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**GAMMA-GAMMA DIRECTIONAL CORRELATION
STUDIES IN ^{77}Ge DECAY**

Edward M. Lent
(Ph.D. Thesis)

November 20, 1974

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Gamma-Gamma Directional Correlation
Studies in ^{77}Ge Decay

ABSTRACT

The decay properties of the levels in ^{77}As were investigated following the decay of 11.3 hour ^{77}Ge . Singles γ -ray energy and intensity measurements were obtained with a 54 cm^3 high resolution Ge(Li) detector. Extensive γ - γ coincidence and directional correlation measurements were obtained with large coaxial Ge(Li) detectors. An improved decay scheme was constructed. From γ - γ correlation measurements, deduced $\log ft$ values, and internal conversion coefficients, the J^π values for the following levels were confirmed: g.s., $3/2^-$; 215.51, $3/2^-$; 264.41, $5/2^-$; 475.45, $9/2^+$; 614.49, $3/2^-$ or $5/2^+$; 631.82, $5/2^+$; 1189.81, $7/2^+$; and 1560.67, $5/2^+$. Further J^π values were proposed as follows: 194.75, $1/2^-$; 1528.32, $5/2^-$; 1538.80, $3/2^-$ or $5/2$; 2000.16, $5/2^+$; 2341.64, $5/2$ or $7/2$; 2354.12, $5/2$ or $7/2$; and 2543.70, $5/2$ or $7/2$. The following $\delta(E2/M1)$ mixing ratios were obtained: 215.51-keV, -0.164 ± 0.16 or -0.321 ± 0.011 ; 558.02-keV, -0.139 ± 0.006 ; 810.35-keV, 0.194 ± 0.030 or 2.37 ± 0.18 ; 928.85-keV, -0.63 ± 0.41 ; 1263.86-keV, -1.41 ± 0.06 or -0.079 ± 0.013 ; 1312.80-keV, -5.86 ± 1.7 or -1.03 ± 0.11 ; and 1368-keV, $-21. \pm 12.$ or 0.519 ± 0.49 .

1) INTRODUCTION

There is at present no nuclear structure theory that can adequately explain the experimental data that are available in the region of odd mass medium weight nuclei. Attempts to understand the level structure in terms of a single particle core coupling model have met with little success. Improvements have been made and are incorporated into the Coriolis coupling model, but the evaluation of the theories is seriously hampered by the lack of reliable experimental data. ^{77}As is a particularly good example of a complex, but poorly understood medium weight nucleus. Studies have been limited by experimental difficulties arising from the high density of ^{77}As levels and by the relatively few means available for populating excited states. This study of ^{77}Ge decay was undertaken, therefore, to accurately delineate and characterize the level structure of ^{77}As .

There have been several previous studies of the ^{77}As levels.⁽¹⁻¹⁶⁾ Smith⁽¹⁾ studied the β spectrum of ^{77}Ge with a magnetic lens spectrometer. Burson et al.⁽²⁾ used a NaI detector to investigate the γ spectrum, and also made γ - γ and β - γ coincidence measurements. A rudimentary decay scheme was proposed on the basis of the results of these works; spin and parity assignments were based on similarities with neighboring nuclei. Martin and Wiedenbeck⁽⁴⁾ used a NaI-NaI system to make angular correlation measurements. Their results were not definitive in that they were only able to show that the assumed spin and parity assignments were not inconsistent with their results. Van der Kooi and Van den Bold⁽⁵⁾ made a detailed investigation of the decay of ^{77}Ge . Their work included γ spectrum, γ - γ coincidence, delayed coincidence (to study the metastable 475-keV level), and angular correlation measurements. These early works involved the use of poor resolution NaI detectors, and hence yielded inadequate and ambiguous results. More recently several investigators⁽⁶⁻¹⁶⁾ studied the decay of ^{77}Ge with high resolution Ge(Li) spectrometers. Many new γ -rays were observed.

Ng et al.⁽⁹⁾ made γ - γ coincidence measurements using a NaI-Ge(Li) system. However they obtained coincidence spectra only for several of the most intense transitions. In addition, due to the poor resolution of the NaI detector, not one, but several γ -rays were present in many of the coincidence gates. This caused a significant uncertainty in the analysis of the data.

The most recent work is that of Poncet⁽¹⁶⁾ who, using a Ge(Li) detector, obtained energies and intensities for 135 transitions. However, the energy uncertainties are considerably larger than those attainable with high quality Ge(Li) spectrometer systems. The decay scheme proposed by Poncet (Figure 1) is based heavily on the results of previous studies, particularly the γ - γ coincidence results of Ng et al.⁽⁹⁾ The complexity of the nuclear structure of ^{77}As , as is evident from the number and extreme density of γ -ray lines observed in Ge(Li) spectra, clearly indicates the need for an accurate (~ 0.1 keV) determination of γ -ray energies, and for detailed high resolution [Ge(Li)-Ge(Li)] γ - γ coincidence studies.

The spin and parity assignments shown in Figure 1 are based for the most part on model predictions and on similarities with neighboring isotopes, ^{73}As and ^{75}As . There are no measurements leading to a strong assignment for any of these spins or parities, with the exception of the $3/2^-$ ground state, which is well established by measured log ft values for the β decay of ^{77}As to levels in ^{77}Se .⁽¹⁷⁾ Aside from log ft values, which in general only place limits on possible spin and parity values, there are three methods commonly used to determine spins and parities of excited levels. These are reaction studies, internal conversion measurements, and angular correlation measurements. There is a scarcity of reaction data on ^{77}As ; this is due in part to the fact that the ^{77}As ground state is not stable. The only reaction study reported is by Beits et al.,⁽¹³⁾ who, using the (^3He , d) reaction on ^{76}Ge , determined the spins of several positive parity states in ^{77}As .

Figure 1

Internal conversion measurements on the decay of ^{77}Ge have yielded meager results.^(1,7) The theoretical conversion coefficients for $Z=32$, as given by Hager and Seltzer,⁽¹⁸⁾ are shown in Figures 2a) and b). As can be seen the magnitudes of the coefficients are small, and in addition there is not a large separation between the coefficients for the different multipole orders. These factors, plus the high density of peaks in the conversion spectrum, make the interpretation of conversion data very difficult in this case. This leaves angular correlation measurements as the remaining method for making spin and parity assignments. The previous angular correlation studies^(4,5) were done with NaI-NaI systems; hence, considering the complexity of the decay scheme were not definitive.

This investigation is an attempt to determine the ^{77}As level structure populated by the β decay of ^{77}Ge . A detailed study of the γ spectrum was made using a large-volume high-resolution Ge(Li) detector. Two large volume, coaxial Ge(Li) detectors were used to make extensive γ - γ coincidence studies. In addition these same two detectors were used to investigate many cascades by the method of directional correlation.

Figure 2

II. NUCLEAR STRUCTURE

A. Interactions In The Nucleus

Nuclei are aggregates consisting of two types of particles, protons and neutrons, together referred to as nucleons. The forces responsible for binding nucleons to form nuclear system belong to the category referred to as "strong interactions". Although a number of general features characterizing the strong interactions are understood, the basic character of these phenomena and the relationship to other known interactions remain a challenging field of investigation. In the structure of nuclei, however, the full complexity of the strong interactions does not come into play, owing to the fact that nuclei are relatively weakly bound systems. The energy required to remove a nucleon from a nucleus is less than 10 MeV, and the average kinetic energy of nucleons in the nucleus is of the order of 25 MeV. These energies are small compared to the nucleon rest energy of 1000 MeV. Therefore, in the analysis of nuclear bound states, it is a good approximation to consider the nucleus as composed of a discrete number of nucleons with properties similar to those of free nucleons, and moving with non-relativistic velocities.

In addition to the strong interaction, the nucleons produce electromagnetic effects and the still weaker type of interactions that manifest themselves in the β -decay processes. Although these weaker interactions play a relatively minor role in the nuclear structure itself, they are of decisive importance for the study of nuclear states, for example, they determine the degree of stability of the bound nuclear states, i.e., the states that cannot emit nucleons and which, therefore, would be completely

stationary if only strong interactions were present. Moreover, because of its relatively simple nature and well-established properties, the electromagnetic processes represent a valuable tool in the study of nuclear phenomena.

B. Symmetry Properties and the Level Diagram

Because of the complexity of the nuclear forces and the difficulties inherent in a detailed description of systems with large number of particles and hence a large number of degrees of freedom, the characterization of nuclear states in terms of symmetry properties and the application of conservations laws play a prominent role in the analysis of nuclear phenomena. The symmetry laws of nuclear physics stem from the invariance of the interactions with respect to transformations of the space-time coordinate system. While the invariance under the continuous transformations, such as rotations, appears to have universal validity, the invariance under discrete transformations, such as reflections, has been found to be only partially valid.

If no external fields act on a system, the Hamiltonian, H , which describes the "motion" of the system, is invariant with respect to rotation of the coordinate axes. This rotational invariance implies the possibility of labeling the stationary (in time) states by the quantum number J representing the eigenvalue of the total angular momentum, or spin, $J^2 (= J(J+1)\hbar^2)$.

A reflection of the three coordinate axes can be associated with a transformation, P , which inverts all spatial coordinates. It is well known that for system of particles governed by nuclear, as well as electromagnetic,

forces, the Hamiltonian is invariant under such a reflection and, hence, can be characterized by the eigenvalue of the parity operator $P (= \Pi)$. Two successive operations with the parity operator leaves the system unchanged, hence $\Pi^2 = 1$ or $\Pi = \pm 1$, i.e., the parity of such a system is either even (+) or odd (-).

We have been considering bound states of a system of particles governed by a Hamiltonian, H . This implies yet another quantum number with which to label these states, that is the eigenvalue of H ($=E$) or the total energy of the state. Each state is characterized by a very narrow level width, $\Delta E \ll 1$ eV, about the energy E . This is an intrinsic property arising from the quantum nature of matter as expressed by the well-known Heisenberg uncertainty principle, $\Delta E \Delta t \geq \hbar/2 \sim 3 \times 10^{-16}$ eV-sec, where Δt is the lifetime of the state. The lifetime of these states is generally short, less than 10^{-9} sec, although there are instances of states with much longer halflives. These long-lived states are known as "isomeric" levels. (For example, ^{77}As has a level at 475 keV with a halflife of 116 μsec , and the halflife of $^{77\text{m}}\text{Ge}$ is 54 sec.)⁽¹⁷⁾ We now have a system which can be described by a set of discrete states, discrete in the sense that each state is characterized by a unique set of quantum numbers. This system can be represented schematically by an energy-level diagram (see Figure 3) where the zero reference for energy is taken to be the energy of the ground state or lowest-energy state.

C. Decay Scheme

If the nucleus exists at a level other than the lowest-energy state, it is in an energetically unstable configuration and will give off the excess energy in the process of reaching the ground state. This is usually

accomplished by one of two competing processes: a) the emission of electromagnetic radiation (a γ -ray); or, b) the ejection of one of the orbital electrons. The later process (internal conversion) will be discussed later. There are "selection rules" which apply to electromagnetic radiation that reflect the fact that transitions between some levels are inhibited. Thus, the decay of an excited (excess energy) state to the ground state may be a multistep process, proceeding through various intermediate levels. These processes are described by a decay scheme, schematically represented in Figure 4. Conservation of energy requires that the energy of a specific γ -ray be the energy difference between the initial and final levels for the transition, e.g., in Figure 4, the energy of γ_1 is E_1 , the energy of γ_2 is $E_2 - E_1$, etc. [An exact energy balance requires including the effect of nuclear recoil; that is, the excited state energy is shared between the emitted γ -ray and the recoiling nucleus, so as to conserve momentum. This is analogous to the familiar action-reaction law of Newtonian physics. Considering the large rest energy of the nucleus (many thousands of MeV) compared to the energies involved in the γ decay (a few MeV), this effect is small and easily accounted for.] Thus, we see that a measure of the γ spectrum (i.e., the energies of the transitions involved in the γ decay) is an indirect measure of the energy differences between the various levels and, hence, a powerful method in the study of nuclear properties.

As we mentioned above, there are selection rules which apply to the electromagnetic radiations. These properties will be stated here, with a brief derivation being given in Appendix A. Gamma rays carry off at least one unit of angular momentum ($\hbar$ with $L \geq 1$). Imposing conservation of angular momentum and noting that angular momentum is a

vector quantity we have, referring to γ_1 in Figure 4, $\vec{J}_1 + \vec{L}_1 = \vec{J}_0$, where the L 's are integral and the J 's are either integral or half-integral depending on whether the total number of nucleons comprising the nucleus is even or odd. Thus, permissible L_1 values are limited to $J_1 + J_0 \geq L_1 \geq |J_1 - J_0|$ ($= 1$ if $J_1 = J_0$). In addition, for electromagnetic transitions of multipole order L , we have $\pi(EL) = (-)^L$ and $\pi(ML) = (-)^{L+1}$ for electric and magnetic operators, respectively. Recalling that parity is conserved in electromagnetic interactions, we have, between states of the same parity, only even electric and odd magnetic transitions, while between states of opposite parity the only multipoles are odd electric and even magnetic.

The effects of multipole order and energy on the γ -ray intensities are given by the theoretical transition rates (see Appendix A). These predict that, from a given level, high-energy low-multipole-order transitions are the most intense, while for the same multiple order electric transitions are more probable than are magnetic.

Gamma-ray energies and intensities can be determined from a γ -spectrum (single-parameter) measurement. The analysis involves energy summing, i.e. referring to Figure 4, the energies of γ_1 and γ_2 sum to that of γ_3 (after including nuclear recoil effects); obtaining several such sums establishes a possible level at E_2 . For complex cases involving many levels and transitions, the γ -spectrum data are generally not sufficient to firmly establish a decay scheme. The nucleus investigated in this study is one example of the complexities one can encounter. There are 172 transitions in the γ spectrum, which we placed between 38 excited levels of ^{77}As . This would not have been possible without

additional information obtained from a γ - γ coincidence measurement. This powerful technique involves obtaining the spectrum of γ rays which are in cascade with each γ -ray of interest. For example (again referring to Figure 4), the coincidence spectrum for γ_2 would contain $\gamma_1, \gamma_4, \dots, \gamma_{m-1}, \gamma_m, \dots$, i.e. only those transitions which populate, either directly or indirectly, the level at E_2 and which depopulate, either directly or indirectly, the level at E_1 . The information thereby derived on γ cascades reduces the large number of decay scheme γ placements which are possible based on single-parameter data alone. In a complicated nucleus, such as is the subject of this work, coincidence data are essential to the unambiguous establishing of the decay scheme.

D. Populating Excited Nuclear Levels

Several methods are used to populate excited nuclear levels. The first method which we will consider is β -decay, the most common mode of decay for unstable neutron-rich nuclei (β^+ - decay and electron capture are closely related processes which apply to proton-rich nuclei). A β -unstable nucleus is usually produced by using a nuclear reaction which leads to a nucleus with a neutron excess (as in this work where neutron capture on ^{76}Ge was used to produce ^{77}Ge). The resulting nucleus decays by the conversion of a neutron into a proton with the emission of a β -ray (electron). This process leaves the nucleus in an excited state. The levels populated by this means are governed by selection rules which depend on the nuclear characteristics.

The halflife of the parent nucleus in β decay ranges from less than a second to more than 10^5 years. As is the case with γ decay, the

half-life gives an indication of the spin and parity change involved in the decay process. The relative intensity of the β feeding to the various levels may be obtained by a detailed γ -transition intensity balance for each level. One compares the total γ and conversion electron intensity feeding and depopulating a level and attributes the difference to β feeding. From this, one can determine for each level a partial half-life, t , (defined as the ratio of the half-life of the parent nucleus for β decay to the percent β intensity for each level). The partial half-lives are then weighted by a statistical rate factor, f , to account for the systematics of the β interaction. This rate factor depends on the Z value of the parent nucleus, the energy difference of the parent and daughter levels, and the classification of the degree of forbiddenness. Due to the extreme range of ft values encountered, the logarithm of this quantity is used to express the relative half-life. Values of the $\log ft$ place certain empirically derived limits on the spin and parity change involved in a β decay (see Table 1⁽¹²⁾).

TABLE 1

allowed	$0 < \log f_0 t$	$\Delta J = 0, 1; \Delta \pi = +$
1st forbidden non-unique	$5.9 \leq \log f_0 t \leq 8.0$ $5.1 \leq \log f_0 t \leq 8.0$	$\Delta J = 0, 1; \Delta \pi = -$
1st forbidden unique	$8.5 \leq \log f_1 t$	$\Delta J = 2; \Delta \pi = -$
2nd forbidden non-unique	$11 \leq \log f_0 t$	$\Delta J = 2; \Delta \pi = +$
2nd forbidden unique	$12.8 \leq \log f_0 t$	$\Delta J = 3; \Delta \pi = +$

Values of $\log f_0 t$ and $\log f_1 t$ can be obtained from tables⁽²⁰⁾ once the partial half-lives, t , are known.

Nuclear reactions provide a second method of populating excited states of a nucleus. In simple scattering experiments (for example, Coulomb excitation), energetic (charged) particles are scattered from a nucleus in its ground state. The scattering particles impart some of their energy to the nucleus, thus leaving it in an excited state. This method is not applicable to our case since the ground-state of ^{77}As is not stable, undergoing β -decay to the levels in ^{77}Se with a half-life of 39 hours. The desired excited nucleus may also be produced by some appropriate nuclear reaction on a nearby stable nucleus, for example, an investigation by Betts, et al⁽¹³⁾ using the $(^3\text{He}, d)$ reaction on ^{76}Ge to produce ^{77}As .

E. Determining Spins and Parities

Up to this point we have dealt mainly with methods of determining the energies of the excited levels of a nucleus. We now consider methods of determining the spins and parities. As was mentioned before, $\log ft$ values give some indication of possible spins and parities of excited levels. As will be shown later, the $\log ft$ values obtained for the ^{77}Ge β decay place only wide limits on the possible spin and parity assignments. This, plus the lack of available reaction data, leave little information with which to determine spins and parities.

There are two remaining methods commonly used to determine spins and parities of excited levels -- the measurement of internal conversion coefficients and of angular correlations. Internal conversion was mentioned previously as one process for de-excitation of the nucleus. In this process

the excitation energy is transferred directly to an orbital electron, which is subsequently ejected from the atom with an energy equal to the excitation energy less the orbital binding energy. The internal conversion coefficient is defined as the ratio of the number of conversion electrons to the number of γ -rays for a particular transition. (This definition does not apply to $J_i = 0$ to $J_f = 0$ transitions since, in this case, γ transitions are absolutely forbidden.)

The conversion coefficient depends on the multipole order of the electromagnetic field, with electric transitions, on the whole, being less probable than magnetic. This dependence on the order and character of the electromagnetic interaction allows the prediction of spins and parities based on the comparison of measured and theoretical⁽¹⁸⁾ conversion coefficients. The mechanism responsible for internal conversion is the electromagnetic field of the nucleus interacting with that of the orbital electrons. Thus, the proximity of the orbital electrons has a direct effect on the conversion process. The conversion coefficients increase with increasing Z , since the orbital electrons are on the average closer to the nucleus. For the same reason, conversion is more probable with K-shell electrons than with L-or higher-shell electrons. The conversion coefficients also depend on transition energy, decreasing rapidly with increasing energy.

For the case of ^{77}Ge , which is the subject of this work, the interpretation of conversion data is very difficult and ambiguous. This is due to the following reasons: (1) a low probability for conversion corresponding to a low Z ; (2) the lack of a clear separation between the coefficients for the various multipoles (see Figures 2a and b); (3) the small energy separation between K-and L-shells; and (4) the extreme density of transitions.

The remaining method of determining spins and parities is the measurement of γ - γ angular correlations. Since it is discussed in detail in Section III, suffice it to say here that angular correlation theory predicts that the γ - γ coincidence counting rate is a function of the relative angle of emission between the two transitions of a cascade, with the functional form being dependent on the spin sequence of the levels and the multipolarities of the transitions participating in the cascade. Thus, by measuring the angular dependence of correlation and comparing with theory, one can place limits on the possible spins and parities of the levels involved. A measurable perturbation of the correlation is possible if large electromagnetic fields interact with the dipole or quadrupole moments for sufficient times. In our case there were no applied external fields and the diamagnetic nature of the material precludes large internal fields. This, in conjunction with the very short lifetimes ($< 10^{-11}$ sec) of the excited states considered, rules out any perturbation effects.

F. Theoretical Models of the Nucleus

The theoretical description of nuclear structure can, at present, be treated only in approximation. The reason for this lies partly in the inherent complexity of all many-bodied systems, and partly in the very sparse amount of information to the nuclear forces. The most familiar approximation is the shell model; it is analogous to the well known orbital description of the electronic states of the atom. Basically, this model considers one arbitrary nucleon in the nucleus and assumes that the influence of all other nucleons on the chosen nucleon can be approximated by an averaged potential. The internal motion of the nucleus can then be described by the superposition of the orbits of each of the individual nucleons comprising the nucleus. Each nucleon orbital can be characterized by a set of quantum numbers and represents a nuclear energy level. Describing the potential by an attractive, central force (e.g., a modified harmonic oscillator well) and including a term arising

from spin-orbit coupling (which couples the spin of a nucleon to the angular momentum in its orbit) leads to the well-known magic number shell closures. Figure 5 shows a schematic diagram of the shell model energy levels. Since the two classes of nucleons, neutrons and protons, are not identical particles, each class separately fills its own set of energy levels, i.e., there is an energy level scheme (like Figure 5) for neutrons and slightly different one for protons. This difference lies mainly in the Coulomb force acting between the protons; neutrons being uncharged exhibit no such interaction.

Evidence for the shell structure of the nucleus can be found by noting that major discontinuities in the nuclear binding energies occur at the nucleon numbers 2, 8, 20, 28, 50 and 82 (these are "magic numbers") indicating that nucleons in a closed shell are tightly bound to the nucleus. This is analogous to the noble gases of atomic physics with their tightly bound closed shell of electrons. Similarly, neutron capture experiments show that for nuclei with a closed neutron shell, the probability of capturing an extra neutron is small, indicating that these extra neutrons are weakly bound. In addition, experiments measuring the energy of the first excited state of even-even nuclei (even numbers of both neutrons and protons) show very large values for nuclei with closed shells in both neutrons and protons, again indicating a very stable configuration.

For nuclei with closed shells of both neutrons and protons, the nuclear shape is spherical. Adding or removing one nucleon represents a relatively small deformation, and so one retains many of the shell model features. However, the system now possesses additional degrees of

freedom corresponding to oscillations in the nuclear shape. This coupling between the particle and the collective motion of the core produces vibrational levels which intermix with the shell model states. The situation becomes complicated very rapidly upon including a few additional particles or holes (a hole is the lack of a particle) outside the closed shells.

With the inclusion of many particles or holes, the situation, rather than getting more complex, becomes simpler. This is because for configurations far removed from closed shells, the nucleus acquires a large permanent deformation and the nuclear coupling scheme takes on the relative simplicity resembling that of a linear molecule. Thus, one obtains an approximate solution by treating independently the particle motion, in the now deformed, rather than spherical potential and the collective motion corresponding to rotation and vibrations of the deformed nuclear shape. This independent motion is verified by the well defined rotational and vibrational bands observed in the strongly deformed nuclei. This approach works well for the heavy nuclei, $155 < A < 185$ and $A > 225$, where A is the total number of nucleons and the gap between 185 and 225 corresponds to the region around the doubly magic nucleus ^{208}Pb where extreme deformations do not occur.

In general, it is assumed that the observed nuclear properties associated with light nuclei ($A \leq 100$) can be understood in terms of the deformed shell model with appropriate residual interactions between the nucleons in the last unfilled shell or shells. One such interaction is the angular momentum of the deformed core coupling with that of the particle outside the core. This coupling, known as Coriolis coupling,

tends to produce more deformation. There is another residual interaction, which depends on the tendency of like nucleons to pair up with their spins opposed; this effect tends to produce more spherically-shaped nuclei. Thus, the shape of nuclei in this region depends on which effect is dominant. Nuclei with $32 \leq Z \leq 36$ are known to be considerably deformed,⁽²¹⁾ though they do not show rotational spectra as clearly as the heavy nuclei in the well-established deformed regions. The structure of the nuclei in this region has been discussed on the basis of several models without substantial success. This, plus the lack of reliable experimental results on the ^{77}As levels, makes the present work of considerable interest.

III. ANGULAR CORRELATION

A. Introduction

The probability of radiative emission by a nucleus is dependent upon the angle between the nuclear spin axis and the propagation direction of the emitted radiation. Since, under normal conditions, the nuclei in a radioactive sample are randomly oriented, the total radiation pattern is isotropic. A non-isotropic distribution can be observed only from a collection of nuclei that are not randomly oriented. One method to achieve this, which we will here be concerned with, consists of selecting those nuclei whose spins happen to lie in a particular direction. This can be accomplished if the nuclei decay through the successive emission of two radiations. The observation of one radiation in a fixed direction selects a set of nuclei that have a non-isotropic distribution of spins. The succeeding radiation then shows a definite angular distribution with respect to the fixed direction.

A simplified description of an angular correlation experiment is as follows: A nucleus emits a γ cascade, i.e., γ_1 followed in rapid succession by γ_2 (see Figure 6a). We wish to determine the relative probability, $W(\theta)d\Omega$, that γ_2 is emitted into the solid angle $d\Omega$ at an angle θ with respect to the propagation direction of γ_1 . The form of $W(\theta)$, the correlation function, can be determined from theory if the spin sequence (J_3, J_2, J_1) and multipole mixing ratios (δ_1, δ_2) are known. Experimentally one measures the number of coincident events $C(\theta)$ between γ_1 and γ_2 as a function of the angle θ subtended by the axes of the two detectors. Due to the finite size of the detectors, $C(\theta)$ represents an integral average of the true correlation

Figure 6

$W(\phi)$ over angles ϕ around θ as determined by the detectors (see Figure 6b). Correcting $C(\theta)$ for this effect yields the measured correlation function $W_m(\theta)$. Finally, the comparison of $W_m(\theta)$ with theory (Figure 6c) provides the desired information about the properties of the nucleus, i.e., the spins and/or parities of the nuclear levels and the multipolarities of the emitted radiation.

Included in the above is the tacit assumption that, apart from their direct interaction with the radiation field, the nuclei are isolated. This situation is seldom realized in practice. The interaction between internal or external electromagnetic fields and the nuclear magnetic and electric moments during the time between formation and decay of the intermediate state of the cascade may cause a perturbation of the observed angular correlation. The magnitude of the perturbation depends on the strength of the fields and the lifetime of the intermediate state. (It is this effect that permits the determination of static magnetic dipole and electric quadrupole moments of excited nuclei.) Detailed discussions of perturbed angular correlation may be found in the works of Abragam and Pound⁽²²⁾ and Steffen and Frauenfelder (survey).⁽²³⁾

The theory of angular correlation is based on very general symmetry relations and, for this reason, is probably the most complete and best understood theory of nuclear phenomena. These symmetries -- three dimensional rotation and reflection -- are intimately connected to the principles of conservation of angular momentum and parity, respectively. Thus, angular correlation measurements have become a valuable method for

determining spins and parities of excited nuclear states. The theory also yields information on the transition matrix elements in the form of multipole mixing ratios. If the transition occurs between sharp angular momentum states which is always true in radioactive decay experiments, then the information obtained from angular correlation measurements is independent of nuclear models. This is an important result for the experimental nuclear spectroscopist.

By the term "polarization correlation", we refer to observations involving linear or circular polarization. Polarization, like angular momentum, is a rotational property whose measurement, although difficult, provides a better understanding of the nuclear process involved. For γ -rays, the angular distribution is independent of multipolarity, but not of linear polarization. Hence, only those correlation experiments sensitive to linear polarization can determine the parity change in γ -transitions*. The bulk of γ -correlation experiments are polarization insensitive. These we refer to as "directional correlation", since the distribution depends only on the angle of the relative propagation directions. Both techniques fall under the general classification of "angular correlation".

B. Historical Development

Dunworth⁽²⁴⁾, in 1940, first pointed out the fact that successive nuclear radiations are correlated in their relative propagation directions. In the same year Hamilton⁽²⁵⁾ investigated γ - γ correlations using second-order time-dependent perturbation theory. His original work was very limited in scope, mainly because of the lack of suitable algebraic techniques; and although certain important features were evident, he could only treat the simplest cases. The necessary transformation theory

* Since observed $E1/M2$ mixing ratios are very rare, we can, in practice, almost surely assign a mixed dipole - quadrupole transition as $E2/M1$.

for angular momenta was developed mainly by Racah⁽²⁸⁾ in 1942. In later years (1949 through 1952), number authors⁽²⁷⁻³³⁾, using the Racah algebra, investigated various types of cascade-decay processes. Subsequently, Fano⁽³⁴⁾ (1952) and Coester and Jauch⁽³⁵⁾ (1953) independently introduced a more elegant approach. Using density matrices and a tensor formalism, they avoided much of the tedious algebra still inherent in the earlier formulations. Since then the theory has been fully developed⁽³⁶⁻³⁹⁾ and is well understood today.

C. A Brief Theoretical Description of The Correlation Process

Consider a simple γ -cascades such as shown in Figure 6. A density matrix⁽⁴⁰⁾ can be used to describe the spin orientation of the initial state. Using first-order perturbation theory, one can determine the transition probability for the emission of the first γ -ray. This, combined with the density matrix for the initial state, yields the density matrix for the intermediate state. This process can be repeated for the second γ -ray to the final state from which desired results, i.e., the probability function for angular correlation $W(\theta)$ can be obtained. A full description of the above method is given in reference 39. A brief derivation, not involving the density matrix formalism, is given in Appendix A.

There has in the past been considerable confusion concerning the multipole mixing ratio. Values of δ have been reported without explanation as to how the term was defined. Since varied phase conventions have been used in defining wave functions and matrix elements, the sign convention used in defining the mixing ratio varies. Even in some theoretical papers,

the definition of the mixing ratio is not clear since the γ -transition operators, and hence the transition matrix elements, are not expressly given. These factors have led to great difficulty in the comparison of experimental results with theory and with other experimental results.

As used in this work, the mixing ratio of two multipole components, $=L$ and $=L'$, where emission operators are used and $\pi = E$ or M for electric or magnetic multipoles, is given by:

$$\delta = \frac{\langle I_f || \pi L' || I_i \rangle}{\langle I_f || \pi L || I_i \rangle} \quad (1)$$

where I_i and I_f represent the initial and final states, respectively. Note that the initial state always appears on the right in the matrix elements. The reduced matrix elements are, in turn, defined by the Wigner-Eckart theorem:

$$\langle I_f m_f | \pi L, -M | I_i m_i \rangle = \frac{(-)^{M+m_f-m_i}}{(2I_f+1)^{1/2}} \langle I_i m_i L, -M | I_f m_f \rangle \langle I_f || \pi L || I_i \rangle \quad (2)$$

The mixing ratio, as defined by Equation (1), is related to that of Rose and Brink⁽³⁸⁾ by a phase term,

$$\delta = (-)^{L'-L} \delta_{RB}$$

the phase difference arising from the fact that Rose and Brink define the mixing ratio in terms of reduced matrix elements involving absorption operators.

D. The Directional Correlation Function

The directional correlation of two mixed multipole γ -radiations, emitted according to the general scheme:

$$I_1 \xrightarrow{\gamma_1} I_2 \xrightarrow{\gamma_2} I_3 \rightarrow \dots \rightarrow I_n \xrightarrow{\gamma_n} I_{n+1}$$

is given by:

$$W(\theta) = \sum_{k \text{ even}} B_k(\gamma_1) U_k(\gamma_2) \dots U_k(\gamma_{n-1}) A_k(\gamma_n) P_k(\cos\theta) \quad (3)$$

where it is assumed that γ_2 through γ_{n-1} are not observed. The B coefficients (orientation parameters) for the initial γ transition are given by:

$$B_k(\gamma_1) = \frac{F_k(L_1 L_1 I_1 I_2) + 2(-)^{L_1 - L_1} \delta_1 F_k(L_1 L_1' I_1 I_2) + \delta_1^2 F_k(L_1' L_1' I_1 I_2)}{(1 + \delta_1^2)} \quad (4)$$

while the A coefficients (angular distribution parameters) for the final γ transition are given by:

$$A_k(\gamma_n) = \frac{F_k(L_n L_n I_{n+1} I_n) + 2\delta_n F_k(L_n L_n' I_{n+1} I_n) + \delta_n^2 F_k(L_n' L_n' I_{n+1} I_n)}{(1 + \delta_n^2)} \quad (5)$$

where the $F_k (LL'II')$ are the usual F coefficients. Note the phase for the multipole interference term in the orientation parameters, as well as the different order for the initial and final-state spins appearing in the F coefficients in the expressions for $B_k (\gamma_1)$ and $A_k (\gamma_n)$. The F coefficients have been extensively tabulated^(41,42) and are also explicitly expressible in terms of Clebsch-Gordan and Racah coefficients, e.g.:

$$F_k (LL'II') = (-)^{I-I'-1} [(2L+1)(2L'+1)]^{\frac{1}{2}} < LIL'; -1 | k0 > W(I'I'LL'; kI) \quad (6)$$

The U coefficients (de-orientation parameters) for the unobserved γ transitions are given by:

$$U_k (\gamma_i) = \frac{G_k (L_i I_i I_{i+1}) + \delta_i^2 G_k (L_i' I_i' I_{i+1})}{(1 + \delta_i^2)}; i = 2, 3, \dots, n-1 \quad (7)$$

Note that since these radiations are not observed, there are no interference terms arising from competing multipolarities. The G coefficients are expressible in terms of the more familiar Racah coefficients.

$$G_k (LII') = (-)^{I+I-L} [(2I+1)(2I'+1)]^{\frac{1}{2}} (III' I'; kL). \quad (8)$$

For the relations given above, we have assumed that only the two lowest possible multipole orders contribute in each transition, i.e. for $i = 1$ to n .

$$\begin{aligned} L_i &= |I_{i+1} - I_i| \\ &= 1 \quad \text{if } I_{i+1} = I_i \end{aligned} \quad (9)$$

and

$$L'_i = L_i + 1 \quad (10)$$

Conservation of parity requires that the competing radiations be of different character, i.e. mixed electric and magnetic multipoles. Hence,

$$\pi'_i \neq \pi_i \quad (11)$$

The directional correlation function (Equation 3) is generally expressed as:

$$W(\theta) = \sum_{k \text{ even}} a_k P_k(\cos \theta) \quad (12)$$

The a_k being the product of the appropriate parameters given above, where each is normalized such that $a_0 = 1$. The maximum value for k is determined by the nonvanishing of the Clebsch-Gordan and Racah coefficients contained in the F and G coefficients. In the bulk of experiments, $k_{\max} < 4$.

The above expression describes an ideal experiment with point detectors. For the real case of finite size detectors, the observed correlation becomes:

$$\bar{W}(\theta) = \sum_{k \text{ even}} Q_k a_k P_k(\cos \theta) \quad (13)$$

where Q_k are the solid angle correction coefficients, also normalized such that $Q_0 = 1$ (see Section IV D 2).

It should be emphasized that a_k is a product of coefficients, each of which depends solely on the properties of a single member of the cascade. Further, there is but one such coefficient for each member of the cascade. With this in mind, consider the case where a transition γ_1 feeds a level which, in turn, is depopulated by two transitions γ_2 and γ_3 and where we seek information on the γ_1 transition. There are two possible correlations for which we obtain the values,

$$a_k(\gamma_1 \gamma_2) = B_k(\gamma_1) A_k(\gamma_2)$$

and

$$a_k(\gamma_1 \gamma_3) = B_k(\gamma_1) A_k(\gamma_3)$$

As is evident, both correlations must yield the same value for $B_k(\gamma_1)$ and, hence, this represents redundancy, not new information on the properties of the γ_1 transition. In contrast, consider the case of a triple cascade, γ_1 followed by γ_2 followed by γ_3 , in which we seek information on the properties of γ_2 . In this case, these are three correlations possible yielding:

$$a_k(\gamma_1 \gamma_2) = B_k(\gamma_1) A_k(\gamma_2)$$

$$a_k(\gamma_2 \gamma_3) = B_k(\gamma_2) A_k(\gamma_3)$$

and

$$a_k(\gamma_1 \gamma_3) = B_k(\gamma_1) U_k(\gamma_2) A_k(\gamma_3)$$

From this, we obtain values for $A_k(\gamma_2)$, $B_k(\gamma_2)$, and $U_k(\gamma_2)$ giving three independent determinations of the properties of the γ_2 transition. Thus, we are more likely able to determine unique values for the mixing ratio and the spins of the initial and final states for the γ_2 transition.

IV. EXPERIMENT AND ANALYSIS

A) Source

The ^{77}Ge activity was produced by the $^{76}\text{Ge} (n, \gamma)$ reaction on approximately 14 mg of germanium oxide with a ^{76}Ge enrichment of 95.55%. The source was irradiated for 10 to 20 minutes in a thermal neutron flux of approximately 2×10^{13} neutrons per second from the Livermore pool type reactor (LPTR) facility. Counting was started about 30 minutes after irradiation, allowing the 54-second isomer to die away without seriously affecting the 11.3-hour ground-state activity.

B) Single Parameter Experiment

1) The Singles Counting System

Figure 7 shows a schematic of the singles counting system. A true coaxial Ge(Li) detector, with an active volume of 54 cm^3 , a resolution of 2.2 keV, and a peak-to-Compton ratio of $\sim 30:1$, was used for the singles measurements. An amplifier (Ortec model 450 Research Amplifier), pile-up rejector (Canberra model 1464 Restorer/Rejector), and pulse-height analyzer (Nuclear Data 2200 Analogue to Digital Converter) completed the system. The entire electronic system was direct coupled and had excellent long-term stability. Magnetic tape was used to record the data for analysis at a later time.

To prevent degradation of the resolution the counting rate was kept below 3000 counts per second. Data were accumulated for 160 minutes, then recorded on tape. The source position was readjusted to maintain the count rate, and counting was resumed. This procedure was repeated through 6 half-lives. This time history of the source facilitated identification of the ^{77}Ge lines.

Figure 7

Care was taken in the analysis of the later spectrum dumps since the ^{77}As (38.8 hr) activity was increasing thus producing lines in the spectra not originating from the ^{77}Ge decay. Fortunately the decay of ^{77}As is simple, containing only a few lines of well known energy⁽¹⁷⁾, hence this effect did not seriously complicate the resulting γ spectra. In addition the smaller source-to-detector distance increased the detector solid angle, thereby increasing the probability of summing effects in the resultant γ spectra. Summing occurs when two distinct γ -rays interact simultaneously in the detector, thus producing a spurious peak in the γ spectrum at an energy corresponding to the sum of the energies of the two γ -rays. This effect was negligible at distances greater than 10 cm and for this reason only the data taken at these larger distances were used in the detailed energy and intensity analysis.

2) Data Analysis

A computer code, GAMANAL⁽⁴³⁾ was used to read and analyze the data tapes. The code uses a gaussian with an exponential leading edge on a modified step-function background to describe the peak shape. A series of strong, single peaks are used to define the FWHM and tailing parameters as a function of energy. The code then searches the spectrum, locates peaks or peak groupings, and fits the data with the predetermined shape function to yield peak centroids and areas. Shown in Figure 8 is one of the strong points of the peak fitting code, i.e. the ability to resolve multiplets. As can be seen, the data depicted in the figure are not properly fit by the single predetermined peak shape, and is in fact a doublet with an energy separation of ~ 1 keV. The fitting is done in a least-squares sense, and the appropriate errors are obtained from the corresponding error matrix.

An energy calibration run was made using the lines of known energy listed in Table 2. A fourth order polynomial fit was made to these energies using the channel positions determined as described above. The system non-linearity is shown in Figure 9. Figure 10 shows the deviations of the calibration points from the fitted curve. The dotted lines, obtained from the error matrix, represent one standard deviation and give the contribution to the energy error due to the calibration. The total energy error for a γ -ray consists

Figure 8

TABLE 2
PRIMARY ENERGY CALIBRATION STANDARDS^(a)

<u>Isotope</u>	<u>Energy</u>	<u>Isotope</u>	<u>Energy</u>
⁷⁵ Se	136.000 ± 0.004	⁸⁸ Y	898.024 ± 0.015
	264.651 0.007		1836.022 0.030
	279.528 0.006		
	400.646 0.006	⁵⁶ Co	846.754 0.015
			1037.818 0.016
^{110m} Ag	446.798 0.008		1238.255 0.017
	657.751 0.012		1360.176 0.019
	677.606 0.016		1771.310 0.028
	706.673 0.017		2015.136 0.030
	763.932 0.015		2034.710 0.030
	818.020 0.018		2598.399 0.036
	884.668 0.015		
	937.481 0.015		
	1384.275 0.020		
	1475.766 0.028		
	1505.010 0.025		

^(a)Energy values from Reference 44.

Figure 9

Figure 10

of this error summed in quadrature with the peak position error obtained from the GAMANAL code. The non-linearity of the system having been determined, a second calibration was made using six standard lines (see Table 3) run simultaneously with the ^{77}Ge source. The energies of these six standard lines, as obtained from the primary calibration, were adjusted to their correct values by making a first-order least-squares fit. The coefficients from the two fits were combined to yield the final energy calibration. As can be seen from Table 3 these data were fit in all cases to better than 10 eV, indicating that the non-linearity of the system was very accurately determined by the primary calibration. In addition the errors from the internal calibration are negligible compared to those resulting from the primary calibration.

An efficiency calibration was made using the sources listed in Table 4. The calibration runs were made using thin sources and with source-to-detector distances ≥ 10 cm. A fifth order log-log fit was made to the data. The results, shown in Figure 11, were used to determine the relative intensities. The errors were obtained by combining the area uncertainties from the GAMANAL code with the efficiency uncertainties from the error matrix involved in the efficiency fit. Since the efficiencies are relative and not absolute, the errors in the relative intensities can not be smaller than the smallest efficiency error, $\sim 5\%$. As a check a Monte Carlo computer code was written to calculate the efficiency (see Appendix B). As can be seen in Figure 11, the agreement is excellent. The deviation at low energy is due to uncertainties in the dead layer thickness.

TABLE 3
INTERNAL ENERGY CALIBRATION STANDARDS^(a)

<u>Isotope</u>	<u>Energy</u>		<u>Energy-Energy Fit</u>
²⁰³ Hg	279.188	± 0.004	-0.006
¹³⁷ Ca	661.646	0.012	0.008
⁸⁸ Y	898.024	0.015	-0.005
	1836.022	0.030	-0.009
⁶⁰ Co	1173.211	0.017	-0.001
	1332.482	0.018	0.006

(a) Energy values from Reference 44.

TABLE 4
EFFICIENCY CALIBRATION STANDARDS

<u>Isotope</u>	<u>Energy</u>	<u>Relative Intensity</u>	
⁷⁵ Se (a)	121	27.9 ±	0.4
	136	95.5	1.2
	265	100	
	280	42.	0.5
	304	2.23	0.09
	401	20.0	0.7
⁸² Br (b)	92	0.887	0.026
	221	2.66	0.08
	273	0.954	0.038
	554	84.8	1.7
	619	51.7	1.1
	698	34.0	0.7
	777	100	
	828	29.1	0.7
	1044	32.8	0.5
	1317	32.2	0.6
	1475	19.9	0.2
¹⁸² Ta (c)	100	40.2	1.0
	152	20.6	0.4
	156	7.68	0.17
	179	8.98	0.25
	222	21.5	0.5
	229	10.5	0.3
	264	10.3	0.3
	1121	100	
	1189	46.9	0.5
	1221	78.3	0.8
	1231	33.1	0.4

TABLE 4 (Cont'd)

<u>Isotope</u>	<u>Energy</u>	<u>Relative Intensity</u>
⁵⁶ Co(d)	847	100
	1038	14.0 ± 0.1
	1238	67.6 0.4
	1360	4.33 0.04
	1771	15.7 0.2
	2015	3.08 0.03
	2034	7.89 0.07
	2598	16.9 0.2
	3202	3.04 0.03
	3253	7.41 0.07
	3273	1.75 0.02
	3451	0.875 0.010
	3548	0.180 0.005

- (a) Weighted average of values from References 45, 46, 47, 48 and 49.
- (b) Weighted average of values from References 45, 50 and 51.
- (c) Weighted average of values from References 52, 53 and 54.
- (d) Reference 55.

Figure 11

C) Three Parameter Coincidence Experiment

1) The Three Parameter Counting System

Figure 12 is a block diagram of the three parameter system. In the usual "slow-fast" coincidence system, the fast (or timing) signal is used simply as a gate for the slow (or energy) signal. The three parameter system modifies this basic approach and extracts the timing signal for subsequent analysis. As will be shown in the next section this system allows for the simultaneous accumulation of all data necessary to analyze a coincidence experiment.

The detectors were positioned in conventional dip sticking mountings. Thus the liquid Nitrogen dewars, plus all additional sources of scattering, were kept far from the source-detectors arrangement. The source was located, on a small polyethylene holder, at the intersection of the detector axes and at a distance of about 4 cm from each detector. Scattering in the source and capsule was negligible. The detector axes were at an angle of 135° . A half-inch lead shield was placed between the detectors to prevent accidental coincidences arising from scattering from one detector to the other.

a) Detectors

Detector A is a true-coaxial Ge(Li) detector of 66-cm^3 active volume with a resolution of 2.3 keV (FWHM) at 1332 keV. Detector B is a very similar true-coaxial Ge(Li) detector of 67-cm^3 active volume with a resolution of 2.3 keV (FWHM) at 1332 keV. These detectors are especially well suited for coincidence studies because of their large volumes (high peak efficiency) and coaxial design (good timing characteristics). Each preamp (PA) has two outputs, one used for event timing and one used for γ -ray energy determination.

Figure 12

b) Energy Determination

The high-rate linear amplifier (HRLA, LRL model 5501 P-1) and pile-up rejector (PUR, LRL model 5550 P-1) combination produces a valid pulse at the coincidence unit under non-pile-up conditions. The circuit rejects a pulse preceded by another by a variable preset time (in this case 12 μ sec) or one which is followed by another during the pulse rise time.

The dual Single-Channel Analyzer (DSCA, Canberra model 1431) is used for energy selection if desired. The Decay Amplifier (DA, Ortec model 427) insures the signal arrives at the coincidence unit at the proper time.

The research amplifier (RA, Ortec model 450) shapes the signal for pulse-height analysis. In the trigger mode, the Linear Gate Stretcher (LGS, Ortec model 442), upon receiving the gate signal from the coincidence unit, allows the amplifier signals to pass.

c) Event Timing

For side A, the timing information is obtained by using a Timing Filter Amplifier (TFA, Ortec model 454) in conjunction with a Constant Fraction Timing Discriminator (CFTD, Ortec model 453) operated in the leading edge mode. The discriminator was set just above the noise for minimum time uncertainty. The output from this combination provides a "start" signal to the Time - to-Amplitude converter (TAC, Ortec model 437A). The "stop" signal, from side B, is obtained with an Extrapolated Zero Strobe (EVS, Canberra model 1426). This unit has two discriminators. When an input signal exceeds both discriminator levels, zero time is obtained by extrapolating back to zero level. If the input signal is essentially linear until the signal crosses the upper level, the time jitter is minimal. About 200 nsec of cable delays the stop signal to correspond to the time range of the TAC. The TAC produces a voltage signal proportional to the time difference between the start and stop signals, thus all the timing information is contained in the TAC output. This output goes to the coincidence unit in addition to a third LGS.

d) Data Acquisition

The slow coincidence unit (Canberra model 1446) has five inputs: The PUR signals from side A and B; the SCA signals from side A and B; and the TAC signal. This unit produces a valid output signal whenever the input signals overlap in time to within the preset resolving time (in this case 0.5 μ sec). The output provides a gate trigger signal for the three LGS's.

The three analogue signals, from the LGS's, are digitized by two analogue to-Digital Converters (ADC, Nuclear Data 160F and 161F Dual Analogue-to-Digital Converters). These units, slightly modified, in conjunction with a multiplexer, fabricated in this laboratory, record the data in the form of coincidence events. Each event is comprised of an energy from side A, an energy from side B, and the time relationship of these signals. These events are stored sequentially in the core memory buffer, which when full is recorded on magnetic tape. The buffer is cleared and the process is repeated. Due to the inherent dead time in the electronics (mainly the PUR's and the TAC) the maximum usable counting rate is 5×10^4 counts per second. This corresponds to about 120 coincidence events per second recorded on tape. A full tape contains about 6000 records for a total of approximately 1.5×10^7 coincident events.

2) Data Analysis

The three parameter system produces a coincidence matrix, $S(E_A, E_B, T)$, representing the number of events as a function of the three parameters. A single parameter spectrum can be obtained by summing the other two parameters over a part or all of their respective ranges. Referring to Figure 13, the shaded area labeled "1" represents spectrum on the A energy axis which is obtained using the B energy and time restrictions indicated, i.e. those events on the A energy axis which are in coincidence with the true-plus-random timing events in window ΔT and with the peak-plus-background events in the window ΔE_B on the B energy axis.

Figure 13

$$S_1 = S(E_A, p + b, t + r)$$

Similarly

$$S_2 = S(E_A, p + b, r)$$

$$S_3 = S(E_A, b, t + r)$$

$$S_4 = S(E_A, b, r)$$

Now subtracting spectrum 2 from spectrum 1 yields those events on the E_A axis which are in coincidence with only the true timing events and with the peak-plus-background events from the E_B axis. Thus

$$S_{1-2} = S(E_A, p + b, t)$$

has been corrected for randoms. Similarly

$$S_{3-4} = S(E_A, b, t).$$

The final result is obtained by subtracting S_{3-4} from S_{1-2} to produce an E_A spectrum in true coincidence with the peak of interest on the E_B axis.

$$\begin{aligned} S(E_A, p, t) &= S_{1-2} - S_{3-4} \\ &= S_1 - S_2 - (S_3 - S_4) \end{aligned}$$

where we have assumed for simplicity that the timing spectrum is flat everywhere except in the region of the coincidence peak and that the width of $\Delta T'$ is the same as that of ΔT so that the number of counts represented by the two shaded regions labeled r are equal. Similar considerations are assumed for the B energy axis, i.e. the counts in the two shaded regions labeled b are equal.

With the usual single-parameter coincidence system only one of the spectra (S_1 through S_4) is obtained. Therefore four experiments, under very similar conditions, must be run in order to obtain a coincidence spectrum corrected for both random and background events. In addition corrections must be made for source decay, varying source strengths, etc. to yield a complete set of coincidence data. And still only the spectrum in coincidence with one peak is obtained. In comparison the three parameter system simultaneously accumulates all the data necessary to analyze a coincidence experiment.

The first phase of the data analysis involved the use of the computer codes C1 and C2⁽⁵⁶⁾, run on the Livermore CDC 6600. The C1 code reads each data tape, sorting the three parameters for each event, to produce three cumulative spectra. (A cumulative spectrum for a single parameter is obtain by summing the other two parameters over their full range.) By direct comparison of the of the cumulative energy spectra with calibrated singles data, the individual peaks were identified as to energy.

The cumulative time spectrum is a composite resulting from a wide range of energies of the coincident γ -rays. This causes the coincidence peak to be seriously broadened due to the energy dependent slewing in the time-pick-off circuitry. For this reason, the C1 code divides each energy axis into 16 regions and individually accumulates the 256 energy dependent time spectra. Figure 14 shows a typical case of superimposing composite and individual time spectra. As is evident, the internal resolving time, ranging from 10 to 20 nsec depending on the energies of the coincident γ -rays, is narrower than the 40 nsec window which would be set with hardware. Thus the random counts, which are directly proportional to the resolving time, are significantly reduced. With the above procedure the maximum random-to-true rate was on the order of 2%. Hence the errors incurred in the random correction, as described above were negligible.

Figure 14

The C1 program chooses appropriate digital windows for each of the 256 individual time spectra for use with the C2 code. The C2 code again searches through all the data, accumulating a spectrum of those events for which two of the three parameters fall into preselected digital windows. For example, one sets two windows on the B energy axis, one around a peak of interest, the other on the background adjacent to the peak. The corresponding windows on the time axis, representing the true-plus-random and random events, are obtained from the compilation of C1. The resulting A energy spectrum has thus been corrected for both random and accidental coincidences. This procedure works very well for the random correction owing to the fact that the random distribution is indeed flat. The situation on the energy axis is not as simple. Ge(Li) detectors exhibit a characteristic step-like background distribution. Additional background fluctuations arise from contributions due to all higher energy peaks. As can be seen in Figure 15 the background obtained from the high side of a peak is too large, while that from the low side is too small. This could cause serious errors in the background correction. For this reason, the C2 code was used to generate two spectra, each corrected for randoms, but with no background correction. The energy windows are set as in Figure 16. We calculate the resultant spectrum as follows.

$$S(E_A, p) = \frac{W2 S_{W1} - W1 S_{W2}}{W2 - W1}$$

where S_{W1} and S_{W2} are the random corrected spectra for windows W1 and W2 respectively. This result is valid under assumption that

$$b + b_{HI} + b_{LO} = b W2/W1$$

This effectively averages the background on either side of the peak. Figures 17 a) and b) show for example the 439.4-keV gate coincidence spectra with randoms having been subtracted, the two figures corresponding to before and after background subtraction. Note that the 928.9- and 1085.2-keV lines, which are the only major transitions in direct coincidence with the 439.4-keV γ -ray, are the only lines remaining in the corrected spectrum. This indicates that the correction for accidentals is being handled properly, and the error in the correction is small enough to be disregarded. The errors in the corrected areas are due almost entirely to statistics and range from less than 0.5% for the large peaks to 5 to 6% for the weak peaks.

Figure 15

Figure 16

Figure 17

D) Directional Correlation Experiment

1) Procedure

The correlation experiment is basically a series of coincidence measurements made at several angles. The experimental arrangement is described in Section IV C1. Five essentially identical sources were used to obtain data at the five angles, 90° , 112.5° , 135° , 157.5° , and 180° . The sources (obtained from the same batch of source material) consisted of ~ 14 mg of GeO_2 encapsulated in thin polyethylene disks, so that scattering in the source material itself or in the capsule was small. Moreover the sources were symmetrical so that any absorption effects were independent of angle. There were no electromagnetic fields to perturb the correlation. A half-inch lead shield was placed between the detectors to prevent scattering from one detector to another. This was possible in all configurations except that at 180° . (Even in this case the backscattering corrections were small and easily accounted for.) The source-to detector distances were 4.15 ± 0.05 cm and 4.30 ± 0.05 cm for the fixed and movable detectors respectively. These same distances were maintained for each run, using a source of fixed intensity (^{60}Co), to insure a proper positioning of the movable detector. The counting rates varied by $\sim 1\%$ from run to run. The experiment at each angle had a duration of ~ 36 hours. Scalers were used to record the total number of counts in each detector for each run. These data, after correcting for electronic dead-time losses, were used to normalize the results at each angle.

2) Data Analysis

The individual coincidence spectra (corrected for randoms and background, Section IV C 2) were searched and peak areas determined. These areas, normalized as outlined above, give the observed correlation $W_m(\theta)$. This measured function approaches the theoretical correlation function only in the geometrical limit of a point source and ideal, point detectors. Corrections for the finite source and for real, finite detectors are available in the form of solid angle attenuation factors.

The solid angle correction factors for a cascade $\gamma_1 \rightarrow \gamma_2$ are given by (57)

$$Q_k = \frac{J_k^A(\gamma_1) J_k^B(\gamma_2)}{J_0^A(\gamma_1) J_0^B(\gamma_2)} \frac{R}{1+R} + \frac{J_k^A(\gamma_2) J_k^B(\gamma_1)}{J_0^A(\gamma_2) J_0^B(\gamma_1)} \frac{1}{1+R}, \quad k = 2, 4, \dots$$

where

$$R = \frac{\epsilon^A(\gamma_1) \epsilon^B(\gamma_2)}{\epsilon^B(\gamma_1) \epsilon^A(\gamma_2)}$$

and the ϵ 's denote photo peak efficiencies. The superscripts A and B refer to the two detectors. The Monte Carlo code used to calculate the efficiencies was used to determine the J_k/J_0 ratios. The code was checked by comparing our results to those of Camp and Van Lehn⁽⁵⁷⁾, and found to be in excellent agreement. A description of the code can be found in Appendix B. For the present geometry we obtained values of $Q_2 \approx 0.8$ and $Q_4 \approx 0.5$ with errors estimated to be less than .5% and 2%, respectively.

Listed below are the factors which contribute to the uncertainties in the measured correlations.

- Uncertainty in peak area, which ranged from $\sim 0.1\%$ for the very strong peaks to $\sim 5\%$ for the weak peaks.
- Uncertainty in count rate normalization, $\sim 0.1\%$.
- Uncertainty in Q_k , $\lesssim 0.2\%$, which depends on a_k and $P_k(\cos\theta)$.
- Uncertainty in θ ($d\theta \approx 1^\circ$), $\lesssim 1.5\%$, which depends on a_k and $P_k(\cos\theta)$.

In most cases the contributions from b), c), and d) were negligible compared to the uncertainties in the peak areas.

In the final stage of analysis a computer code GGDC (Appendix C) was used to perform a linear least-squares fit of the measured correlation to the function

$$W_{\text{fit}}(\theta) = 1 + Q_2 a_2 P_2(\cos\theta) + Q_4 a_4 P_4(\cos\theta) \quad (14)$$

The "best" values of a_k as well as of the reduced chi-squared, $\chi^2_{\nu}(a_k)$, were thereby obtained. $P_{\nu}(\chi^2)$ gives the probability that a random set of data (with ν degrees of freedom) when compared to the parent function would yield a value of chi-squared as large or larger than χ^2 , i. e.

$$P_{\nu}(\chi^2) = \int_{\chi^2}^{\infty} p(\nu, \chi^2) d(\chi^2)$$

where $p(\nu, \chi^2)$ is the probability distribution for chi-squared. Thus we define $\chi^2_{\nu}(p)$ as the value of χ^2_{ν} such that $P_{\nu}(\chi^2) = p$, i.e. the probability of obtaining an experimental value of the reduced chi-squared as large or larger than $\chi^2_{\nu}(p)$ is p . If $\chi^2_{\nu}(a_k) \leq \chi^2_{\nu}(p = 0.5)$, then the fitting function (the measured correlation) is a good approximation to the parent function (eq. 14 with the appropriate a_k values).

The analysis of the spins and mixing ratios was performed directly on the measured correlation rather than indirectly on the a_k values obtained in the above mentioned fit. The method consists of comparing, for a given spin sequence and for given mixing ratios, the predicted and measured correlations.

A value of the reduced chi-squared, $\chi^2_{\nu}(J, \delta)$, is obtained, where the spin sequence (J) and mixing ratios (δ) are parameters to be determined. All spin combinations whose fits produce a minimum reduced chi-squared greater than $\chi^2_{\nu}(p = 0.1)$ are rejected. For those spin combinations judged acceptable, the criterion that $\chi^2_{\nu}(J, \delta) \leq \chi^2_{\nu}(p = 0.5)$ determines the ranges over which the mixing ratios have acceptable values. In the above is assumed the $\chi^2_{\nu}(a_k) \leq \chi^2_{\nu}(p = 0.5)$, i.e. the measured correlation is well approximated by a function of the proper form (eq.(14)). If $\chi^2_{\nu}(a_k) > \chi^2_{\nu}(p = 0.5)$, then the values of $\chi^2_{\nu}(p = 0.1)$ and $\chi^2_{\nu}(p = 0.5)$ used above are increased by the factor $R = \chi^2_{\nu}(a_k) / \chi^2_{\nu}(p = 0.5)$. This effectively renormalize the chi-squared probability scale to account for cases where the correlation or its uncertainties are not adequately determined. Examples of this analysis are available in the chi-squared plots shown in Section V E.

There is one final factor to consider in the analysis of correlations data. That is the possible perturbation of the correlation due to other radiations present in the coincidence gate. Since in general this effect can not be adequately corrected for, a satisfactory interpretation of a measured correlation is possible only if the decay scheme of the isotope under consideration is very well known, as in the present case.

V. RESULTS

A) Singles Data

Figure 18 shows a typical Ge(Li) γ spectrum covering the range 0 to 2600 keV. Ten singles runs were made over a period of several days. The energies obtained for the individual lines agreed to within errors for each run. Several impurities were easily reconized, ^{24}Na , which was present in the source, and ^{60}Co , which was a contaminant in the vicinity of the counting apparatus. (This contaminant was not present in the coincidence measurements.) Lines arising from the ^{77}As decay were also clearly evident, especially in the spectra taken at later times. Approximately 120 lines had sufficient intensity to be definitely assigned to the decay of ^{77}Ge on the basis of halflife considerations. The remaining lines could not be ascribed to any known possible impurity, hence were also assigned to the decay of ^{77}Ge . In Table 5 we list the energies and intensities for the 172 observed transitions which were assigned to the decay of ^{77}Ge .

B) Coincidence Data

Using the experimental procedure and analysis methods described in Section IV C, we obtained coincidence spectra corresponding to each prominent line across the entire spectrum. In some cases it was not possible to resolve multiplets within the gating region. Figures 51 to 100 (in Appendix D) show the coincidence spectra obtained. Results are shown in Table 6 in the form of a coincidence matrix. Listed in the table are all transitions found to be in coincidence with a gated transition.

C) Decay Scheme

The most complete decay scheme previously proposed was that of Poncet⁽¹⁶⁾, shown in Figure 1. This decay scheme contains 135 transitions and 41 levels. Figure 19 shows the decay scheme resulting from our work. It is based on energy summing of the transitions listed in Table 5 as well as the γ - γ coincidence results presented in Table 6 (and in Figures 51 through 100 in Appendix D). Our decay scheme, containing 38 levels to account for all 172 observed transitions, is in qualitative agreement with the work of Poncet, having 27 levels in common with that decay scheme. We observed 37 new transitions and propose 11 new levels (of which 4 are only weakly established). The log ft values are calculated, using the tables by Gove and Martin⁽²⁰⁾, from a detailed transition intensity balance and a Q value of 2740 keV⁽¹⁷⁾. A discussion of the spin assignments will be given in Section V E).

[illegible]

⁹ Transmission placed twice based on survey error and coincidence samples

$$\frac{1}{2} \leq \beta_{\text{max}} \leq 1 \quad \text{for } 1 \leq p \leq 2 \text{ and } \text{any } \alpha \in \mathbb{R} \quad (1)$$

This work represents a significant improvement over previous studies for two important reasons. First, the uncertainties in the transition energies are small. As can be seen in Table 5, these uncertainties range from 0.012 keV up to, in some cases, as high as 0.65 keV, with the majority (119 out of 172) being less than 0.1 keV. In comparison Poncet obtained a minimum energy uncertainty of 0.5 keV. Energy sum rules alone are generally not sufficient to establish the existence of levels in a complex decay⁽⁵⁸⁾. This is especially true when dealing with large energy uncertainties, which tend to produce many accidental sum relationships. Even with additional information, for example γ - γ coincidence data, accurate transition energies are essential in adequately defining a complete decay scheme.

Second, this work includes extensive γ - γ coincidence data, acquired using two high resolution Ge(Li) detectors. The three parameter coincidence method allowed the accumulation of a "complete" set of γ - γ coincidence spectra, i.e. we obtained coincidence information for almost every prominent transition. Poncet's work did not include coincidence studies but rather made a re-assessment of previous coincidence results.^(5,9) This previous work was not nearly as extensive as the present work, and in addition was obtained using NaI-NaI or Ge(Li)-NaI systems, in which the poor resolution [compared to Ge(Li)-Ge(Li)] tended to mask the available information.

We will now outline the logic used to establish the present decay scheme.

a) Levels at 215, 264 and 631 keV

Referring to Table 5 we see that there are two pairs of intense γ -rays 215-416 and 264-367, whose energies sum to that of the 631-keV transition. From Table 6 we note that both pairs of γ -rays were in coincidence with the many other γ -rays which were interpreted as transitions to the 631-keV level. In the two cascades the 215- and 264-keV γ -rays were the more intense and hence were assumed to populate the ground state. Additional coincidences with these two γ -rays confirmed these placements.

The determination of the complete decay scheme (Figure 19) will be discussed with the aid of a series of partial decay schemes shown in Figures 20 to 26. Each figure has a number of dark lines indicating established levels - either the levels at 215, 264, or 631 keV as given above or levels which have been established by a previous figure in this series. The light lines represent the levels to be established with the aid of the included pertinent transitions.

b) Levels at 475, 1457, 1971, 2000, 2110, 2341, and 2354 keV (Figure 20)

i) The 475-keV level

This level was established by the fact that the 211-keV γ -ray was in coincidence with only the 264-keV transition. This level is known to have a halflife of $116 \mu\text{sec}^{(17)}$, thus, as expected, cascades passing through this level were not observed in the prompt coincidence spectra.

ii) The 1457-keV level

This level was required to explain the fact that the 1193- and 1242-keV γ -rays were observed to be in coincidence with only the 264- and 215-keV transitions, respectively.

iii) The 1971- and 2000-keV levels.

The 1339- and 1368-keV transitions were found to be in coincidence with only those transitions depopulating the 631-keV level, thus establishing the levels at 1971 and 2000 keV, respectively.

iv) The 2110-keV level.

The fact that the 1846-keV γ -ray was in coincidence with only the 264-keV transition, and that the 1479-keV transitions was found to directly feed the 631-keV level, establish the level at 2110 keV.

v) The 2341-keV level.

The 2126- and 2077-keV γ -rays were in coincidence with only the 215- and 264-keV transitions, respectively. In addition the 1709-keV γ -ray was found to populate the 631-keV level. These facts established the level at 2341 keV.

Figure 20

vi) The 2354-keV level.

The 2089-keV γ -ray was in coincidence with only the 264-keV transition. And the 1722-keV transition was found to directly feed the level at 631 keV. These facts established the level at 2354 keV.

These results agree with those of Poncet with the exception of the 1971-keV level. Poncet showed a level at 1971 keV but interpreted the 1339-keV transition as depopulating a level at 1555 keV. Furthermore transitions at 680 and 1080 keV were shown to depopulate the 1555-keV level. We found no evidence for the existence of a level at 1555 keV, in fact our coincidence results clearly place each of these transitions- 680, 1080, and 1339 keV - elsewhere in the decay scheme.

c) Levels at 1189, 1398, 1528, and 2513 keV (Figure 21)

i) The 1189-keV level.

This level was established to explain the following observed coincidences: 558-631 keV; 925-264 keV; and 974-215 keV. Further evidence for the existence of this level was provided by the coincidences involving the 714-, 781-, 810-, 921-, and 1151-keV transitions.

ii) The 1398-keV level.

The 209-558 keV coincidence suggested the existence of the level at 1398 keV. Further evidence was provided by the observed coincidences involving the 712-, 766-, and 923-keV γ -rays.

iii) The 1528-keV level.

This level was required to explain the observed coincidences involving the 1338-, 582-, 813-, 986-, 1263-, 1312-, and 1528-keV transitions.

iv) The 2513-keV level.

The 1114-keV γ -ray depopulating the level at 2513 keV accounted for the observed coincidences involving the 1114-keV γ -ray with each of the 209-, 766-, and 923-keV transitions.

Figure 21

Poncet placed the 673-keV transition between a level at 1862 keV and the 1189-keV level. We found no evidence for the level at 1862 keV. Our results clearly showed the 673-keV γ -ray as not being in coincidence with the 558-keV transition. In addition Poncet showed the 1114- and 813-keV transitions as depopulating levels at 1378 and 1077 keV, respectively. We found no evidence for the existence of either of these levels.

d) Levels at 194, 614, 634, 1319, 1573, and 2543 keV (Figure 22)

i) The 194-keV level.

The 194- and 419-keV γ -rays were in coincidence; their energies sum to that of the 614-keV level (established below). The 194-keV transition was the more intense member of the cascade, and was therefore assumed to populate the ground state. Additional coincidences confirmed this placement.

ii) The 614-keV level.

The 614-keV γ -ray was in coincidence with the 843-, 913-, and 1727-keV transitions. This established the level at 614-keV. The 350-264 keV and 399-215 keV cascades provided further evidence for the existence of this level.

iii) The 634-keV level.

This level was established to explain the observed coincidences between the 634-keV γ -ray and the 823-, 1476-, and 1719-keV transitions.

iv) The 1319-keV level.

The 705-keV γ -ray was found to directly populate the level at 614 keV. The 685-634 keV and 1104-215 keV cascades confirmed the existence of the 1319-keV level.

v) The 1573-keV level.

The 939-keV γ -ray was in coincidence with only the 634-keV γ -ray. The 959-614 keV and 1309-264 keV cascades confirmed the existence of the 1573-keV level.

Figure 22

vi) The 2543-keV level.

The existence of this level was based on the fact that the 1929-keV transition was found to populate the level at 614 keV.

Poncet showed the 1929 keV γ -ray as depopulating a level at 2145 keV. In addition the 823- and 1727-keV transitions were shown as originating from a level at 2202-keV. These placements were not substantiated by our results; we found no evidence for levels at 2145 or 2202 keV.

e) Levels at 784, 875, and 889 keV (Figure 23)

i) The 784-keV level.

The 784-keV γ -ray was found to be in coincidence with transitions at 535, 614 (doublet); 673 (doublet), 788, 1186, 1215, 1326, and 1569 keV. These facts, along with the observed 520-264 keV cascade, established the level at 784 keV.

ii) The 875-keV level.

The 875-keV γ -ray was found to be in coincidence with transitions at 444, 698, 1125, 1465 (doublet), and 1479 (doublet) keV. These facts, along with the observed 680-194 keV cascade, established the level at 875 keV.

iii) The 889-keV level.

The γ -rays at 1452 and 1465 (doublet) keV were in coincidence with transitions at 254, 624, and 673 (doublet) keV. These fact established the level at 889 keV.

Poncet proposed a level at 1716 keV, in part to account for the 624-1452 keV cascade. Since the level at 889 keV accounts for this cascade and since we found no other evidence to substantiate the existence of a level at 1716 keV, we concluded that there is no such level.

Figure 23

f) Levels at 1201, 1221, 1538, and 1560 keV (Figure 24)

i) The 1201-keV level.

This level was established to explain the observed 569-631 keV and 985-215 keV cascades as well as the coincidences between the 798-keV γ -ray and the 569- and 985-keV transitions.

ii) The 1221-keV level. The 745- and 749-keV γ -rays were found to be in coincidence. Their energies sum to the energy difference between the 1971- and 475-keV levels. These facts indicated a level at either 1221 or 1225 keV. The 177-keV γ -ray was found to be in coincidence with either the 745- or 749-keV transition. This γ -ray was placed between the 1398-keV level and the level at 1221 keV.

iii) The 1538-keV level.

This level was established to explain the observed coincidence between the 461-keV γ -ray and the 907-, 924-, 1323-, and 1538-keV transitions.

iv) The 1560-keV level.

The level was established to explain the observed coincidence between the 439-keV γ -ray and the 928- and 1085-keV transitions.

Poncet showed the 461-keV γ -ray depopulating a level at 1092 keV. Furthermore the 745- and 749-keV transitions were shown as originating from levels at 1378 and 1381 keV, respectively. These placements were not consistent with our results.

g) Levels at 1058, 1164, 1279, 1345, 1397, and 1837 keV (Figure 25)

i) The 1058-keV level.

This level was established on the basis of coincidences between the 794-keV γ -ray and transitions at 264, 1295 (doublet), and 1454 keV.

Figure 24

Figure 25

ii) The 1164-keV level.

The 901-264 keV and 970-194 keV cascades established the level at 1165 keV.

iii) The 1280-and 1837-keV levels.

The 1061-1280 keV cascade established a level at either 1061 or 1280 keV. Moreover the 503- and 557-keV γ -rays were found to be in coincidence with each other as well as with the 1280-keV transition. (The 557-keV γ -ray was not observed in the singles data due to masking by the very intense 558-keV transition.) Considering energy summing the 1061-keV γ -ray was assumed to be the crossover transition corresponding to the 503-557 keV cascade. Thus a level at 1061 keV required a level at either 503 or 557 keV. Since such a weakly populated low-lying level seemed very improbable, we ruled out the possibility of a level at 1061 keV and propose the level at 1280 keV. With this result the 503-557 keV cascade implies the existence of a level at 1784 or 1837 keV. The choice of 1837 keV was made on the basis of a weak 1643-keV transition (see Figure 19) between the 1837- and 194-keV levels.

iv) The 1345-keV level.

The existence of this level was based on the fact that the 470-keV γ -ray was in coincidence with only the 875-keV transition. The 1080-keV γ -ray in coincidence with the 264- and 996-keV transitions provided further evidence for this level.

v) The 1397-keV level.

This level was proposed to explain the fact that the 794-keV transition, which depopulates the 1058-keV level, was observed in the 337/338-keV coincidence spectrum, yet neither the 337- nor 338-keV transitions feed this level. In addition a weak 1397-keV transition was observed in the singles data. We propose a possible 339-keV transition between the 1397- and 1058-keV levels.

h) Levels at 1350, 1732, 2195, 2424, and 2463 keV (Figure 26)

The previous levels are based on the extensive coincidence data. These remaining levels are based almost entirely on energy sums and correspond to those transitions which we are unable to place within the already established levels. The evidence for the 2424-keV level is strong, involving energy summing of seven transitions. There is weak evidence for a 1581-419 keV cascade which is explained by a proposed level at 2195 keV. The levels at 1350, 1732, and 2463 keV are based on only two transitions.

Figure 26

D. Directional Correlation

In Table 7 we list the a_k values (obtained by least-squares fitting in the GGDC code) for 58 cascades considered in this study. Accurate correlations could not be measured for the remaining cascades; either the small intensities of the peaks involved prohibited accurate area determinations, or the distribution was perturbed by other correlations observable in the same coincidence gate (note the large number of multiple peaks in the spectrum). Table 8 shows a comparison of our results to the previous directional correlation studies (4,5). There is, on the whole, reasonable agreement; the discrepancies are probably attributable to the fact that the earlier work was done with poor resolution NaI-NaI systems. As was mentioned in Section IV D2 and in Appendix C, the correlation analysis was performed directly on the measured correlations rather than indirectly with the fitted a_k values.

TABLE 7

γ_2	γ_1	a_2	a_4	χ^2_{ν}
194.8	419.8	-0.014±0.025		0.698
		-0.012±0.030	-0.008±0.056	1.035
215.5	416.3 ^(a)	-0.014±0.003		0.989
		-0.014±0.003	0.000±0.005	1.480
	461.4 ^(a)	-0.011±0.015		0.849
		-0.011±0.017	0.001±0.031	1.274
	558.0 ^(a)	-0.030±0.004		1.402
		-0.029±0.004	-0.003±0.008	2.050
	766.7	0.077±0.021		0.838
		0.084±0.023	-0.032±0.044	0.990
	907.0	-0.045±0.020		0.673
		-0.040±0.023	-0.018±0.042	0.919
	928.9	0.069±0.019		0.900
		0.082±0.021	-0.059±0.039	0.250
	1242.2	0.035±0.023		0.706
		-0.031±0.026	-0.013±0.048	1.017
	1312.8	-0.092±0.025		1.105
		-0.090±0.029	-0.005±0.052	1.653
	1368 ^(a)	-0.012±0.012		0.212
		-0.012±0.013		0.318
	2126.1	-0.036±0.045		0.927
		-0.065±0.051	0.107±0.091	0.702

TABLE 7

γ_2	γ_1	a_2	a_4	χ^2_{ν}
264.4	211.0	-0.166 ± 0.004		0.440
		-0.168 ± 0.004	0.008 ± 0.008	0.178
	367.4 ^(a)	-0.374 ± 0.003		0.697
		-0.374 ± 0.004	0.001 ± 0.007	1.043
	461.4 ^(a)	0.039 ± 0.020		0.359
		0.041 ± 0.023	-0.009 ± 0.043	0.519
	520	0.195 ± 0.029		0.703
		0.216 ± 0.033	-0.082 ± 0.063	0.198
	558.0 ^(a)	0.185 ± 0.005		0.039
		0.184 ± 0.005	0.004 ± 0.010	0.880
	624.8	-0.359 ± 0.026		0.385
		-0.366 ± 0.033	0.021 ± 0.057	0.507
	766.7	-0.347 ± 0.026		0.674
		-0.349 ± 0.031	0.004 ± 0.055	1.008
	794.3	-0.334 ± 0.023		0.667
		-0.342 ± 0.029	0.022 ± 0.050	0.905
	1080.8	0.073 ± 0.030		0.441
		0.057 ± 0.035	0.059 ± 0.064	0.239
	1193.3	-0.373 ± 0.008		0.952
		-0.372 ± 0.010	-0.002 ± 0.019	1.425
	1263.9	-0.440 ± 0.017		0.965
		-0.430 ± 0.020	-0.027 ± 0.034	1.149

TABLE 7

γ_2	γ_1	a_2	a_4	X_v^2
264.4	1309.3	-0.367±0.022		0.495
		-0.374±0.027	0.022±0.047	0.636
	1368 ^(a)	0.045±0.016		0.458
		0.051±0.018	-0.023±0.032	0.441
	1846.4	0.140±0.046		0.512
		0.170±0.058	-0.106±0.096	0.163
	2077.2	-0.391±0.042		0.668
		-0.379±0.051	-0.035±0.088	0.923
	2089.6	0.115±0.044		0.788
		0.084±0.051	0.112±0.091	0.434
367.4	461.4 ^(a)	-0.030±0.022		1.033
		-0.042±0.024	0.052±0.045	0.861
	558.6 ^(a)	-0.143±0.005		1.052
		-0.147±0.006	0.016±0.011	0.534
	766.7	0.259±0.035		0.811
		0.264±0.037	-0.032±0.070	1.115
	781.3	0.148±0.039		0.992
		0.129±0.042	0.100±0.077	0.639
	810.4	-0.258±0.023		1.075
		-0.257±0.026	-0.002±0.046	1.611
	907.0	-0.160±0.033		0.382
		-0.170±0.036	0.041±0.066	0.381

TABLE 7

Y_2	Y_1	a_2	a_4	χ^2_v
416.3	928.9	0.289 ± 0.033		1.003
		0.275 ± 0.035	0.089 ± 0.065	0.570
	1368 ^(a)	-0.084 ± 0.018		0.243
		-0.090 ± 0.020	0.022 ± 0.035	0.165
	461.4 ^(a)	0.021 ± 0.017		1.006
		0.024 ± 0.019	-0.012 ± 0.035	0.894
	558.0 ^(a)	0.125 ± 0.004		0.728
		0.124 ± 0.005	0.003 ± 0.009	1.024
	766.7	-0.217 ± 0.026		0.583
		-0.234 ± 0.031	0.055 ± 0.054	0.363
	781.3	-0.151 ± 0.028		0.634
		-0.142 ± 0.034	-0.028 ± 0.059	0.841
	810.4	0.201 ± 0.021		0.975
		0.201 ± 0.023	0.000 ± 0.002	1.462
	907.0	0.126 ± 0.027		0.919
		0.107 ± 0.030	0.075 ± 0.054	0.425
419.8	928.9	-0.264 ± 0.024		0.850
		-0.262 ± 0.029	-0.006 ± 0.050	1.267
	1368 ^(a)	0.045 ± 0.015		0.728
		0.048 ± 0.017	-0.011 ± 0.030	1.018
	913.8	-0.119 ± 0.037		1.165
		-0.151 ± 0.041	0.129 ± 0.073	0.196

TABLE 7

γ_2	γ_1	a_2	a_4	χ^2_v
558.0	209.0	0.288 ± 0.026		0.697
		0.270 ± 0.030	0.070 ± 0.057	0.289
	781.3	-0.248 ± 0.021		0.516
		-0.257 ± 0.025	0.029 ± 0.043	0.554
	810.4	0.390 ± 0.016		0.630
		0.399 ± 0.017	-0.039 ± 0.032	0.192
631.8	461.4 ^(a)	0.086 ± 0.030		0.894
		0.064 ± 0.034	0.092 ± 0.060	0.160
	558.0 ^(a)	0.116 ± 0.008		0.981
		0.115 ± 0.009	0.005 ± 0.016	1.420
	810.4	0.154 ± 0.039		0.121
		0.150 ± 0.043	0.018 ± 0.077	0.154
	1368 ^(a)	0.130 ± 0.027		0.313
		0.129 ± 0.030	0.004 ± 0.054	0.467
634.4	823.1	-0.145 ± 0.027		0.898
		-0.170 ± 0.031	0.086 ± 0.053	0.022
	939.4	-0.134 ± 0.042		0.746
		-0.125 ± 0.049	-0.030 ± 0.083	1.053
745.7	749.9 ^(a)	-0.085 ± 0.019		4.748
		-0.122 ± 0.022	0.128 ± 0.037	1.088
907.0	461.4	0.010 ± 0.022		0.589
		0.004 ± 0.025	0.024 ± 0.045	0.744

TABLE 7

γ_2	γ_1	a_2	a_4	χ^2_v
1085.2	439.4	-0.110 ± 0.058		0.769
		-0.121 ± 0.003	0.049 ± 0.111	1.055
2754	1368 ^(a)	0.115 ± 0.021		0.389
		0.114 ± 0.024	0.004 ± 0.041	0.578

(a) Average of γ_2/γ_1 gate correlation and γ_1/γ_2 gate correlation.

TABLE 8

Cascade	Present Work		Reference 5		Reference 4	
	a_2	a_4	a_2	a_4	a_2	a_4
211-264	-0.166±0.004					
	-0.168±0.004	0.008±0.008	-0.173±0.005	-0.011±0.008	-0.190±0.004	-0.009±0.004
416-215	-0.014±0.003					
	-0.014±0.003	0.000±0.005	0.003±0.015	0.024±0.029	-0.031±0.007	0.017±0.009
367-264	-0.374±0.003					
	-0.374±0.004	0.001±0.007	-0.360±0.012	-0.023±0.019	-0.321±0.009	-0.007±0.012
558-367	-0.143±0.005					
	-0.147±0.006	0.016±0.011	-0.14± 0.05	-0.05± 0.07	-0.035±0.020	0.068±0.020
558-416	0.125±0.004					
	0.124±0.005	0.003±0.009	0.16± 0.04	-0.01± 0.05	0.163±0.016	0.052±0.024
558-531	0.116±0.008					
	0.115±0.009	0.005±0.016	0.13± 0.05	-0.28± 0.10	0.085±0.027	0.065±0.030
810-558	0.39± 0.016					
	0.399±0.017	-0.039±0.032	0.37± 0.13	-0.03± 0.16		

E) Spin Assignments

As a starting point for making spin assignments, we use the following information:

- 1) Relative halflife measurements of Van der Kooi and Van den Bold⁽⁵⁾ and internal conversion measurements of Imanishi, et al⁽⁷⁾ have established an E3 multipolarity for the 159-keV isomeric transition in ^{77m}Ge , in agreement with similar isomeric transitions in the neighboring $N = 45$ nuclei, ^{79}Se and ^{81}Br .⁽¹⁷⁾
- 2) Van der Kooi and Van den Bold, and Meeker and Tucker⁽¹²⁾ determined log ft values for ^{77m}Ge decay and established that the β feeding to the ground state and 215-keV level in ^{77}As is allowed.
- 3) The log ft values for the β decay of ^{77}As to ^{77}Se require the spin/parity of the ^{77}As ground state to be $3/2^-$.⁽¹⁷⁾
- 4) The 211-keV γ -ray, from the 475- to the 264-keV level in ^{77}As , is an M2 transition. This was established by Smith⁽¹⁾ from internal-conversion studies and confirmed by Schardt⁽³⁾ with relative half-life measurements.

Spin and parity assignments are based primarily on the experimentally determined directional correlations and log ft values presented in earlier sections. The empirical rules proposed by Raman and Grove⁽¹⁹⁾ are used for making spin and parity assignments based on log ft values. As explained earlier, it is assumed that E1 transitions have no M2 admixture. It is further assumed that the lower or intermediate members of a cascade for which coincidence data was obtained cannot be of octupole or higher-order multipolarity. Weisskopf's estimates (see Appendix A) give halflives of >10 μsec

for these higher-order transitions. These times are not compatible with the experimental coincidence resolving time, hence observed coincidences require that the lower or intermediate members of a cascade be dipole or quadrupole transitions.

a) The ^{77m}Ge and ^{77g}Ge Levels

The allowed β transition (item 2 above) to the $3/2^-$ ground state of ^{77}As (item 3) indicates that ^{77m}Ge is $5/2^-$, $3/2^-$ or $1/2^-$. Also, the E3 transition from ^{77m}Ge (item 1) indicates that ^{77g}Ge has a corresponding assignment of $11/2^+$, $9/2^+$ or $7/2^+$. Figures 27a), b), and c) show three partial decay schemes containing only the pertinent levels and transitions, for each of the possibilities given above. We will discuss each case in detail. In Figure 27a) we see that the 631-keV level has a spin of $9/2$, $11/2$ or $13/2$ (based on the log ft), which requires that the 631-keV transition be of octupole or higher multipolarity. However, a short halflife is indicated by the observation of coincident γ -rays which would not be consistent with an octupole or higher assignment (see above). Thus, we can rule out the $5/2^-$ and $11/2^+$ assignments for ^{77m}Ge and ^{77g}Ge , respectively.

Now consider the second possibility: $3/2^-$ for ^{77m}Ge and $9/2^+$ for ^{77g}Ge . Consider further the levels shown in Figure 27b) which have possible spin and parity values of: (1) $3/2^-$ for the ground state; (2) $5/2^-$ or $7/2$ for the 264-keV level (from the log ft), $9/2$, $11/2$ and $13/2^-$ values being ruled out by the short halflife (see above) of the 264-keV transition; (3) $9/2^+$ or $11/2$ for the 475-keV level based on the M2 nature of the γ -ray transition (both are consistent with the log ft value); and (4) $7/2$ for the 631-keV level (from the log ft), $9/2$

Figure 27

and $11/2$ values being excluded by the short half-life of the 631-keV transition. We can now proceed to show that the measured correlation data is not consistent with the spin and parity possibilities which we have deduced for this case.

Note that there are four simple cascades in Figure 27b) and four transitions having unknown mixing ratios. The cascades and associated mixing ratios are:

- 1) 211-264 keV δ_{264}
- 2) 367-264 keV δ_{367} and δ_{264}
- 3) 558-367 keV δ_{558} and δ_{367}
- 4) 558-631 keV δ_{558} and δ_{631}

Directional correlation data has been obtained for each of the four cascades from which mixing ratios are deduced in the following manner. Analyzing the first three correlations in turn, i.e., incorporating the results of 1) into 2) and 2) into 3), yields a value for δ_{558} . The value of δ_{558} from 4) must agree with that obtained from 1), 2) and 3).

Figures 28a) and b) show the results of analyzing the $J_{475}(211,0)$ $J_{264}(264, \delta_{264})$ $3/2$ correlation for the two spin possibilities given in Figure 27b). As can be seen, only the $9/2^+$ and $5/2^-$ assignments for the 475- and 264-keV levels, respectively, are consistent with the data. A mixing ratio of -0.32 is obtained for the 264-keV transition. Using these results, we simultaneously analyze the $7/2(367, \delta_{367})$ $5/2(264, -0.32)$ $3/2$ and $J_{1189}(558, \delta_{558})$ $7/2(367, \delta_{367})$ $5/2$ correlations for each of the three possible spins of the 1189-keV level (Figures 29a), b) and c). The shaded areas in these figures represent regions in the

Figure 28

Figure 29

Figure 30

$\delta_{558} - \delta_{367}$ plane (i.e., values of δ_{558} and δ_{367}) which are consistent with the correlation data. Finally Figures 30a), b), and c show the results of analyzing the $J_{1189}(558, \delta_{558}) 7/2(631, \delta_{631}) 3/2$ correlation for the three possible spins of the 1189-keV level, where again the shaded areas are possible values for δ_{558} and δ_{631} . The abscissas in both sets of figures is the same, i.e., $\tan^{-1}(\delta_{558})$, so by comparing the two sets we can determine the values of δ_{558} which are consistent with the data for each possible spin. We can see from Figure 29a) that there is no value of δ_{558} consistent with a 7/2 assignment for the 1189-keV level. By comparing Figures 29b) and 30b), we find that the values of δ_{558} are mutually exclusive for the two sets of data, hence the 1189-keV level cannot be 9/2. Similarly, the 1189-keV level cannot be 11/2 [Figures 29c) and 30c)]. Thus, none of the possible spins for the 1189-keV level is consistent with the data. From this we can rule out the possibility of $3/2^-$ and $9/2^+$ assignments for the ^{77}Ge levels.

We are left with, as the only remaining possibility, $1/2^-$ and $7/2^+$ assignment for ^{77m}Ge and ^{77g}Ge , respectively. As will be shown below, this result is consistent with the present correlation data. In addition, this spin sequence is in accord with known spins in the neighboring $N = 45$ nuclei, ^{79}Se and $^{81}\text{Br}^{(17)}$ which exhibit similar isomerism.

b) The 215-, 264-, and 475-keV Levels

Consider the ^{77}As levels shown in Figure 27c). Possible spin and parity values for this case are: (1) $3/2^-$ for the ground state; (2) $3/2^-$ for the 215-keV level, since it is fed by both the $1/2^-$ and $7/2^+$ states of ^{77}Ge ; (3) $3/2^-$ or $5/2$ for the 264-keV level (from the log ft)

higher spins being ruled out by the spin of the 475-keV level and the M2 nature of the 211-keV transition; (4) $7/2^+$ or $9/2$ for the 475-keV level (from the log ft) , the $5/2$ value being ruled out by the spin of the 264-keV level and the M2 nature of the 211-keV transition; (5) $5/2$ or $7/2$ for the 631-keV level based on the log ft and the half-life of the 631-keV transition; and (6) $5/2$, $7/2$ or $9/2$ for the 1189-keV level from the log ft.

Consider the 211-264 keV cascade. We can see from Figure 31 that $7/2^+$ and $3/2^-$ assignments for the 475- and 264-keV levels, respectively, are not consistent with the measured correlation. This leaves only the $9/2$ and $5/2$ spin possibilities. This case was examined earlier [Figure 28a)] and a finite mixing ratio for the 264-keV γ -ray ($\delta_{264} = -0.32$) was obtained. Combining this with our assumption of no M2/E1 mixing provides the needed parity information. Hence, we make assignments of $9/2^+$ for the 475-keV level and $5/2^-$ for the 264-keV level. These results are in agreement with the neighboring isotopes ^{73}As and ^{75}As which exhibit M2 transitions between low-lying $9/2^+$ and $5/2^-$ levels.⁽¹⁷⁾

c) The 631-keV Level

Spin possibilities for the 631-keV level are limited to $5/2$ and $7/2$ as explained above. A spin of $7/2$ can be ruled out by consideration of correlations involving the 1189-keV level. We have already shown [Figures 29a) and b) and 30a) and b)] that $7/2$ and $9/2$ spin assignments for the 1189-keV level are inconsistent with a $7/2$ spin for the 631-keV level. The $5/2$ case, which is the only remaining possibility for the 1189-keV state, can also be ruled out. Consider the three cascades:

Figure 31

- 1) 367-264 keV δ_{367}
- 2) 558-367 keV δ_{558} and δ_{367}
- 3) 558-631 keV δ_{558} and δ_{631}

A simultaneous analysis of the first two is shown in Figure 32. Comparing this with an analysis of the third, shown in Figure 33, indicates that no consistent value of δ_{558} can be obtained. Hence, we are left with a 5/2 spin for the 631-keV level. A correlation analysis based on this spin value confirms the assignment and indicates a positive parity for this state.

We have four cascades:

- 1) 367-264 keV δ_{367}
- 2) 558-367 keV δ_{558} and δ_{367}
- 3) 558-416 keV δ_{558} and δ_{416}
- 4) 558-631 keV δ_{558} and δ_{631}

In analyzing this correlation data, we assume a 5/2 spin for the 631-keV level. Figure 34 shows a chi-squared plot for the first cascade, from which we conclude that $\delta_{367} = 0$. The remaining three cascades must be analyzed for each of the three spin possibilities (5/2, 7/2, and 9/2) for the 1189-keV level. Figures 35a), b), and c) show this analysis for the 5/2 possibility. In the figures the arrows indicate values of δ_{558} which are consistent with the assumed spins and with a zero value for δ_{367} (Figure 34). As can be seen, these values of δ_{558} correspond to zero mixing for both the 416- and 631-keV γ -ray. Similar analyses for the two other possibilities [Figures 36a), b), and c) and 37a), b), and c)] also yield $\delta_{416} = \delta_{631} = 0$.

$$\begin{aligned} &7/2(367, \delta_2)5/2(264, -0.321)3/2 \\ &+ 5/2(558, \delta_1)7/2(367, \delta_2)5/2 \end{aligned}$$

Figure 32

$$5/2(558, \delta_1)7/2(631, \delta_2)3/2$$

Figure 33

Figure 34

Figure 35

$$7/2(558, \delta_1) 5/2(367, \delta_2) 5/2$$

$$7/2(558, \delta_1) 5/2(416, \delta_2) 3/2$$

$$7/2(558, \delta_1) 5/2(631, \delta_2) 3/2$$

Figure 36

Figure 37

A positive parity for the 631-keV level is indicated by noting that the 367-, 416-, and 631-keV transitions are all essentially pure dipole and all populate negative parity states. This parity is confirmed by comparing the relative intensities of the 367-, 416-, and 631-keV γ -rays to that of the 156-keV transition. From the decay scheme (Figure 19) we note that the 156-keV γ -ray, which goes between the 631-keV level and the $9/2^+$ 475-keV level, strongly competes with the 367-, 416- and 631-keV transitions as a means of depopulating the 631-keV level. A positive parity for this level means we have an E2 transition competing strongly with E1 transitions, this phenomenon is fairly common. On the other hand, a negative parity for the 631-keV level requires a strong competition between M2 and M1 transitions which has not been previously observed.

Having determined the spin and parity of the 631-keV level, we can analyze the 416-215 keV correlation to obtain a value for the mixing ratio of the 215-keV transition. This analysis, shown in Figure 38, yields $\delta_{215} = -0.164 \pm 0.016$ or 11.4 ± 2.0 . The latter value can be ruled out since the corresponding internal conversion coefficient would upset the intensity balance of the 215-keV level.

The final value for the mixing ratio of the 264-keV transition is obtained from a simultaneous analysis of 211-264 keV and 367-264 keV correlations, $\delta_{264} = -1.46 \pm 0.02$ or -0.321 ± 0.011 .

Figure 39 shows a more complete decay scheme containing all the cascades investigated in this work. The spins are based on the work outlined above and on $\log ft$ values, including those for ^{77m}Ge decay.^(5,12)

Figure 38

Figure 39

d) The 1189-keV Level

To gain information on the spin and parity of the 1189-keV level, we must consider correlations involving the 558-keV transition. As can be seen in Figure 39, the 215-, 264-, 367-, 416-, and 631-keV δ -rays are all in coincidence with the 558-keV transition. The intensities of these transitions were such as to permit a correlation measurement in each case. But since the 558-keV γ -ray is, in each case, the first member of the cascade, the five measured correlations represent redundancy rather than new information. This redundancy is, however, a valuable tool to check the internal consistency of the data, a point which we will return to later. To obtain new information on the 558-keV γ -ray, we must consider cases where it is the final or intermediate member of a cascade. The two cascades we will consider are 810-558 keV and 810-558-367 keV. We then have three independent determinations of δ_{558} . The above two cascades must be analyzed for all possible spin combinations corresponding to $J_{2000} = 5/2, 7/2$ or $9/2$ and $J_{1189} = 5/2, 7/2$, or $9/2$. Figures 40a) and b) show the results of this analysis for $J_{2000} = J_{1189} = 5/2$. Both plots describe the same space, i.e., the δ_{810} - δ_{558} plane. As can be seen in Figure 40b), there are no values for δ_{810} and δ_{558} which are consistent with the assumed spin sequence. Figures 41a) and b) through Figures 48a) and b) show the results corresponding to the other possible spin combinations. Considering Figures 41a) and b) $J_{2000} = 7/2$ and $J_{1189} = 5/2$, we see that neither correlation by itself rules out this spin sequence but the shaded areas, representing regions of acceptable values for δ_{810} and δ_{558} are mutually exclusive, indicating

Figure 40

Figure 41

Figure 42

Figure 43

Figure 44

$$9/2(810, \delta_1)7/2(558, \delta_2)5/2$$

$$9/2(810, \delta_1)7/2(558, \delta_2)5/2(367, 0)5/2$$

Figure 45

Figure 46

Figure 47

Figure 48

that this spin sequence is not compatible with both correlations. Proceeding in this manner through Figures 48a) and b), we find that the 1189-keV level must be $7/2^+$, where the parity is established by the finite mixing of the 558-keV transition, $\delta_{558} \sim -0.14$. In addition, the 2000-keV level is either $5/2^+$ with $\delta_{810} \sim 0.20$ or 2.2 , or it is $9/2^+$ with $\delta_{810} \sim -3.5$ or 0.40 . As a check on the value of the 558-keV mixing ratio, we simultaneously analyzed the five correlations involving this γ -ray in coincidence with the 215-, 264-, 367-, 416- and 631-keV transitions. This resulted in a value of $\delta_{558} = -0.139 \pm 0.006$, which is in good agreement with the value obtained above.

e) The 2000-keV Level

As explained above, the spin assignment for the 2000-keV level is limited to either $5/2^+$ or $9/2^+$. The $9/2^+$ spin can be ruled out by comparing the transition intensities for the 810- and 2000 keV γ -rays to theoretical predictions. The 810-keV transition from above is either $0.9 M1 + 0.1 E2$ or $0.1 M1 + 0.9 E2$, while the 2000-keV transition (to the $3/2^-$ ground state) is $E3$. We find that the ratio of the experimental intensities are $\sim 5 \times 10^4$ times larger than that predicted using Weisskopf's estimates. For this reason, we exclude the $9/2^+$ case, leaving the 2000-keV level to be $5/2^+$.

Having determined the spin of the 2000-keV level, a value of $\delta_{810} = 0.194 \pm 0.030$ or 2.37 ± 0.18 was obtained by simultaneously analyzing the four correlations involving the 810-keV γ -ray and the 367-, 416-, 558-, and 631-keV transitions. Similarly, $\delta_{1368} = -21 \pm 12$ or 0.519 ± 0.049 from

the simultaneous analysis of the four correlations involving the 1368-keV γ -ray and the 215-, 264-, 367-, and 416-keV transitions.

f) The 1528-keV Level

Figures 49a) and b) show the results of the analysis of the 1264-264 keV correlation for the two possible spins, 5/2 and 7/2 for the 1528-keV level. As can be seen from the figures, neither spin is ruled out, but the 5/2 case, being more probable, is tentatively proposed. With this spin, a value of $\delta_{1264} = -1.41 \pm 0.68$ or -0.079 ± 0.019 is obtained. The 1312-215 keV correlation yields $\delta_{1312} = -6.3 \pm 4.9$ or -0.69 ± 0.38 .

g) The 1560-keV Level

The analysis of the 929-416 keV correlation is shown in Figures 50a), b), and c). Based on this analysis, an assignment of $5/2^+$ is made for the 1560-keV level. The 929-keV γ -ray is in coincidence with the 215-, 367-, and 416-keV transitions, a simultaneous analysis of these three cascades yields a value for δ_{929} of -0.63 ± 0.41 .

The remainder of the cascades investigated in this study did not yield unique spin assignments and, hence, will not be discussed in detail. The resulting spin possibilities are given in the decay scheme (Figure 19).

As is evident from the above discussion there are several cases where there is considerable redundancy involved in the analysis. For example, there are five independent determinations of the mixing ratio of the 558-keV transition corresponding to the five measured correlations involving this γ -ray in cascade with the 215-, 264-, 367-, 416-, and 631-keV transitions. In Table 9 we list this as well as the other cases of redundancy; the table includes the individual values plus that obtained in the simultaneous analysis. As can be seen, in all cases, the

Figure 49

Figure 50

TABLE 9

CASCADE

		δ_{215}	
416-215 keV		-0.164 ± 0.016	
		δ_{264}	
211-264 keV		-0.318 ± 0.022	
367-264	-1.46 ± 0.02	-0.322 ± 0.007	
Simultaneous	-1.46 ± 0.02	-0.321 ± 0.011	
		δ_{367}	
		0.	
		δ_{558}	
558-416-215 keV		-0.107 ± 0.031	
558-367-264		-0.136 ± 0.008	
558-367		-0.144 ± 0.009	
558-416		-0.145 ± 0.010	
558-631		-0.128 ± 0.015	
Simultaneous		-0.139 ± 0.006	
		δ_{631}	
		0.	
		δ_{810}	
810-558-367-keV		0.215 ± 0.041	2.24 ± 0.24
810-558-416		0.166 ± 0.040	2.57 ± 0.29
810-558		0.201 ± 0.023	2.32 ± 0.11
810-558-631		0.09 ± 0.12	3.9 ± 1.7
Simultaneous		0.194 ± 0.030	2.37 ± 0.18

TABLE 9

		δ_{929}	
929-416-215 keV		-0.76 ± 0.71	
929-367		-0.66 ± 0.45	
925-416		-0.60 ± 0.32	
Simultaneous		-0.63 ± 0.41	.
		δ_{1264}	
1264-264 keV	-1.41 ± 0.06	-0.079 ± 0.013	
		δ_{1313}	
1313-215 keV	-5.86 ± 1.7	-1.03 ± 0.11	
		δ_{1368}	
1368-416-215 keV	<-4.1	0.54 ± 0.23	>9.2
1368-367-264	-10.8 ± 3.6	0.476 ± 0.042	
1368-367	$<-18.$	0.625 ± 0.091	$>14.$
1368-416	$-23. \pm 14.$	0.524 ± 0.050	
Simultaneous	$-21. \pm 12.$	0.519 ± 0.049	

individual results agree to within errors. This reflects favorably on all phases of the precorrelation analysis including the normalization factors for each angle, the random and background corrections, the area determinations and their associated errors and the calculated values for the solid angle attenuation factors. Further proof of the accuracy of the data is available from the 4 (2754, 0) 2 (1368, 0) 0 correlation arising from the ^{24}Na contaminant. As can be seen from Table 7, the values of A_x obtained from the measured correlation are in good agreement with the theoretical values of $A_2 = 0.102$ and $A_4 = 0.009$.

VI) DISCUSSION

There have been several theoretical studies^(11,59-61) on the level structure of ^{77}As . Kisslinger and Sorensen⁽⁵⁹⁾ used a spherical core approximation and considered residual interactions of the nucleons outside the core, including pairing effects. Kisslinger and Kumar⁽⁶⁰⁾ modified the previous model in order to properly predict the ground state spin, but on the whole the agreement with experiment was poor. The failure of these two models can in part be attributed to the spherical core approximation, since it is known that the ^{77}As nucleus is considerably deformed⁽²¹⁾.

Scholz and Malik⁽⁶¹⁾ devised a model based on a deformed core shell model with residual interactions between the extracore nucleons. These interactions included Coriolis coupling, whereby the angular momentum of the extracore nucleons is coupled to that of the core, and pairing effects, to account for the tendency of the nucleons outside the core to pair up with the spins opposed. This model has been used with success in the region of light nuclei, particularly for $A \leq 60$,⁽⁶²⁾ to correctly predict the low-lying level structure, including the ground state spin, of most of the light odd nuclei. Moreover the model predicts magnetic and quadrupole moments in reasonable agreement with observation without the use of an effective charge, and properly explains the observed enhanced E2 transition rates.

The magnitude of the Coriolis coupling is inversely proportional to the moment of inertia of the nucleus, which in turn is roughly proportional to $A^{5/3}$. For light nuclei the strength of this term becomes comparable with the difference between two band head energies. And so, rotational bands, so apparent in heavy nuclei, where this term is small enough to be treated as a perturbation, are not readily observable in the level structure of the light nuclei.

Imanishi and Nishi⁽¹¹⁾ used this model to predict the negative parity states of ^{77}As . Their results along with those of Kisslinger and Kumar, and Scholz and Malik, are compared with our experimental results in Figure 51. As can be seen the agreement between the latter theories^(11, 61) and experiment for the low-energy levels is good. In particular these theories predict a low-lying $1/2^-$, $3/2^-$, $5/2^-$ triplet as the first three excited states. Such low-energy triplets are observable in the neighboring ^{75}As and ^{79}Br nuclei⁽¹⁷⁾, although in the latter case these levels have a different order and are not as clearly defined. The experimental results show the 215- and 264-keV levels as having assignments of $3/2^-$ and $5/2^-$, respectively, with the 194-keV level as the obvious third member of a possible triplet.

An assignment of $1/2^-$ for the 194-keV level is not consistent with our calculated log ft value for this level, which by itself would rule out this possibility. The log ft values for the β of ^{77}Ge indicate a spin of $1/2$ or $3/2$ for this level^(5,12), our correlation results are consistent with both these possibilities. However, the coincidence results show that the 416-keV γ -ray is in coincidence with the 194-keV γ -ray. This is possible only if there is a transition between the 215- and 194-keV levels, as indicated in the decay scheme, Figure 19. By comparing the area of the 416- and 419-keV lines in the 194-keV coincidence spectrum we determine that the relative intensity of this 21-keV transition is 0.5 ± 0.2 . This value is in reasonable agreement with the value of 0.86 ± 0.18 which is required to achieve an intensity balance for the 194-keV level necessary to account for the spin/parity change of 3^- involved in the β decay from the $7/2^+$ ground state of ^{77}Ge to the $1/2^-$ 194-keV level of ^{77}As . Considering internal conversion coefficients at 21 keV we find that the relative γ intensity is on the order of 0.02 to 0.04. We in fact did not observe this 21-keV transition in the single parameter studies, but here we were faced with a region of very intense background which would mask the existence of such a weak line.

In Table 10 we list experimental hinderance factors for E1 transitions and enhancement factors for E2 transitions. These factors are calculated relative to M1 transition rates which are assumed to be hindered by a factor of 10. E1 hinderance factors of 10^5 are common. However, E2 enhancement factors of 200 are consistent with the results for deformed nuclei, and hence, lend support to the hypothesis that nuclei in the region of A=77 are deformed.

TABLE 10

Relative To M1 With Hinderance Factor of 100

<u>E(keV)</u>	<u>E1 Hinderance</u>	<u>E2 Enhancement</u>
215		200
264		600
558		30
810		20
896	400	
925	3×10^4	
929		200
1263		500
1295	9×10^3	
1312		300
1368		60
1735	5×10^3	
1784	5×10^5	
2000	6×10^3	
average	10^5	200

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APPENDIX A: A BRIEF OUTLINE OF ELECTROMAGNETIC THEORY LEADING TO THE DIRECTIONAL CORRELATION FORMULAE

The fundamental starting point is the transition probability as given by first order perturbation theory

$$W_{ab}(\vec{k}, \varepsilon) = \frac{2\pi}{\hbar} |\langle b | H(\vec{k}, \varepsilon) | a \rangle|^2 dn/dE \quad (A1)$$

where $|a\rangle$ and $|b\rangle$ represent the initial and final states, and $H(\vec{k}, \varepsilon)$ is the electromagnetic interaction Hamiltonian

$$H(\vec{k}, \varepsilon) = -\beta g_e (\vec{p} \cdot \vec{A}_0 + \vec{A}_0 \cdot \vec{p}) - \beta g_s \vec{s} \cdot (\nabla \times \vec{A}_0) \quad (A2)$$

with

$$\begin{aligned} \beta &= e\hbar/2mc \\ \vec{p} &= -i\nabla \\ \vec{s} &= \frac{1}{2} \vec{\sigma} \\ g_e &= 0, 1 \quad ; \text{neutron, proton} \\ g_s &= -3.826, 5.585; \text{neutron, proton} \end{aligned}$$

and $\vec{A}_0$ is the vector potential, which when normalized to one quantum per unit volume per unit solid angle is given by⁽¹⁾

$$\vec{A}_0 = (\hbar c/2k)^{1/2} \left[\hat{\varepsilon} e^{i(\vec{k} \cdot \vec{r} - \omega t)} + \hat{\varepsilon}^* e^{-i(\vec{k} \cdot \vec{r} - \omega t)} \right] \quad (A3)$$

where $\hat{\varepsilon}$, the polarization, and $\hat{k}$, the direction of propagation, are orthogonal, i.e. $\hat{\varepsilon} \cdot \hat{k} = 0$. As can be seen from the form of $\vec{A}_0$, the vector potential is expressed in terms of an absorption component plus an emission component. The final term in Equation (A1) is the number of final photon states per unit energy interval per unit volume and is given by⁽¹⁾

$$\begin{aligned} dn/dE &= (2\pi)^{-3} 4\pi k^2 dk/dE \\ &= k^2/2\pi^2 \hbar c \quad ; \text{using } E = \hbar \omega c \end{aligned} \quad (A4)$$

Rewriting Equation (A1), the transition probability per unit solid angle is

$$W_{ab}(\vec{k}, \hat{e}) = (k/2\pi\hbar) |\langle b | H_e(\vec{k}, \hat{e}) | a \rangle|^2 \quad (A5)$$

where the emission operator, $H_e(\vec{k}, \hat{e})$, is given by Equation (A2) with $\vec{A}_0$ replaced by $\vec{A}^* = \hat{e}^* e^{-i\vec{k} \cdot \vec{r}}$.

The next step is to make a multipole expansion of $H_e(\vec{k}, \hat{e})$. The expansion of a plane wave into spherical waves is given by the well known relation

$$e^{-i\vec{k} \cdot \vec{r}} = 4\pi \sum_{\ell m} (-i)^\ell J_\ell(kr) Y_\ell^m(\hat{r}) Y_\ell^{m*}(\hat{k}) \quad (A6)$$

where the J_ℓ are spherical Bessel functions and the Y_ℓ^m are spherical harmonics, phase defined as in Condon and Shortley,⁽²⁾ i.e. $Y_\ell^{m*} = (-)^m Y_\ell^{-m}$. Expressing $\hat{e}^*$ in terms of the unit spherical vector bases,

$$\hat{e}^* = \sum_{\mu} (\hat{e}^* \cdot \hat{e}_\mu^*) \hat{e}_\mu \quad (A7)$$

and combining Equations (A6) and (A7), the vector potential is given by

$$\vec{A}^* = 4\pi \sum_{\ell m \mu} (\hat{e}^* \cdot \hat{e}_\mu^*) \hat{e}_\mu (-i)^\ell J_\ell(kr) Y_\ell^m(\hat{r}) Y_\ell^{m*}(\hat{k}) \quad (A8)$$

Equation (A8) may be rewritten more compactly as

$$\vec{A}^* = 4\pi \sum_{\ell m \lambda} (-i)^\ell J_\ell(kr) \bar{Y}_{\lambda \ell}^{m+\mu}(\hat{r}) \hat{e}_\mu^* \bar{Y}_{\lambda \ell}^{m+\mu*}(\hat{k}) \quad (A9)$$

where the vector spherical harmonics are given by⁽³⁾

$$\bar{Y}_{\lambda \ell} = \sum_{\mu} \langle \ell, m-\mu, 1\mu | \lambda m \rangle \hat{e}_\mu Y_\ell^{m-\mu} \quad (A10)$$

$$\hat{e}_\mu Y_\ell^{m-\mu} = \sum_{\lambda} \langle \ell, m-\mu, 1\mu | \lambda m \rangle \bar{Y}_{\lambda \ell}^m \quad (A11)$$

$$; \lambda = \ell+1, \ell, \ell-1$$

and the $\langle \ell, m-\mu, 1\mu | \lambda m \rangle$ are Clebsch-Gordon coefficients, again phase defined as in Condon and Shortley. Fixing $\lambda = L$ and $m + \mu = M$, and taking parity into consideration the vector potential may be split into two multipole components, electric with parity $(-)^{L-1}$ and magnetic with parity $(-)^L$.

$$\bar{A}_{LM}^{e*}(\hat{k}, \varepsilon) = 4\pi \sum_{\ell} (-i)^{\ell} J_{\ell}(kr) \bar{Y}_{L\ell}^M(\hat{r}) \varepsilon^* \cdot \bar{Y}_{L\ell}^{M*}(\hat{k}) \quad ; \ell = L \pm 1 \quad (A12)$$

$$\bar{A}_{LM}^{m*}(\hat{k}, \varepsilon) = 4\pi (-i)^L J_L(kr) \bar{Y}_{LL}^M(\hat{r}) \varepsilon^* \cdot \bar{Y}_{LL}^{M*}(\hat{k}) \quad (A13)$$

All the angular information is contained in the terms $\varepsilon^* \cdot \bar{Y}_{L\ell}^{M*}(\hat{k})$, for $\ell = L-1, L, L+1$. At this point we define the polarization correlation amplitude

$$g_{LM}(\hat{k}, \varepsilon) \equiv \varepsilon \cdot \bar{Y}_{LL}^M(\hat{k}) \quad (A14)$$

It is straightforward to show that

$$\varepsilon^* \cdot \bar{Y}_{L, L-1}^{M*}(\hat{k}) = -i \left(\frac{L+1}{2L+1} \right)^{\frac{1}{2}} g_{LM}^*(\hat{k}, \varepsilon') \quad (A15)$$

and

$$\varepsilon^* \cdot \bar{Y}_{L, L+1}^{M*}(\hat{k}) = -i \left(\frac{L}{2L+1} \right)^{\frac{1}{2}} g_{LM}^*(\hat{k}, \varepsilon') \quad (A16)$$

where ε' is the "other" polarization, i.e. $\hat{k} \times \varepsilon = \varepsilon'$. With this result the vector potential may be rewritten as

$$\bar{A}_{LM}^{e*}(\hat{k}, \varepsilon) = \frac{-4\pi(-i)^{L-1}}{k[L(L+1)]^{\frac{1}{2}}} \nabla \times \left[J_L(kr) Y_L^M(\hat{r}) \right] g_{LM}^*(\hat{k}, \varepsilon') \quad (A17)$$

$$\bar{A}_{LM}^{m*}(\hat{k}, \varepsilon') = \frac{4\pi(-i)^{L-1}}{k[L(L+1)]^{\frac{1}{2}}} \nabla \times \left[J_L(kr) Y_L^M(\hat{r}) \right] g_{LM}^*(\hat{k}, \varepsilon) \quad (A18)$$

$$\bar{A}_{LM}^{e*}(\hat{k}, \varepsilon) = \frac{4\pi(-i)^L}{[L(L+1)]^{\frac{1}{2}}} \left[J_L(kr) Y_L^M(\hat{r}) \right] g_{LM}^*(\hat{k}, \varepsilon) \quad (A19)$$

$$\bar{A}_{LM}^{m*}(\hat{k}, \varepsilon') = \frac{4\pi(-i)^L}{[L(L+1)]^{\frac{1}{2}}} \left[J_L(kr) Y_L^M(\hat{r}) \right] g_{LM}^*(\hat{k}, \varepsilon') \quad (A20)$$

where we have used the following relations,

$$\begin{aligned} \nabla \times [J_L(kr) Y_L^M(\hat{r})] &= ik [L(L+1)]^{\frac{1}{2}} \left(\frac{L+1}{2L+1} \right)^{\frac{1}{2}} J_{L-1}(kr) \bar{Y}_{L,L-1}^M(\hat{r}) \\ &\quad - \left(\frac{L}{2L+1} \right)^{\frac{1}{2}} J_{L+1}(kr) \bar{Y}_{L,L+1}^M(\hat{r}) \end{aligned} \quad (A21)$$

and

$$\nabla [J_L(kr) Y_L^M(\hat{r})] = [L(L+1)]^{\frac{1}{2}} J_L(kr) \bar{Y}_{LL}^M(\hat{r}) \quad (A22)$$

The vector potentials as given above satisfy the following relations,

$$\nabla \cdot \bar{A}_{LM}^{e*}(\hat{k}, \hat{\varepsilon}) = \nabla \cdot \bar{A}_{LM}^{e*}(\hat{k}, \hat{\varepsilon}') = \nabla \cdot \bar{A}_{LM}^{m*}(\hat{k}, \hat{\varepsilon}) = \nabla \cdot \bar{A}_{LM}^{m*}(\hat{k}, \hat{\varepsilon}') = 0 \quad (A23)$$

$$\nabla \times \bar{A}_{LM}^{e*}(\hat{k}, \hat{\varepsilon}) = -ik \bar{A}_{LM}^{m*}(\hat{k}, \hat{\varepsilon}') \quad (A24a)$$

$$\nabla \times \bar{A}_{LM}^{e*}(\hat{k}, \hat{\varepsilon}') = ik \bar{A}_{LM}^{m*}(\hat{k}, \hat{\varepsilon}) \quad (A24b)$$

$$\nabla \times \bar{A}_{LM}^{m*}(\hat{k}, \hat{\varepsilon}) = -ik \bar{A}_{LM}^{e*}(\hat{k}, \hat{\varepsilon}') \quad (A24c)$$

$$\nabla \times \bar{A}_{LM}^{m*}(\hat{k}, \hat{\varepsilon}') = ik \bar{A}_{LM}^{e*}(\hat{k}, \hat{\varepsilon}) \quad (A24d)$$

In addition, using Equations (A23) and (A24), these fields satisfy, as they must, the vector wave equation

$$\nabla^2 \bar{A} + k^2 \bar{A} = 0 \quad (A25)$$

where $\bar{A}$ is any of the vector potentials described in Equations (A17) through (A18).

We are now in a position to give an expression for the multipole components of the emission operator. Defining

$$\alpha_L(k) \equiv \frac{4\pi(-1)^{L+1} k^L}{(2L+1)!!} \left(\frac{L+1}{L} \right)^{\frac{1}{2}} \quad (A26)$$

where $(2L+1)!! = (2L+1)(2L-1) \dots (3)(1)$. We have

$$H_e(L, M, \vec{k}, \epsilon) = \alpha_L(k) \{ [Q_{LM} + Q'_{LM}] g_{LM}^*(k, \epsilon) + [M_{LM} + M'_{LM}] g_{LM}^*(\hat{k}, \epsilon) \} \quad (A27)$$

and

$$H_e(L, M, \vec{k}, \epsilon') = \alpha_L(k) \{ -[Q_{LM} + Q'_{LM}] g_{LM}^*(k, \epsilon) + [M_{LM} + M'_{LM}] g_{LM}^*(\hat{k}, \epsilon') \} \quad (A28)$$

where

$$Q_{LM} = 2Bg_L \frac{(2L+1)!!}{k^{L+1}(L+1)} \nabla \times \overline{[J_L(kr) Y_L^M(\hat{r})] \cdot \vec{p}} \quad (A29)$$

$$Q'_{LM} = -Bg_s \frac{(2L+1)!!}{k^{L-1}(L+1)} \overline{[J_L(kr) Y_L^M(\hat{r})] \cdot \vec{s}} \quad (A30)$$

$$M_{LM} = -2iBg_s \frac{(2L+1)!!}{k^L(L+1)} \overline{[J_L(kr) Y_L^M(\hat{r})] \cdot \vec{p}} \quad (A31a)$$

$$= 2Bg_L \frac{(2L+1)!!}{k^L(L+1)} \nabla [J_L(kr) Y_L^M(\hat{r})] \cdot \vec{z} \quad (A31b)$$

$$M'_{LM} = -iBg_s \frac{(2L+1)!!}{k^L(L+1)} \nabla \times \overline{[J_L(kr) Y_L^M(\hat{r})] \cdot \vec{s}} \quad (A32)$$

The forms for the operators given above were chosen such that in the long wavelength approximation they reduce to the usual expressions^{4,5}.

The polarization insensitive transition rate for either electric or magnetic multipoles is arrived at by summing the rates obtained using the appropriate terms from Equations (A27) and (A28)

$$W_{ab}(\pi LM, \vec{k}) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{k^{2L+1}}{\hbar} |\langle b | \pi LM | a \rangle|^2 G_{LM}(\hat{k}) \quad (A33)$$

where $\pi LM = Q_{LM} + Q'_{LM}$ or $M_{LM} + M'_{LM}$ for electric or magnetic respectively, and

$$\begin{aligned} G_{LM}(\hat{k}) &= |g_{LM}(\hat{k}, \epsilon)|^2 + |g_{LM}(\hat{k}, \epsilon')|^2 \\ &= \sum_{\mu} \langle L, M-\mu, 1\mu | LM \rangle^2 |Y_L^{M-\mu}(\hat{k})|^2 \end{aligned} \quad (A34)$$

The total rate is obtained from Equation (A33) by integrating over the solid angle of the emitted radiation, summing over the substates M , summing over the final state projections B , and averaging over the initial state projections a . The final result is

$$W_{ab}(\pi L, k) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{k^{2L+1}}{\pi} B_{ab}(\pi L) \quad (A35)$$

$B_{ab}(\pi L)$ is the reduced transition strength and is given by

$$B_{ab}(\pi L) = \frac{1}{2a+1} |\langle b || \pi L || a \rangle|^2 \quad (A36)$$

where $\langle b || \pi L || a \rangle$ is the reduced matrix element, defined by the Wigner - Eckart Theorem

$$\langle bB | \pi LM | aa \rangle = (-)^{M-B-\alpha} \frac{1}{(2b+1)^{1/2}} \langle a \alpha L M | bB \rangle \langle b || \pi L || a \rangle \quad (A37)$$

Making the Weisskopf⁴ single particle approximations the reduced transition strengths are

$$B(EL) = \frac{(1.2)^{2L}}{4\pi} \left(\frac{3}{L+3} \right)^3 A^{2L/3} (10^{-13} \text{ cm})^{2L} e^2 \quad (A38)$$

$$B(ML) = 10 \frac{(1.2)^{2(L-1)}}{\pi} A^{2(L-1)/3} (10^{-13} \text{ cm})^{2(L-1)} \mu_p^2 \quad (A39)$$

where

A is the number of nucleons in the nucleus

e is the electronic charge ($e^2 = 1.44 \times 10^{-13} \text{ MeV cm}$)

μ_p is the nuclear magneton ($\mu_p^2 = 1.81 \times 10^{-28} \text{ MeV cm}^3$)

Substituting these expressions into eq. A35 we obtain the Weisskopf single particle transition probabilities (we use the symbol $T(\pi L)$ to conform to convention).

$$T(E1) = 7.7 \times 10^{13} E^3 A^{2/3} \text{ sec}^{-1}$$

$$T(M1) = 5.6 \times 10^{13} E^3 \text{ sec}^{-1}$$

$$T(E2) = 4.4 \times 10^7 E^5 A^{4/3} \text{ sec}^{-1}$$

$$T(M2) = 6.2 \times 10^7 E^5 A^{2/3} \text{ sec}^{-1}$$

$$T(E3) = 1.7 \times 10^{11} E^7 A^2 \text{ sec}^{-1}$$

$$T(M3) = 4.1 \times 10^{11} E^7 A^{4/3} \text{ sec}^{-1}$$

$$T(E4) = 4.6 \times 10^{-6} E^9 A^{8/3} \text{ sec}^{-1}$$

$$T(M4) = 1.8 \times 10^{-6} E^9 A^2 \text{ sec}^{-1}$$

where E is the energy in MeV of the transition. Because of the approximations involved these expressions are not intended to be exact but only to serve as guidelines for estimating the transition rates.

Returning to the directional correlation theory, we require the total transition probability per unit solid angle for going from magnetic substate α of state $\bar{a}$ to magnetic substate β of state $\bar{b}$ by the emission of a photon $\bar{k}$

$$W_{ab}(\bar{k}) = (k/2\pi\hbar) [|\langle b | \sum_{LM} H_e(L, M, \bar{k}, \bar{e}) | a \rangle|^2 + |\langle b | \sum_{LM} H_e(L, M, \bar{k}, \bar{e}') | a \rangle|^2] \quad (A40)$$

The algebra necessary to obtain the desired result is both intricate and tedious, hence only the final form of this transition rate is given

$$W_{ab}(\bar{k}) = \sum_{\pi' L' \lambda} [1 + (-1)^\lambda] (-)^{M+L+L'-\lambda-1} (2b+1)^{-1} (2L+1)^{1/2} (2L'+1)^{1/2} \langle L L' -1 | \lambda 0 \rangle \quad (A41)$$

$$\langle L M L' -M | \lambda 0 \rangle \langle a L M | b \beta \rangle \langle a L' M' | b \beta \rangle \langle b | | T_L^{<\pi>} | | a \rangle \langle b | | T_{L'}^{<\pi'>} | | a \rangle^* P_\lambda(\cos\theta)$$

where the P_λ are legendre polynomials and the $\langle b || T_L^{<\pi>} || a \rangle$ are the reduced matrix elements corresponding to the operators

$$T_{LM}^{<0>} = \frac{1}{4\pi} (k/\hbar)^{\frac{1}{2}} \alpha_L(k) [Q_{LM} + Q_{LM}'] \quad (A42)$$

$$T_{LM}^{<1>} = \frac{1}{4\pi} (k/\hbar)^{\frac{1}{2}} \alpha_L(k) [M_{LM} + M_{LM}'] \quad (A43)$$

With this transition probability the population parameters for state $\bar{b}$ can be expressed in terms of the population parameters of state $\bar{a}$, i.e.

$$P(\beta) = \sum_{\alpha} P(\alpha) W_{\alpha\beta}(\bar{k}) \quad (A44)$$

where $P(\alpha)$ is the probability that state $\bar{a}$ is in magnetic substate α , similarly for $P(\beta)$.

Now consider the simple cascade: state $\bar{a}$ goes to state $\bar{b}$ with the emission of a photon $\bar{k}_1$ and state $\bar{b}$ in turn immediately decays to state $\bar{c}$ by the emission of a photon $\bar{k}_2$. Treating the second member of the cascade first, the total probability of being in state $\bar{c}$ is obtained by summing $P(\gamma)$ over all possible γ . Using (A44) and (A41) yields

$$\begin{aligned} P &= \sum_{\gamma} P(\gamma) \\ &= \sum_{\substack{\gamma \pi_2 L_2 M_2 \\ \beta \pi_2' L_2' \lambda_2}} P(\beta) [1 + (-)^{\lambda_2}] (-)^{M_2 + L_2 + L_2' - \lambda_2 + 1} (2C+1)^{-1} (2L_2+1)^{\frac{1}{2}} (2L_2'+1)^{\frac{1}{2}} \langle L_2' 1 L_2' -1 | \lambda_2 0 \rangle \\ &\quad \langle L_2 M_2 L_2' -M_2 | \lambda_2 0 \rangle \langle b \beta L_2 M_2 | c \gamma \rangle \langle b \beta L_2' M_2' | c \gamma \rangle \\ &\quad \langle c || T_{L_2}^{<\pi>} || b \rangle^* \langle c || T_{L_2}^{<\pi>} || b \rangle^* P_{\lambda_2}(\cos \theta_2) \end{aligned} \quad (A45)$$

Using the Racah algebra⁽³⁾ to perform the sum over γ and M_2 , we have

$$\begin{aligned} P &= \sum_{\substack{\pi_2 L_2 \beta \\ \pi_2' L_2' \lambda_2}} P(\beta) [1 + (-)^{\lambda_2}] (-)^{-\lambda_2 + \beta + 1} \langle b \beta b, -\beta | \lambda_2 0 \rangle \langle L_2' 1 L_2' -1 | \lambda_2 0 \rangle \\ &\quad W(b \beta L_2 L_2'; \lambda_2 c) \langle c || T_{L_2}^{<\pi_2>} || b \rangle \langle c || T_{L_2}^{<\pi_2'>} || b \rangle P_{\lambda_2}(\cos \theta_2) \end{aligned} \quad (A46)$$

where the $W(bbL_2L_2';\lambda_2c)$ are Racah coefficients. Restricting the sum over L_2 and L_2' to the lowest two terms, i.e.

$$L_2 = |b-c|, = 1 \text{ if } b = c$$

and

$$L_2' = L_2 + 1$$

we have, neglecting unimportant multiplicative factors, for the angular distribution

$$W(\theta_2) = \sum_{\beta\lambda_2} P(\beta)(-)^{b+\beta} \langle b\beta b, -\beta | \lambda_2 0 \rangle A_{\lambda_2}(\gamma_2) P_{\lambda_2}(\cos\theta). \quad (A47)$$

where the directional distribution coefficient

$$A_{\lambda_2}(\gamma_2) = \frac{\{F_{\lambda_2}(L_2L_2'cb) + 2\delta_2 F_{\lambda_2}(L_2L_2'cb) + \delta_2^2 F_{\lambda_2}(L_2L_2'cb)\}}{1 + \delta_2^2} \quad (A48)$$

with

$$F_{\lambda_2}(L_2L_2'cb) = (-)^{b+c+1}(2L_2+1)^{\frac{1}{2}}(2L_2'+1)^{\frac{1}{2}}(2b+1)^{\frac{1}{2}} \langle L_2 1 L_2', -1 | \lambda_2 0 \rangle W(bbL_2L_2';\lambda_2c) \quad (A49)$$

and

$$\delta_2 = \frac{\langle c || T_{L_2}^{<\pi_2>} || b \rangle}{\langle c || T_{L_2}^{<\pi_2>} || b \rangle} \quad (A50)$$

where the phases for the nuclear wavefunctions are chosen to insure that the reduced matrix elements are real. In addition the term $[1 + (-)^{\lambda_2}]$ implies λ_2 is even.

Considering the first member of the cascade we have

$$P(\beta) = \sum_{\alpha} P(\alpha) \psi_{ab}(\bar{k}_1)$$

and since the state $\bar{a}$ is unobserved $P(\alpha) = (2a+1)^{-1}$ for all α . So

$$P(\beta) = \sum_{\alpha} (2a+1)^{-1} [1 + (-)^{\lambda_1}] (-)^{M_1+L_1+L_1'-\lambda_1-1} (2b+1)^{-1} (2L_1+1)^{\frac{1}{2}} (2L_1'+1)^{\frac{1}{2}} \\ \pi_{11}^{L_1' L_1' \lambda_1} \langle L_1' | L_1' -1 | \lambda_1 0 \rangle \langle L_1' M_1' L_1' -M_1' | \lambda_1 0 \rangle \langle a \alpha L_1 M_1 | b \beta \rangle \langle a \alpha L_1' M_1' | b \beta \rangle$$

$$\langle b | | T_{L_1}^{<\pi_1>} | | a \rangle \langle b | | T_{L_1'}^{<\pi_1'>} | | a \rangle^* P_{\lambda_1}(\cos \theta_1) \quad (A51)$$

Again the Racah algebra⁽¹⁾ performs the sum over α and M_1

$$P(\beta) = \sum_{\alpha} (2a+1)^{-1} [1 + (-)^{\lambda_1}] (-)^{a+L_1+L_1'+\beta-1} \langle L_1' | L_1' -1 | \lambda_1 0 \rangle \langle b \beta b, -\beta | \lambda_1 0 \rangle \\ \pi_{11}^{L_1' L_1' \lambda_1} W(bb L_1 L_1'; \lambda_1 a) \langle b | | T_{L_1}^{<\pi_1>} | | a \rangle \langle b | | T_{L_1'}^{<\pi_1'>} | | a \rangle^* P_{\lambda_1}(\cos \theta_1) \quad (A52)$$

Taking $L_1 = |a-b|$, $=1$ if $a = b$ and $L_1' = L_1 + 1$, and defining the mixing ratio

$$\delta_1 = \frac{\langle b | | T_{L_1}^{<\pi_1>} | | a \rangle}{\langle b | | T_{L_1'}^{<\pi_1'>} | | a \rangle} \quad (A53)$$

and the directional orientation coefficients

$$B_{\lambda_1}(\gamma_1) = \frac{\{F_{\lambda_1}(L_1 L_1 ab) - 2\delta_1 F_{\lambda_1}(L_1 L_1' ab) + \delta_1^2 F_{\lambda_1}(L_1' L_1' ab)\}}{1 + \delta_1^2} \quad (A54)$$

we have

$$P(\beta) = \sum_{\lambda_1} (-)^{b+\beta} \langle b \beta b, -\beta | \lambda_1 0 \rangle B_{\lambda_1}(\gamma_1) P_{\lambda_1}(\cos \theta_1) \quad (A55)$$

where λ_1 is also even. Combining Equations (A55) and (A47), using the orthogonality of the Clebsch-Gordon coefficients, we obtain the desired result.

$$W(\theta) = \sum_{\lambda} B_{\lambda}(\gamma_1) A_{\lambda}(\gamma_2) P_{\lambda}(\cos\theta), \lambda_{\text{even}} \quad (\text{A56})$$

with θ as the relative angle, i.e. $\cos\theta = \hat{k}_1 \cdot \hat{k}_2$.

Very similar arguments lead to the general directional correlation formula for the case of unobserved transitions

$$W(\theta) = \sum_{\lambda \text{ even}} B_{\lambda}(\gamma_1) U_{\lambda}(\gamma_2) \dots U_{\lambda}(\gamma_{n-1}) A_{\lambda}(\gamma_n) P_{\lambda}(\cos\theta) \quad (\text{A57})$$

where

$$U_{\lambda}(\gamma_i) = \frac{G_{\lambda}(L_i I_i I_{i+1}) + \delta_i^2 G_{\lambda}(L_i I_i I_{i+1})}{1 + \delta_i^2}; i = 2, 3, \dots n-1 \quad (\text{A58})$$

are the deorientation coefficients for the unobserved transitions $\gamma_2, \gamma_3, \dots \gamma_{n-1}$, and

$$G_{\lambda}(L_i I_i I_{i+1}) = (-)^{I_i + I_{i+1} - L_i} [(2I_i + 1)(2I_{i+1} + 1)]^{1/2} W(I_i I_i I_{i+1} I_{i+1}; \lambda L_i) \quad (\text{A59})$$

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Appendix B: SOLID ANGLE CORRECTION FACTORS

The solid angle correction factors have been calculated by Camp and Van Lehn⁽¹⁾ for Ge(Li) detectors of varying size and shape. They present tables from which one can interpolate to find the corrections for your particular detector and experimental configuration. The detectors used in this study were not included in these tables, necessitating interpolations for size and shape as well as energy and distance. For this reason, we decided to develop our own code to calculate the solid angle corrections.

For axially symmetric detectors, it can be shown^(2,3) that the measured correlation function has the form:

$$W(\theta) = 1 + Q_2 a_2 P_2(\cos \theta) + Q_4 a_4 P_4(\cos \theta) + \dots \quad (B1)$$

where θ is the angle between the axes of the two detectors, P_k are the Legendre polynomials, a_k are the theoretical correlation coefficients as previously defined, and Q_k are the solid angle attenuation coefficients. The Q_k are a direct result of the finite solid angle subtended by the detectors and can be expressed as:

$$Q_k = \frac{J_k^A(\gamma_1)}{J_0^A(\gamma_1)} \frac{J_k^B(\gamma_2)}{J_0^B(\gamma_2)} \frac{R}{1+R} + \frac{J_k^A(\gamma_2)}{J_0^A(\gamma_2)} \frac{J_k^B(\gamma_1)}{J_0^B(\gamma_1)} \frac{1}{1+R} \quad (B2)$$

with

$$R = \frac{J_0^A(\gamma_1)}{J_0^A(\gamma_2)} \frac{J_0^B(\gamma_2)}{J_0^B(\gamma_1)} \quad (B3)$$

and where the superscripts A and B refer to the two detectors. As can be seen, Q_k is expressed entirely in terms of ratios of the J_k for each detector. This is an important fact that we will return to later. The form for J_k is given by

$$J_k^x(\gamma_j) = \int_{\Omega^x} p_k(\cos \alpha) \epsilon_I^x(\alpha, E_j) d\Omega^x \quad (B4)$$

where the integral extends over Ω^x the solid angle subtended by detector x, α is the off-axis entrance angle for γ_j into detector x and ϵ_I^x is the intrinsic efficiency of detector x at angle α and energy E_j .

The absolute efficiency is just the product of the detector solid angle and the intrinsic efficiency averaged over this same solid angle, i.e.

$$\epsilon^x(\gamma_j) = \Omega^x \frac{\int_{\Omega^x} \epsilon_I^x(\alpha, E_j) d\Omega^x}{\int_{\Omega^x} d\Omega^x} = J_0^x(\gamma_j) \quad (B5)$$

So, the J_0 term is just the efficiency and

$$R = \frac{\epsilon^A(\gamma_1)}{\epsilon^B(\gamma_1)} \frac{\epsilon^B(\gamma_2)}{\epsilon^A(\gamma_2)} \quad (B6)$$

The J_k integrals were evaluated using the Monte Carlo approach, that is, a computer simulates the interactions of the primary photons and all secondary radiation produced in the detectors. To greatly simplify the problem, we

assumed the secondary electrons, which are produced by the primary γ interactions (photoelectric, Compton, and pair-production), have a zero range. This assumption ignores energy loss due to electron escape, bremsstrahlung production, etc. But for the primary γ energies considered (below 4 MeV), these effects are small, producing negligible errors. Sampling is done in such a way as to produce the required probability distributions for each event. In this way, the efficiency and J_k/J_0 ratios can be obtained for Ge(Li) detectors of any size and configuration, operating at any distance and for any size source.

The code was checked in two steps: first, we compared the calculated and measured efficiencies. The results are shown in Figure 11. The deviation at low energy is due to uncertainty in the dead layer thickness. The slight deviation at high energy is probably due to our assumption of zero range electrons. But for the energy range of interest (200 to 3000 keV), the agreement is seen to be excellent. Next we compared our results for the solid angle corrections with those of Yates⁽⁴⁾ for NaI detectors and with those of Camp and Van Lehn⁽¹⁾ for Ge(Li) detectors, in both cases the agreement was excellent. In addition, there are five measured correlations involving the 558-keV transition (see section V). The other (observed) member of the cascades have energies ranging from 215- to 631-keV; this is the energy region having the largest variation in the J_k/J_0 ratios. These five correlations yielded similar results under analysis (see Table 9) indicating the values of Q_k used were correct. One reason for this excellent agreement is the fact that the Q_k consist of ratios of the form J_k/J_0 and, hence, are relatively insensitive to small uncertainties and to the method of calculation.

The errors are estimated by repeating the calculation several times, each time sampling from different set of random numbers, to produce statistically independent results. These results are averaged to yield the final result and the errors are obtained by determining the deviations of the individual results from the mean. In this way, errors of less than 5%, .4%, and 1.4% are obtained for the efficiency, J_2/J_0 , and J_4/J_0 terms, respectively. Thus, the errors in Q_2 and Q_4 are found to be less than .5% and 2%, respectively. These errors are negligible compared to the experimentally determined errors in the Q_2 and Q_4 coefficients and, hence, were ignored.

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Appendix C: A COMPUTER CODE FOR γ - γ DIRECTIONAL CORRELATION ANALYSIS

A computer code GGDC was developed to properly analyze the directional correlation data. As a first step, the measured data is fit, by the method of least squares, to the theoretical correlation function, i.e.

$$W_m(\theta) = \sum_{k \text{ even}} Q_k a_k P_k(\cos \theta) \quad (C1)$$

Following the usual procedure, one obtains values and uncertainties for the a_k . The a_k represents the data and, in fact, it is the only practical way to present the data for publication. But since these parameters only represent the data, extracting further information from the a_k can be misleading. This is especially true of the a_4 values, which, in general, are smaller than the a_2 terms. Examples can be seen in Table 7, the finite a_4 terms for the 929-367 keV and 907-416 keV correlations would indicate some quadrupole-dipole mixing for the 367 and 416-keV transitions which are, in fact, pure E1. A proper analysis requires the use of the correlation itself.

The reduced chi-squared, measuring the "goodness of fit", is defined as

$$\chi^2_\nu = \frac{1}{\nu} \chi^2 = \frac{1}{\nu} \sum_{\theta} \frac{[W_m(\theta) - \sum_{k \text{ even}} Q_k a_k P_k(\cos \theta)]^2}{[dW_m(\theta)]^2} \quad (C2)$$

where ν is the number of degrees of freedom, the a_k are the coefficients obtained in the fit, and $dW_m(\theta)$ is the uncertainty in $W_m(\theta)$. The chi-squared probability distribution yields

$$P_v(x^2) = \int_x^\infty \frac{1}{2^{v/2} \Gamma(v/2)} x^{2(v/2-1)} e^{-x^2/2} dx \quad (C3)$$

as the probability of obtaining a value of chi-squared greater than x^2 . A good fit is defined by having the x_v^2 of the fit (equation C2) be less than or equal to that value of x_v^2 which makes $P_v(x^2)$ (equation C3) equal to 50%. A too-large value of x_v^2 indicates the fit was made to the wrong function.

The theoretical correlation function is completely determined when the spin sequence and multipole mixing ratios are given. The correlation analysis is performed by fixing the spin sequence, either with known or assumed values, and evaluating the reduced chi-squared for all possible values of the mixing ratio, i.e.

$$x_v^2(\delta) = \frac{1}{v} \sum_{\theta} \frac{[W_m(\theta) - \sum_{k \text{ even}} Q_k a_k(\delta) P_k(\cos \theta)]^2}{[dW_m(\theta)]^2} \quad (C4)$$

Values of $x_v^2(\delta) \leq x_v^{2*}$ represent permissible ranges for δ for the assumed spin sequence. Where x_v^{2*} is the maximum of the x_v^2 of the a_k fit and that value of the reduced chi-squared corresponding to a value of $P_v(x^2)$ of 50%. Example of these one dimensional chi-squared plots are shown in Section V. Analysis for two unknown mixing ratios is performed in a similar way, with the plot of the result given as a $\delta_1 - \delta_2$ space with shading to indicate those values of δ_1 and δ_2 which satisfy the above-mentioned conditions. Analysis for three or more δ 's is a simple extension of the above methods, the problem lies in displaying the results.

The above methods of analysis can be extended to consider more than one correlation simultaneously.

$$x_v^2(\delta_1, \delta_2) = \frac{1}{v} \sum \frac{[w_{mi}(\theta) - \sum_{k \text{ even}} Q_{ki} a_{ki}(\delta_1, \delta_2) P_{ki}(\cos \theta)]^2}{[dw_{mi}(\theta)]^2} \quad (C5)$$

where the i index runs over the various correlations. This method generally has the advantage of further restricting the permissible ranges of the unknown deltas so as to more easily define the possible solutions. Examples of this form of analysis is given in Section V.

Appendix D: COINCIDENCE SPECTRA

Figures 52 through 101 show the coincidence spectra for the 50 gates used in this study.

Figure 52

Figure 56

Figure 57

Figure 58

Figure 59

Figure 60

Figure 61

Figure 62

Figure 63

Figure 64

Figure 65

Figure 66

Figure 67

Figure 68

Figure 69

Figure 70

Figure 71

Figure 72

Figure 73

Figure 74

Figure 75

Figure 80

Figure 81

Figure 82

Figure 83

Figure 84

Figure 85

Figure 86

Figure 87

Figure 88

Figure 89

Figure 90

Figure 91

Figure 100

Figure 101

NOTICE

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