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DYNAMIC RESPONSE OF TWO PARALLEL CIRCULAR CYLINDERS IN A LIQUID

by

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PREFACE

The work reported herein was performed as part of the base technology activity under the Flow Induced Vibration Programs (189a Nos. 02659 and 02683) sponsored by ERDA/RRD. The overall objective of the activity is to develop new and/or improved analytical methods and guidelines for designing LMFBR components to avoid detrimental flow induced vibration.

Heat exchanger tubes and reactor fuel pins are long, slender, beam-like components typically arranged in bundles and immersed in a flowing liquid. As such, they are susceptible to flow induced vibration. The excitation mechanism may be associated with vortex-shedding, fluidelastic interaction, or random pressure fluctuations in the turbulent flow. Designing to avoid large amplitude motion, that is, to avoid a resonance condition or instability condition, and the prediction of component response, require knowledge of the dynamic behavior of the components. However, cylinders in a closely spaced bundle do not respond as single cylinders immersed in a liquid, rather, interaction with the liquid causes coupled motion of groups of cylinders. The fundamental natural frequency of the coupled system will be lower than that of a single cylinder immersed in a liquid.

Understanding and modeling fluid/structure interaction in cylinder bundles is a basic requirement in the development of analytical methods and guidelines for designing heat exchanger and reactor fuel assemblies that are free from component vibration problems. As a first step toward satisfying this requirement, in this report, two parallel cylinders vibrating in a liquid are studied analytically. A method of analysis is presented for free and forced vibrations. Steady-state responses to flow noises are included. The results illustrate the significance of the interaction of two structures in a liquid, and show that an analysis which does not account for fluid coupling effect is, in general, not conservative.

The analysis method presented will be extended to account for fluid/structure coupling in bundles and will be used in the development of sets of design curves corresponding to various values of displaced-fluid-mass to cylinder-mass, and gap to radius ratios. These curves will be included in a future design guide and will permit determination of the added mass coefficients and natural frequencies of coupled cylinder bundle systems.

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NOMENCLATURE

a_n = arbitrary constants in Eq. 8

a_{nm} = coupling coefficient given by Eq. 18

A_{mn} = matrix elements given by Eq. 22

B_{mn} = matrix elements given by Eq. 22

c_j = coefficient of viscous damping

C_{mn} = matrix elements given by Eq. 22

D_{mn} = matrix elements given by Eq. 22

E_j = modulus of elasticity

f_j = excitation force

f_v = vortex shedding frequency

F_j = hydrodynamic force

g_j = axial distribution of force

G = gap between cylinders

h, h_1, h_2 = parameter given by Eq. 4

I_j = moment of inertia of cylinder

ℓ = rod length

m_j = mass of cylinder per unit length

M_j = mass of fluid displaced by cylinder per unit length

p_n = nth root of Eq. 9

q_{jn} = generalized coordinate

$\bar{q}_{jn}$ = generalized coordinate

Q_{jn} = generalized force

r_n = parameter defined by Eq. 10

R = distance between cylinder centers

R_j = radius

t = time

u_j = cylinder displacement

$\bar{u}_1$ = dimensionless cylinder displacement

V = flow velocity

x = axial coordinate

α_j = parameter given by Eq. 18

β_j = M_j/m_j

δ_{mn} = Kronecker delta

ζ_{jn} = modal damping ratio

λ = 1 for in-plane motion and -1 for out-of-plane motion

μ_k = added mass coefficient ($k = 1,2,3$)

ρ = fluid density

ϕ_{jn} = orthonormal functions of cylinder in vacuo

ψ = phase angle

ω = circular frequency

ω_{jn} = natural frequency of nth mode of cylinder j in liquid

$\bar{\omega}_{jn}$ = natural frequency of nth mode of cylinder j in vacuo

Ω_{jn} = natural frequency of coupled mode

Subscripts

j denotes rod 1 ($j = 1$) or rod 2 ($j = 2$)

n denotes mode number

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by

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ABSTRACT

The problem of two parallel circular cylinders vibrating in a liquid is studied analytically. First, the equations of motion including fluid coupling are formulated using the added mass concept. Then, a closed form solution and an approximate solution are obtained for free vibration. Finally, the steady state responses of two cylinders subjected to harmonic excitations are presented. The results of this study illustrate the significance of the interaction of two structures in a liquid.

I. INTRODUCTION

The vibration response of rod bundles in liquids to various types of excitations, including earthquakes, fluid flows, and acoustic noises, is of importance in the design of heat exchangers and reactor internal components such as fuel assemblies. Several studies have been made on the coupled motion of multiple rods in a liquid. Livesey and Dye [1]* presented an experimental investigation on the possible vibration modes of a single row of rods mounted normal to an air flow; Mazur [2] considered the fluid forces acting on two cylinders moving in an ideal fluid; Wilson and Caldwell [3] utilized a water flow tank to observe conditions for vortex-shedding induced vibrations of two parallel pipes, parallel to a flat plane; and Shimogo and Inui [4] analyzed the vibrations of two and four identical circular cylinders vibrating in water. In general, the analysis of the vibration modes and responses of a group of cylinders in a liquid is difficult because of the difficulty in accounting for the coupling effect of the surrounding fluid.

As a first step in the development of a model for the vibration of heat-exchanger tubes and nuclear fuel assemblies, two parallel cylinders vibrating in a liquid are studied analytically. First, the equations of motion including fluid coupling are derived using the added mass concept. Then, a closed form solution and an approximate solution are obtained for free vibration; some important conclusions are drawn from the analyses. Finally, steady state responses of two cylinders subjected to flow excitations are presented. The results of this study illustrate the significance of the interaction of two structures in a liquid and show that an analysis which does not account for the coupling effect is not conservative.

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*Numbers in brackets designate References at the end of paper.

II. EQUATIONS OF MOTION

Consider two parallel circular cylindrical rods (designated 1 and 2), immersed in a liquid, as illustrated in Fig. 1. Rod motions consist of an in-plane displacement along the y axis and an out-of-plane displacement along the z axis. The equation of motion for either in-plane or out-of-plane motions can be written

$$E_j I_j \frac{\partial^4 u_j}{\partial x^4} + c_j \frac{\partial u_j}{\partial t} + m_j \frac{\partial^2 u_j}{\partial t^2} = F_j + f_j \quad , \tag{1}$$

where the index j denotes rod 1 (j = 1) and 2 (j = 2), x is axial coordinate, t is time, u_j is rod displacement, m_j is mass per unit length of the rods, $E_j I_j$ is flexural rigidity, c_j is damping coefficient, F_j is hydrodynamic force and f_j is excitation force.

The hydrodynamic forces associated with two vibrating cylinders were considered by Mazur [2] using a two dimensional theory:

$$F_1 = -M_1 \mu_1 \frac{\partial^2 u_1}{\partial t^2} + M_1 \mu_3 \left(\frac{R_2}{R}\right)^2 \frac{\partial^2 u_2}{\partial t^2} \quad ,$$

and (2)

$$F_2 = -M_2 \mu_2 \frac{\partial^2 u_2}{\partial t^2} + M_2 \mu_3 \left(\frac{R_1}{R}\right)^2 \frac{\partial^2 u_1}{\partial t^2} \quad ,$$

for in-plane motion; and

$$F_1 = -M_1 \mu_1 \frac{\partial^2 u_1}{\partial t^2} - M_1 \mu_3 \left(\frac{R_2}{R}\right)^2 \frac{\partial^2 u_2}{\partial t^2} \quad ,$$

and (3)

$$F_2 = -M_2 \mu_2 \frac{\partial^2 u_2}{\partial t^2} - M_2 \mu_3 \left(\frac{R_1}{R}\right)^2 \frac{\partial^2 u_1}{\partial t^2} \quad ,$$

for out-of-plane motion. M_1 and M_2 are the displaced masses of fluid by the two rods, R_1 and R_2 are rod radii, R is the distance between the centers of the two rods, and μ_1, μ_2 and μ_3 are added mass coefficients given by

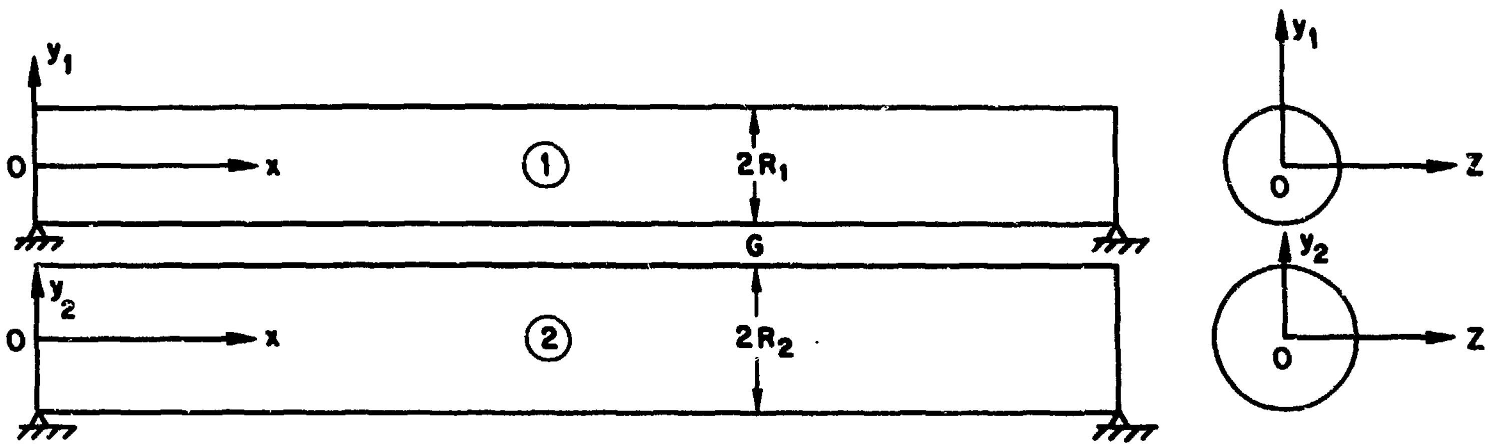

Fig. 1. Schematic of two parallel circular cylindrical rods vibrating in a liquid

$$\mu_1 = 1 + \frac{R^4 - 2R^2(R_1^2+R_2^2) + (R_2^2-R_1^2)^2}{R^2R_1^2} \sum_{k=1}^{\infty} k \frac{\exp[-k(h+h_1)]}{\sinh(kh)} ,$$

$$\mu_2 = 1 + \frac{R^4 - 2R^2(R_1^2+R_2^2) + (R_2^2-R_1^2)^2}{R^2R_2^2} \sum_{k=1}^{\infty} k \frac{\exp[-k(h+h_2)]}{\sinh(kh)} ,$$

and

$$\mu_3 = 1 + \frac{R^4 - 2R^2(R_1^2+R_2^2) + (R_2^2-R_1^2)^2}{R_1^2R_2^2} \sum_{k=1}^{\infty} h \coth(kh)\exp(-2kh) ,$$

where (4)

$$h = \ln\left\{\frac{R^2 - R_1^2 - R_2^2}{2R_1R_2} + \left[\left(\frac{R^2 - R_1^2 - R_2^2}{2R_1R_2}\right)^2 - 1\right]^{1/2}\right\} ,$$

$$h_1 = 2\ln\left\{\frac{R^2 + R_1^2 - R_2^2}{2RR_1} + \left[\left(\frac{R^2 + R_1^2 - R_2^2}{2RR_1}\right)^2 - 1\right]^{1/2}\right\} ,$$

and

$$h_2 = 2\ln\left\{\frac{R^2 - R_1^2 + R_2^2}{2RR_2} + \left[\left(\frac{R^2 - R_1^2 + R_2^2}{2RR_2}\right)^2 - 1\right]^{1/2}\right\} .$$

The values of μ_k ($k = 1, 2, 3$) depend on the dimensionless parameters R_2/R_1 and G/R_1 ($G = R - R_1 - R_2$). Figure 2 presents the added mass coefficients as functions of G/R_1 for $R_2/R_1 = 0.5$ and 1.

It should be mentioned that Mazur obtained the hydrodynamic forces based on a two dimensional theory. In the case of two vibrating rods in liquid, the fluid field is not two dimensional. However, the three dimensional effect is very small for large wave lengths [5]. Consequently, the two dimensional hydrodynamic forces will be employed in this analysis.

Substituting Eqs. (2) and (3) into (1) yields

$$E_1I_1 \frac{\partial^4 u_1}{\partial x^4} + c_1 \frac{\partial u_1}{\partial t} + (m_1 + \mu_1 M_1) \frac{\partial^2 u_1}{\partial t^2} - \lambda M_1 \mu_3 \left(\frac{R_2}{R}\right)^2 \frac{\partial^2 u_2}{\partial t^2} = f_1 , \quad (5)$$

and

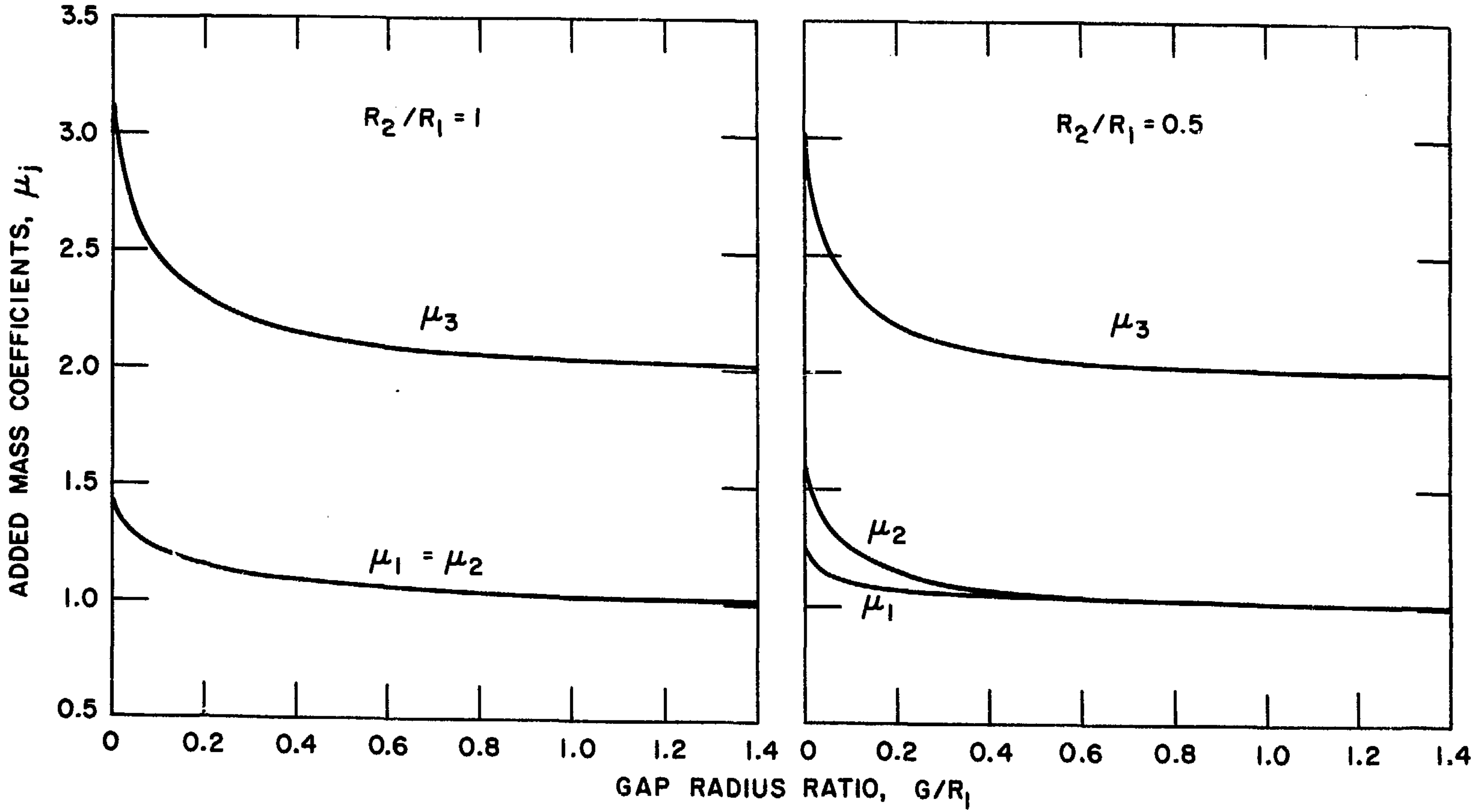

Fig. 2. Calculated added mass coefficients of two rods

$$E_2I_2 \frac{\partial^4 u_2}{\partial x^4} + c_2 \frac{\partial u_2}{\partial t} + (m_2 + \mu_2 M_2) \frac{\partial^2 u_2}{\partial t^2} - \lambda M_2 \mu_3 \left(\frac{R_1}{R}\right)^2 \frac{\partial^2 u_1}{\partial t^2} = f_2 \quad , \qquad (5) \text{ (contd.)}$$

where $\lambda = 1$ for in-plane motion and $\lambda = -1$ for out-of-plane motion. Rod motions in the two planes are uncoupled.

III. FREE VIBRATION

A. Exact Solution

Neglect the damping terms and forcing functions in Eqs. (5) and let

$$u_j = R_j \bar{u}_j \exp(i\omega t) \quad , \tag{6}$$

where $i = \sqrt{-1}$ and ω is the vibration frequency. Substituting Eq. (6) into (5) yields

$$\frac{d^4\bar{u}_1}{dx^4} - \frac{m_1 + \mu_1 M_1}{E_1 I_1}\omega^2\bar{u}_1 + \frac{\lambda M_1 \mu_3 \left(\frac{R_2}{R}\right)^2 \omega^2}{E_1 I_1}\bar{u}_2 = 0 \quad ,$$

and (7)

$$\frac{d^4\bar{u}_2}{dx^4} - \frac{m_2 + \mu_2 M_2}{E_2 I_2}\omega^2\bar{u}_2 + \frac{\lambda M_2 \mu_3 \left(\frac{R_1}{R}\right)^2 \omega^2}{E_2 I_2}\bar{u}_1 = 0 \quad .$$

The solution of Eqs. (7) is

$$\bar{u}_1 = \sum_{n=1}^{8} a_n \exp(p_n x) \quad ,$$

and (8)

$$\bar{u}_2 = \sum_{n=1}^{8} a_n r_n \exp(p_n x) \quad ,$$

where a_n's are arbitrary constants and p_n's are the eight roots of the equation

$$p^8 - \left(\frac{m_1 + \mu_1 M_1}{E_1 I_1} + \frac{m_2 + \mu_2 M_2}{E_2 I_2}\right)\omega^2 p^4$$

$$+\left[\left(\frac{m_1 + \mu_1 M_1}{E_1 I_1}\right)\left(\frac{m_2 + \mu_2 M_2}{E_2 I_2}\right) - \frac{\mu_3^2 M_1 M_2 \left(\frac{R_1 R_2}{R^2}\right)^2}{E_1 I_1 E_2 I_2}\right]\omega^4 = 0 \quad , \tag{9}$$

and

$$r_n = \frac{(m_1 + \mu_1 M_1)\omega^2 - E_1 I_1 p_n^4}{\lambda M_1 \mu_3 \left(\frac{R_2}{R}\right)^2 \omega^2} \quad . \tag{10}$$

Substitution of Eqs. (8) into the boundary conditions at $x = 0$ and $x = \ell$ (ℓ is rod length) yields

$$[b_{mn}]\{a_n\} = \{0\} \quad . \tag{11}$$

The elements b_{mn} depend on frequency, fluid density ρ, material and geometric properties of rods, and end conditions. Therefore, from Eqs. (11), the frequency equation may be written

$$F(\omega,\ E_jI_j,\ m_j,\ R_j,\ \ell,\ \rho,\ G) = 0 \quad . \tag{12}$$

The frequency can be calculated by Eq. (12) and mode shapes by Eqs. (8).

B. Approximate Solution

It is straightforward to obtain the frequencies from the exact frequency equation. However, it is instructive to examine approximations to the exact frequency. First, consider a limiting case: one of the rods is rigid. The equation for free vibration in this case is

$$E_jI_j \frac{\partial^4 u_j}{\partial x^4} + (m_j + \mu_j M_j) \frac{\partial^2 u_j}{\partial t^2} = 0 \quad . \tag{13}$$

Let the natural frequency of the nth mode of the rod j in vacuo be denoted by $\bar{\omega}_{jn}$. It is easily shown from Eq. (13) that the frequency for the rod close to a rigid rod is

$$\omega_{jn} = \frac{\bar{\omega}_{jn}}{\sqrt{1 + \mu_j \beta_j}} \quad , \tag{14}$$

where

$$\beta_j = \frac{M_j}{m_j} \quad .$$

The corresponding modal functions satisfy the relation

$$E_jI_j \frac{\partial^4 \phi_{jn}}{\partial x^4} - \omega_{jn}^2 (m_j + \mu_j M_j) \phi_{jn} = 0 \quad . \tag{15}$$

Thus, the natural frequencies are reduced in proportion to $1/\sqrt{1+\mu_j\beta_j}$, and the modal functions are exactly the same as those in vacuo.

Let

$$u_j = R_j \sum_{n=1}^{\infty} q_{jn}(t)\phi_{jn}(x) \quad , \tag{16}$$

where $\phi_{jn}(x)$ are the orthonormal functions of rods in vacuo. Substituting Eq. (16) into (5) and using the orthogonality condition yield

$$\ddot{q}_{1n} + 2\zeta_{1n}\omega_{1n}\dot{q}_{1n} + \omega_{1n}^2 q_{1n} - \lambda\alpha_1 \sum_m a_{nm}\ddot{q}_{2m} = Q_{1n}(t) \quad ,$$

and (17)

$$\ddot{q}_{2n} + 2\zeta_{2n}\omega_{1n}\dot{q}_{2n} + \omega_{2n}^2 q_{2n} - \lambda\alpha_2 \sum_m a_{mn}\ddot{q}_{1m} = Q_{2n}(t) \quad ,$$

where

$$\alpha_j = \frac{\beta_j\mu_3(R - R_j - G)^3}{R_j R^2(1 + \mu_j\beta_j)} \quad ,$$

$$\zeta_{jn} = \frac{c_j}{2\omega_{jn}(m_j + \mu_j M_j)} \quad ,$$

(18)

$$a_{nm} = \frac{1}{\ell}\int_0^{\ell} \phi_{1n}\phi_{2m}dx \quad ,$$

and

$$Q_{jn} = \frac{1}{\ell R_j(m_j + \mu_j M_j)}\int_0^{\ell} f_j(x,t)\phi_{jn}dx \quad .$$

For free vibration, neglect the damping and forcing terms and let

$$q_{jn} = \bar{q}_{jn}\exp(i\omega t) \quad . \tag{19}$$

Substitution of Eqs. (19) into (17) gives

$$(\omega^2 - \omega_{1n}^2)\bar{q}_{1n} - \lambda\alpha_1\omega^2 \sum_m a_{nm}\bar{q}_{2m} = 0 \quad ,$$

and (20)

$$(\omega^2 - \omega_{2n}^2)\bar{q}_{2n} - \lambda\alpha_2\omega^2 \sum_m a_{mn}\bar{q}_{1m} = 0 \quad .$$

Equations (20) consist of an infinite number of ordinary equations. However, typically, only a finite number of equations is taken from case to case according to the desired accuracy. The frequency equation obtained from Eqs. (20) is

$$\left|\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right| = 0 \quad , \tag{21}$$

where the elements of the matrices A, B, C, and D are

$$\begin{aligned} A_{mn} &= (\omega^2 - \omega_{1n}^2)\delta_{mn} \quad , \\ B_{mn} &= -\lambda\alpha_1\omega^2 a_{nm} \quad , \\ C_{mn} &= -\lambda\alpha_2\omega^2 a_{mn} \quad , \end{aligned} \tag{22}$$

and

$$D_{mn} = (\omega^2 - \omega_{2n}^2)\delta_{mn} \quad .$$

When the two rods have the same type of boundary conditions,

$$\phi_{1n} = \phi_{2n} \quad ,$$

and (23)

$$a_{mn} = \delta_{mn},$$

thus, Eqs. (20) become

$$\begin{bmatrix} \omega^2 - \omega_{1n}^2 & -\lambda\alpha_1\omega^2 \\ -\lambda\alpha_2\omega^2 & \omega^2 - \omega_{2n}^2 \end{bmatrix} \begin{Bmatrix} \bar{q}_{1n} \\ \bar{q}_{2n} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad . \tag{24}$$

The frequency equation becomes

$$(1 - \alpha_1\alpha_2)\omega^4 - (\omega_{1n}^2 + \omega_{2n}^2)\omega^2 + \omega_{1n}^2\omega_{2n}^2 = 0 \quad . \tag{25}$$

Eq. (25) gives two frequencies, Ω_{1n} and Ω_{2n},

$$\Omega_{1n} = \left\{(\omega_{1n}^2 + \omega_{2n}^2) - [(\omega_{1n}^2 - \omega_{2n}^2)^2 + 4\alpha_1\alpha_2\omega_{1n}^2\omega_{2n}^2]^{1/2}\right\}^{1/2} \Big/ [2(1-\alpha_1\alpha_2)]^{1/2} ,$$

and (26)

$$\Omega_{2n} = \left\{(\omega_{1n}^2 + \omega_{2n}^2) + [(\omega_{1n}^2 - \omega_{2n}^2)^2 + 4\alpha_1\alpha_2\omega_{1n}^2\omega_{2n}^2]^{1/2}\right\}^{1/2} \Big/ [2(1-\alpha_1\alpha_2)]^{1/2} .$$

The amplitude ratio $\bar{q}_{2n}/\bar{q}_{1n}$ is given by

$$\frac{\bar{q}_{2n}}{\bar{q}_{1n}} = \frac{\Omega_{jn}^2 - \omega_{1n}^2}{\lambda\alpha_1\Omega_{jn}^2} . \tag{27}$$

From Eq. (25) it is easily shown that if

$$\alpha_1\alpha_2 < 1 , \tag{28}$$

the following relations are satisfied:

$$\Omega_{1n} < \omega_{1n}, \omega_{2n} ,$$

and (29)

$$\Omega_{2n} > \omega_{1n}, \omega_{2n} .$$

Equation (28) is found to be satisfied in all cases. Thus, the frequencies of the coupled modes may be lower or higher than those of the uncoupled modes.

When the two tubes are identical, $\omega_{1n} = \omega_{2n}$ and $\alpha_1 = \alpha_2$; therefore, Eqs. (26) and (27) reduce to

$$\Omega_{1n} = \frac{\omega_{1n}}{\sqrt{1+\alpha_1}} , \qquad \frac{\bar{q}_{2n}}{\bar{q}_{1n}} = -\lambda ;$$

and (30)

$$\Omega_{2n} = \frac{\omega_{1n}}{\sqrt{1-\alpha_1}} , \qquad \frac{\bar{q}_{2n}}{\bar{q}_{1n}} = \lambda .$$

It is obvious that Ω_{1n} and Ω_{2n} satisfy the inequality given in Eqs. (29).

C. Qualitative Results and Numerical Examples

Some general conclusions can be drawn from the analyses:

(1) From Eqs. (16), (19), (20), (23), and (24), it is seen that if the boundary conditions of the two rods are different, the axial mode shapes of the individual rods during coupled rod motion will be different from those of the individual rods in vacuo. However, if the rods have the same type of boundary and are of the same length, the axial mode shapes of the coupled modes will be the same as the individual rods in vacuo.

(2) Equations (21) and (25) reveal that the frequencies are independent of λ; therefore, the frequencies of in-plane motion and out-of-plane motion are the same. In Ω_{1n}, $\bar{q}_{2n}/\bar{q}_{1n}$ is negative for in-plane motion ($\lambda = 1$) and positive for out-of-plane motion ($\lambda = -1$); while, in Ω_{2n}, $\bar{q}_{2n}/\bar{q}_{1n}$ is positive for in-plane motion and negative for out-of-plane motion. That is, there exist out-of-phase modes, in which the rods move in opposite directions, and in-phase modes, in which the rods move in the same direction (see Fig. 3). Furthermore, the frequency of the out-of-phase mode of in-plane motion is the same as that of in-phase mode of out-of-plane motion, while, the frequency of the in-phase mode of in-plane motion is the same as that of the out-of-phase mode of out-of-plane motion.

(3) Equations (29) show that when two rods are not identical, the frequency obtained by assuming that the rod with higher natural frequency is rigid is the upper bound of the frequency of out-of-phase mode of in-plane motion and in-phase mode of out-of-plane motion, while the frequency obtained by assuming that the rod with lower natural frequency is rigid is the lower bound of the frequency of in-phase mode of in-plane motion and out-of-phase mode of out-of-phase motion.

(4) The coupling between two rods depends on the radius ratio R_2/R_1, gap-radius ratio G/R_1 and mass ratio β_j. When β_j is small and G/R_1 is large, the two rods will respond independently.

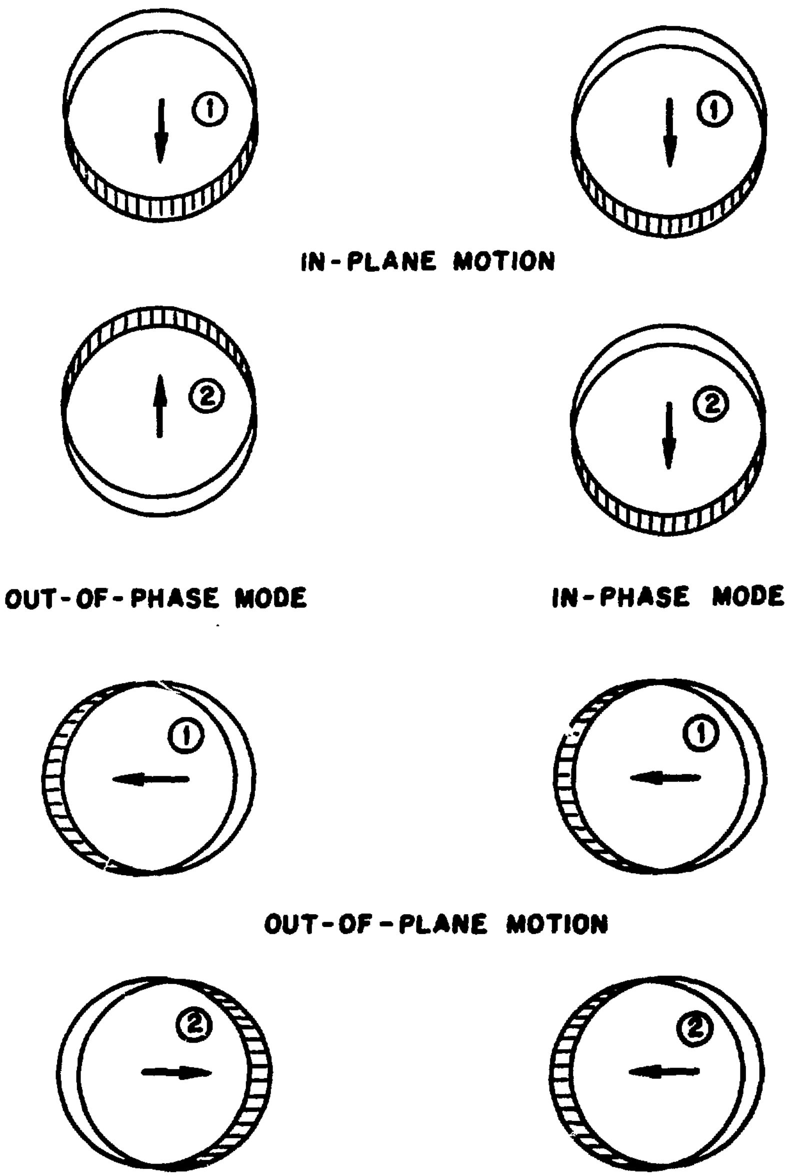

Fig. 3. Four normal modes of two rods vibrating in a liquid

(5) When two rods have the same type of boundary conditions, the exact solution for frequencies is obtained in closed form as given in Eqs. (26). For rods with different end conditions, the frequencies may be computed from the exact frequency equation (12) or the approximate equation (21).

Figure 4 shows the frequencies as functions of gap-radius ratio (G/R_1) for two identical rods vibrating in water. The rods are two identical steel tubes whose outside radius is 1.270 cm (0.5 in.), wall thickness 0.1588 cm (0.0625 in.), and length 1.27 m (50 in.), and are simply supported at both ends. For in-plane motion, the odd-numbered modes are associated with out-of-phase motion and even-numbered modes are associated with in-phase motion, as illustrated in Fig. 4. For out-of-plane motion, the odd-numbered modes correspond to in-phase modes, and the even-numbered modes correspond to out-of-phase modes. From the figure, it is clear that the frequencies of the coupled modes deviate from those of the individual rods considerably when G/R_1 is small. As G/R_1 increases, the interactions between the two rods vs. fluid become small and for large G/R_1, they will respond independent of each other.

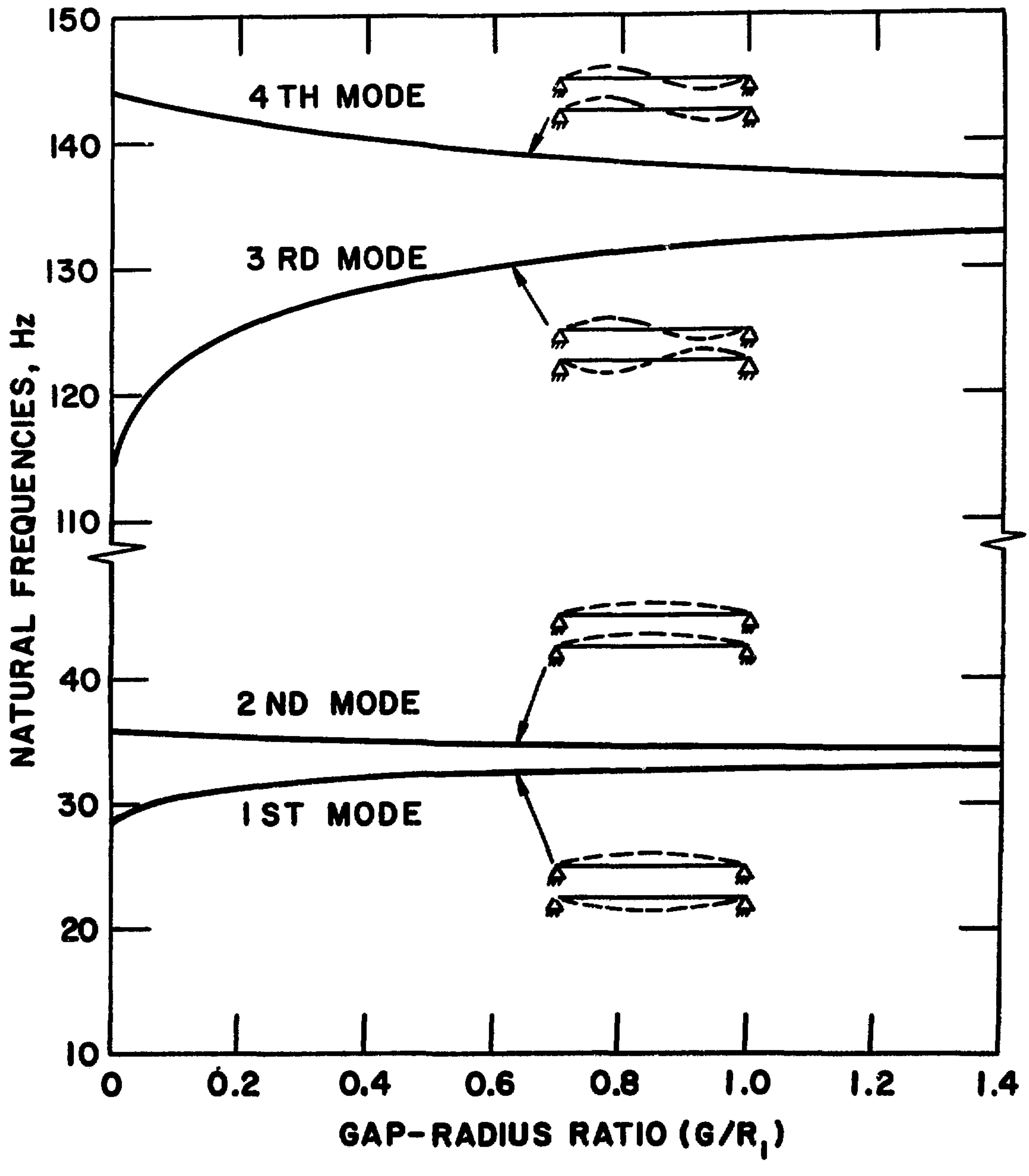

Fig. 4. Natural frequencies of two identical rods vibrating in water (Illustrations are for in-plane motion)

IV. FORCED VIBRATION

The steady state response of two rods is considered. It is assumed that two simply-supported rods are subjected to an excitation of the following form:

$$f_1(x,t) = g_1(x)\sin\omega t \quad ,$$

and (31)

$$f_2(x,t) = g_2(x)\sin(\omega t+\psi) \quad .$$

The response can be obtained from Eqs. (16) and (17). In this case, Eqs. (17) become

$$\ddot{q}_{1n} + 2\zeta_{1n}\omega_{1n}\dot{q}_{1n} + \omega_{1n}^2 q_{1n} - \lambda\alpha_1\ddot{q}_{2n} = \bar{Q}_{1n}\sin\omega t \quad ,$$

and

$$\ddot{q}_{2n} + 2\zeta_{2n}\omega_{2n}\dot{q}_{2n} + \omega_{2n}^2 q_{2n} - \lambda\alpha_2\ddot{q}_{1n} = \bar{Q}_{2n}\sin(\omega t+\psi) \quad , \tag{32}$$

where

$$\bar{Q}_{jn} = \frac{1}{\ell R_j(m_j+\mu_j M_j)}\int_0^\ell g_j(x)\phi_{jn}dx \quad .$$

The solutions of Eqs. (32) are easily obtained:

$$q_{1n}(t) = \alpha_{1n}\sin(\omega t) + \beta_{1n}\cos(\omega t) \quad ,$$

and (33)

$$q_{2n}(t) = \alpha_{2n}\sin(\omega t) + \beta_{2n}\cos(\omega t) \quad ,$$

where α_{1n}, α_{2n}, β_{1n}, and β_{2n} are solutions of the following equations:

$$\begin{bmatrix} \omega_{1n}^2-\omega^2 & -2\zeta_{1n}\omega_{1n}\omega & \lambda\alpha_1\omega^2 & 0 \\ 2\zeta_{1n}\omega_{1n}\omega & \omega_{1n}^2-\omega^2 & 0 & \lambda\alpha_1\omega^2 \\ \lambda\alpha_2\omega^2 & 0 & \omega_{2n}^2-\omega^2 & -2\zeta_{2n}\omega_{2n}\omega \\ 0 & \lambda\alpha_2\omega^2 & 2\zeta_{2n}\omega_{2n}\omega & \omega_{2n}^2-\omega^2 \end{bmatrix} \begin{bmatrix} \alpha_{1n} \\ \beta_{1n} \\ \alpha_{2n} \\ \beta_{2n} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{1n} \\ 0 \\ \bar{Q}_{2n}\cos\psi \\ \bar{Q}_{2n}\sin\psi \end{bmatrix} \quad . \tag{34}$$

Substituting Eqs. (33) into (16) yields

$$u_j = R_j \Phi_j(x) \sin[\omega t + \psi_j(x)] \quad ,$$

where

$$\Phi_j(x) = \left\{ \left[\sum_{n=1}^{\infty} \alpha_{jn} \phi_{jn}(x) \right]^2 + \left[\sum_{n=1}^{\infty} \beta_{jn} \phi_{jn}(x) \right]^2 \right\}^{1/2} \quad , \tag{35}$$

and

$$\psi_j(x) = \tan^{-1} \left[\frac{\sum_{n=1}^{\infty} \alpha_{jn} \phi_{jn}(x)}{\sum_{n=1}^{\infty} \beta_{jn} \phi_{jn}(x)} \right] \quad .$$

The rod displacements and other quantities of interest can be calculated from Eqs. (35).

For illustration, the two steel rods considered earlier in free vibration study are subjected to in-plane harmonic forces of the form given by Eq. 31 with magnitudes g_1 = 0.179 gm/cm (0.001 lb/in.) and $g_2 = 0$. It is assumed that $\zeta_{1n} = \zeta_{2n} = 0.05$. The responses of the two rods at midspan are shown in Figs. 5 and 6 where the phase angle is plotted for $\psi_j(\ell/2)$ and the amplification factor is defined as the ratio of the displacement amplitude to that of the deflection of rod 1 subject to a static load of the same magnitude. In Fig. 6, the gap-radius ratio (G/R_1) is equal to 0.1, the two rods are strongly coupled, and the two peaks of the response occur close to the frequencies of the first out-of-phase and in-phase modes. Note that the peak response of the rod 2 is almost the same as rod 1, although it is not subject to any direct excitation. It is also noted at small frequencies, the two rods vibrate out-of-phase, while at frequencies higher than the first two natural frequencies, they vibrate in-phase. In Fig. 5, G/R_1 = 1.0, and from a comparison with results in Fig. 6, it can be seen that as two rods move farther apart, the coupling effect becomes small

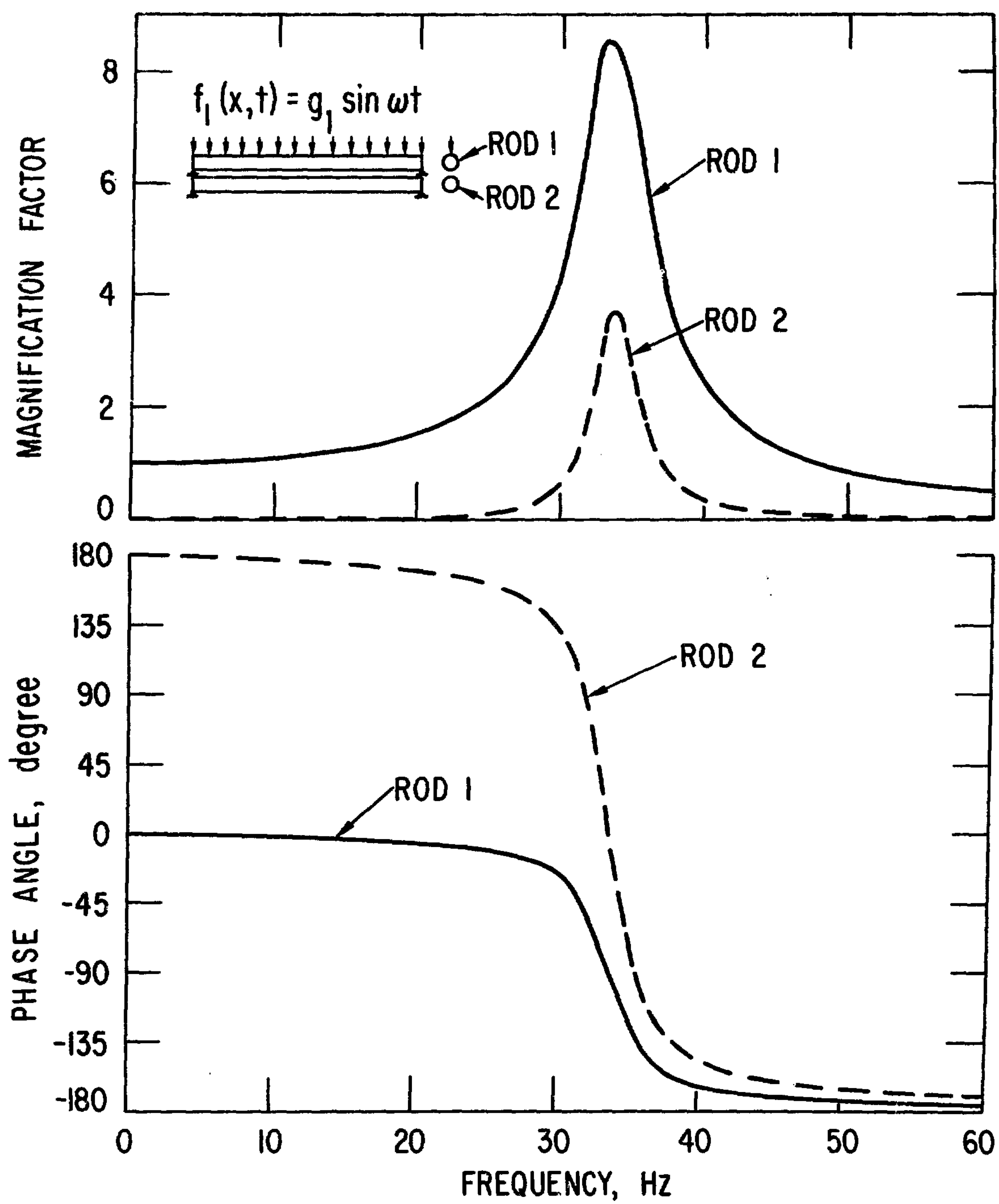

Fig. 5. Frequency response of two identical rods in liquid for $G/R_1 = 1.0$

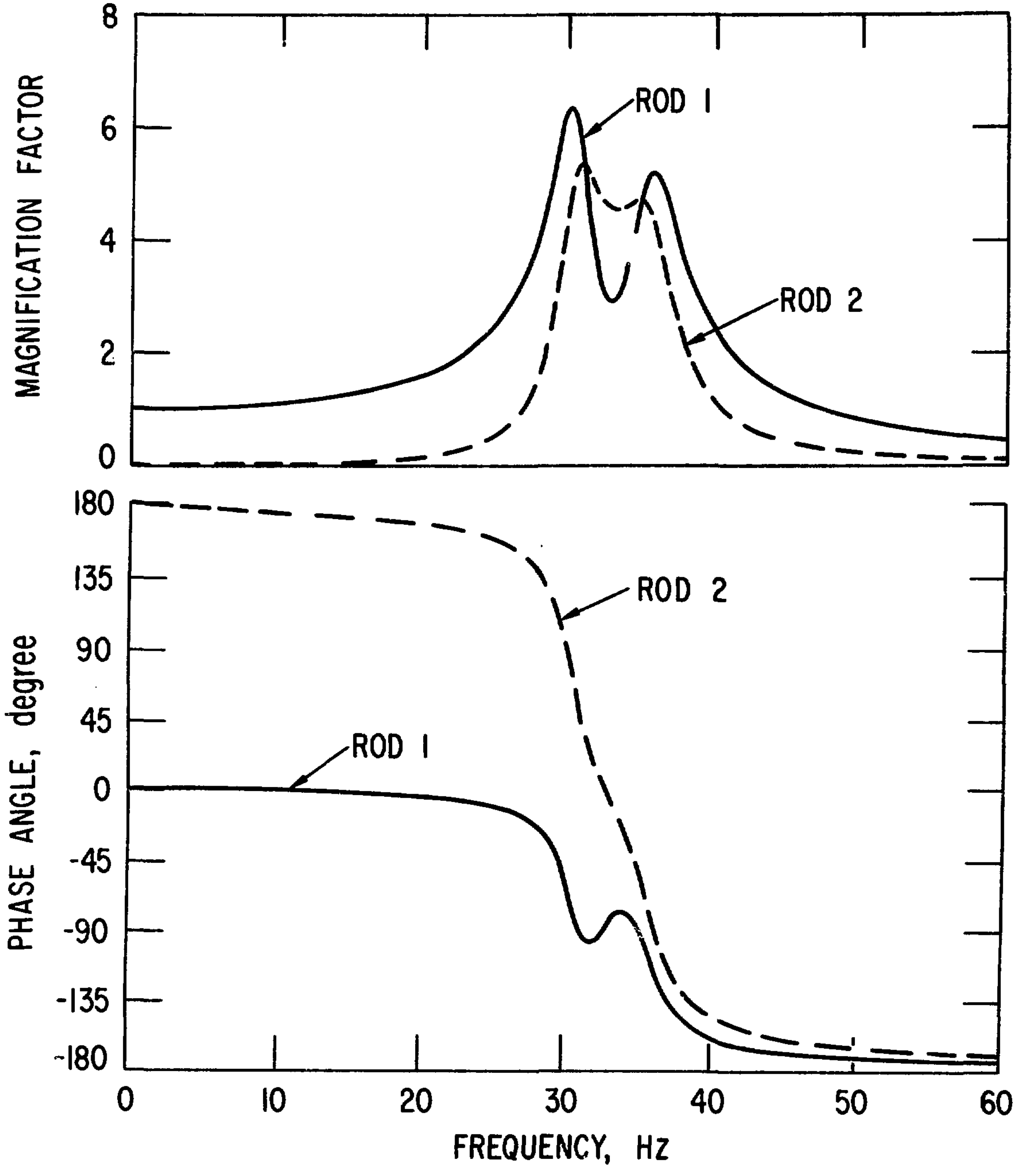

Fig. 6. Frequency response of two identical rods in liquid for $G/k_1 = 0.1$

and the response of rod 2 is much smaller than that of rod 1. It is interesting to note that the peak response of rod 1 is much larger when the two rods are far apart than when they are close to each other.

V. RESPONSE TO FLOW EXCITATIONS

The vibration and stability of two cylinders subjected to parallel and cross flows have been considered. Details from the parallel flow study will not be presented in this paper. However, two important conclusions were reached: (1) due to the fluid coupling effect, the lowest frequency of a two-cylinder system is lower than that of the individual cylinders subjected to the same flow condition; and (2) the critical flow velocity of a coupled cylinder system is reduced due to the interactions of the two cylinders. The implication of these results is that using an isolated cylinder as a model for fuel assembly or heat exchanger tube vibration study is not conservative.

A few experimental studies have been performed on the flows around a pair of parallel cylinders at various separations (e.g., [6] and [7]). The main conclusion of these studies is as follows: When the two cylinders are separated by a gap larger than the cylinder diameter, the frequency of vortex shedding is the same as that for a single cylinder, while for a gap less than the radius of the cylinder, the shedding frequency appears to be associated with a body width equal to twice the cylinder diameter. At the intermediate gap width, two shedding frequencies are found. The experimental data give the following results for the main sequence of vortex shedding frequency f_v:

$$
\begin{aligned}
f_v &= \frac{0.1\,V}{R} && \text{for} \quad \frac{G}{2R} \geq 1 \;, \\
f_v &= \frac{V}{2R}\left[0.09 + \left(\frac{G/2R - 0.2}{G/2R + 2}\right)^2\right] && \text{for} \quad G/2R \leq \frac{11}{10} \;, \\
f_v &= \frac{0.1\,V}{2R} && \text{for} \quad \frac{G}{2R} \sim 0 \;,
\end{aligned}
\tag{36}
$$

where V is flow velocity. These experimental results are utilized to find the response of two cylinders to crossflow excitation.

For illustration, the two steel rods considered in free vibration study are subjected to a crossflow for $x < \ell/15$. The lift forces acting on the cylinders are assumed to be

$$
\begin{aligned}
f_1(x,t) &= \rho V^2 R \sin(2\pi f_v t) \\
f_2(x,t) &= \rho V^2 R \sin(2\pi f_v t + \psi)
\end{aligned}
\tag{37}
$$

where ψ is equal to 0 or 180° [6]. Two cases are presented: (a) $G/R = 2.0$; and (b) $G/R = 0.2$. The rod displacements and phase angles at midspan are shown in Figs. 7 and 8 as functions of flow velocity for $\psi = 180°$. It is seen that the responses for the two gaps are completely different. For $G/R = 2.0$, the two cylinders are only weakly coupled, such that the responses are practically the same as that of an isolated cylinder. For $G/R = 0.2$, the cylinders are strongly coupled; resonance occurs at a flow velocity approximately twice that for a single cylinder and the responses are much larger.

When $\psi = 180°$, the two cylinders always move out-of-phase. For $\psi = 0°$, similar results are obtained except the two cylinders vibrate in-phase. Other calculations have been made for two cylinders with different flexural rigidity. In this case, both in-phase and out-of-phase modes are excited and there are two peaks in the response curves.

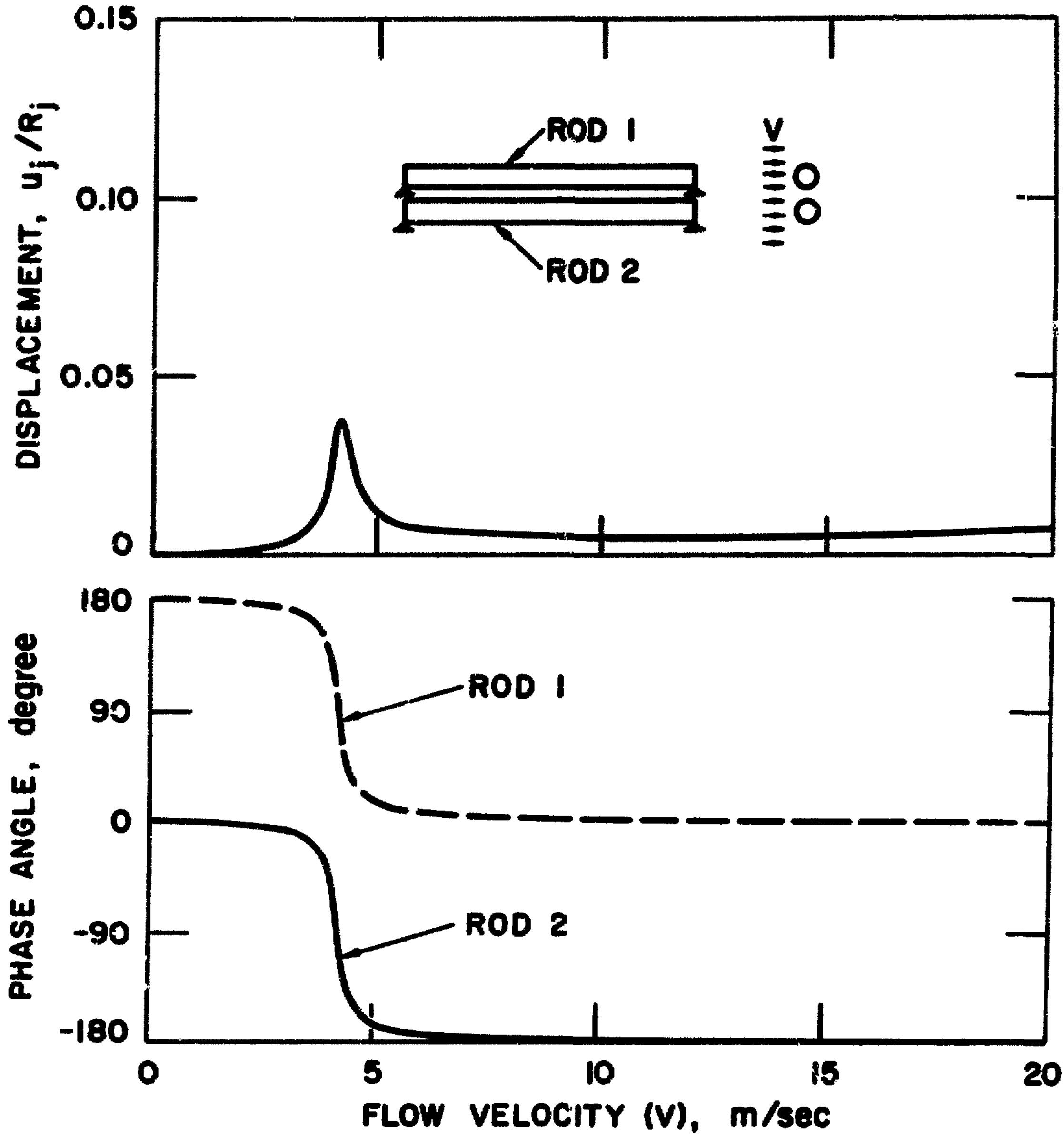

Fig. 7. Response of two identical rods subjected to cross flow for $G/R_1 = 2.0$

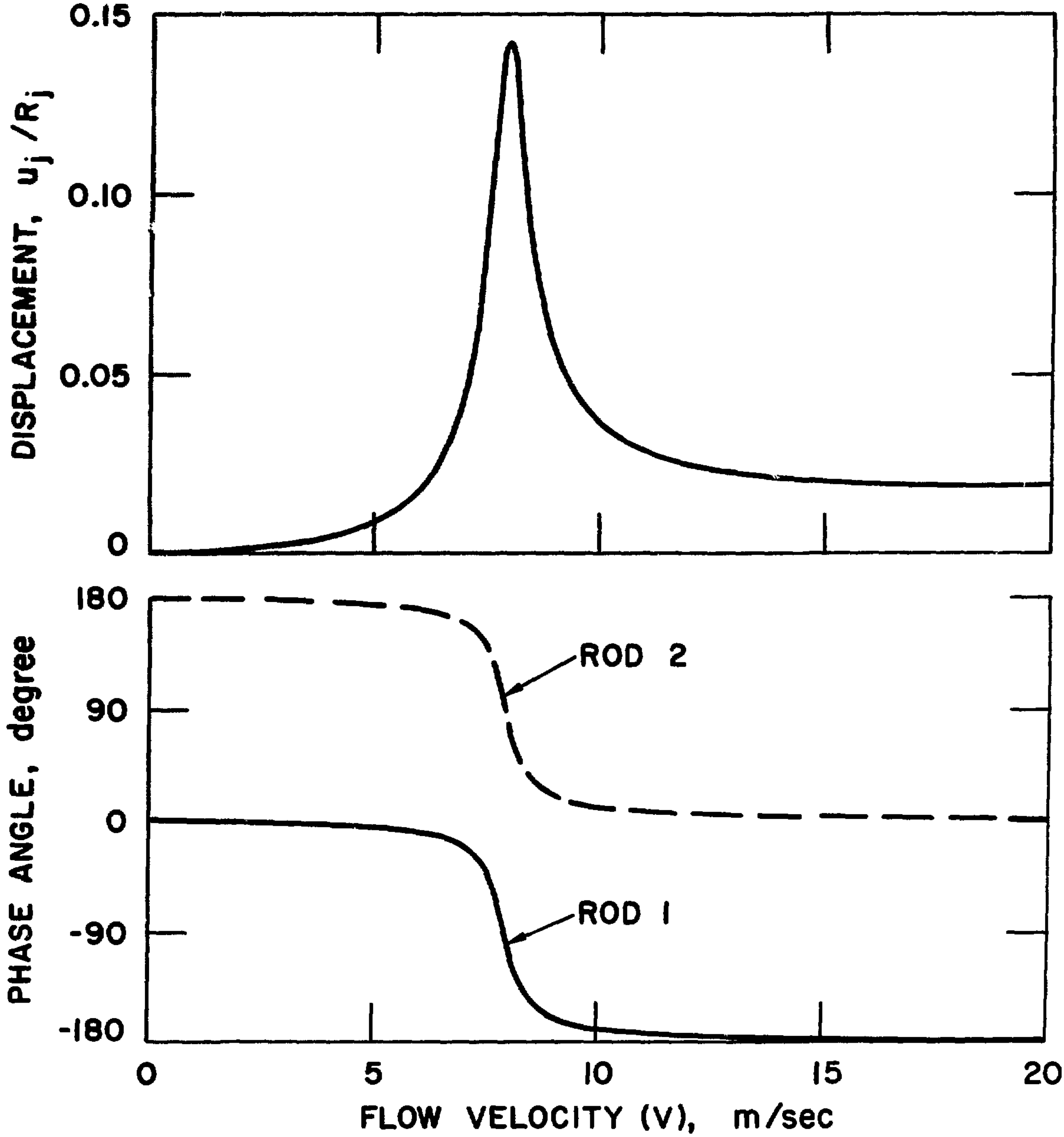

Fig. 8. Response of two identical rods subjected to cross flow for $G/R_1 = 0.2$

VI. CONCLUDING REMARKS

Various models of parallel-flow-induced vibration of fuel rods and crossflow-induced vibration of heat exchanger tubes were developed without considering fluid coupling effects. The results of this analysis illustrate that neglecting the fluid coupling is, in general, not conservative. Therefore, fluid coupling effects should be included in the development of mathematical models for vibrations of fuel bundles and heat exchanger tube banks. Several specific conclusions can be drawn from this study:

1. A two-cylinder system can vibrate in plane or out of plane. In free vibration, motions in two planes are not coupled.

2. Natural frequencies of in-plane motion and out-of-plane motion are the same. However, mode shapes are different. The frequency of in-phase mode of out-of-plane motion is equal to that of the out-of-phase mode of in-plane motion, while the frequency of the out-of-phase mode of out-of-plane motion is equal to that of in-phase mode of in-plane motion.

3. The lowest frequency of the system is associated with the in-phase mode of out-of-plane motion and out-of-phase mode of in-plane motion and is lower than the lowest frequency of uncoupled modes associated with the constituent cylinders.

4. Fluid coupling effects depend on the gap-radius ratio. As the gap decreases, fluid coupling effects increase.

5. The dynamic characteristics of two cylinders subjected to fluid flows are different from those of a single cylinder.

An experimental study to verify the theory has been performed and will be reported in the future. Primary results show that analytical results and experimental data are in good agreement. Efforts also are being made to improve the models for vibrations of reactor fuel bundles and heat exchanger tube banks including fluid coupling effects.

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