**CHARACTERIZATION OF IN-SITU STRESS AND
PERMEABILITY IN FRACTURED RESERVOIRS**

SEMI-ANNUAL TECHNICAL PROGRESS REPORT

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by

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Abstract

Using a 3-D finite difference method with a rotated-staggered-grid (RSG) scheme we generated synthetic seismograms for a reservoir model consisting of three horizontal layers with the middle layer containing parallel, equally spaced fractures. By separating and analyzing the backscattered signals in the FK domain, we can obtain an estimate of the fracture spacing. The fracture spacing is estimated by taking one-half of the reciprocal of the dominant wavenumber of the backscattered energy in data acquired normal to the fractures. FK analysis for fracture spacing estimation was successfully applied to these model results, with particular focus on PS converted waves. The method was then tested on data from the Emilio Field. The estimated fracture spacing from the dominant wavenumber values in time windows at and below the reservoir level is 25-40m. A second approach for fracture spacing estimation is based on the observation that interference of forward and backscattered energy from fractures introduces notches in the frequency spectra of the scattered wavefield for data acquired normal to the fracture strike. The frequency of these notches is related to the spacing of the fractures. This Spectral Notch Method was also applied to the Emilio data, with the resulting range of fracture spacing estimates being 25-50m throughout the field. The dominant spacing fracture spacing estimate is about 30-40 m, which is very similar to the estimates obtained from the FK method.

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I. Executive Summary

The purpose of this project is to develop and implement large-scale numerical models to quantify the effects of fracture parameter variations on seismic reflection signals and in-situ stress variations on flexural modes in boreholes. These models will be used as the basis of data analysis and inversion routines for estimating the heterogeneous fracture distribution in fractured reservoirs from seismic and borehole field data. Fracture property distributions estimated from seismic data can be used to estimate the permeability tensor in the reservoir for input to reservoir simulators.

During the past six month period we have continued to adapt and test numerical codes for modeling seismic scattering from fractures, and to refine spectral analysis methods for estimating fracture corridor spacing in reservoirs. Additional tests of the accuracy and stability of the 3-D finite difference modeling scheme were carried out. Following these tests we generated results for a reservoir model consisting of three horizontal layers with the middle layer containing parallel, equally spaced fractures. By separating and analyzing the backscattered signals in the FK domain, we can obtain an estimate of the fracture spacing. The fracture spacing is estimated by taking one-half of the reciprocal of the dominant wavenumber of the backscattered energy in data acquired normal to the fractures. FK analysis for fracture spacing estimation was tested on these model results, with particular focus on PS converted waves. Because S wave arrivals have shorter wavelengths than P waves, these PS arrivals have higher resolution and therefore can provide estimates of finer scale fracture spacing. The method was tested on data from the Emilio Field. The estimated fracture spacing from the dominant wavenumber values in time windows at and below the reservoir level is 25-40m. A second approach for fracture spacing estimation is based on the observation that interference of forward and backscattered energy from fractures introduces notches in the frequency spectra of the scattered wavefield for data acquired normal to the fracture strike (Willis et al., 2005). The frequency of these notches is related to the spacing of the fractures (Willis et al., 2005). This Spectral Notch Method was also applied to the Emilio

data. The range of fracture spacing estimates is 25-50m, with a dominant spacing of about 30-40 m, very similar to the estimates from the FK method.

II. Introduction

The purpose of this project is to develop and implement large-scale numerical models to quantify the effects of fracture parameter variations on seismic reflection signals and in-situ stress variations on flexural modes in boreholes. These forward modeling efforts will be used to develop data analysis and inversion routines for estimating the heterogeneous fracture distribution in fractured reservoir from seismic and borehole field data. These fracture property distributions can be used to estimate the permeability tensor in the reservoir by using statistical and discrete feature network techniques, with the resulting values used as inputs to reservoir simulators. Figure 1 shows the schematic workflow.

Figure 1. Schematic display of the project objectives. Surface 3-D seismic data is used to estimate the spatial distribution of fracture parameters (e.g., fracture density, spacing, orientation). A model is used to convert such fracture parameters into permeability values, which are input to reservoir simulators for history matching.

III. Results and Discussion - Fracture properties from seismic data

During the past six month period we have continued to adapt and test numerical codes for modeling seismic scattering from fractures, and to refine spectral analysis methods for estimating fracture corridor spacing in reservoirs. Additional tests of the accuracy and stability of the 3-D finite difference modeling scheme were carried out. Following these tests we generated results for a model of three horizontal layers with the middle layer containing parallel, equally spaced fractures. FK analysis for fracture spacing estimation was tested on these model results, with particular focus on PS converted waves. The FK and the spectral notch (Willis et al., 2005) methods were then tested on data from the Emilio Field. Both methods generated similar fracture spacing estimates.

III.A Numerical modeling

The mechanical properties of fractures are much weaker than the surrounding background media, resulting in high contrasts between the elastic properties of the rock matrix and fracture zones. Standard-staggered-grid (SSG) finite difference schemes, even with 4th order accuracy in space and 2nd order accuracy in time, may generate artificial numerical noise or become unstable when very high contrast interfaces are present (Nihei et al., 2004). This is because the SSG needs to average the shear moduli of cells around the high contrast interface at each time step. To address this issue we use the rotated-staggered-grid method (RSG) (Saenger et al., 2000), which is 2nd order accurate both in space and time. To illustrate, Figure 2 compares the snapshots of seismic wave propagation calculated by each of these two schemes for the same model: a single large fracture filled with gas embedded in a homogeneous and isotropic medium. Numerical noise is present when the SSG scheme is used, but not with the RSG scheme. Additional tests of the accuracy of the RSG method, as well as its implementation, are given in the Appendix.

Figure 2 Wavefront snapshots of seismic wave propagation calculated by the standard staggered grid (SSG) scheme (left) and rotated staggered grid scheme (right) for the same model: a single large fracture filled with gas embedded in a homogeneous and isotropic medium. Numerical noise is present along the fracture when the SSG scheme is used, but not with the RSG scheme.

III.B Fracture spacing estimation from FK analysis of backscattered energy

Fracture spacing is an important parameter in reservoir development. As described in our previous report, we have developed two approaches for estimating fracture spacing from scattered seismic energy: spectral notch analysis and FK analysis. We have continued to develop the FK method by investigating the use of PS converted waves for fracture spacing estimation.

In the FK domain it is relatively easy to identify (and therefore separate) events with different apparent velocities in the seismic data. In the numerical model results we observe the presence of both forward and backscattered signals. The backscattered waves (that is, the energy propagating back towards the source) are maximized when the acquisition direction is normal to the fractures and minimized when the direction is parallel to the fractures. Since these signals are generated at the fractures, they contain information related to the spacing of the fracture corridors. By separating and analyzing the backscattered signals in the FK domain, we can obtain an estimate of the fracture spacing. As described in our previous report, the fracture spacing is estimated by taking

one-half of the reciprocal of the dominant wavenumber of the backscattered energy in data acquired normal to the fractures. Specifically, the fracture spacing (d) is given by:

$$d = \frac{1}{2 \cdot |k|} = \frac{v_a}{2 \cdot f} = \frac{\lambda_a}{2}$$

Local wavefield decomposition (LWD) is used to extract the coherent backscattered energy from the data window. Details on the overall approach are provided in the Appendix.

Figure 3 shows an example of the process applied to the numerical model data for a case with fracture spacing of 30m. In this case a window is chosen to analyze the PP scattered energy in the reservoir interval. The resulting spacing estimate is 31.8m. Another option is to analyze the PS energy that arrives later in the section. In Figure 4 we extract data from two different time windows and look at the divergence (that is, PP energy) and curl (PS energy) of the wavefield. We can see that PS energy is present at later times. Because S wave arrivals have shorter wavelengths (due to lower propagation velocity) than P waves, these PS arrivals have the added benefit of higher resolution and therefore the ability to provide estimates of finer scale fracture spacing. Figure 5 shows the results of using the PS energy to estimate fracture spacing in such a case (20m fracture spacing).

Figure 3 Determination of fracture spacing in f-k domain for model with 30 m spacing. (a) Seismogram of vertical component at near offset for direction perpendicular to fracture strike. Red window contains PP scattered energy between PP reflected at top and bottom interfaces of fractured reservoir; (b) Seismogram of energy in red window after being zoomed in (6 times scale as in (a)). (c) Normalized energy distribution in dip value p after applying LWD method on seismogram in (b); (d) Reconstructed seismic data by using profile with energy above 80% in (c), which is indicated by the red line; (e) Result of f-k transformation of reconstructed data in (d). The value of (k, f) of maximum energy is $(-0.0157 \text{ 1/m}, 78 \text{ Hz})$. The estimated spacing is about 31.8 m.

Figure 4 Divergence and curl energy in two successive time windows for model with 20 m fracture spacing. (a) Seismogram of vertical component at near offset in the direction perpendicular to fracture strike. Windows indicated in red and blue are chosen whose divergence and curl energy are compared; (b) Divergence energy contained in the red window; (c) Curl energy contained in the red window at the same scale as in (b); (d) Divergence energy contained in the blue window; (e) Curl energy contained in the blue window at the same scale as in (d).

Figure 5 Determination of fracture spacing from the f-k domain for the model with 20 m fracture spacing. (a) Seismogram of vertical component at near offset in the direction perpendicular to fracture strike. Blue window contains PS scattered energy between the PS reflected at top and bottom interfaces of the fractured reservoir; (b) Seismogram of energy in the blue window after being zoomed in (3 times scale as in (a)). (c) Normalized energy distribution in dip value p after applying LWD method on seismogram in (b); (d) Reconstructed seismic data by using profile with energy above 80% in (c), which is indicated by the red line; (e) Result of f-k transformation of reconstructed data in (d). The value of (k, f) of maximum energy is $(-0.0252 \text{ 1/m}, 84.7 \text{ Hz})$. The estimated fracture spacing is about 19.8m.

III.C Field data results

We continue to work on two field data sets from fractured carbonate reservoirs provided by our industry sponsors—the offshore Emilio Field data (provided by ENI-AGIP), and an onshore reservoir from the Middle East (provided by Shell). Here we present briefly some new results on fracture spacing estimation from the Emilio data.

The Emilio Field is a fractured carbonate reservoir located in the Adriatic Sea in about 80m of water. The field is an area of complex folding and faulting at a depth of approximately 2800m. A high quality 3D/4C seismic survey was acquired using ocean bottom cables (OBC). Scattered wave analyses of these data have been presented in previous reports and papers (e.g., Willis et al., 2004a, b, 2005, 2006). We have completed some additional analyses on fracture spacing estimation from FK analysis and spectral notch analysis (Willis et al, 2005; Rao et al., 2005).

III.C.1 FK Analysis Results

Sorting of the Emilio data into profiles perpendicular and parallel to the dominant fracture direction show effects similar to those we have seen in numerical model results (Figure 6). In particular, the data perpendicular to the fractures display less coherence and more backscattered energy than the data parallel to fractures. In Figure 7 we extract data from four different time windows – two above the fractured reservoir interval and two within or below the fractured interval. Significant backscattered energy is identified from the negative wavenumber portion of the FK plot in the bottom two windows that are at and below the fractured reservoir. The estimated fracture spacing from the dominant wavenumber values in these windows is 25-40m.

Figure 6 Azimuthal variation of scattered energy from discrete fracture zones in Emilio field data. (a) Seismic profile in the direction perpendicular to fracture strike; (b) Seismic profile in the direction parallel to fracture strike.

Figure 7 Energy distribution in f-k domain at different time windows for Emilio Field Data shown in Figure 12c. Four successive time windows with time duration 0.25 s are used.

III.C.2 Spectral Notch Results

The interference of forward and backscattered energy from fractures introduces notches in the frequency spectra of the scattered wavefield for data acquired normal to

the fracture strike (Willis et al., 2005). The frequency of these notches is related to the spacing of the fractures (Willis et al., 2005). Details of the method are given in our previous report. This approach was applied to the Emilio data and a fracture spacing estimate was calculated for each CDP point in the field. The results are shown in Figure 8. The range of spacing estimates is 25-50m, with a dominant spacing of about 30-40 m. These values are consistent with the estimates obtained by the FK method.

Figure 8 Scattered wavefield analysis of 3-D seismic data volume in a fractured carbonate reservoir (Emilio Field). The left panel shows the seismic faults and well locations. The right panel shows the spatial distribution of fracture density and orientation (the direction of each line segment gives the orientation estimate). The center panel shows the estimated spacing of fracture corridors, with the spacing values given by the color bar on the far right.

IV. Technology Transfer

We have been actively presenting our results to industry through presentations and publications. During the past six months papers were presented at the Society of Exploration Geophysicists annual meeting in Houston. Presentations have also been made at oil and gas producing companies in Houston and Dallas.

V. Conclusions

Our methods for estimating fracture orientation, fracture spacing, and fracture density from the seismic scattering signals have been tested with numerical modeling data and field data from two different fields. The fracture orientation results have been validated against well log measurements (FMI logs), other seismic analysis methods, and existing geologic models of the fields. Two different fracture spacing estimation methods (FK and Spectral Notch) give similar results at the Emilio Field.

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APPENDIX A

Characterization of Discrete Fracture Zones in Reservoir with Seismic Scattering Energy

Yang Zhang

Abstract

Contrary to the assumption of effective media theory that cracks evenly distribute in the subsurface with length scales much smaller than seismic wavelength, cracks usually cluster along fracture zones in real geological situations with scales comparable to or larger than seismic wavelength. These large fracture zones form discrete features with a preferred orientation because of the nature of the stress field in the subsurface of the Earth. Such large parallel features will induce some frequency-dependent phenomena, the seismic scattering off discrete fractures, when a seismic wave propagates through this kind of fractured media. To study the seismic responses to discrete fracture zones we use a 3-D finite difference method with a rotated-staggered-grid (RSG) scheme, which is good at handling media with high contrast like fractures. We only consider the vertical component of seismic data in this paper. In time-space domain we analyze the azimuthal variation of scattered energy from the fracture strike direction to the direction perpendicular to the fractures. Models show variations of coherence of seismic events and interference between backward and forward scattered energy. Backscattered energy at near offset for a shot-gather oriented perpendicular to fracture strike shows that PS converted energy has larger amplitude, shorter wavelength, and higher signal-to-noise-ratio (SNR) than PP scattered energy, and therefore can be used to illuminate the finer structure of fracture zones. By using the Local Wavefield Decomposition (LWD) method, we extract the backscattered energy from fracture zones. Looking at the backscattered energy in the f-k domain, we conclude that: (1) for the fundamental mode,

the wavenumber value provides a good constraint on the estimate of spacing between discrete fractures, while the frequency is sensitive to the time window chosen (there is a frequency drop when the time window is moved from earlier to later time due to the attenuation); (2) when the wavelength of the dominant scattered energy approaches the fracture spacing, more modes are generated. The method is applied to data from the Emilio Field. Seismic scattering signatures are similar to the model results and can be used to estimate the fracture spacing in the field.

Introduction

In subsurface of the Earth's crust, fractures and faults are common and usually have preferred orientation due to the nature of the stress field. They control the mechanical properties of the solid material, affect the seismic wave propagation and impact the fluid flow and production of oil and gas in reservoir. As a result, any information about the fractures, such as density, orientation and properties, is of great interest. Significant efforts have been made to understand the physical behavior of fractures by theoretical and experimental research.

Fracture induced anisotropy and its effect on seismic wave propagation has been a primary method for characterizing fracture properties. Studies on effective representation of fractured media (Hudson, 1980, 1981; Schoenberg and Douma, 1988; Schoenberg and Sayers, 1995; Liu, et al., 2000; etc.) have been done to understand the seismic anisotropy. In these studies, fractures or cracks are assumed to be isolated and sparsely distributed in the media, with scales much shorter than seismic wavelength. Because the velocities of seismic waves propagating in fractured media are azimuth-dependent, so are the reflection amplitudes. AVOaz (amplitude variation with offset and azimuth) in 3D seismic data provides a means of characterizing the fractured reservoir by analyzing the PP reflected energy to obtain the information about fracture orientation, density and properties (Lynn et al., 1996; Shen, et al., 2002; Vetri, et al., 2003; Lynn, 2004). Besides using PP energy, the PS converted energy also can be used as a tool for fractured reservoir characterization by analyzing the shear wave splitting (Crampin 1985; Lynn et

al., 1996; Gaiser, et al., 2002). These two methods have become the most common techniques for seismic anisotropy analysis and fractured reservoir characterization in the oil industry.

Often, however, AVOaz analysis, which is based on effective media theories, does not adequately explain the magnitude of seismic anisotropy in field data (Lynn, 2004; Zhang, et al., 2005). People realize that in real geological situations, large-scale fracture corridors or fault zones are dominant discontinuities in the subsurface, and microscale cracks or fractures usually cluster onto these large fracture zones (Bai and Pollard, 2000; Liu, et al., 2000), and these fracture zones are distributed discretely on a macroscopic scale. Rather than an effective media model, a discrete fracture model should be considered to account for the large-scale fractures, which would better explain what is seen in field data (Lynn, 2004). Because the large fractures have scales comparable to seismic wavelengths, frequency-dependent behavior due to seismic wave scattering off the discrete fractures is expected. In experiments, the effects of single or multiple fractures on seismic waves have been studied (Pyrak-Nolte et al., 1990; Roy and Pyrak-Nolte, 1997; Nolte et al., 2000; Nakagawa et al., 2000, 2002; Pyrak-Nolte et al., 2000; Xian et al., 2001). The linear slip interface condition that describes the behavior of a seismic wave traveling through a single fracture has been verified by experimental observation (Pyrak-Nolte et al., 1990). Frequency-dependent seismic scattering phenomena such as the interface wave (Gu, et al., 1996, 1997; Roy and Pyrak-Nolte 1997; Pyrak-Nolte and Roy, 2000), the channel wave (Nihei et al., 1999; Xian et al., 2001), resonance behavior (Nolte et al., 2000) and seismic dispersion (Nakagawa, et al., 2000, 2002) in discrete fracture zones have been studied in laboratory experiments. Theoretical studies of these phenomena have also been undertaken. Based on these studies we expect seismic scattering off discrete fractures in 3D surface seismic surveys to provide a new way to characterize fractured reservoirs, because the amplitude and frequency content of scattered seismic energy carries information about fracture orientation and spacing. Numerical modeling of the seismic responses to discrete fractures has been completed by Nakawaga et al. (2003, 2004), Vlastos et al. (2003), and Willis et al. (2003, 2004a, b, 2005, 2006). Based on the analysis of the numerical results, methods to estimate the orientation and spacing of the

discrete fractures have been developed and applied to field data (Willis, et al., 2005, 2006; Grandi, et al., 2005).

The analysis of seismic scattering off discrete large fractures in surface seismic surveys is just at its beginning stage. Although such scattered energy is assumed to be of low amplitude and difficult to separate from the background noise, in this paper we show seismic signatures of this kind of scattered energy, both in time-space and frequency domains, in field data. This indicates that the existence of the scattered energy in field data should not be ignored. By using the finite difference method, we first construct 3D forward numerical models to understand the seismic scattering phenomena from discrete fractures. Detailed analysis of the backscattered energy from fractures at near offset on a shot gather in the direction perpendicular to the fracture strike is then completed. We then analyze field data from Emilio Oil Field and compare the results to the numerical model output.

Numerical Modeling Method

Representation of Discrete Fracture in Finite Difference Code

Fractures with finite length in the subsurface form nonwelded interfaces with mechanical properties weaker than the surrounding background medium. When a seismic wave propagates through fractures, the displacement across the fracture surface is discontinuous while the stress traction remains continuous. A linear slip interface condition that accounts for the effect of the nonwelded interface on seismic waves relates the discontinuous displacement and continuous stress traction by a fracture compliance matrix $\mathbf{Z}$ (Hudson, 1980; Pyrak-Nolte et al., 1990; Schoenberg and Sayers, 1995). In numerical modeling, first we need to choose a method that is able to handle complex strata, then we need to implement the linear slip displacement discontinuity condition of the fracture. To satisfy the first requirement, we choose the finite difference method in our modeling. The implementation of the linear slip interface condition in the finite

difference code is accomplished by using the effective media theory developed by Coates and Schoenberg (1995).

Fractures considered in our model are discrete and parallel with scales in spacing and length comparable to the seismic wavelength. In the implementation, fractures can be thought of as compliant zones that take up a single cell thickness in the finite difference grid, the properties of which are described by the anisotropic elastic constants of the fractures. As described by Schoenberg and Sayers (1995), the effective elastic constants can be obtained by calculating the compliance tensor that relates the average strain over a representative volume to the average stress components through the constitutive relation:

$$\varepsilon_{ij}^{cell} = s_{ijkl}^b \sigma_{kl} + \frac{1}{2L} ([u_i] n_j + [u_j] n_i) \quad (1)$$

ε_{ij}^{cell} is the average strain, σ_{kl} is the average stress, s_{ijkl}^b is the compliance tensor of the background media, $[u]$ is the discontinuous displacement across fracture surface, n is the normal direction to fracture surface and L is the length of the cell. Fracture properties described by compliance matrix tensor $\mathbf{Z}$ is taken into account by the linear slip interface condition.

$$[u_i] = \mathbf{Z}_{ij} \sigma_{jk} n_k \quad (2)$$

substituting this condition into the constitutive relation in (1), we obtain:

$$\varepsilon_{ij}^{cell} = s_{ijkl}^b \sigma_{kl} + \frac{1}{4L} (\mathbf{Z}_{ik} n_l n_j + \mathbf{Z}_{jk} n_l n_i + \mathbf{Z}_{il} n_k n_j + \mathbf{Z}_{jl} n_k n_i) \sigma_{kl} \quad (3)$$

So inverting the compliance tensor can give the anisotropic elastic constants of the cell containing the fractures.

3D Finite Difference Modeling with Rotated-Staggered-Grid (RSG)

Materials within fracture zones are damaged so that the mechanical properties of fractures are much weaker than the surrounding background media. The contrast between them should be higher than that between two geological layers separated by a welded interface. The presence of fluids, such as gas or oil, within fractures contributes to this contrast.

Standard-staggered-grid (SSG) finite difference schemes, even with 4th order accuracy in space and 2nd order accuracy in time, may generate artificial numerical noise or become unstable when very high contrast interfaces are present (Nihei et al., 2004). This is because the SSG needs to average the shear moduli of cells around the high contrast interface at each time step. To accurately model the seismic responses from discrete fractures, we have chosen another finite difference scheme, the rotated-staggered-grid method (RSG) (Saenger et al., 2000), which is 2nd order accurate both in space and time (see Appendix A). To illustrate, Figure 1 compares the snapshots of seismic wave propagation calculated by each of these two schemes for the same model: a single large fracture filled with gas embedded in a homogeneous and isotropic medium. Artificial numerical noise is generated when the SSG scheme is used, but not with the RSG scheme.

Even though the RSG method can efficiently model the seismic wave propagating through fractures without generating artificial noise, we still need to make sure method is accurate. Figure 2 shows the model used for the accuracy test. A point source is excited at a shallow depth near the surface of a homogeneous, isotropic half space, with a receiver located 1km away from source along the surface. Figure 3 shows the comparison of the generated surface wave results from the RSG method and the discrete wavenumber method (DWN). The RSG results are almost identical to those from DWN verifying the accuracy of the RSG modeling. Besides implementation of RSG in our finite difference code, we also use perfect match layer (PML) absorbing boundary conditions to reduce the unwanted reflected waves from the boundaries of the model.

Numerical Modeling and Data Analysis

The model we use is a 3-layered model, where a fractured layer with parallel, discrete fracture zones is sandwiched between two homogeneous, isotropic layers. Source and receiver arrays are located at the surface. Figure 4 shows the schematic configuration of the model. The detailed modeling parameters are listed in Table 1. The finite difference code is 2nd order accuracy both in space and time.

Azimuthal Variation of Scattered Energy

Since the space and length scales of these discrete compliant fracture zones are comparable or larger than seismic wavelengths, seismic waves traveling through them can be scattered strongly, in both forward and backward directions. Figure 5 shows the seismic responses to the 3-layered model without (Figure 5a) and with (Figures 5b and 5c) discrete fractures in the middle layer (the direct waves are muted in both figures). Without discrete fractures (Figure 5a) all events are coherent and easily identified. With discrete fractures Figure 5b (normal to fractures) and 5c (parallel to fractures) show seismic scattered energy contaminating the events arriving at later times, especially for PS energy converted at the top and bottom of the reservoir. Strong scattered waves are the dominant energy in this case, and they show azimuthal variation. For data acquired in the direction perpendicular to fractures (Figure 5b), most events are not coherent. We can see prominent oscillations of the scattered energy due to the interference of forward and back scattered arrivals. But in the direction parallel to fractures (Figure 5c), most energy is coherent forward scattered arrivals with space-time behavior similar to the reflected waves top and bottom of the reservoir. The orientation of the vertical discrete parallel fractures in the reservoir can be estimated from these azimuthal variations of coherence and backscattered energy (Willis, et al., 2005, 2006; Grandi, et al., 2005).

Use P to S Converted Energy in Fracture Spacing Determination

In addition to knowing the orientation of fractures, the spacing between fracture corridors is also an important parameter impacting fluid mobility. The frequency-dependent seismic responses should contain this information. Grandi, et al. (2005) analyzed backscattered energy at near offsets of shot-gathers acquired in the direction perpendicular to fracture strike to estimate fracture spacing in the f-k domain. Figure 6 shows the results of using the same method on the numerical modeling data generated with 30m fracture spacing. In this case the method has been improved by using the Local Wavefield Decomposition (LWD) method (Sacchi, 2004; see Appendix B). Figure 6a shows a near offset shot gather perpendicular to fracture strike. The numerical data is windowed to pick out the scattered energy between the PP reflected energy from the top and bottom interfaces of the reservoir. The scattered energy in this window is magnified and shown in Figure 6b. LWD is then applied to capture the local coherent scattered energy. Figure 6c shows the normalized energy distribution of the decomposed seismic data at each dip value p . We can see that there are two main parts of the scattered energy corresponding to backscattered (negative dip values) and forward scattered (positive dip values) energy respectively. Energy above 80% at negative values of p is used to reconstruct a new seismic section as shown in Figure 6d. This new section represents the backscattered energy from the fractures. In the f-k domain the location of the backscattered energy can be used to estimate fracture spacing using the relation:

$$d = \frac{1}{2 \cdot |k|} = \frac{v_a}{2 \cdot f} = \frac{\lambda_a}{2} \quad (4)$$

The underlying theory behind this is to capture the fundamental mode information of the repeat pattern of the backscattered energy. In the fracture zone, because of seismic waves bouncing between fractures, there might be resonance occurring, which gives rise to the formation of modes. The modes in this paper refer to the repeat pattern of upcoming backscattered waves received at surface, which might be different from the modes in the fracture zone. Here d is the estimated fracture spacing, k and f are values of

wavenumber and frequency of fundamental mode, v_a is the apparent velocity and λ_a is apparent wavelength. As indicated in Figure 6e, the (k, f) values for the location of the maximum energy are $(-0.0157 \text{ 1/m}, 78 \text{ Hz})$. So the estimated fracture spacing by equation (4) is 31.8 m. The actual spacing between fractures zones in this model is 30 m.

In Figure 7a-7e, the same process is applied to data with 20 m fracture spacing. In this case there is almost no backscattered energy in this window because the frequency content of the seismic source is too low. Most energy locates at positive values of p , as shown in Figure 7c. For this case, however, if we move the window to later times to capture the scattered energy containing PS converted energy from the fracture zone we may be able to resolve the finer structure because of the shorter wavelengths of the shear waves. Figure 8 compares the divergence and curl energy computed in two windows, the previously used shallow (PP) window and a deeper (PS) window. We can see that in the shallow (red) window there is strong divergent energy (Figure 8b) but almost no curl energy (Figure 8c). In the deeper (blue) window, both divergent (Figure 8d) and curl (Figure 8e) energy are strong, the curl energy is especially larger at these later time. By taking advantage of PS energy arriving at later time, we might illuminate the finer structure of the reservoir because PS converted energy has smaller wavelengths than PP energy. Another issue we need to consider is the SNR (Signal to Noise Ratio) of the scattered energy. The SNR of backscattered energy in the deeper (blue) window is higher than that from the shallow (red) window. Strong PS converted waves can be generated at interfaces with high contrast (Toksöz et al., 2005), such as fracture zones, and it is therefore more advantageous to choose scattered energy arriving later in time.

Variation of Scattered Energy in F-K Domain with Moving Windows

The analysis of the previous section shows that where the window is located is a factor that might affect the results of our fracture spacing estimates. Since in real data analysis it is hard to know in advance where the window should be located to capture the PP or PS scattered energy, it is important to understand the sensitivity of the results to the time windows we choose.

To test the sensitivity four windows are chosen to cover the full time series of the model data with fracture spacings of 30 and 40 m. The time windows are: 0.41-0.46 s, 0.5-0.6 s, 0.6-0.7 s and 0.7-0.8 s. Figure 10 shows the distributions of backscattered energy in the f-k domain after being processed with LWD method and F-K transformation as was done previously. For the 30m fracture spacing case, when the window is moved from earlier to later times the location of the backscattered energy in the f-k domain migrates to lower frequencies but with almost the same wavenumber value. For the 40m fracture spacing case, however, when the window is moved to later times the backscattered energy in the f-k domain splits into two maxima. Because the data were filtered with the LWD operators at certain dips, these two peaks are almost colinear on a line passing through the origin of f-k plot. The peaks with lower and higher frequency content correspond to the fundamental and higher modes of the scattered energy respectively. The fundamental mode in this case follows the same trend seen for the case with 30 m spacing. The higher mode generated might be a resonance effect of the PS energy between fractures. It should be noted that when the time window is moved down from the earliest (0.41-0.46 s, which contains only PP energy) to the latest (0.7-0.8s, where PS energy dominates), the seismic wavelengths that are analyzed become shorter because of the PS converted energy. Therefore, PS converted energy bouncing back and forth within discrete fracture zones will generate higher modes than PP energy does. Once these different modes propagate out of the fractured layer they will interfere with each other, which contributes to the repeat pattern received at surface. For a given discrete fracture zone, the fracture spacing estimation should not be too sensitive to the time window chosen because the wavenumber value of the backscattered energy is quite stable compared to the frequency values.

Field Data Analysis

4.1 Scattered Energy of Emilio Field Data in Time-Space Domain

Numerical modeling is a useful tool for helping illustrate and predict seismic scattering from fracture corridors. Others have also used numerical models for the same purpose (Nakawaga, et al., 2003, 2004; Vlastos, et al., 2003). More recently numerical results have been used to develop analysis methods that have been applied to field data to estimate fracture orientation and spacing (Willis, et al., 2004, 2005, 2006; Grandi, et al., 2005). By applying these f-k methods to data from the Emilio Field we first show the existence of the scattered energy by comparisons with the numerical model results in both time-space and frequency domain, and then generate estimates of the fracture spacing.

Emilio Field is a fractured carbonate reservoir at a depth of 2850 m. Two sets of fractures with orthogonal orientation have been interpreted by traditional PP and PS anisotropic analyses (Gaiser, et al., 2002; Vetri, et al., 2003), and scattered wave analysis (Willis et al., 2003, 2004, 2005, 2006). Figure 11 shows two near offset seismic shot-gathers (vertical component) from an area with one set of fractures (orientated NNW) taken in the directions perpendicular and parallel to the fracture strike (Grandi et al., 2005). A strong reflected event above the reservoir at about 1.9 s can be seen in both profiles. Comparing these profiles with those in Figure 5, we can see some similarities in the seismic phenomena between real data and numerical results, such as the variation of the coherence of energy from the direction perpendicular to fracture strike to the parallel direction. In the profile perpendicular to fracture strike (Figure 11, left panel), energy within 2.0-2.4 s is quite incoherent and few events can be identified. But in the profile parallel to fracture strike (Figure 11, right panel), events become much more coherent and easily identified (for example, around 2.1 s where strong coherent events can be seen). This is consistent with what was seen in the numerical modeling results, as shown in Figures 5b and 5c.

The comparison between the real data and numerical results in the time-space domain shows us that it is reasonable to model such reservoir fractures as vertical discrete parallel zones and that the seismic scattered energy from such fractures zones is strong enough to be seen in the real data. Next we will apply the fracture spacing estimation methods to the field data.

4.2 Variation of Backscattered Energy of Emilio Data in F-K Domain

When processing real data with f-k analysis we always encounter the problem of irregular spacing between receivers. A shot gather from Emilio data in the direction perpendicular to fracture strike (Figure 12a) shows that the traces at near offset are distributed irregularly in space. This spacing needs to be regularized before f-k transformation. One way to regularize these data is to pad zeros between the traces, as shown in Figure 12b. Although such dead traces can help regularize data and avoid aliasing during f-k transformation, a better approach is to interpolate traces from nearby traces. The LWD method can accomplish this by capturing the local wavefield according to the local trend of coherence. The sampling matrix (see Appendix B) is a special controlling factor to deal with aliased data, which can interpolate traces not only with nearby traces but also by considering the coherence of local wavefield. The resulting reconstructed seismic data by LWD is shown in Figure 12c.

After data regularization, four time windows, each 0.25s in length, are applied successively on this seismic section. The actual time windows are 1.5-1.75 s, 1.75-2.0 s, 2.0-2.25 s and 2.25-2.5 s. F-k transformation is directly applied to the seismic section in each time window. The energy distribution in the f-k domain for each time window is shown in Figure 13. Backscattered energy is located on the left side of the f-k window (negative wavenumbers). We can see some features of the backscattered energy in the different windows: (1) the energy in the shallow window (the section above the major reflector) is smaller than that below both in forward and backward scattering; (2) backscattered energy becomes stronger when it enters into the fractured layers below the major reflector; (3) the frequency content of the backscattered energy drops with time

while the wavenumber values are the same, which can provide a useful constraint on the results of the spacing estimation. 4) the fracture spacing estimate based on the backscattered energy wavenumber values is 25-40m.

Discussion and Conclusions

For all the numerical cases we considered fractures with compliance values of $Z_N = Z_T = 2 \times 10^{-10}$ (m/Pa), which is a suitable value for gas-filled fractures. In addition to the fluid saturation, fracture compliance is also a function of the structure of the fractures, such as connectivity, population, size, clustering of cracks, fluid distribution and so on. Therefore, the fracture compliance we can measure will depend on what the scale of the experiment which is carried out. Usually the fracture compliance increases when measured at lower frequency ranges (Lubbe 2005). The fracture compliance values definitely will affect the seismic response to discrete fracture zones. More compliant fracture zones will generate higher amplitude scattering and higher attenuation as well. Our results show that the seismic scattering response to reservoirs with discrete fracture zones is measurable and a strong function of fracture orientation and spacing.

In this study, we only showed the case of evenly spaced fracture corridors. For cases with a non-uniform distribution (such as a Gaussian distribution), the estimated fracture spacing will be an average value as shown by Grandi, et al. (2005). The characteristics of such systems are consistent with those discussed in this paper.

The frequency drop we saw in the f-k domain is a more complicated question that needs to be addressed. In the numerical results, since the model is purely elastic without attenuation, the observed frequency drop is purely due to seismic scattering. This scattering attenuation only depends on the distribution and scale of the heterogeneities in model. The frequency drop observed in the real data (Figure 13), however, includes scattering and intrinsic attenuation. Discriminating between these two kinds of attenuation mechanisms will require additional study.

Our results show that when we model fractures as parallel, discrete features (rather than using an effective media model), the seismic response has many similarities with field data. The azimuthal variation of seismic scattered energy in data from the fractured Emilio Field is consistent with numerical results using such a model. In particular, strong multiple-scattered energy from fracture zones results in a complex wave pattern of interference that contaminates coherent arrivals. PS converted energy from the fractures has higher amplitude and shorter wavelength than PP scattered energy, which can illuminate finer scale fracture spacing. In the f-k domain, the wavenumber value of scattered fundamental mode can provide an estimate of fracture spacing. The frequency content, however, is quite sensitive to the chosen analysis window length, with the frequency values as the time window is moved from earlier to later times.

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Appendix A

Rotated Staggered Grid Finite Difference Method

In the seismic propagation modeling, the finite difference method with velocity-stress formulations is popularly used. The standard staggered grid (SSG) scheme usually used has 2nd order accuracy in time and 4th order accuracy in space, as shown in Figure A1a. The shear stress components are put a half-grid away from the normal stress components, as are the velocity components. When the velocity and stress components are updated at each time step, the average of densities and shear moduli of the nearby cells are taken. When there is a high property contrast between those cells, the numerical calculation gets unstable, which will generate artificial numerical noise or sometimes even blow up at the high contrast interface. This problem becomes serious when we model the seismic response to fractures because fractures are usually weaker areas with high contrast to the background media. In our numerical modeling we choose a new finite difference scheme, the rotated-staggered-grid (RSG) (Saenger, et al., 2000), as shown in Figure A1b. This scheme puts all the stress components at the center of a cell and all velocity components at the corners. In this method the shear moduli and density are at the same grid positions as the stress components, so there is no need to average the shear moduli during the calculation.

As shown in Figure A1b in two dimensions, RSG needs first to calculate the derivatives in the diagonal directions ($\tilde{x}$ and $\tilde{z}$) within each cell, and then rotate them back to the directions x and z . In a 2D rectangular cell with length Δx and Δz , the transformation between these two sets of directions are:

$$\tilde{x} = \frac{\Delta x}{\Delta r} x - \frac{\Delta z}{\Delta r} z \quad (\text{A-1})$$

$$\tilde{z} = \frac{\Delta x}{\Delta r} x + \frac{\Delta z}{\Delta r} z \quad (\text{A-2})$$

$$\Delta r = \sqrt{\Delta x^2 + \Delta z^2}$$

And the derivatives in the directions $\mathbf{x}$ and $\mathbf{z}$ can be expressed as derivatives in the directions $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{z}}$ as follows:

$$\frac{\partial}{\partial x} = \frac{\Delta r}{2\Delta x} \left(\frac{\partial}{\partial \tilde{z}} + \frac{\partial}{\partial \tilde{x}} \right) \quad (\text{A-3})$$

$$\frac{\partial}{\partial z} = \frac{\Delta r}{2\Delta z} \left(\frac{\partial}{\partial \tilde{z}} - \frac{\partial}{\partial \tilde{x}} \right) \quad (\text{A-4})$$

The differentiation operators $D_{\tilde{x}}$ and $D_{\tilde{z}}$ for the derivatives $\frac{\partial}{\partial \tilde{x}}$ and $\frac{\partial}{\partial \tilde{z}}$ in the new directions for 2nd order accuracy in space are:

$$D_{\tilde{x}} u(x, z, t) = \frac{1}{\Delta r} \left[u\left(x + \frac{\Delta x}{2}, z - \frac{\Delta z}{2}, t\right) - u\left(x - \frac{\Delta x}{2}, z + \frac{\Delta z}{2}, t\right) \right] \quad (\text{A-5})$$

$$D_{\tilde{z}} u(x, z, t) = \frac{1}{\Delta r} \left[u\left(x + \frac{\Delta x}{2}, z + \frac{\Delta z}{2}, t\right) - u\left(x - \frac{\Delta x}{2}, z - \frac{\Delta z}{2}, t\right) \right] \quad (\text{A-6})$$

Then by using (A-3) and (A-4), we can get the spatial derivatives in directions $\mathbf{x}$ and $\mathbf{z}$:

$$\frac{\partial}{\partial x} u(x, z, t) \approx \frac{\Delta r}{2\Delta x} (D_{\tilde{z}} u(x, z, t) + D_{\tilde{x}} u(x, z, t)) \quad (\text{A-7})$$

$$\frac{\partial}{\partial z} u(x, z, t) \approx \frac{\Delta r}{2\Delta z} (D_{\tilde{z}} u(x, z, t) - D_{\tilde{x}} u(x, z, t)) \quad (\text{A-8})$$

For the 3-D case in a cell with length Δx , Δy and Δz , as shown in Figure A2, we have the new derivative directions:

$$\tilde{\mathbf{d}}_1 = \frac{\Delta x}{\Delta l} \mathbf{x} + \frac{\Delta y}{\Delta l} \mathbf{y} + \frac{\Delta z}{\Delta l} \mathbf{z} \quad (\text{A-9})$$

$$\tilde{\mathbf{d}}_2 = \frac{\Delta x}{\Delta l} \mathbf{x} + \frac{\Delta y}{\Delta l} \mathbf{y} - \frac{\Delta z}{\Delta l} \mathbf{z} \quad (\text{A-10})$$

$$\tilde{\mathbf{d}}_3 = \frac{\Delta x}{\Delta l} \mathbf{x} - \frac{\Delta y}{\Delta l} \mathbf{y} + \frac{\Delta z}{\Delta l} \mathbf{z} \quad (\text{A-11})$$

$$\tilde{\mathbf{d}}_4 = \frac{\Delta x}{\Delta l} \mathbf{x} - \frac{\Delta y}{\Delta l} \mathbf{y} - \frac{\Delta z}{\Delta l} \mathbf{z} \quad (\text{A-12})$$

$$\Delta l = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

As in the 2-D case, the differentiation operators for spatial derivatives are:

$$\frac{\partial}{\partial x} u(x, y, z, t) \approx \frac{\Delta l}{4\Delta x} (D_{\tilde{d}_1} + D_{\tilde{d}_2} + D_{\tilde{d}_3} + D_{\tilde{d}_4}) u(x, y, z, t) \quad (\text{A-13})$$

$$\frac{\partial}{\partial y} u(x, y, z, t) \approx \frac{\Delta l}{4\Delta y} (D_{\tilde{d}_1} + D_{\tilde{d}_2} - D_{\tilde{d}_3} - D_{\tilde{d}_4}) u(x, y, z, t) \quad (\text{A-14})$$

$$\frac{\partial}{\partial z} u(x, y, z, t) \approx \frac{\Delta l}{4\Delta z} (D_{\tilde{d}_1} - D_{\tilde{d}_2} + D_{\tilde{d}_3} - D_{\tilde{d}_4}) u(x, y, z, t) \quad (\text{A-15})$$

Appendix B

Local Wavefield Decomposition

The Radon transform is a powerful technique used in data processing to extract coherent or incoherent signals and filter unwanted events. However, no matter what integration path the Radon transform uses, parabolic or hyperbolic, it has problems of resolution when the integration path through the whole data space mismatches with the seismic events in space and time. A solution to this problem is to use a local wavefield operator to capture the data for only a small data aperture.

Scattered waves from discrete fracture zones is not a totally random wavefield such as noise. Its moveout and curvature are different from the primaries and even multiples, but it shows coherence for waves at local scales. Because of these features of scattered energy, it is problematic to use this local Radon transform on data to extract the scattered energy.

An alternative solution is to use the Local Wavefield Decomposition (LWD) method (Sacchi, 2004). It is based on a generalized convolution, which allows use of any class of integration path in the Radon transform. The integration path here is referred to as the Local Wavefield Operator (LWO) with the formulation:

$$\hat{b}(\omega, x, p) = \hat{s}(\omega)h(x)e^{-i\omega\phi(p,x)}, \quad -a \leq x \leq a \quad (\text{B-1})$$

Here a defines the half-length of the operator; $h(x)$ is spatial taper function (usually a Hamming window is used); $\hat{s}(\omega)$ is the Fourier transform of the source wavelet; and $\phi(p, x)$ is the parametric integration path defining the local wavefield. For example, we can define the local operator by choosing $\phi(p, x)$. $\phi(p, x) = px$ defines a local linear operator and $\phi(p, x) = px^2$ defines a local parabolic operator. $b(t, x, p) = F^{-1}[\hat{b}(\omega, x, p)]$ is the inverse Fourier transformation of (B-1).

To apply the operator $b(t, x, p)$ on data, we need a scaled and shifted version like $Ab(t - t_0, x - x_0, p)$, where A is an amplifying factor. In general, for each LWO with a given dip value p , at every (t_0, x_0) there is an amplifying factor $f(t_0, x_0)$ corresponding to it. So we can consider reconstructing a new 2D data slice by the combination of $f(t_0, x_0)$ and $b(t - t_0, x - x_0, p)$ as:

$$\sum_{t_0} \sum_{x_0} f(t_0, x_0) b(t - t_0, x - x_0, p) \quad (\text{B-2})$$

Usually seismic data $d(t, x)$ is composed of the superposition of seismic events described by many LWOs. So we can generalize this idea by the following expression:

$$d(t, x) = \sum_p \sum_{t_0} \sum_{x_0} f(t_0, x_0) b(t - t_0, x - x_0, p) \quad (\text{B-3})$$

or in matrix form as:

$$D = \sum_p \mathbf{F}(p) \otimes \mathbf{B}(p) \quad (\text{B-4})$$

This defines a generalized convolution that describes a seismic data decomposition process with temporal and spatial invariant operators. It represents a transformation from Local Wavefield Decomposition domain $\mathbf{F}(p)$ to the data space $\mathbf{D}$. The only task to do here is to estimate $\mathbf{F}(p)$ from the data $\mathbf{D}$. To accomplish this, we can use an inversion process to minimize the cost function:

$$J = \left\| \mathbf{D} - \sum_p \mathbf{F}(p) \otimes \mathbf{B}(p) \right\| + R(\mathbf{F}) \quad (\text{B-5})$$

Here $\mathbf{W}$ is the sampling matrix that is useful for handling aliased data, and $R(\mathbf{F})$ is the regularization term. The best way to obtain the optimal solution is to use the conjugate gradient method.

Figure 1a: Snapshot from SSG. Numerical noise is generated

Figure 1b: Snapshot from RSG. No numerical noise is generated

Figure 2: Surface wave modeling as accuracy test for RSG method.

Figure 3: Comparison of results of surface wave modeling from RSG (red) and DWN (blue). (a) V_x component; (b) V_z components.

Table 1: Modeling parameters for 3D-layered fractured reservoir.

	V _p (m/s)	V _s (m/s)	Density (kg/m ³)	Nominal Wavelength (m)	
				P	S
Top Layer	2460	1230	2300	60	30
Fractured Layer*	3300	1800	2200	80	45
Bottom Layer	2460	1230	2300	60	30
Fracture Compliance	$Z_N = Z_T = 2 \times 10^{-10}$ (m/Pa)				
Source	Point Source, Ricker Wavelet, $f_0 = 40$ (Hz)				
Fracture spacing	20m, 30m, 40m

Figure 4: Schematic configuration of 3-layered model. A fractured layer with vertical and parallel large fracture corridors is embedded between two isotropic and homogeneous layers. 3-D seismic data is collected at the surface. The source is indicated with a red star and receiver arrays are indicated with black dots.

Figure 5: Comparison of seismograms of the vertical component between 3-layered models without and with discrete fractures. (a) Seismogram of homogeneous, isotropic model without discrete fractures; (b) Seismogram of the model with discrete fractures (30 m spacing case) in the direction perpendicular to the fracture strike; (c) Seismogram of the model with discrete fractures in the direction parallel to the fracture strike.

Figure 6 Determination of fracture spacing in f-k domain for model with 30 m spacing. (a) Seismogram of vertical component at near offset for direction perpendicular to fracture strike. Red window contains PP scattered energy between PP reflected at top and bottom interfaces of fractured reservoir; (b) Seismogram of energy in red window after being zoomed in (6 times scale as in (a)). (c) Normalized energy distribution in dip value p after applying LWD method on seismogram in (b); (d) Reconstructed seismic data by using profile with energy above 80% in (c), which is indicated by the red line; (e) Result of f-k transformation of reconstructed data in (d). The value of (k, f) of maximum energy is $(-0.0157 \text{ 1/m}, 78 \text{ Hz})$. The estimated spacing is about 31.8 m.

Figure 7: Determination of fracture space in f-k domain for model with 20 m spacing. (a) Seismogram of vertical component at near offset for direction perpendicular to fracture strike. Red window contains PP scattered energy between PP reflected at top and bottom interfaces of fractured reservoir; (b) Seismogram of energy in red window after being zoomed in (26 times scale as in (a)). (c) Normalized energy distribution in dip value p after applying LWD method on seismogram in (b); (d) Reconstructed seismic data by using profile with energy above 80% in (c), which is indicated by the red line; (e) Result of f-k transformation of reconstructed data in (d). Forward scattered energy dominates the total energy. No backscattered energy can be seen in the f-k domain.

Figure 8: Divergence and curl energy in two successive time windows for model with 20 m fracture spacing. (a) Seismogram of vertical component at near offset in the direction perpendicular to fracture strike. Windows indicated in red and blue are chosen whose divergence and curl energy are compared; (b) Divergence energy contained in the red window; (c) Curl energy contained in the red window at the same scale as in (b); (d) Divergence energy contained in the blue window; (e) Curl energy contained in the blue window at the same scale as in (d).

Figure 9: Determination of fracture spacing from the f-k domain for the model with 20 m fracture spacing. (a) Seismogram of vertical component at near offset in the direction perpendicular to fracture strike. Blue window contains PS scattered energy between the PS reflected at top and bottom interfaces of the fractured reservoir; (b) Seismogram of energy in the blue window after being zoomed in (3 times scale as in (a)). (c) Normalized energy distribution in dip value p after applying LWD method on seismogram in (b); (d) Reconstructed seismic data by using profile with energy above 80% in (c), which is indicated by the red line; (e) Result of f-k transformation of reconstructed data in (d). The value of (k, f) of maximum energy is $(-0.0252 \text{ 1/m}, 84.7 \text{ Hz})$. The estimated fracture spacing is about 19.8m.

Figure 10: Distribution of backscattered energy in f-k domain with moving windows for cases with fracture spacing of 30 m and 40 m.

Figure 11: Azimuthal variation of scattered energy from discrete fracture zones in Emilio field data. (a) Seismic profile in the direction perpendicular to fracture strike; (b) Seismic profile in the direction parallel to fracture strike.

Figure 12: Regularization of Emilio Field Data by LWD. (a) One section of the shot gather at near offsets in the direction perpendicular to fractures, which shows irregular spacing of traces; (b) Regularization of data by just padding zeros between traces; (c) Reconstructed data with LWD by capturing the local scattered wavefield.

Figure 13: Energy distribution in f-k domain at different time windows for Emilio Field Data shown in Figure 12c. Four successive time windows with time duration 0.25 s are used.

Figure A1: Comparison of finite difference schemes of SSG and RSG in velocity-stress formulation. (a) Elementary cell of SSG scheme; (b) Elementary cell of RSG scheme.

Figure A2: New four directions for derivatives in 3D RSG scheme.

APPENDIX B

Fracture Spacing and Orientation Estimation from Spectral Analyses of Azimuth Stacks

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Summary

Discrete, vertically aligned fracture systems impart one or more notches in the spectral ratios of stacked reflected seismic traces. This apparent attenuation is due to the azimuth dependant scattering introduced by the fractures. The most prominent notch is located at the frequency where the P wavelength is about twice the fracture spacing. The frequency location of the notches can be used to determine the fracture spacings. Azimuth stacks with an orientation parallel to the fractures tend not show these spectral notches – allowing for another way to detect the fracture orientation. An analysis of the vertical component of the 3D, ocean bottom cable seismic survey data from the Emilio field, offshore Italy, shows a prominent set of fractures with a spacing of about 30 to 40 meters with orientations that agree with previous studies.

Introduction

There is a large body of the literature devoted to analyzing rocks with fracture sizes and spacings which are much less than one wavelength. For these systems the rock will behave elastically as an effective anisotropic medium displaying the aggregate properties of a distribution of small fractures. Lynn (2004a) gives a good summary of the differences between effective medium and discrete fractures.

Much of our recent work has been devoted to deriving fracture properties from vertical sets of discrete fractures which have dimensions and spacings on the order of the seismic wavelength (e.g. Willis et al, 2004a and 2004b). This study builds upon our recent work looking at spectral characteristics of azimuth stacks from modeled fractured systems (Willis and Rao, 2005). In this paper we apply and expand these methods to seismic field data.

Background Model Study – the Spectral Notch Method

In Willis and Rao (2005) we analyzed the surface seismic reflection traces created from a 3D, anisotropic finite difference algorithm. The models consisted of a sandwich of five layers. The center layer is an isotropic reservoir containing discrete sets of gas-filled, vertical fractures represented by vertical planes made up of a single grid cell, with equivalent anisotropic medium properties determined using the method of Coates and Schoenberg (1995). We studied models with several different fracture spacings.

We created azimuth stacks of the shot records at 10 degree increments from normal to parallel to the fractures. We found that scattered energy from the fractures is reinforced on the stacks parallel to the fractures and is reduced in the direction normal to the fractures.

From these azimuth stacks we created their associated transfer functions, as described by Willis et al. (2004). Briefly, this process entails for each azimuth stack trace: 1) identifying a target depth range (the fractured layer) to investigate 2) extracting a window of data containing reflections which are above this fractured layer, 3) extracting a window of data below the fractured layer, containing reflections which includes the scattered energy, 4) taking the autocorrelation of the extracted windows, and 5) computing the spectra ratio of the lower and upper window autocorrelations.

We then converted the frequency axis to wavelength using the average P-wave velocity in the depth range of investigation. Figure 1 shows the modeled average spectrum for the 50m fracture spacing. The vertical axis is amplitude spectrum displayed in dB.

Figure 1. Spectral ratios for the 50m fracture spacing model showing notches at 100m and 200m wavelengths, corresponding to twice and four times the fracture spacing.

Clearly evident on this spectrum are notches (i.e. zeros or low amplitudes) at 100m and 200m, corresponding to twice and four times the fracture spacing of 50m. These distances correspond to particular frequencies in the source function which happen to cancel out due to the time delay of a P wave traveling between two fractures and creating a null. The notch at about twice the fracture spacing is characteristic of all the models studied. However, not all models showed the notch at four times the fracture spacing.

Further, azimuths within 50 degrees of the normal to the fracture strike were used in computing the average spectral ratio. We excluded the spectral ratios in the remaining directions as the notch effect is reduced close to the fracture strike direction. This is due to the fractures acting like mechanical polarizers, channeling energy away from the normal direction, towards the parallel direction. Hence, in the parallel direction, there is in fact an amplification, or a peak in the spectral ratios. Thus, this azimuth sensitive behavior of the spectral ratios also provides fracture orientation. Lynn (2004b) has noted

holes in the spectra shear components and attributed them to interference of backscattered energy.

Emilio Field Data Study

In 2000, a 3D/4C seismic survey was collected over the Emilio Field, located in the central part of the Adriatic Sea, near the eastern coast of Italy. The reservoir unit is a fractured carbonate with two orthogonal fracture sets oriented ENE and NNW (Angerer et al, 2002). This field has been investigated using PP and PS wave anisotropy to identify fracture characteristics of the reservoir level (Minsley et al, 2004; Vetri et al, 2003; Gaiser et al, 2002).

We apply the spectral notch method, described above for the model data, to analyze the seismic data from the Emilio field. We selected the near to mid range offsets of the preprocessed PP data (Vetri et al, 2003) and created eighteen different azimuth stack volumes from East to West using 20 degree wide overlapping ranges. We created two superbins to collect the stacked traces around wells 4 and 8. The size of each superbin was 11x11 bins (125m x 250m). For each of the 18 azimuth stacks, the traces which fell within the two bins were collected. For each of these traces, a time window above, and another window below the reservoir were extracted and their autocorrelations computed.

While the time domain transfer function could be computed as an intermediate step using Weiner deconvolution, we directly computed the spectral ratios by spectral division of the fast Fourier transforms of the autocorrelations. We then averaged the spectral ratio values for all the traces with the same azimuth stack direction within the same superbin.

It should be noted that the signal-to-noise level in the data will dictate the viable frequency bandwidth, which in combination with P velocity of the layers under consideration defines an observable range of wavelengths and consequently fracture spacings for a given data set. For the current data set, observable fracture spacings are expected to be in the range 20 to 120 m. The spectral ratio amplitudes are presented in Figure 2 and 3.

Fig 2 shows the spectral ratio amplitudes as a function of azimuth and wavelength for well 4 (left) and 8 (right). The vertical axis shows each of the 18 azimuths from 0 degrees East, 90 degrees North and 170 degrees nearly West.

Clearly evident on the figure is a significant azimuth variation of spectral ratio amplitudes. Further, prominent notches (blue) across a broad range of azimuths can be observed, particularly at fracture spacings of about 30m. From the model study, the notches are expected over a range of azimuths that are nearly perpendicular to the fracture direction. Additional features are the peaks (red) in both plots. Well 4 shows large amplitudes at about 70 and 120 degrees. Well 8 shows a band of large amplitudes at a range of azimuths from about 90 to 120 degrees. These larger amplitudes occur at spacings of 50 to 60 m, and are expected indicate directions that are close to the fracture

strike directions. Thus the peaks and troughs are expected over perpendicular azimuths if only a single, equally spaced fracture set is present.

Figure 2. Spectral ratio functions for well 4 (left) and well 8 (right). The vertical axis is the direction of the azimuth stack where 0 degrees is East and 90 degrees it North. The horizontal axis is wavelength/2, which is interpreted as fracture spacing

Figure 3 shows the median spectral ratio, averaging over azimuths, for wells 4 (left) and 8 (right). A linear trend in the spectral ratio amplitude, attributable to attenuation (decreasing amplitude with increasing frequency), was removed. This curve amounts to a projection of the contours in Fig 2 along the x-axis.

Figure 3. Averaged spectral ratios in the area around well 4 (left) and well 8 (right). The wavelength axis has been scaled by 2 so that fracture spacing may be read directly from the x-axis.

Troughs are evident in both plots, consistent with discrete fractures spacing of about 30 to 40 m. A second, weaker trough around 60 to 80 m is discernable, corresponding to a twice fracture spacing, along with the peak around 50 to 60 m.

Some of these results can be validated with prior studies on this data. Figure 4 shows the borehole derived measurements for horizontal stress directions from Vetri et al. (2003).

We've added the red arrow in the left rose diagram to highlight the direction of maximum horizontal stress, since the breakouts appear in the minimum horizontal stress direction. Generally fractures tend to align subparallel to the maximum horizontal stress direction.

Estimates of the aggregated fracture orientations using these volumes from the analysis of scattering indices are shown as rose plots in Figure 5, modified from Willis et al. (2004). The rose plots, which estimate the distribution of fractures in the area around well 4 (left) and well 8 (right), agree with the ENE and NNW directions derived in the studies mentioned above and shown in Figure 4.

For comparison, we derive a polar plot of the spectral ratio amplitudes for the bins corresponding to well 4 and 8 in Figure 6. Spectral ratio amplitudes are plotted versus angle, with the peak amplitudes identifying fracture directions. There is a reasonable consistency between the present results and prior work (Figs 4 and 5) in overall fracture directions. It should be noted that the three measures are at different scales: stress direction measurements in Fig 4 are essentially "point" measurements localized around a borehole, whereas the scattering index measures (Fig 5) are for an area covering about 0.5km x 1km while the spectral notch method measures are for an area of 0.125 km x .25 km.

Figure 4. Borehole Measurements: Stress measurements from borehole break outs in well 4 (left) and from natural fractures observed in the borehole in well 8 (right) (modified from Vetri et al, 2003). Vertical fractures tend to be aligned with the maximum horizontal stress direction.

Figure 5. Scattering Index: Rose diagrams showing fracture orientations in the area surrounding well 4 (left) and well 8 (right) from an analysis of scattering indices (Willis et al, 2004).

Figure 6. Spectral Notch Method: Rose diagrams showing fracture orientations using the method described in this paper, for the area surrounding well 4 (left) and well 8 (right).

Discussion and Conclusions

We have presented a spectral notch method for extracting fracture characteristics from surface seismic data. The data volume is subdivided into 18 azimuth stack volumes. Spectral ratios are computed using windows of seismic data above and below the target reservoir for all azimuth stacks. The azimuth variation of the spectral ratio values can be used to detect both the spacing and orientation of fractures in the reservoir. For azimuths about 50 degrees or closer to the direction normal to fracturing, we expect to see spectral ratio notches (or zeros) indicating the absorption of seismic energy. The notches are located at wavelengths which are two and four times the fracture spacing. We expect that there is overall more energy, and thus more amplitude, in the spectral ratio for azimuths parallel to the fracture strike. These trends are clearly evident on the finite difference derived model data.

We applied the spectral notch method to the PP data derived from the 3D/4C Emilio field data. We observed significant variation with azimuth of the spectral ratios over the entire survey. We frequently observed notches corresponding to fracture spacings of 30 to 40 meters over the survey. Fracture orientations were also extracted using this approach and they agreed with prior studies, including borehole breakout and natural fracture analyses. They also agreed with estimates of seismically derived fracture orientations from scattering indices. These three results are consistent with each other even though they were derived from measurements averaged at three different scales – point (well based), 0.04 km^2 (spectral ratio notches), and 2.5 km^2 (scattering index).

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