MASTER

APAE Memo No. 184

15

REC I & II, PREV I
PROGRAMS NO. 304, 30I, 30I A
REFLECTOR & PRESSURE VESSEL
COOLING & THERMAL STRESS ANALYSIS
ON THE IBM 650

ALCO PRODUCTS, INC.
POST OFFICE BOX 414
SCHENECTADY, N. Y.

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ON THE IBM 650

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ISSUED MARCH 26, 1959

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NOMENCLATURE

Symbol		Unit
a	inside radius	ft
b	outside radius	ft
D_A	equivalent diameter of inner water gap	ft
D_B	equivalent diameter of outer water gap	ft
E	modulus of elasticity	psi
h	heat transfer coefficient	Btu/ft ² hr °F
k	thermal conductivity	Btu/ft hr °F
P_r	Prandtl number	
Q_0	heat generation rate at inside surface	Btu/ft ³ hr
Q_x	heat generation rate at distance (x) from the inside surface	Btu/ft ³ hr
r	radius	ft
r_m	radius at which the maximum compressive stress occurs in a reflector	ft
T_w	bulk coolant temperature	°F
T_i	inside surface temperature of wall	°F
T_o	outside surface temperature of wall	°F
t	temperature at radius (r)	°F
u	heat generation rate attenuation coefficient	ft ⁻¹
V_A	velocity in inner water gap	ft/hr
V_B	velocity in outer water gap	ft/hr

W	thickness of reflector or vessel wall	ft
x	distance through the wall from the inner surface	ft
α	modulus of thermal expansion	$^{\circ}\text{F}^{-1}$
ρ	density	$\#/ft^3$
μ	viscosity	$\#/ft\text{-hr}$
ν	Poisson's ratio	
σ_{ta}	tangential thermal stress at inner wall surface	psi
σ_{tb}	tangential thermal stress at outer wall surface	psi
σ_{tm}	maximum compressive tangential thermal stress in reflector	psi

Subscripts

1, 2, 3, etc.	reflector no. 1, 2, 3 etc., no. 1 being the reflector furthest from the core.
A	water gap adjacent to inner surface of wall
B	water gap adjacent to outer surface of wall
a	inner surface of wall
b	outer surface of wall
i	inner surface of wall
o	outer surface of wall
s	properties of the reflector or vessel wall material
w	properties of bulk coolant

INTRODUCTION

These programs are intended to assist in establishing reflector cooling requirements, as well as thermal stresses in the reflectors and pressure vessel.

The main program (304 - Rec I) establishes required velocities between reflectors as a function of geometry, heat generation rates, material and fluid properties, and allowable surface temperature. This is done by iterating on velocities for successive water gaps. When required velocities have been established, corresponding thermal stresses are calculated.

Rec II (301) and Prev I (301A) are supplement programs. Rec II is utilized for establishing surface temperatures of a reflector as a function of velocities, geometry, heat generation rates, and material and fluid properties. It handles only one reflector per case. This program also calculates thermal stresses in the reflector.

Prev I is intended for calculations concerning a pressure vessel, perfectly insulated at the outer surface. It calculates surface temperature and thermal stresses in the vessel as a function of velocity, geometry, heat generation rates, and material and fluid properties.

All calculations are performed at the midplane, where maximum heat generation rates occur.

ANALYSIS

Following are the equations and calculation methods utilized by the three programs:

Velocity Determination

Starting with reflector #1:

The value of V_{B1}^* is the only input velocity. The corresponding heat transfer coefficient is calculated by:

$$(1) \quad h_{B1} = 0.021 \frac{k_w}{D_{B1}} \left(\frac{\rho_w V_{B1} D_{B1}}{\mu_w} \right)^{0.8} (Pr_w)^{0.4}$$

Assume:

$$V_{A1} = 1.5 V_{B1}$$

$$(2) \quad h_{A1(\text{assumed})} = 0.021 \frac{k_w}{D_{A1}} \left(\frac{\rho_w V_{A1} D_{A1}}{\mu_w} \right)^{0.8} (Pr)^{0.4}$$

* Under most circumstances, the value of V_{B1} will be determined by the cooling requirements of the pressure vessel.

Surface temperatures corresponding to V_{B1} and the assumed value of V_{A1} are calculated by the following equations: *

$$(3) \quad (T_o - T_i)_1 = \frac{\frac{Q_{o1}}{u_1^2 W_1} \left[\frac{1 - (1 + u_1 W_1) e^{-u_1 W_1}}{h_{B1}} + \frac{1 - u_1 W_1 - e^{-u_1 W_1}}{h_{A1}} \right]}{\left[1 + \frac{k_s}{W_1} \left(\frac{h_{A1} + h_{B1}}{h_{A1} h_{B1}} \right) \right]}$$

$$(4) \quad T_{i1} = T_w + \frac{Q_{o1}}{u_1^2 W_1 h_{A1}} (u_1 W_1 + e^{-u_1 W_1} - 1) + (T_o - T_i)_1 \frac{k_s}{W_1 h_{A1}}$$

$$(5) \quad T_{o1} = T_{i1} + (T_o - T_i)_1$$

If $(T_o - T_i)_1$ is positive, T_{o1} is matched against T_s allowable; if $(T_o - T_i)_1$ is negative, T_{i1} is matched against T_s allowable.

If T_{o1} or $T_{i1} > T_s$ allowable, V_{A1} is increased; if T_s allowable $> T_{o1}$ and T_{i1} , V_{A1} is decreased. The program iterates on V_{A1} until T_{o1} or T_{i1} , whichever is larger, matches T_s allowable within $\pm 1/40^\circ\text{F}$.

Calculation of Thermal Stresses

Thermal stresses in the reflector are calculated corresponding to V_{B1} and the final calculated value of V_{A1} .

The maximum tangential tensile stress ** in the reflector occurs at the inner or the outer surface:

$$(6) \quad \sigma_{\theta a_1} = \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left[\left\{ \frac{Q_{o1}}{k_s u_1^2} \right\} \left\{ 1 + \frac{2(u_1 b_1 + 1)e^{-u_1 W_1} - 2(u_1 a_1 + 1)}{u_1^2 (b_1^2 - a_1^2)} \right\} \right. \\ \left. + C_1 \left\{ \frac{2(b_1^3 - a_1^3)}{3(b_1^2 - a_1^2)} - a_1 \right\} \right]$$

where:

$$(7) \quad b_1 = a_1 + W_1$$

* Derivations are included in the Appendix.

** Tangential stresses only are calculated, since these are larger than the radial or axial stresses.

and

$$(8) \quad C_1 = \frac{(T_o - T_i)_1}{W_1} + \frac{Q_{o1}}{k_s u_1^2} \left(\frac{e^{-u_1 W_1} - 1}{W_1} \right)$$

$$(9) \quad \sigma_{t b_1} = \sigma_{t a_1} - \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left[\left(\frac{Q_{o1}}{k_s u_1^2} \right) (1 - e^{-u_1 W_1}) + C_1 (b_1 - a_1) \right]$$

$$\sigma_{t b_1} = \sigma_{t a_1} - \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] [T_o - T_i]_1$$

The maximum tangential compressive stress occurs within the reflector at a distance "m" from the inner surface:

$$(10) \quad m_1 = -\frac{1}{u_1} \ln \left[\frac{1 - e^{-u_1 W_1}}{u_1 W_1} - \frac{(T_o - T_i)_1}{W_1} \left(\frac{k_s u_1}{Q_{o1}} \right) \right]$$

$$(11) \quad r_{m_1} = a_1 + m_1$$

$$(12) \quad \sigma_{t m_1} = \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left[\left\{ \frac{Q_{o1}}{k_s u_1^2} \right\} \left\{ \frac{u_1^2 r_{m_1}^2 + u_1 r_{m_1} + 1}{u_1^2 r_{m_1}^2} e^{-u_1 (r_{m_1} - a_1)} \right. \right.$$

$$\left. - \frac{(u_1 a_1 + 1)(b_1^2 + r_{m_1}^2)}{u_1^2 r_{m_1}^2 (b_1^2 - a_1^2)} + \frac{(r_{m_1}^2 + a_1^2)(u_1 b_1 + 1)}{u_1^2 r_{m_1}^2 (b_1^2 - a_1^2)} e^{-u_1 W_1} \right\}$$

$$\left. + C_1 \left\{ \frac{-2 r_{m_1}}{3} - \frac{a_1^3}{3 r_{m_1}^2} + \frac{(b_1^3 - a_1^3)(r_{m_1}^2 + a_1^2)}{3 r_{m_1}^2 (b_1^2 - a_1^2)} \right\} \right]$$

Reflector #2:

$$(13) \quad V_{B_2} = V_{A_1}$$

$$(14) \quad D_{B_2} = D_{A_1}$$

Equations (1) thru (12) are then evaluated for reflector #2, and a similar procedure followed for each reflector in succession. Since for the first

assumption V_{A_n} is set equal to $1.5 V_{B_n}$, reflectors should always be numbered from the pressure vessel in.

Rec II (301)

The equations utilized by Rec II are identical with those utilized by Rec I. This program does not, however, iterate on velocity. It handles only one reflector per case, and establishes surface temperature, as well as thermal stresses, corresponding to input values of V_A and V_B . The thermal stress calculations are optional in this program.

Prev I (301A)

The equations programmed for Prev I represent a pressure vessel, perfectly insulated at its outer surface. The program calculates surface temperature of the wall at the inner surface, with an optional calculation of thermal stress in the wall.

Surface Temperature

Given V_A as input:

$$(1) \quad h_A = 0.021 \frac{K_W}{D_A} \left(\frac{\rho_W V_A D_A}{\mu_W} \right)^{0.8} (Pr_W)^{0.4}$$

$$(2) \quad T_i = T_w + \frac{Q_0}{u h_A} (1 - e^{-uw})$$

Thermal Stresses

In an insulated vessel the maximum tangential tensile stress occurs at the inner surface:

$$(3) \quad \sigma_{ta} = \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left[\frac{Q_0}{K_s u^2} \right] \left[\frac{2}{b^2 - a^2} \right] \left[\frac{(1 + aue^{-uw})}{2} (b^2 - a^2) \right. \\ \left. + \frac{e^{-uw}}{u^2} (ub + 1) - \frac{(ua + 1)}{u^2} - \frac{ue^{-uw}}{3} (b^3 - a^3) \right]$$

The maximum tangential compressive stress occurs at the outer surface:

$$(4) \quad \sigma_{tb} = \sigma_{ta} - \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left[\frac{Q_0}{k_s u^2} \right] \left[1 - e^{-uw} (1 + uw) \right]$$

IBM 650 CODING

The coding for calculating the thermal stress in reflectors is done in For Transit. The identification given to this code is *Program 304.

Program 304 is designed to iterate on flow rate and can be used for an infinite number of reflectors. The user must make up a set of input for each reflector to be calculated. After the initial input has been read in, and results for first reflector have been punched out, the only values needed to continue calculation for the next ring are An, Wn, Un, and Qn.

If no iteration is required, two other codes are available. **Programs 301, Calculation of Thermal Stress in Reflectors; ***Program 301-A, Calculation of Thermal Stress in Pressure Vessel. The input and output formats appear in this writeup.

Floating Point Number Form

Number Form:

Example:

+ .1234567800	1234567850 +
-1. 234567800	1234567851 -
+ .0012345600	1234560048 +

* REC I

** REC II

***PREV I

Input Format

The input for Program 304 is entered into the machine on For Transit type 1 cards. Data cards of this type are identified by a 12-punch in column 3. Four pieces of data may be punched on each type 1 card. Negative values are indicated by an X-punch in the low order column of the data field, i. e., columns 20, 40, 60, 80. Associated with each item of data on a type 1 card are a code, a numerical identification, and an arbitrary numerical name as indicated in type 1 card diagram. This information must be punched in the ten card columns to the left of each data field. If less than four pieces of data are entered on a card, the ten digit field following the last entry must contain all zeros; any remaining fields may be left blank. The last card of each set of input must have an x-punch in column 10. The code, as referred to above, is either an 01 or 02. The 01 code denotes a fixed point value, and an 02 code denotes a floating point value. All input for Program 304 is in floating point with the exception of the identification word, which appears as the second word of the first card for each set of input.

TABLE I
INPUT FORMAT
REC I (304)

COLUMNS	1———10	11———20	21———30	31———40	41———50	51———60	61———70	71———80
CARD NO. 1	0100020000	IDENT.*	0200090000	DB	0200050000	VB	0200020000	A1
CARD NO. 2	0200030000	W1	0200080000	Kw	0200100000	P	0200110000	W
CARD NO. 3	0200130000	Prw	0200180000	U1	0200200000	Q1	0200320000	E
CARD NO. 4	0200330000	α	0200340000	✓	0200240000	K	0200160000	D1
CARD NO. 5	0200300000	Ts	0200260000	Tw	0200440000	0000000000	0000000000	BLANK

***Identification Word 304XXXXOa**

Where 304 Program name

XXXX Problem identification

a Reflector number; i. e., 1, 2, 3 etc.

Note: Card number 5 must contain an x-punch in column 10 since it is the last card in the first set. In order to run another set, or another reflector, only the values affected by a change from one reflector to another need be punched and read in before resuming calculation. Again, an x-punch must be punched into column 10 of last card.

Identification word should be read in with each reflector input in order for user to check for proper values for each reflector. Only "a" requires changing; i. e., from 1 to 2, etc.

**** Test Word**

Should be set to zero for calculation of initial reflector and reset to zero before calculation of successive reflectors.

Output Format

The output from Program 304 is on two cards per reflector. All output values are in floating point.

TABLE II
OUTPUT FORMAT

REC I (304)

Card No. 1 of Output

**Words 1, 3 and 5 contain the numerical identification
of word to the right.**

Word 2 = Identification

Word 4 = VA

Word 6 = σ a

Card No. 2 of Output

**Words 1, 3 and 5 contain the numerical identification
of word to the right.**

Word 2 = σ b

Word 4 = γ m

Word 5 = σ m

3. Running Time:

TABLE III
INPUT FORMAT
REC II (301)

COLUMNS	1———10	11———20	21———30	31———40	41———50	51———60	61———70	71———80
CARD NO. 1	0100020000	* IDENT.	0100030000	** TEST WORD	0200090000	Kw	0200020000	p
CARD NO. 2	0200050000	uw	0200070000	Prw	0200040000	DA	0200130000	DB
CARD NO. 3	0200300000	A	0200290000	B	0200030000	VA	0200120000	VB
CARD NO. 4	0200120000	VB	0200210000	K	0200310000	E	0200320000	a
CARD NO. 5	0200160000	W	0200180000	Qo	0200150000	U	0200240000	Tw
CARD NO. 6	0200330000	v	0000000000	BLANK	BLANK	BLANK	BLANK	BLANK

* Identification is 3010XXXX00

Where 301 = Program number.

XXXX = Problem identification number. Number can be obtained from IBM 650 Serial Number file.

**Test Word is used because program is designed to calculate the stresses at a, b, and m. If user is concerned only with the surface temperatures of reflector, the test word must contain all zeros. If the stresses are required, test word must be any ten digit number > 0 i. e., 1000000000.

Note: If more than one reflector is to be calculated, only values affected by change from one reflector to another is required to be read into the machine, in order for calculation to continue. If only one card is required for changes, an x-punch must be punched in column 10. If more than one card is required, an x-punch must be punched in column 10 of last card in set only.

For operating instructions, see page 15.

Output Format

The output from Program 301 is on two cards if only the temperatures reflector are required, and four cards if the stresses are also required.

TABLE IV

OUTPUT FORMAT

REC II (301)

Card No. 1 of Output:

Words 1, 3 and 5 contain the numerical identification of word to the right.

Word 2 = Identification

Word 4 = To-Ti

Word 6 = Ti

* Each reflector is a separate problem

The REC II Code has been modified to calculate the temperature at m. This program will be known as REC II-A, and the number of the Code is 301-B.

The REC II-A Code is written in Fortran II and the only required special attention to this code is the input and output formats.

The REC II Code is still in use and the user should concern himself with REC II-A only if he desires the calculation of the temperature at m. The required input is the same as REC II and user should use the REC II write-up as a guide in making up the input to REC II-A.

The input is on four (4) cards, and each card must contain a high x in column 73.

Input Format

Card 1	Card 2	Card 3	Card 4
Identification	A	VB	TW
KW	B	K	Z
P	VA	E	TEST WORD
AW	BLANK	Q	BLANK
Prw	BLANK	W	BLANK
DA	BLANK	Qo	BLANK
DB	BLANK		BLANK
0000000001	0000000002	0000000003	0000000004

Card No. 2 of Output:

Words 1, 3 and 5 contain the numerical identification of word to the right.

Word 2 = To

Word 4 = Zeros

Word 6 = Zeros

Card No. 3 of Output:

Words 1, 3 and 5 contain the numerical identification of word to the right.

Word 2 = σ a

Word 4 = σ b

Word 6 = r m

Card No. 4 of Output:

Words 1, 3 and 5 contain the numerical identification of word to the right.

Word 2 = σ m

Word 4 = Zeros

Word 6 = Zeros

The output for REC II-A (301-B) is on two (2) cards

Output Format

Card 1

Identification

To-Ti

Ti

To

ZEROS

ZEROS

ZEROS

ZEROS

Card 2

σ_a

σ_b

RM

Temp. M

ZEROS

ZEROS

ZEROS

ZEROS

TABLE V
INPUT FORMAT
PREV I (301A)

COLUMNS	1———10	11———20	21———30	31———40	41———50	51———60	61———70	71———80
CARD NO. 1	0100020000	*IDENT.	0100030000	** TEST WORD	0200160000	Qo	0200020000	U
CARD NO. 2	0200260000	A	0200250000	B	0200030000	W	0200190000	E
CARD NO. 3	0200200000	<i>d</i>	0200210000	<i>✓</i>	0200080000	<i>p</i>	0200060000	Kw
CARD NO. 4	0200100000	<i>Mw</i>	0200120000	Prw	0200230000	K	0200070000	DA
CARD NO. 5	0200090000	VA	0200150000	0000000000	BLANK	BLANK	BLANK	BLANK

*Identification is 3011XXXX00

Where 3011 = 301A = Program number

XXXX = Problem identification number. Number can be obtained from IBM 650 Serial Number file.

**Test word is used because program is designed to calculate stresses at a and b. If user is concerned only with surface temperature of vessel, the test word must contain all zeros. If the stresses are required, test word must be any ten digit number >0 ; i. e., 1000000000.

Note: If more than one vessel is to be calculated, only values affected by change from one vessel to another is required to be read into the machine in order for calculation to continue. If only one card is required for changes, an x-punch must be punched in column 10. If more than one card is required, an x-punch must be punched in column 10 of last card in set only.

For Operating Instructions, see page 18.

Output Format

The output from Program 301A is on one card if only surface temperature of vessel is required, and two cards if stresses are also required.

TABLE VI OUTPUT FORMAT PREV I (301A)

Card No. 1 of Output:

Words 1, 3 and 5 contain the numerical identification of word to the right.

Word 2 = Identification

Word 4 = T_i

Word 6 = Zeros

Card No. 2 of Output:

Words 1, 3 and 5 contain the numerical identification of word to the right.

Word 2 = σ a

Word 4 = σ b

Word 6 = Zeros

Operating Instructions

1. Input Makeup Decks:
 - a. Program 304, 301, 301A
 - b. Input sets
2. Console Setting:
 - a. 7019529999 + in storage entry switches.
 - b. Half-cycle switch to Run.
 - c. Control switch to Run.
 - d. Display switch to Program Register.
 - e. Overflow switch to sense.
 - f. Programmed switch to stop.
 - g. Error switch to stop.

APPENDIX

DERIVATIONS

A. Reflectors

1. Temperature Distribution

The temperature distribution equation for the reflector was derived for slab geometry, rather than for cylindrical geometry. This assumption is valid for large diameter/thickness ratios ($\frac{a}{b} \geq 0.8$), as encountered in reflector rings and pressure vessels.

(1) $Q_x = Q_0 e^{-ux}$

(2) $-k_s \frac{d^2 t}{dx^2} = Q_x$

(3) $-k_s \frac{d^2 t}{dx^2} = Q_0 e^{-ux}$

Integrating:

(4)
$$t = \frac{-Q_0}{k_s u^2} e^{-ux} + C_1 x + C_2$$

Boundary conditions:

at $x=0$, $t = T_i$
 at $x=W$, $t = T_o$

(5)
$$C_2 = T_i + \frac{Q_0}{k_s u^2}$$

(6)
$$C_1 = \frac{(T_o - T_i)}{W} + \frac{Q_0}{k_s u^2} \left(\frac{e^{-uW} - 1}{W} \right)$$

The maximum wall temperature occurs within the wall at distance m from the inner surface:

$$(4) \quad t = \frac{-Q_0}{k_s u^2} e^{-ux} + C_1 x + C_2$$

Boundary condition:

$$\text{at } x = m, \quad \frac{dt}{dx} = 0$$

$$(7) \quad C_1 = \frac{-Q_0}{k_s u^2} u e^{-um}$$

Combining equation (7) and (6):

$$(8) \quad m = -\frac{1}{u} \ln \left[\frac{(1 - e^{-uW})}{uW} - \frac{(T_o - T_i)}{W} \left(\frac{k_s u}{Q_0} \right) \right]$$

$$(9) \quad T_i - T_w = \frac{\phi_A}{h_A} ; \quad T_o - T_w = \frac{\phi_B}{h_B}$$

$$(10) \quad \phi_A = \int_0^m Q_0 e^{-ux} dx = -\frac{Q_0}{u} (e^{-um} - 1)$$

$$(11) \quad \phi_A = \frac{Q_0}{u} \left[1 + \frac{e^{-uW} - 1}{uW} + \frac{(T_o - T_i)}{W} \left(\frac{k_s u}{Q_0} \right) \right]$$

$$(12) \quad \varphi_B = \int_0^W Q_0 e^{-ux} dx - \varphi_A = \frac{Q_0}{u} (1 - e^{-uW}) - \varphi_A$$

$$(13) \quad \varphi_B = \frac{Q_0}{u} \left[\frac{1 - e^{-uW}}{uW} - e^{-uW} - \frac{(T_0 - T_i)}{W} \left(\frac{K_s u}{Q_0} \right) \right]$$

$$(14) \quad (T_0 - T_i) = \frac{\varphi_B}{h_B} - \frac{\varphi_A}{h_A}$$

$$(15) \quad (T_0 - T_i) = \frac{\frac{Q_0}{u^2 W} \left[\frac{1 - (1 + uW) e^{-uW}}{h_B} + \frac{1 - uW - e^{-uW}}{h_A} \right]}{\left[1 + \frac{K_s}{W} \left(\frac{h_A + h_B}{h_A h_B} \right) \right]}$$

$$(16) \quad T_i = T_w + \frac{\varphi_A}{h_A} = T_w + (T_0 - T_i) \frac{K_s}{W h_A} + \frac{\frac{Q_0}{u^2 W h_A} (uW + e^{-uW} - 1)}$$

$$(17) \quad T_0 = T_w + \frac{\varphi_B}{h_B} = T_w - (T_0 - T_i) \frac{K_s}{W h_B} + \frac{Q_0}{u^2 W h_B} [1 - e^{-uW} (1 + uW)]$$

2. Tangential Thermal Stress

The tangential thermal stress in a long hollow cylinder, nonuniformly heated, is represented by (Ref. 1):

$$(18) \quad \sigma_t = \frac{E}{1-\nu} \left[\frac{1}{r^2} \int_a^r \alpha t r dr + \frac{r^2 + a^2}{r^2(b^2 - a^2)} \int_a^b \alpha t r dr - \alpha t \right]$$

Substituting the equation for temperature distribution in a slab:

$$(19) \quad \frac{1}{r^2} \int_a^r \alpha t r dr = \frac{\alpha}{r^2} \int_a^r \left(\frac{-Q_0}{k_s u^2} e^{-u(r-a)} + C_1(r-a) + C_2 \right) r dr$$

$$(20) \quad x = r - a ; \quad W = b - a$$

$$(21) \quad \frac{1}{r^2} \int_a^r \alpha t r dr = \frac{Q_0 \alpha}{k_s u^2} \left[\frac{(ur+1)}{u^2 r^2} e^{-u(r-a)} - \frac{(ua+1)}{u^2 r^2} \right] + C_1 \alpha \left[\frac{2r^3 + a^3 - 3ar^2}{6r^2} \right] + C_2 \alpha \left[\frac{r^2 - a^2}{2r^2} \right]$$

$$(22) \quad \frac{r^2 + a^2}{r^2(b^2 - a^2)} \int_a^b \alpha t r dr = \frac{(r^2 + a^2)\alpha}{r^2(b^2 - a^2)} \left(\int_a^b \frac{-Q_0}{k_s u^2} e^{-u(r-a)} r dr + C_1(r-a)r dr + C_2 r dr \right)$$

$$(23) \quad \frac{r^2 + a^2}{r^2(b^2 - a^2)} \int_a^b \alpha t r dr = \left[\frac{Q_0 \alpha}{k_s u^2} \right] \left[\frac{r^2 + a^2}{b^2 - a^2} \right] \left[\frac{(ub+1)}{u^2 r^2} e^{-uW} - \frac{(ua+1)}{u^2 r^2} \right] + C_1 \alpha \left[\frac{r^2 + a^2}{r^2} \right] \left[\frac{2b^3 - 3ab^2 + a^3}{6(b^2 - a^2)} \right] + C_2 \alpha \left[\frac{r^2 + a^2}{2r^2} \right]$$

$$(24) \quad -\alpha t = \frac{Q_0 \alpha}{k_s u^2} e^{-u(r-a)} - C_1 \alpha (r-a) - C_2 \alpha$$

$$(25) \quad \sigma_t = \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left\{ \left[\frac{Q_0}{k_s u^2} \right] \left\{ \frac{ur^2 + ur + 1}{u^2 r^2} e^{-u(r-a)} - \frac{(ua+1)(b^2+r^2)}{u^2 r^2 (b^2-a^2)} + \frac{(r^2+a^2)(ub+1)}{u^2 r^2 (b^2-a^2)} e^{-uW} \right\} + C_1 \left\{ \frac{-2r}{3} - \frac{a^3}{3r^2} + \frac{(b^3-a^3)(r^2+a^2)}{3r^2(b^2-a^2)} \right\} \right\}$$

At the inner wall surface ($r=a$):

$$(26) \quad \sigma_{ta} = \left[\frac{E_s \alpha_s}{1 - \nu_s} \right] \left\{ \left[\frac{Q_0}{k_s u^2} \right] \left\{ 1 + \frac{2(ub+1)e^{-uW} - 2(ua+1)}{u^2(b^2-a^2)} \right\} + C_1 \left\{ \frac{2(b^3-a^3)}{3(b^2-a^2)} - a \right\} \right\}$$

At the outer wall surface ($r=b$):

(27)

$$\sigma_{tb} = \left[\frac{E_s \alpha_s}{1-\nu_s} \right] \left[\left(\frac{Q_0}{k_s u^2} \right) \left\{ e^{-uw} + \frac{2(ub+1)e^{-uw} - 2(ua+1)}{u^2(b^2-a^2)} \right\} + C_1 \left\{ \frac{2(b^3-a^3)}{3(b^2-a^2)} - b \right\} \right]$$

(28)

$$\begin{aligned} \sigma_{tb} &= \sigma_{ta} - \left[\frac{E_s \alpha_s}{1-\nu_s} \right] \left[\left(\frac{Q_0}{k_s u^2} \right) (1 - e^{-uw}) + C_1(b-a) \right] \\ &= \sigma_{ta} - \left[\frac{E_s \alpha_s}{1-\nu_s} \right] [T_0 - T_i] \end{aligned}$$

The tangential compressive stress (σ_{tm}) is calculated by substituting r_m in equation (25):

(29)

$$r_m = a + m$$

B. Pressure Vessel

1. Temperature Distribution

Assuming slab geometry and a perfectly insulated outer surface:

$$(1) Q_x = Q_0 e^{-ux}$$

$$(2) -k_s \frac{d^2 t}{dx^2} = Q_x = Q_0 e^{-ux}$$

Integrating:

$$(3) -k_s t = \frac{Q_0}{u^2} e^{-ux} + Cx + D$$

Boundary conditions:

at $x=0$, $t = T_i$; at $x=w$, $Q/A = 0$.

$$(4) D = -k_s T_i - \frac{Q_0}{u^2}$$

$$(5) C = \frac{Q_0}{u} e^{-uw}$$

$$(6) t - T_i = \frac{Q_0}{k_s u} \left(\frac{1}{u} - \frac{e^{-ux}}{u} - x e^{-uw} \right)$$

at $x=w$:

$$(7) \quad T_o - T_i = \frac{Q_o}{k_s u^2} [1 - e^{-uw}(1 + uw)]$$

$$(8) \quad T_i - T_w = \frac{\varphi_A}{h_A}$$

$$(9) \quad \varphi_A = \int_0^w Q_o e^{-ux} dx = \frac{Q_o}{u} (1 - e^{-uw})$$

$$(10) \quad T_i = T_w + \frac{Q_o}{u h_A} (1 - e^{-uw})$$

2. Tangential Thermal Stress

The tangential thermal stress in a non-uniformly heated, long, hollow cylinder is given by (Ref. 1):

$$(11) \quad \sigma_t = \frac{E_s}{1-\nu_s} \left[\frac{1}{r^2} \int_a^r \alpha t r dr + \frac{r^2 + a^2}{r^2(b^2 - a^2)} \int_a^b \alpha t r dr - \alpha t \right]$$

Substituting the value of "t" from equation (6) in equation (11) and integrating:

$$(12) \quad \sigma_t = \left[\frac{E_s}{1-\nu_s} \right] \left[\frac{\alpha_s Q_0}{K_s u^2} \right] \left[\left\{ \frac{1 + a u e^{-uW}}{2} \left(1 - \frac{a^2}{r^2} \right) \right\} + \left\{ e^{-u(r-a)} \frac{(ur+1)}{u^2 r^2} \right\} - \left\{ \frac{ua+1}{u^2 r^2} \right\} - \left\{ \frac{ue^{-uW}}{3} \left(r - \frac{a^3}{r^2} \right) \right\} + \left\{ \frac{r^2 + a^2}{r^2(b^2 - a^2)} \right\} \left\{ \frac{1 + a u e^{-uW}}{2} (b^2 - a^2) + \frac{e^{-uW}}{u^2} (ub+1) - \frac{(ua+1)}{u^2} - \frac{ue^{-uW}}{3} (b^3 - a^3) \right\} - \left\{ 1 - e^{-u(r-a)} - (r-a) u e^{-uW} \right\} \right]$$

at $r = a$:

$$(13) \quad \sigma_{ta} = \left[\frac{E_s}{1-\nu_s} \right] \left[\frac{\alpha_s Q_0}{k_s u^2} \right] \left[\frac{2}{b^2 - a^2} \right] \left[\frac{1 + aue^{-uW}}{2} (b^2 - a^2) \right. \\ \left. + \frac{e^{-uW}}{u^2} (ub+1) - \frac{(ua+1)}{u^2} - \frac{ue^{-uW}}{3} (b^3 - a^3) \right]$$

at $r = b$:

$$(14) \quad \sigma_{tb} = \left[\frac{E_s}{1-\nu_s} \right] \left[\frac{\alpha_s Q_0}{k_s u^2} \right] \left[\left\{ \frac{2}{b^2 - a^2} \right\} \left\{ \frac{1 + aue^{-uW}}{2} (b^2 - a^2) \right. \right. \\ \left. \left. + \frac{e^{-uW}(ub+1)}{u^2} - \frac{(ua+1)}{u^2} - \frac{ue^{-uW}(b^3 - a^3)}{3} \right\} - \{1 - e^{-uW} - Wu e^{-uW}\} \right]$$

$$(15) \quad \sigma_{tb} = \sigma_{ta} - \left[\frac{E_s}{1-\nu_s} \right] \left[\frac{\alpha_s Q_0}{k_s u^2} \right] [1 - e^{-uW} - Wu e^{-uW}]$$

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