Colour Annealing — A Toy Model of Colour Reconnections

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Abstract
We present a simple toy model for colour reconnections at the non-perturbative level. The model resembles an annealing-type algorithm and is applicable to any collider and process type, though we argue for a possible enhancement of the effect in hadron-hadron collisions. We present a simple application and study of the consequences for semi-leptonic $\mathrm{t\bar{t}}$ events at the Tevatron.

1. Introduction

Among the central objectives of collider physics is the precise measurement of the elementary particle masses and couplings. Striking recent examples are the measurements both at LEP and at the Tevatron of the mass of the W boson to a precision better than one per mille [1, 2] — a precision giving truly valuable insight into the mechanism of electroweak symmetry breaking as well as in probing for the quantum effects of New Physics.

At present, with the top quark in focus at the Tevatron and the physics programme of the LHC only a few years distant, the solid understanding of QCD phenomena beyond leading-order perturbation theory is becoming increasingly more important, with a large range of both experimental and theoretical methods and tools being developed. The aim, to achieve theoretical and systematic uncertainties capable of matching the expected statistical precision of the large data samples becoming available.

Apart from developments in flavour physics and lattice QCD, essentially all of these approaches focus on the perturbative domain of QCD — in brief: including more legs/loops/logarithms in the calculations. The point we wish to stress here is that, even assuming these approaches to one day deliver predictions with negligible uncertainties associated with uncalculated perturbative orders, there still remains the non-perturbative aspects, for which current understanding cannot be called primitive, but certainly not crystal clear either.

Recently, the structure and physics of the underlying event has received some attention [3–6], but again the main theoretical thrust, with few exceptions [7, 8], has taken place in the perturbative modeling, in the form of more sophisticated models for multiple perturbative interactions [9–11]. While non-perturbative aspects certainly play a significant role, and enter into the descriptions in the form of various phenomenological parameters, they generally suffer from being hard to quantify, hard to calculate, and hard to test. In this study, we shall focus on precisely such a source of potential uncertainty: colour reconnection effects in the final state, in particular in the context of measurements made at hadron colliders.

In Section 2. we briefly discuss some previous cerebrations on colour reconnections, and in Section 3. present our own toy model, for use in the present study. In Section 4. we give a few explicit examples and show some results for $\mathrm{t\bar{t}}$ events at the Tevatron. Section 5. contains a summary and outlook.

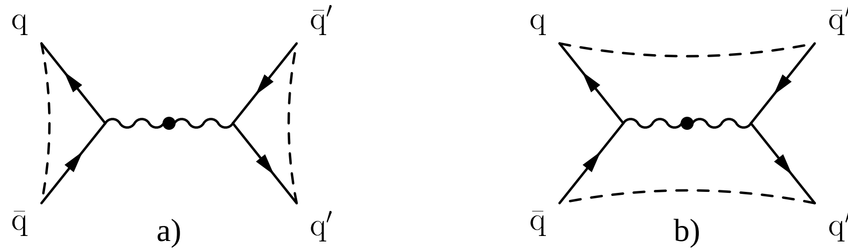

Fig. 1: a) the original colour topology in hadronic $\mathrm{e^+e^-} \to \mathrm{WW}$ events, and b) a reconnected version. Note that these are not Feynman diagrams but rather spatial diagrams depicting the situation after the annihilation, with the production point at the origin. Arrows pointing against the direction of motion signify antifermions.

2. Colour Reconnections

The subject of colour rearrangements was first studied by Gustafson, Pettersson, and Zerwas (GPZ) [12], there in a mainly qualitative way, and in the context of rearrangements taking place already at the perturbative level. They observed that, e.g. in hadronic $\mathrm{WW} \to (\mathrm{q_1\bar{q}_2})(\mathrm{q_3\bar{q}_4})$ events at LEP, illustrated in Fig. 1a with colour connections traced by dashed lines, interference effects and gluon exchanges between the decay products could lead to a reconfiguration of the colour topology into the one depicted in Fig. 1b. In the reconnected topology, both the perturbative QCD cascade and the subsequent hadronisation phase would be substantially different, leading to very large effects.

Sjöstrand and Khoze (SK) [13, 14] subsequently argued that such large effects were most likely unrealistic. A reconnection already at the perturbative level requires at least two perturbative gluon vertices, leading to an α_s^2 suppression. Moreover, the relevant reconnection diagram is colour suppressed by $1/N_c^2$ with respect to the leading (non-reconnected) $\mathcal{O}(\alpha_s^2)$ diagrams. Finally, for the decay products of the two W bosons to radiate coherently, they must, in the language of wave mechanics, be in phase, which only occurs for radiation at energies smaller than the W width. In other words, gluons with wavelengths smaller than the typical separation of the two W decay vertices will be radiated (almost) incoherently. For these reasons, SK considered a scenario where reconnections occur as part of the non-perturbative hadronisation phase.

The SK model is based on the standard Lund string fragmentation model [15], in which the chromo-electric flux lines formed between colour charges separated at distances larger than $\sim 1\mathrm{fm}$ are represented by simple massless strings. SK argued that, if two such strings overlap in space and time, there should be a finite possibility for them to ‘cut each other up’ and rearrange themselves, much as has been recently discussed for the case of cosmic and mesonic superstrings [16, 17]. However, since we do not yet know whether QCD strings behave more like flux tubes in a Type II or a Type I superconductor (roughly speaking whether the topological information is stored in a small core region or not), SK presented two distinct models, commonly referred to as SK-II and SK-I, respectively. As would be expected, both models resulted in effects much smaller than in the GPZ model, leading to a predicted total uncertainty on the W mass from this source of $\sigma_{M_\mathrm{W}} < 40\ \mathrm{MeV}$. SK also performed a study of QCD interconnection effects in $\mathrm{t\bar{t}}$ production [18], but only in the context of $\mathrm{e^+e^-}$ collisions.

Subsequently, a number of alternative models have also been proposed, most notably the ones proposed by the Lund group, based on QCD dipoles [19–21], and one based on clusters by Webber [22]. Apart from WW physics, colour reconnections have also been proposed to model rapidity gaps [23–25] and quarkonium production [26].

Returning to $\mathrm{e^+e^-}$, experimental investigations at LEP II have not found conclusive evidence of the effect [27, 28], but were limited to excluding only the more dramatic scenarios, such as GPZ and versions of SK-I with the recoupling strength parameter close to unity. Hence, while colour reconnection effects cannot be arbitrarily large, there is room for further speculation. In addition, as we shall argue below, it may be possible that the effect is enhanced in hadron collisions over $\mathrm{e^+e^-}$ — with the added complication that the environment at hadron colliders is necessarily much less benign to this sort of measurement than was the case at LEP.

3. Our Toy Model — Colour Annealing

In electron–positron annihilation, the two incoming states carry electromagnetic charge — giving rise to a dilute cloud of virtual photons surrounding them — but no strong charge. From the QCD point of view, the vacuum state is thus undisturbed in the initial state, at least up to effects of order α^2, i.e. $\mathrm{e} \to \mathrm{e}'\gamma^* \to \mathrm{e}'\mathrm{q}\bar{\mathrm{q}}$. After the production of, say, a WW pair, e.g. with both W bosons decaying hadronically, $\mathrm{e^+e^-} \to \mathrm{W^+W^-} \to (\mathrm{q}_1\bar{\mathrm{q}}_2)(\mathrm{q}_3\bar{\mathrm{q}}_4)$, further QCD radiation and hadronisation then develops, in the background of this essentially pure vacuum state. As discussed above, the final state colour topology during the perturbative part of the QCD cascade, at least down to energies of order the W width, in all likelihood is the one depicted in Fig. 1a. For gluon energies smaller than the W width, however, the question is still relatively open.

Going to (inelastic, non-diffractive) hadron-hadron collisions, the initial state already contains strong charges. Using a simple bag model for illustration, the vacuum at the collision point and in the space-time area immediately surrounding it would not be the undisturbed one above, but would rather correspond to the vacuum *inside* the hadronic bag. Though detailed modeling is beyond the scope of the present discussion, we note that soft colour fields living inside this bag, with wavelengths of order the hadron size $\sim$ hadronisation length, could impact in a non-trivial way the formation of colour strings at the time of hadronisation [23, 24], effects that would not have been present in $\mathrm{e^+e^-}$ collisions.

We are not aware of any detailed studies, neither experimental nor theoretical at this time. Several of the models mentioned above would still be more or less directly applicable, but the noisier environment of hadron colliders makes it daunting to attempt to look for any effect. In this paper, we propose a simple toy model, to give a first indication of the possible size of the effect, in particular for $\mathrm{t}\bar{\mathrm{t}}$ production at the Tevatron.

Since we do not expect the difference in background vacuum to affect the short-distance physics, we take the arguments of SK concerning the absence of colour reconnections at the perturbative level to still be valid. Though one could still imagine reconnections below the relevant resonance widths, we shall not consider this. That is, we let the entire perturbative evolution remain unchanged, and implement our model at the hadronisation level only. Having no explicit model for how the presence of soft background fields would affect the collapse of the colour wave functions at hadronisation time, we consider an extreme case, where the quarks and gluons completely forget their colour 'history'. Instead, what determines between which partons hadronising strings form is a minimization of the total potential energy stored in strings. Specifically, we propose that the partons, regardless of their formation history, will tend to be colour connected to the partons closest to them in momentum space, hence minimizing the string length and thereby the average particle multiplicity produced by the configuration, as measured by the so-called 'Lambda measure' [7, 29], here given for massless partons for simplicity:

$$\Lambda = \Pi_{i=1}^{N} \frac{m_{\mathrm{i}}^2}{M_0^2} \quad , \tag{1}$$

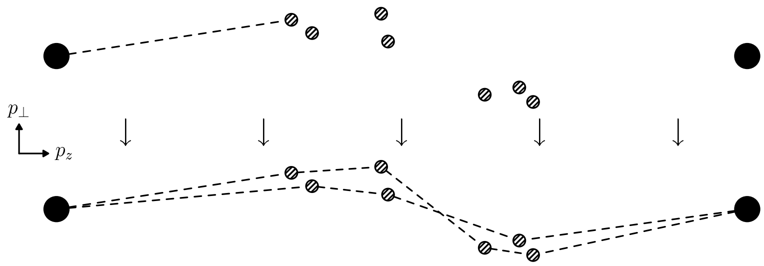

Fig. 2: Type I colour annealing in a schematic $\mathrm{gg} \to \mathrm{gg}$ scattering. Black dots: beam remnants. Smaller dots: gluons emitted in the perturbative cascade. All objects here are colour octets, hence each dot must be connected to two string pieces. Upper: the first connection made. Lower: the final string topology.

where i runs over the number of colour-anticolour pairs (dipoles) in the event, N, m_{i} is the invariant mass of the i'th dipole, and M_0 is a constant normalisation factor of order the hadronisation scale. The average multiplicity produced by string fragmentation is proportional to the logarithm of Λ. Technically, the model implementation starts by erasing the colour connections of all final state coloured partons, including ones from W decays etc. It then begins an iterative procedure (which unfortunately can be quite time-consuming):

1. Loop over all final state coloured partons.
2. For each such parton with a still unconnected colour or anticolour charge,
   (a) Compute the Λ measure for each possible string connection from that parton to other final state partons which have a compatible free colour charge.
   (b) Store the connection with the smallest Λ measure for later comparison.
3. Compare all the possible 'minimal string pieces' found, one for each parton. Select the largest of these to be carried out physically. That parton is in some sense the one that is currently furthest away from all other partons.
4. If any 'dangling colour charges' are left, repeat from 1.
5. At the end of the iteration, if the last parton is a gluon, and if all other partons already form a complete colour singlet system, the remaining gluon is simply attached between the two partons where its presence will increase the total Λ measure the least.

This procedure will find a local minimum of the Λ measure. More aggressive models could still be constructed, most noticeably by refining the algorithm to avoid being trapped in shallow local minima. As a side remark, we note that the above procedure, which we shall refer to as Type II below, as it stands would tend to result in a number of small closed gluon loops. Hence, we also consider a variant (Type I) where closed gluon loops are suppressed, if other possibilities exist, see illustration in Fig. 2. Both variants of the annealing algorithm are implemented in PYTHIA 6.326, and are carried over to PYTHIA 6.4, where they can be accessed using the `MSTP(95)` switch, see also the update notes [30] and the PYTHIA 6.4 manual [31].

4. Results

As a first application of the new models, we consider their effects on semileptonic $\mathrm{t\bar{t}}$ events at the Tevatron. Specifically, whether an effect could be observable in the light-quark jet system from the hadronic W decay. This is closely related to the work presented in [32].

For any fragmentation model, the first step is to make a (re)tune of the minimum-bias and underlying-event (UE) parameters. Ideally, the whole range of model parameters should come

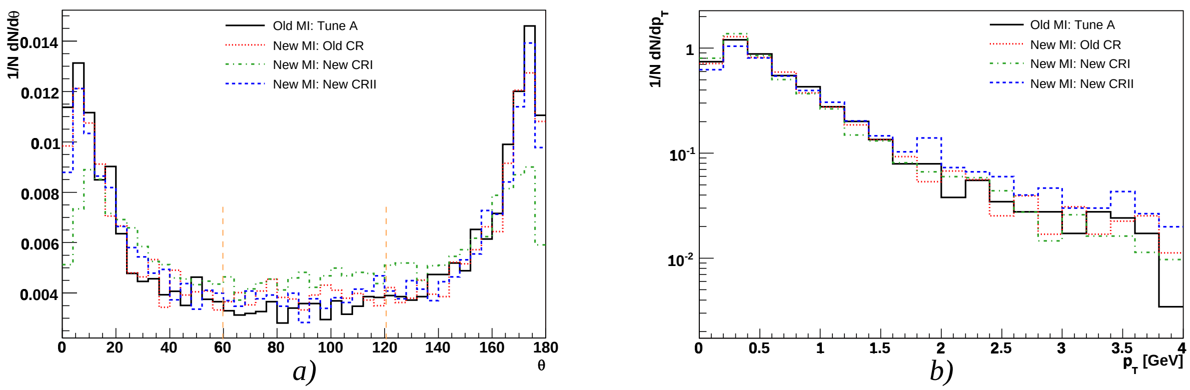

Fig. 3: Semi-leptonic top events at the Tevatron (see text). *a)* Charged particle density between the W jets (note the zero suppression) and *b)* $p_\perp$ spectra for charged particles in the region $60^\circ < \theta < 120^\circ$.

under scrutiny, however for the present study we limit ourselves to a one-parameter retuning of the multiple interactions colour-screening cutoff in PYTHIA (`PARP(82)`), requiring the re-tuned models to agree with the average charged particle multiplicity of Tune A [3]. Below, we compare Tune A to a preliminary tune of the new UE framework (Old CR) adapted from the Low FSR tune in [11], and to the same model with Type I and Type II colour reconnections applied. For the 4 models, `PARP(82)=2.0`, `2.1`, `2.2`, `1.55`, respectively.

Next, for each of the tuned models, 50000 $\mathrm{t\bar{t}}$ events were generated at $E_{CM} = 1960$ GeV, corresponding to approximately $8\mathrm{fb}^{-1}$ of integrated luminosity. Out of the semi-leptonic fraction of this sample, events with exactly four charged particle jets were selected (clustered with an exclusive kT jet algorithm [33] with $d_{\mathrm{cut}} = 150\ \mathrm{GeV}^2$). Finally, the jets have to be uniquely identified to the correct parton. This was done requiring that the (and only the) dedicated jet has a minimal ΔR between its axis and the initial parton.

In the undisturbed colour topology, three string pieces are relevant; one spanned between the W jets, one between the b quark and the p beam remnant, and one between the $\mathrm{\bar{b}}$ and the $\mathrm{\bar{p}}$ remnant. To maximise the overlap of these strings, and hence create a bias towards situations where colour reconnections should be enhanced, we reject events that do not fulfill either condition A) $\eta_{\mathrm{q}} > \eta_{\mathrm{\bar{q}}} > \eta_{\mathrm{b}}$ or B) $\eta_{\mathrm{\bar{b}}} > \eta_{\mathrm{q}} > \eta_{\mathrm{\bar{q}}}$.

For each accepted event, we perform a boost to the rest frame of the hadronic W, then a polar rotation to line up the decay jets along the z axis (for condition A (B), the quark is rotated to 0° (180°)), and finally an azimuthal rotation to bring the b jet from the associated top decay into the (x, z) plane, in the positive-x hemisphere. We then reject events where the other b jet is not also in the positive-x hemisphere, so that the negative-x hemisphere between the W jets should, at least to some extent, be free from extraneous hadronic activity.

We consider two observables, in both cases only including particles in the negative-x hemisphere. First, in Fig. 3a, the charged particle multiplicity between the jets, $1/N_{\mathrm{ch}}\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\theta$, and second, in Fig. 3b, the transverse momentum distribution $1/N_{\mathrm{ch}}\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}p_\perp$ for particles in the inter-jet region, $60^\circ < \theta < 120^\circ$, indicated in Fig. 3a by dashed vertical markers.

In Fig. 3a, the asymmetry between the left and right peak sizes is due to the rapidity constraints and to the way we performed the rotations; conditions A and B then both force the associated b quark to be closer to the right-hand jet. Given the subtle nature of the effect, and the noisy hadronic environment, the variations in Fig. 3 are quite large (the distortion of the peak shape at small angles for Type I is, however, probably too large to be realistic). However, notice that the reconnected scenarios do *not* lead to a significant reduced charged particle density in the inter-jet region, which would have been the effect we should naively have been looking for, by comparison to the $\mathrm{e^+e^-}$ studies. We note, however, that the most agressive of the new models, Type II (blue dashed curve), does produce fewer particles in the fragmentation region than its sister Type I (green dot-dashed), and also (as shown in figure 3b) that the charged particles produced in Type II have a higher average $p_\perp$.

What is going on is that, as for so many aspects of hadron-hadron physics, the end result is not controlled by one effect alone, but by a combination of factors. Multiplicity will be increased by allowing more underlying-event activity and will be decreased by allowing more colour reconnections. Hence the same multiplicity can be arrived at through different mixes of these. By first tuning to the min-bias data we are to some extent cancelling these effects against each other. This illustrates an essential point: in a hadron-hadron environment, the multiplicity alone may not be a discriminating variable. However, the mixes are not completely equivalent. While they may lead to the same result in one distribution, they will differ for another. Specifically, by combining the particle flow with the energy flow, some discriminating power can be gained. One way of realising this is to consider that the underlying activity is pumping energy into the event. To maintain the same multiplicity distribution, the particle hardness must then be a function of the underlying activity, as is illustrated by Fig. 3. While we shall terminate our discussion here, the subject of disentangling these effects certainly merits further consideration.

5. Conclusions

We have presented a few simple toy models of colour reconnections, based on an annealing-like algorithm. These models are quite general and are directly applicable to any process, unlike many previous models for which only implementations specific to WW events exist.

As a first application, we have studied the effects on two simple observables in semileptonic $\mathrm{t\bar{t}}$ events at the Tevatron. We find that, while we cannot discern the presence or absence of a classical string effect in the multiplicity distributions alone, it may still be possible to distinguish between different models by including energy-flow information. The natural next step would be to consider the extent to which measurements of the top mass at the Tevatron and LHC are influenced by these effects. For instance, an attractive possibility is to use the hadronically reconstructed W mass in these events to set the jet energy scale, hence the degree to which the hadronic W mass reconstruction is affected by the effects discussed here would be interesting to examine.

We intend this study mostly for illustration and for communicating a few essential remarks. As such, we have freely (ab)used Monte Carlo truth information and have skipped lightly over a number of aspects, which would have to be more carefully addressed in a real analysis. We hope that this work may nevertheless serve to stimulate further efforts in this exciting and presently little understood field.

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References

[1] t. L. E. W. Group,, **OPAL** Collaboration hep-ex/0511027.

[2] V. M. Abazov *et. al.*,, **CDF** Collaboration *Phys. Rev.* **D70** (2004) 092008, [hep-ex/0311039].

[3] R. D. Field, hep-ph/0201192 CDF Note 6403; further recent talks available from webpage http://www.phys.ufl.edu/∼rfield/cdf/.

[4] T. Affolder *et. al.*,, **CDF** Collaboration *Phys. Rev.* **D65** (2002) 092002.

[5] C. M. Buttar, D. Clements, I. Dawson, and A. Moraes, *Acta Phys. Polon.* **B35** (2004) 433–441.

[6] R. Field and R. C. Group, hep-ph/0510198.

[7] T. Sjöstrand and P. Z. Skands, *Nucl. Phys.* **B659** (2003) 243, [hep-ph/0212264].

[8] T. Sjöstrand and P. Z. Skands, *JHEP* **03** (2004) 053, [hep-ph/0402078].

[9] T. Sjöstrand and M. van Zijl, *Phys. Rev.* **D36** (1987) 2019.

[10] J. M. Butterworth, J. R. Forshaw, and M. H. Seymour, *Z. Phys.* **C72** (1996) 637–646, [hep-ph/9601371].

[11] T. Sjöstrand and P. Z. Skands, *Eur. Phys. J.* **C39** (2005) 129–154, [hep-ph/0408302].

[12] G. Gustafson, U. Pettersson, and P. M. Zerwas, *Phys. Lett.* **B209** (1988) 90.

[13] T. Sjöstrand and V. A. Khoze, *Phys. Rev. Lett.* **72** (1994) 28–31, [hep-ph/9310276].

[14] T. Sjöstrand and V. A. Khoze, *Z. Phys.* **C62** (1994) 281–310, [hep-ph/9310242].

[15] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, *Phys. Rept.* **97** (1983) 31.

[16] M. G. Jackson, N. T. Jones, and J. Polchinski, *JHEP* **10** (2005) 013, [hep-th/0405229].

[17] A. L. Cotrone, L. Martucci, and W. Troost, hep-th/0511045.

[18] V. A. Khoze and T. Sjöstrand, *Phys. Lett.* **B328** (1994) 466–476, [hep-ph/9403394].

[19] G. Gustafson and J. Hakkinen, *Z. Phys.* **C64** (1994) 659–664.

[20] L. Lonnblad, *Z. Phys.* **C70** (1996) 107–114.

[21] C. Friberg, G. Gustafson, and J. Hakkinen, *Nucl. Phys.* **B490** (1997) 289–305, [hep-ph/9604347].

[22] B. R. Webber, *J. Phys.* **G24** (1998) 287–296, [hep-ph/9708463].

[23] W. Buchmuller and A. Hebecker, *Phys. Lett.* **B355** (1995) 573–578, [hep-ph/9504374].

[24] A. Edin, G. Ingelman, and J. Rathsman, *Phys. Lett.* **B366** (1996) 371–378, [hep-ph/9508386].

[25] R. Enberg, G. Ingelman, and N. Timneanu, *Phys. Rev.* **D64** (2001) 114015, [hep-ph/0106246].

[26] A. Edin, G. Ingelman, and J. Rathsman, *Phys. Rev.* **D56** (1997) 7317–7320, [hep-ph/9705311].

[27] G. Abbiendi *et. al.*,, **OPAL** Collaboration *Phys. Lett.* **B453** (1999) 153–168, [hep-ex/9901019].

[28] G. Abbiendi *et. al.*,, **OPAL** Collaboration hep-ex/0508062.

[29] B. Andersson, G. Gustafson, and B. Söderberg, *Z. Phys.* **C20** (1983) 317.

[30] T. Sjöstrand *et. al.*, PYTHIA update notes, see http://www.thep.lu.se/∼torbjorn/pythia/pythia6326.update.

[31] T. Sjostrand, S. Mrenna, and P. Skands, hep-ph/0603175.

[32] M. Sandhoff,. Diploma Thesis, Bergische Universität Wuppertal, WU D 05-08.

[33] J. M. Butterworth, J. P. Couchman, B. E. Cox, and B. M. Waugh,, *KtJet: A C++ implementation of the* $K_\perp$ *clustering algorithm*. October, 2002. MAN/HEP/2002/02, UCL/HEP 2002-02.