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THE NONEQUILIBRIUM, TWO-PHASE
CRITICAL DISCHARGE OF NEARLY SATURATED
AND SUBCOOLED CCl_3F THROUGH SHORT TUBES

by

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NOMENCLATURE—▶

NOMENCLATURE

<u>Symbol</u>		<u>Units</u>
A	Cross-sectional area	ft^2
C	Compressibility	$\text{ft}^3/\text{lb}_f\text{-sec}$
C_d	Coefficient of discharge	unitless
c	Specific heat of liquid	$\text{Btu}/\text{lb}_m\text{-}^\circ\text{R}$
D	Diameter of tube	ft
E	Interphase energy transport	$\text{Btu}/\text{ft-sec}$
	Expansion rate	$\text{ft}^3/\text{lb}_f\text{-sec}$
F	Interphase momentum transport	lb_m/sec^2
G	Mass flow rate per unit area	$\text{lb}_m/\text{ft}^2\text{-sec}$
g_c	Dimensional constant	$32.1739 \text{ lb}_m\text{-ft}/\text{lb}_f\text{-sec}^2$
h	Specific enthalpy	Btu/lb_m
J	Unit conversion factor	$778.16 \text{ ft-lb}_f/\text{Btu}$
K	Ratio of specific heats, c_p/c_v	unitless
k	Slip ratio, u_g/u_l	unitless
L	Length of tube	ft
M	Mach number	unitless
m	Interphase mass transport	$\text{lb}_m/\text{ft-sec}$
N	Empirical parameter indicative of degree of nonequilibrium present	unitless
P	Static pressure	lb_f/ft^2
R	Radius of liquid jet	ft
r	Radial spatial coordinate or radius	ft
S	Interphase entropy transport	$\text{Btu}/\text{lb}_m\text{-}^\circ\text{R-ft-sec}$
S_π	Entropy production rate per unit length	$\text{Btu}/\text{lb}_m\text{-}^\circ\text{R-ft-sec}$
s	Specific entropy	$\text{Btu}/\text{lb}_m\text{-}^\circ\text{R}$
t	Time	sec

NOMENCLATURE →

<u>Symbol</u>		<u>Units</u>
T	Temperature	$^{\circ}\text{F}$ or $^{\circ}\text{R}$
u	Velocity	ft/sec
v	Specific volume	ft ³ /lb _m
x	Quality	unitless
z	Axial spatial coordinate	ft
<u>Greek</u>		
α	Void fraction, $A_g / (A_g + A_l)$	unitless
Δ	Change or increment	
$\Delta h_{\text{CCl}_3\text{F}}$	Change in CCl ₃ F level	ft
κ	Thermal conductivity	Btu/sec-ft- $^{\circ}\text{R}$
η	Critical pressure ratio, P_{cr}/P_o	unitless
ρ	Density	lb _m /ft ³
μ	Viscosity	lb _m /ft-sec
<u>Subscripts</u>		
cp	Critical-flow plane or choking plane	
cr	Critical	
e	Exit plane	
E	Equilibrium	
fg	Saturated liquid property subtracted from saturated vapor property	
g	Vapor phase	
gl	Transport from vapor to liquid	
hvp	High-pressure vessel	
i	Interface between liquid and vapor phase	
l	Liquid phase	
lg	Transport from liquid to vapor	
o	Stagnation property	
R	Low-pressure vessel	
s	Saturation property	

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ABSTRACT

Two-phase critical flow data, both critical flow rates and critical pressure ratios, were obtained for sharp-edged entrance tubes for which $2.82 \leq L/D \leq 12.78$. The testing fluid was CCl_3F . The ranges of stagnation pressures and temperatures were: $9.72 \text{ psia} \leq P_o \leq 16.74 \text{ psia}$ and $35^\circ\text{F} \leq T_o \leq 72^\circ\text{F}$.

Test sections of two major types were fabricated. The first type, termed the gradually diverging type, was a constant-area tube with a 7° included-angle diverging section affixed to the tube exit. The second type, termed the rapidly diverging type, was like the first, only the affixed diverging section had an included angle of 120° or 180° . The rapidly diverging test sections were made to be representative of test sections used to obtain critical flow data prior to 1968. By comparing runs of identical P_o , T_o , and L/D between the two types of test sections, it was determined that the expansion of the fluid at the exit plane of the rapidly diverging test section has caused investigators to misinterpret the actual critical-flow phenomenon. From such comparisons, it was concluded that a steep pressure gradient does not exist near the exit plane of a short tube nor is the exit-plane pressure in any way dependent on the receiver pressure while critical flow persists, contrary to the findings of previous investigators who used rapidly diverging test sections.

Comparison of the data to various critical-flow models published in the literature revealed that no equilibrium model is representative of the observed behavior. All equilibrium models underpredicted the critical flow rates obtained in this experiment. The most favorable predictions of critical flow rate and critical pressure ratio were rendered by nonequilibrium critical-flow models.

Visual observations of a rectangular test section with glass sides, $L/D = 2.82$, interpreted in light of axial pressure measurements, discharge-coefficient calculations, and visual studies of previous investigators, has afforded some insight into determining the flow patterns in short tubes.

An estimate of the individual phase velocities has been made. These are much lower than the respective acoustic velocities of each phase; accordingly, "vapor choking" is not the mechanism by which critical flow of a two-phase mixture occurs.

I. INTRODUCTION AND BACKGROUND

What is Critical Flow?

Critical flow is a phenomenon wherein it has been observed that, for fixed stagnation conditions, there is a maximum flow rate that can be obtained through a straight pipe, nozzle, or other duct, such as shown in Fig. 1.1, regardless of how great a pressure difference might be imposed across the pipe. In other words, if $P_o - P_R$ begins at zero and increases by holding P_o constant and lowering P_R , the flow rate also increases, and P_e , the pressure at the exit plane of the pipe, decreases until a condition is reached in which P_e is constant with respect to changes in P_R ; the flow rate thus reaches a maximum value. This condition is called *critical flow*; the value of P_e at critical flow is termed the *critical pressure*. The criterion for critical flow to be present can be stated mathematically by saying that for fixed P_o and T_o and, in general, all other independent variables fixed, that $dG/dP_e = 0$, that is, the flow rate reaches an extreme (maximum in this case) value with respect to the exit pressure at the *critical flow plane*, or *choking plane*, as it is sometimes called.

The critical-flow phenomenon is governed by the compressibility of the fluid. As shown in Appendix D, and as will be shown for the single-phase case, the maximum flow rate, that is, the critical flow rate, may be expressed as

$$G_{cr} = \left[\frac{-g_c}{C_{cr}} \right]^{1/2}, \quad (D.1)$$

Fig. 1.1. Fluid Flowing from a Reservoir at Pressure P_o through a Constant-area Pipe into a Reservoir at Pressure P_R . ANL Neg. No. 900-1877.

where C_{cr} is the compressibility at critical flow. In the English system, C has units of $\text{ft}^3/\text{lb}_f\text{-sec}$. C is inherently negative and the compressibility is said to increase as the absolute value of C increases. Figure 1.2 depicts the relationship between C_{cr} and G_{cr} . From this it is seen that as the compressibility increases, G_{cr} decreases. In fact, it may be deduced that a truly incompressible substance, one for which $C_{cr} = 0$, would have an infinite flow rate for its maximum value. Practically speaking, this means that the rate at which an incompressible substance can flow is limited only by the driving force applied to cause flow. At the other extreme is a substance that is infinitely compressible. Its maximum or critical flow rate would be zero, and it would not flow at all.

The question of understanding why a compressible fluid exhibits a maximum flow rate and in incompressible fluid does not then become a question of examining the difference in the expression of the extreme value of the flow rate for each situation. To make this comparison, then, the momentum

Fig. 1.2. Critical Flow Rate as a Function of Compressibility: High Values of C_{cr} and Low Values of G_{cr} are Typical of Single-phase Gas Flow; Low Values of C_{cr} and High Values of G_{cr} are Typical of Single-phase Liquid Flow. ANL Neg. No. 900-1858.

equation written along a streamline for single-phase, frictionless, steady flow--the so-called Euler equation of motion--may be written, neglecting gravity effects, as

$$v(z) \frac{dP(z)}{dz} = -u(z) \frac{du(z)}{dz} . \quad (1.1)$$

Integrating from the stagnation point to the exit plane of the tube, one obtains

$$\int_{z_0}^{z_e} v(z) \frac{dP}{dz} dz = - \int_{z_0}^{z_e} u(z) \frac{du}{dz} dz . \quad (1.2)$$

Carrying out the integration on the right-hand side, Eq. 1.2 becomes

$$\int_{z_0}^{z_e} v(z) \frac{dP}{dz} dz = \frac{u(z_0)^2 - u(z_e)^2}{2} . \quad (1.3)$$

Since at the stagnation point $u(z_0) = 0$, Eq. 1.3 becomes

$$\int_{z_e}^{z_0} v(z) \frac{dP}{dz} dz = \frac{u(z_e)^2}{2} . \quad (1.4)$$

The mass rate of flow per unit area is defined as

$$G \equiv \rho u = u/v . \quad (1.5)$$

Substituting Eq. 1.4 into Eq. 1.5, one obtains

$$G_e = \frac{1}{v_e} \sqrt{2 \int_{z_e}^{z_0} v(z) \frac{dP}{dz} dz} . \quad (1.6)$$

If the fluid is truly incompressible along the streamline, $v \neq v(z)$ and Eq. 1.6 becomes

$$G_e = \sqrt{\frac{2(P_o - P_e)}{v_e}} . \quad (1.7)$$

If G_e has a maximum, it is necessary that $dG/dP_e = 0$. Differentiating Eq. 1.7 with respect to P_e and equating the result to zero yield

$$\frac{1}{\sqrt{P_o - P_e}} = 0 . \quad (1.8)$$

Equation 1.7 says that if G_e has a maximum for incompressible flow, P_o must be infinite. Practically speaking, this means that the maximum flow rate for truly incompressible flow is unbounded, and the flow rate is limited only by the pressure difference across the pipe.

On the other hand, if the fluid is compressible, $v = v(z)$ and Eq. 1.6 becomes

$$G_e = \frac{1}{v_e} \sqrt{2 \int_{P_e}^{P_o} v(P) dP} . \quad (1.9)$$

If G_e has a maximum, it is necessary that $dG/dP_e = 0$. Applying this condition to Eq. 1.9 results in

$$-v_e^2 = \left[2 \int_{P_e}^{P_o} v(P) dP \right] \frac{dv}{dP_e} . \quad (1.10)$$

At critical flow, this expression becomes, using Eq. 1.9,

$$G_{cr}^2 = - \frac{1}{\frac{dv}{dP_e}} . \quad (1.10a)$$

This is equivalent to Eq. D.1, where $C_{cr} = dv/dP_e$. When v is known as a function of P , Eq. 1.10 can be solved to obtain the value of $P_e = P_{cr}$ for which G_e is a maximum. For example, for isentropic flow of a perfect gas, this occurs when

$$P_e = P_{cr} = P_o \left(\frac{2}{K+1} \right)^{\frac{K}{K-1}} , \quad (1.11)$$

where K is the ratio of the specific heat at constant pressure to the specific heat at constant volume, c_p/c_v .

In the case of multiphase flows, the compressibility of the fluid is affected not only by the manner in which the specific volume varies with pressure, but it is also affected by interaction between the phases. The effects of pressure upon the ratio of the individual phase velocities and

the change-of-phase process are examples of such interactions. Thus, compressibility should not be viewed as simply "compression." It should be viewed as compression plus interphase interaction.

Single-phase critical flow of liquids and gases has been studied for some time. The relative simplicity with which such flows can be analyzed comes largely from the fact that the manner in which fluid properties change is known.

Of more recent interest is the subject of two-phase critical flow. The presence of an additional phase complicates the problem for two main reasons. The first is that there are interactions between the phases which are extremely difficult to account for. The second reason is that one must be able to determine the configuration of the phases, that is, what parts of the tube volume are occupied by vapor, what parts are occupied by liquid, what is the shape, and movement of such regions. The question must be answered: "What is the flow pattern?" In the single-phase case, the flow pattern is one of a single phase occupying the entire volume. In two-phase flow, one must determine if the flow pattern is one of bubbly flow, slug flow, annular flow, droplet flow, a jet of liquid breaking up, stratified flow, some completely chaotic flow pattern, or one which is a combination of the preceding flow patterns.

Applications of Critical Flow

In an effort to investigate the safety of a nuclear reactor system under various hypothetical accidents, it is necessary to describe what will happen if a primary-coolant line carrying the reactor coolant breaks. In order to design proper containment for the coolant and to determine the rate of coolant loss, it is necessary to determine the critical flow rate for this situation.

Critical two-phase flow is encountered in other engineering and industrial applications. It occurs commonly in cascade drain pipes of boilers and turbines, in traps of steam-heating plants, in low-temperature flow of refrigerants and condensed gases, and in the high-temperature flow of rocket propellants.¹ Attention must be paid to the critical flow rate in the design of flash evaporator-type seawater desalting plants and supersonic condensing injectors.²⁴ An analysis of the ruptured oxygen tank on the Apollo 13 moon mission incorporated a critical-flow analysis to calculate the maximum possible discharge rate. Critical flow may also occur when flow discharges to a region of zero absolute pressure.

To summarize, critical flow is of concern when $\Delta p/\rho$ approaches unity, that is, when density changes are large. More specifically, critical flow often occurs when:

- 1) Large pressure differences are forcing a liquid through a constriction, or moderate-to-large pressure differences are forcing a gas or a liquid-gas mixture through a constriction.
- 2) Fluid is discharged to a zero pressure environment.
- 3) A maximum possible flow rate is desired for design purposes--critical flow represents the upper limit.

Requirements of a Short-tube Critical-flow Model

A useful critical-flow model to predict two-phase critical flow in short tubes should contain the following desirable features:

- 1) The model must reasonably predict both G_{cr} and P_e , for if it does not, one cannot say that such a model correctly portrays the observed behavior, even though it may give reasonable results over a limited range.
- 2) The nonequilibrium nature of the phase change must be accounted for.
- 3) A model that is not based on stagnation conditions and requires *a priori* knowledge of P_e is of limited practical value, for in the case of rupture in the coolant line of a water-cooled, thermal nuclear reactor, the pressure at the break, P_e , is not known. Thus, it is most desirable to have a model based on parameters which are obtainable *a priori*, such as the stagnation conditions.
- 4) The model should incorporate:
 - i) appropriate simplifications, and
 - ii) postulated thermodynamic paths which are descriptive of various regions of the flow field,so that the mathematical model might be solved subject to this information obtained by experimental investigation.
- 5) If empirical relationships are used, the scope of applicability of the model is largely dependent on the applicability of the empirical relationships over a range of fluids and over a range of temperatures and pressures. Therefore, the use of empirical relationships which apply over a range of fluids and conditions is advantageous.
- 6) The model should be as mathematically simple as possible.

II. PREVIOUS WORK

Although a description of the mechanics of two-phase critical flow has been sought since about 1890, the work most pertinent to this investigation is of more recent vintage. Reviews of pertinent investigations are presented in Refs. 1 and 4, providing much background information.

Experimental and theoretical investigations which have contributed directly to the understanding of critical two-phase flow through short tubes are discussed in this chapter.

EXPERIMENTAL INVESTIGATIONS

Worthy of note are several experimental investigations which have contributed to the understanding of the nonequilibrium nature of the phenomenon, the establishment of the flow pattern at various values of L/D , and the determination of the roles of entrance configuration and L/D . Comparisons of the various experiments have made possible a meaningful evaluation of the various data presented as well as showing the proper manner in which to obtain exit plane pressure measurements.

Pasqua

Pasqua^{10,11} studied the flow of Freon-12 through sharp-edged entrance tubes for which $2.3 \leq L/D \leq 23.5$. The contribution of Pasqua's work was threefold. He:

- 1) investigated the problem as a nonequilibrium process,
- 2) determined the effect of entrance configuration on flow rate, and
- 3) made a visual study of the flow pattern.

Pasqua found that changing the sharpness of the entrance by as little as 0.0005 in., equivalent to breaking the edge with a whetstone, can increase the flow rate by as much as 15%. Actually, rounding the entrance 1/32 to 1/8 in. resulted in an increase of 22% in flow rate for subcooled flow and 40% for saturated flow with respect to the sharp-edged condition. It was pointed out in Pasqua's work that extreme care must be taken in keeping the entrances the same if several tubes are used in an experiment so that the investigator knows whether or not he may justifiably ascribe a change in flow rate to an L/D effect, or whether it is merely a result of a difference in entrance configuration.

By photographic means, Pasqua verified that the flow pattern was much like the flow pattern observed in nozzle flow by Bailey,¹⁴ that is, the flow could be represented as a core of nonequilibrium liquid surrounded by a two-phase-mixture annulus.³⁶ For short tubes, Pasqua observed the behavior as pictured in Fig. 2.1 for a tube of semicircular cross section for which $L/D = 4.64$ (D is taken as the hydraulic diameter).

Fig. 2.1. Flow Pattern Observed by Pasqua;¹⁰ Saturated Flow through a Short Tube with a Sharp-edged Entrance for which $L/D = 4.64$. ANL Neg. No. 900-1845.

Fauske and Min

Fauske and Min³ were the first to investigate experimentally the critical flow of CCl_3F through tubes in the L/D range from 2 to 55. In this investigation, it was shown that L/D is, indeed, one of the variables determining the critical flow rate: the flow rate increases as L/D decreases, and increases more rapidly as L/D approaches 0.

Steel test sections were utilized. For short L/D s, it was postulated that the liquid stream did not touch the tube wall, being separated from it by vapor. This is in close agreement with the findings of Pasqua;¹⁰ however, Pasqua stated that the region separating the liquid stream from the wall was a two-phase mixture, whereas Fauske and Min stated that it was vapor. They also observed that the tube had essentially the same effect on the liquid that an orifice has on a single-phase fluid, such as the contraction witnessed by Pasqua¹⁰ and pictured in Fig. 2.1.

That the flow rate could be described by an orifice-type flow-rate expression such as

$$G_{cr}^2 = 2C_d^2 \rho_{l0} (P_o - P_e),$$

where C_d has the value of the orifice discharge coefficient, 0.61, adds credibility to the observation of Fauske and Min. In general, C_d is the discharge coefficient which reflects the effects of the entrance on the fluid. In trying to turn the sharp corner at the inlet, the fluid acquires a radial component of velocity and contracts to an area less than that of the duct. Also, due to viscosity, the fluid, in passing through the entrance, is subjected to frictional interactions which cause the velocity to be less than if such interactions did not take place. Thus, C_d compensates for the fact that a one-dimensional, frictionless momentum equation is being used to describe a two-dimensional viscous flow.³⁹

Fauske and Min also pointed out that they believed the departure from thermodynamic equilibrium increased as the L/D decreased below 20, where the deviation of the predictions of Fauske's equilibrium model¹ from the data increased as L/D decreased. The fluid in going immediately from the

stagnation pressure to a very low pressure at the sharp inlet needs a short but definite amount of time for molecular readjustment to take place before it is again at an equilibrium state. The authors pointed out that the shorter the tube, the less time a fluid particle spends in the tube and the less time it has in which to relax the nonequilibrium generated at the entrance.

Henry

In a critical two-phase flow study of water in a long tube ($L/D = 144$), Henry⁴ explained the apparent dependency of exit-plane pressure on receiver pressure as a misinterpretation of a two-dimensional effect. As fluid flowing through a tube approaches the exit plane of the tube, it acquires a radial component of velocity and an accompanying radial pressure gradient as it expands into the receiver. Large radial pressure gradients were acquired by dense fluids, and small radial profiles were acquired by fluids of low density. Thus, the pressure at the wall in the vicinity of the exit plane is less than the pressure at the centerline of the tube, which is very nearly the one-dimensional pressure characteristic of the behavior at a given cross section. This is pictured in Fig. 2.2.

Fig. 2.2. Pressure Profiles Measured at the Centerline and at the Wall in Both Rapidly and Gradually Diverging Test Sections. ANL Neg. No. 900-1780.

To verify this reasoning, Henry constructed two test sections hydrodynamically identical upstream of the exit plane. At the exit, one was affixed with a 120° included-angle diverging portion, characteristic of critical-flow studies prior to 1968, and the other was affixed with a 7° included-angle diverging portion to "geometrically restrict the expansion of the fluid." The use of a 7° angle can be inferred from Ref. 40; it is the angle at which losses for such a shape become minimum for incompressible flow. By this approach, it was shown that much care must be taken when trying to obtain a measurement of P_e which is characteristic of the behavior of the flow when using a wall pressure tap. The use of 7° included-angle diverging section is necessary to achieving this end in most cases. Exceptions to this are discussed in Ch. IV.

Kelly

In 1968, Kelly⁸ studied the two-phase critical flow of water through tubes for which $L/D = 100$. In this investigation, the effect of the included angle in a downstream diverging portion, such as used by Henry,⁴ was examined. Results are depicted in Fig. 2.3.

Fig. 2.3. Effect of Downstream Geometry on Flow Rate when P_0 is Adjusted to Give the Same Value of P_e in Each Test Section. The data are the two-phase water data of Kelly.⁸ ANL Neg. No. 900-1856.

Figure 2.3 should be viewed in a qualitative sense, since P_0 is adjusted so that the measured value of P_e is the same for each downstream geometry. Of most direct bearing in evaluating a given measurement of P_e would be a comparison of P_e measured using various diverging sections for the same stagnation conditions. Kelly calculated the equilibrium exit-plane quality, x_{Ee} , from the energy balance

$$x_{Ee} = \frac{\frac{q}{W} + h_i = h_{fe}}{h_{fge}},$$

where h_i is the inlet enthalpy, q represents heat transfer of energy, and W represents the mass flow rate. Using the tabulated values of x_{Ee} given in Ref. 8, it is possible to calculate the stagnation state, since the kinetic energy term is about 0.01% of the other terms,⁸ as fixed by h_o and a given value of G_{cr} via the equation

$$h_o = (1 - x_{Ee})h_{fe} + x_{Ee} h_{ge}.$$

Thus, for the same h_o and G_{cr} , the values of P_e in the various types of diverging sections can be compared. These results are presented in Table 2.1. From this table, it can be concluded that, for low qualities, an included angle of 28° gives a value of P_e measured by a wall tap that is about halfway between a value measured in a 7° diverging test section and a measurement taken with a 180° diverging test section (no diverging section at all).

TABLE 2.1. Critical-pressure Measurements of Two-phase Water, Ref. 8: P_e measured in test sections with various included-angle diverging sections for constant stagnation conditions and L/D

h_o , Btu/lb _m	G_{cr} , lb _m /ft ² -sec	$P_e(7^\circ), P_e(14^\circ)$, psia psia		$P_e(28^\circ)$, psia	$P_e(180^\circ)$, psia
247	832	36.7		31.2	
260	832	38.6	33.0	34.5	
286	832	46.0	41.2	38.95	
317 ^a	832	52.0	48.0	44.6	
253	1110	39.0	36.0		30.9
272	1110	46.4	41.4	40.9	37.7

The variation in h_o is no more than ± 2 Btu/lb_m in corresponding runs.

^a h_o varies ± 5 Btu/lb_m between corresponding runs.

Fauske

In 1965, Fauske² investigated critical two-phase flow of water through short tubes for which $0 < L/D < 40$, where $L/D = 0$ corresponds to orifice flow. In this study, entrance conditions were of the sharp-edged type. Fauske's investigation was oriented toward developing a greater understanding of the role of L/D . For the various ranges of L/D tested the following conclusions were made:

$0 < L/D < 3$

It was deduced from pressure measurements at the wall that for $0 < L/D < 3$ the liquid "breaks immediately from the wall" and remains as a nonequilibrium jet, with flashing from the jet surface taking place.

$3 < L/D < 12$

Due to significant pressure fluctuations measured by pressure transducers along the length of the test section, Fauske hypothesized that the nonequilibrium liquid jet breaks up.

$12 < L/D < 40$

Fauske believed that the nonequilibrium begins to relax as L/D approaches 12, since in the range $12 < L/D < 40$ the flow rate became a much weaker function of L/D than in the range $0 < L/D < 12$. Fauske further stated that for $L/D > 12$ he believed the thermodynamic path to be very nearly one of thermodynamic equilibrium, that is, most of the nonequilibrium generated at the inlet has relaxed.

In an effort to confirm the use of L/D as the variable of influence, Fauske cited the work of Friedrich²² who, for sharp-edged entrance tubes of differing ID (1.5, 3, and 4 mm) and identical $L/D = 1.6$, obtained

essentially the same critical flow rate of saturated water through the different tubes.

The critical-pressure ratios reported by Fauske in this investigation are low for the reason pointed out by Henry,⁴ that is, the presence of a significant radial pressure gradient in the vicinity of the tube exit causes a wall pressure tap near the exit to read low. In Fauske's experiment, the exit-plane pressure tap was located 1/8 in. from the exit of the tube; thus, the reported value of P_e is subject to this error.

Uchida and Nariiai

In 1966, Ichida and Nariiai¹² studied two-phase critical flow of saturated and subcooled water through sharp-edged entrance tubes for which $1.25 \leq L/D \leq 500$. They observed that the appearance of a vapor phase decreased the flow rate from that in the all-liquid case. It was also noted that for tubes for which $1.25 \leq L/D \leq 5$, the axial pressure profiles were quite constant after the entrance pressure drop took place, save for the case of very short tubes, in which a pressure rise was detected near the exit.

Exit-plane pressures were measured by a painstaking method of taking small cuts off the end of the tube and taking a pressure reading after each cut until the last pressure tap was situated half on the tube and halfway past the exit of the tube. However, this method also is subject to the error caused by the presence of the radial pressure gradient near the tube exit, as pointed out by Henry.⁴ That the fluid acquires a radial pressure profile near the exit was also pointed out by Klingebiel;³⁰ Uchida and Nariiai themselves recognized that the expansion at the exit of the pipe was two-dimensional for, as they noted, "the steam-water mixture...is discharged to the atmosphere and disperses as a water spray spreading vertically to the flow direction." By failing to interpret their measurements in light of this, it is believed that Uchida and Nariiai were led to the erroneous conclusion that a steep pressure gradient exists near the exit of a short tube. This point will be taken up in more detail in Ch. IV.

Using a high-speed camera, Uchida and Nariiai observed that the liquid and vapor exhibited little movement relative to each other and therefore concluded that the slip ratio, $k = u_g/u_l$, seemed to be very nearly 1. This agrees well with the values of k determined by Henry⁴ and suggests that critical-flow models giving large values of k are somewhat suspect.

For one who is interested in comparing the data of Ref. 12 with a critical-flow model, the reported values of flow rate, calculated by measuring the decreasing rate of water level, are considered to be reliable, as well as the reported stagnation conditions, termed "initial" temperature and pressure. However, one must view the reported values of P_e and, thus, η as being lower than is actually characteristic of the phenomenon.

Howard

Howard⁷ studied the critical discharge of CCl_3F through tubes for which $25 \leq L/D \leq 300$. His results contribute to the understanding of the problem at hand for two reasons. We have attempted to learn the effect of

L/D over a wider range than was covered in the present study and, second, to compare both the data of Ref. 7 and data taken in the present study to a critical-flow model⁵ in an effort to evaluate its applicability.

The data of concern in Howard's experiment were taken with a glass test section affixed with a 22° included-angle diverging section. As in the case of other investigations,^{2,12} the exit-pressure measurements are low. As seen from Refs. 4, 8, and 40, a 7° included-angle diverging section is most proper for minimizing the two-dimensional effects of the exit-plane expansion. One would have to assess the reported values of P_e as being low by approximately half the deviation between taking the measurement with a 7° diverging section affixed to the tube exit and taking this measurement with no diverging section at all.

The glass test sections in Howard's investigation had four different entrance configurations--sharp-edged, rounded, fire-polished, and rounded and glass-blasted. No difference in flow rate was noticed in this investigation for the different entrance configurations when two phases were present. The data taken at $L/D = 25$ and 50 were from sharp-edged entrance tubes; the data taken at $L/D = 100$ are associated with both the sharp-edged and fire-polished entrances.

A plot of Howard's data and the data from the present study are compared to Henry's model⁵ in Ch. IV.

THEORETICAL INVESTIGATIONS

Most theoretical studies of two-phase critical flow to date have been one-dimensional, that is, the equations are written in terms of one spatial variable and variations are considered only with respect to that variable. For this reason, this section will begin with a discussion of one-dimensional considerations and a presentation of a general model for one-dimensional, two-phase flow.

Of specific interest are four theoretical investigations resulting in critical-flow models as well as the predecessor to these models, the homogeneous equilibrium model. Two of the models to be subsequently discussed^{1,9} might be termed "general models," that is, they are not restricted to any specific L/D or entrance configuration, although the model presented in Ref. 1 is restricted to constant-area tubes. The other two models to be treated were derived specifically for sharp-edged entrance tubes for which $L/D \leq 12^5$ and for which $0 \leq L/D \leq 3.^6$

Although other critical-flow models exist,^{10,14,15,20,23,31,34,35} the models covered here have been more readily accepted and do a reasonable job of predicting certain types of data over some range. An analysis such as presented in Ref. 34, while the author is to be commended for solving separate momentum equations for each phase so that the slip ratio may be calculated (although it is unclear from Ref. 34 how the interphase terms are specified), is so mathematically unmanageable that it tends to be somewhat impractical.

One-dimensional Considerations

It is appropriate to consider the applicability of one-dimensional models to describe critical flow in short tubes for, earlier in this chapter, it was discussed how various investigators have taken unreliable measurements of exit-plane pressure, since their reported pressures are not the one-dimensional pressure characteristic of the flow behavior.

When a fluid, initially all liquid, is adiabatically discharged through a short tube, there is little time for vaporization to occur and the quality, even at the exit of the tube, is very low. Since less liquid vaporizes in this process than would be expected if sufficient relaxation time occurred, $x \leq x_F$. Thus, the mass rate of flow is essentially the liquid flow rate; however, the vapor phase and how it appears control the thermodynamic process and, thus, the critical-flow phenomenon.

By observing the flow pattern (see Fig. 4.7 below), it is seen that the flow pattern is one of a liquid jet, surrounded by a two-phase-mixture annulus, which breaks up into a dense, two-phase mixture. A Reynolds number, Re , based on measured values of G and a mean diameter chosen at the cross-sectional area where the jet shrinks to about 60%, as in the case of orifice flow, can be calculated. It is found that Re is typically of the order of 10^5 . If one views the Reynolds number as strictly the ratio of inertial to viscous forces, one would have to ascribe the inertial forces to the liquid and the viscous forces to the vapor phase, since mostly vapor contacts the wall. Typical values of Re calculated by $Re = \rho u D/\mu$ are of the order of 10^5 . Such values are recognized as being indicative of turbulent flow. Due to the flatness of the radial pressure profiles in fully developed turbulent flow, the centerline pressure at a location z is quite representative of the pressure averaged over the cross-sectional area of the jet at z .

In light of this apparent appropriateness, then, it seems that the mathematical description of two-phase flow in short tubes is amenable to the simplicity of one-dimensional modeling.

General Mathematical Model

A general mathematical model will now be set down to describe two-phase flow through short tubes. The experimental investigations mentioned above suggest that several conditions hold:

- 1) The flow is adiabatic and steady state through a constant-area tube.
- 2) The process is nonequilibrium in nature, that is, the liquid is superheated. While this nonequilibrium state persists, the quality of the mixture is less than if the fluid were at an equilibrium state (see Fig. 2.4). As explained in Ref. 33:

"[For equilibrium flows] It has been assumed that the fluid in flow passes from one equilibrium state to another without lag from molecular readjustment (zero *relaxation time*). Actually, this assumption is not true, since time is required to redistribute

Fig. 2.4. The thermodynamic path for isothermal flow through a nozzle. When initiated by equilibrium $P-v-T$ properties: beginning with slightly subcooled liquid at state A, a sudden external pressure drop to state B is sustained by the fluid at a constant temperature. An equilibrium $P-v-T$ surface indicates that an amount of liquid should evaporate as dictated by the equilibrium quality (the distance between the estimated liquid line and point B divided by the distance between the estimated liquid and estimated vapor line). In reality, less liquid evaporates, indicating that the process is not one that is equilibrium in nature. AEC Rep. No. 900-1091.

the energy among the molecules...If the fluid passes quickly from one temperature (or pressure) to another, the relaxation time may be greater than the time of the process, and the (properties) cannot adjust to the equilibrium values.

When the fluid is all gas or all liquid, the effects of relaxation are usually relatively insignificant. But, when the flowing fluid reaches the state where a phase change is demanded, relatively great changes must be accomplished, and the lag in reaching equilibrium may be quite apparent."

- 3) The flow can be represented with one-dimensional equations.
- 4) Wall friction is negligible.
- 5) The effects of gravity are negligible.

First, extensive property balances are written with reference to Fig. 2.5:

Momentum balances:

For the liquid phase:

$$A_L P \Big|_z - A_L P \Big|_{z+\Delta z} + \rho_L A_L u_L^2 \Big|_z - \rho_L A_L u_L^2 \Big|_{z+\Delta z} + \Delta z F_{lg} = 0,$$

where F_{lg} is the rate at which momentum is transferred from the liquid to the vapor phase per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$-A_L \frac{dP}{dz} = \frac{d}{dz} (\rho_L A_L u_L^2) - F_{lg}. \quad (2.1a)$$

Fig. 2.5. Control Volume for Derivation of General Two-phase-flow Model. ANL Neg. No. 900-1888.

For the vapor phase:

$$A_g P \Big|_z - A_g P \Big|_{z+\Delta z} + \rho_g A_g u_g^2 \Big|_z - \rho_g A_g u_g^2 \Big|_{z+\Delta z} + \Delta z F_{gl} = 0 ,$$

where F_{gl} is the rate at which momentum is transferred from the vapor to the liquid phase per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$-A_g \frac{dP}{dz} = \frac{d}{dz} (\rho_g A_g u_g^2) - F_{gl} . \quad (2.1b)$$

Mass balances:

For the liquid phase:

$$\rho_l A_l u_l \Big|_z - \rho_l A_l u_l \Big|_{z+\Delta z} + \Delta z m_{lg} = 0 ,$$

where m_{lg} is the rate at which liquid changes to vapor per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$\frac{d}{dz} (\rho_l A_l u_l) = m_{lg} . \quad (2.2a)$$

For the vapor phase:

$$\rho_g A_g u_g \Big|_z - \rho_g A_g u_g \Big|_{z+\Delta z} + \Delta z m_{gl} = 0 ,$$

where m_{gl} is the rate at which vapor changes to liquid per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$\frac{d}{dz} (\rho_g A_g u_g) = m_{gl} . \quad (2.2b)$$

Energy balances (adiabatic):

For the liquid phase:

$$\rho_l A_l u_l \left(h_l + \frac{u_l^2}{2} \right) \Big|_z - \rho_l A_l u_l \left(h_l + \frac{u_l^2}{2} \right) \Big|_{z+\Delta z} + \Delta z E_{lg} = 0 ,$$

where E_{lg} is the rate at which energy is transferred from the liquid to the vapor per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$\frac{d}{dz} \left[\rho_l A_l u_l \left(h_l + \frac{u_l^2}{2} \right) \right] = E_{lg} . \quad (2.3a)$$

For the vapor phase:

$$\rho_g A_g u_g \left(h_g + \frac{u_g^2}{2} \right) \Big|_z - \rho_g A_g u_g \left(h_g + \frac{u_g^2}{2} \right) \Big|_{z+\Delta z} + \Delta z E_{gl} = 0 ,$$

where E_{gl} is the rate at which energy is transferred from the vapor to the liquid per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$\frac{d}{dz} \left[\rho_g A_g u_g \left(h_g + \frac{u_g^2}{2} \right) \right] = E_{gl} . \quad (2.3b)$$

Entropy balances:

For the liquid phase:

$$\rho_l A_l u_l s_l \Big|_z - \rho_l A_l u_l s_l \Big|_{z+\Delta z} + \Delta z S_{lg} - \Delta z S_{\pi l} - \Delta z S_{\pi il} = 0 ,$$

where S_{lg} is the rate at which entropy is transferred from the liquid to the vapor per unit length, $S_{\pi il}$ is the rate at which entropy produced at the liquid-vapor interface is transported to the liquid per unit length, and $S_{\pi l}$ is the rate at which entropy is produced within the liquid per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$\frac{d}{dz} (\rho_l A_l u_l s_l) + S_{\pi l} + S_{\pi il} = S_{lg} . \quad (2.4a)$$

For the vapor phase:

$$\rho_g A_g u_g s_g \Big|_z - \rho_g A_g u_g s_g \Big|_{z+\Delta z} + \Delta z S_{gl} - \Delta z S_{\pi g} - \Delta z S_{\pi ig} = 0 ,$$

where S_{gl} is the rate at which entropy is transferred from the vapor to the liquid phase per unit length, $S_{\pi ig}$ is the rate at which entropy produced at the liquid-vapor interface is transported to the vapor per unit length, and $S_{\pi g}$ is the rate at which entropy is produced within the vapor per unit length. Dividing through by Δz and taking the limit as $\Delta z \rightarrow 0$:

$$\frac{d}{dz} (\rho_g A_g u_g s_g) + S_{\pi g} + S_{\pi ig} = S_{gl} . \quad (2.4b)$$

Next, appropriate relationships between thermodynamic properties must be found. These may be expressed as:

$$\rho_l = \rho_l(P, T) ; \quad (2.5)$$

$$\rho_g = \rho_g(P, T) ; \quad (2.6)$$

$$h_l = h_l(P, T) ; \quad (2.7)$$

$$h_g = h_g(P, T) ; \quad (2.8)$$

$$s_l = s_l(P, T) ; \quad (2.9)$$

$$s_g = s_g(P, T) . \quad (2.10)$$

However, there do not exist constitutive equations to describe the interphase-transport or the entropy-production terms. Therefore, at this point, one is confronted with 18 unknowns: $A_l, A_g, P, T, u_l, u_g, F_{lg}, F_{gl}, m_{lg}, m_{gl}, E_{lg}, E_{gl}, S_{lg}, S_{gl}, S_{\pi l}, S_{\pi g}, S_{\pi lg}, S_{\pi gl}$, and only 15 equations.

One could choose to eliminate the interphase-transport terms by adding the individual phase equations together since it is true that

$$-F_{lg} = F_{gl} ;$$

$$-m_{lg} = m_{gl} ;$$

$$-E_{lg} = E_{gl} ;$$

$$-S_{lg} = S_{gl} .$$

However, by so doing, the means by which interphase transports are taken into account is lost. In particular, adding the two momentum equations together discards the means by which the individual phase velocities are calculated. Yet, since the difficulty in expressing the interphase-transport terms is quite great, the trade-off between losing the means to account for interphase transport and the elimination of eight unknowns is usually made. Therefore, adding Eqs. 2.1a and 2.1b, one obtains an overall momentum balance:

$$-(A_g + A_l) \frac{dP}{dz} = \frac{d}{dz} [\rho_l A_l u_l^2 + \rho_g A_g u_g^2] . \quad (2.1c)$$

Summing Eqs. 2.2a and 2.2b yields the overall mass balance:

$$\frac{d}{dz} [\rho_l A_l u_l + \rho_g A_g u_g] = 0 . \quad (2.2c)$$

The overall energy balance is obtained by adding Eqs. 2.3a and 2.3b:

$$\frac{d}{dz} \left[\rho_l A_l u_l \left(h_l + \frac{u_l^2}{2} \right) + \rho_g A_g u_g \left(h_g + \frac{u_g^2}{2} \right) \right] = 0 . \quad (2.3c)$$

The addition of Eqs. 2.4a and 2.4b gives the overall entropy balance:

$$\frac{d}{dz} [\rho_g A_g u_g s_g + \rho_l A_l u_l s_l] + S_{\pi l} + S_{\pi g} + S_{\pi i} = 0 , \quad (2.4c)$$

where $S_{\pi i}$ is the total entropy production at the liquid-vapor interface.

Finally, at this point there are nine unknowns: A_l , A_g , P , T , u_l , u_g , $S_{\pi l}$, $S_{\pi g}$, and $S_{\pi i}$, and 11 equations. What is lacking, then, is

- 1) a representation of the nonequilibrium nature of the phase change so that the mass-transfer rate and other interphase terms can be calculated;
- 2) a representation of how the phases are configured as well as the interphase momentum transfer so that the relationships between A_g and A_l , and u_g and u_l are known;
- 3) a means of calculating $S_{\pi l}$, $S_{\pi g}$, and $S_{\pi i}$.

How the various investigators have met with these difficulties can be seen in the following sections.

It should be pointed out that in two-phase flow literature, it is common to define a *define* fraction, $\alpha \equiv A_g / (A_g + A_l)$, a *slip ratio*, $k \equiv u_g / u_l$, and a *quality*, $x \equiv \rho_g A_g u_g / (\rho_g A_g u_g + \rho_l A_l u_l)$.

Homogeneous Equilibrium Model

The homogeneous equilibrium model is, perhaps, the simplest approach to two-phase-flow analysis. It is derived subject to three major assumptions:

- 1) The liquid and vapor phases are assumed to travel at the same local velocity ($k = 1$).
- 2) Suitable average properties are determined as mass averages of the individual phase properties, and the two-phase mixture is treated as a pseudo-single-phase fluid that obeys the usual equations of single-phase flow.⁴¹
- 3) The process is one of thermodynamic equilibrium.

The homogeneous equilibrium model is easily derived by the analysis resulting in Eq. 1.10a:

$$G_{cr}^2 = - \frac{1}{\frac{dv}{dP_e}} , \quad (1.10a)$$

where the specific volume of the two-phase mixture, v , is defined as

$$v \equiv v_l (1 - x_E) + v_g x_E .$$

Substituting the definition of v and differentiating, Eq. 1.10a becomes

$$G_{cr} = \left[\frac{-1}{x_E \frac{dv_g}{dP_e} + (v_g - v_l) \frac{dx_E}{dP_e} + (1 - x_E) \frac{dv_l}{dP_e}} \right]^{1/2} . \quad (2.11)$$

Data taken in this experiment are compared to this model in Ch. IV.

Fauske

The critical-flow model of Fauske¹ is one of the most often-cited models of two-phase critical flow. The interest in this model comes largely from the fact that it gives the critical flow rate in an explicit, closed-form expression. In addition, a means has been devised for calculating the slip ratio k at critical flow.

This model is applied to situations for which:

- 1) thermodynamic equilibrium exists,
- 2) the flow is one-component, adiabatic, and steady-state through a constant-area pipe, and
- 3) the flow can be represented by one-dimensional equations.

One needs to know P_e to use this model. In the sense that Fauske's model is not restricted to any specific entrance configuration or L/D range it might be termed a "general model."

In an effort to make up for the information lost by not writing separate momentum equations, Fauske imposed a constraint equation that allowed calculation of the slip ratio. At the plane of critical flow it was stated that

$$\frac{\partial}{\partial k} \left[\frac{dP}{dz} \right] = 0 . \quad (2.12)$$

This can be rewritten as

$$\frac{dz}{dk} \frac{d}{dz} \left[\frac{dP}{dz} \right] = 0 . \quad (2.13)$$

As long as $dk/dz \neq \infty$, Eq. 2.12 implies that the pressure gradient reaches a maximum at the location of the critical flow plane. As will be discussed in Ch IV, this statement is questionable for critical flow through short tubes. Further, values of k calculated from Fauske's model are higher than those measured experimentally.^{4,30} This is graphically illustrated in Fig. 12 of Ref. 28. Measured values of k range from 1 to 5, whereas k calculated from Fauske's model ranges from 20 to 30 for CCl_3F .

Fauske's expression for the critical flow rate is

$$G_{cr} = \left\{ \left[\frac{-g_c k /}{\frac{dv_g}{dP}} + [v_g (1 + 2kx - 2x) + v_\ell (2xk - 2k - 2xk^2 + k^2)] \frac{dx}{dP} + [k \{1 + x(k - 2) - x^2(k - 1)\}] \frac{dv_\ell}{dP} \right] \right\}^{1/2}, \quad (2.14)$$

where

$$k = \begin{cases} \sqrt{\frac{v_g}{v_\ell}}, & 0 < x < 1 \\ 1, & x = 0, x = 1. \end{cases} \quad (2.15)$$

An evaluation of the applicability of this model is in order. The term $\left\{ [(1-x)v_\ell + x/k v_g]x - x/k^2 v [1 + x(k-1)] \right\} dk/dP$ has been eliminated from the denominator of Eq. 2.14, for when Eq. 2.15 is substituted for k , the coefficient of dk/dP vanishes. Thus, if one views Eq. 2.15 as yielding an overprediction of k , and Eq. 2.14 as missing the dk/dP term in the denominator, some conclusions can be drawn.

If one examines the order of magnitude of the terms in the denominator of Eq. 2.14, it is seen that the dv_ℓ/dP term is negligible, being four orders of magnitude smaller than the dv_g/dP term for CCl_3F calculations. The dk/dP term, had it appeared, is positive. Thus, the magnitude of the denominator is too large because a dk/dP term does not appear due to the expression for k . The magnitude of the numerator is too large due to the overprediction of k . One would expect, then, that for conditions for which equilibrium flow exists, Fauske's model will give reasonable predictions of the flow rate if the errors in the numerator and denominator of Eq. 2.14 compensate for each other.

Data taken in the present investigation are compared with Fauske's model in Ch IV. In a sense, it is a somewhat unfair comparison, since it is known that the data are representative of a highly nonequilibrium situation. However, the comparison is made to show that a model derived under the assumption of thermodynamic equilibrium is not descriptive of the actual process occurring at low qualities.

Moody

Perhaps the critical-flow model that has been most widely applied to reactor-safety analysis is that of Moody.⁹ The attractive feature of this model is that G_{cr} and P_e can be calculated by knowing only the stagnation

conditions. Since this model, too, is not restricted to a specific entrance configuration or L/D , and is applicable to nozzles as well as constant-area ducts, it is also termed a "general model."

Moody's model is derived subject to the restrictions that:

- 1) both phases are at the same static pressure,
- 2) both phases are at local equilibrium at entrance and exit,
- 3) the exit slip ratio is an independent variable, that is, k and P are independent,
- 4) the flow is isentropic, and
- 5) fluid properties are calculated as saturation properties, which are functions of P alone.

Combining energy and mass balances, thermodynamic property relations, and definitions of k , x , and α , Moody derived the following expression for the mass flow rate per unit area:

$$G(P,k) = \left\{ \frac{2g_c J \left[h_o - h_f - \frac{h_{fg}}{s_{fg}} (s_o - s_f) \right]}{\left[\frac{k(s_g - s_o)v_f}{s_{fg}} + \frac{(s_o - s_f)v_g}{s_{fg}} \right]^2 \left[\frac{s_o - s_f}{s_{fg}} + \frac{s_g - s_o}{k^2 s_{fg}} \right]} \right\}^{1/2} \quad (2.16)$$

To express the critical flow rate, the function $G(P,k)$ must be maximized. If $G(P,k)$ has a maximum, it is necessary that

$$\frac{\partial G(P,k)}{\partial P} = 0 \quad (2.17a)$$

and that

$$\frac{\partial G(P,k)}{\partial k} = 0 \quad (2.17b)$$

Solving Eq. 2.17b for k gives G for the extreme value of G ,

$$k = \left(\frac{v_g(P')}{v_f(P')} \right)^{1/3}, \quad (2.18)$$

where P' is the value of P which maximizes Eq. 2.16 when Eq. 2.18 is substituted for k .

However, Eq. 2.18 gives values of k that range from 7 to 10 for CCl_3F . These are higher than the experimentally measured values given in Ref. 4 and the observation of Ref. 12 that k is very nearly 1. Figure 12 of Ref. 28 shows Eq. 2.18 plotted against water data, illustrating the

overprediction of k by Moody's model. Assumptions 2, 3, and 5 are instrumental in deriving Eq. 2.18. As previously discussed, assumption 2 is contrary to observed behavior--the process is not an equilibrium process. Assumption 3, that k and P are independent, has not been verified by experimental evidence. Calculating fluid properties as saturation properties of P alone, assumption 5, is difficult to justify for short-tube flow, for the liquid (superheated for short-tube critical flow) properties are better portrayed as functions of temperature. Therefore, it is believed that values of k obtained from Eq. 2.18 are subject to the apparent inappropriateness of these three assumptions.

By examining the relative magnitudes of the terms in Eq. 2.16, it is seen that $(s_g - s_o)/(s_{fg} k^2)$ is the controlling term in the denominator. Thus, one would expect, for instances where the assumptions are valid (high-quality flows), that Moody's model overpredicts the critical flow rate. However, in the low quality range, the equilibrium assumption is not valid and Moody's model underpredicts the flow rate considerably, for the flow is actually less compressible than what this model predicts.

Data taken in the present experiment are compared to Moody's model in Ch IV. Once again, this is a case of comparing nonequilibrium data to an equilibrium model. However, the comparison is made to illustrate the consequences of applying such models to short-tube data.

Henry ($L/D \geq 12$)

The next critical-flow model to be discussed is that derived by Henry in 1970.⁵ The salient features of this model are that one needs only to know the stagnation conditions to predict the critical flow rate and critical pressure ratio; it incorporates a description of the nonequilibrium behavior characteristic of critical flow through short tubes. The model has been successfully applied over a range of fluids, and, in addition, it incorporates the length-to-diameter ratio L/D as an independent variable.

Contrary to the previous models discussed, Henry's model is a very specific model, intended to describe only the two-phase critical discharge of initially saturated or subcooled liquid through sharp-edged-inlet, constant-area ducts for which $L/D \geq 12$.

To derive this model, Henry assumed that between the inlet and $L/D = 12$, the fluid experiences an entrance pressure drop to a constant value which it maintains until $L/D = 12$. This assumption of constant axial pressure is substantiated by the pressure profile data, as will be shown in Ch IV and in App A. Therefore, the pressure drop from the stagnation point to the point where $L/D = 12$ is only an entrance pressure loss.

It is further assumed that the flow can be described by one-dimensional equations. Thus, the momentum equation may be written as

$$G_{cr}^2 = 2C_d^2 \rho_{lo} (P_o - P_e) , \quad (2.19)$$

where C_d is taken to be 0.61.

A general one-dimensional expression for the two-phase, one-component critical flow rate has been derived by Henry⁴ as

$$G_{cr}^2 = -g_c \left\{ k \left[\{ [1 + x(k-1)]x \} \frac{dv_g}{dP} + \{ v_g [1 + 2x(k-1)] + kv_l [2(x-1) + k(1-2x)] \} \frac{dx}{dP} + \{ k[1 + x(k-2) - x^2(k-1)] \} \frac{dv_l}{dP} + x(1-x) \left(kv_l - \frac{v_g}{k} \right) \frac{dk}{dP} \right]^{-1} \right\}_e \quad (2.20)$$

This equation may be viewed as the constraint ($dG/dP = 0$) by which Eq. 2.19 is solved, for the solution is the intersection of Eqs. 2.19 and 2.20. Thus, the thermodynamic path assumed in this model is a fabricated path in which an incompressible process is constrained by a compressible constraint at the exit plane of the duct.

Equation 2.20 may be simplified by assuming the liquid phase to be incompressible ($dv_l/dP = 0$), that $k = 1$ everywhere, and that $dk/dP = 0$, which is approximately true for low-quality or high-pressure flows. Thus, Eq. 2.20 becomes

$$G_{cr}^2 = \frac{-g_c}{\left[x \frac{dv_g}{dP} + (v_g - v_l) \frac{dx}{dP} \right]_e} \quad (2.21)$$

In the region of the exit plane, the amount of vapor formed is negligible, while the rate of formation is not. Thus, to describe the nonequilibrium mass transfer rate, a correlation based on Ref. 4 is introduced:

$$\frac{dx}{dP} = \frac{N dx_e}{dP}, \quad (2.22)$$

where N may be viewed as a measure of the difference between the actual and equilibrium processes at the critical flow plane. From experimental data,⁴ N is evaluated as

$$N = \begin{cases} 20x_E & \text{for } x_E < 0.05 \\ 1 & \text{for } x_E \geq 0.05 \end{cases} \quad (2.23)$$

and, for $s_o = \text{constant}$,

$$x_E = \left[\frac{s_o - s_l}{s_g - s_l} \right]_E \quad (2.24)$$

Implicit in Eq. 2.23 is the assumption that, for equilibrium qualities greater than 5%, the process is taking place at an equilibrium rate at the exit plane. Equation 2.21 reduces to

$$G_{cr} = \frac{-g_c}{\left[(v_g - v_l) \frac{Ndx_E}{dP} \right]_e} \quad (2.25)$$

The above, then, describes the critical-flow phenomenon in ducts for which $L/D = 12$. For $L/D > 12$, a momentum pressure drop between $L/D = 12$ and the exit plane must be included. Since this study does not encompass long- L/D ducts, this aspect of Henry's model will not be discussed here. However, the details may be found in Ref. 5.

A comparison of Henry's model to data taken in this study is discussed in Ch IV.

Henry ($0 \leq L/D \leq 3$)

The final critical-flow model to be presented concerns critical flow through very short, sharp-edged entrance tubes. This model, derived by Henry,⁶ is similar to Silver's "heat transfer limited" model for nozzles,³⁵ that is, "the rate at which evaporation can occur from a surface is limited by the rate of heat flow into the surface".³⁵ Silver makes an approximation to solve the energy equation, while Henry makes use of the exact solution.

The only other short-tube model known to the author is that of Ref. 20, which includes the effect of bubbles and postulates that the same number of droplets will be formed (see Fig. 4.6 below) as the original number of bubbles present. This nonequilibrium model, however, is extremely complex. Since the model has failings for nearly saturated conditions, this model has not been accepted as being particularly attractive.

The model derived by Henry is subject to the following assumptions:

- 1) The flow pattern is as pictured in Fig. 2.6.
- 2) The flow can be described by one-dimensional equations.
- 3) Axial temperature gradients are small compared to radial gradients.
- 4) $k = 1$ at all times.
- 5) $dk/dP = 0$.
- 6) The vapor phase is in thermodynamic equilibrium at the local pressure.

Fig. 2.6. Flow Pattern Assumed by Henry⁶ for Critical Flow in Sharp-edged Entrance Tubes for which $0 \leq L/D \leq 3$. ANL Neg. No. 900-445.

This model is derived much the same way as the model for $L/D \geq 12$ except that the compressibility term is evaluated differently to account for the different flow pattern. The momentum equation is again Eq. 2.19:

$$G_{cr}^2 = 2C_d^2 \rho_{l0} (P_o - P_e) . \quad (2.19)$$

The energy balance results in the following equation:

$$\frac{1}{u_l r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{\rho_l c_l}{\kappa} \frac{\partial T}{\partial z} . \quad (2.26)$$

The boundary conditions are:

$$r = 0; \quad \frac{\partial T}{\partial r} = 0 ; \quad (2.27a)$$

$$r = R; \quad T = T_s(P_e) ; \quad (2.27b)$$

$$z = 0; \quad T = T_1 , \quad (2.27c)$$

where

$$T_1 = T_o - (G_{cr}/\rho_{l0})^2 / 2c_l .$$

Solution of Eqs. 2.26 and 2.27 gives an expression for the average temperature at the exit plane:

$$T_e = T_s(P_e) + 2[T_1 - T_s(P_e)] \sum_{n=1}^{\infty} \frac{2}{\lambda_n^2} e^{-\frac{4\lambda_n^2 \kappa L}{c_l G_{cr}^2 D^2}} , \quad (2.28)$$

where λ_n is the n^{th} root of the equation $J_0(\lambda_n) = 0$, which may be found in Ref. 17. Here J_0 is the Bessel function of order zero.

The exit-plane quality is given by

$$x_e = \frac{h_l - h_e}{h_{gEe} - h_e} , \quad (2.29)$$

where

$$h_1 = h_o - G_{cr}^2 / 2\rho_{lo}^2 \quad (2.30)$$

and

$$h_e = c_l T_e \quad (2.31)$$

Differentiating Eq. 2.29 and making appropriate substitutions, the mass-transfer rate is determined to be

$$\left. \frac{dx}{dP} \right|_e = \frac{x_e}{T_o - T_s(P_e)} \frac{T_s(P_e) v_{fgE}}{h_{fgE}} \quad (2.32)$$

The previous assumptions and Eq. 2.32 reduce Eq. 2.20 to

$$G_{cr}^2 = \left[\frac{g_c}{\frac{xv_g}{P} + \frac{T_s(P_e) xv_{fgE} (v_g - v_{lo})}{[T_1 - T_s(P_e)] h_{fgE}}} \right]_e \quad (2.33)$$

Note that in Eq. 2.33, temperatures are absolute temperatures.

Thus, Eq. 2.19 can be solved in conjunction with Eq. 2.32 to obtain a prediction of η and G_{cr} .

This model is compared to data from the present study in Ch IV.

III. EXPERIMENTAL INVESTIGATION

Critical flow rate, exit-plane pressure, and pressure-profile measurements were obtained from the experimental apparatus shown in Fig. 3.1. The various test sections were attached between the glass high-pressure vessel and a flexible bellows mounted on the low-pressure vessel, as pictured in Fig. 3.2. The basic apparatus, operating procedure, test sections, and measurement techniques are discussed in this chapter. The experimental data are tabulated in Appendix A.

*Fig. 3.1a. Experimental Apparatus.
ANL Neg. No. 900-1590.*

Basic Apparatus

High-pressure Vessel

The high-pressure vessel was a vertical section of 6-in.-dia Pyrex pipe capped at the bottom with a hemispherical section. At the bottom of the cap was a 3-in.-dia inlet port, and at the side was a 2-in.-dia discharge port. The glass vessel had $21/64$ -in.-thick walls, and was rated to withstand temperatures from 0 to 400°F and pressures from full vacuum to 30 psig.

Fig. 3.11. Schematic diagram of the Experimental Apparatus. AMI Reg. No. 9-10-1493.

Fig. 3.2a. Transparent Test Section Mounted in Place Between High-pressure Vessel and Low-pressure Vessel by Means of a Bellows Arrangement. ANL Neg. No. 900-1591.

Fig. 3.2b. Test Section of Rectangular Cross Section with Glass Sides. Pertinent dimensions (in inches) are shown. ANL Neg. No. 900-1846.

Low-pressure Vessel

The low-pressure vessel was a 46-ft³ cylindrical stainless-steel drum, 58 in. in height and 42 in. in diameter, with 1/4-in.-thick walls oriented as in Fig. 3.1b. The need for maintaining a low pressure in this vessel required the ability to also keep the temperature low. To this end, 800 ft of 1/2-in. copper tubing, used as a cooling coil, was wrapped around the outside of the tank. One inch of asbestos mud was then pressed on as a coating, over which was put a layer of muslin pipe covering. Finally, the entire vessel was covered with 4 in. of foil-backed fiberglass insulation.

Refrigeration System

To keep the low-pressure vessel cold, a refrigeration system was connected to the cooling coil wrapped around the vessel. This system consisted of a tank containing a 50% water-50% ethylene glycol mixture to which 100 to 150 lb of dry ice was added. The mixture was pumped by a 1/2-hp pump from the tank through the coil and back into the tank. The mixture typically reached -40°F, which was sufficient to cool the empty low-pressure vessel to -20°F after several hours of operation.

Operating Procedure

Preparation

Preparation of the test facility consisted of calibrating the pressure transducers against a precision Heise brand, temperature-compensated, bourbon-tube gauge, scale 0 to 25 psia; the atmospheric pressure was set on the gauge after reading this value from a mercury barometer. The zero point of the chart recorders was reset each day. The pressure transducer and the nitrogen-inlet valve were located beneath the high-pressure vessel and were packed in dry ice to prevent boiling of the CCl₃F. This was necessary since CCl₃F boils at room temperature at atmospheric pressure.

The low-pressure vessel was pumped out with a vacuum pump and cooled by circulating ethylene glycol at about -40°F around the vessel. The ethylene glycol was brought to this low temperature by placing dry ice in the refrigeration unit through which the ethylene glycol circulated.

Prior to the first filling of the high-pressure reservoir each day, the surge tank, high-pressure vessel, test section, and pressure transducers were evacuated to approximately 0.3 psia to remove the bulk of the atmospheric air in them. This was unnecessary in subsequent runs, since

the CCl_3F level was not allowed to fall low enough for the test section and pressure transducers to empty. The high-pressure vessel was filled by pressurizing a CCl_3F drum with dry nitrogen and flowing the CCl_3F through a filter of fine-mesh wire cloth.

Making Runs

While filling the high-pressure vessel, the CCl_3F was brought to the desired temperature by placing the heat exchanger, a coil of 1/2-in. copper tubing, in either a bucket of hot water or a bucket of cold ethylene glycol. Dry nitrogen was bubbled through the CCl_3F to prevent stratification of cold and warm fluid. The desired stagnation pressure was obtained by pumping down the surge tank, and consequently the volume above the CCl_3F level, until within 1 psi of the desired pressure. When the valve to the low-pressure vessel was opened and the run begun, compressed air was fed in at the top of the high-pressure vessel at a rate sufficient to maintain a constant stagnation pressure at the inlet to the test section.

Data Map

This study was intended to investigate systematically the effects of the assumed independent variables P_o , T_o , and L/D on the dependent variables G_{cr} and η as well as to determine the role of downstream geometry. To this end, the 15 data points tabulated in Table 3.1 were taken for each L/D for the rapidly diverging and for the gradually diverging test sections. All points taken were within $\pm 1^\circ\text{F}$ and ± 0.05 psia of the points mapped out in Table 3.1.

TABLE 3.1. Data Points Taken for Both Rapidly and Gradually Diverging Test Sections at the Four L/D 's Investigated

T_o ($^\circ\text{F}$)	P_o (psia)				
72	16.74				
65	16.74	15.73			
55	16.74	15.73	12.73		
45	16.74	15.73	12.73	10.72	
35	16.74	15.73	12.73	10.72	9.72

Test Sections

Data were taken from four test sections, all made of Type 304 stainless steel, in this study. Sections C7 and C120 initially had constant-area sections with $L/D = 12.8$. They were subsequently cut down to values of 8 and 2.8 for L/D . Test sections C7S and C180 were used for the data taken at $L/D = 5$. Figure 3.3 shows the test sections and their pertinent dimensions. Location of the centerline of each pressure tap and sizes are summarized in Table 3.2.

Although the initial intent was to take data from test sections C7 and C120 at the four L/D s of interest, unfortunate circumstances dictated that

Fig. 3.3a. Test Section C7 at $L/D = 12.78, 7.96, \text{ and } 2.82$. ANL Neg. No. 900-1844.

$L/D = 7.96$

$L/D = 2.82$

Fig. 3.3b. Test Section C120 at $L/D = 12.78, 7.97, \text{ and } 2.82$. ANL Neg. No. 900-1852.

$L/D = 12.78$

$L/D = 7.97$

$L/D = 2.82$

$L/D = 5.00$

RIGHT END VIEW

$L/D = 5.08$

RIGHT END VIEW

Fig. 3.3c. Test Sections C7S at $L/D = 5.00$ and C180 at $L/D = 5.08$. ANL Neg. No. 900-1882.

new test sections be made to investigate $L/D = 5$. As can be seen from Fig. 3.3, C7S and C7, termed the gradually diverging test sections, are not identical downstream of the throat, nor are C120 and C180, termed the rapidly diffusing test sections. However, meaningful conclusions can still be drawn if, for the gradually diverging test sections, it is kept in mind that:

- 1) C7 and C7S both have a 7° diverging section beginning at the exit plane.
- 2) The diverging section in C7 extends the remaining length of the test section, while in C7S the diverging section continues 0.704 in. downstream of the exit plane and is followed by a 0.75-in.-ID constant-area section.
- 3) Critical pressure ratios and critical flow rates are unaffected by the differences in C7 and C7S, since these are governed by the flow in the constant-area section upstream of the exit plane (so long as critical flow is present at the exit plane of the constant-area section) and since the two-dimensional effects in the vicinity of the exit plane are equally minimized in the two test sections.
- 4) Axial pressure profiles are not comparable far downstream of the exit plane.

and if, for the rapidly diverging test sections, it is kept in mind that:

- 1) C120 has a 120° diverging section and C180 has a 180° expansion.
- 2) Mass flow rates per unit area through the constant-area sections upstream of the exit plane are unaffected by the different diverging sections while critical flow persists at the exit plane. This is demonstrated by comparing flow-rate data between rapidly and gradually diverging test sections at the same L/D .
- 3) The "exit" pressure, and, consequently, critical pressure ratio, will have a higher value in C180 than in C120 since the "exit" tap in C180 is 0.03 in. upstream of the throat, while in C120, it is 0.01 in. upstream.
- 4) Axial pressure profiles will be slightly different in the vicinity of and downstream of the exit plane.

Details of how the test sections were constructed are given in Appendix C.

Condition for Sharp-edged Entrance

Care was taken to prepare the entrance of each test section with a sharp edge, yet without a burr. This was done by taking a small face cut from the entrance on a lathe. If a burr was present, it was scraped from the inside with a scraping tool.

TABLE 3.2. Pressure-tap Locations in Test Sections

Test Section	Position	Distance Down-stream of Throat, in.	Pressure-tap Diameter, in.	L/D
C7	S1	-2.747	1/16	12.78,
	S2	-1.498	0.010	
	S3	-0.502	0.010	
	S4	-0.032	0.010	7.96,
	S5	-0.011	0.010	
	exit	0.000	0.010	2.82,
	D7	0.123	0.010	
	D8	0.500	0.010	and
	D9	1.002	0.010	
	D10	2.5	1/16	1.01
C120	S1	-2.747	1/16	12.78,
	S2	-1.500	0.010	
	S3	-0.505	0.010	
	S4	-0.031	0.010	7.97,
	"exit"	-0.010	0.010	
	D6	0.001	0.010	2.82,
	D7	0.068	0.010	
	D8	0.129	0.010	and
	D9	0.247	0.010	
C180	S1	-1.27	0.041	5.08
	S2	-0.91	0.041	
	"exit"	-0.03	1/32	
	D4	0.12	0.041	
	D5	3.10	0.041	
	D6	5.79	0.041	
C7S	S1	-0.50	1/16	5.00
	S2	-0.30	1/16	
	exit	0.0	1/32	
	D4	0.125	1/16	
	D5	0.50	1/16	
	D6	1.06	1/16	
	D7	5.69	1/16	
	P _{flange}	7.44	1/8	

A sharp edge was needed for reasons of uniformity. As pointed out by Pasqua,¹⁰ the sharpness of the entrance significantly influences the flow rate--by as much as 10%: the sharper the edge, the lower the flow rate. Since the effects of variable entrance conditions were not to be studied, a sharp edge was maintained so that all entrance conditions could be kept as nearly identical as possible.

Experimental Measurements

Flow Rate

The mass flow rate was measured by timing the CCl_3F head drop by means of a tape measure mounted on the high-pressure vessel. The fluid density was evaluated at the stagnation temperature. The time of a given run was known to ± 0.2 sec, and the CCl_3F head drop was known to ± 1 in. For the apparatus of this experiment, the range of flow rates (1000 to 2000 $\text{lb}/\text{ft}^2\text{-sec}$), running times (40 to 75 sec), and head drops (20 to 40 in.), it was determined (see Appendix B) that the critical mass flow rate per unit area, G_{cr} , was known to within ± 100 $\text{lb}/\text{ft}^2\text{-sec}$. The reproducibility of flow-rate measurements was well within this limit.

Stagnation Temperature

The stagnation temperature was measured by a sheathed chromel-alumel thermocouple inserted through the bottom of the glass high-pressure vessel at the level of the centerline of the test section. Although nitrogen was vigorously bubbled through the CCl_3F prior to each run, inhomogeneities in temperature sometimes produced a slight drift in the value of the stagnation temperature. In such cases, the arithmetic average of the high and low temperatures was reported. In no instance was a run retained wherein this drift was greater than 1°F . Calibration of the thermocouple indicated that the reported stagnation temperature was known to within $+0.5^\circ\text{F}$ or -0.0°F .

Pressure

All pressures were measured by temperature-compensated, strain-gauge pressure transducers having ranges of 0-15, 0-25, 0-30, or 0-50 psia.

During the runs, fluctuations occurred in some of the axial pressure measurements even after reaching steady state on a "time-smoothed" scale, analogous to steady-state turbulence. Because the flow pattern is one of a two-phase, mixture-surrounded jet of liquid, which vaporizes and finally breaks up (see Fig. 4.7 below), the fluctuating velocity component due to this flow pattern caused local variations in the pressure field.

In these cases, the arithmetic average of the high and low pressures occurring during the run was reported. By reasoning that a representative steady-state pressure lay between the high and low values, one could assess the uncertainty of the reported pressure as being no more than half the maximum fluctuation in pressure during the run from this representative value because of "unsteady effects" (see Table B.1, Appendix B).

Since CCl_3F boils at room temperature at atmospheric pressure, a two-phase mixture was sometimes formed in the copper tubing connecting the pressure transducers to the test section (see Fig. 3.4). This was dependent on the temperature of the tubing. Since a two-phase mixture is more compressible and less dense than liquid alone, the volume required to flow out through the 0.010-in.-dia pressure tap to depressurize the transducer line is more than if the mixture were totally liquid, which, for all practical purposes, can be considered incompressible. Because of this

Fig. 3.4. Test Section C120 at $L/D = 1.01$ with Pressure Transducers Mounted in Place. ANL Neg. No. 900-1589.

phenomenon, some pressure transducers never recorded a steady-state value during the runs, which ranged from 40 to 75 sec in duration. Such conditions are designated by asterisks(*) in the tables of data found in Appendix A. To remedy this situation, pieces of dry ice were placed in contact with the copper tubing running from the test section to the pressure transducers. Thus, the essentially incompressible liquid depressurized the transducer line very quickly by ejecting only a very small volume of liquid.

Test section C7S was made with larger-diameter pressure taps, as designated in Table 3.2. Adequate response was obtained without cooling the copper tubing. Therefore, it is suggested that a 1/32-in.-dia pressure tap be used at the exit plane, where location of the tap is crucial, and 1/16-in.-dia taps be used for all other taps.

The stagnation pressure was held constant during the runs by manually feeding in air through the top of the high-pressure vessel by means of a regulator. The stagnation pressure was measured by a temperature-compensated, strain-gauge pressure transducer located 23 in. below the centerline of the test section. The reading of this transducer was corrected for the 23-in. difference in level. It was also found necessary to pack this pressure transducer in dry ice, since boiling from it caused disturbances in the pressure trace recorded on the chart recorder. Calibrations were made after this transducer was in contact with dry ice for

1 hr. The accuracy of the stagnation-pressure measurement was taken to be ± 0.05 psia--the uncertainty caused by the instrumentation alone.

The reproducibility of the remaining pressure measurements were found to be within the experimental accuracy tabulated in Table B.1, Appendix B. It should also be pointed out that when runs were checked for critical flow, the accuracy of these measurements, since only comparison between recorded pressures are of concern, were not subject to uncertainties caused by instrumentation or head. Thus, the critical-flow tests were subject only to uncertainties caused by "unsteady effects."

Testing for Critical Flow

In this experiment, it was necessary to determine whether or not a particular run was undergoing the critical-flow phenomenon at the exit plane of the tube. To make this determination, the critical-flow criterion was indirectly invoked experimentally.

To repeat, the critical-flow criterion is, for T_o and P_o fixed, $dG/dP_e = 0$. This means that G reaches an extreme value at a choking plane with respect to changes in exit-plane pressure. In fact, this extreme value is the maximum flow rate obtainable for the given stagnation conditions and duct geometry. In order to make the desired determination, one might, at a first glance, be inclined to take flow-rate measurements while changing the pressure downstream of the exit plane slightly. However, even though this would be an exact application of the critical-flow criterion, the relative insensitivity of G to P_e when P_e approaches P_e^{cr} is beyond what the experimental apparatus could detect. To demonstrate this, one can write the one-dimensional, steady-state incompressible momentum equation modified by a discharge coefficient which characterized the results of this study:

$$G = \sqrt{2g_c C_d^2 \rho_l (P_o - P)} . \quad (3.1)$$

At the exit plane of the tube this becomes

$$G_e = \sqrt{2g_c C_d^2 \rho_l (P_o - P_e)} . \quad (3.2)$$

Differentiation with respect to P_e gives

$$\frac{dG_e}{dP_e} = - \sqrt{\frac{g_c C_d^2 \rho_l}{2(P_o - P_e)}} . \quad (3.3)$$

For changes in P_e of the order of 0.3 psia (typical of the experimental test), substitution into Eq. 3.3 gives a change in G_e of about 33 lb_m/ft²-sec. This, however, is beyond the range of the experimental accuracy (see Appendix B).

Even if the experimental apparatus were such that very small changes in flow rate could be measured, it is not possible to locate exactly *where* critical flow is occurring, for there is no guarantee that it will occur at the exit plane or even in the test section itself. For example, in flows that are well approximated by a steady-state, equilibrium, isentropic model, such as single-phase gas flow, critical flow may occur at one or more relative minimums of area.

Thus, to circumvent these difficulties, small pressure changes were observed in the pressure tap furthest downstream of the exit plane, yet upstream of the bellows, as the valve to the low-pressure vessel was gradually closed. Thus, attention was focused only on the exit plane of the tube as being the possible location of critical flow, since all other relative minima in area were downstream of the furthest downstream tap.

Since the force to push fluid through the constant-area tube is $P_o - P_e$, one can infer minute changes in the flow rate as a result of noting small changes in P_e . To determine the location of the critical-flow plane, the location is sought at which a small rise in local pressure causes all upstream pressures to also rise. In general, the location may very well be between two pressure taps. If a pressure rise is sensed by a pressure tap while the tap immediately upstream of it does not change, this indicates that a critical flow plane exists either at the location of the immediately upstream tap or between the two taps, but not at the tap where the pressure rise was sensed. Of interest here, in contrast, in answering the specific question of whether or not a given run is choked at the exit plane. With a pressure tap mounted exactly at this location, therefore, a run for which P_e did not change when a change was sensed in the pressure tap immediately downstream of the exit and a further increase caused all upstream taps to measure a rise in pressure was determined to be choked at the exit plane. A run for which P_e did change when a change in the next downstream tap was sensed was determined to be unchoked, that is, critical flow was not present. After a run was determined to be choked by performing this test on the gradually diverging test section, it could be determined if the corresponding run in the rapidly diverging test section was choked by simply determining whether or not the flow rates matched; if they did, the rapidly diverging test section was known to have sustained critical flow at the exit plane. This is so since, as will be discussed in Ch IV, the one-dimensional exit-plane pressure for both types of test sections is the same.

For each set of 15 data points, the 72°F run was tested for critical flow, as well as the highest and lowest pressure runs at 35°F. These points correspond to the three corners of the data map (see Table 3.1). If these three points were found to be undergoing critical flow, it was concluded that all other data points in the group of 15 were critical-flow points. Runs that were unchoked were tested individually save for the case where test section C7 was run at $L/D = 1.01$. In this case, all 15 data points were determined to be unchoked, since the critical-flow test failed at the three corner points of the data map. These data are discussed in Ch IV.

Typical chart traces for critical and noncritical runs are shown in Fig. 3.5. Pressure profiles taken for a critical flow run at discrete

times are superimposed in Fig. 3.6; such profiles are presented for both rapidly and gradually diverging test sections.

Fig. 3.5a(1). Chart Trace for the Critical Flow Test of Run 3-1-11, Furthest Downstream Pressure Tap. Small step changes in pressure are sensed in this pressure tap. ANL Neg. No. 900-1873.

Fig. 3.5a(2). Chart Trace for the Critical Flow Test of Run 3-1-11, Exit Plane Pressure Tap. The small step changes in pressure sensed by the furthest downstream tap are not sensed by the exit-plane pressure tap. Thus, this flow was choked at the exit plane. ANL Neg. No. 900-1862.

Fig. 3.5b(1). Chart Trace for the Critical Flow Test of Run 4-1-11, Furthest Downstream Pressure Tap. Pressure changes are sensed in this pressure tap. ANL Neg. No. 900-1887.

Fig. 3.5b(2). Chart Trace for the Critical Flow Test of Run 4-1-11, Exit-plane Pressure Tap. Every pressure change sensed in the furthest downstream pressure tap is also sensed by the exit plane pressure tap. Thus, the flow was not choked at the exit plane. ANL Neg. No. 900-1885.

Fig. 3.6a. Critical Flow Test for Run 3-1-1: This shows that critical flow is occurring at the exit plane. In going from profile A to profile B, pressures upstream of the exit plane do not change. However, in profile C, when a small rise in pressure is experienced at the exit plane, all upstream pressures also rise. ANL Neg. No. 900-1777.

Fig. 3.6b. Critical Flow Test for Run 3-2-1. If the results of this test are not interpreted in light of the two-dimensional expansion occurring at the exit plane, one is led to the incorrect conclusion that P is dependent on the pressure in the low-pressure vessel, even while critical flow persists. Such a conclusion was drawn in Refs. 21, 28, 29, and 41. ANL Neg. No. 900-1865.

IV. DISCUSSION OF RESULTS

Effect of Downstream Geometry on Experimental Measurements

When one-dimensional equations are used to model a physical phenomenon, the success of such an approach is determined by either the phenomenon actually being one-dimensional or the measurements taken are describable as one-dimensional even if the behavior is not entirely one dimensional.

In the study of critical flow through tubes, experimental investigations have been confronted with this problem.^{2,4,7,12,13} The fluid approaching the exit plane of the tube acquires a radial velocity component and an accompanying radial pressure gradient as it expands upon issuing from the tube. Clearly, the behavior in the vicinity of the exit is at least two-dimensional, and *the problem is to obtain a pressure measured by a wall tap that characterizes the behavior of the flow.* Henry⁴ showed that this can be done by affixing a 7° included-angle diverging section to the exit plane of the tube, for such an addition minimizes the two-dimensional effect of the expansion.

It is believed that this method of obtaining P_e is the best available technique. (Although actually positioning a pressure probe at the center-line of the tube has been tried,^{21,30} it mechanically disturbs the flow pattern and may introduce sites from which flashing might be initiated had the probe not been there.) (The term *flashing* is usually used to describe vapor formation caused by lowering the pressure, whereas *boiling* usually refers to vapor formation as a result of a heat addition of energy.)⁴¹ It must be considered, however, that this approach still yields slightly low pressure measurements because the radial pressure gradient has been *minimized*, not eliminated.

In this study, test sections were fabricated of two major types which had identical constant-area sections. One type, termed the gradually diverging test sections, were short tubes with 7° included-angle diverging sections affixed to the tube ends. The other type, termed the rapidly diverging test sections, were short tubes with 120° or 180° included-angle diverging sections affixed to the tube ends (cf. Fig. 3.3).

A rapidly diverging and a gradually diverging test section were compared for the same T_o , P_o , and L/D . The quantitative effect of downstream geometry is illustrated in Figs. 4.1, 4.2, and 4.3. Figure 4.1 shows how the measurement of η is affected by downstream geometry. That the value of η , since $\eta = P_e/P_o$ at critical flow, is consistently low in the rapidly diverging test section is clear from the figure. For the rapidly diverging test section used at $L/D = 5$, P_e was measured approximately 0.02 in. further upstream than for the other rapidly diverging test sections. Therefore, it measures a pressure somewhat higher than it would were it nearer the exit plane (cf. Fig. 4.13).

Figure 4.2 shows that the measurement of G_{cr} is unaffected by the downstream geometry when critical flow persisted at the exit plane. From this it can be deduced that the overall behavior upstream of the exit plane and the driving force for flow in both types of test sections are the same for identical stagnation conditions and L/D . This is a significant result,

Fig. 4.2. Effect of Downstream Geometry on Measurements of Critical Flow Rate. ANL Neg. No. 900-1781.

Fig. 4.1. Effect of Downstream Geometry on Measurements of Critical Pressure Ratio. ANL Neg. No. 900-1775.

Fig. 4.3a. Effect of Downstream Geometry on Measurements of Axial Pressure Profile. The assumed behavior is deduced by studying the pressure profile plots of all runs. ANL Neg. No. 900-1859.

Fig. 4.3b. Effect of Downstream Geometry on Measurements of Axial Pressure Profile. The assumed behavior is deduced by studying the pressure profile plots of all runs. ANL Neg. No. 900-1860.

Fig. 4.3c. Effect of Downstream Geometry on Measurements of Axial Pressure Profile. The assumed behavior is deduced by studying the pressure profile plots of all runs. ANL Neg. No. 900-1880.

Fig. 4.3d. Effect of Downstream Geometry on Measurements of Axial Pressure Profile. The assumed behavior is deduced by studying the pressure profile plots of all runs. ANL Neg. No. 900-1847.

for it implies that the one-dimensional exit-plane pressure for both types of test sections, for identical stagnation conditions and L/D , is the same.

The axial pressure profiles presented in Fig. 4.3 show a striking difference in measured values in the vicinity of the exit plane. Pressures measured near the exit plane of the gradually diverging test sections are more indicative of the one-dimensional pressure than the corresponding measurements taken in the rapidly diverging test sections.

The slight "dip" in the pressure profile in the region of the exit plane is due to the exit-plane pressure tap being situated on a corner. One might improve upon this pressure reading by drawing a smooth line between the readings at D5 and D7. Mindful of the fact that this "dip" is a systematic error, it can be concluded from the gradually diverging test section that no steep pressure gradient exists at the exit of a short tube, contrary to the conclusions drawn in Refs. 12 and 13, who reported steep pressure gradients near the exit plane.

Inferring Flow Pattern Information from Axial Pressure Profiles and C_d Calculations

Although fabricating test sections from stainless steel greatly facilitated accurate machining and the ability to instrument the test sections with many pressure taps, the ability to observe the flow visually was eliminated.

To facilitate visual observation, a test section of rectangular cross section with glass sides was built (see Fig. 3.2). The constant-area section had $L/D = 2.82$, based on the hydraulic diameter. Direct comparison between this test section and the stainless steel test section was not possible on a quantitative basis. Cavitation occurred only on the top and bottom edges of the transparent test section, whereas in the stainless steel test section cavitation occurred around the entire entrance. Discharge coefficients as high as 0.75 were calculated for the transparent test section, and the pressures were greater than in the stainless steel test section, although the shape of the profile was the same. It must also be considered that there is no guarantee that the hydraulic diameter is the appropriate means of comparing critical two-phase flow through ducts of different cross section. In light of this, then, most of the flow-pattern information had to be deduced by indirect means, although qualitative observations could be made from the transparent test section.

It has been observed both visually and by pressure measurements by Fauske and Min,³ Pasqua,¹⁰ Fauske,² and the author in the present study that the sharp-edged entrance of a tube has essentially the same effect on the liquid as a sharp-edged orifice has on a single-phase fluid. It is known from experiment, as well as from potential flow theory, that C_d for a sharp-edged orifice has a value of 0.61.⁴⁰ The coefficient of discharge is defined as

$$C_d \equiv \frac{A_{\text{actual}}}{A_{\text{duct}}} \frac{u_{\text{actual}}}{u_{\text{pf}}}, \quad (4.1)$$

where A_{actual} represents the actual area the fluid occupies after passing through the orifice, u_{actual} is the actual fluid velocity, A_{duct} is the area of the duct, and u_{pf} represents the velocity calculated from potential flow theory. For an orifice, $u_{\text{actual}}/u_{\text{pf}} = 0.98$.⁴⁰ Therefore, Eq. 4.1 may be approximated by

$$C_d \approx \frac{A_{\text{actual}}}{A_{\text{duct}}}, \quad (4.2)$$

and C_d may be viewed as the ratio of the area to pass liquid to the cross-sectional area of the duct. By rearranging Eq. 2.19, C_d may be calculated:

$$C_d = \left[\frac{G^2}{2\rho_{\text{lo}}(P_o - P_e)} \right]^{1/2}. \quad (4.3)$$

Implicit in Eq. 4.3 is the fact that the only pressure drop is an entrance pressure drop (since $P(z) = P_e$), and the axial pressure profile is constant, such as seen in Fig. 4.3. If this assumption is good, then for z greater than approximately one tube diameter downstream of the entrance (where the vena contracta occurs) $P(z) = P_e$ and $C_d = 0.61$. Such a flow pattern would be as pictured in Fig. 4.4.

Hence, significant deviations of a calculated discharge coefficient from 0.61 may be interpreted as a deviation from the flow pattern pictured in Fig. 4.4. Such deviations would be measured if, for example, the shape of the jet changed, if the jet broke up, or if a two-dimensional flow field caused a pressure tap to measure a component of the dynamic pressure.

Within the limits of the experimental measurements, the calculated values of C_d are considered good to $\pm 15\%$.

Fig. 4.4. Flow Pattern Inferred from Calculating $C_d = 0.61$ for All Axial Locations in the Tube.
ANL Neg. No. 900-2848.

Considerations of the Flow Pattern

Figure 4.5 shows the basic characteristics of the four L/D s from which critical flow data was taken. Although determination of the flow pattern over the entire range of L/D was beyond the scope of this investigation, some of the measurements taken can be interpreted in light of visual studies^{2,10,12,13} and observations of the rectangular test section with glass sides (see Fig. 4.7 below) obtained in the present study via photographs and high-speed motion pictures.

$L/D = 12.78$

Data taken with $L/D = 12.78$ were characterized by having an axial pressure profile upstream of the exit plane nearly coincident with $P(T_o)$. In 10 of the 15 runs taken in the gradually diverging test section, P_e was greater than $P_s(T_o)$. Thus, since the measured values of G_{cr} were much lower than if only liquid were present, this suggests that two phases existed before the liquid, essentially at temperature T_o throughout the tube, reached the saturation pressure. Since the solubility of nitrogen in CCl_3F decreases as the pressure decreases, a reasonable explanation for the observed behavior at this L/D is that upon experiencing the entrance

Fig. 4.5. Measurements Characterizing the Four L/D s at which Critical Flow Was Obtained. ANL Neg. No. 900-1883.

pressure drop to a pressure near $P_s(T_0)$, nitrogen came out of solution, bringing CCl_3F vapor with it in sufficient amount to pressurize the two-phase mixture annulus above $P_s(T_0)$. It is most probable that the very low pressure at the sharp-edged entrance (perhaps even a negative absolute pressure, Ref. 25--that is, the liquid is in tension) caused some nitrogen to come out of solution at that location as well as causing cavitation of the CCl_3F .

Measurements taken in this test section suggest that the flow pattern is similar to that deduced by Henry⁵ from Refs. 2, 11, 12, 13, and 16, as shown in Fig. 4.6. Flat profiles such as this were also reported by Uchida and Nariai¹² for water. Thus, it seems reasonable to say that the fluid entered the tube as a liquid and immediately began vaporizing as it contacted the sharp edge, forming a two-phase mixture annulus, probably containing nitrogen and CCl_3F vapor, surrounding the jet of superheated (nonequilibrium) liquid. Flashing of the liquid took place from entrained liquid droplets as well as from the jet of liquid, which began breaking up in the region of $L/D = 4$.

Fig. 4.6. Flow Pattern Deduced by Henry⁵ for $L/D > 12$.

$$L/D = 7.96$$

The pressure profiles at $L/D = 7.96$ are very nearly the same as those obtained at $L/D = 12.78$. Thus, it is believed that the flow pattern is quite similar to the flow pattern at $L/D = 12.78$, that is, a liquid jet is surrounded by a two-phase mixture annulus. The small difference in G measured at $L/D = 12.78$ and at $L/D = 7.96$ indicates that the compressibility has remained relatively unchanged, adding credibility to the statement that the flow pattern has, likewise, remained relatively unchanged.

L/D = 5.00

The flow pattern present for the $L/D = 5.00$ test section is of particular interest due to the fact that, between $5 \leq L/D \leq 8$, P_e decreases, yet G remains relatively unchanged (see Fig. 4.5). It might be concluded that if the driving force is greater and the flow rate has remained relatively unchanged between $L/D = 5$ and $L/D = 8$, the compressibility increased as the L/D decreased, perhaps indicating a change in flow pattern. Thus, a change in G does not always accompany a change in P_e . Beginning at about $L/D = 3.5$, a rise in pressure was detected, indicating, perhaps, that flashing from the jet was occurring at a more rapid rate than in the previously discussed test sections, causing radial flow of the two-phase mixture indicative of breaking up of the liquid jet. Such a pressure rise was also observed in Ref. 12 at short L/D . The flow pattern observed in Ref. 10 at $L/D = 4.64$ is shown in Fig. 2.1, where D is taken as the hydraulic diameter of the test section of semicircular cross section.

L/D = 2.82

At the shortest L/D for which critical flow was obtained, the flow pattern resembled that obtained at $L/D = 5.00$. The flow rate significantly increased from $L/D = 5.00$ to $L/D = 2.82$, indicating the presence of a lower-quality, less compressible mixture in the short tube. A rectangular test section with glass sides was built to augment the study of the $L/D = 2.82$ test section. The photographs in Fig. 4.7 show the rapid shrinking of the jet due to the breakup beginning at about $L/D = 12.5$, which is also indicated by the rise in the pressure profile. The breakup is complete before the fluid reaches the exit plane. The light areas in the photograph represent the liquid; the dispersed two-phase mixture is represented by the dark areas. High-speed motion pictures taken of the transparent test section revealed that changes in the shape and length of the liquid jet occurred in the region of the breakup. These observations agree closely with the flow pattern observed in Ref. 13, pictured in Fig. 4.8.

The Roles of P_o , T_o , and L/D

The effects of three variables P_o , T_o , and L/D were examined. It is believed that these three variables are, in fact, three of the independent variables governing the phenomenon, for quantities such as x , α , k , dx/dP , and the degree of nonequilibrium is dependent on these. The entrance configuration significantly affects the system compressibility; however, in this study, this was kept constant. The amount of dissolved gas also affects the measurements of interest; however, this will be treated separately in a subsequent section of this chapter for it is a difficult variable to quantify and control systematically.

The Role of P_o

If L/D and T_o are constant, raising or lowering P_o correspondingly raises or lowers the magnitude of the measured pressures, but does not appreciably change the shape of the pressure profile. An increase in P_o increases the driving force for flow, which is reflected by an increase in flow rate. At the same time, an increase in P_o causes an increase in P_e , but since P_e is essentially bounded from above by $P_s(T_o)$ (the process

Fig. 4.7a. Transparent Test Section of Rectangular Cross Section, $L/D = 2.82$: $P_o = 16.74$ psia, $T_o = 72.7^\circ\text{F}$. ANL Neg. No. 900-1585.

being nearly adiabatic), P_e increases more slowly than P_o and the ratio of P_e/P_o decreases with increasing P_o .

Figure 4.9 shows the effect of P_o for constant T_o and L/D .

The Role of T_o

If L/D and P_o are held constant, changing T_o changes the amount of subcooling. As the subcooling increases, the amount of internal energy stored within the liquid decreases and less energy exists to cause a phase change. Thus, the greater the subcooling, the less is the amount of liquid that will flash to vapor. As the amount of vapor comprising the mixture becomes

Fig. 4.7b. Transparent Test Section of Rectangular Cross Section, $L/D = 2.82$: $P_o = 16.74$ psia, $T_o = 73^\circ\text{F}$. ANL Neg. No. 900-1588.

less, the compressibility decreases, which causes the flow rate to increase. Therefore, dropping T_o increases the flow rate (see Fig. 4.10a). As T_o increases, the critical pressure ratio increases; the decreased subcooling causes an increase in quality and, therefore, an increase in compressibility; the increased compressibility dictates a lower flow rate (see Appendix D) and a higher critical pressure ratio in most cases (see Fig. 4.10b) (cf. "Flow Pattern Considerations" for an exception to this). One might also view T_o as dictating the exit-plane pressure at runs of high subcooling, for, as previously mentioned, P_e approaches $P_s(T_o)$ as the subcooling increases. Thus, the most crucial test of a critical-flow model is in comparison of the predictions of data taken at nearly saturated conditions. A model such as presented in Ref. 23 essentially takes Eq. 2.19 and puts

Fig. 4.7c. Transparent Test Section of Rectangular Cross Section, $L/D = 2.82$: $P_o = 16.74$ psia, $T_o = 73^\circ\text{F}$. ANL Neg. No. 900-1587.

$P_o = P_o(T_o)$, which succeeds in predicting G_{cr} for highly subcooled conditions, but predicts $G_{cr} = 0$ for saturated stagnation conditions, that is, $P_o = P_s(T_o)$.

The Role of L/D

The inclusion of L/D as an independent variable was first suggested in Ref. 3 and later confirmed in Refs. 2 and 22. The role of L/D is, indeed, a complex one; however, for the conditions studied in this investigation, it can be succinctly stated that L/D determines the flow pattern which persists. Assuming wall friction to be negligible, one is led to

Fig. 4.7d. Transparent Test Section of Rectangular Cross Section, $L/D = 2.82$: $P_o = 16.74$ psia, $T_o = 72.7^\circ\text{F}$. ANL Neg. No. 900-1586.

this conclusion, for if it is not a frictional interaction between wall and fluid that changes the flow rate as the L/D is changed, changes in flow rate are reflected by the changing flow patterns.

When critical flow exists at the exit plane, the L/D of the tube has a direct bearing on:

- 1) the amount of time the fluid spends in the tube, which may affect:
 - i) the ratio of residence time to relaxation time;
 - ii) the quality of the mixture;
 - iii) the degree of nonequilibrium remaining.

Fig. 4.8. Flow Pattern as Seen by Zaloudek¹³ at $L/D = 2.5$. ANL Neg. No. 900-1890.

Fig. 4.9a. The Effect of P_o on G for Constant Values of T_o and L/D . ANL Neg. No. 900-1853.

Fig. 4.9b. The Effect of P_o on η for Constant Values of T_o and L/D . ANL Neg. No. 900-1868.

Fig. 4.10a. The Effect of T_o and G for Constant Values of P_o and L/D . ANL Neg. No. 900-1864.

Fig. 4.10b. The Effect of T_0 on η for Constant Values of P_0 and L/D . ANL Neg. No. 900-1872.

2) The flow pattern that persists, which affects the compressibility.

Fauske and Min³ also proposed that the degree of nonequilibrium remaining increased with decreasing L/D .

To reiterate, the L/D determines the flow pattern, which, in turn, greatly affects the compressibility. As discussed previously, the compressibility determines the critical flow rate.

The effect of L/D for constant P_0 and T_0 is presented in Fig. 4.11.

Figure 4.12 is a plot of the change in flow rate between the $L/D = 12.78$ test section and the $L/D = 2.82$ test section for constant stagnation conditions versus subcooling. (In other words, it is a plot of the difference between G_{cr} measured for Run 4-1-n and G_{cr} measured for Run 1-1-n, $n = 1, 2, \dots, 15$.) The point of this plot is to investigate which runs were most influenced by the change in L/D from 12.78 to 2.82. As seen in the figure, the runs experiencing the greatest amount of subcooling were affected least by the change in L/D , while the runs experiencing the least subcooling were affected most by changing the L/D of the test section. Similarly, the values of η changed most for the least subcooling and least for the greatest subcooling. It is believed that this is due to the fact that the higher subcooling runs, since the internal energy associated with the liquid is comparatively low, are the runs of lowest quality and lowest compressibility, being less affected by changes in L/D than are the runs of higher quality. At high subcoolings, less mass transfer of liquid to vapor takes place, and the mixture is quite incompressible and relatively insensitive to changes in L/D . Since less mass transfer takes place at high subcooling, the flow pattern is one in which minimal surface breakup of the liquid jet takes place for a specified L/D , giving a smaller surface area from which flashing can take place than would be present if much surface breakup occurred. Therefore, the "inefficient" surface for vaporization renders such a flow pattern relatively insensitive to changes in L/D .

The Role of Dissolved Gas

The amount of dissolved gas was not covered in the previous section because in this experiment it was not possible to determine quantitatively the amount coming out of solution, as was also the case for the critical-

Fig. 4.11. The Effect of L/D on G_{cr} and η for Constant Values of P_0 and T_0 . ANL Neg. No. 900-1881.

Fig. 4.12. Change in Critical Flow Rate Experienced by Changing L/D from 12.78 to 2.82 as Affected by Subcooling. ANL Neg. No. 900-1863.

flow experiments discussed in Refs. 4, 18, and 30. In addition, no effort was made to degas the CCl_3F used due to the difficulty involved in removing nitrogen from CCl_3F . Also since fluid systems encountered in engineering practice will generally contain dissolved gas, it was believed that the data reported would be more representative of actual behavior.

Nitrogen entered the CCl_3F upon contact with atmospheric air at approximately 70°F ; nitrogen was also used to pressurize the CCl_3F drums so that the liquid might be pumped from place to place. Nitrogen was bubbled through the CCl_3F as it stood in the high-pressure vessel prior to a run to insure homogeneity of temperature; this also produced dissolution of the nitrogen into the CCl_3F . As a check to see if the type of gas used affected the measured values of G_{cr} and η , argon was substituted for several trial runs; no difference was detected.

Basically, the presence of any noncondensable gas serves to increase the compressibility of the mixture. In actuality, the system observed in this study was a two-component, two-phase flow-- CCl_3F liquid and vapor plus nitrogen gas. Then, for mathematical models containing a dv/dP term, the term should be evaluated as $d/dP [x_{\text{CCl}_3\text{F}} v_{g\text{CCl}_3\text{F}} + x_{\text{N}_2} v_{g\text{N}_2}]$.⁸ However,

the models analyzed in this report were not evaluated in this manner since there was no way to assess this term quantitatively. Thus, the mixture was more compressible than it would have been had it been degassed, and, consequently, the critical-flow rates are lower.

It is of concern, then, to consider the effect that dissolved gas coming out of solution has on the experimental results. The solubility of nitrogen in CCl_3F depends on the temperature and pressure, yet the nearly isothermal behavior of the liquid renders the temperature dependence inconsequential. However, the solubility of nitrogen decreases when the pressure decreases. Thus, one would suspect gas to be coming out of solution in the runs wherein large pressure drops at the entrance occurred. It is believed that gas came out of solution in nearly all runs in varying amounts proportional to the entrance pressure drop. In cases for which the axial pressure profile was in the vicinity of $P(T)$, this phenomenon manifested itself by producing values of P_e higher than $P(T)$. This occurred in Runs 1-1-4, 1-1-5, 1-1-7 through 1-1-15, 2-1-11, and 2-1-12. Such a situation makes a calculation of the equilibrium quality meaningless, for calculating a quality using a value of P_e greater than $P(T)$ gives a negative quality. In the same manner, dissolved gas coming out of solution adds a degree of uncertainty to all evaluations of x_E , which is not possible to assess.

Dissolved gas coming out of solution may also add to the compressibility of the mixture by bringing CCl_3F vapor with it. Consider a bubble at P_b . If the liquid surrounding it is at pressure P_l and temperature T_l , P_b must equal P_l , neglecting surface tension effects, and the vapor pressure of the CCl_3F , P_v , equals $P_s(T_l)$. If P_l is only slightly higher than $P_s(T_l)$, that is, the liquid is very nearly saturated, the difference $P_l - P_v$ represents a potential for vaporization of CCl_3F . This situation causes the bubble, beginning at $P_b = P_{\text{gas}}$, to expand until $P_b = P_{\text{gas}} + P_v - P_l$, where P_{gas} is the partial pressure of the noncondensable gas. Thus, the vapor occupies a larger volume, adding considerably to the compressibility of the mixture.

Therefore, it is concluded that higher flow rates would have been measured at the runs of high subcooling had the noncondensable nitrogen not been present. In fact, Zaloudek¹³ showed that by degassing water critical flow rates of 5-8% higher were obtained.

Conditions in which the Characteristic Value of P_e May Be Measured without Using a 7° Diverging Section

Since most investigators who measured P_e did so without the benefit of a 7° diverging section, it is of interest to inquire whether or not a situation might arise for which failing to use such an aid is of no consequence. Two situations are considered in which this might be the case in two-phase flow.

The first involves a matter of chance. Consider Fig. 4.13. If the exit-plane pressure tap were placed between location z' and e , the value measured for P_e would be low; the nearer to the exit is the location of the tap, the further is the deviation of the measured value of P_e from the characteristic pressure. Thus, a spectrum of measured values of P_e can be

Fig. 4.13. Situation in Which the Exit-plane Pressure Tap Might Measure the Characteristic One-dimensional Pressure without the Aid of a 7° Diverging Section. ANL Neg. No. 900-1884.

obtained, depending on the locating of the pressure tap. Such is the case with test sections C120 and C180 used in this study. With C180 measurements closer to the one-dimensional exit-plane pressure were obtained since the exit-plane tap was located 0.02 in. further upstream than in C120. If one were fortunate enough to situate the exit-plane pressure tap at location z' , the value of P_e characteristic of the overall behavior of the exit plane would be measured.^e Since the location z' is not known, however, and since it is quite likely to change for different flow conditions, one could not justify using values of P_e measured without a 7° diverging section because of this fortuitous situation.^e

Of more importance is the case, as pointed out by Henry,⁴ in which P_e approaches the pressure in the low-pressure vessel. If P_e were equal to P_e the pressure in the low-pressure vessel, no expansion would take place and the fluid would issue from the tube as a constant-area jet; a wall pressure tap would then measure the one-dimensional pressure without regard to the presence or absence of an exit-plane diverging section. As P_e approaches the pressure of the low-pressure vessel, the value of P_e measured in a rapidly diverging test section approaches the value of P_e measured in a gradually diverging test section. If the flow pattern is such that vapor touches the walls of the exit plane rather than liquid or a dense, two-phase mixture, one would expect the measurements of P_e to come even closer together, since the radial pressure gradient is less in a low density fluid. Henry⁴ showed this by noting no appreciable difference in P_e measured in both types of test sections for all-gas flows. Therefore, since P_e approached the pressure in the low-pressure vessel as the L/D decreased, the effect of not using a 7° diverging section became of less and less consequence. The data presented in Fig. 4.14 illustrates this.

Fig. 4.14. The Effect of L/D and Subcooling on the Agreement of η Measured in Both Rapidly and Gradually Diverging Test Sections. ANL Neg. No. 900-1849.

Therefore, from the behavior observed in this study, it can be concluded that for conditions for which $L/D \leq 3$ and the fluid is subcooled sufficiently, values of η taken from either rapidly or gradually diverging test sections are essentially the same.

Comparison with Models

An important question still unanswered is: What critical-flow models, if any, reasonably predict the critical-flow phenomenon through short tubes? A comparison of data from this study with the predictions of various models can help to answer this question; perhaps just as importantly valuable insight into the actual behavior will be gained if it can be determined why a given model succeeds or fails in predicting the data.

Homogeneous Equilibrium Model

The homogeneous equilibrium model is derived assuming that the process occurring is one of thermodynamic equilibrium, which implies that $k = 1$.

Although it has been shown in Refs. 4, 12, and 30 that k is not much different than 1, the homogeneous equilibrium model still fails to predict the data taken in this experiment (see Fig. 4.15). The resolution of this disagreement lies in the nonequilibrium nature of the phase change that characterizes the data. Since the actual process is not one of equilibrium, either $x < x_E$ or $dx/dP < dx_E/dP$, or both. Therefore, the magnitude of the denominator of Eq. 2.11 is too large, and G_{cr} predicted by the homogeneous equilibrium model is consistently low. This is demonstrated in Fig. 4.15.

The "flatness" of the model's predictions at very low qualities is typical of all equilibrium models, due to the fact that the nonequilibrium nature of the phase change is not included. In other words, the model is rather insensitive to change in P_0 when predicting η and G_{cr} . At low qualities, dx/dP dominates the compressibility of the mixture, as can be illustrated by setting $k = 1$ in Eq. 2.20:

Fig. 4.15. Comparison of Critical-flow-rate Data Taken from Test Section C7S, $L/D = 5$, to the Prediction of the Homogeneous Equilibrium Model. ANL Neg. No. 900-1855.

$$G_{cr}^2 = \left[\frac{-g_c}{x \frac{dv_g}{dP} + (v_g - v_l) \frac{dx}{dP} + (1-x) \frac{dv_l}{dP}} \right]_e$$

In this equation, dv_l/dP is negligible, and at low qualities, $x dv_g/dP$ approaches zero, leaving the dx/dP term as the only term having any effect at low quality.

An equilibrium evaluation of the quality x_E is given by

$$x_{E_e} = \frac{s_o - s_f(P_e)}{s_{fg}(P_e)}$$

The derivative of x_E with respect to P_e is given by

$$\frac{dx_E}{dP_e} = \frac{-s_{fg} \frac{ds_f}{dP_e} - (s_o - s_f) \frac{ds_{fg}}{dP_e}}{s_{fg}^2}$$

For fixed T_o and varying P_o , the conditions for which the "flatness" is seen, all terms in the preceding expression are functions of P_e except for s_o . From the data tabulated in Appendix A, it is seen that for constant T_o , varying P varies P_e only slightly. Thus, the terms evaluated as functions of P_e remain relatively unchanged when P_o is varied and T_o is fixed. Since s_o is a rather weak function of P_o , the entire expression for dx_E/dP_e is quite insensitive to variations in P_o when T_o is constant. Since dx_E/dP_e controls the compressibility, the critical flow rate at low qualities becomes insensitive to changes in P_o as calculated by equilibrium models.

From this, it is seen that at low qualities it is crucial to describe mathematically the mass transfer process quite carefully.

Fauske's Model

Fauske's model¹ is another equilibrium model that has been used to predict critical-flow-rate data (this model does not predict critical pressure). To obtain the maximum flow rate, the value of k at critical flow was calculated from the measured exit pressure and substituted into the flow-rate expression. The value of k at critical flow was calculated by maximizing the pressure gradient at the plane of critical flow; however, the pressure profiles obtained in this study (see Fig. 4.3) show that for sharp-edged entrance tubes for which $2.82 \leq L/D \leq 12.78$, the maximum in the pressure gradient exists at the entrance of the tube rather than at the critical-flow plane. Thus, the application of such a constraint to calculate k for short tubes is questionable. The details of Fauske's model are discussed in Ch. II. Figure 4.16 compares data taken in this study of Fauske's model. When using measured values of P_e , Fauske's model predicts decreasing flow for decreasing L/D , which is opposite of the actual behavior, indicating that the predictions worsen with increasing nonequilibrium. It predicts consistently low values of flow rate, with the characteristic "flat" behavior typical of equilibrium models. These results imply that Fauske's model is not descriptive of the phenomenon occurring in short tubes.

Fig. 4.16. Comparison of Fauske's Model¹ to Data from Test Section C7 at $L/D = 12.78$ and C7S at $L/D = 5.00$. ANL Neg. No. 900-1892.

Moody's Model

The model proposed by Moody⁹ is another equilibrium critical-flow model. Moody also calculated the maximum flow rate by solving for the value of k at critical flow and substituting into the flow-rate expression. However, as pointed out in Ch. II, Moody arrives at his method of calculating this value of k via three questionable assumptions. Moody's model is discussed in more detail in Ch. II. Figure 4.17 shows a comparison of data taken in this study to Moody's model. Since this model can be employed by knowing only the stagnation conditions and since the expression is not an explicit function of L/D , the model predictions do not change with change of L/D . The predictions of η given by this model are somewhat low, yet exhibit the behavior of the data; however, the predictions of G_{cr} are quite low, exhibiting the usual "flat" behavior of equilibrium models. That Moody's model does not correctly predict both η and G_{cr} is reason to conclude that it, too, does not correctly portray the behavior of critical flow through short tubes.

Fig. 4.17a. Comparison of Moody's Model⁹ to Data from Test Section C7 at $L/D = 12.78$ and C7S at $L/D = 5.00$. Since the model is based on stagnation conditions and the role of L/D is not considered, the model predicts the same critical pressure ratio for both sets of data. ANL Neg. No. 900-1783.

Fig. 4.17b. Comparison of Moody's Model⁹ to Data from Test Section C7 at $L/D = 12.78$ and C7S at $L/D = 5.00$. Since the model is based on stagnation conditions and the role of L/D is not considered, the model predicts the same critical flow rate for both sets of data. ANL Neg. No. 900-1782.

In a subsequent modification of his model,³² Moody introduced the effect of L/D by including a friction-factor term in the momentum equation in which the friction factor is evaluated as a function of L/D . Since the version of Moody's model presented in Ref. 32 takes into account frictional losses, the flow-rate predictions will be even lower than those of the model as presented in Ref. 9.

Henry's Model ($L/D \geq 12$)

A critical-flow model which has attempted to account for the nonequilibrium nature of the interphase mass transfer is Henry's model.⁵ This model was derived with the intent of predicting critical flow through sharp-edged entrance tubes for which $L/D \geq 12$. The details of this model are covered in Ch. II.

Since the longest L/D dealt with in this study was 12.78, long-tube data from Ref. 7 was used so that this model might be evaluated. Figure 4.18 shows predictions of this model compared with data taken in this study and data from Ref. 7. The model predictions for G_{cr} are quite good in the region $12.78 \leq L/D \leq 35$, overpredicting only slightly. For $35 \leq L/D \leq 100$, the model predicts a nearly constant G_{cr} , while the data shows a gradually decreasing flow rate.

The predictions of η are, likewise, slightly high. However, if one considers Table 2.1 and corrects the reported values of P_e from Ref. 7 by adding half the difference between P_e measured in a gradually diverging

test section and P measured in a rapidly diverging test section from this study (about 1 psia^e), the agreement between model and data becomes better. In Fig. 4.18, these values are termed "Howard's data, modified."

Since this study concerns short tubes, it is of more concern to inquire as to the applicability of Henry's model for $L/D \leq 12$.

As can be seen in Figs. 4.19 and 4.20, this model overpredicts both G and η slightly for the high-temperature runs at $L/D = 12.78$. As the L/D is cut to 7.96, the model underpredicts G and overpredicts η for the high-temperature runs, and at $L/D = 5.00$, the deviation from the data increases for these runs. Yet, for $5.00 \leq L/D \leq 12.78$, the model predictions are very good for the lower-temperature runs. Examination of the amount of subcooling associated with each run reveals that the smallest amount of subcooling is associated with runs that are not predicted well by Henry's model. This is graphically illustrated in Fig. 4.21. Since these runs are affected most by changing the L/D , this is an expected result.

Thus, it may be concluded that Henry's model reasonably predicts critical-flow data for sharp edged entrance tubes for which $L/D = 12$. Further, it may be used in situations for which $2.8 \leq L/D \leq 12$ if the subcooling is substantial (greater than 20°F for CCl_3F). One might conjecture that for these conditions, the flow pattern is close to that considered in the model. This nonequilibrium model is the only model examined that predicts the observed behavior of the critical flow of CCl_3F through short tubes.

Fig. 4.18a. Critical-flow-rate Data Compared to Henry's $L/D \geq 12$ Model.⁵

Fig. 4.18b. Critical-pressure-ratio Data Compared to Henry's $L/D \geq 12$ Model.⁵ ANL Neg. No. 900-1785, Rev. 1.

Fig. 4.19a. Comparison of Critical-flow-rate Data, $L/D = 12.78$, to Henry's Model Derived for $L/D = 12$.⁵ ANL Neg. No. 900-1778, Rev. 1.

Fig. 4.19b. Comparison of Critical-flow-rate Data, $L/D = 7.96$, to Henry's Model Derived for $L/D = 12$.⁵ ANL Neg. No. 900-1871.

Fig. 4.19c. Comparison of Critical-flow-rate Data, $L/D = 5.00$, to Henry's Model Derived for $L/D = 12$.⁵ ANL Neg. No. 900-1870.

Fig. 4.20b. Comparison of Critical-pressure-ratio Data, $L/D = 7.96$, to Henry's Model Derived for $L/D = 12.5$ ANL Neg. No. 900-1874.

Fig. 4.20c. Comparison of Critical-pressure-ratio Data, $L/D = 5.00$, to Henry's Model Derived for $L/D = 12.5$ ANL Neg. No. 900-1867.

Fig. 4.21a. Applicability of Henry's $L/D = 12$ Model⁵ for Predicting Critical Flow Rates in Tubes for Which $L/D < 12$. ANL Neg. No. 900-1851.

Fig. 4.21b. Applicability of Henry's $L/D = 12$ Model⁵ for Predicting Critical Pressure Ratios in Tubes for Which $L/D < 12$. ANL Neg. No. 900-1850.

Henry's Model ($0 \leq L/D \leq 3$)

Henry's model⁶ for very short tubes is derived in much the same way as the $L/D \geq 12$ model;⁵ the difference lies in the attempt to interpret the terms of Eq. 2.20 in light of the assumed flow pattern for very short L/D .

The predictions of this model have been compared to data taken from the test section for which $L/D = 2.82$. As seen in Fig. 4.22, this model overpredicts G_{CR} and underpredicts η . What is most striking, however, is the relative insensitivity of the model to changes in T_0 . It is believed that the assumed flow pattern is the cause of such behavior.

It is assumed (see Fig. 2.6) that the jet remains intact and is surrounded by a vapor annulus. If it is true that the rate of vaporization is limited by the rate at which energy can be transported to the surface, it is reasonable to say that this model does not account for enough

Fig. 4.22b. Comparison of Critical-pressure-ratio Data, $L/D = 2.82$, to Predictions of Henry's $0 < L/D < 3$ Model.⁶ ANL Neg. No. 900-1854.

vaporization. Since a cylinder is the geometric volume which offers the least surface area per unit volume, except for a sphere, this model has depicted the "least efficient" configuration for energy transfer. However, Fig. 4.7 shows that at short L/D that the annulus is not a vapor annulus, but a two-phase mixture annulus of vapor and fine droplets of liquid. In addition, the jet does not remain intact, but breaks up before reaching the exit plane. Thus, more surface area of liquid is exposed than the model accounts for and the model calculates less vaporization than actually occurs, as well as a lower vaporization rate. This is the same as saying that the model calculates a less compressible mixture than the actual case, and so predictions of critical flow rate are high and predictions of critical pressure ratio are low. The representation of the conduction process in this manner of minimum surface area is suggestive of the model's insensitivity to changes in T_0 .

Figure 4.23 compares this model to three data points taken from test section C120 at $L/D = 1.01$. It could not be ascertained whether or not the flow rates measured were actually critical flow rates; however, it is believed that the reported values are very close to the critical flow rates, if, in fact, they are not. At any rate, these three points are, at worst, low. The agreement between model predictions and data is better than at $L/D = 2.82$, indicating that at $L/D = 1.01$ the flow pattern may be more nearly the pattern assumed by Henry (see Fig. 2.6).

Fig. 4.23. Comparison of Flow-rate Data, $L/D = 1.01$, to Predictions of Henry's $0 < L/D < 3$ Model.⁶ ANL Neg. No. 900-1857.

Observation of Critical Flow in the Diverging Downstream Section

An attempt was made to obtain critical-flow data at $L/D = 1.01$. However, it was found by the method described in Ch. III that critical flow could not be obtained at the exit plane of the constant-area portion of test section C7. The fact that in two runs the flow rates measured in test section C120 at $L/D = 1.01$ were essentially equal to those obtained from C7, as seen in Table 4.1, suggested that test section C120 was not choked at the exit plane, since it was determined by the critical-flow test that C7 was unchoked at the exit plane. However, it was possible that test section C120 was undergoing critical flow at the exit plane when the flow rate was significantly higher than the flow rate through C7. The proof of these statements is not possible, since a rapidly diverging test section is of concern, yielding a critical-flow test much like the critical-flow test seen in Fig. 3.6b. Further, there is no corresponding gradually diverging test section which is known to be choked at the exit plane to which the flow rates can be compared to determine if run 5-2-1 is truly choked at the exit plane.

The significant pressure drop occurring between pressure tap D10 and the tap mounted in the flange (see Fig. 4.24) suggested that three more taps be installed downstream of D10. To this end, 1/16-in.-dia pressure taps were installed at locations D11, 3.76 in. downstream of the exit plane, D12, 4.32 in. downstream, and D13, 5.18 in. downstream. By performing the critical-flow test (see Fig. 4.24), it was determined that the plane of critical flow was somewhere between D8 and D10, that is, between 0.5 and 2.5 in. into the 7° diverging section. As far as is known to the author, the location of the critical-flow plane in a diverging conduit has not been reported prior to this, although Grolmes²⁴ had indications that this occurred in his condensing-injector experiment, also a highly non-equilibrium phenomenon. Thus, with critical flow occurring in the diverging portion, P_e could not drop to the critical pressure, and critical flow could not take place at the exit plane of the constant-area section.

A description of the mechanism of critical flow of a nonequilibrium, two-phase mixture in a diverging conduit is needed to explain this phenomenon, for it is believed that the nonequilibrium nature of the flow was the cause of such an occurrence. At present, no such description exists, nor does any mathematical model which takes into account both nonequilibrium and nonconstant area effects.

Table 4.1. Comparison of Flow Rates Measured in Both Rapidly and Gradually Diverging Test Sections at $L/D = 1.01$

Run	P_o (psia)	T_o (°F)	G (lb _m /ft ² -sec)	Run	P_o (psia)	T_o (°F)	G (lb _m /ft ² -sec)
5-1-1 ^a	16.76	71.5	1655	5-2-1 ^b	16.76	71.9	1825
5-1-11 ^a	16.72	35.4	1983	5-2-11 ^c	16.72	35.0	2004
5-1-15	9.72	35.0	1360	5-2-15 ^c	9.72	35.0	1414

^a - not choked at the exit plane as determined by test.

^b - choked at the exit plane, author's opinion.

^c - not choked at the exit plane, author's opinion.

Fig. 4.24. Observation of Critical Flow in the Diverging Portion of Test Section C7 at $L/D = 1.01$. In going from profile A to profile B, it is seen that the critical-flow plane lies somewhere between D8 and D10 (no pressure was measured at D9 during this test), since, within experimental accuracy, pressures upstream of D8 did not change in going from A to B. Going from profile B to C demonstrates that the entire pressure profile has changed upstream of D8. Thus, critical flow was occurring between D8 and D10. ANL Neg. No. 900-1861.

Phase Velocities at Critical Flow

A final conclusion that can be drawn from these data is an estimate of the individual phase velocities at critical flow. Those who are familiar with single-phase critical flow are aware that for isentropic, adiabatic, or isothermal flow the fluid velocities reaches $M = 1$, that is, the fluid travels at the acoustic velocity, the so-called "speed of sound."

For liquid flow, this corresponds to values of approximately 6000 ft/sec for water, 4000 ft/sec for ethyl alcohol, 4800 ft/sec for castor oil, and 4300 ft/sec for liquid mercury. Typical acoustic velocities through gases at 0°C are: 1100 ft/sec for air, 850 ft/sec for carbon dioxide, 1000 ft/sec for oxygen, and 1600 ft/sec for saturated steam at 134°C.²⁶

By using data gathered in this experiment, as well as some simplifying assumptions, it is possible to obtain a reasonable estimate of the individual phase velocities at the critical-flow plane. This can be accomplished by substituting the definition of quality into Eq. 2.2c to obtain

$$\frac{d}{dz} \left[\frac{\rho_l A_l u_l}{(1-x)} \right] = 0 . \quad (4.11)$$

By invoking the definition of void fraction α , Eq. 4.11 can be rewritten as

$$\frac{d}{dz} \left[\frac{\rho_l u_l (1-\alpha)(A_g + A_l)}{(1-x)} \right] = 0 . \quad (4.12)$$

For purposes of calculating approximate results for low qualities, set $1-x=1$. (However, dx/dz is not necessarily negligible.) Further, one can assume that the fraction of the volume occupied by the liquid jet is about 60%, inferred from calculating the discharge coefficients (see Appendix A), that is, the discharge coefficient is a measure of the percent of the original area to which the cross section shrinks. Thus, $\alpha = 0.4$.

To show that α remains relatively constant, integration of Eq. 2.2c gives

$$M = \rho_l A_l u_l + \rho_g A_g u_g , \quad (4.13)$$

where M , the total mass flow rate, appears as a constant of integration. Also, define $G \equiv M/A$, where A , the total area, is equal to $A_g + A_l$. Substitution of the definitions of α , x , and G into Eq. 4.13 gives

$$xG = \alpha u_l \rho_g \quad (4.14a)$$

as well as

$$(1-x)G = (1-\alpha)u_l \rho_l . \quad (4.14b)$$

Equation 2.1c and the definition of quality can be combined to give

$$-(A_l + A_g) \frac{dP}{dz} = \frac{d}{dz} \left[(\rho_l A_l u_l + \rho_g A_g u_g) \left\{ (1-x)u_l + xu_g \right\} \right] . \quad (4.15)$$

In terms of the previously defined symbols, this is rewritten as

$$-\frac{dP}{dz} = G \frac{d}{dz} \left[(1-x)u_l + xu_g \right] . \quad (4.16)$$

Substitution from Eq. 4.14 gives

$$-\frac{dP}{dz} = G \frac{d}{dz} \left[\frac{(1-x)^2 G}{(1-\alpha)\rho_l} + \frac{x^2 G}{\alpha\rho_g} \right] . \quad (4.17)$$

Experimental measurements indicate that dP/dz may be taken to be zero. Thus, Eq. 4.17 becomes

$$0 = G \frac{d}{dz} \left[\frac{(1-x)^2 G}{(1-\alpha)\rho_l} + \frac{x^2 G}{\alpha\rho_g} \right] . \quad (4.18)$$

For constant-area tubes G is constant and ρ_l may be considered constant. The fact that $dP/dz = 0$ also suggests that ρ_g is nearly constant. Therefore, Eq. 4.18 can be integrated to give

$$G^2 \left[\frac{(1-x)^2}{(1-\alpha)\rho_l} + \frac{x^2}{\alpha\rho_g} \right] = \text{constant} . \quad (4.19)$$

For low qualities, again put $1-x = 1$ and $x = 0$. This gives

$$\frac{G^2}{(1-\alpha)\rho_l} = \text{constant}, \quad (4.20)$$

which implies that α is a constant.

Thus, if $dP/dz = 0$, $\alpha = \text{constant}$. This is very nearly the case observed experimentally (see Ch. IV).

To recapitulate, it is seen that, if an effort to approximate the individual phase velocities at the critical flow plane, α must be approximated. It has been shown that the upper limit on α for this investigation is about 0.4 and that α remains nearly constant.

Now, integration of Eq. 4.12 gives the same result as the integration of Eq. 2.2c, since they are, indeed, the same equation:

$$\rho_l u_l \frac{(1-\alpha)}{(1-x)} = G . \quad (4.21)$$

For low qualities,

$$u_l = \frac{G}{\rho_l(1-\alpha)} . \quad (4.22)$$

At this point, however, there is still no reliable relationship available between u_l and u_g , save for experimental observation. Henry⁶ has measured values of the slip ratio for low-quality, two-phase water flow in the range from 1 to 2.4; Uchida and Nariai¹² also determined that the value of k was

very nearly unity. This argument can be strengthened by pointing out that the vapor is formed *at the liquid velocity* as it flashes from the liquid jet. Since the axial pressure is nearly constant, there is little potential to accelerate the vapor. Thus, the vapor velocity is very nearly the liquid velocity. Thus, for illustrative purposes, it will be assumed that

$$u_g = 2u_l . \quad (4.23)$$

By using Eqs. 4.22 and 4.23, the individual phase velocities in two-phase critical flow can be estimated. The results for several runs from this study are tabulated in Table 4.2.

It is most significant to point out that, at critical flow, the liquid is not traveling at anywhere near the acoustic velocity of the liquid, and that the vapor is, likewise, not traveling at the acoustic velocity of the

TABLE 4.2. Estimation of Individual Phase Velocities at Critical Flow for Various Runs ($\alpha = 0.4$)

Run	G_{cr} (lb _m /ft ² -sec)	u_l (ft/sec)	u_g (ft/sec)
1-1-1	1020	18.4	36.8
1-1-11	1795	31.3	62.6
1-1-15	1045	18.2	36.4
3-1-1	1290	23.2	46.4
3-1-11	1813	31.6	63.2
3-1-15	1143	19.9	39.8

vapor. This result illustrates the inadequacy of attempting to explain the mechanism by which two-phase critical flow occurs as "vapor choking",³⁸ that is, critical flow is said to occur when the velocity of the vapor reaches the acoustic velocity. This is definitely not the case; for CCl₃F, the acoustic velocity in the vapor phase is approximately 460 ft/sec, and the acoustic velocity in the liquid phase is approximately 3000 ft/sec for the temperature range covered in this study.

To put an upper and lower limit on the acoustic velocity through the two-phase mixture, values of the acoustic velocity obtained from the homogeneous equilibrium model (dx_E/dP calculated in an equilibrium manner [Ch. II]) and the homogeneous frozen model ($dx/dP = 0$) may be calculated. These represent the lower and upper limits, respectively, since $0 < dx/dP < dx_E/dP$. Such calculations reveal that the acoustic velocity through a two-phase CCl₃F mixture is between 15 and 100 ft/sec.²⁷

Thus, for critical two-phase flow, one does not always associate the high velocities characteristic of single-phase critical flow.

V. CONCLUSIONS

- 1) The critical flow of CCl_3F through sharp-edged entrance tubes for which $L/D \leq 12.78$ is a process of thermodynamic nonequilibrium. The nonequilibrium is caused when the fluid sustains a sudden pressure drop as it passes through the sharp-edged entrance. The liquid becomes superheated and, before sufficient time has passed for the fluid to transist to an equilibrium state, the fluid has issued from the tube.
- 2) Equilibrium critical-flow models are not descriptive of this phenomenon. Such models underpredict the critical flow rate and are quite insensitive to changes in P_o .
- 3) Henry's $L/D = 12$ model best predicts the data taken in the $L/D = 12.78$ test section, slightly overpredicting G_{cr} and slightly underpredicting η at nearly saturated conditions. The nearly saturated condition is the most severe test of a critical-flow model due to the sensitivity of G_{cr} and η to changes in stagnation conditions and L/D . This model may also be used to predict G_{cr} and η down to $L/D = 5$ if the subcooling is greater than 20°F . This is fortuitous, since G_{cr} and η are relatively insensitive to changes in L/D as the subcooling becomes large.
- 4) Although Henry's $0 \leq L/D \leq 3$ model exhibits the trend of the data at short L/D , it overpredicts G_{cr} and underpredicts η due to the assumption of the flow pattern used in deriving the model. The fact that the predictions of this model are very close to flow-rate data taken at $L/D = 1.01$ indicates that, perhaps, as the L/D approaches 0, the actual flow pattern becomes more like that assumed in the derivation of this model.
- 5) For CCl_3F , the superheated liquid jet is surrounded by a two-phase-mixture annulus, not a vapor annulus, for $L/D \geq 2.82$.
- 6) A change in the critical flow rate does not always accompany a change in P_e , for in going from $L/D = 8$ to $L/D = 5$, the critical flow rate remained relatively unchanged while P_e decreased. The change in compressibility in shortening the L/D , which is believed to be caused by a change in flow pattern, is a plausible explanation for this.
- 7) For the range of parameters tested, it was determined that it is necessary to use a 7° diverging section to obtain the one-dimensional value of P_e . Comparison of data taken with a 7° diverging section versus data taken with a 120° or 180° diverging section has shown that:
 - a) no steep pressure gradient exists at the exit of a short tube, and
 - b) the critical flow test reveals no dependency of P_e on P_R .

- 8) By comparing axial pressure profiles between test sections C7 and C120, it was found that axial pressures measured by a wall tap were affected by the two-dimensional expansion in a rapidly diverging test section within 0.1 of a tube diameter upstream of the exit plane; measurements 1.5 diameters upstream were not affected; the region between 0.1 and 1.5 diameters could not be assessed. This statement should not be applied to long tubes, since pressure profiles near the exit are not the same.
- 9) The method used in this study to obtain values of P_e only minimizes the effects of the exit-plane expansion. The location of the pressure tap on a corner also causes a lower value of P_e to be recorded than is characteristic of the phenomenon. Thus, although it is believed that such a method is the best available, it still gives slightly low values of P_e . One might improve upon this by situating pressure taps immediately upstream and immediately downstream of the exit plane of a 7° included-angle test section. One could then draw a smooth line through these points of the pressure profile to obtain a even better representation of P_e . Such an augmentation of this method would give values of P_e 0.2 to 0.5 psia higher than those tabulated in Appendix A. Since all taps did not record steady-state values in this experiment, such a correction to the values of P_e was not made.
- 10) In the range $2.82 \leq L/D \leq 12.78$, G_{cr} :
 - i) increases with decreasing L/D ;
 - ii) increases with decreasing T_o ;
 - iii) increases with increasing P_o .
- 11) In the range $2.82 \leq L/D \leq 12.78$, η :
 - i) increases with increasing L/D ;
 - ii) increases with increasing T_o ;
 - iii) increases with decreasing P_o .
- 12) The liquid velocity at critical flow is very much lower than the acoustic velocity of the liquid. Likewise, the vapor velocity is very much lower than the acoustic velocity of the vapor. Thus, the mechanism of two-phase critical flow is not one of "vapor choking."

APPENDIX A

DATA

Table A.1.4 Critical-flow Data for Test Sections C7 and C10 at $L/p = 12.78$

C120										
Rapidly Diverging Downstream Section (120°)										
Run	L/D	θ (°)	P (psia)	G (lbm/ft ² -sec)	v/v ₀	Run	L/P	θ (°)	P (psia)	P/P ₀
1-1-1	12.78	71.0	16.76	1020	0.739	1-2-1	12.78	72.3	16.76	0.609
1-1-2	12.78	64.7	16.70	1166	0.687	1-2-2	12.78	65.7	16.73	0.567
1-1-3	12.78	64.9	15.73	1129	0.725	1-2-3	12.78	64.8	15.70	0.573
1-1-4	12.78	54.8	16.70	1511	0.604	1-2-4	12.78	55.5	16.68	0.518
1-1-5	12.78	55.9	15.74	1343	0.635	1-2-5	12.78	55.0	15.72	0.522
1-1-6	12.78	54.4	12.74	989	0.732	1-2-6	12.78	54.7	12.74	0.611
1-1-7	12.78	46.0	16.73	1594	0.538	1-2-7	12.78	45.2	16.71	0.440
1-1-8	12.78	45.5	15.73	1508	0.563	1-2-8	12.78	45.5	15.73	0.457
1-1-9	12.78	46.1	12.67	1239	0.625	1-2-9	12.78	44.9	12.73	0.542
1-1-10	12.78	44.5	10.66	1008	0.713	1-2-10	12.78	44.1	10.73	0.601
1-1-11	12.78	34.8	16.77	1795	0.414	1-2-11	12.78	35.4	16.72	0.377
1-1-12	12.78	35.0	15.70	1701	0.444	1-2-12	12.78	36.0	15.72	0.391
1-1-13	12.78	34.7	12.70	1368	0.516	1-2-13	12.78	35.3	12.67	0.450
1-1-14	12.78	34.8	10.67	1170	0.613	1-2-14	12.78	35.4	10.67	0.513
1-1-15	12.78	34.6	9.72	1045	0.648	1-2-15	12.78	34.3	9.69	0.537

Table A.1b Pressure Profiles (psia) and Discharge Coefficients for Test Sections C7 and C120 at L/D = 12.78

		C7								Discharge Coefficients Based on Upstream Taps				
Pressure-tap Distance from Exit (in.)	Run	S1	S2	S4	S5	T6	D7	D8	D10	S1	S2	S4	S5	T6
		-2.747	-1.498	-0.032	-0.011	0.000	0.123	0.500	2.500					
	1-1-1	13.10	13.40	***	13.00	12.39	13.00	***	6.77	0.576	0.601	***	0.568	0.527
	1-1-2	11.90	12.35	11.60	11.20	11.48	11.00	10.65	7.20	0.573	0.602	0.556	0.535	0.549
	1-1-3	11.70	12.00	11.60	11.20	11.41	11.00	10.80	7.00	0.605	0.629	0.598	0.571	0.584
	1-1-4	10.10	10.70	10.45	10.30	10.09	***	9.80	7.40	0.630	0.661	0.648	0.640	0.630
	1-1-5	10.20	10.50	10.40	10.20	10.00	10.20	9.50	7.10	0.612	0.629	0.623	0.612	0.601
	1-1-6	9.70	9.90	***	***	9.33	***	***	5.60	0.608	0.629	***	***	0.574
	1-1-7	8.80	9.10	***	***	9.00	***	***	6.60	0.604	0.616	***	***	0.612
	1-1-8	***	9.20	9.20	***	8.85	***	8.60	6.50	***	0.630	0.630	***	0.613
	1-1-9	8.30	8.60	8.50	8.70	8.18	***	8.10	6.05	0.638	0.661	0.653	0.669	0.629
	1-1-10	7.80	8.00	8.00	8.10	7.60	7.90	7.50	5.20	0.636	0.659	0.659	0.672	0.615
	1-1-11	6.80	6.90	7.40	***	6.95	***	6.80	5.15	0.604	0.607	0.623	***	0.609
	1-1-12	6.80	6.90	7.20	7.00	6.97	***	6.70	5.25	0.606	0.609	0.620	0.613	0.612
	1-1-13	6.70	6.90	7.05	6.90	6.56	***	6.65	5.05	0.593	0.603	0.611	0.603	0.586
	1-1-14	6.60	6.80	6.80	***	6.54	***	6.20	4.85	0.616	0.632	0.632	***	0.612
	1-1-15	6.40	6.50	6.80	6.60	6.30	***	***	4.65	0.609	0.619	0.650	0.629	0.600

		C120								Discharge Coefficients Based on Upstream Taps			
Pressure-tap Distance from Exit (in.)	Run	S1	S2	S4	T5	D6	D7	D8	D9	S1	S2	S4	T5
		-2.747	-1.500	-0.031	-0.010	0.001	0.068	0.129	0.247				
	1-2-1	13.60	13.60	10.70	10.20	7.90	***	***	4.35	0.602	0.602	0.435	0.418
	1-2-2	12.40	12.40	10.10	9.48	7.40	***	5.20	***	0.594	0.594	0.480	0.459
	1-2-3	12.00	12.00	9.60	9.00	7.20	***	***	4.35	0.611	0.611	0.476	0.454
	1-2-4	10.20	10.30	***	8.65	6.30	***	***	4.60	0.605	0.610	***	0.544
	1-2-5	10.20	10.40	9.10	8.20	6.30	4.50	***	4.40	0.598	0.609	0.546	0.513
	1-2-6	9.80	9.80	8.20	7.78	5.80	4.00	***	3.70	0.620	0.620	0.499	0.477
	1-2-7	8.80	8.90	8.20	7.35	5.60	***	***	4.60	0.616	0.619	0.593	0.566
	1-2-8	8.70	8.70	8.00	7.19	5.60	***	***	4.50	0.613	0.613	0.584	0.556
	1-2-9	8.50	8.50	7.60	6.90	5.20	***	***	3.90	0.622	0.622	0.565	0.530
	1-2-10	8.00	8.00	7.00	6.45	4.90	4.30	4.00	3.40	0.640	0.640	0.547	0.511
	1-2-11	7.10	7.20	6.60	6.30	***	4.80	4.10	4.40	0.604	0.607	0.589	0.580
	1-2-12	7.10	7.10	6.40	6.15	***	4.60	4.00	3.95	0.602	0.602	0.579	0.572
	1-2-13	6.80	6.80	5.90	5.70	4.50	4.20	3.50	3.70	0.610	0.610	0.568	0.560
	1-2-14	6.70	6.70	5.70	5.47	4.25	***	***	3.30	0.609	0.609	0.545	0.532
	1-2-15	6.30	6.30	***	5.20	4.10	3.80	***	3.10	0.610	0.610	***	0.530

Table A.2a Critical-flow Data for Test Sections C7 at L/D = 7.96 and C120 at L/D = 7.97

C7											
Gradually Diverging Downstream Section (7°)						C120					
Rapidly Diverging Downstream Section (120°)											
Run	L/D	T (°F)	P (psia)	G (lbm/ft ² -sec)	v/v _o	Run	L/D	T (°F)	P (psia)	G (lbm/ft ² -sec)	v/v _o
2-1-1	7.96	71.3	16.76	1192	0.710	2-2-1	7.97	71.3	16.72	1240	0.541
2-1-2	7.96	64.5	16.75	1367	0.648	2-2-2	7.97	64.5	16.75	1385	0.501
2-1-3	7.96	64.5	15.75	1278	0.676	2-2-3	7.97	65.0	15.75	1285	0.524
2-1-4	7.96	55.3	16.74	1498	0.555	2-2-4	7.97	54.8	16.74	1564	0.421
2-1-5	7.96	54.8	15.74	1463	0.575	2-2-5	7.97	55.0	15.74	1473	0.464
2-1-6	7.96	55.0	12.74	1142	0.687	2-2-6	7.97	55.0	12.74	1157	0.543
2-1-7	7.96	45.6	16.70	1706	0.464	2-2-7	7.97	45.8	16.69	1647	0.374
2-1-8	7.96	45.8	15.73	1581	0.496	2-2-8	7.97	45.0	15.73	1612	0.400
2-1-9	7.96	45.0	12.73	1273	0.593	2-2-9	7.97	45.8	12.73	1300	0.471
2-1-10	7.96	44.8	10.73	1051	0.666	2-2-10	7.97	44.4	10.73	1152	0.517
2-1-11	7.96	34.7	16.72	1759	0.379	2-2-11	7.97	35.5	16.72	1798	0.318
2-1-12	7.96	35.8	15.72	1700	0.413	2-2-12	7.97	35.8	15.72	1707	0.344
2-1-13	7.96	35.0	12.72	1420	0.491	2-2-13	7.97	35.3	12.72	1472	0.413
2-1-14	7.96	35.5	10.72	1199	0.578	2-2-14	7.97	35.3	10.72	1219	0.478
2-1-15	7.96	35.0	9.72	1112	0.617	2-2-15	7.97	35.8	9.67	1091	0.511

Table A.2b Pressure Profiles (psia) and Discharge Coefficients for Test Sections C7 at L/D = 7.96 and C120 at L/D = 7.97

Pressure-tap Distance from Exit (in.)	Run	C7								Discharge Coefficients Based on Upstream Taps				
		S2	S3	S4	S5	T6	D7	D8	D10	S2	S3	S4	S5	T6
		-1.498	-0.502	-0.032	-0.011	0.000	0.123	0.500	2.500					
2-1-1		12.30	12.35	12.10	12.05	11.90	11.90	11.10	8.00	0.609	0.613	0.596	0.593	0.584
2-1-2		11.10	11.10	11.00	10.90	10.85	10.70	10.00	7.90	0.619	0.619	0.614	0.608	0.606
2-1-3		10.90	10.90	10.70	10.80	10.65	10.60	9.80	7.70	0.624	0.624	0.612	0.618	0.609
2-1-4		9.70	9.60	9.50	9.70	9.30	9.40	8.60	7.10	0.605	0.601	0.597	0.605	0.589
2-1-5		9.20	9.30	9.20	9.50	9.05	9.20	8.35	6.95	0.613	0.618	0.613	0.628	0.606
2-1-6		9.30	9.10	9.10	***	8.75	***	8.30	6.40	0.659	0.641	0.641	***	0.612
2-1-7		8.20	8.00	8.10	***	7.75	***	7.20	6.10	0.624	0.617	0.621	***	0.609
2-1-8		8.60	8.80	8.40	***	7.80	***	7.50	6.05	0.632	0.641	0.624	***	0.599
2-1-9		***	***	8.00	***	7.55	7.60	7.30	5.90	***	***	0.624	***	0.597
2-1-10		7.50	***	7.60	***	7.15	***	7.10	5.60	0.624	***	0.634	***	0.593
2-1-11		7.40	***	6.90	***	6.34	6.60	6.20	5.10	0.626	***	0.610	***	0.593
2-1-12		6.50	6.50	***	***	6.50	***	***	5.00	0.595	0.595	***	***	0.595
2-1-13		6.40	6.20	***	***	6.25	***	***	4.80	0.600	0.591	***	***	0.593
2-1-14		6.40	6.20	***	***	6.20	***	***	4.80	0.613	0.599	***	***	0.599
2-1-15		6.20	5.90	6.50	6.10	6.00	6.10	6.00	4.70	0.630	0.605	0.659	0.621	0.613

Pressure-tap Distance from Exit (in.)	Run	C120								Discharge Coefficients Based on Upstream Taps			
		S2	S3	S4	T5	D6	D7	D8	D9	S2	S3	S4	T5
		-1.500	-0.505	-0.031	-0.010	0.001	0.066	0.129	0.247				
2-2-1		11.50	11.70	9.80	9.05	6.90	5.40	***	4.80	0.586	0.597	0.509	0.483
2-2-2		10.50	10.50	8.80	8.40	6.30	***	4.90	4.80	0.596	0.596	0.529	0.516
2-2-3		10.40	10.40	8.70	8.25	6.20	***	4.70	4.70	0.598	0.598	0.521	0.505
2-2-4		8.90	8.90	7.80	7.05	5.20	5.00	4.90	4.95	0.598	0.598	0.560	0.538
2-2-5		8.80	8.80	7.50	7.30	5.40	4.60	4.75	4.60	0.599	0.599	0.550	0.543
2-2-6		8.50	8.50	7.30	6.92	5.20	4.10	4.10	4.00	0.602	0.602	0.531	0.514
2-2-7		7.40	7.60	6.70	6.25	5.00	4.80	4.70	***	0.577	0.583	0.556	0.544
2-2-8		7.50	7.70	6.70	6.30	4.80	4.60	4.55	4.80	0.600	0.607	0.572	0.560
2-2-9		7.20	7.40	6.00	6.00	4.70	4.20	4.00	4.20	0.590	0.601	0.535	0.535
2-2-10		6.70	6.90	6.20	5.55	4.60	4.20	4.00	4.10	0.612	0.628	0.577	0.540
2-2-11		6.10	6.10	5.80	5.32	4.50	4.70	4.50	***	0.586	0.586	0.578	0.566
2-2-12		6.00	6.20	5.50	5.41	4.60	4.75	4.50	4.80	0.582	0.588	0.567	0.565
2-2-13		5.90	5.90	5.60	5.25	4.25	4.38	4.30	4.20	0.599	0.599	0.586	0.573
2-2-14		5.90	6.10	5.40	5.12	4.30	4.20	4.00	***	0.590	0.603	0.562	0.547
2-2-15		5.70	5.90	4.90	4.94	4.20	4.30	3.95	4.30	0.582	0.597	0.531	0.533

Table A.3a Critical-flow Data for Test Sections C7S at L/D = 5.00 and C180 at L/D = 5.08

C7S Gradually Diverging Downstream Section (7°)						C180 Rapidly Diverging Downstream Section (180°)					
Run	L/D	T _o (°F)	P _o (psia)	G _{cr} (lbm/ft ² -sec)	P _e /P _o	Run	L/D	T _o (°F)	P _o (psia)	G _{cr} (lbm/ft ² -sec)	P _e /P _o
3-1-1	5.00	72.3	16.78	1290	0.664	3-2-1	5.08	72.0	16.76	1238	0.573
3-1-2	5.00	65.0	16.75	1417	0.600	3-2-2	5.08	64.8	16.75	1385	0.507
3-1-3	5.00	64.7	15.76	1314	0.619	3-2-3	5.08	64.6	15.73	1296	0.531
3-1-4	5.00	55.5	16.74	1557	0.523	3-2-4	5.08	55.3	16.74	1554	0.448
3-1-5	5.00	55.3	15.74	1458	0.549	3-2-5	5.08	55.4	15.77	1430	0.472
3-1-6	5.00	54.8	12.74	1189	0.632	3-2-6	5.08	55.3	12.74	1156	0.569
3-1-7	5.00	45.1	16.73	1706	0.442	3-2-7	5.08	45.0	16.73	1715	0.403
3-1-8	5.00	45.2	15.73	1600	0.467	3-2-8	5.08	45.3	15.73	1611	0.426
3-1-9	5.00	45.2	12.73	1296	0.554	3-2-9	5.08	45.0	12.73	1326	0.495
3-1-10	5.00	45.0	10.73	1091	0.629	3-2-10	5.08	44.3	10.65	1148	0.559
3-1-11	5.00	35.2	16.72	1813	0.362	3-2-11	5.08	35.3	16.72	1828	0.353
3-1-12	5.00	35.0	15.72	1724	0.388	3-2-12	5.08	35.2	15.71	1750	0.363
3-1-13	5.00	35.4	12.72	1444	0.472	3-2-13	5.08	35.3	12.72	1460	0.423
3-1-14	5.00	34.7	10.72	1250	0.536	3-2-14	5.08	34.9	10.72	1232	0.480
3-1-15	5.00	35.2	9.72	1143	0.576	3-2-15	5.08	35.5	9.71	1130	0.517

Table A.3b Pressure Profiles (psia) and Discharge Coefficients for Test Sections C7S at L/D = 5.00 and C180 at L/D = 5.08

Pressure-tap Distance from Exit. (in.)	Run	C7S								Discharge Coefficients Based on Upstream Taps		
		S1 -0.50	S2 -0.30	T3 0.00	D4 0.125	D5 0.50	D6 1.06	D7 5.69	P (flange) 7.440	S1	S2	T3
	3-1-1	11.20	11.80	11.15	10.40	9.90	9.80	7.00	3.00	0.590	0.624	0.587
	3-1-2	10.10	10.60	10.05	9.40	9.10	8.30	7.20	3.20	0.592	0.615	0.589
	3-1-3	9.95	10.30	9.75	9.20	8.90	8.10	6.90	3.00	0.586	0.605	0.577
	3-1-4	8.70	9.30	8.75	8.10	8.00	7.50	6.90	3.40	0.589	0.612	0.591
	3-1-5	8.60	9.10	8.65	7.90	7.80	7.30	6.70	3.20	0.585	0.606	0.587
	3-1-6	8.10	8.30	8.05	7.40	7.40	6.80	5.80	3.65	0.591	0.604	0.588
	3-1-7	7.25	8.10	7.40	6.80	6.85	6.40	6.00	3.40	0.591	0.620	0.596
	3-1-8	7.30	8.00	7.35	6.70	6.80	6.35	5.95	3.25	0.588	0.614	0.590
	3-1-9	7.15	7.50	7.05	6.40	6.50	6.10	5.50	2.80	0.586	0.605	0.580
	3-1-10	6.85	7.10	6.75	6.00	6.20	5.80	4.90	2.50	0.591	0.611	0.583
	3-1-11	5.90	6.60	6.05	5.50	5.70	5.40	5.10	3.30	0.586	0.606	0.590
	3-1-12	5.85	6.50	6.10	5.40	5.70	5.30	5.10	3.20	0.583	0.603	0.591
	3-1-13	5.80	6.40	6.00	5.30	5.60	5.45	4.95	2.90	0.583	0.610	0.592
	3-1-14	5.70	6.10	5.75	***	5.40	5.10	4.65	2.70	0.593	0.618	0.596
	3-1-15	5.65	6.05	5.60	4.90	5.30	5.10	4.50	2.60	0.602	0.634	0.598

Pressure-tap Distance from Exit (in.)	Run	C180						Discharge Coefficients Based on Upstream Taps		
		S1 -1.23	S2 -0.91	T3 -0.03	D4 0.12	D5 3.10	D6 5.79	S1	S2	T3
	3-2-1	11.40	11.60	9.61	4.90	4.95	4.90	0.577	0.588	0.500
	3-2-2	10.35	10.40	8.50	5.10	5.10	4.95	0.589	0.592	0.519
	3-2-3	10.05	10.05	8.35	4.95	4.85	4.90	0.585	0.585	0.513
	3-2-4	8.70	8.80	7.50	5.10	5.10	5.00	0.587	0.591	0.548
	3-2-5	8.90	8.90	7.45	4.90	4.85	4.70	0.585	0.585	0.531
	3-2-6	8.50	8.50	7.25	4.30	4.30	4.15	0.602	0.602	0.529
	3-2-7	7.50	7.40	6.75	4.95	4.90	4.80	0.602	0.599	0.579
	3-2-8	7.40	7.30	6.70	4.85	4.80	4.95	0.596	0.592	0.572
	3-2-9	7.20	7.20	6.30	4.50	4.40	4.30	0.602	0.602	0.558
	3-2-10	6.90	6.85	5.95	4.20	4.15	4.10	0.633	0.628	0.565
	3-2-11	6.20	6.20	5.90	4.95	4.70	4.60	0.599	0.599	0.591
	3-2-12	6.10	6.00	5.70	***	4.30	4.35	0.600	0.597	0.588
	3-2-13	6.00	5.90	5.38	3.70	3.80	3.80	0.599	0.594	0.573
	3-2-14	5.90	5.80	5.15	3.40	3.45	3.45	0.596	0.590	0.555
	3-2-15	5.80	5.60	5.02	3.25	3.35	3.35	0.607	0.592	0.554

Table A.4a Critical-flow Data for Test Sections C7 and C120 at $L/D = 2.82$

C7 Gradually Diverging Downstream Section (7°)						C120 Rapidly Diverging Downstream Section (120°)					
Run	L/D	T_o (°F)	P_o (psia)	G_{cr} (lbm/ft ² -sec)	P_e/P_o	Run	L/D	T_o (°F)	P_o (psia)	G_{cr} (lbm/ft ² -sec)	P_e/P_o
2-1-1	2.82	72.0	16.76	1436	0.600	4-2-1	2.82	71.5	16.71	1503	0.491
4-1-2	2.82	64.7	16.75	1534	0.534	4-2-2	2.82	64.5	16.76	1573	0.471
4-1-3	2.82	64.8	15.75	1455	0.581	4-2-3	2.82	65.2	15.73	1533	0.470
4-1-4	2.82	55.0	16.74	1672	0.451	4-2-4	2.82	55.5	16.74	1725	0.440
4-1-5	2.82	55.5	15.74	1574	0.488	4-2-5	2.82	55.0	15.74	1660	0.400
4-1-6	2.82	54.7	12.68	1377	0.576	4-2-6	2.82	54.9	12.71	1376	0.456
4-1-7	2.82	45.0	16.73	1798	0.380	4-2-7	2.82	45.3	16.73	1824	0.388
4-1-8	2.82	45.0	15.73	1748	0.381	4-2-8	2.82	45.1	15.73	1737	0.367
4-1-9	2.82	45.3	12.73	1475	0.452	4-2-9	2.82	45.4	12.73	1487	0.434
4-1-10*	2.82	44.6	10.73	1376	0.517	4-2-10*	2.82	44.3	10.73	1332	0.483
4-1-11*	2.82	35.4	16.72	1918	0.305	4-2-11*	2.82	35.5	16.68	1933	0.306
4-1-12*	2.82	35.4	15.70	1834	0.316	4-2-12*	2.82	35.1	15.72	1850	0.312
4-1-13*	2.82	35.1	12.72	1567	0.388	4-2-13*	2.82	35.8	12.72	1583	0.397
4-1-14*	2.82	35.0	10.72	1380	0.457	4-2-14*	2.82	35.8	10.72	1388	0.448
4-1-15*	2.82	35.0	9.70	1318	0.484	4-2-15*	2.82	35.3	9.72	1298	0.478

Table A.4b Pressure Profiles (psia) and Discharge Coefficients for Test Sections C7 and C120 at L/D = 2.82

Pressure-tap Distance from Exit (in.)		C7								Discharge Coefficients Based on Upstream Taps			
		S3	S4	S5	T6	D7	D8	D9	D10	S3	S4	S5	T6
	Run	-0.502	-0.032	-0.011	0.000	0.123	0.500	1.002	2.500				
4-1-1		10.00	***	***	10.05	10.20	***	10.00	8.50	0.597	***	***	0.599
4-1-2		8.90	9.80	9.20	8.95	9.50	9.00	8.90	7.80	0.589	0.626	0.601	0.591
4-1-3		8.90	9.80	9.30	9.15	9.50	9.10	9.00	7.70	0.598	0.642	0.617	0.610
4-1-4		7.80	8.50	8.00	7.55	8.20	8.00	7.70	6.80	0.599	0.624	0.606	0.591
4-1-5		7.60	8.40	7.90	7.68	8.20	7.90	7.70	6.80	0.591	0.623	0.603	0.594
4-1-6		7.10	7.70	7.40	7.30	7.80	7.30	7.50	6.40	0.625	0.661	0.642	0.636
4-1-7		6.40	7.40	***	6.35	***	6.60	6.80	5.70	0.597	0.628	***	0.596
4-1-8		6.10	6.90	6.10	6.00	6.50	6.40	6.20	5.50	0.601	0.628	0.601	0.598
4-1-9		5.90	6.40	5.90	5.75	6.30	6.00	6.00	5.60	0.602	0.626	0.602	0.596
4-1-10*		5.65	6.00	5.80	5.55	5.90	5.70	5.90	5.40	0.651	0.675	0.661	0.645
4-1-11*		5.00	5.80	5.50	5.10	***	5.50	5.20	***	0.595	0.617	0.609	0.598
4-1-12*		4.85	5.60	5.00	4.96	5.30	5.30	5.20	4.60	0.592	0.613	0.596	0.595
4-1-13*		5.10	0.60	5.15	4.94	5.40	5.30	5.20	4.60	0.603	0.624	0.605	0.597
4-1-14*		5.00	5.55	5.20	4.90	5.60	5.30	5.20	4.60	0.613	0.645	0.624	0.608
4-1-15*		4.70	5.30	4.90	4.70	5.30	5.10	4.90	4.70	0.626	0.668	0.639	0.626
Pressure-tap Distance from Exit (in.)		C120							Discharge Coefficients Based on Upstream Taps				
		S3	S4	T5	D6	D7	D8	D9	S3	S4	T5		
	Run	-0.505	-0.031	-0.010	0.001	0.068	0.129	0.247					
4-2-1		9.10	9.20	8.20	5.97	5.70	5.60	5.90	0.588	0.592	0.556		
4-2-2		8.90	8.80	7.90	6.00	5.50	5.60	5.70	0.604	0.600	0.569		
4-2-3		7.95	8.40	7.40	5.60	***	5.60	5.60	0.592	0.609	0.572		
4-2-4		6.10	7.90	7.37	6.20	5.80	***	5.50	0.567	0.622	0.604		
4-2-5		6.60	7.70	6.30	***	5.80	***	5.80	0.588	0.627	0.579		
4-2-6		6.90	7.00	5.80	4.80	4.60	4.80	4.70	0.612	0.617	0.561		
4-2-7		6.20	6.90	6.50	5.20	5.10	5.10	5.30	0.600	0.621	0.609		
4-2-8		5.90	6.90	5.78	4.90	4.90	5.10	5.20	0.591	0.624	0.588		
4-2-9		6.10	6.50	5.53	4.60	4.50	4.60	4.70	0.616	0.636	0.592		
4-2-10*		5.70	6.00	5.18	4.30	4.20	4.10	4.40	0.633	0.653	0.603		
4-2-11*		5.20	6.20	5.10	4.70	4.70	4.80	4.90	0.607	0.635	0.604		
4-2-12*		4.70	5.90	4.90	4.20	4.70	4.60	5.20	0.592	0.627	0.598		
4-2-13*		4.95	5.80	5.05	4.20	***	***	4.50	0.604	0.640	0.608		
4-2-14*		4.70	5.40	4.80	3.90	3.90	4.10	4.10	0.601	0.640	0.606		
4-2-15*		4.90	5.20	4.65	3.70	3.50	***	***	0.628	0.649	0.613		

ERROR ANALYSIS

Maximum Suspected Error in the Measurement of the Mass Flow Rate per Unit Area

The continuity equation can be written between the high-pressure vessel and the exit plane of the test section as

$$\rho Au]_{\text{hpv}} = \rho Au]_e \quad (\text{B.1})$$

or

$$\rho Au]_{\text{hpv}} = GA]_e . \quad (\text{B.2})$$

Substituting for u_{hpv} , A_{hpv} , and A_e and solving for G_e ,

$$G_e = \rho u]_{\text{hpv}} \frac{A_{\text{hpv}}}{A_e} = \rho_{\text{hpv}} \frac{\Delta h_{\text{CCl}_3\text{F}}}{t} \frac{\pi r_{\text{hpv}}^2}{\pi r_e^2} = \rho_{\text{hpv}} \frac{\Delta h_{\text{CCl}_3\text{F}}}{t} \left[\frac{r_{\text{hpv}}}{r_e} \right]^2 . (\text{B.3})$$

For the case of $r_{\text{hpv}} = 3$ in., $r_e = 0.1565$ in., $\rho_{\text{hpv}} = \rho_l$, and with proper unit conversion factors, Eq. B.3 becomes

$$G_e = \frac{30.62 \rho_l \Delta h_{\text{CCl}_3\text{F}}}{t} , \quad (\text{B.4})$$

where

$$G_e = \text{lb}_m / \text{ft}^2\text{-sec}$$

$$\rho_l = \text{lb}_m / \text{ft}^3$$

$$\Delta h_{\text{CCl}_3\text{F}} = \text{inches}$$

$$t = \text{seconds}.$$

To assess the uncertainty in G_e caused by measuring t and $\Delta h_{\text{CCl}_3\text{F}}$, we have

$$\Delta G_e = \frac{30.62 \rho_l [\Delta h_{\text{CCl}_3\text{F}} + \Delta(\Delta h_{\text{CCl}_3\text{F}})]}{t + \Delta t} - G_e . \quad (\text{B.5})$$

Runs wherein G_e was large, t was small, and the error in reading the CCl_3F level $\Delta h_{\text{CCl}_3\text{F}}$ and time t opposite sign, are subject to the greatest error. Sample calculations 1 and 2 below are chosen for the worst conditions previously mentioned. It should be noted, however, that the errors in other runs are no more than these, and usually much less.

Sample calculation 1: Run 3-1-11

For

$$G_e = 1813 \text{ lb}_m/\text{ft}^2\text{-sec}$$

$$\Delta(\Delta h_{\text{CCl}_3\text{F}}) = -1 \text{ in.}$$

$$\rho_l = 95.5 \text{ lb}_m/\text{ft}^3$$

$$\Delta t = +0.2 \text{ sec}$$

$$\Delta h_{\text{CCl}_3\text{F}} = 30 \text{ in.}$$

$$t = 48.6 \text{ sec}$$

Eq. B.5 yields

$$\Delta G_e = \frac{30.62(95.5)(31)}{48.8} - 1813 = -75 \text{ lb}_m/\text{ft}^2\text{-sec}.$$

Sample calculation 2: Run 2-2-11

For

$$G_e = 1798 \text{ lb}_m/\text{ft}^2\text{-sec}$$

$$\Delta(\Delta h_{\text{CCl}_3\text{F}}) = -1 \text{ in.}$$

$$\rho_l = 95.5 \text{ lb}_m/\text{ft}^3$$

$$\Delta t = +0.2 \text{ sec}$$

$$\Delta h_{\text{CCl}_3\text{F}} = 30 \text{ in.}$$

$$t = 48.8 \text{ sec}$$

Eq. B.5 yields

$$\Delta G_e = \frac{30.62(95.5)(29)}{50} - 1798 = -102 \text{ lb}_m/\text{ft}^2\text{-sec}.$$

**Table B.1 Reliability of Axial Pressure
Measurements for All Runs (psia)**

Runs	Pressure Tap	Sources of Error				
		Head	Instrumentation	Unsteady Effects	Maximum Suspected Error	
					+	-
1-1-1 thru 1-1-75	S1	-0.1	<u>+0.1</u>	<u>+0.1</u>	0.2	0.3
	S2	-0.1	<u>+0.1</u>	<u>+0.3</u>	0.4	0.5
	S4	+0.1	<u>+0.1</u>	<u>+0.2</u>	0.4	0.3
	S5	0	<u>+0.1</u>	<u>+0.1</u>	0.2	0.2
	exit	-0.1	<u>+0.05</u>	<u>+0.15</u>	0.2	0.3
	D7	0	<u>+0.1</u>	<u>+0.1</u>	0.2	0.2
	D8	+0.1	<u>+0.1</u>	<u>+0.35</u>	0.55	0.45
	D10	-0.1	<u>+0.1</u>	<u>+0.25</u>	0.35	0.45
1-2-1 thru 1-2-15	S1	-0.1	<u>+0.1</u>	<u>+0.1</u>	0.2	0.3
	S2	-0.1	<u>+0.1</u>	<u>+0.25</u>	0.35	0.45
	S4	0	<u>+0.1</u>	<u>+0.3</u>	0.4	0.4
	"exit"	-0.1	<u>+0.05</u>	<u>+0.15</u>	0.2	0.3
	D6	-0.1	<u>+0.1</u>	<u>+0.2</u>	0.3	0.4
	D7	+0.1	<u>+0.1</u>	<u>+0.1</u>	0.3	0.2
	D8	0	<u>+0.1</u>	<u>+0.05</u>	0.15	0.15
	D9	-0.1	<u>+0.1</u>	<u>+0.2</u>	0.3	0.4
2-1-1 thru 2-1-15	S2	-0.1	<u>+0.1</u>	<u>+0.3</u>	0.4	0.5
	S3	0	<u>+0.1</u>	<u>+0.15</u>	0.25	0.25
	S4	+0.1	<u>+0.1</u>	<u>+0.3</u>	0.5	0.4
	S5	0	<u>+0.1</u>	<u>+0.1</u>	0.2	0.2
	exit	-0.1	<u>+0.05</u>	<u>+0.15</u>	0.2	0.3
	D7	0	<u>+0.1</u>	<u>+0.1</u>	0.2	0.2
	D8	+0.1	<u>+0.1</u>	<u>+0.3</u>	0.5	0.4
	D10	-0.1	<u>+0.1</u>	<u>+0.2</u>	0.3	0.4
2-2-1 thru 2-2-15	S2	-0.1	<u>+0.1</u>	<u>+0.5</u>	0.6	0.7
	S3	-0.1	<u>+0.1</u>	<u>+0.4</u>	0.5	0.6
	S4	0	<u>+0.1</u>	<u>+0.4</u>	0.5	0.5
	"exit"	-0.1	<u>+0.05</u>	<u>+0.2</u>	0.25	0.35
	D6	-0.1	<u>+0.1</u>	<u>+0.3</u>	0.4	0.5
	D7	+0.1	<u>+0.1</u>	<u>+0.2</u>	0.4	0.3
	D8	0	<u>+0.1</u>	<u>+0.15</u>	0.25	0.25
	D9	-0.1	<u>+0.1</u>	<u>+0.05</u>	0.15	0.25

**Table B.1 Reliability of Axial Pressure
Measurements for All Runs (psia)
(cont'd)**

Runs	Pressure Tap	Sources of Error			Maximum Suspected Error	
		Head	Instrumentation	Unsteady Effects		
					+	-
3-1-1 thru 3-1-15	S1	-0.1	± 0.1	± 0.15	0.25	0.35
	S2	+0.1	± 0.1	± 0.45	0.65	0.55
	exit	-0.1	± 0.05	± 0.2	0.25	0.35
	D4	+0.1	± 0.1	± 0.1	0.3	0.2
	D5	-0.1	± 0.1	± 0.05	0.15	0.25
	D6	-0.1	± 0.1	± 0.1	0.2	0.3
	D7	-0.1	± 0.1	± 0.1	0.2	0.3
	P _{flange}	-0.1	± 0.1	± 0.1	0.2	0.3
3-2-1 thru 3-2-15	S1	-0.1	± 0.1	± 0.3	0.4	0.5
	S2	+0.1	± 0.1	± 0.3	0.4	0.5
	"exit"	-0.1	± 0.05	± 0.3	0.35	0.45
	D4	+0.1	± 0.1	± 0.1	0.3	0.2
	D5	-0.1	± 0.1	± 0.15	0.25	0.35
	D6	-0.1	± 0.1	± 0.15	0.25	0.35
4-1-1 thru 4-1-15	S3	0	± 0.1	± 0.2	0.3	0.3
	S4	+0.1	± 0.1	± 0.4	0.6	0.5
	S5	0	± 0.1	± 0.2	0.3	0.3
	exit	-0.1	± 0.05	± 0.25	0.3	0.4
	D7	0	± 0.1	± 0.2	0.3	0.3
	D8	+0.1	± 0.1	± 0.35	0.55	0.45
	D9	0	± 0.1	± 0.15	0.25	0.25
	D10	-0.1	± 0.1	± 0.2	0.3	0.4
4-2-1 thru 4-2-15	S3	-0.1	± 0.1	± 0.4	0.5	0.6
	S4	0	± 0.1	± 0.3	0.4	0.4
	"exit"	-0.1	± 0.05	± 0.35	0.4	0.5
	D6	-0.1	± 0.1	± 0.1	0.2	0.3
	D7	+0.1	± 0.1	± 0.1	0.3	0.2
	D8	0	± 0.1	± 0.2	0.3	0.3
	D9	-0.1	± 0.1	± 0.15	0.25	0.35

CONSTRUCTION OF TEST SECTIONS

C7

Test section C7 was made in two pieces due to the difficulties arising from attempting to accurately machine a 7° diverging cone over nearly 6 in. and still maintain a good finish. The two pieces were welded together as shown in Fig. C.1. The ID of the upstream portion was polished with emery paper. All 0.010-in.-dia taps (see Table 3.2) were installed by electrical-discharge drilling which insured:

- 1) accurate location,
- 2) precise control of the diameter, and
- 3) the burrs would not be formed.

The remaining taps were drilled and deburred.

Fig. C.1. Assembly of Test Section C7 with Mounting Flanges. ANL Neg. No. 900-1875.

Test section C7 was mounted by welding flanges to the ends. The entrance flange was bored to the OD of the test-section entrance, the test section inserted so that the flange and entrance were flush, and the test section finally welded to the flange. A face cut across the flange and test-section entrance was taken on a lathe to insure a sharp-edged entrance. When C7 was cut down to $L/D = 7.96$ and 2.82 , the weld on the entrance flange was cut away, and the test section was cut down, rewelded, and refaced.

C120

For simplicity of machining, test section C120 was also made in two pieces. It was welded as shown in Fig. C.2. The ID of the upstream portion was polished with emery paper. All 0.010-in.-dia taps (see Table 3.2) were installed by electrical-discharge drilling--the remaining taps were drilled and deburred.

C120 was mounted and cut down as described for C7. Test section C120 with its mounting flanges welded on is shown in Fig. 3.4.

C7S

Test section C7S was also made in two pieces and welded together, as shown in Fig. C.3. Due to the difficulties caused by the small taps in

Fig. C.2. Assembly of Test Section C120 with Mounting Flanges. ANL Neg. No. 900-1879.

Fig. C.3. Assembly of Test Section C7S with Mounting Flanges. ANL Neg. No. 900-1869.

Fig. C.4. Assembly of Test Section C180 with Mounting Flanges. ANL Neg. No. 900-1876.

C7 and C120 (see Ch. III), larger pressure taps were used in test section C7S (see Table 3.2). Test section C7S was mounted as described for C7 and was used only for $L/D = 5$.

C180

For convenience, test section C180 was made in three pieces, as shown in Fig. C.4. Once again, larger pressure taps were used (see Table 3.2). The test section was mounted in the same manner as the others.

APPENDIX D

DEMONSTRATING THAT DURING CRITICAL FLOW THE FLUID UNDERGOES ITS MAXIMUM EXPANSION RATE AND REACHES ITS MAXIMUM COMPRESSIBILITY

The critical mass rate of flow per unit area may be written as

$$G_{cr} = \left[\frac{-g_c}{C_{cr}} \right]^{1/2}, \quad (D.1)$$

where C_{cr} is the compressibility, or the ability of the fluid to be compressed, at critical flow. Inherently C is negative and is said to increase as the absolute value of C increases. For example, C_{cr} for Henry's model⁵ is given by

$$C_{cr} = \left[\frac{-xv}{P} g + (v_g - v_{lo}) N \frac{dx_E}{dP} \right]_e, \quad (D.2)$$

and for the homogeneous equilibrium model, discussed in Ch. IV, C_{cr} is given by

$$C_{cr} = x_E \frac{dv_g}{dP} + (v_g - v_l) \frac{dx_E}{dP} + (1 - x_E) \frac{dv_l}{dP}. \quad (D.3)$$

It will now be illustrated that the critical-flow criterion implies that the compressibility of the fluid becomes maximum at critical flow; in other words, the rate of expansion of the fluid also becomes maximum, since $\mathcal{E} = -C$.

To illustrate this, the functions $G[P(z)]$, $C[P(z)]$, and $\mathcal{E}[P(z)]$ are sketched (see Fig. D.1) for $P_{cr} \leq P(z) \leq P_0$, since no one-dimensional axial pressure in a tube can be below the critical pressure. In Fig. D.1, $G[P(z)]$ is the variation of the product ρu along an isentropic path, $C[P(z)]$ is the variation of dv/dP along an isentropic path, and $\mathcal{E}[P(z)]$ is $-C[P(z)]$. When $dG/dP = 0$, both C and $\mathcal{E}$ take their maximum values.

Fig. D.1. Illustration of the Fact that at Critical Flow the Rate of Expansion and Compressibility of the Fluid are Maximum. ANL Neg. No. 900-1889.

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