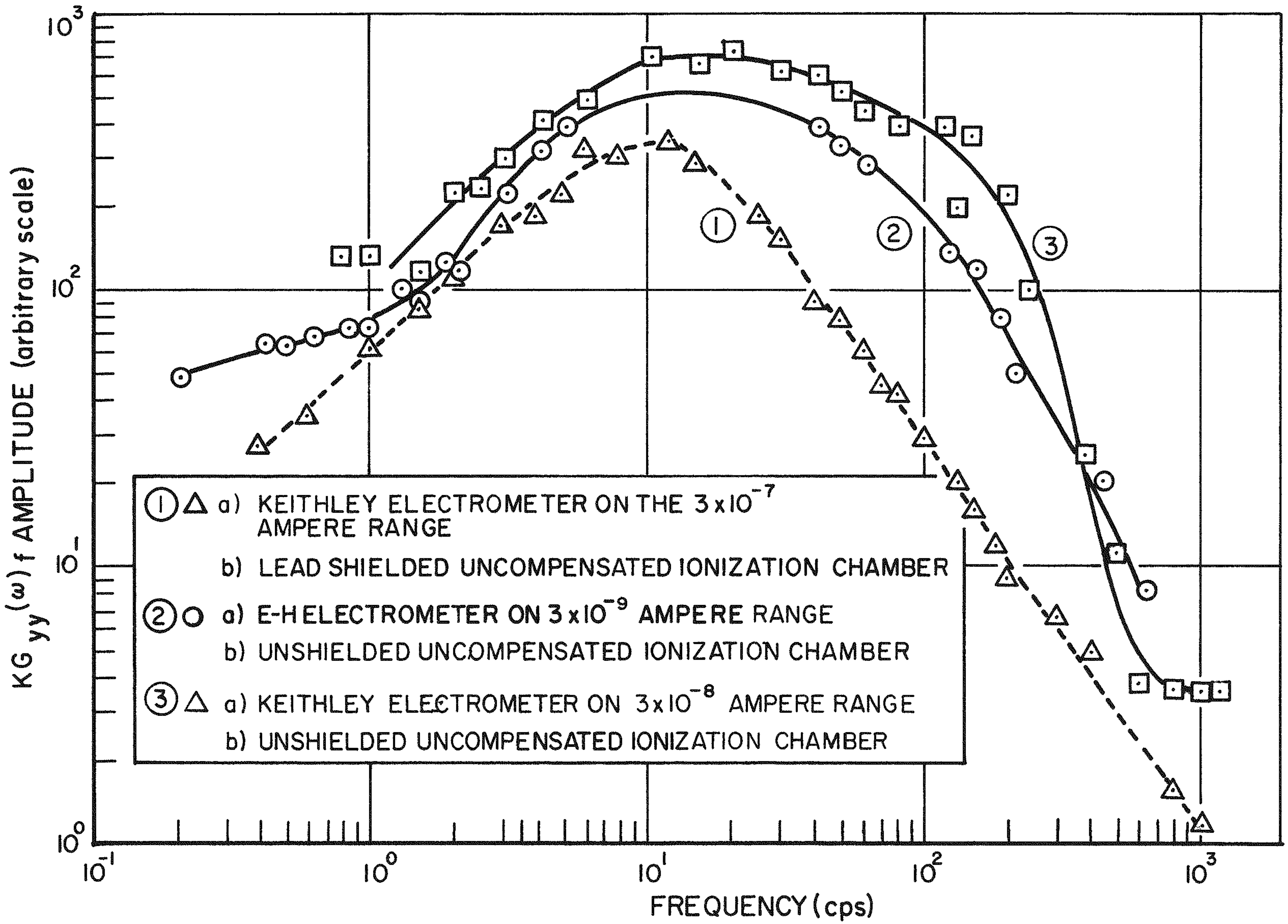


Figure 6. Measured Reactor Power Spectral Density of the KEWB Multiplied by Frequency (December 4, 1958)

b) An unshielded, uncompensated ionization chamber lay next to the outer layer of the graphite reflector.

c) 10% band width.

d) Reactor power level was 7 watts.

A lead-shielded ionization chamber was used in the two experiments which resulted in curve 1. When this curve was corrected for system noise and replotted in Figure 9, it was seen to have approximately a −30 db/decade slope rather than the −20 db/decade slope as required by the square root of equation (7). The other four measurements, which resulted in curves 2 and 3, were made with a bare unshielded ionization chamber and had the correct slope. All six measurements, however, indicated the same break frequency for ℓ/β. There is, at present, no explanation to account for the difference in the responses, but lead-shielded ionization chambers are not recommended.

Curves 2 and 3 result in the same transfer function when their respective amplitudes are normalized to coincide.

B. CALIBRATION OF EQUIPMENT

Two calibrations were made of the instrumentation system frequency response, for two different current range settings of the Keithley electrometer, by using an iridium gamma source as a white noise generator. Both response curves are shown in Figure 7.

The straight line approximation of the calibration with the least scatter was extended to the indicated break frequency of the electrometer range used. This approximation was then used as the power spectral density response of the instrumentation. This was a good approximation since frequencies over 100 cps were of no interest.

Actually, the straight line approximation is good at any time that the measuring circuit break frequency is at least one decade better than the break frequency "ℓ/β" of the reactor. It can be seen that, if the amplitude of this line

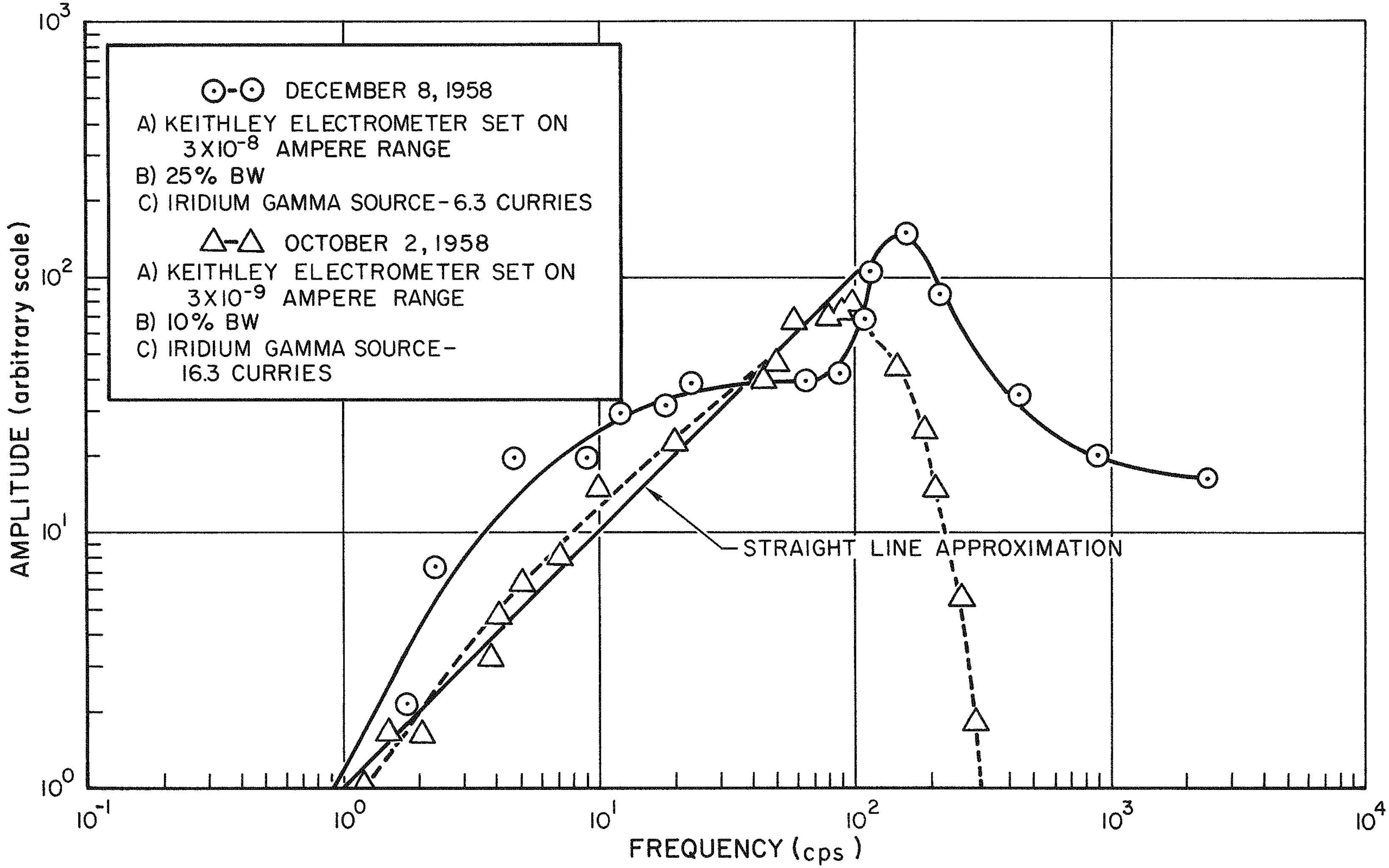


Figure 7. Measured Gamma Power Spectral Density Multiplied by Frequency

is shifted to a point where it is equal to 1 at 1 cps, the transformation from equation (4) to equation (6) is equivalent to dividing points of the curves of Figure 6 by the frequency at that point. The result of operating upon the smoothed data of curve 3 to obtain equation (5) is plotted as the squared modulus of the transfer function in Figure 8. The square root of this curve is the high frequency portion of the frequency response of the reactor and is shown as the upper curve of Figure 8. The lower curve would have resulted in a curve such as is shown in Figure 5 if measurement of the frequency response had been continued to a high enough frequency.

C. EXPERIMENTAL RESULTS

The measured frequency response of Figure 8 is compared in Figure 9 with two calculated curves for ℓ/β equal to 5×10^{-3} and 10^{-2} sec. The indicated ℓ/β for the unshielded ionization chamber runs is about 9.7×10^{-3} which, for an assumed β of 8×10^{-3}, gives a prompt neutron lifetime of $(7.8 \pm 0.3) \times 10^{-5}$ sec. This is in reasonable agreement with both the energy independent calculated value of 6.6×10^{-5} sec and the value of 6.2×10^{-5} sec obtained from the experimental inhour equation which holds for periods down to 7 msec. For shorter periods, no conventional inhour equation is consistent with the period data. This is attributed to neutrons being delayed by the graphite reflector.[7] Analysis has not been carried to the point of refinement required to obtain a suitable description of this effect.

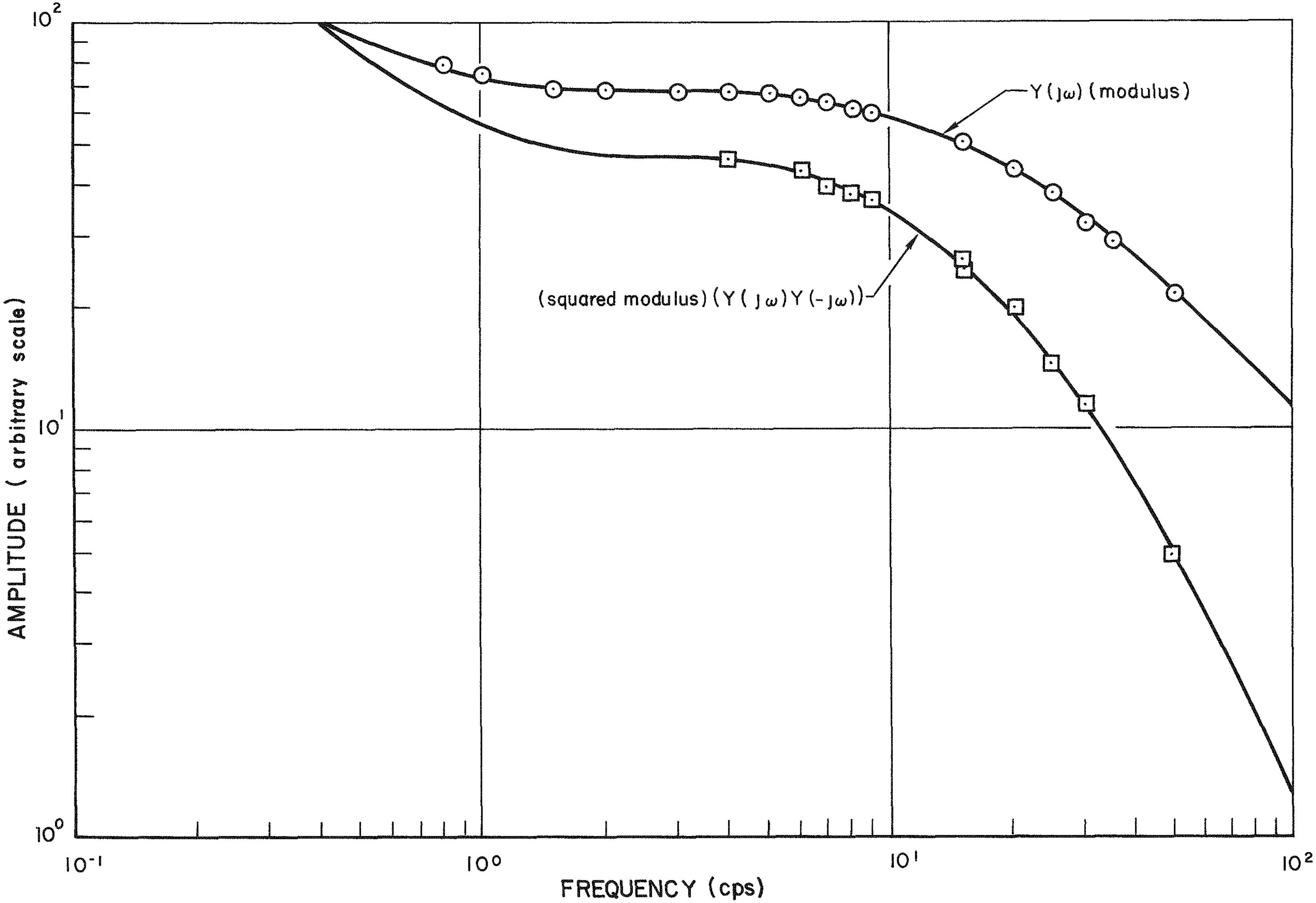


Figure 8. Modulus of the KEWB Reactor Frequency Response
(from curve 3 in Figure 6)

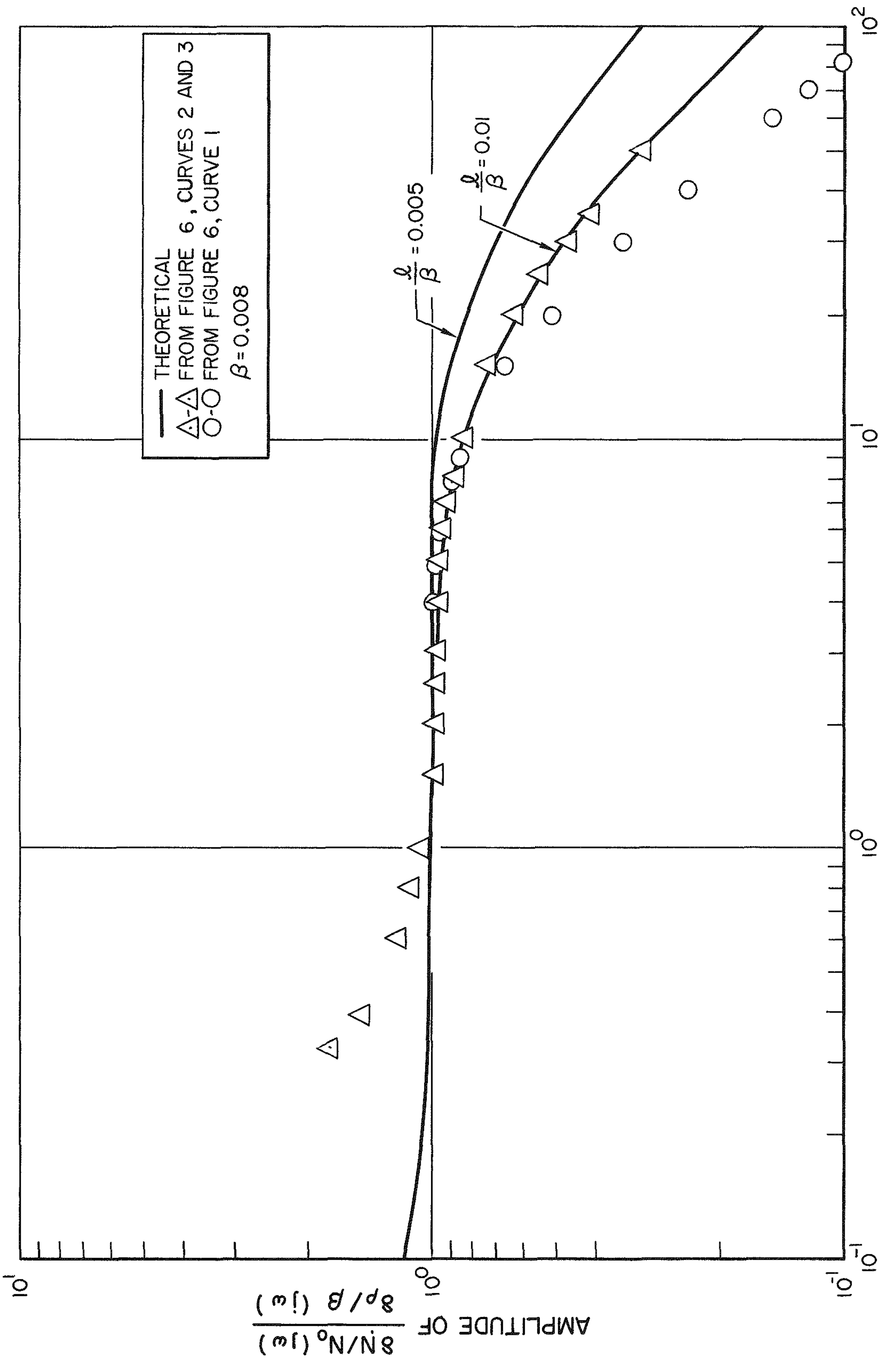


Figure 9. Measured and Theoretical Frequency Response Amplitude Curves for the KEWB

FREQUENCY (cps)

Figure 10. Measured Reactor Power Spectral Density of the SRE Multiplied by Frequency (March 5, 1959)

V. SRE NOISE EXPERIMENT

A. PROBLEMS AND EXPECTED RESULTS AS COMPARED TO THE KEWB

Since the lifetime of the SRE had already been measured by the reactor oscillation method and had checked well with the calculated value, it was decided

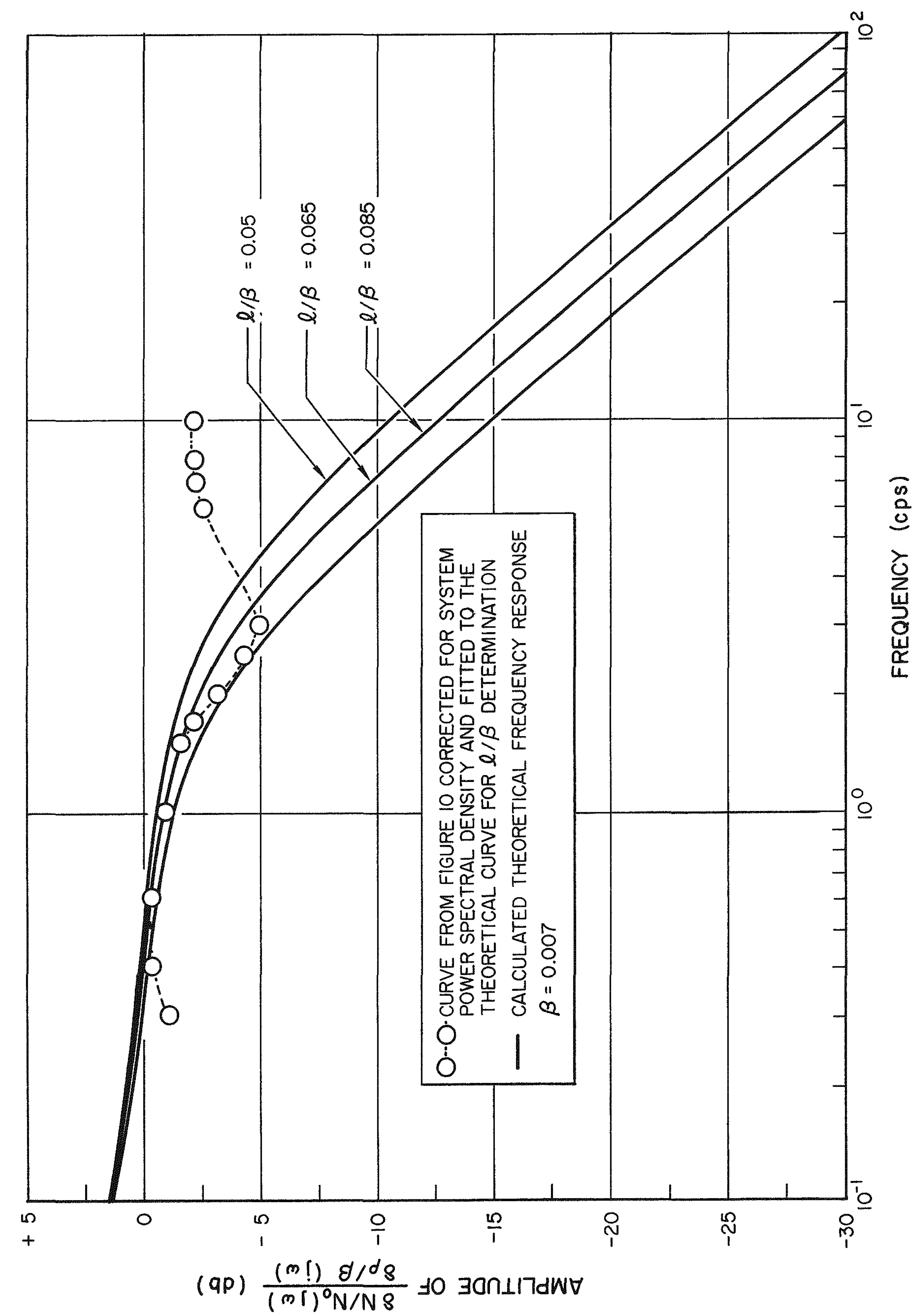


Figure 11. Measured and Theoretical SRE Frequency Response Amplitude Curves

TABLE I

KEWB AND SRE COMPARISONS

Reactor	Prompt Neutron Lifetime (sec)			
	Experimental			Calculated
	Oscillation	Noise	Period	
SRE	$5.25 \pm 0.35 \times 10^{-4}$	$5.25 \pm 0.7 \times 10^{-4}$	-	5×10^{-4}
KEWB	-	$7.8 \pm 0.3 \times 10^{-5}$	6.2×10^{-5}	6.6×10^{-5}

VI. CONCLUSIONS

A. COMPARISON OF THE TWO TECHNIQUES

While experience in the use of either oscillation or noise techniques for the experimental determination of the prompt neutron lifetime of a nuclear reactor is very limited, it is felt that the results of the experiments described by this report justify certain conclusions. The experiments on the SRE indicate that both methods of measuring the prompt neutron lifetime may be considered valid since both agreed to within 5% of each other and of the calculated lifetime. Conversely, increased confidence may be assumed in the method of theoretical calculation.

The oscillation technique is better suited for low-frequency response measurements since a discrete frequency of known amplitude is available. In noise measurements, the low-frequency randomness is very spasmodic and may make instantaneous changes of a factor of 10 or more. Since a very high gain is necessary in these measurements, such changes tend to overload the instrumentation. They also require a long measuring time to approximate the required white noise power spectral density. Thus, although noise measurements may be made much more rapidly, it is felt that oscillation techniques should be used for reactors which have a prompt neutron break frequency of 5 cps or less and noise analysis for those having break frequencies greater than 20 cps. The relative merit of the two methods in the intermediate zone is primarily dependent upon the physical installation and environment of the reactor being investigated. As shown in Figure 1, the oscillator has the added advantage at low frequencies of affording a second check on the validity of the experiment through a plot of its phase shift data.

B. CALIBRATION REQUIREMENTS

Insofar as instrumentation calibration is concerned, it was shown that calibration was unnecessary if the frequency response of the measuring circuitry was known to be satisfactory.

C. ℓ/β FROM SUPERPOSITION

The actual determination of ℓ/β by either method is best accomplished by comparing the measured response of the reactor to previously calculated frequency response curves for several values of ℓ/β in the region of expectation.

D. SUGGESTED IMPROVEMENTS

It is recognized that there exists a considerable area for improvement in the instrumentation which was used for these noise experiments. Among those to be considered are: diode limiting to prevent burnout of the vacuum thermocouple due to low frequency surge effects; improvements in methods of nullifying the effects of reactor drift while taking measurements; investigation of different squaring devices such as electronic multipliers; and investigation of commercial equipment which use a constant two-cycle band pass width in conjunction with a magnetic tape recorder.

A completely different method of analysis, which is under consideration, makes use of a magnetic tape to record the reactor fission noise and a digital computer to analyze the tape in order to determine the auto-correlation function of the noise. The power spectral density is then obtained from the auto-correlation function.

E. FUTURE EFFORT

There are several areas in the random noise technique of analysis which the authors feel are in need of further investigation. These are:

1) The difference, if any, for measurements using compensated or uncompensated ionization chambers should be clearly determined. No compensation was used on the KEWB measurements, but compensation was used on the SRE. Both appeared to give valid results.

2) Measurements should be made under identical conditions using both lead-shielded and unshielded ionization chambers. Clarification of the odd effects noticed in run one of section IV-6, when the lead-shielded ionization chamber was used, should result.

3) A large number of measurements should be made on a small experimental reactor at various power levels in order to determine the exact mathematical relationship of reactor noise to power level and ionization chamber noise to power level. From such knowledge both the optimum and the maximum power levels at which noise measurement can be made might be determined. Such information would also indicate, to some degree, whether or not it might be possible to extend the use of either the power spectral density or auto-correlation function to analysis of the relative stability of nuclear reactors.

REFERENCES

1. J. G. Lundholm, Jr., C. W. Griffin, and E. R. Meise, "Measurement of the SRE Zero Power Frequency Response," NAA-SR-3762 (1959)

2. J. H. Laning and R. H. Battin, Random Processes in Automatic Control, (New York: McGraw-Hill Book Co., 1956)

3. M. A. Schultz, Control of Nuclear Reactors and Power Plants, (New York: McGraw-Hill Book Co., 1955) Chapter 4

4. G. R. Keepin, and T. F. Wimett, "Reactor Kinetic Functions: A New Evaluation," Nucleonics (October 1958)

5. C. E. Cohn, "Determination of Reactor Transfer Functions by Pile Noise Analysis," ANS Annual Meeting, Paper 21-3 (June 1958)

6. M. N. Moore, "The Determination of Reactor Transfer Functions from Measurements at Steady Operation," Nuclear Science and Engineering, 3, 387 (1958)

7. D. L. Hetrick and D. P. Gamble, "Transient Reactivity During Power Excursions in a Water Boiler Reactor," Transactions of the American Nuclear Society, Paper 6-5, (December 1958)