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**PARTICLE IDENTIFICATION BY PROCESSING OF
SIGNALS FROM MULTIPLE DETECTORS***

Emilio Gatti[†], Veljko Radeka, and Martin Rosenblum

Brookhaven National Laboratory
Upton, New York 11973

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PARTICLE IDENTIFICATION BY PROCESSING OF SIGNALS FROM MULTIPLE DETECTORS*

Emilio Gatti[†], Veljko Radeka, Martin Rosenblum
Instrumentation Division, Brookhaven National Lab.

Some interesting instrumentation problems concerning particle recognition have arisen in connection with the development of new detectors especially transition radiation detectors. The use of likelihood estimation has been suggested by Ellsworth et al.¹. The majority logic technique has been proposed by Atac and Wang². Analog computation of the geometric mean of the signal amplitudes from several detectors has been used by Uto et al.³.

This paper attacks the problem quite generally in order to compare different methods and to provide suggestions for more convenient instrumentation for different cases. Treatment is not confined to transition radiation problems but can be applied to identification problems for which not only the amplitude of a radiation detector pulse but also its pulse shape is observed.

Some of the ideas discussed in this paper have been published recently⁴.

Let us suppose that a set of radiation detectors gives an output described by a vector $\vec{v}$ when a particle is detected. The vector $\vec{v}$ might have scalar components v_k , where v_k is the pulse amplitude of the k th detector, or it can be more generally formed by several vectors $\vec{v}_k$ each representing a waveform at the output of the k th detector.

Our aim is to find the best operator $\vec{A}$ for processing the output $\vec{v}$ in order to obtain a discrimination parameter S suitable for estimating according to its amplitude which of two different kinds of particles F and G might have produced the signal $\vec{v}$.

Let us process $\vec{v}$ linearly so that S will be given by

$$S = \vec{A} \cdot \vec{v} \quad (1)$$

as shown in fig. 1 a.

The average signal $\bar{S}$ given by particle F will be

$$\langle S_F \rangle = \int (\vec{A} \cdot \vec{v}) D(\vec{v}|F) d\vec{v} \quad (2)$$

where $D(\vec{v}|F)$ is the conditional probability density of an output $\vec{v}$ when the set of counters detects a particle F . The integral spans over the whole space in which $\vec{v}$ is defined. For $D(\vec{v}|F)$ holds:

$$\int D(\vec{v}|F) d\vec{v} = 1. \quad (3)$$

Similarly

$$\langle S_G \rangle = \int (\vec{A} \cdot \vec{v}) D(\vec{v}|G) d\vec{v} \quad (4)$$

being $D(\vec{v}|G)$ the normalized conditional probability density referred to particle G .

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We look for an operator $\vec{A}$ which produces the largest separation of the average discrimination signals S_F and S_G with respect to the sum of variances of the signals $S = S_F$ and $S = S_G$ due respectively to F and G only. $\vec{A}$ should therefore maximize the following expression:

$$R_S^2 = \frac{[\langle S_F \rangle - \langle S_G \rangle]^2}{\int (\vec{A} \cdot \vec{v})^2 D(\vec{v}|F) d\vec{v} + \int (\vec{A} \cdot \vec{v})^2 D(\vec{v}|G) d\vec{v}} \quad (5)$$

By standard variational analysis it is easily shown that R_S^2 is maximized by $\vec{A}$ chosen so that

$$S = \vec{A} \cdot \vec{v} = \frac{D(\vec{v}|F) - D(\vec{v}|G)}{D(\vec{v}|F) + D(\vec{v}|G)} \quad (6)$$

this expression does not give an explicit value for $\vec{A}$ but the result of its operation on $\vec{v}$ (fig. 1 b). By the choice of $\vec{A}$ suggested by (6), the figure of merit R_S^2 given by (5) becomes:

$$R_S^2 = \frac{2 \int \frac{[D(\vec{v}|F) - D(\vec{v}|G)]^2}{D(\vec{v}|F) + D(\vec{v}|G)} d\vec{v}}{2 \int \frac{[D(\vec{v}|F) - D(\vec{v}|G)]^2}{D(\vec{v}|F) + D(\vec{v}|G)} d\vec{v}} \quad (7)$$

The instrument should code the output $\vec{v}$ of the detectors in order to produce a signal S given by (6).

Recalling the Bayes relationships:

$$\begin{aligned} D(\vec{v}|F) d\vec{v} P(F) &= P(F|\vec{v}) D(\vec{v}) d\vec{v} \\ D(\vec{v}|G) d\vec{v} P(G) &= P(G|\vec{v}) D(\vec{v}) d\vec{v} \end{aligned} \quad (8)$$

(where $P(F)$ and $P(G)$ are the a priori probabilities for a particle to be respectively F or G , $P(F|\vec{v})$ and $P(G|\vec{v})$ are the conditional probabilities for a particle to be respectively F or G , when $\vec{v}$ is produced by the detectors and $D(\vec{v})$ is the probability density for an output $\vec{v}$) remembering also the fact that a particle must be either F or G , so that

$$P(F) + P(G) = 1 \quad P(F|\vec{v}) + P(G|\vec{v}) = 1, \quad (9)$$

we can write S , as given by (6), as:

$$S = \frac{P(F|\vec{v}) - P(F)}{P(F|\vec{v}) (1 - 2P(F)) + P(F)} \quad (10)$$

and solving for $P(F|\vec{v})$:

$$P(F|\vec{v}) = P(F) \frac{1+S}{1-S(1-2P(F))}. \quad (11)$$

This expression shows that the probability with which a particle is recognized as F is a single valued monotonic function of S, so that to an interval ΔS , selected by a single channel discriminator, correspond particles identified with the same degree of uncertainty. A threshold discriminator selects all the particles identified with an uncertainty lower than a given value.

A parameter like S is optimum in that it allows the separation of particles identified with the same degree of uncertainty. However every other signal, having this property of S, is theoretically equivalent and optimum. Any monotonic function of $P(F|\bar{v})$, and consequently of S, belongs to this class.

We could choose for instance $Q=P(F|\bar{v})$ itself which taking into account (11) and (6) can be written:

$$Q=P(F|\bar{v}) = \frac{\frac{P(F)}{1-P(F)} \frac{D(\bar{v}|F)}{D(\bar{v}|G)}}{\frac{P(F)}{1-P(F)} \frac{D(\bar{v}|F)}{D(\bar{v}|G)} + 1} \quad (12)$$

or we can choose

$$T = \ln \frac{D(\bar{v}|F)}{D(\bar{v}|G)} \quad (13)$$

which as apparent from (12) is a monotonic function of Q. This parameter T is the easiest for physical implementation (fig. 1 c) when the response of the detectors can be thought as independent and therefore

$$\frac{D(\bar{v}|F)}{D(\bar{v}|G)} = \prod_k \frac{D(v_k|F)}{D(v_k|G)} \quad (14)$$

where v_k is the amplitude of the pulse given by the kth detector. If this amplitude is coded as:

$$T_k = \ln \frac{D(v_k|F)}{D(v_k|G)} \quad (15a)$$

(we note that $D(v_k|F)$ and $D(v_k|G)$ are experimentally known*), T becomes the sum of the T_k s (fig. 1 d)

$$T = \ln \prod_k \frac{D(v_k|F)}{D(v_k|G)} = \sum_{k=1}^n T_k \quad (15b)$$

$P(F|\bar{v})$ becomes as a function of T:

*More precisely $p(v_k|F)$ and $p(v_k|G)$ are approximately known generally by means of experiments in which the spectra $P(v_k|F)$ and $P(v_k|G)$ are recorded by means of multichannel analyzers, being:

$$P(v_k^i|F) = \int_{v_k^i - \Delta v_k/2}^{v_k^i + \Delta v_k/2} p(v_k|F) dv_k \quad (17)$$

(and similarly for $P(v_k^i|G)$) where v_k^i represents the mean position of channel i and Δv_k the width of the channels.

$$P(F|\bar{v}) = \frac{e^T}{\frac{1-P(F)}{P(F)} + e^T} \quad (16)$$

When considering discrete values v_k^i eq. (15a) becomes:

$$T_k^i = \ln \frac{P(v_k^i|F)}{P(v_k^i|G)} \quad (15c)$$

We note that $P(v_k^i|F) = P(T_k^i|F)$ that is, for corresponding values v_k^i and T_k^i of the discrete variables, the conditional probabilities are obviously the same.

Therefore the spectrum $P(T_k^i|F)$ of the variable T_k^i , when the kth detector is traversed by a F particle, is known, and by (15b), noting that the T_k are assumed independent stochastic variables, the spectrum of $P(T|F)$ is given by the n-fold convolution of $P(T_k^i|F)$:

$$P(T|F) = P(T_k^1|F) * P(T_k^2|F) * \dots \quad (18)$$

It is convenient to consider the generating function associated to $P(T_k^i|F)$:

$$p_k(s|F) = \sum_{i=1}^n P(T_k^i|F) s^{T_k^i} \quad (19)$$

and the corresponding generating function $p(s|F)$ associated with $P(T|F)$.

Assuming equivalence of all the counters, the convolution (18) becomes for the generating functions a product of n identical factors:

$$p(s|F) = (p_k(s|F))^n \quad (20)$$

and similarly for the G particles

$$p(s|G) = (p_k(s|G))^n \quad (21)$$

By the well known properties of generating functions it is easily shown that the average $\langle T_F \rangle$ of the F particle spectrum is related to that of $\langle T_{kF} \rangle$ by

$$\langle T_F \rangle = \sum_i T_k^i P(T_k^i|F) = \sum_i T_k^i P(T_k^i|F) = n \langle T_{kF} \rangle \quad (22)$$

and similarly

$$\langle T_G \rangle = \sum_i T_k^i P(T_k^i|G) = \sum_i T_k^i P(T_k^i|G) = n \langle T_{kG} \rangle \quad (23)$$

The variances are given for the F and G spectral peaks respectively by:

$$\begin{aligned} \varepsilon_{TF}^2 &= \sum_i P(T_k^i|F) (T_k^i - \langle T_F \rangle)^2 = \\ &= n \sum_i P(T_k^i|F) (T_k^i - \langle T_{kF} \rangle)^2 = n \varepsilon_{TkF}^2 \\ \varepsilon_{TG}^2 &= n \varepsilon_{TkG}^2 \end{aligned} \quad (24)$$

A figure of merit similar to R_S^2 defined by (5) but related to parameter T , is:

$$R_T^2 = \frac{(\langle T_F \rangle - \langle T_G \rangle)^2}{\epsilon_{TF}^2 + \epsilon_{TG}^2} = n R_{Tk}^2 \quad (25)$$

where R_{Tk} is the figure of merit of the single detector given by

$$R_{Tk}^2 = \frac{(\langle T_{kF} \rangle - \langle T_{kG} \rangle)^2}{\epsilon_{kF}^2 + \epsilon_{kG}^2} \quad (26)$$

Eq. (24) allows estimating of the improvement in discrimination as a function of the number of detectors.

If we select, with the output discriminator, particles for which $T > T_0$, the yield for F particles counting will be

$$Y_F(T_0) = \sum_i P(T_i | F) \quad \text{for all } T_i \geq T_0 \quad (27)$$

and the contamination yield of G particles will be

$$C_G(T_0) = \sum_i P(T_i | G) \quad \text{for all } T_i \geq T_0 \quad (28)$$

If the a priori probabilities of F and G particles are respectively $P(F)$ and $P(G)$, the percentage contamination, when selecting F particles, $E(T_0)$ will be

$$E(T_0) = \frac{P(G) C_G(T_0)}{P(F) Y_F(T_0) + P(G) C_G(T_0)} = \frac{1}{\frac{P(F)}{P(G)} \frac{Y_F(T_0)}{C_G(T_0)}} \quad (29)$$

The percentage contamination is so tied to the function $Y_F(T_0)/C_G(T_0)$ which, once $Y_F(T_0)$ is known, can be considered, by eliminating T_0 , a function of Y_F :

$$\frac{Y_F}{C_G} = h(Y_F) \quad (30)$$

The contamination E can be finally expressed as a function of the yield Y_F of the particles of interest and their a priori probability P_F as follows:

$$E = \frac{1}{1 + h(Y_F) \frac{P(F)}{1 - P(F)}} \quad (31)$$

$P(T|F)$ and $P(T|G)$ can be calculated from the generating functions (19) and (20). For instance:

$$P(T|F) = P(m_1 T_k^1 + m_2 T_k^2 + \dots m_l T_k^l | F) =$$

$$= \frac{n!}{m_1! m_2! \dots m_l!} (P(T_k^1 | F))^{m_1} (P(T_k^2 | F))^{m_2} \dots (P(T_k^l | F))^{m_l}$$

where l is the number of assumed different values of the discrete variable T_k , n is the number of detectors, m_i is the number of detectors out of n which have given the same coded output T_k^i . The number of different values for T arising from all possible sets of m_i such as $1 \leq m_i \leq n$ and $\sum_{i=1}^l m_i = n$, is $(n+1)^l - 1$.

In order to reduce the complexity of the coding instrumentation it is worthwhile to evaluate how much resolving power is lost by analyzing each detector (fig. 2) by a single threshold set at v_{k0} so that each detector gives only two discrete levels with the following probabilities (when detecting a particle F):

$$P(v_k < v_{k0} | F) = \int_0^{v_{k0}} P(v_k | F) dv_k \quad (32)$$

$$P(v_k > v_{k0} | F) = 1 - \int_0^{v_{k0}} P(v_k | F) dv_k \quad (33)$$

For this evaluation we shall refer to the figure of merit R_{Sk}^2 as given by (7) but referred to a single detector:

$$R_{Sk2}^2 = \frac{2M_2}{2 - M_2} \quad (34)$$

where M_2 is given by

$$M_2 = \frac{P(v_k < v_{k0} | F) - P(v_k < v_{k0} | G)}{P(v_k < v_{k0} | F) + P(v_k < v_{k0} | G)} + \frac{P(v_k > v_{k0} | F) - P(v_k > v_{k0} | G)}{P(v_k > v_{k0} | F) + P(v_k > v_{k0} | G)} \quad (35)$$

We can minimize R_{Sk2} by maximizing M_2 choosing the most convenient threshold v_{k0} . Remembering the expressions (32) and (33), $dM_2/dv_{k0} = 0$ corresponds to the v_k which satisfies the following expression:

$$\frac{P(v_k | F) - P(v_k | G)}{P(v_k | F) + P(v_k | G)} = \frac{\int_0^{v_k} (P(v_k | F) - P(v_k | G)) dv_k (1 - \int_0^{v_k} (P(v_k | F) + P(v_k | G)) dv_k)}{\int_0^{v_k} (P(v_k | F) + P(v_k | G)) dv_k (2 - \int_0^{v_k} (P(v_k | F) + P(v_k | G)) dv_k)} \quad (36)$$

The so chosen optimum v_{ko} allows to calculate M_2 and by (41) the figure of merit R_{Sk}^2 to be compared with R_{Sk}^2 as given by eq. (7) written for a single detector.

When this binary coding is used, the instrument becomes identical to the one based on a majority logic selection: in fact the choice of the output trigger threshold T_{bo} in fig. 2 means that, to accept a particle as an F particle, a given number m of detectors, out of n , have triggered their own trigger circuit set at v_{ko} .

A non optimum coding which has been used³ consists in taking the geometrical mean of the detector outputs as a discriminating parameter: this is equivalent to coding each detector to give a signal proportional to the logarithm of its original output and to sum the coded outputs of the n detectors.

Introducing the variable $L_k = \ln v_k$ we can consider the discrimination parameter $L = \frac{1}{n} \sum L_k$ and through the same steps used dealing the discrimination parameter T , we can define corresponding variables, parameters, and figures of merit of those defined by expressions from (20) to (29).

Approximate calculations

When the evaluation of the function $h(Y_F)$ through the calculation of $Y_F(T_0)$ and $C_G(T_0)$ by mean of the $P(T|F)$ and $P(T|G)$ is considered too involved, a rough estimate of the resolving power of a given system can be made by approximating $P(T|F)$ and $P(T|G)$ by gaussian distributions with mean and variances given by (23) and (24). Obviously the same holds for $P(L|F)$ and $P(L|G)$. Y_F and C_G are then easily estimated as follows:

$$Y_F(T_0) = \int_{T_0}^{\infty} \frac{1}{\sqrt{2\pi} \epsilon_F} e^{-\frac{(T - \langle T_F \rangle)^2}{2 \epsilon_F^2}} dT = \frac{1}{2} - \Phi\left(\frac{T_0 - \langle T_F \rangle}{\epsilon_F}\right) \quad (37)$$

$$C_G(T_0) = \int_{T_0}^{\infty} \frac{1}{\sqrt{2\pi} \epsilon_G} e^{-\frac{(T - \langle T_G \rangle)^2}{2 \epsilon_G^2}} dT = \frac{1}{2} - \Phi\left(\frac{T_0 - \langle T_G \rangle}{\epsilon_G}\right) \quad (38)$$

$$\frac{Y_F(T_0)}{C_G(T_0)} = \frac{\frac{1}{2} - \Phi\left(\frac{T_0 - \langle T_F \rangle}{\epsilon_F}\right)}{\frac{1}{2} - \Phi\left(\frac{T_0 - \langle T_G \rangle}{\epsilon_G}\right)} \quad (39)$$

This latter introducing explicitly the number of detectors n becomes:

$$\frac{Y_F(T_0)}{C_G(T_0)} = \frac{\frac{1}{2} - \Phi\left(\frac{T_0 - n \langle T_{kF} \rangle}{\sqrt{n} \epsilon_{kF}}\right)}{\frac{1}{2} - \Phi\left(\frac{T_0 - n \langle T_{kG} \rangle}{\sqrt{n} \epsilon_{kG}}\right)} \quad (40)$$

The gaussian approximation should become increasingly better as the number n of the detectors is increased because the final distribution of the parameter T becomes that of the sum of an increasing number of independent random variables T_k ; however as, when using a large number of detectors, the interest is focused in high resolving power and in high yields, that is in high Y_F/C_G ratios and values of Y_F near to 1, it happens that the function $Y_F/C_G = h(Y_F)$ has to be considered in a range for which the shape of the "tails" of $P(T|G)$ and $P(T|F)$, now approximated by gaussian distributions, is dominant: this explains the increasing discrepancies of the approximating curves for large Y_F/C_G values or for $Y_F \rightarrow 1$, as it appears from fig. 3(d).

Following what has been discussed, comparison has been made among different methods of discrimination by simulating with the computer experiments of detection of 10^4 electrons (F) or 10^4 pions (G) by a set of n identical detectors ($n=4, 8, 16, 32$) having response to electrons and pions defined by the data of Table 1 corresponding to the histogram of fig. 3(a) taken from the experimental curves of paper⁵.

Results, under the form of the function (30)

$Y_F/C_G = h(Y_F)$, are given for

- 1) discrimination according to parameter T , in fig. 3(b) (quantization in 10 channels: Table 1);
- 2) discrimination according to a signal obtained by taking the geometrical mean of the signals v_k from the detectors (which corresponds to coding the detectors output according $L_k = \ln v_k$ and to consider as discriminating variable L equal to the sum of the coded outputs) in Table 1 and fig. 3(c) and 3(d);
- 3) discrimination according parameter T in fig. 3(c) and 3(d) (quantization in 3 channels grouped as in table 2);
- 3) discrimination according parameter T in fig. 3(c) and 3(d) (quantization in 2 channels grouped as in Table 3; v_{ko} as given by eq. (36) turns out to be 4, 4).

For comparison gaussian estimates are also given in fig. 3(c) and 3(d). Curves and simulated points for case 1) state the theoretical limits of resolving power.

Similar results are given in fig. 4(b) for a set of $n (=1, 2, 4, 8)$ identical transition radiation detectors having much larger resolving power: the probability distribution for electrons and pions is shown in fig. 4(a) taken from references⁶ and 7. From the results it may be noted that differences in discrimination methods become important only for large numbers of counters and/or for high values of Y_F/C_G . The discrimination based on the geometrical mean signal is only slightly better (as it also appears from the figures of merit), for the numerical case chosen, to the binary (two channel) codification of the detectors, when the discrimination threshold v_{ko} of the associated trigger circuits is suitably chosen.

The implementation of this binary codification seems quite simple and convenient.

Conclusion

It has been shown that for particle identification it is convenient to develop instrumentation based on coding the output of the detectors according the ratio of probabilities that a particular signal is given by the two kinds of particles considered.

Particular emphasis has been given to uncorrelated responses of each detector of a set, but the case of correlated responses leads to obvious extensions of the ideas presented.

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Table 1

v_k^i	1	2	3	4	5	6	7	8	9	10
$P(v_k^i F)=P(T_k^i F) =$ $= P(L_k^i F)$	.0652	.2409	.1954	.1104	.0822	.0793	.0764	.0624	.0510	.0368
$P(v_k^i G)=P(T_k^i G) =$ $= P(L_k^i G)$	.123	.460	.223	.0860	.0390	.0230	.0150	.0124	.0093	.0093
$T_k^i = \ln \frac{P(v_k^i F)}{P(v_k^i G)}$	-.6347	-.6468	-.1321	.2497	.7455	1.237	1.628	1.6158	1.7018	1.375
$L_k^i = \ln v_k^i$	0	0.6931	1.0986	1.3862	1.6094	1.7917	1.9459	2.0794	2.1972	2.3026

$$R_{T10}^2 = 0.2943 \quad R_{L10}^2 = 0.2520$$

Table 2

v_k^i	1 $\rightarrow$ (1+2)	2 $\rightarrow$ (3+4)	3 (5+6+7+8+9+10)
$P(v_k^i F)=P(T_k^i F)$	0.3061	0.3058	0.3881
$P(v_k^i G)=P(T_k^i G)$	0.5830	0.3090	0.1080
$T_k^i = \ln \frac{P(v_k^i F)}{P(v_k^i G)}$	-0.644	-0.0104	1.279

$$R_{T3}^2 = 0.2783$$

Table 3

v_k^i	1 $\rightarrow$ (1+2+3) 1 $\rightarrow$ (1+2+3+4)	2 $\rightarrow$ (4+5+6+7+8+9+10) 2 $\rightarrow$ (5+6+7+8+9+10)
$P(v_k^i F)=P(T_k^i F)$	0.5015 0.6119	0.4985 0.3881
$P(v_k^i G)=P(T_k^i G)$	0.806 0.8920	0.194 0.1080
$T_k^i = \ln \frac{P(v_k^i F)}{P(v_k^i G)}$	-0.4745 -0.3768	0.9437 1.279

$$R_{T2}^2 = 0.2282$$

$$R_{T2}^2 = 0.2350$$

Figure Captions

Fig. 1 a - Array of n detectors connected to the device $\bar{A}$ which will provide the recognition signal S .

Fig. 1 b - Optimum device $\bar{A}$ is specified by its output pulse S when fed by the input v_k .

Fig. 1 c - An equivalent optimum device specified by its output T when fed by the input v_k .

Fig. 1 d - Implementation of the device of fig. 1 c for the case of uncorrelated detector outputs. Each detector output is coded according to the logarithm of the likelihood ratio.

Fig. 2 - Implementation of a suboptimum device. Each detector output is coded deviding the output range in two channels. The coded circuit output of the k th detector is:

$$\ln \frac{\int_0^{v_{ko}} D(v_k|F) dv_k}{\int_0^{v_{ko}} D(v_k|G) dv_k} \quad \text{if } v_k < v_{ko}$$

$$\ln \frac{\int_0^{v_{ko}} D(v_k|F) dv_k}{\int_{v_{ko}}^{\infty} D(v_k|G) dv_k} \quad \text{if } v_k > v_{ko}$$

Fig. 3(a) - 10 channel histogram of the probability distribution for electrons and pions (particles F and G); data for mylar radiators, reference⁵.

(b) - Yield/contamination ratio Y_F/C_G versus yield Y_F for different number of detectors. R_{Tk10} is the figure of merit of a single detector.

(c) - Comparison of different discrimination methods for the case of $n=16$ detectors (sets of 1 each transition radiation foil stack and proportional chamber).

(d) - Comparison of different discrimination methods for the case of $n=32$ detectors for (c) and (d): $\times$ binary coding in groups of 3 and 7 channels; $\circ$ binary coding in groups of 4 and 6 channels; $\square$ coding into three levels in groups of 2, 2 and 6 channels.

Fig. 4 (a) - Histogram of the probability distribution for electrons and pions (curve 1 for e^- , curve 2 for π^-) for lithium radiators; data from references⁶ and⁷.

(b) - Yield/contamination ratio Y_F/C_G versus yield Y_F for different number of detectors. The results are shown for coding into 20 channels; $\times$ binary coding in groups of 3 and 17 channels; $\square$ coding into three levels in groups of 3, 2 and 15 channels. (F denotes electrons

and G pions), R_{Tk} are figures of merit of a single detector.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 1d

FIGURE 1

Neg. #4-900-75

FIGURE 2

Neg. #4-901-75

FIGURE 3(a)

Neg. #4-801-75

FIGURE 3 (b)

Neg. #4-803-75

FIGURE 3 (c)

Neg. #4-798-75

FIGURE 3 (d)

Neg. #4-802-75

FIGURE 4 (a)

FIGURE 4 (b)

NEG. #4-799-75