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PRESTRESSED CONCRETE REACTOR VESSELS: REVIEW OF THE DESIGN AND FAILURE CRITERIA

by

Elton G. Endebrook

ABSTRACT

The design and failure criteria of prestressed concrete reactor vessels (PCRVs) are reviewed along with the analysis methods. The mechanical properties of concrete under multiaxial stresses are not adequately quantified or described to permit an accurate analysis of a PCRV. Structural analysis of PCRVs almost universally utilizes a finite element which encounters difficulties in numerical solution of the governing equations and in treatment of fractured elements.

I. INTRODUCTION

The structural phase of the Los Alamos Scientific Laboratory (LASL) high temperature gas-cooled reactor (HTGR) safety program began with a study of the extent of current technology on design of prestressed concrete reactor vessels (PCRVs). Many available works address analysis methods, test results, and design techniques. These show a large uncertainty of the predictability of concrete vessel behavior, because very low allowable stresses are used in design, and testing usually shows that failure or cracking occurs at significantly higher loads than normal design techniques predict. It is also evident that refined computer analysis methods cannot be blamed for this inability to predict failure or to maintain a high degree of confidence, without using large safety factors.

Part of the reason for using large safety factors in PCRV design is attributed to great variability in the properties of construction grade concrete; another significant part is the lack of knowledge about concrete behavior and properties.

II. PCRV PERFORMANCE CRITERIA

PCRVs for HTGRs are unconventional, thick-walled concrete structures with embedded prestressing tendons and reinforcing steel. They differ from

usual concrete structures in being more massive, in having to withstand higher temperatures, and in experiencing multiaxial stress states. In addition, an internal steel liner functions as a gas barrier and is an integral part of the vessel. PCRVs are designed to satisfy the following performance criteria¹

(1) Elastic response to all load combinations under all operating conditions.

(2) Gradual failure mode under internal pressure overload.

(3) A specified margin of safety against failure.

An elastic analysis determines whether criterion (1) is satisfied; an ultimate strength analysis determines whether criterion (3) is met; and satisfaction of criterion (2) depends on the type, amount, and arrangement of the prestressing tendons and reinforcing steel.

III. MATERIALS TECHNOLOGY

A. Steel

Steel components in PCRVs are the prestressing tendons, reinforcing bars, and liner. The prestressing tendons induce compressive stresses in the concrete vessel walls and resist the tensile forces resulting from internal pressures. The reinforcing steel helps to carry tensile stresses and controls the cracking

pattern in the vessel walls when overloaded. Unbonded prestressing tendons are normally used. At high load levels, a few large cracks usually develop in prestressed concrete structures if all prestressing tendons are unbonded. Use of reinforcing steel controls this tendency and provides a more uniform distribution of smaller cracks. A few large cracks can cause sudden failures, whereas many small cracks will usually produce a gradual or progressive failure. The steel liner is a barrier to prevent hot coolant from penetrating into cracks in the vessel walls. It is an integral part of the PCRV; hence, its effect on the structural behavior of the PCRV must be considered.

Analysis of this effect is complicated by the multiaxial nature of liner loadings. The prestressing tendons and reinforcing steel are slender members that, in a structural analysis, can be treated as uniaxially loaded. In contrast, the liner is a steel shell subjected to a triaxial stress state. Fortunately, the pressure applied to the interior surface of the liner is small in comparison to the yield stress of the steel, so the liner can be considered to be under biaxial stress. The short-term strength behavior of steel under uniaxial stress is well known. Further, there are yield theories and plastic flow rules for steel when subjected to multiaxial stresses.² Examples of such yield theories are Tresca's and Von Mises' yield conditions. Plastic flow expressions (stress-strain relations in the plastic range) are obtained from the particular yield condition used. For stress combinations below the yield surface, the stress-strain relationship is normally assumed to be linear (Hooke's law).

Knowledge of short-term behavior, however, is not enough. PCRVs are designed for an operational life of 30-40 yr. During the PCRV's operational life, its margin of safety must be maintained at or above a specified value. A reasonably accurate estimate of the safety factor of PCRVs at or near the end of their operational life is necessary. This estimate requires a knowledge of the long-term structural characteristics of the materials.

The long-term strength characteristics of steel in all environments have not been completely established. For example, it is known that steel degrades as a result of corrosion and radiation, but these degradation factors have not been adequately quantified. The combined effects of creep in the concrete and relaxation in the prestressing tendons reduce the prestress force with time. In the usual prestressed structures, these effects can be estimated with sufficient accuracy from existing experience with other prestressed structures. However, there has been little experience in prestressing massive concrete, so there are uncertainties about its creep and relaxation in this use.

Furthermore, the temperature and radiation effects on concrete and steel are relatively unknown. These uncertainties are presently circumvented by using unbonded prestressing tendons. Unbonded tendons can be retensioned or replaced when necessary. As the long-term structural characteristics of the PCRV materials become better known, it may be possible to use bonded tendons, thus reducing the amount of reinforcing steel required.

B. Concrete

Concrete is the major material used in PCRVs. Its tensile strength is low and is usually neglected. The compressive strength of concrete is its chief asset as a structural material. In HTGR vessels, the concrete is prestressed so that it is subjected only to compressive stresses under operating conditions. Compressive stresses are induced by prestressing in the longitudinal and tangential directions (for cylindrical vessels) and radially by the internal pressure and the confinement effect of the liner. The effect of the internal pressure on concrete behavior cannot be neglected, as is possible for the steel liner. Tests indicate that any confinement of concrete in a triaxial compressive stress state noticeably increases its ultimate strength.³

In PCRVs, the concrete is under multiaxial stress. Also, it is subjected to nuclear radiation, heat, and pressures in HTGRs, so its environment is more severe than that of normal concrete structures.

The structural properties of concrete depend on many variables and conditions, including stress, loading rate, loading history, moisture content, temperature, and radiation. The properties also vary with aggregate composition, gradation, proportioning and mixing, consolidation, and curing methods. Concrete properties upon which PCRV structural behavior depends are shrinkage, creep, stress-strain relationship, thermal expansion, heat conductivity, and thermal diffusivity, all of which vary nonlinearly. It has been said that concrete is a nonisotropic, heterogeneous, viscoelastic-plastic material with properties depending on many variables.⁴ The mathematical formulation of the properties of such a complex material is very difficult. Progress in overcoming these difficulties has been slow and piecemeal, as the record shows.

The structural properties of concrete under uniaxial stress have been extensively investigated for many years. Most investigations were limited to a particular concrete's behavior as influenced by one or two factors such as creep as a function of stress and water: cement ratio. Effects of the other significant variables were often disregarded. The results of these investigations have been that many empirical

formulas for predicting one-dimensional concrete behavior have been proposed, but none has gained general usage. Many of the proposed expressions include multiple variables, making the resulting mathematical expressions so complicated that they are useless except in evaluating relative effects of the variables. Other proposed empirical formulas do not include the effects of all significant variables; hence, these prediction formulas apply only to the concrete upon which they were based.

Considerable test data on the uniaxial properties of concrete have been published. Although documentation of the material properties and the test procedures is often incomplete, there is enough information to establish bounds on the uniaxial concrete properties. A PCRV could be analyzed using the uniaxial properties of concrete, and the results would be considered conservative. The usual concrete structural members are slabs, columns, and beams, relatively small members that could realistically be considered to be under uniaxial stress.

Available analytical methods and computational tools were once inadequate for treating three-dimensional structures; there was no need to know the behavior of concrete under multiaxial stress states. This is no longer true. Much research is now done to determine the behavior of concrete subjected to multiaxial stress states. Properties which have been considered include strength, creep, Poisson's ratio, modulus of elasticity, and stress-strain relationships. This research concentrated mainly on the biaxial case or on a special triaxial stress state in which two of the three principal stresses are equal.

Concrete subjected to a compressive triaxial stress exhibits a significant increase above its uniaxial strength. The ultimate strength along the major stress axis, the direction of greatest stress, is approximately three and one-half-times the uniaxial strength, with optimal magnitudes of the minor and intermediate stresses (those in the directions of least and intermediate stress, respectively).³ The strain at the point of maximum major stress is much greater in triaxial than in uniaxial stress. If the minor stress is tensile, the major compressive stress is less than the uniaxial compressive ultimate strength. The strain in the major direction at failure is less than that at the peak stress of the uniaxial case. Fortunately, in PCRVs, the loading is such that the structural strength will not be greatly affected by tensile stresses in the concrete. The major compressive stress is the internal pressure, which is <50% of the concrete's uniaxial ultimate strength, even under overload conditions.

The strength increase for the biaxial stress state is less than for the triaxial stress.^{3,5} The strength depends on the ratio of the principal stresses and exceeds the uniaxial ultimate strength by 30% or less.

Cylindrical and cubical test specimens are used to determine the behavior of concrete under multiaxial stress. Cylindrical test specimens are loaded in a commercially available triaxial fluid-pressure load cell. General multiaxial concrete behavior cannot be examined using cylindrical specimens, because two of the three principal stresses applied must be equal. Stresses can be applied independently in the three principal directions of a cube.

Two types of testing machines are used with cubical test specimens, neither of which is commercially available. These are called the rigid platen and the fluid cushion testing machines. In the rigid platen machine, a steel plate or brush transmits the applied loads from a hydraulic ram to the test specimen.^{3,5} The loads are transmitted by fluid pressures in the fluid cushion machine.⁶ A flexible membrane is used between the fluid and the test specimen. Electrical capacitive or inductive extensometers must be used for strain measurements made with the fluid cushion machine.⁶ These extensometers must be located within the pressurized fluid and must be unaffected by both the fluid and the fluid pressure.

A combination of mechanical and electrical resistance strain gages is usually used in rigid platen machines.³ Friction between the load platens and the test specimens greatly influences the test results, so provisions must be made to reduce this friction whenever the rigid platen machine is used. Friction is reduced by using brush platens or two plastic pads with grease between them.^{3,5} The plastic pads are placed between the platen and test specimen. Specimens tested in a rigid platen machine must have plane surfaces, because a plane rigid platen in contact with an uneven surface produces stress concentrations. Rigid platen test specimens must therefore be conditioned in a surface grinder.

Friction is no problem in the fluid cushion machine nor do the test specimen surfaces require planing and smoothing. Specimen preparation for the fluid cushion machine is slight compared to that of the rigid platen machine, but testing in a fluid cushion machine is limited to static loads at ambient temperatures. Both dynamic and static loads are possible in a rigid platen machine, as is testing at high temperatures.

Because multiaxial testing of concrete is relatively new, there is little uniformity in the design of the test programs and in methods of reporting results. In many previous investigations, only ultimate

strength information was reported. Strain information is required to prepare mathematical formulas defining the stress-strain relationships of concrete. Publications on multiaxial investigations now generally include strain information; in biaxial investigations, strains in the unrestrained direction usually were not recorded or reported. The probability of discovering a failure theory for concrete and of formulating its constitutive relations would increase if all investigators would record and report the strains in all principal directions for all loading combinations.

Several mathematical expressions have been proposed for predicting the maximum or failure stresses. These expressions (failure envelopes)^{7,10} are empirically determined relationships between the principal stresses at failure. Examples of suggested failure envelopes are as follows.

$$\sigma_1 = f'_c + C_1 \sigma_2 + C_2 \sigma_3, \quad (1)$$

$$\tau_o = f(\sigma_o), \quad (2)$$

$$\tau_o = g(\sigma_o, \alpha), \quad (3)$$

$$\begin{aligned} & (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + C_3 (\sigma_1 \sigma_2 + \sigma_1 \sigma_3 + \sigma_2 \sigma_3) \\ & + C_4 f'_c (\sigma_1 + \sigma_2 + \sigma_3) + C_5 (f'_c)^2 = 0, \end{aligned} \quad (4)$$

where

f'_c = uniaxial ultimate strength,

C_i = constants,

σ_i = principal stresses,

$$\tau_o = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2},$$

$$\sigma_o = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3),$$

$$\alpha = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

and

$$\sigma_1 > \sigma_2 > \sigma_3.$$

The failure envelopes are relationships between the principal stresses at failure and provide no information on the stress-strain relationships. Originally, a linear stress-strain relationship was assumed for all stresses below the failure envelope. This approach evolved from the yield theories developed for metals. In two cases, stress-strain relationships were presented along with failure envelopes;^{11,12} both were for biaxial stress only. In other instances, only stress-strain relationships were presented.^{3,13}

Models and empirical mathematical expressions for predicting the stress-strain behavior and ultimate strength of concrete under multiaxial stress have been suggested, but many apply to biaxial stress only. The basis of several proposed stress-strain expressions is Hooke's law with nonlinearities introduced by varying the modulus of elasticity and Poisson's ratio, which are usually taken as functions of the hydrostatic and/or deviatoric parts of the stresses and strains.¹³ The relating functions are determined empirically.

One suggested model includes all multiaxial stress states (compressive and tensile) and can predict the initiation of cracking and the direction in which the crack forms.⁴ Crack initiation is based on a maximum tensile strain. This model consists of nodal points connected by bars or springs. The nodes are located at the points of an octahedron. The model is developed with reference to principal stress and strain coordinates. The general form of the stress-strain relations is:

$$\begin{aligned} d\sigma_1 = & (K_{11} + K_{12} + K_{13}) d\epsilon_1 + K_{12}d\epsilon_2 \\ & + K_{13}d\epsilon_3, \end{aligned} \quad (5)$$

$$\begin{aligned} d\sigma_2 = & K_{12}d\epsilon_1 + (K_{12} + K_{22} + K_{23}) d\epsilon_2 \\ & + K_{23}d\epsilon_3, \end{aligned} \quad (6)$$

$$\begin{aligned} d\sigma_3 = & K_{13}d\epsilon_1 + K_{23}d\epsilon_2 \\ & + (K_{13} + K_{23} + K_{33}) d\epsilon_3, \end{aligned} \quad (7)$$

where

$d\sigma_i$ = incremental stresses,

$d\epsilon_i$ = incremental strains,

$$K_{ij} = K_{ij} (\epsilon_1, \epsilon_2, \epsilon_3, \sigma_1, \sigma_2, \sigma_3).$$

Equations (5)-(7) were formulated in terms of incremental stresses and strains. Other proposed

stress-strain relationships are stated in terms of total stress and strains. The major stress reaches a maximum and then decreases. At the maximum major stress, the value of the coefficient of the stress-strain relationship becomes zero. Most of the proposed stress-strain expressions do not predict beyond the point of maximum stress, thus avoiding the singularity. In incremental stress-strain computations, there are no problems in the treatment at the point of maximum major stress. Owing to the nonlinear behavior of concrete's structural properties, the validity of superposing total stresses and strains is questionable. The expressions for the various concrete properties are linear functions of the incremental stresses and strains, so superposition of incremental stresses and strains is valid. The incremental values must be kept within certain magnitudes to guarantee a stable, accurate solution.

All the proposed models and mathematical expressions for predicting the stress-strain behavior of concrete incorporate test data, but the data have not included all factors influencing concrete's behavior. Therefore, all the proposed stress-strain relationships depend upon the concrete from which the data were obtained; thus, none of the proposed stress-strain relationships can be classified as general.

The high temperatures at which HTGRs operate led to research on temperature effects on concrete behavior. Besides producing stresses by thermal expansion and contraction, high temperatures can degrade the strength, creep, and shrinkage properties and may increase the water migration and reduce the moisture content of the concrete mass. Strength, creep, and shrinkage are influenced by moisture content. Temperature cycling can seriously reduce the strength. The chief cause of strength loss owing to temperature cycling and high temperatures was thought to be the different thermal expansion coefficients of the mortar and aggregates.¹⁴ There is now evidence that other factors may be more significant.^{15,16} One possible factor is the chemical instability of the aggregate over a wide temperature range.¹⁵

Many investigations of temperature effects on concrete have been reported. Test temperatures have ranged from 19 to 800°C, but usually from 20 to 90°C. The latter range has given little agreement in the various results. Some investigators report no significant degradation of concrete in this temperature range, whereas others report severe degradation. Obviously, the basic mechanism and factors which lead to changes in concrete properties at high temperatures have not been isolated. There are other uncertainties concerning temperature

effects, obtained experimentally from small test specimens. Tests of such specimens may not be applicable to the massive PCRVs. Because of the uncertainties of temperature effects on concrete, high safety factors are used, and the temperature in PCRVs is controlled by cooling the liner.

High-temperature effects on the long-term behavior of concrete are still relatively unknown. One investigation has continued for 13 yr.¹⁶ Its results indicate that the mechanical properties of concrete deteriorate most at first heating. Subsequent heat cycling causes progressive deterioration in the mechanical properties. The extent of deterioration markedly depends upon the aggregate type and is associated with loss of bonding between the mortar and aggregate.¹⁵ More information is needed in this area.

It is known that the mechanical properties of concrete may be degraded by irradiation. Irradiation at integrated fluxes of $\sim 2 \times 10^{19}$ n/cm² seems to have little effect.^{18,19} Larger integrated fluxes may cause severe effects.¹⁸ Irradiation causes growth of aggregate particles and breaks down the water within the paste. Test results may be difficult to apply to reactor vessels. Irradiation damage depends on the irradiation energy spectrum to which the concrete is subjected. The spectra emitted from experimental reactors used in test programs differ from those in operating reactors.

A degree of confidence will be established if analyses using refined methods verify that design by less refined methods is safe. To permit more refined analysis, the voids in knowledge must be eliminated. Some of these voids are the following.

(1) The "elastic limit" of concrete under multiaxial stress states. This is the allowable stress for use in the elastic analysis of a PCRv. In the uniaxial case, it is $\sim 30\%$ of the uniaxial ultimate strength. It would be larger under multiaxial stresses.

(2) The complete stress-strain relationship of concrete under multiaxial stresses. This is needed for PCRv load-history analysis.

(3) The effect of temperature on the mechanical properties of concrete under multiaxial stress.

(4) The effect of moisture migration, accentuated by temperature, on mass concrete creep and shrinkage.

(5) The influence of coolant leakage.

(6) The influence of present and future higher irradiation levels on all properties.

To fill the voids, extensive and coordinated research on concrete, is required. A critical analysis of existing behavior information is needed to determine research areas and to reduce duplication of effort.

IV. PCRVS ANALYSIS

The PCRVS is a unique type of structure, because the prestressed (and reinforced) concrete is much more massive than the usual concrete structures. Analysis of PCRVSs becomes more complicated than that of typical concrete structural members. Moisture gain or loss occurs more slowly, and heat of hydration is a problem in massive PCRVSs. In ordinary concrete structures, the surface areas of members are large compared with their volumes, thereby permitting dissipation of the heat of hydration. Heat cannot dissipate quickly from massive structures. Excessive heat may cause thermal damage, which may be recoverable with time owing to concrete's self-healing property.¹⁹ Concrete can heal itself if it is moist enough for continued hydration, and if the surfaces of the cracks are in contact. A moisture gradient will form because of the thick PCRVS wall and the temperature gradient. Moisture content influences the mechanical properties of concrete, so they will vary radially in a PCRVS.

PCRVSs are highly redundant structures. Properly designed, highly redundant structures exhibit a gradual mode of failure (structural distress starts with large, visible deformations well before total collapse). In highly redundant structures, the load originally carried by a fractured element is transmitted to less stressed adjacent elements. Structural analysis of PCRVSs normally requires the use of the displacement method, in which displacements of preselected points become the unknowns. Also, the PCRVSs must be idealized because of their complexity.

A PCRVS safety evaluation has usually consisted of an elastic analysis and an ultimate strength analysis. A PCRVS should remain elastic under operating loads for its entire operational life. An ultimate strength analysis is used to estimate the PCRVS's reserve strength.

Originally, multiaxial effects in PCRVSs were ignored; uniaxial concrete properties were used in elastic analyses.²⁰ Later, it was realized that using one-dimensional material properties in three-dimensional structural analyses was not satisfactory. Further, creep and shrinkage, and temperature effects thereon, were included in an elastic analysis by using a reduced elastic modulus. This procedure completely ignores time as a variable. Strength deterioration was considered by using a lower allowable stress or elastic limit.

The present trend is to use more refined analysis techniques and the latest information on material properties.^{21,22} There is also a trend toward using

load-history analyses instead of determining bounds on PCRVS structural behavior.

An ultimate load analysis of a PCRVS is undertaken to determine its margin of safety. Such analyses have been applied successfully to concrete beams and columns for years. Methods such as the yield line method have been used to estimate the ultimate load of the massive PCRVSs.²³ In using the usual ultimate load methods, a mode of failure is assumed, and the load that would cause that failure is then computed. There is no procedure or method of analysis to determine whether the assumed failure mode will occur. In the United States, practice has been to base ultimate load calculations on methods which predict the structural behavior of the vessel mathematically. The trend is toward using the finite element method for determining load factors.

The basic parts of a finite element solution are as follows.

- (1) Finite element modeling of the structure,
- (2) Constitutive modeling of material behavior, such as plasticity and fracture.
- (3) Numerical solution of the nonlinear structural equations.

Finite element modeling requires selection of a finite element model and the number of elements needed to represent the structure adequately. Accuracy and computational effort usually increase as the number of degrees of freedom (number of unknowns) and the order of the assumed displacement field increase.

Finite element models are derived on the basis of continuous displacement fields; therefore, they must include continuous strains and stresses. Cracks introduce discontinuities into the models and cause stress and strain incompatibilities. Several procedures have been suggested for circumventing this problem. One is to assume a crack location and length and then compute the load that caused that crack. The cracks are assumed to occur between adjacent finite elements, thus conserving continuity within these elements. Different crack locations and lengths are assumed, and the corresponding loads are computed. The correct crack location and length are considered to be those corresponding to the lowest computed load.

Another procedure that predicts the crack locations and lengths on the basis of principal stresses has been reported.²⁴ The rupture surface is defined specifically in terms of the stress invariants. Tensile cracking is accounted for by introducing a local orthotropy oriented along the plane of cracking and placed discontinuously in the model at the integration points of the finite elements. Stresses incompatible with the local orthotropy are corrected

and redistributed in the structure through imaginary equivalent node forces.

Another method accounts for cracking within the finite elements.²⁵ Element continuity is preserved by setting the material stiffness to zero in a direction normal to the crack. Crack initiation depends upon the principal stresses.

Multiaxial stresses influence the ultimate strength of the structure. Several expressions for predicting the ultimate concrete strength under multiaxial stresses have been suggested.⁸⁻¹² These expressions, called fracture, rupture, yield, or ultimate strength surfaces, generally are very nonlinear relationships between the principal stresses. A few surfaces consist of piecewise linear parts.^{12,26} Most of these expressions are valid for the biaxial case only. For example, the failure surface used in designing the Fort St. Vrain Nuclear Station was²³

$$(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3) - 0.25 f_c'(\sigma_1 + \sigma_2 + \sigma_3) - 0.75 (f_c')^2 = 0, \quad (8)$$

where

f_c' = uniaxial compressive strength

σ_i = principal stresses,

$$\sigma_1 < 3f_c',$$

and

$$\sigma_1 < \sigma_2 < \sigma_3.$$

This expression permits all combinations of compressive and tensile stresses. Compression is negative, and tension is positive.

Numerical solutions of nonlinear finite element equations generally require incremental or iterative procedures. The numerical technique used depends on the formulation of the structural problem.

V. SUMMARY

The basic theory for determining PCRv stresses, strains, and deformation is available, as are the structural analysis theory and the finite element models for three-dimensional structures. Temperature diffusion theory with accompanying

numerical techniques is available, also. Methods for determining the temperature distributions, creep and shrinkage effects, and load effects in PCRvs have been demonstrated. There is sufficient technology to perform load-history analyses of PCRvs. The voids in PCRv technology are in mathematical descriptions of the materials. A computer program for structural analysis that can be used for a comprehensive PCRv load-history analysis is not available yet; however, the technology exists to develop such a program. Development of this program for load-history analysis would be a significant effort. Considering the nonlinearities in PCRv analysis, an incremental solution may be required, in which superposition of the different effects would be valid. More information on the multiaxial, temperature, and long-term behavior of concrete is required, as is increased knowledge of the degradation factors of corrosion of the prestressing tendons.

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