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Simulation of a D-T Burn Using D^{3}He in TCT

Part I:

Superthermal Ion and Fusion Product Pressure Considerations

by

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Abstract

Helium-3 is substituted for a tritium background in a two component system in order to minimize the neutronics problems. It is found that, under appropriate conditions, helium-3 can be expected to simulate rather well several aspects of injected deuterium and fusion product pressures that are anticipated for the proposed D-T experiments with the TCT.

I. Introduction

The use of a D-T fuel in the early stages of the TCT experiment[1] will undoubtedly require a massive effort directed toward the 14-MeV neutron problem. The expense in providing the necessary shielding, along with the related induced radioactivity and the problem of tritium handling, could retard other important scientific objectives (e.g. experiments to study the physics of fusion burns). Thus, in the initial stages of the TCT program, serious thought should be given to the possibility of running many of the physics experiments using a less troublesome fuel, such as D-^{3}He ($D + {}^3He \rightarrow p + \alpha$) to simulate D-T. The neutron problems from this approach are not nearly as serious as those encountered in the D-T design and tritium is virtually eliminated.

This report will deal with some preliminary studies of the feasibility of simulating the superthermal and fusion product pressures associated with a two-component D-T system* via injection of deuterium into a ^{3}He background. The study of these pressures, by simulation or otherwise, is considered important due to the possible effect on plasma equilibrium and stability such as, for instance, the incremental pressure anisotropy[2] introduced by preferential α-losses because of the unconfined α-banana orbits caused by the large $V_{\perp\alpha}$.

One advantage of using a D-^{3}He fuel for the simulation is that measurements of α-confinement and energy deposition are relatively easy to extrapolate to D-T systems since α-particles produced in a D-^{3}He reaction have

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*It should be noted that, unlike 50-50 Maxwellian D-^{3}He systems, two-component D-^{3}He plasmas will produce fewer neutrons through the D-D reaction, since the deuterium concentration which thermalizes is maintained at a low level compared with background ^{3}He concentration. As the deuterium and background ion (^{3}He or T) leak out of the plasma, the species are separated and circulated back into the plasma.

approximately the same energy as those from a D-T reaction. The importance in exploring these areas is, of course, clear when one considers that fusion particle confinement and heating characteristics in large part determine the efficiency in extracting useful energy from the plasma. However, these particular simulation problems will be treated in a subsequent report.

At present, ^3He is produced by the decay of tritium ($T \rightarrow {}^3He + e^- + \overline{\nu}$) at the Savannah River (Ga.) nuclear facility. Since the quantity of tritium is classified, one has no certain way of determining the precise inventory of ^3He on hand. Nevertheless, we may reasonably assume that sufficient amounts of ^3He for our limited purposes are in storage and may be obtained from the government for worthwhile projects.

II. Calculation*

The defining parameters for each of the cases to be considered are:

a) deuterium injection rate per cubic centimeter ($\dot{n}$);

b) deuterium injection energy E_{inj};

c) electron and ion background densities (average [$\overline{n}_e$, $\overline{n}_i$] and central [n_{eo}, n_{io}]);

d) electron and ion background temperatures (average [$\overline{T}_e$, $\overline{T}_i$] and central [T_{eo}, T_{io}]).

The principal assumptions in this analysis are:

a) neoclassical particle confinement;

b) one-half of the α-particles ($q_\alpha = 1/2$) and one-fifth of the protons ($q_p = 1/5$) are confined beyond the first bounce;

c) thermalization time much greater than randomization time for injected deuterium;

*A complete listing and explanation of the symbols to be employed in this paper are found in Appendix A.

d) density profiles $\sim 1 - (\frac{r}{a})^4$ and temperature profiles $\sim 1 - (\frac{r}{a})^2$;

e) injected electrons are assumed to thermalize instantaneously;

f) the slowing down of high energy ions follows Sivukhin's approach[3];

g) for convenience, $\dot{n}$ for both D-T and and D-^{3}He cases are equated.

With regard to assumption (b), there is no precise technique for determining what fraction of alphas (or protons) will be contained in the plasma for times longer than a bounce time ($\sim 10^{-6}$ sec). The fraction $q_\alpha = 1/2$ is, more or less, consistent with McAlees[4] approximations for given (TCT) aspect ratio and plasma current. Unfortunately, no work has been done on proton containment fractions (with $E_{p\alpha} = 14.7$ Mev). However, since the banana width of the proton at birth is roughly four times that of the α, one may reason that q_p ($\sim \frac{q_\alpha}{4}$) lies somewhere in the vicinity of 0.2.

With respect to assumption (f), the peak densities and temperatures are used in the deuterium slowing-down and F-value calculations, as the deuterium injection will more likely occur in the denser (central) regions of the plasma. On the other hand, since the alpha and proton orbits may encompass large radial distances in the plasma, average density and temperature values are used in the alpha and proton slowing down calculations.

The parameters which will be assigned to the D-T base system are:

$E_{inj} = 80$ Kev, $T_{eo} = T_{io} = 6.0$ Kev,

$\overline{T}_e = \overline{T}_i = 3.0$ Kev, $n_{eo} = n_{io} = 8.0 \times 10^{13}/cm^3$,

$\overline{n}_e = \overline{n}_i = 5.6 \times 10^{13}/cm^3$.

These specifications for the DT base case are comparable to the proposed TCT values.[(1)]

The pressure ($P_{DT(1)}$ and $P_{D^3He(1)}$) exerted by the injected deuterium and the fusion products is:

For DT:

$$\frac{P_{DT(1)}}{\dot{n}} = (1-p)\left\{\overbrace{\tfrac{2}{3}\tau_{th}\langle W_D\rangle}^{A} + \overbrace{\tau_f\, M\, \overline{T}_i}^{B}\right.$$

$$+\overbrace{\tfrac{2}{3}q_\alpha\, M^2\, \tau_{th\alpha}\,\langle W_\alpha\rangle}^{C} + \overbrace{q_\alpha\, M^2\, (\tau_{c\alpha} - \tau_{th})\, \overline{T}_i}^{D}$$

$$\left. + \overbrace{(1 - q_\alpha M)\, M\, \tau_{B\alpha}\, E_{\alpha o}}^{E}\right\}$$

$$+ p\left\{\overbrace{\tfrac{2}{3}q_\alpha \tau_{th\alpha}\,\langle W_\alpha\rangle}^{F} + \overbrace{q_\alpha\,(\tau_{c\alpha} - \tau_{th\alpha})\,\overline{T}_i}^{G} + \overbrace{(1-q_\alpha)\,\tau_{B\alpha}\,E_{\alpha o}}^{H}\right\}$$

$$+ (1-p)\,\overline{T}_e\left\{\overbrace{\tau_{th}}^{AA} + \overbrace{M\tau_f}^{BB} + \overbrace{q_\alpha\, M^2\tau_{c\alpha}}^{CD}\right.$$

$$\left. + \overbrace{(1 - q_\alpha M)\, M\tau_{B\alpha}}^{EE}\right\}$$

$$+ p\overline{T}_e\left\{\overbrace{q_\alpha\,\tau_{c\alpha}}^{FG} + \overbrace{(1-q_\alpha)\,\tau_{B\alpha}}^{HH}\right\} \tag{1}$$

For D-^{3}He:

$$\frac{P_{D^3He(1)}}{\dot{n}} = (1 - p)\{\tfrac{2}{3}\tau_{th} \langle W_D\rangle + \tau_f M\overline{T}_i + \tfrac{2}{3} q_\alpha M^2 \tau_{th\alpha} \langle W_\alpha\rangle$$

$$+ \frac{2}{3} q_p M^2 \tau_{thp} \langle W_p\rangle + q_\alpha M^2 (\tau_{c\alpha} - \tau_{th\alpha}) \overline{T}_i$$

$$+ q_p M^2 (\tau_{cp} - \tau_{thp}) \overline{T}_i + (1 - q_\alpha M) M\tau_{B\alpha} E_{\alpha o}$$

$$+ (1 - q_p M) M \tau_{Bp} E_{po}\} + p \{\tfrac{2}{3} q_\alpha \tau_{th\alpha} \langle W_\alpha\rangle$$

$$+ \frac{2}{3} q_p \tau_{thp} \langle W_p\rangle + q_\alpha (\tau_{c\alpha} - \tau_{th\alpha}) \overline{T}_i$$

$$+ q_p (\tau_{cp} - \tau_{thp}) \overline{T}_i + (1 - q_\alpha) \tau_{B\alpha} E_{\alpha o}$$

$$+ (1 - q_p) \tau_{Bp} E_{po}\} + (1 - p) \overline{T}_e \{ \tau_{th} + M\tau_f$$

$$+ \frac{2}{3} q_\alpha M^2 \tau_{c\alpha} + \frac{1}{3} q_p M^2 \tau_{cp}$$

$$+ (1 - q_\alpha M) M \tau_{B\alpha} + (1 - q_p M) M \tau_{Bp}\}$$

$$+ p\ \overline{T}_e \{ \frac{2}{3} q_\alpha \tau_{c\alpha} + \frac{1}{3} q_p \tau_{cp} + (1 - q_\alpha) \tau_{B\alpha}$$

$$+ (1 - q_p) \tau_{Bp} \} \qquad (2)$$

Let us examine Eq. (1) [Eq. (2) is of the same form as (1), except that for every α contribution term, there is an analogous proton term counterpart].

Term (A) refers to the pressure resulting from the deuterium which will ultimately thermalize. Term (B) corresponds to the pressure from the thermalized deuterium (this presumes that the deuterium relaxation time is considerably less than the deuterium confinement time; in the cases discussed, this is roughly true). Term (C) is the pressure from the α's produced from the thermalized D-T reaction, while (D) is the thermalized α-pressure (from (C)), and (E) is the pressure of the α's escaping within one bounce time $\tau_{B\alpha}$. Term (F) represents the α-pressure produced while the injected deuterium is slowing down; (G) is the pressure due to the thermalization of these α's; (H) is the pressure from the portion of these created α's escaping on the first bounce. The remaining terms correspond to those electrons associated with the injected and product particles. For example, term AA is the electron pressure associated with term (A), (BB) refers to the electron pressure connected with (B), (CD) refers likewise to both (C) and (D), etc.

The pressure due to the fusion product terms alone (i.e., without including the injected deuterium) also may be evaluated using these equations. This pressure ($P_{DT(2)}$ and $P_{D^3He(2)}$) for the tritium and ^3He cases, respectively, is found by simply omitting (A), (B), (AA), and (BB) from formulas (1) and (2).

The F-values and slowing down kinematics are evaluated using the plasma codes of H. Towner[5]; the fusion probability is then determined: $p = F \frac{E_{inj}}{E_{fus}}$; the time averaged superthermal energies (i.e. $\langle W_\alpha \rangle$, $\langle W_p \rangle$, and $\langle W_D \rangle$) are figured by breaking up the slowing down process into small time segments and averaging (e.g., $\langle W_\alpha \rangle = \frac{\Sigma W_{\alpha i} \Delta t_i}{\tau_{th\alpha}}$). The neoclassical confinement times are found* by employing $\tau_c = \frac{\int_0^a n\, r dr}{J_{\perp e}\, r|_{r=a}}$ and by assuming the neoclassical diffusion coefficient. The thermalized ions are also assumed to have approximately the same radial speeds, although a vigorous treatment utilizing a multi-specied, ambipolar model would very likely differentiate between their radial speeds slightly.

III. Results

There are four cases** which we shall compare with the D-T base case I. Case II uses approximately the same parameter values as the proposed TCT experiment (i.e., $\overline{T}_e = \overline{T}_i = 3$ Kev, $\overline{n}_i \approx 5 \times 10^{13}$, etc.). Case III is like (II), except that the background densities are doubled. Case IV results when the temperatures of (III) are roughly tripled. Finally, Case V resembles IV, except that the background densities have been tripled.

Figure 1 shows $\frac{P_{D^3He(1)}}{P_{DT(1)}}$ $(\equiv R_1)$ plotted as a function of E_{inj}***. The solid lines represent the contribution of all the particles involved in the fusion process for each case (i.e., injected and thermalized deuterium, superthermal and thermalized fusion products, along with their "associated"

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* Calculation carried out by S. Justiss, graduate assistant, Univ. of Ill.

** The parameters for these D-^{3}He trials are summarized in Appendix B.

*** For all the pressure ratios R, P_{DT} is a constant determined by Case I parameters, unless otherwise specified. P_{DT} serves as a normalization factor for variations in D_{D^3He}.

electrons). As one expects, the important terms in this process are (A), (B), and (BB); the remaining terms can be neglected for $E_{inj} \lesssim 200$ Kev.

When the contributions from electron terms are omitted from consideration, the results are the dashed lines in Figure 1 (the cross-hatching shows the differences between the two types of calculation for each case). The fact that the thermalized deuterium ions and their electrons dominate both $P_{D^3He(1)}$ and $P_{DT(1)}$ about equally, omitting the electron contribution from the formulae will not alter the R_1 ratio appreciably for $E_{inj} \lesssim 200$ Kev.

The pressure ratios for all the D-^{3}He trials yield curves which exceed 0.1 in the energy range considered. Suppose we wish to raise R_1; the results given in Figure I suggests the following: (1) increase the injection energy E_{inj} for the D-^{3}He cases, consistent with the stability criteria; (2) raise the temperature of the background species (compare curves (III) with (IV); (3) lower the background densities (compare (II) with (III) and (IV) with (V)).

The last point is interesting, as it intimates that the effect of increasing the background ^{3}He densities actually decreases $P_{D^3He(1)}$. The effect of the increased background electrons is to slow down the injected deuterium at a faster rate than a less dense plasma; consequently, while the fusion probability remains essentially constant, the injected superthermal ions contribute less to the pressure in the denser plasma.

Figure (2) demonstrates this in greater detail. Log $\frac{P_{D^3He(2)}}{P_{DT(2)}}$ ($\equiv R_2$) is plotted versus the log of average background ion density. The respective pressures are comprised only of the fusion product particles; the effect of the "associated" electrons is negligible. For D-^{3}He, $E_{inj} = 200$ Kev, $\overline{T}_e = \overline{T}_i = 3$ Kev, $T_{eo} = T_{io} = 6$ Kev; for D-T, $E_{inj} = 80$ Kev, $\overline{T}_e = \overline{T}_i = 3$ Kev, $T_{eo} = T_{io} = 6$ Kev, $n_{eo} = n_{io} = 5.6 \times 10^{13}/cm^3$. Evidently, in the energy region considered, the graph appears as a straight line. This is not unreasonable. The only significant contributing terms are: p $(q_\alpha \tau_{th\alpha} \langle w_\alpha \rangle + q_p \tau_{thp} \langle w_p \rangle)$. The F-values (hence p) are rather insensitive to changes in density and remains fairly constant over the density range considered (<9% variation). This is not unexpected; since $\frac{dE}{dt} \propto$ background density n, the formula $F = \frac{nQ_f}{E_{inj}} \int_{E_{inj}}^{E_{th\alpha}} \frac{\langle\sigma v\rangle dE}{\frac{dE}{dt}}$ suggests F will be fairly independent of n.[5] Similarly $\langle w_\alpha \rangle$ and $\langle w_p \rangle$ stay approximately unchanged.* What does change significantly is the thermalization time, which is determined by $\tau_{th} = \int_{E_{inj}}^{E_{th\alpha}} \frac{dE}{\langle \frac{dE}{dt} \rangle}$; as above, $\langle \frac{dE}{dt} \rangle \propto$ background density; this implies $\tau_{th} \propto n^{-1}$; hence $R_2 \propto n^{-1}$. Under such reasoning, the curve in Figure 2 can be expected to have a slope of -1; the measured slope is -0.91.

Figure 3 shows the ratio R_3 of the pressure due only to the fusion product ions and injected superthermal ions. The simulation D-^{3}He cases compare favorably with the D-T base case, with $R_3 \gtrsim 1$ for Case II at $E_{inj} < 150$ Kev. Indeed the combination of lower densities, and/or higher temperatures may be used to simulate the D-T case quite well.

When only the superthermal fusion products are considered (Figure (IV)), the comparison with D-T is less favorable, although $\frac{P_{D^3He}(4)}{P_{DT}(4)}$ ($\equiv R_4$) is still not insignificant when compared with unity.

Since it depends on what aspects of the particle injection one wishes to stress, the choice of favorable D-^{3}He conditions is somewhat arbitrary. For example, to study the effects of superthermal fusion products, the R_4 approach is more relevant than the other calculations. On the other hand, to examine the total pressure resulting from all aspects of injection and fusion considerations, R_1 must be employed.

* That $\langle w_\alpha \rangle$ is approximately independent of n (and T) may be demonstrated using appropriate approximations to Sivukhin's formulae[3] for the region $V_e \gg V \gg V_i$.

Consideration is presently being directed toward extending the above analysis to include the effects of impurities. Energy deposition and plasma rotation simulations are also under study. Optimized conditions will be examined in future work, if these simulation techniques continue along a promising vein.

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Appendix A

Nomenclature

n_{eo}	≡	peak electron density (background)
n_{io}	≡	peak ion density (background)
$\bar{n}_e$	≡	average electron density (background)
$\bar{n}_i$	≡	average ion density (background)
T_{eo}	≡	peak electron temperature (background)
T_{io}	≡	peak ion temperature (background)
$\bar{T}_e$	≡	average electron temperature (background)
$\bar{T}_i$	≡	average ion temperature (background)
E_{inj}	≡	deuterium injection energy
E_{fus}	≡	energy released via a fusion reaction.
$P_{D^3He(1)}$	≡	pressure due to deuterium injection, fusion particles, and associated electrons with a ^{3}He background.
$P_{DT(1)}$	≡	pressure due to deuterium injection, fusion particles, and associated electrons with a T background
$P_{D^3He(2)}$	≡	pressure due to fusion particles and associated electrons with a ^{3}He background.
$P_{DT(2)}$	≡	pressure due to fusion particles and associated electrons with a T background
R_1	≡	$P_{D^3He(1)}/P_{DT(1)}$
R_2	≡	$P_{D^3He(2)}/P_{DT(2)}$
τ_{th}	≡	deuterium thermalization time
$\tau_{th\alpha}$	≡	α-particle thermalization time
τ_{thp}	≡	proton thermalization time
τ_f	≡	thermalized D-^{3}He fusion time
τ_c	≡	thermalized D confinement time

$\tau_{c\alpha}$ $\equiv$ thermalized α confinement time

τ_{cp} $\equiv$ thermalized proton confinement time

$\tau_{B\alpha}$ $\equiv$ α-bounce time (neoclassical)

$\tau_{B\alpha}$ $\equiv$ proton bounce time (neoclassical)

$E_{\alpha o}$ $\equiv$ α-energy immediately after birth

E_{po} $\equiv$ proton energy immediately after birth

M $\equiv$ $\dfrac{\frac{1}{\tau_f}}{\frac{1}{\tau_f} + \frac{1}{\tau_c}}$

q_α $\equiv$ α-containment fraction after first bounce

q_p $\equiv$ proton containment fraction after first bounce

$\langle W_D \rangle$ $\equiv$ average deuterium speed in the slow down phase

$\langle W_\alpha \rangle$ $\equiv$ average α-speed in the slow down phase

$\langle W_p \rangle$ $\equiv$ average proton speed in the slow down phase

$P_{D^3He(3)}$ $\equiv$ pressure due only to superthermal deuterium and fusion particles with a ^3He background

$P_{DT(3)}$ $\equiv$ pressure due only to superthermal deuterium and fusion particles with a T background

$P_{D^3He(4)}$ $\equiv$ pressure due only to superthermal fusion products in a ^3He background

$P_{DT(4)}$ $\equiv$ pressure due only to superthermal fusion products in a T background

V_e $\equiv$ background electron speed

V_i $\equiv$ background ion speed

V $\equiv$ superthermal particle speed

p $\equiv$ fusion probability

Q_f $\equiv$ energy released per reaction due to fusion

Appendix B

D-T BASE CASE (I):

$n_{eo} = n_{io} = 8.0 \times 10^{13}/cm^3$

$\bar{n}_e = \bar{n}_i = 5.6 \times 10^{13}/cm^3$

$T_{eo} = T_{io} = 6.0$ kev

$\bar{T}_e = \bar{T}_i = 3.0$ kev

CASE II (D-^{3}He)

$n_{io} = 7.0 \times 10^{13}/cm^3$, $n_{eo} = 1.4 \times 10^{14}/cm^3$

$\bar{n}_i = 5.0 \times 10^{13}/cm^3$, $\bar{n}_e = 1.0 \times 10^{14}/cm^3$

$T_{eo} = T_{io} = 6.0$ kev

$\bar{T}_e = \bar{T}_i = 3.0$ kev

CASE III (D-^{3}He)

$n_{io} = 1.4 \times 10^{14}/cm^3$, $n_{eo} = 2.8 \times 10^{14}/cm^3$

$\bar{n}_i = 1.0 \times 10^{14}/cm^3$, $\bar{n}_e = 2.0 \times 10^{14}/cm^3$

$T_{eo} = T_{io} = 6.0$ kev

$\bar{T}_e = \bar{T}_i = 3.0$ kev

CASE IV (D-^{3}He)

$n_{io} = 1.4 \times 10^{14}/cm^3$, $n_{eo} = 2.8 \times 10^{14}/cm^3$

$\bar{n}_i = 1.0 \times 10^{14}/cm^3$, $\bar{n}_{eo} = 2.0 \times 10^{14}/cm^3$

$T_{eo} = T_{io} = 20.0$ kev

$\bar{T}_e = \bar{T}_i = 10.0$ kev

CASE V (D-^{3}He)

$n_{io} = 4.3 \times 10^{14}/cm^3$, $n_{eo} = 8.6 \times 10^{14}/cm^3$

$\bar{n}_i = 3.0 \times 10^{14}/cm^3$, $\bar{n}_e = 6.0 \times 10^{14}/cm^3$

$T_{io} = T_{eo} = 20.0$ kev

$\bar{T}_i = \bar{T}_e = 10.0$ kev

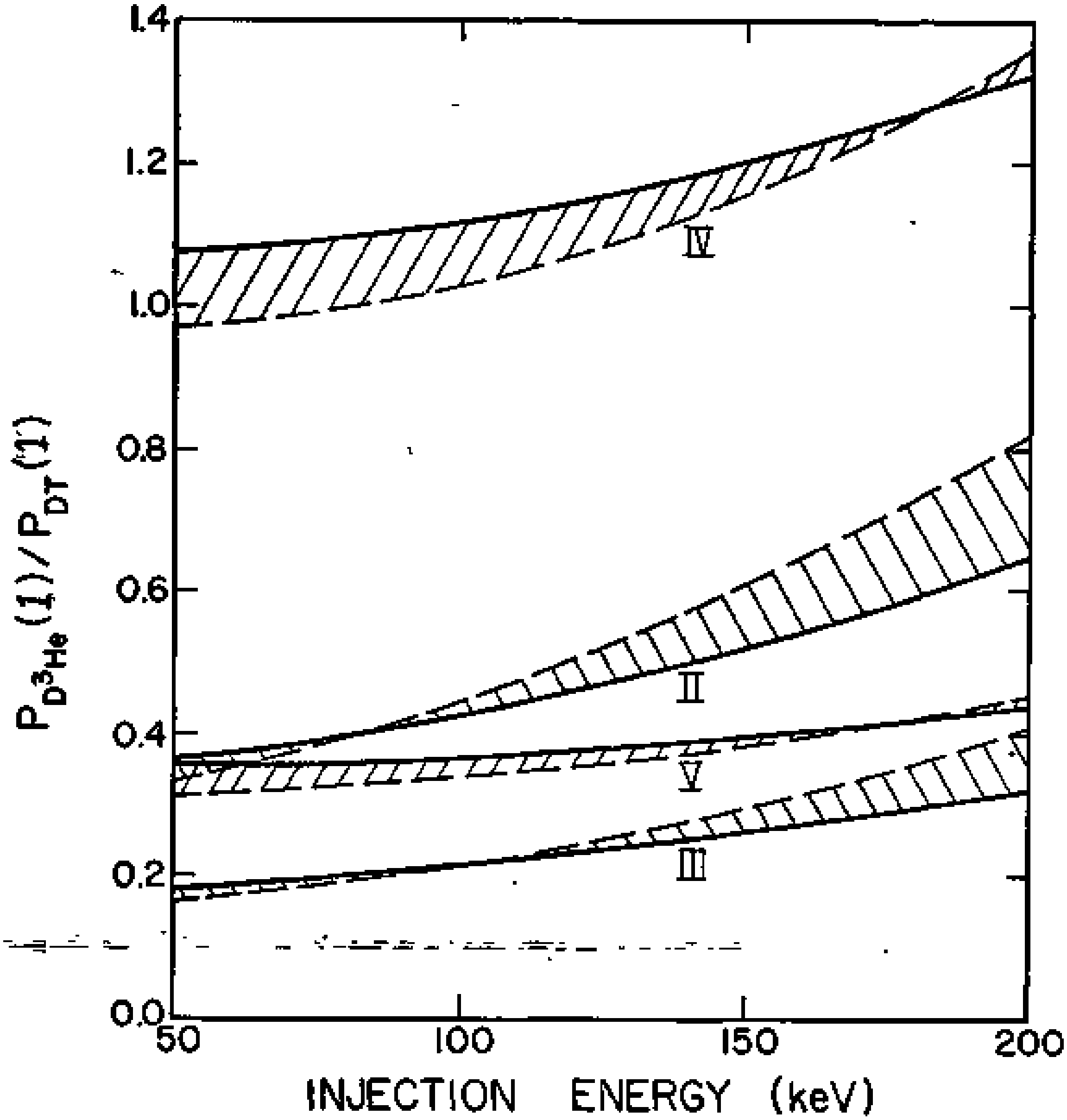

Figure (1): $\frac{P_{D^3He(1)}}{P_{DT(1)}}$ ratio plotted as a function of injection energy. Heavy lines represent calculations including "associated" electrons; dotted lines exclude these electrons.

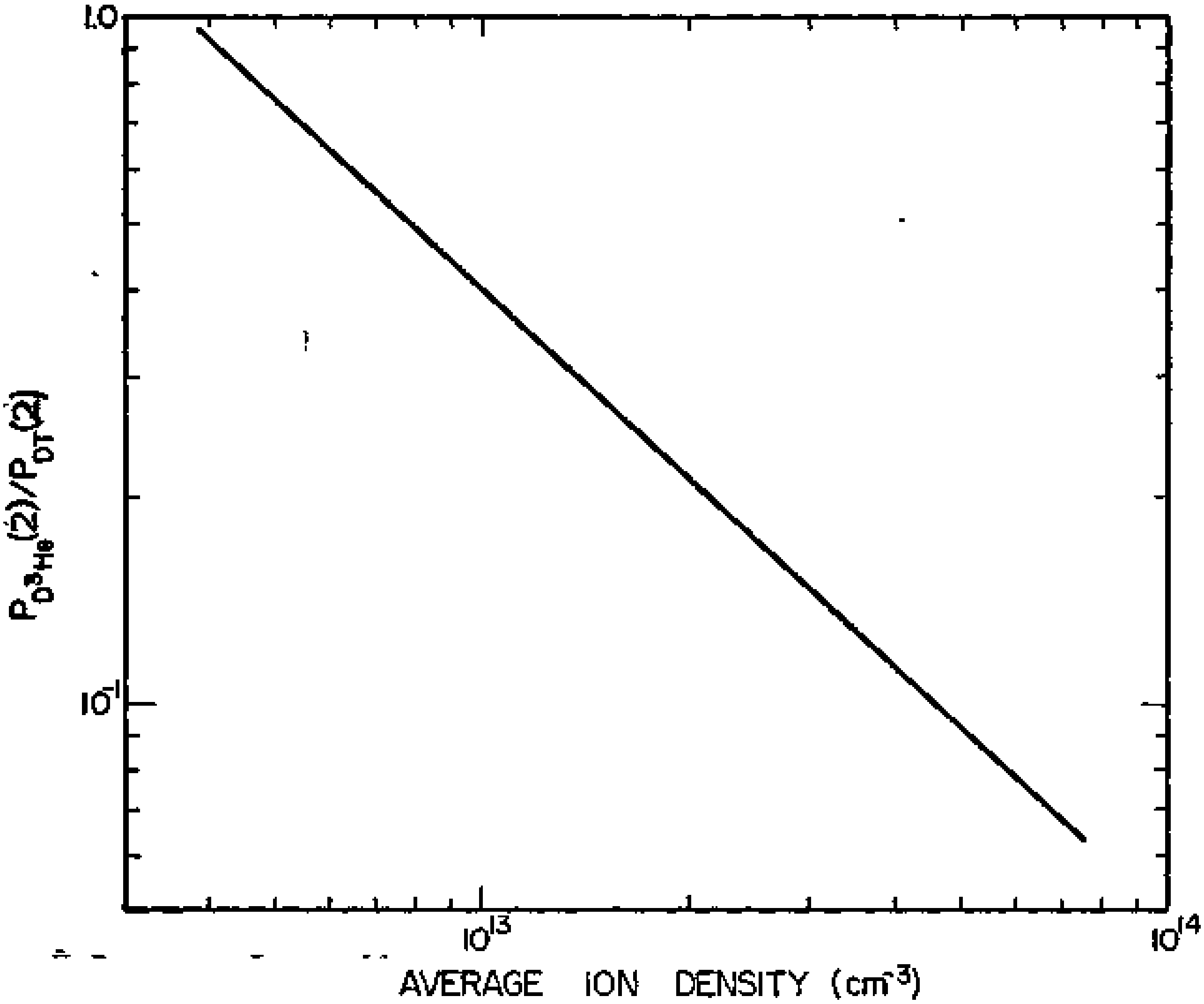

Figure (2): The effect of background helium-3 density on the ratio of pressures due to the fusion products. The curve in the region considered is a straight line for all practical purposes. The constant D-T normalization pressure ($P_{DT(2)}$), as well as the above variable D^3He pressure ($P_{D^3He(2)}$), is evaluated using the parameters given on page 8.

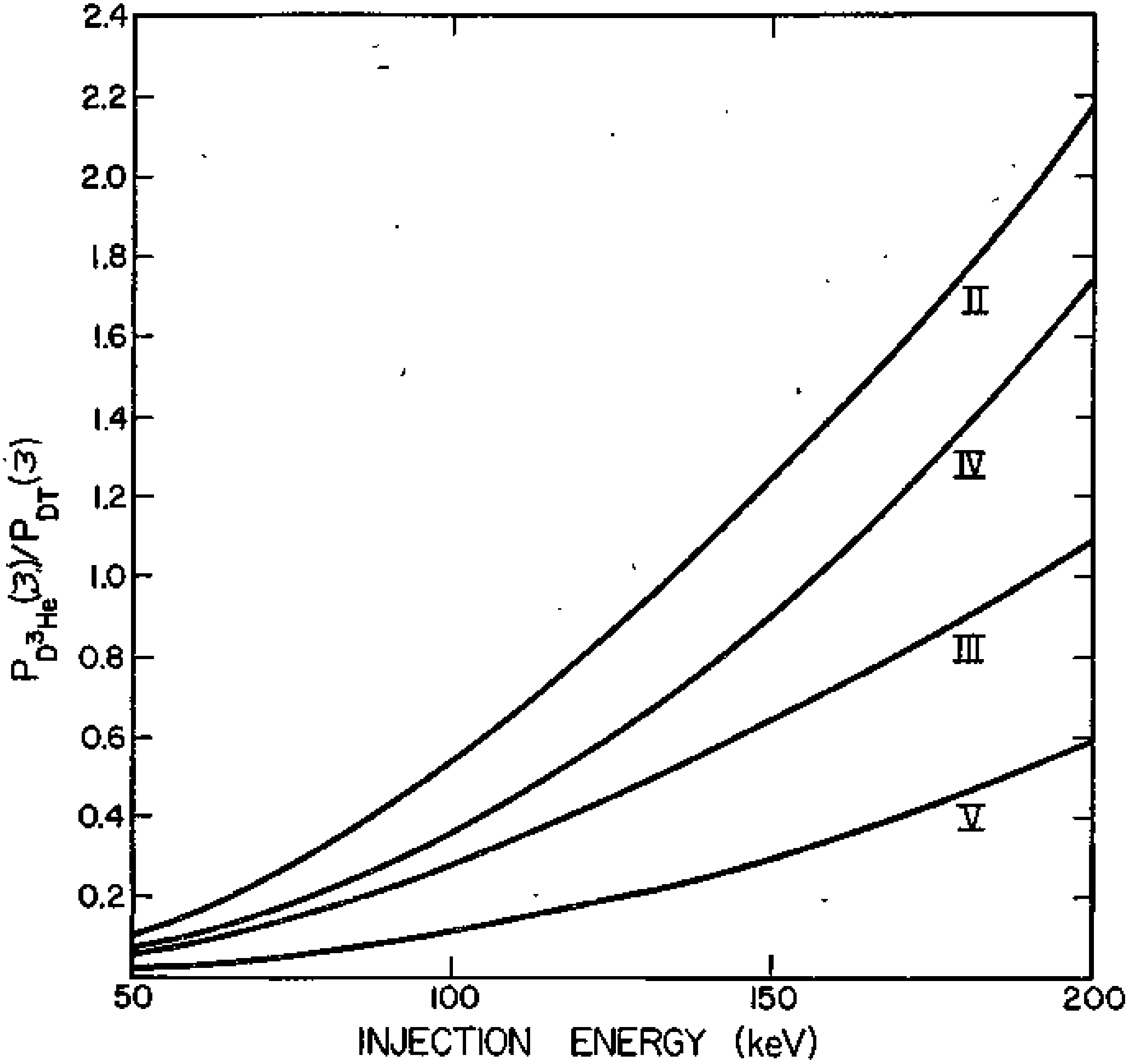

Figure (3): The effects of injection energy on the ratio of pressures due only to superthermal particles.

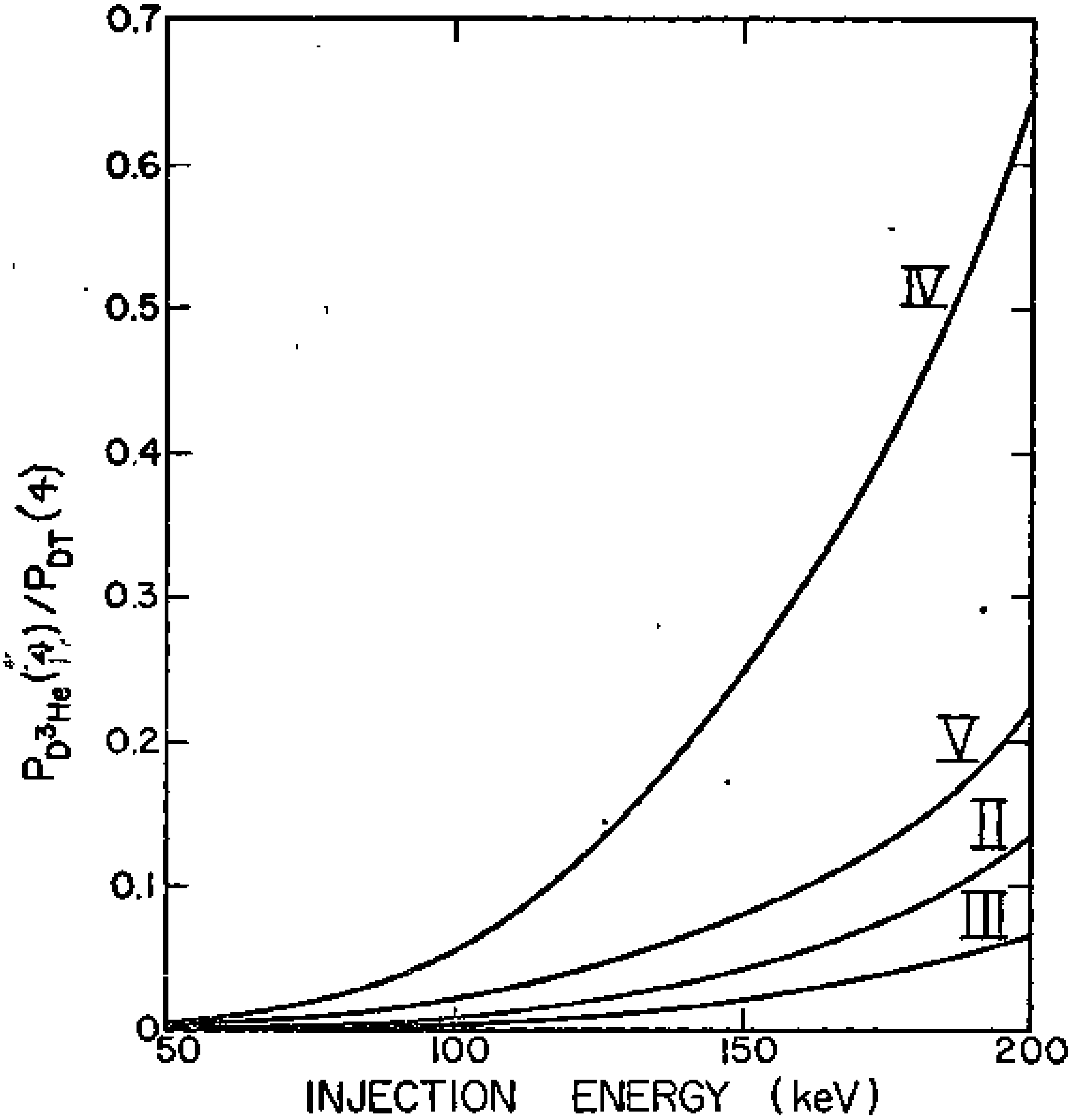

Figure (4): The effects of injection energy on the ratio of pressures due only to the fusion products.