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**Knolls
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Critical Flow Velocities for Collapse of Reactor Parallel-Plate Fuel Assemblies

D. R. Miller
August 26, 1958

Operated for the
United States Atomic
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CRITICAL FLOW VELOCITIES FOR COLLAPSE OF REACTOR
PARALLEL-PLATE FUEL ASSEMBLIES

Daniel R. Miller

August 26, 1958

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Date

General Electric Company
KNOLLS ATOMIC POWER LABORATORY
Schenectady, New York
Operated for the
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ABSTRACT

Theoretical formulas are presented for prediction of the flow velocity at which collapse occurs in long parallel-plate assemblies. Beyond the critical velocity the pressure-unbalance forces developed as a consequence of a small deflection exceed the corresponding elastic restraining forces, and the plates collapse. Both flat- and curved-plate assemblies are considered, and the applicability of the formulas to design of reactor fuel-plate assemblies is discussed.

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The author wishes to acknowledge assistance from his colleague, Mr. E. B. Johansson, who performed the analysis of plates with longitudinally spaced supports and the analysis of the effect of external compressive loads on a complete flat-plate assembly, added to the author's understanding of some aspects of the problem and reviewed the author's derivations.

CRITICAL FLOW VELOCITIES FOR COLLAPSE OF REACTOR PARALLEL-PLATE FUEL ASSEMBLIES

Daniel R. Miller

INTRODUCTION

The collapse of reactor-fuel-plate mockups was observed in tests conducted at Oak Ridge ten years ago (1)*, and this problem has been encountered more recently in the original fuel elements for the Engineering Test Reactor (ETR) (2). When the flow velocities in the Oak Ridge tests were raised to sufficiently high values, flat plates were deformed plastically, and curved plates were buckled. The flow channels between plates were reported to have been "almost or even completely closed through part or all of the length of the plates." The ETR reactor was initially operated below its rated flow velocity to avoid collapse of its fuel plates.

The analysis which follows applies the Bernoulli theorem for incompressible flow to determine the pressure differences which would be developed across the plates by modification of local fluid velocities as a result of plate deflections. Equating these pressure differences to those which would be required to produce the corresponding deflections gives an expression for the critical velocity in terms of plate and fluid parameters.

ASSUMPTIONS USED IN ANALYSIS

Unless otherwise stated, the following assumptions were made in the derivation of the formulas for critical velocities:

1. The plates are homogeneous, isotropic, elastic, initially flat or uniformly curved, uniform in spacing and dimensions and free of unidentified sources of deformation.
2. The coolant is incompressible; all channels have the same mass flow; at any cross section normal to the longitudinal axis the flow within any channel is uniform; and leakage between channels is suppressed.

* Numbers in parentheses refer to similarly numbered references in Bibliography at end of paper.

3. Plates are broad enough (in comparison with their thickness) so that shear deformation is negligible, and are long enough (in comparison with their breadth) so that they can deflect locally without significant redistribution of flow among the coolant channels.
4. Side plates or supports are rigid.

NOMENCLATURE

The following nomenclature is used in this report:

- a = plate thickness
- b = width of flat plate or chord of curved plate
- g = gravitational acceleration
- h = initial flow channel thickness at midspan
- l = longitudinal distance between plate-edge supports
- P = pressure difference tending to bend or stretch fuel plate
- p = pressure difference imposed across plates due to irregularity in configuration or flow distribution
- x = longitudinal distance from edge support
- y = deflection of plate relative to supports
- y' = deflection of curved plate relative to midspan tangent
- A = area of beam cross section per unit width of beam
- C = a function of α defined by equation (41)
- E = Young's modulus of elasticity
- I = moment of inertia of beam cross section per unit width of beam
- M = moment tending to straighten curved plate per unit width of beam
- N = membrane tensile load in curved plate per unit width of beam
- P = membrane compressive load in flat plate per unit axial length of plate

- P_c = critical value of P in absence of other loads
 Q_o = original cross-sectional area of coolant channel
 ΔQ = change in Q_o
 R = initial mean radius of curvature of curved plate
 V_1, V_2 = linear velocity of coolant in deformed channels
 V_o = linear velocity of coolant in undeformed channel
 V_c = value of V_o for which collapse occurs due to flow-induced effects alone
 V'_c = collapse velocity with superimposed transverse compression
 V_{rf} = ratio of V_c for curved-plate assembly to V_c for flat-plate assembly for the case of fixed longitudinal edges
 V_{rh} = same as V_{rf} except applicable to hinged longitudinal edges
 V_{rfh} = ratio of V_c for curved-plate assembly with fixed longitudinal edges to V_c for curved-plate assembly with hinged longitudinal edges
 $\frac{X}{2}$ = shear force at edge of curved plate per unit width of beam
 2α = curved-plate arc between supports
 γ, ϕ = angles of curved-plate arc measured from midspan
 σ_h, ϵ_h = membrane stress and strain in curved plate in direction of its width
 ρ = weight density of coolant
 ν = Poisson's ratio

DERIVATION OF FORMULAS FOR CRITICAL FLOW VELOCITY

Case A - Flat Plates with Built-In Edges

This case represents a configuration which has been used or proposed for many reactors. On the order of ten similar plates with length large in comparison with width are assembled with uniform spacing and with their long edges restrained by attachment to side plates. Figure 1 shows schematically the shape of the original transverse cross section of such an assembly and also shows the nature of the distortion which is postulated at the lowest critical flow velocity. It should be noted

that the deflections of adjacent plates are in opposite directions. This condition corresponds to a lower critical velocity than would be characteristic of the case of a single plate which divides a rectangular duct into two channels, with the same plate and channel dimensions as the multiple-plate assembly.

At the critical velocity the modifications of local linear velocity resulting from a deflection of the plates produce across the plates a pressure difference sufficient to maintain the deflection. If the plates are deflected over a longitudinal distance several times the width of the plates, then near the center of the deflected portion of a plate the longitudinal variations in deflection can be neglected, and the deflection curve of the plate shown in Figure 2, modified from the narrow-beam deflection curve from (3) by the use of the factor $1 - \nu^2$ for a wide beam is

$$y = \frac{p x^2 (1 - \nu^2) (x^2 - 2bx + b^2)}{24 E I} \quad (1)$$

Integrating Equation (1) and multiplying by two to determine the total change in channel cross-sectional area gives

$$\Delta Q = \frac{(1 - \nu^2) p b^5}{360 E I} \quad (2)$$

Substitution of the moment of inertia of a strip of unit width in Equation (2) and division by bh gives the per-unit change in channel cross-section,

$$\frac{\Delta Q}{Q_0} = \frac{(1 - \nu^2) p b^4}{30 E a^3 h} \quad (3)$$

The pressure differential developed across the plate away from the upstream and downstream ends of the deformed regions, derived from Bernoulli's theorem, is

$$p = \frac{\rho}{2g} \left[\left(\frac{V_0}{1 - \frac{\Delta Q}{Q_0}} \right)^2 - \left(\frac{V_0}{1 + \frac{\Delta Q}{Q_0}} \right)^2 \right]$$

which, as $\Delta Q/Q_0$ approaches zero, reduces to

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FIGURE 1. OUTLINES OF PLATE ASSEMBLY

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FIGURE 2. LOAD DEFLECTION DIAGRAM

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FIGURE 3. MODE OF DISTORTION WITH
LONGITUDINALLY-SPACED
EDGE SUPPORTS

$$p = \frac{2\rho V_o^2 \Delta Q}{g Q_o} \quad (4)$$

Substitution of (3) in (4) determines the critical velocity,

$$V_c = \left[\frac{15 g E a^3 h}{\rho b^4 (1 - \nu^2)} \right]^{1/2} \quad (5)$$

Case B - Flat Plates with Simply-supported Edges

The configuration considered here is the same as for Case A except that the long edges of the plates are simply supported. It is assumed that means are provided to prevent leakage from one channel to another without additional restraint on the plates.

Use of the expression for the deflection curve of a uniformly loaded wide beam with simply-supported ends and the procedures of Case A leads to the following expression for the critical velocity:

$$V_c = \left[\frac{5 g E a^3 h}{2 \rho b^4 (1 - \nu^2)} \right]^{1/2} \quad (6)$$

Case C - Flat Plates With Longitudinally-spaced Supports

Fuel plates might be supported only at uniform intervals along their long edges. This arrangement would introduce the possibility that the lowest critical velocity will correspond to deflections which vary along the length instead of across the width of the plate. The case of this type which is most easily analyzed is that in which the plate deflection varies only in the axial direction, and the simple supports only provide local control of spacing. This mode of collapse would be approximated if the supports were spaced at longitudinal intervals long in comparison with the plate width, so that transverse curvatures would be small in comparison with longitudinal curvature. Figure 3 shows a sketch of the plates and their deflection curves. In this case all plates would tend to deform in a similar manner over their full length with adjacent plates deflecting in opposite directions at any longitudinal position.

The velocity at any cross section in an enlarged channel is

$$V_1 = \frac{V_o h}{h + 2y} \quad (7)$$

and in a constricted channel is

$$V_2 = \frac{V_o h}{h - 2y} \quad (8)$$

The pressure difference across the plates, from Bernoulli's theorem and Equations (7) and (8) is

$$p = \frac{\rho V_o^2}{2g} \left[\left(\frac{1}{1 - \frac{2y}{h}} \right)^2 - \left(\frac{1}{1 + \frac{2y}{h}} \right)^2 \right] \quad (9)$$

In the limit as y/h approaches zero, (9) becomes

$$p = \frac{4\rho V_o^2 y}{gh} \quad (10)$$

The equation for the deflection curve of one of the plates is

$$\left[\frac{EI}{(1 - \nu^2)} \right] \left[\frac{d^4 y}{dx^4} \right] = p \quad (11)$$

Equating (10) and (11) gives

$$\left[\frac{EI}{(1 - \nu^2)} \right] \left[\frac{d^4 y}{dx^4} \right] = \frac{4\rho V_o^2 y}{gh} \quad (12)$$

The solution of (12) corresponding to the simplest mode of deflection and the lowest critical velocity is

$$y = B \sin \frac{\pi x}{\ell} \quad (13)$$

where B is an arbitrary constant.

Substitution of (13) and the expression for I in terms of plate thickness in (12) gives the lowest critical velocity

$$V_c = \left[\frac{\pi^4 g E a^3 h}{48 \rho \ell^4 (1 - \nu^2)} \right]^{1/2} \quad (14)$$

It is of interest to note that in this case the local differential-pressure loading on the beam is proportional to the local deflection of the beam, as it is for the free vibration of a beam. Because of this similarity, the critical velocities for various deflection modes have the same ratios as the corresponding natural frequencies of beam vibrations. Using natural frequency formulas from Den Hartog (6), the critical velocity for a cantilever and for a beam on fixed end supports can be obtained by multiplying Equation (14) by 3.52/9.87 and 22.0/9.87 respectively.

Case D - Curved Plates With Fixed Edges

The assembly considered here consists of several plates whose long edges are fixed and whose cross sections normal to the direction of flow have an initially uniform curvature. Cross sections of such an assembly are illustrated in Figure 4 in the original and in the deformed conditions.

Before deriving an expression for the deflection of the curved plate under the action of a uniform pressure differential, an expression is needed for the bending moment per unit of beam width. The latter can be derived by using Castigliano's first theorem, following the procedure given by Timoshenko (4) for the case of centrifugal loading of a flywheel. The present case is analogous to that of a flywheel with rigid spokes and an elastic rim. Figure 5 shows a representation of half of a plate cross section and identifies some of the nomenclature used in the analysis.

The following expressions were obtained by the author:

$$x = \frac{pR}{\beta_1} \quad (15)$$

$$M_o = \frac{pR^2}{2\beta_1} \left(\frac{1}{\alpha} - \csc \alpha \right) \quad (16)$$

$$N_o = pR \left(1 - \frac{\csc \alpha}{2\beta_1} \right) \quad (17)$$

$$M_1 = \frac{pR^2}{2\beta_1} \left(\frac{1}{\alpha} - \cot \alpha \right) \quad (18)$$

$$N_1 = pR \left(1 - \frac{\cot \alpha}{2\beta_1} \right) \quad (19)$$

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FIGURE 4. OUTLINE OF PLATE ASSEMBLY

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FIGURE 5. DIAGRAM FOR CURVED-PLATE
DEFLECTION ANALYSIS

$$M = \frac{p R^2 \sin^2 \frac{\phi}{2}}{\beta_1 \sin \alpha} - \frac{p R^2}{2 \beta_1} \left(\frac{1}{\sin \alpha} - \frac{1}{\alpha} \right) \quad (20)$$

$$N = p R \left(1 - \frac{\cos \phi}{2 \beta_1 \sin \alpha} \right) \quad (21)$$

where

$$\beta_1 = \frac{A R^2}{I} f_2(\alpha) + f_1(\alpha) \quad (22)$$

$$f_1(\alpha) = \frac{1}{2 \sin^2 \alpha} \left(\frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right) \quad (23)$$

$$f_2(\alpha) = f_1(\alpha) - \frac{1}{2\alpha} \quad (24)$$

The solution shown by Timoshenko includes the coefficient $2/3$ multiplying the right-hand side of Equation (15). The author's solution checks the flat-beam expression for X where α approaches zero, whereas the solution shown by Timoshenko does not.

The displacements of the curved plate in a direction normal to a midspan tangent define an approximate deflection curve from which the change in flow channel cross section can be determined. This procedure neglects the relatively small displacements parallel to the specified tangent. Neglecting deformation due to shear and assuming the plate thickness to be very small in comparison with the radius of curvature, it can be shown that the following defines the displacements normal to and relative to the midspan tangent to the plate as shown in Figure 5:

$$y' = - \int_0^\gamma \frac{2 R^2 M (1 - \nu^2)}{E I} \sin \left(\frac{\gamma - \phi}{2} \right) \cos \phi d\phi \\ + \int_0^\gamma \frac{R N (1 - \nu^2)}{E A} \sin \gamma d\phi \quad (25)$$

For small angles the second integral may be neglected and the first can

be simplified to

$$y' = - \int_0^{R\gamma} \frac{M(1-\nu^2)}{EI} x' ds \quad (26)$$

which represents the moment-area method (5), modified by use of ds instead of dx as suggested by Roark (7). Equation (26) was used here to simplify the analysis. To obtain the displacement function y below, the y' -function was shifted as shown in Figure 5.

Thus

$$y = \frac{6pR^4(1-\nu^2)}{\beta_1 E a^3} \left(\frac{\gamma \sin \gamma}{\alpha} + \frac{\cos \gamma}{\alpha} - \frac{\sin^2 \gamma}{2 \sin \alpha} - \frac{\sin \alpha}{2} - \frac{\cos \alpha}{\alpha} \right) \quad (27)$$

Integrating Equation (27) with respect to distance along the mid-span tangent over the distance between midspan and one side plate and multiplying by four gives the change in coolant-channel cross section. Dividing by bh , which for small α and normal plate-thickness to coolant-channel-thickness ratios is a close approximation to the initial coolant-channel area, gives

$$\frac{\Delta Q}{Q_0} = \frac{pb^4(1-\nu^2) \left(\frac{1}{6} - \frac{\sin 2\alpha}{8\alpha} + \frac{\cos 2\alpha}{12} \right)}{16 h E I \beta_1 \sin^5 \alpha} \quad (28)$$

Substitution of Equation (28) in Equation (4) gives, for the critical velocity,

$$V_c = \left[\frac{2 g h a^3 E \beta_1 \sin^5 \alpha}{3 \rho b^4 (1-\nu^2) \left(\frac{1}{6} - \frac{\sin 2\alpha}{8\alpha} + \frac{\cos 2\alpha}{12} \right)} \right]^{1/2} \quad (29)$$

The ratio of the above critical velocity to that for the flat plate with fixed edges is

$$V_{rf} = \left[\frac{48 \beta_1 \sin^5 \alpha}{45 (4-3 \frac{\sin 2\alpha}{\alpha} + 2 \cos 2\alpha)} \right]^{1/2} \quad (30)$$

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FIGURE 6. CRITICAL VELOCITY VS α FOR CURVED-PLATE ASSEMBLY
RELATIVE TO THAT OF FLAT-PLATE ASSEMBLY. EDGES FIXED.

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FIGURE 7. CRITICAL VELOCITY VS b/a FOR CURVED-PLATE ASSEMBLY
RELATIVE TO THAT OF FLAT-PLATE ASSEMBLY. EDGES FIXED.

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Using $A = a$ and $I = a^3/12$ in Equation (22) and expanding trigonometric functions in power series of α with sufficient terms to give good accuracy for $\alpha < 0.5$ gives

$$V_{rf} = \left[\frac{b^2 \alpha^2}{15 a^2} \left(1 - \frac{5 \alpha^2}{42} \right) + 1 - \frac{9 \alpha^2}{14} \right]^{1/2} \quad (31)$$

This critical-velocity ratio is plotted in Figures 6 and 7.

Case E - Curved Plates With Hinged Edges

The assembly considered here is identical with that of Case D except that the longitudinal edges of the plates are hinged to rigid side plates. It is assumed that flow channels are isolated from each other between the ends of the assembly.

Following the procedure used for the fixed-edge curved plates gives the following equations:

$$X = \frac{p R}{\beta_2} \quad (32)$$

$$M_o = - \frac{p R^2}{2 \beta_2} (\csc \alpha - \cot \alpha) \quad (33)$$

$$N_o = p R \left(1 - \frac{\csc \alpha}{2 \beta_2} \right) \quad (34)$$

$$N_1 = p R \left(1 - \frac{\cot \alpha}{2 \beta_2} \right) \quad (35)$$

$$M = \frac{p R^2}{\beta_2 \sin \alpha} \left(\sin^2 \frac{\phi}{2} - \sin^2 \frac{\alpha}{2} \right) \quad (36)$$

$$N = p R \left(1 - \frac{\cos \phi}{2 \beta_2 \sin \alpha} \right) \quad (37)$$

where

$$\beta_2 = \frac{A R^2}{I} f_3(\alpha) + f_1(\alpha) \quad (38)$$

$$f_3(\alpha) = \frac{1}{2 \sin^2 \alpha} \left(\alpha - \frac{3}{4} \sin 2\alpha + \frac{\alpha \cos 2\alpha}{2} \right) \quad (39)$$

and $f_1\alpha$ is defined by Equation (23).

Use of Equation (36) and the procedure used for the fixed-edge assembly gives the following expression for the displacements of the plate relative to the supports and in the direction normal to the midspan tangent:

$$y = \frac{p R^4 (1 - \nu^2) C}{2 E I \beta_2 \sin \alpha} \quad (40)$$

where

$$C = -\frac{2}{3} + \sin 2\alpha \left(\frac{3}{4} \cot \alpha - \alpha \right) - \frac{4}{3} \cos 2\alpha + \alpha \cot \alpha \left(1 - \frac{\cos 2\alpha}{2} \right) \quad (41)$$

Determining the fractional change in coolant channel cross sectional area as for the fixed-edge curved-plate assembly gives

$$\frac{\Delta Q}{Q_0} = \frac{3 p b^4 (1 - \nu^2) \csc^5 \alpha}{8 E a^3 \beta_2 h} \left[\begin{array}{l} -\frac{2}{3} + \sin 2\alpha \left(\frac{3}{4} \cot \alpha - \alpha \right) \\ -\frac{4}{3} \cos 2\alpha + \alpha \cot \alpha \left(1 - \frac{\cos 2\alpha}{2} \right) \end{array} \right] \quad (42)$$

Substitution of Equation (42) in Equation (4) gives the following expression for the critical velocity:

$$V_c = \left[\frac{4 g h E a^3 \beta_2 \sin^5 \alpha}{3 \rho b^4 (1 - \nu^2) C} \right]^{1/2} \quad (43)$$

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FIGURE 8. CRITICAL VELOCITY VS. α FOR CURVED-PLATE ASSEMBLY
RELATIVE TO THAT OF FLAT-PLATE ASSEMBLY. EDGES HINGED.

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FIGURE 9. CRITICAL VELOCITY VS. b/a FOR CURVED-PLATE ASSEMBLY
RELATIVE TO THAT OF FLAT-PLATE ASSEMBLY. EDGES HINGED.

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FIGURE 10. CRITICAL VELOCITY OF FIXED-EDGE CURVED-PLATE
ASSEMBLY RELATIVE TO THAT OF HINGED-EDGE
CURVED-PLATE ASSEMBLY

The ratio of this critical velocity to that for the simply-supported flat plate assembly is:

$$V_{rh} = \left(\frac{8\beta_2 \sin^5 \alpha}{15 C} \right)^{1/2} \quad (44)$$

Expanding Equation (44) in power series in α with sufficient terms for good accuracy for $\alpha < 0.5$ gives

$$V_{rh} = \left[\frac{2 b^2 \alpha^2}{5 a^2} \left(1 - \frac{\alpha^2}{21} \right) + 1 - \frac{11 \alpha^2}{21} \right]^{1/2} \quad (45)$$

This critical-velocity ratio is plotted in Figures 8 and 9.

Dividing Equation (29) by Equation (43) gives the following ratio of the critical velocity of a fixed-edge curved-plate assembly to that of an assembly which is identical except for having hinged edges:

$$V_{rfh} = \left\{ \frac{6 \left[\frac{b^2 \alpha^2}{15 a^2} \left(1 - \frac{5 \alpha^2}{42} \right) + 1 - \frac{9 \alpha^2}{14} \right]}{\frac{2 b^2 \alpha^2}{5 a^2} \left(1 - \frac{\alpha^2}{21} \right) + 1 - \frac{11 \alpha^2}{21}} \right\}^{1/2} \quad (46)$$

This critical-velocity ratio is plotted in Figure 10.

EFFECT OF UNBALANCED CHANNEL PRESSURES

If pressure differences are imposed in opposite directions across adjacent plates due to unbalanced conditions, the deflections due to the imposed pressure difference will be magnified by an approach to critical flow velocity. The significance of unbalanced pressures can be illustrated by assuming that in the plate assembly of Case A, at velocities below critical, there are superimposed pressure differences (p_0 across each plate) which cause small deflections of the form shown in Figure 2. Adding p_0 to Equation (4) gives the total pressure differential acting on the plates in the deformed region:

$$p = p_0 + \frac{2 \rho V_0^2 \Delta Q}{g Q_0} \quad (47)$$

Combining Equations (3) and (47) to eliminate p gives an equation which reduces to

$$\frac{\Delta Q}{Q_0} = \frac{p}{\frac{30 E a^3 h}{(1 - \nu^2) b^4} \left[1 - \frac{V_0^2 \rho b^4 (1 - \nu^2)}{15 g E a^3 h} \right]}$$

which on substitution of Equation (4) becomes

$$\frac{\Delta Q}{Q_0} = \frac{p_c}{\frac{30 E a^3 h}{(1 - \nu^2) b^4} \left[1 - \left(\frac{V_0}{V_c} \right)^2 \right]} \quad (48)$$

Equation (48) shows that deflections due to initial pressure differentials across the plates are increasingly amplified as the flow velocity approaches the critical value. Note that the term $[1 - (V_0/V_c)^2]$ is the ratio of the net stiffness due to combined fluid and elastic effects to the elastic stiffness, and that the term $(V_0/V_c)^2$ can be visualized as a normalized negative spring constant or stiffness factor. Actually the assumption of alternately high and low pressures in successive channels is academic. If only a single plate in the assembly is subject to an imposed pressure differential, the deflection will be amplified to an extent smaller than that indicated by Equation (48).

EFFECT OF MEMBRANE COMPRESSIVE STRESSES

If flat fuel plates are subjected to transverse loadings which cause membrane compressive stresses in the plates, collapse can occur due to these alone. Such a condition can result from pressures on the outside of the side plates exceeding the pressure in the channels between plates. The existence of a transverse compressive loading in combination with the effects of coolant flow allows collapse of the plates with less extreme conditions than those which individually would cause collapse. The analysis of this problem can be simplified by assuming that the local pressure difference across a plate is proportional to the local deflection instead of the previous assumption of uniform velocity at any transverse cross section of a channel. The result of analysis of the combined effect of fluid flow and transverse compressive loading is

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FIGURE 11. COMBINED EFFECT OF FLOW AND MEMBRANE COMPRESSION OF FLAT PLATES

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$$\frac{P}{P_c} + \left(\frac{V'_c}{V_c} \right)^2 = 1 \quad (49)$$

where P/P_c is the ratio of the actual transverse compressive load to that which alone would cause elastic instability, and V'_c/V_c is the ratio of critical coolant velocity to that which corresponds to flow-induced collapse alone. The critical velocity V_c for the velocity distribution assumed for this analysis is 10% lower than that predicted for flat plates with uniform local velocity in the deformed channels.

Equation (49) can also be deduced by visualizing that the two instability phenomena each produce a component of negative bending stiffness, just as an axial compression of a column increases the transverse deflection of a column under the action of a transverse force. When the sum of the negative stiffness due to fluid effects (proportional to V_o^2) plus the negative stiffness due to transverse compression (proportional to P) is equal and opposite to the positive stiffness of the plate, the system is unstable.

If only one plate is in compression due to thermal expansion or welding residual stresses, the situation is approximately equivalent to the case of a single plate centrally located within a rectangular duct. The critical velocity of the single plate without the compressive load is easily seen to be higher by the factor $\sqrt{2}$ than the critical velocity for the assembly of several plates with the same coolant-channel dimensions. The condition for incipient collapse of the single plate under compression is, in the region where the plate is near collapse due to membrane compression alone, approximately

$$\frac{P}{P_c} + \frac{1}{2} \left(\frac{V'_c}{V_c} \right)^2 = 1 \quad (50)$$

where V_c is computed for the assembly without consideration of the compressive load on a single plate. Figure 11 shows plots of Equations (49) and (50) and a curve which shows a plot based on judgment to represent the stability of the single plate in the region where both phenomena contribute to instability.

DISCUSSION

The assumptions made for derivation of the critical velocities are considered not to impair the applicability of the critical-velocity formulas to practical fuel-plate assemblies with the exception of factors discussed below.

The assumptions of initial flatness of plates, uniformity of plate thickness, spacing, mass flow, and pressure distribution lead to an overestimate of the collapse velocity if nominal parameters are used for calculation. The assumption for Cases B and C that there is no leakage from one channel to another may lead to an underestimate of the collapse velocity, but it is not obvious that such leakage will have a strong effect on the critical velocity.

The assumption of incompressibility of the coolant does not impair the applicability of the derivations to systems using liquid coolants and perfect gases insofar as the inception of instability is concerned. Compressibility in a gaseous coolant may limit the elastic deflections developed by velocities which are slightly above that point at which instability begins.

As the plate length-to-width ratio is reduced from large values, it appears that the critical velocity will increase. For most nuclear reactor applications, however, this ratio is likely to be so large that its effect on the critical velocity will be small.

It may be possible to have a plate assembly in which the collapse modes of Case B and Case C would occur at velocities on the same order. Under this condition it appears that the lower of these two critical velocities, predicted on the basis of formulas presented here, would be a substantial overestimate of the actual lowest critical velocity.

A small error was introduced in the curved-plate critical-velocity formulas by eliminating the second integral in Equation (25). For $\alpha = 0.5$, the fixed-edge curved-plate critical velocity indicated by the formulas derived is overestimated by about 6%, and the hinged-edge curved-plate critical velocity is overestimated by about 4% by using this approximation. The error is smaller with smaller values of α .

It has been shown that transverse compressive loads on flat plates reduce the critical velocity. It is also clear that transverse tensile loads on all plates would increase the critical velocity.

There may be other sources of instability which may interact with column instability and flow-induced instability. One of these is thermoelastic instability which has been analyzed by the author (8, 9) and independently by Illiffe (10). Thermoelastic instability may arise due to the effects of fuel-plate deflection upon flow distribution and heat transfer, which in turn can cause temperature gradients which tend to maintain the deflection. It may also be possible that high neutron-flux gradients introduce an additional contribution to thermoelastic instability. In the idealized case a straight beam is unstable thermoelastically if, due to its temperature distribution, it tends to acquire a sufficiently high local curvature proportional to the local deflection. If this local

curvature per unit of deflection of a beam of length L is denoted K , the critical value of K is $(\pi/L)^2$ for a beam with simple end supports and $(2\pi/L)^2$ for a beam with fixed ends (9).

In the case of curved plates the presence of unbalanced pressures imposed across the plates introduces the possibility of buckling independent of the effects of plate deflection on the pressure difference existing across the plates. There is also an asymmetrical mode of flow-induced collapse which for large values of α may have lower critical velocities than those presented here. In this mode of collapse the two half-widths of plate would deflect in opposite directions. This mode of collapse at worst would limit the minimum critical velocity of a simply-supported curved-plate assembly to about four times the critical velocity of a similar flat-plate assembly, because the more complex mode loses the membrane restraints which are present in the simple mode of distortion. The second mode of instability would, at worst, limit the critical velocity of a fixed-edge curved-plate assembly to somewhat more than four times the critical velocity of a similar simply-supported flat-plate assembly or somewhat less than four times the critical velocity of a similar fixed-edge flat-plate assembly. Actually, the effect of this mode of collapse is probably much less than is indicated by the "at-worst" limits indicated above.

Since the critical velocities are proportional to Young's modulus of elasticity, it is apparent that the occurrence of yielding or creep of fuel plates due to unbalanced pressure, thermal stresses, and growth of fuel meat reduces the plate stiffness and the critical velocity. Roberts and Cottrell (11) have shown that alpha-uranium creeps during exposure to neutrons under external loads which, without neutron flux, would not produce significant creep. Willis (12) has suggested that alloys and dispersions of fissionable material will have a similar loss of resistance to creep deformation under external load. Thus it appears likely that in plate assemblies which are not afforded stability against collapse by the stiffness of cladding alone (neither shear nor bending stiffness attributed to the fuel meat), collapse may proceed slowly over a period of months or years instead of occurring almost instantaneously. If creep collapse is a possibility, it would be prudent to remove a fuel-plate assembly from the reactor at intervals for inspection until it is confirmed that creep collapse does not occur. If the fuel meat were assumed to have no shear or bending stiffness, the critical velocity of the assembly of Case A would be

$$V_c = \left[\frac{30 g E t_c^3 h}{\rho b^4 (1 - \nu^2)} \right]^{1/2}$$

where t_c is the thickness of cladding on each surface.

There would probably be a slight loss of critical velocity of fixed-edge plates due to local yielding in cladding at the longitudinal boundary of the fuel meat due to growth of the meat with burn-up of fissionable material. The zone of yielding would probably be so localized as to make a slight deviation of the fixed-plate critical velocity toward that for simply-supported plates. It is interesting to note that the significance of this problem is smaller for curved plates than for flat plates, since for the former the critical velocities coincide at large value of α .

The assumption of uniform coolant velocity in a channel across a plane normal to the flow is slightly optimistic for Cases A, B, D and E. This is indicated by the analysis leading to Equation (49), which assumed that the local velocity was inversely proportional to the local spacing between plates. This assumption gave a critical velocity for flat plates 10% lower than did the assumption of uniform velocity. Actually, the velocity at the region of minimum plate separation may be smaller than the average velocity in the channel. This condition would make the critical velocities somewhat higher than the derived expressions indicated.

In some fuel-plate assemblies collapse has occurred at velocities on the order of one-half of that predicted by the formulas presented here. The calculated critical velocity for the ETR fuel-plate assembly is about 54 ft/sec for water at room temperature. Observed collapse velocities are reported by Doan (2) to be as low as 30 ft/sec. The fuel element consists of 19 aluminum-clad uranium-aluminum alloy plates 0.050 in. thick, about 2.62 in. span, with water channel thickness of 0.108 in. and length of 36 in. Doan cites lack of flatness of some plates as contributing to collapse and also points out that the plates would deform plastically with pressure differences across the plates of 1-1/2 to 2 psi for the outermost plates and about 3 psi for the inner plates. He also reports that use of work-hardened plates or perforation of the side plates allowed application of flow at 25% or more above the design flow rate of 35 ft/sec with little or no distortion. With the 25% variation in flow distribution indicated by Doan, the work-hardened plates have attained, without collapse, velocities almost as high as the theoretical collapse velocity.

The data on the Oak Ridge collapse observations (1) are not sufficiently complete to provide a check on the derived collapse-velocity formulas. The MTR plates are made of aluminum-clad uranium-aluminum alloy, 0.059 in. plate thickness, 0.117 in. water channel, 2.67 in. free span, 5-1/2 in. radius of curvature. The plates are brazed into side plates. The theoretical collapse velocity for these plates with water at room temperature is about 240 ft/sec. With an assembly having the same dimensions except for plates being flat, the theoretical critical velocity would be about 80 ft/sec. Reference (1) reports collapse of 1.0-mm (0.039-

in.) and 1.5-mm (0.059-in.) curved plates and 1.0-mm flat plates 2.68-in. wide. Water-channel thicknesses were not identified for specific assemblies, but a normal dimension of 3 mm was specified. Collapse velocities were not specified, but the test series, including assemblies which did not collapse, was run up to velocities of about 45 ft/sec. The theoretical critical velocity for the 1.0-mm flat plate assembly is about 44 ft/sec, so it is not surprising that it collapsed. Collapse of the other plates may have been influenced by deviations from ideal conditions.

The previously mentioned experimental data lend some support to the formulas presented here. It would be of considerable interest to have tests conducted to determine the validity of the critical-velocity formulas for idealized assemblies and to investigate the effect of deviations from conditions assumed for derivation of the formulas. It is also essential that the effect of irradiation upon the resistance of the fuel meat to distortion be evaluated.

In development of fuel-plate assemblies it is appropriate to conduct flow tests at velocities substantially above rated flow to confirm the existence of a substantial margin against collapse at rated flow. This margin should be made large enough to allow for differences between test conditions and in-pile conditions including fuel burnup.

In view of the reduced plate stiffness introduced by the approach to collapse velocity, it would be of interest to know the effect of this reduced stiffness upon the natural frequencies of vibration of the plates. In the lowest frequency mode of bending vibration of the plates, the flow-induced negative stiffness would be a factor only for the case where adjacent plates vibrate in phase opposition.

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