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COMPUTATION OF THE EFFECT OF PIPE PLASTICITY ON
PRESSURE-PULSE PROPAGATION IN A FLUID SYSTEM

by

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ABSTRACT

A simple computational model is developed for incorporating the effect of elastic-plastic deformation of piping on pressure-transient propagation in a fluid system. A computer program (PLWV) is described that incorporates this structural interaction model into a one-dimensional method-of-characteristics procedure for fluid-hammer analysis. Computed results are shown to be in good agreement with available experimental data. The most significant effect of plastic deformation is to limit the peak pressure of a pulse leaving a pipe to approximately the yield pressure of the pipe, if the pipe is sufficiently long.

1. INTRODUCTION

Severe pressure transients in LMFBR heat-transport systems may result from such sources as a sodium/water reaction in a steam generator, a fuel/coolant interaction in the reactor core, valve slam, or pipe rupture. If the peak pressure of the transient exceeds the yield pressure of the piping, portions of the piping system will be plastically deformed. Complex interactions will then occur between the dynamic plastic deformation of the pipe wall and the pressure-wave transmission in the fluid.

Essentially what happens is that a wave with a peak at the pipe yield pressure is transmitted at a wave speed corresponding to elastic pipe-wall deformation. It is followed by a higher pressure wave moving at a much lower wave speed, which varies with pressure and corresponds to plastic pipe-wall deformation. This in turn is followed by an unloading wave from the back of the original pulse traveling at a wave speed corresponding to elastic pipe-wall deformation. As this last wave catches up with the more slowly moving second wave, complex interactions occur which result in the pressure peak being wiped out and dispersed. Consequently, if the pipe is sufficiently long, only the first wave is propagated out of the far end of the pipe, and the peak pressure that may be transmitted to a loop component such as a heat exchanger or pump at the end of the pipe is thus limited to approximately the yield pressure of the pipe. As a result, plastic deformation of the piping acts to protect the critical components in a heat-transport loop, provided the design pressures of the components exceed the yield pressure of the piping.

Fox and Stepnewski¹ point out an analogy between these wave interactions in a fluid contained in a plastically deforming pipe and longitudinal elastic-plastic wave transmission down a solid bar, as discussed by Kolsky.² They hypothesize that the effect of pipe deformation on the propagation of fluid transients can be accounted for by locally modifying the fluid wave speed, and a qualitative description of the resultant wave-interaction phenomena in the fluid is presented in Ref. 1. This hypothesis is the basis for the computational model developed and validated here.

The purposes of this report are (1) to describe a computational model for incorporating the effects of plastic pipe deformation into the fluid-hammer procedures commonly used in pressure-transient propagation analysis, and (2) to demonstrate the practicality and accuracy of this computational model as incorporated into a simple computer program, PLWV. Comparison of computed results with an experiment performed at Stanford Research Institute³ indicates that the accuracy of the procedure is well within the accuracy of standard one-dimensional fluid-hammer techniques and experimental error.

The next section presents a computational model for incorporating the effects of plastic pipe-wall deformation into pressure-transient propagation analysis and summarizes the governing equations. Section III contains a brief description of the PLWV computer program, and Section IV compares computed with experimental results. Some typical results for pressure-transient propagation through fluid contained in plastically deforming pipes are given in Section V, and the conclusions of the report are summarized in Section VI. The appendixes contain information on fitting curves to stress-strain relations for strain-hardening materials and a listing of the PLWV program.

II. COMPUTATIONAL MODEL

A. Analysis Method and Assumptions

One-dimensional fluid-hammer analysis is usually employed for treating hydraulic transients in piping systems caused by pressure pulses of the size and duration considered here.⁴ Among the standard assumptions made in this type of analysis are:

1. The axial velocity u is the only nonzero velocity component. This assumption is modified slightly here in that one-dimensional flow is assumed in deriving the governing fluid-motion equations, but fluid movement in the radial direction is accounted for in determining wall-deformation effects.
2. The pressure p and axial velocity u are functions of axial position x and time t only.
3. Changes in fluid density are negligible compared to the density itself. In the governing differential equations, the fluid density is assumed to

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be a constant and, consequently, independent of position and time, but the bulk compressibility of the fluid is taken into account in computing the wave speed c .

4. Viscous losses in the fluid are neglected.

5. Frictional losses at the pipe wall are included through the Darcy-Weisbach friction coefficient f .

The above assumptions all pertain to the treatment of the fluid. To these must be added some assumptions on the influence of pipe-wall deformation on hydraulic-transient propagation. Various modelings of the pipe deformation are possible, ranging from a rigid pipe-wall model with no structure-fluid interaction effects to a detailed modeling of dynamic deformations and stresses in the piping and the resultant interactions of the stress waves and pipe vibrations with the fluid motion. The model used here is essentially the simplest pipe-response model that incorporates some influence of plastic wall deformation on transients in the fluid. It has the advantages of being readily incorporated into standard fluid-hammer analysis procedures and giving results that are conceptually plausible and agree well with available experimental evidence on plastic wall deformation interaction with fluid-transient propagation. The model is so simple, however, that it could not be reasonably expected to accurately predict dynamic stress and strain histories in the piping.

The additional assumptions involved in this pipe response model include:

6. The pipe response is quasi-static; i.e., the pipe deformation is in equilibrium with the fluid-pressure distribution, which varies with x and t . This eliminates all waves traveling through the pipe material.

7. Bending moments in the pipe wall are neglected, and pipe deformations are not required to be continuous functions of x . This implies that the pipe is treated as a series of rings which act independently of each other.

8. Axial stresses and strains in the pipe are neglected.

9. The pipe material is incompressible; this is the usual assumption in plasticity problems.

10. The pipe wall is thin enough so that circumferential stress variations across the thickness can be neglected.

11. Circumferential strains are small.

The result of these assumptions on pipe response is that the only influence of pipe deformation on transient propagation in the fluid is through its effect on local wave speed. The wave speed, which is equal to the sound speed

in the fluid if wall-deformation effects are neglected, is no longer just a function of fluid properties, but now depends on fluid properties, pipe properties, and pipe-deformation history. Consequently, it can vary with time and position along the pipe, and provision must be made in the computational scheme to accommodate this variation. To illustrate the magnitude of the pipe-deformation effect on local wave speed, the speed of sound in sodium at 700°F is about 7800 ft/sec; if the sodium is contained in a Schedule 30, Type 304 stainless steel pipe, the wave speed drops to about 5200 ft/sec when the pipe deforms elastically; the wave speed in a region of the same pipe that is undergoing plastic deformation depends on the deformation history, but typically varies between 850 and 3500 ft/sec.

A number of computational procedures can be used for fluid-hammer analysis. With the availability of modern computer facilities, the one that appears to offer the best accuracy and versatility in treating complex fluid systems is the one-dimensional nonlinear method of characteristics in a fixed time-space grid.⁴⁻⁶ The governing equations will be given first for strictly one-dimensional fluid motion, i.e., no radial flow resulting from wall deformation; wall-deformation effects will be incorporated later as a modification to the wave speed.

B. Governing Equations for Fluid Motion

The derivation of governing equations for fluid motion under assumptions 1-5 and their conversion to characteristic form are given in Ref. 4 and other standard textbooks; only the results are repeated here. The equation for one-dimensional flow in a rigid pipe of inner diameter D_0 is

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} + \frac{f \rho u |u|}{2D_0} = 0, \quad (1)$$

where x and t are axial position and time, $p(x, t)$ is pressure, $u(x, t)$ is fluid velocity in the x direction, ρ is density, and f is the Darcy-Weisbach friction coefficient. The continuity equation is equivalent to

$$c_0^2 \rho \frac{\partial u}{\partial x} + \frac{\partial p}{\partial t} + \mu \frac{\partial p}{\partial x} = 0, \quad (2)$$

where c_0 is the sound speed, given by

$$c_0^2 = \frac{dp}{d\rho} \quad (3)$$

at constant entropy. For slightly compressible liquids such as subcooled water or sodium, the sound speed is given by

$$c_0 = \sqrt{K/\rho} \quad (4)$$

where K is the bulk modulus of the liquid.

Applying the method of characteristics, Eqs. 1 and 2 can be converted to equivalent differential equations that involve only total derivatives with respect to time and apply only along characteristic curves; these are:

$$\frac{du}{dt} + \frac{1}{\rho c_0} \frac{dp}{dt} + g(u) = 0, \quad (5)$$

which holds along the positive characteristic C^+ , given by

$$dx = (u + c_0)dt, \quad (6)$$

and

$$\frac{du}{dt} - \frac{1}{\rho c_0} \frac{dp}{dt} + g(u) = 0, \quad (7)$$

which holds along the negative characteristic C^- , given by

$$dx = (u - c_0)dt. \quad (8)$$

The function $g(u)$ is defined as

$$g(u) = \frac{fu|u|}{2D_0}. \quad (9)$$

C. Effect of Pipe Deformation on Wave Speed

Under the assumptions of this report, the effect of pipe deformation is to convert the strictly one-dimensional fluid motion to a slightly two-dimensional motion. The combined fluid-structure two-dimensional response is modeled as being equivalent to a one-dimensional fluid response with a reduced wave speed, since the wall flexibility makes the fluid appear to be more compressible than it actually is. The reduced wave speed is derived in this section.

Fig. 1. Fluid Element. ANL Neg. No. 113-5880.

Consider a fluid element of mass m_0 in a pipe of deformed diameter D and deformed wall thickness H (see Fig. 1), where D_0 and H_0 are the initial values of these dimensions. The length L of rigid pipe this fluid element would occupy is given by

$$L = \frac{m_0}{A_0 \rho}, \quad (10)$$

with

$$A_0 = \frac{\pi}{4} D_0^2.$$

For strictly one-dimensional fluid motion, i.e., no wall deformation, changes in L are related to changes in density through

$$dL = -\frac{m_0}{A_0 \rho^2} d\rho = -\frac{L}{\rho} d\rho, \quad (11)$$

and Eq. 3 then becomes

$$c_0^2 = -\frac{L}{\rho} \frac{d\rho}{dL}. \quad (12)$$

Consequently, the wave speed in fluid contained in a rigid pipe (which is equal to the sound speed in the fluid in this case) can be related to the change in length of a fluid element; for strictly one-dimensional response, this change in L is associated with a change in density of the element.

The modified wave speed c , taking into account the combined fluid-structure response, will be defined analogously to Eq. 12 to be

$$c^2 = -\frac{L}{\rho} \frac{d\rho}{dL}, \quad (13)$$

where changes in L now correspond to both changes in density and changes in cross-sectional area. By the definition of the bulk modulus K ,

$$\begin{aligned} d\rho &= -\frac{K}{V} dV \\ &= -K \left(\frac{2}{D} \frac{dD}{D} + \frac{dL}{L} \right), \end{aligned} \quad (14)$$

where the volume V is given by $V = (\pi/4)D^2L$. Under the assumptions of this report, D is a function of pressure determined by pipe properties, so that

$$d\rho = -\frac{2K}{D} \frac{dD}{dp} dp - \frac{K}{L} dL. \quad (15)$$

Then, from Eqs. 13 and 15,

$$c^2 = \frac{K/\rho}{1 + \frac{2K}{D} \frac{dD}{dp}}. \quad (16)$$

The compatibility, equilibrium, strain-displacement, and stress-strain relations for the pipe are used to determine dD/dp . By assumptions 8 and 9, compatibility requires

$$HD = H_0 D_0 \quad (17)$$

or

$$H dD + D dH = 0. \quad (18)$$

By assumptions 6 and 7, the equilibrium equation to be satisfied is

$$2H\sigma = pD \quad (19)$$

or

$$2H d\sigma + 2\sigma dH = p dD + D dp, \quad (20)$$

where σ is the circumferential stress. The circumferential strain ϵ is related to the diameter D through

$$d\epsilon = \frac{dD}{D}. \quad (21)$$

Finally, a relation between σ and ϵ , which may depend on strain history, is assumed to be known. Eliminating p between Eqs. 19 and 20 yields

$$2H \frac{d\sigma}{d\epsilon} d\epsilon + 2\sigma dH = \frac{2H\sigma}{D} dD + D dp. \quad (22)$$

Using Eq. 21 to eliminate $d\epsilon$ and Eq. 18 to eliminate dH gives

$$2H \left(\frac{d\sigma}{d\epsilon} - 2\sigma \right) dD = D^2 dp, \quad (23)$$

and substitution of this result into Eq. 16 gives¹

$$c^2 = \frac{K/\rho}{1 + \frac{KD}{H \left(\frac{d\sigma}{d\epsilon} - 2\sigma \right)}}. \quad (24)$$

By assumption 11, the changes in diameter and wall thickness are small compared to their initial values, so that

$$c^2 = \frac{K/\rho}{1 + \frac{KD_0}{H_0 \left(\frac{d\sigma}{d\epsilon} - 2\sigma \right)}}. \quad (25)$$

If the pipe wall is rigid, $d\sigma/d\epsilon \rightarrow \infty$, and the wave speed reduces to the sound speed as given by Eq. 4. If the pipe deformation is linear elastic, then $d\sigma/d\epsilon = E$, where E is the Young's modulus of the material. Since $E \gg 2\sigma$ for structural materials, the elastic wave speed is given by

$$(c^2)_{\text{elastic}} = \frac{K/\rho}{1 + \frac{KD_0}{H_0E}} \quad (26)$$

This is the modified wave speed given in Ref. 4 for metal pipes undergoing elastic deformation. If the wall material is highly deformable compared to the fluid, $K \gg E$ and $c^2 \approx EH_0/\rho D_0$; this relation is also derived in Ref. 4.

The modified wave speed as given by Eq. 25 is used in this report to characterize the effect of elastic-plastic pipe deformation on transient propa-

Fig. 2. Stress-Strain Relation for an Elastic-Plastic Material, ANL Neg. No. 113-5881.

gation in the fluid. For elastic-plastic deformation, $d\sigma/d\epsilon$ depends not only on the current value of σ (and p through Eq. 19) but also on prior strain history and the sign of dp (see Fig. 2). If there has been plastic deformation at a pipe cross section followed by elastic unloading (path 123 in Fig. 2), the yield stress, which was originally σ_1 , will be increased by strain hardening to σ_2 ; stresses such as σ_4 , which would have produced plastic deformation originally, will now produce elastic deformation with its correspondingly higher wave speed. If a pipe cross section is deforming plastically (point 2 in Fig. 2), a further pressure increase will produce additional plastic deformation,

corresponding to a low wave speed; on the other hand, a pressure decrease will produce elastic unloading, corresponding to a higher wave speed.

In the computer program, the maximum stress experienced at each axial node point is monitored. If the previous maximum stress at a particular point is less than the original yield stress σ_1 , the current stress is compared with σ_1 to determine whether the corresponding deformation is elastic or plastic. If the previous maximum stress is σ_2 , which is greater than σ_1 , then the current stress is compared with σ_2 to determine whether the current deformation is elastic or plastic.

It is assumed throughout that loading and unloading are never so severe as to produce reverse plastic strains. Since the fluid cannot sustain negative pressures, pipe deformations are assumed to be small, and pipe inertia is neglected so that oscillatory deflections do not occur, it is difficult to conceive of circumstances under which reverse plastic straining could occur.

D. Finite-difference Solution along Characteristics

If the solution for pressure and fluid velocity is known at a time t_0 for every position x along a pipe, the solution at a later time $t_0 + \Delta t$ can be found through the relations between du/dt and dp/dt that hold along the characteristic curves. Expressing Eqs. 5 and 7 in finite-difference form for C^+ and C^- characteristics intersecting at point P gives

$$\left. \begin{aligned} & \frac{u_P - u_A}{\Delta t} + \frac{1}{\rho c_{AP}} \frac{p_P - p_A}{\Delta t} + g(u_{AP}) = 0 \\ \text{and} & \\ & \frac{u_P - u_B}{\Delta t} - \frac{1}{\rho c_{BP}} \frac{p_P - p_B}{\Delta t} + g(u_{BP}) = 0 \end{aligned} \right\}, \quad (27)$$

where u_{AP} and c_{AP} are appropriately averaged values of the velocity and wave speed along the C^+ characteristic, and u_{BP} and c_{BP} are corresponding values along the C^- characteristic. If these average velocities and wave speeds could be determined exactly, the above finite-difference equations also would be exact. However, the average quantities can be estimated only from information at points A, B, and P.

Since the frictional losses at the pipe wall are small, $g(u_{AP})$ and $g(u_{BP})$ are conveniently approximated by $g(u_A)$ and $g(u_B)$, respectively. However, the wave speed can vary significantly along the characteristics if the pipe is deforming plastically in the region being analyzed. We will take

$$\left. \begin{aligned} & c_{AP} \approx \frac{1}{2}(c_A + c_P) \\ \text{and} & \\ & c_{BP} \approx \frac{1}{2}(c_B + c_P) \end{aligned} \right\}. \quad (28)$$

Since c_P depends on p_P , it is not known a priori; consequently, an iterative technique will be used. A value of c_P is assumed and p_P calculated; using this pressure and the stress history, a new value of c_P is calculated and the procedure repeated until it converges.

Equations 27 are solved for u_P and p_P to give

$$\left. \begin{aligned} & u_P = \frac{1}{c_{AP} + c_{BP}} \left\{ c_{AP} u_A + c_{BP} u_B + \frac{1}{\rho} (p_A - p_B) - \Delta t [c_{AP} g(u_A) + c_{BP} g(u_B)] \right\} \\ \text{and} & \\ & p_P = \frac{c_{AP} c_{BP}}{c_{AP} + c_{BP}} \left\{ c(u_A - u_B) + \frac{p_A}{c_{AP}} + \frac{p_B}{c_{BP}} + \rho \Delta t [g(u_B) - g(u_A)] \right\} \end{aligned} \right\}. \quad (29)$$

E. Interpolation in Fixed Time-Space Grid

In the fixed-time-space-grid procedure^{4,5} for the method of characteristics, a convenient time step Δt is chosen; an axial grid spacing Δx is then determined for each pipe in the system such that the numerical stability criterion

$$\frac{\Delta t}{\Delta x} [c + |u|]_{\max} \leq 1 \quad (30)$$

is satisfied. The numerical solution will diverge if this inequality is violated; on the other hand, smearing of the propagated pressure pulses results if the left-hand side is considerably less than one. An appropriate strategy for elastic or rigid pipe walls, because c is constant for each pipe and $|u| \ll c$, is to select Δx for each pipe such that

$$1 - \epsilon_1 < c\Delta t/\Delta x < 1 - \epsilon_2, \quad \epsilon_1 > \epsilon_2, \quad (31)$$

where ϵ_1 and ϵ_2 are both small positive numbers. The length of each pipe is then adjusted to become an integer multiple of Δx for the pipe; if inequality 31 is violated during the computation because of increasing magnitudes of fluid velocity u , the time step is reduced to regain satisfaction of the stability criterion.

The variable wave speed corresponding to plastic pipe deformation is considerably less than that associated with elastic deformation. Consequently, the axial grid spacing for the pipe is governed by the wave speed for elastic deformation, and the spacing in the plastically deforming region will be much larger than is necessary for convergence. Therefore, some smearing of the pulse in this region is to be expected. There are two features of the solution, however, that compensate somewhat for this effect. First, neglecting bending moments in the pipe wall eliminates a pulse-dispersing mechanism. Therefore, the error resulting from this assumption is in the opposite direction from the error caused by the grid spacing. Second, because of the chopping off of the peak of the slow-moving wave corresponding to plastic wall deformation by its interaction with the fast-moving unloading wave from the back of the pulse, plastic deformation occurs only over a short portion of the pipe and only for a short time. The accompanying wave smearing caused by a locally inappropriate grid size has only limited effect on the complete solution for the piping system, since subsequent wave propagation is at wave speeds that are appropriate to the grid. Since agreement with available experimental data is good, it may be concluded that the fixed-grid procedure is probably sufficiently accurate to account for the phenomena being investigated here.

Two other procedures that could be used with the method of characteristics were considered briefly: One procedure would be to employ the irregular mesh formed by the actual characteristics. This would result in a

minimum of wave distortion. However, the x, t intersections of the characteristics would have to be computed and stored, and, in order to obtain pressure histories at a given axial position or pressure profiles at a given time, two-dimensional interpolations would have to be performed in the irregular grid. Computer running times would be longer and storage requirements larger than for the fixed-grid procedure.

An alternative procedure would be to use the regular fixed grid appropriate to elastic deformation for most of the pipe and a smaller axial grid spacing in the plastically deforming region, both with the same Δt . Again, wave smearing would be minimized. Pressure profiles at a fixed time could be determined without any interpolation, and pressure histories at a given axial position would require only one-dimensional interpolation. However, as the plastically deforming region moved along the pipe, the finer-meshed axial grid would have to move along with it. Keeping track of this moving grid and assuring that the characteristics did not improperly intersect grid lines would require a more complex program structure and longer running times than the fixed-grid procedure. Consequently, the fixed-time-space-grid procedure was selected for the numerical work described in this report because of its simplicity and because its pulse-smearing errors are at least partially offset by inaccuracies associated with increased interpolation requirements for the other two procedures.

Fig. 3. Finite-difference Grid and Local Characteristics Passing through Point P. ANL Neg. No. 113-5882.

The interpolations required in the fixed-time-space-grid procedure have two aspects: The locations of the intersections of the characteristics with the constant time line (points A and B of Fig. 3) must be determined; and then values of the desired quantities must be computed at these locations in terms of their values at the grid points.

From Fig. 3 and Eqs. 6 and 8 (with c_0 replaced by c) we obtain

$$\left. \begin{aligned} x_Q - x_A &= v_{AP} \Delta t \\ x_B - x_Q &= -v_{BP} \Delta t \end{aligned} \right\}, \quad (32)$$

where v_{AP} is an appropriately averaged value of $u + c$ along the C^+ characteristic and v_{BP} is an appropriately averaged value of $u - c$ along the C^- characteristic. Here, v_{AP} will be assumed to be approximately equal to the arithmetic average of the values of $u + c$ at points A and P; i.e.,

$$v_{AP} = \frac{1}{2}(u_A + c_A + u_P + c_P). \quad (33)$$

Similarly, v_{BP} will be assumed to be given by

$$v_{BP} = \frac{1}{2}(u_B - c_B + u_P - c_P). \quad (34)$$

Since u_P and c_P are unknown quantities at the beginning of the time step, they must be determined by iteration, using Eqs. 29 and 25 and the stress-strain relation.

Linear interpolation between grid points R and Q to estimate values of u and p at point A and interpolation between grid points Q and S to estimate values of u and p at point B gives

$$\left. \begin{aligned} u_A &= u_Q - v_{AP}(u_Q - u_R)\Delta t/\Delta x, \\ p_A &= p_Q - v_{AP}(p_Q - p_R)\Delta t/\Delta x, \\ u_B &= u_Q - v_{BP}(u_S - u_Q)\Delta t/\Delta x, \\ p_B &= p_Q - v_{BP}(p_S - p_Q)\Delta t/\Delta x. \end{aligned} \right\} \quad (35)$$

and

The wave speeds c_A and c_B could be interpolated for in a similar way in terms of c_R , c_Q , and c_S . However, the wave speeds are then likely to be inconsistent with the pressures at the interpolated points. For example, for p_A corresponding to a stress σ_A in the plastic range, the interpolated value for c_A might correspond to elastic deformation. Therefore, the value of c_A that is associated with p_A will be used. To do this, it is necessary to store the maximum pressure that each axial grid point has experienced. Let p_{MQ} be the maximum value of p_Q up to time t_0 , etc. Then, using linear interpolation for the maximum pressure, we have

$$\left. \begin{aligned} p_{MA} &= p_{MQ} - v_{AP}(p_{MQ} - p_{MR})\Delta t/\Delta x \\ p_{MB} &= p_{MQ} - v_{BP}(p_{MS} - p_{MQ})\Delta t/\Delta x \end{aligned} \right\} \quad (36)$$

and

If $p_A < p_{MA}$, the deformation at point A is elastic and c_A is computed from Eq. 26; if $p_A \geq p_{MA}$ and p_A also exceeds the pipe yield pressure, the deformation is plastic and c_A is computed from Eq. 25 and the stress-strain relation.

The computational procedure involves iterations at points A, B, and P. To get the procedure started, initial guesses of c_A , u_A , c_B , u_B , c_P , and u_P

are taken to be c_R , u_R , c_S , u_S , c_Q , and u_Q , respectively. Equation 33 is used to estimate v_{AP} from these initial guesses, and u_A , p_A , and p_{MA} are computed from Eqs. 35 and 36. The corresponding value of c_A is then computed. With these new values of u_A and c_A , the quantity v_{AP} is estimated again from Eq. 33 and the process repeated until successive estimates of v_{AP} converge. If the values of v_{AP} oscillate because p_A is very close to the yield pressure, an average of two successive values is used; this rarely happens, however.

A similar iteration is used at point B to determine u_B , p_B , and c_B .

Using the initial guesses for c_p and u_p and values of u_A , p_A , c_A , u_B , p_B , and c_B obtained in the iterations at points A and B, estimates of u_p and p_p are determined from Eqs. 29 with c_{AP} and c_{BP} computed from Eqs. 28. The wave speed c_p corresponding to the pressure p_p is then computed. These new values of c_p and u_p are used in predicting new values of v_{AP} and v_{BP} from Eqs. 33 and 34. The iterations at points A and B are then repeated, followed by new estimates of u_p , p_p , and c_p until the process converges.

The description of the iteration procedure might make it appear that considerable computer time would be consumed in solving a problem. This is actually not the case, however. Usually convergence occurs in one step if an elastic region of the pipe is being considered, since the initial guesses of the wave speed are exact and the local fluid speed $|u|$ is much less than the wave speed. Only a few iterations are required in the interior of the plastically deforming region, since the initial guesses are quite reasonable. Consequently, the only points that require a number of iterative steps are in the vicinity of elastic-plastic interfaces.

III. PLWV COMPUTER PROGRAM

The PLWV computer program is modeled after the NATRANSIENT program⁵ in its general structure and numerical procedure. NATRANSIENT has a large system capability; it can treat a network of up to 50 pipes, connecting a variety of types of junctions, such as tees, rupture discs, surge tanks, closed ends, etc. Plastic-deformation effects are not included in NATRANSIENT. PLWV, on the other hand, can handle only two pipes connected in series, but either one or both of them can undergo plastic deformation. The effect of plastic deformation on pressure-transient propagation is computed according to the computational model described in the previous section. An arbitrary pressure pulse can be prescribed at one end of the two pipe system, while the other end can be closed, nonreflecting, or a constant-pressure tank.

Various forms of stress-strain relations are available in the material-response subroutine. These include (1) temperature-dependent stress-strain relation for Type 304 stainless steel;⁷ (2) temperature-dependent stress-strain relation for Type 316 stainless steel;⁷ (3) stress-strain relation

with linear elastic and plastic regions separated by an elliptic transition, for fitting stress-strain data (see Appendix A); (4) same as number 3, except with different material properties; (5) bilinear stress-strain relation; and (6) rigid pipe wall. The two pipes are permitted to have the same or different material properties.

PLWV has a modular structure similar to that of NATRANSIENT; i.e., much of the computation is performed in subroutines, which can be altered without affecting the remainder of the program. These subroutines include (1) WVSP, which computes fluid wave speed as a function of pressure for the pipe materials listed above; (2) FLPRP, which determines fluid properties, such as sound speed, density and viscosity, as a function of temperature (subroutines for sodium and water properties are available); (3) FRTRM, which computes the Darcy-Weisbach pipe friction factor as a function of velocity using the Colebrook-White correlation; (4) STRAIN, which computes elastic and plastic strains in the pipe as a function of pressure for the pipe materials listed above; and (5) PULSE, which gives input pressure as a function of time using linear interpolation between input pressure-time values.

The PLWV program listing and input requirements are given in Appendix B.

IV. COMPARISON OF NUMERICAL RESULTS WITH EXPERIMENT

Among the simple piping tests performed by Stanford Research Institute (SRI) as part of an experimental program³ dealing with the effects of a

Fig. 4. Schematic of SRI Test Arrangement.
ANL Neg. No. 113-5583.

hypothetical core-disruptive accident in the Fast Flux Test Facility (FFTF), one test, Simple Primary Piping Experiment No. 260, is particularly suitable for comparison with PLWV computed results. Figure 4 is a sketch of the test arrangement. The 3-in.-OD, 40-mil wall-thickness test pipe was made of annealed Nickel 200 because of the similarity of its stress-strain curve at room temperature to that of

annealed Type 316 stainless steel at reactor operating temperature. The pulse gun and test pipe were filled with room-temperature water. A pressure pulse was initiated in the pulse gun and propagated out of the gun and down the test pipe. Oscilloscope traces of pressure-transducer measurements were recorded at one location in the gun and three locations in the test pipe. The peak pressure was sufficiently high to plastically deform a portion of the test pipe.

The pressure-time history at point 1 in the pulse gun was used as the pressure source for the PLWV computation. Unfortunately, the oscilloscope

trace reproduced in the SRI report contains considerable noise in the vicinity of the pulse peak, and the pulse shape cannot be determined very accurately. Figure 5 shows the estimate of the experimental curve used as input to PLWV; the noise band on the oscilloscope trace is also included in the figure.

Fig. 5. Pressure History at Location 1 in Pulse Gun. ANL Neg. No. 113-5883.

The stress-strain curve for Nickel 200 (Fig. 6) can be accurately fitted by linear elastic and plastic regions connected by a transitional plastic region that is a portion of an ellipse. The curve-fitting procedure is described in Appendix A. PLWV computations were made using the fitted stress-strain curve and a bilinear approximation to it, both shown in Fig. 6.

Fig. 6. Stress-Strain Curve for Annealed Nickel 200. ANL Neg. No. 113-5581.

Figure 7 shows the PLWV results for pressure-time histories at pressure-transducer locations 1A, 2, and 3 of the test pipe, along with approximate redrawings of the oscilloscope traces from the SRI experiment. Computer results were also obtained for a strictly elastic pipe response; i.e., plasticity effects were ignored. These results are also shown in Fig. 7.

If the pipe only deformed elastically, the pressure pulse would be essentially undistorted as it

propagated down the test pipe, and the peak pressure at location 3 would be about 1400 psi. However, the experimental measurement at this location

Fig. 7. Comparison of PLWV Computations with SRI Experiment; Location-1 Pulse Used as Input. ANL Neg. No. 113-5582.

shows a peak of about 800 psi and a considerably different pulse shape than the input pulse. The PLWV results for location 3 with plastic pipe response included match the experimental results very closely, both with respect to peak pressure and pulse shape, and lie well within the noise band on the oscilloscope trace. The small differences in location and magnitude between the computed and experimental pulse peaks at the various transducer locations are probably caused by inaccuracies in determining the experimental pulse shapes from the oscilloscope traces, particularly for the source pulse. The difference between the computed results for the exact stress-strain curve and the bilinear approximation is small, but the pulse shapes corresponding to the exact stress-strain relation are somewhat closer to the experimental pulse shapes.

PLWV results also were obtained using the experimental pressure pulse at location 1A as the source and computing pressure-time histories at locations 2 and 3; these results are shown in Fig. 8. We see that there is almost exact agreement between computed and experimental pulses. Consequently, the discrepancies shown in Fig. 7 must be caused primarily by an inaccurate determination of the input pulse at location 1.

The overall consequence of the plastic deformation of the test pipe is a flattening of the

pulse as it travels down the pipe. For a sufficiently long pipe, the pulse peak would be expected to be reduced eventually to about the yield pressure of the pipe (~600 psi).

Fig. 8. Comparison of PLWV Computations with SRI Experiment; Location-1A Pulse Used as Input. ANL Neg. No. 113-5584.

results are always qualitatively similar. A flattened wave with a peak at the pipe yield pressure travels at a wave speed corresponding to elastic pipe response, followed by a higher-peaked wave traveling at slower wave speeds corresponding to plastic pipe response. The unloading wave, associated with the back of the pulse and traveling at the higher speed, catches up with the slower wave, and the resulting wave interactions reduce the peak pressure transmitted down the pipe. Consequently, the peak pressure experienced at axial locations along the pipe is a decreasing function of distance measured down the pipe. If the pipe is long enough, the highest pressure transmitted out the far end is roughly the yield pressure.

Some simple results are shown in Fig. 9. A triangular pulse with a 1500-psi peak and 1-msec duration is introduced at $x = 0$ into a 2-ft-dia Schedule 30 pipe containing 700°F sodium. First, the pipe was assumed to be perfectly rigid, so that the wave speed corresponded to the sound speed in the fluid, as given by Eq. 4. Second, the pipe was assumed to be made of Type 304 stainless steel, but only elastic response was considered so that the wave speed was given by Eq. 26. Third, the actual elastic-plastic stress-strain relation for Type 304 stainless steel⁷ was used, with the wave speed given by Eq. 25. Pressure-time histories are shown at various locations down the pipe. For the first two cases (Figs. 9a and 9b), the pulse is transmitted in undistorted form along the pipe, traveling more slowly in the second case because of the elastic wall deformation. The results in the third case (Fig. 9c) are completely different, however. The plastic deformation causes severe truncation of the pulse and some dispersion from wave-interaction effects.

From the good agreement between computed and experimental results, it can be concluded that the rather simple model used in this report for incorporating the effects of plastic deformation of the pipe on pressure-pulse propagation in the fluid gives results well within experimental errors and computational inaccuracies inherent in fluid-hammer analysis.

V. TYPICAL RESULTS

Fig. 9

Pulse Transmission for (a) Rigid, (b) Elastic, and (c) Elastic-Plastic Pipe Response. ANL Neg. No. 113-5884.

VI. SUMMARY AND CONCLUSIONS

A simple computational model has been devised to incorporate the effect of elastic-plastic pipe deformation on pressure-transient propagation in a fluid system. In this model, the pipe is considered to be a series of massless disconnected rings such that bending moments and inertial response are neglected; the structure interacts with the fluid only through the influence of pipe deformation on local wave speed. Despite the neglecting of many complex aspects of structural deformation and structure-fluid interaction, this computational model leads to results that are in good agreement with available experimental evidence. Because of its simplicity, the computational model can be incorporated into the fixed time-space grid, one-dimensional method-of-characteristics procedure commonly used in fluid-hammer analysis.

It is important to include the effects of plastic deformation of the piping on pressure-pulse propagation in the fluid because the plastic deformation limits the peak pressure transmitted out of a pipe to approximately its yield pressure if the pipe is sufficiently long. Consequently, plastic pipe deformation can have a significant effect on the analysis of transient propagation through a complex piping system and on the prediction of transient loadings on system components.

The feasibility of using the computational model described in this report has been demonstrated through the development of the PLWV program for a simple two-pipe system. A similar capability for treating the effect of elastic-plastic pipe deformation on fluid-transient propagation soon will be incorporated into the NATRANSIENT program with its elaborate system capability.

APPENDIX A

Curve Fitting for Stress-Strain Relations

The WVSP subroutine for computing wave speed as a function of material response includes among its stress-strain representations a standard functional form for curve fitting to material data. This standard form consists of linear elastic and plastic regions connected by a transitional plastic region that is a portion of an ellipse (see Fig. 6). The stress-strain relation is expressed as

$$\left. \begin{aligned} \sigma &= E\epsilon, & 0 \leq \epsilon \leq \epsilon_1 & \text{(linear elastic);} \\ C_1(\sigma/\sigma_0)^2 + C_2(\sigma/\sigma_0) + C_3(\epsilon/\epsilon_0) + C_4(\epsilon/\epsilon_0)^2 + C_5 &= 0, \\ \epsilon_1 < \epsilon < \epsilon_2 & \text{(elliptic plastic);} \\ \sigma &= \sigma_0 + E_p(\epsilon - \epsilon_0), & \epsilon_2 \leq \epsilon & \text{(linear plastic);} \end{aligned} \right\} \quad (\text{A.1})$$

where σ and ϵ are stress and strain, E and E_p are the elastic modulus and slope of the linear plastic region, and the point (ϵ_0, σ_0) is the intersection of the extended linear elastic and plastic regions. The constants C_i are chosen so that the curves and their slopes are continuous at the points (ϵ_1, σ_1) and (ϵ_2, σ_2) ; they are given by

$$\left. \begin{aligned} C_1 &= (\gamma_2 - 1)^2 - R_m^2(1 - \gamma_1)^2, \\ C_2 &= -2[\gamma_1(\gamma_2 - 1)^2 - R_m(\gamma_2 - \gamma_1)(\gamma_2 - 1)(1 - \gamma_1) - R_m^2\gamma_2(1 - \gamma_1)^2], \\ C_3 &= -2R_m(\gamma_2 - \gamma_1)[(\gamma_2 - 1)(1 - \gamma_1) + R_m(1 - \gamma_1\gamma_2)], \\ C_4 &= R_m^2(\gamma_2 - \gamma_1)(2 - \gamma_1 - \gamma_2), \\ C_5 &= (1 - R_m^2)\gamma_1^2(\gamma_2 - 1)^2, \end{aligned} \right\} \quad (\text{A.2})$$

where

$$R_m = E_p/E, \quad \gamma_1 = \sigma_1/\sigma_0, \quad \gamma_2 = \sigma_2/\sigma_0. \quad (\text{A.3})$$

An essentially exact fit to the Nickel 200 data given in Ref. 3 is obtained by taking $E = 30 \times 10^6$ psi, $\sigma_0 = 29,000$ psi, $R_m = 0.0135$, $\gamma_1 = 3/4$, and $\gamma_2 = 5/4$. Then, from Eqs. A.2, we have $C_1 = 0.06249$, $C_2 = -0.09288$, $C_3 = -0.0008551$, $C_4 = 0$, and $C_5 = 0.03515$. The bilinear approximation is given by $E = 30 \times 10^6$ psi, $\sigma_0 = 29,000$ psi, $R_m = 0.0135$, $\gamma_1 = \gamma_2 = 1$; all the C_i 's are then zero and the elliptic-transition region vanishes. Since $C_4 = 0$ for the fit to the Nickel-200 data, the elliptic transition is reduced to a parabola.

The WVSP subroutine also contains temperature-dependent functions for the stress-strain relations of Types 304 and 316 stainless steel. These functions are given in the LMFBR Materials Handbook.⁷

APPENDIX B

Input Requirements and Program Listing for PLWV

The following cards are required as input data to PLWV:

<u>Card No.</u>	<u>FORTTRAN Name</u>	<u>Format</u>	<u>Description</u>
1	NPROB	I3	Number of problems in run
2		7E10.5, I3, I4, I3	
	D		Inner diameter of pipes, in.
	PIN		Initial fluid pressure, psi
	UIN		Initial fluid velocity, ft/sec
	DZ(2)		Axial node spacing in second pipe, in.
	TF		Final time at which calculation terminates, sec
	STBFAC		Numerical stability factor, usually 1.0001
	TEMP		Fluid temperature, °F
	NE		Far end condition: NE = 1, closed end NE = 2, constant pressure NE = 3, nonreflecting
	NOPRIN		Printout frequency
	KFR		KFR = 1, pipe friction is included KFR ≠ 1, pipe friction is not included
3		3E10.5, I5	
	ALEN(1)		Length of first pipe, ft
	H(1)		Wall thickness of first pipe, in.
	RF(1)		Wall roughness of first pipe, in.
	MAT(1)		Material number of first pipe: MAT(1) = 1, Type 304 stainless steel; MAT(1) = 2, Type 316 stainless steel; MAT(1) = 3, linear elastic and plastic regions with elliptic transition; MAT(1) = 4, same as 3, except for different material constants; MAT(1) = 5, bilinear stress-strain relation; MAT(1) = 6, rigid pipe wall
4		3E10.5, I5	
	ALEN(2)	}	Parameters for second pipe
	H(2)		
	RF(2)		
	MAT(2)		
5	IPUL	I3	Number of pressure-time input points
6	AP(I)	(8E10.5)	Input pulse pressure values, I = 1, IPUL, psi
7	AT(I)	(8E10.5)	Input pulse time values, I = 1, IPUL, sec

```

C          *****PROGRAM PLWV (PLASTIC WAVE)*****
C          THE EFFECT OF ELASTIC-PLASTIC PIPE WALL DEFORMATION ON
C          PRESSURE TRANSIENT PROPAGATION IN A FLUID IS COMPUTED.
C          THE PIPE IS DIVIDED INTO A SOURCE SECTION AND A TEST
C          SECTION, WHICH MAY HAVE DIFFERENT MATERIAL PROPERTIES.
C          A PRESSURE PULSE IS PRESCRIBED AT THE INLET END OF THE
C          SOURCE SECTION, AND VARIOUS BOUNDARY CONDITIONS CAN BE
C          PRESCRIBED AT THE FAR END OF THE TEST SECTION. THE ONE-
C          DIMENSIONAL, FIXED-GRID, METHOD OF CHARACTERISTICS
C          PROCEDURE IS APPLIED TO FLUID HAMMER ANALYSIS OF TRANSI-
C          ENT PROPAGATION. NONLINEAR CONVECTIVE TERMS AND PIPE
C          FRICTION LOSSES ARE INCLUDED.
C          *****
C
C**** MAIN PROGRAM
COMMON /RF/ D,DENS,VISC,KFR,RF(2)
1  /RP/ AP(99),AT(99),KPUL,PIN,IPUL
2  /RT/TEMP
3  /RW/MAT(2),BB(2),BC(2),CO,CEL(2),KW
   DIMENSION P(2,100,2),U(2,100,2),C(2,100,2),PMX(2,100),KITR(2,100),
1  DZ(2),H(2),CIN(2),ALEN(2),NNOD(2)
   DATA NIN,NOUT/5,6/,EPSI/1.0E-04/,RFR/1.0001/,
1  PFAC/144./,XFAC/12./
C**** READ IN NUMBER OF PROBLEMS IN RUN
   NPRO=1
   READ(NIN,1030) NPROB
1030 FORMAT(I3)
   5 CONTINUE
C**** READ PROBLEM PARAMETERS
   READ(NIN,1000) D,PIN,UIH,DZ(2),TF,STBFAC,TEMP,NE,NOPRIN,KFR
1000 FORMAT(7E10.5,I3,I4,I3)
C**** CALCULATE SPECIFIC WEIGHT, DENSITY, VISCOSITY, SOUND SPEED
   CALL FLPRP(SPWT,DENS,VISC,SNDSPO)
   CO=SNDSPO
C**** READ SOURCE PIPE AND TEST PIPE PARAMETERS
   READ(NIN,1001) (ALEN(J),H(J),RF(J),MAT(J),J=1,2)
1001 FORMAT(3E10.5,I5)
   DO 10 J=1,2
   BB(J)=2.0*H(J)/D
   BC(J)=D*CO**2*DENS/H(J)
10 CONTINUE
C**** PRINT SYSTEM PARAMETERS
   WRITE(NOUT,1002)
1002 FORMAT(1H1,/,/,10X,'PLASTIC WAVE PROGRAM',/,/,
1  10X,'PIPE AND PROBLEM PARAMETERS',/,/,/)
   WRITE(NOUT,1003) TEMP,SPWT,DENS,VISC,CO
1003 FORMAT(5X,'FLUID PROPERTIES',/,/,
1  5X,'TEMPERATURE =',E12.5,' DEGREES F',/,/,
2  5X,'SPECIFIC WEIGHT =',E12.5,' LBS/FT3',/,/,
3  5X,'DENSITY =',E12.5,' LBS-SEC2/FT4',/,/,
4  5X,'VISCOSITY =',E12.5,' LBS-SEC/FT2',/,/,
5  5X,'SOUND SPEED =',E13.6,' FT/SEC',/,/,/)
   WRITE(NOUT,1004)

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1004 FORMAT(5X,'SOURCE PIPE PARAMETERS',/)
      WRITE(NOUT,1005) D,H(1),ALEN(1),RF(1),MAT(1)
1005 FORMAT(5X,'DIAMETER =',E12.5,' INCHES',/;
1      5X,'WALL THICKNESS =',E12.5,' INCHES',/,
2      5X,'LENGTH =',E12.5,' FEET',/,
3      5X,'ROUGHNESS =',E12.5,' INCHES',/,
4      5X,'MATERIAL NUMBER =',I2)
      KW=-1
      PINX=PIN*PFAC
      CALL WVSP(1,PINX,PINX,CIN(1))
      WRITE(NOUT,1006)
1006 FORMAT(5X,'TEST PIPE PARAMETERS',/)
      WRITE(NOUT,1005) D,H(2),ALEN(2),RF(2),MAT(2)
      CALL WVSP(2,PINX,PINX,CIN(2))
      KW=+1
C**** COMPUTE NODE SPACING IN SOURCE PIPE
      DZ(1)=DZ(2)*CEL(1)/CEL(2)
      WRITE(NOUT,1007) PIN,UI,NOPRIN,STBFAC,TF,DZ(1),DZ(2)
1007 FORMAT(5X,'ADDITIONAL PROBLEM PARAMETERS',/;
1      5X,'INITIAL PRESSURE =',E12.5,' LBS/IN2',/,
2      5X,'INITIAL VELOCITY =',E12.5,' FT/SEC',/,
3      5X,'PRINTOUT FREQUENCY =',I3,/,
4      5X,'STABILITY FACTOR =',E13.6,/,
5      5X,'END COMPUTATION AT T =',E12.5,' SEC',/,
6      5X,'NODE SPACING IN SOURCE PIPE =',E13.6,' INCHES',/,
7      5X,'NODE SPACING IN TEST PIPE =',E13.6,' INCHES',/)
      IF(KFR.EQ.1) GO TO 26
      WRITE(NOUT,1008)
1008 FORMAT(5X,'FRICTIONAL LOSSES ARE NOT INCLUDED',/)
      GO TO 27
      26 WRITE(NOUT,1009)
1009 FORMAT(5X,'FRICTIONAL LOSSES ARE INCLUDED',/)
      27 IF(NE.LT.1.CR.NE.GT.3) GO TO 34
      IF(NE.EQ.1) WRITE(NOUT,1010)
1010 FORMAT(5X,'FAR END IS CLOSED',////)
      IF(NE.EQ.2) WRITE(NOUT,1011)
1011 FORMAT(5X,'FAR END IS AT CONSTANT PRESSURE',////)
      IF(NE.EQ.3) WRITE(NOUT,1012)
1012 FORMAT(5X,'FAR END IS NON-REFLECTING',////)
      GO TO 40
      34 WRITE(NOUT,1013)
1013 FORMAT(5X,'FAR END CONDITION IS UNSPECIFIED',/)
      STOP
C**** READ PRESSURE PULSE DATA
      40 READ(NIN,1030) IPUL
      READ(NIN,1018) (AP(I),I=1,IPUL)
      READ(NIN,1018) (AT(I),I=1,IPUL)
1018 FORMAT(8E10.5)
      KPUL=-1
      CALL PULSE(0.0,1.0)
      KPUL=+1
C**** CONVERT UNITS TO FOOT-POUND-SECOND SYSTEM
      PIN=PIN*PFAC
      DO 45 I=1,IPUL
      45 AP(I)=AP(I)*PFAC

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      D=D/XFAC
      DO 46 J=1,2
      DZ(J)=DZ(J)/XFAC
      H(J)=H(J)/XFAC
      RF(J)=RF(J)/XFAC
46 CONTINUE
C**** COMPUTE NUMBER OF NODES IN SOURCE PIPE AND TEST PIPE
      DO 50 J=1,2
      ZNOD=ALFN(J)/DZ(J)+0.5
      JNOD=ZNOD
      NNOD(J)=JNOD+1
      IF(NNOD(J).GT.100) GO TO 52
50 CONTINUE
      GO TO 55
52 WRITE(NOUT,1014) J,NNOD(J)
1014 FORMAT(1H0,5X,'NUMBER OF NODES EXCEEDS LIMIT',/,
1      5X,'J =',I3,5X,'NNOD =',I5)
      STOP
C**** INITIALIZATIONS
55 T=0.0
      JPRIN=NOPRIN
      JT=0
      DO 60 J=1,2
      NN=NNOD(J)
      DO 60 I=1,NN
      P(J,I,2)=PIN
      U(J,I,2)=UIN
      PMX(J,I)=PIN
      KITR(J,I)=0
      C(J,I,2)=CIN(J)
60 CONTINUE
      CALL PULSE(T,P1)
      P(1,1,2)=P1
      CALL MVSP(1,P1,P1,C1)
      C(1,1,2)=C1
      IF(P1.GT.PIN) PMX(1,1)=P1
      IF(NE.GT.1) GO TO 80
      U(2,NNOD(2),2)=0.0
      GO TO 8J
C**** DETERMINE INITIAL TIME STEP FOR NUMERICAL STABILITY
80 UINA=ABS(UIN)
      DT1=DZ(1)/(CEL(1)+UINA)/STBFAC
      DT2=DZ(2)/(CEL(2)+UINA)/STBFAC
      DT=AMIN1(DT1,DT2)
      GO TO 1J0
C**** PRINT RESULTS
100 IF(JPRIN.EQ.NOPRIN) GO TO 101
      JPRIN=JPRIN+1
      GO TO 120
101 JPRIN=1
      WRITE(NOUT,1015) T,DT,JT
1015 FORMAT(1H1,/,15X,'T =',E13.6,' SEC, DT =',E13.6,' SEC, JT =',
1      13,/,2X,'PIPE',2X,'NODE',9X,'X, FT',11X,'P, PSI',
2      9X,'U, FT/SEC',8X,'PMAX, PSI',8X,'C, FT/SEC',5X,'KIT')
      DO 104 J=1,2

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        WRITE(NDOUT,1016) J
1016  FORMAT(3X,13)
        NN=NNCC(J)
        DO 104 I=1,NN
            PPP=P(J,I,2)/PFAC
            PXX=AMAX1(PMX(J,I),P(J,I,2))/PFAC
            XI=1-I
            DIS=XI*DZ(J)
            WRITE(NDOUT,1017) I,DIS,PPP,U(J,I,2),PXX,C(J,I,2),KITR(J,I)
1017  FORMAT(8X,14,5E17.5,2X,14)
104  CONTINUE
        IF(T.GE.TF) GO TO 400
C**** SET VARIABLES FOR NEXT TIME STEP
120  T=T+DT
        JT=0
        DO 122 J=1,2
            NN=NNCC(J)
            DO 122 I=1,NN
                P(J,I,1)=P(J,I,2)
                U(J,I,1)=U(J,I,2)
                PMX(J,I)=AMAX1(PMX(J,I),P(J,I,2))
                C(J,I,1)=C(J,I,2)
122  CONTINUE
C***** NCDE CALCULATIONS AT EACH TIME STEP
200  DO 270 J=1,2
        NA=NNCC(J)
        THTA=DT/DZ(J)
        THTB=THTA
        JA=J
        JB=J
202  DO 270 I=1,NN
        KN=0
        KIT=0
        KPA=0
        KPB=0
        IF(J.EQ.1.AND.I.EQ.1) KN=1
        IF(J.EQ.1.AND.I.EQ.NN) KN=2
        IF(J.EQ.2.AND.I.EQ.1) GO TO 375
        IF(J.EQ.2.AND.I.EQ.NN) KN=3
        IF(KN.EQ.3.AND.NE.EQ.3) GO TO 475
C**** VALUES AT PREVIOUS TIME
C        NCDE C
        UQ=U(J,I,1)
        PQ=P(J,I,1)
        CQA=C(J,I,1)
        CQB=CQA
        PMXQ=PMX(J,I)
        PP1=PQ
        UP1=UQ
        CPA=CQA
        CPB=CQB
        UP=UQ
204  IF(KN.EQ.1) GO TO 300
C        NODE R
        UR=U(J,I-1,1)

```

```

      PR=P(J,I-1,1)
      CR=C(J,I-1,1)
      PMXR=PMX(J,I-1)
      CA=CR
      UA=UR
      UPL=0.5*(UA+UP)
      CPL=0.5*(CA+CPA)
      VPL=UPL+CPL
C     NODE S
      IF(KN.EQ.2) GO TO 350
      IF(KN.EQ.3) GO TO 210
205  US=U(J,I+1,1)
      PS=P(J,I+1,1)
      CS=C(J,I+1,1)
      PMXS=PMX(J,I+1)
206  CB=CS
      UB=US
      UMI=0.5*(UB+UP)
      CMI=0.5*(CB+CPB)
      VMI=UMI-CMI
C**** ITERATION AT POINT A
209  IF(KN.EQ.1) GO TO 216
210  KCA=0
211  KCA=KCA+1
      IF(KCA.GT.20) GO TO 920
      VPL1=VPL
      UA=UQ-THTA*VPL*(UQ-UR)
      PA=PQ-THTA*VPL*(PQ-PR)
      IF(CR.LT.CQA*RRR.AND.CR.GT.CQA/RRR) GO TO 213
      IF(KCA.GT.10) GO TO 213
      CA1=CA
      PMXA=PMXQ-THTA*VPL*(PMXQ-PMXR)
      CALL WVSP(JA,PA,PMXA,CA)
      IF(KCA.EQ.10) CA=0.5*(CA+CA1)
212  UPL=0.5*(UA+UP)
      CPL=0.5*(CA+CPA)
      VPL=UPL+CPL
      IF(VPL.LT.VPL1*RRR.AND.VPL.GT.VPL1/RRR) GO TO 214
      GO TO 211
214  CALL FRTM(JA,UA,GA)
C**** ITERATION AT POINT B
      IF(KN.EQ.3) GO TO 400
216  KCB=0
217  KCB=KCB+1
      IF(KCB.GT.20) GO TO 925
      VMI1=VMI
      UB=UQ-THTB*VMI*(US-UQ)
      PB=PQ-THTB*VMI*(PS-PQ)
      IF(CS.LT.CQB*RRR.AND.CS.GT.CQB/RRR) GO TO 219
      IF(KCB.GT.10) GO TO 219
      CB1=CB
      PMXB=PMXQ-THTB*VMI*(PMXS-PMXQ)
      CALL WVSP(JB,PB,PMXB,CB)
      IF(KCB.EQ.10) CB=0.5*(CB+CB1)
219  UMI=0.5*(UB+UP)

```

```

      CMI=0.5*(CB+CPB)
      VMI=UPI-CMI
      IF(VMI.GT.VMI1*RRR.AND.VMI.LT.VMI1/RRR) GO TC 220
      GO TO 217
220  CALL FRTRM(JP,UB,GB)
      IF(KN.EQ.1) GO TO 305
C**** ITERATION AT POINT P
230  UP=(CPL*UA+CMI*UB+(PA-PB)/DENS-DT*(CPL*GA+CMI*GB))/(CPL+CMI)
      PP=(DENS*(UA-UB+DT*(GB-GA))*CPL+CMI+PA*CMI+PB*CPL)/(CPL+CMI)
      CPA1=CPA
      CPB1=CPB
      JKP=0
      IF(KPA.GT.10) GO TO 240
      CALL WVSP(JA,PP,PMXQ,CPA)
      IF(KPA.EQ.10) GO TO 234
      IF(CPA.GT.CPA1/RRR.AND.CPA.LT.CPA1*RRR) GO TO 240
      GO TO 235
234  CPA=0.5*(CPA+CPA1)
235  JKP=1
      KPA=KPA+1
240  IF(KPB.GT.10) GO TO 250
      CALL WVSP(JB,PP,PMXQ,CPB)
      IF(KPB.EQ.10) GO TO 244
      IF(CPB.GT.CPB1/RRR.AND.CPB.LT.CPB1*RRR) GO TO 250
      GO TO 245
244  CPB=0.5*(CPB+CPB1)
245  JKP=1
      KPB=KPB+1
250  IF(JKP.GT.0) GO TO 251
      IF(ABS((PP-PP1)/PP1).LT.EPSI) GO TC 260
251  VPL=0.5*(UA+UP+CA+CPA)
      VMI=0.5*(UB+UP-CB-CPB)
      IF(KIT.EQ.15) GO TO 950
      IF(KIT.GT.15) GO TO 955
252  KIT=KIT+1
      IF(KIT.GT.20) UP=(UP+UP1)/2.0
      IF(KIT.GT.25) GO TO 960
      PP1=PP
      UP1=UP
      GO TO 209
C**** STORE RESULTS OF COMPUTATION AT NODE P
260  CALL WVSP(J,PP,PMXQ,CP)
      IF(ABS(UP).LT.0.00001) UP=0.0
      U(J,I,2)=UP
      P(J,I,2)=PP
      C(J,I,2)=CP
      KITR(J,I)=KIT
270  CONTINUE
      GO TO 600
C**** PRESSURE SOURCE AT FIRST NODE
300  CALL PULSE(T,P1)
      GO TO 205
305  CPL=CMI
      UA=UB
      PA=2.0*P1-PB

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      GA=GB
      GO TO 230
C**** LAST NODE IN SOURCE PIPE
350 JR=2
      THTB=DT/DZ(2)
      CQB=C(2,1,1)
      CPB=CPB
      US=U(2,2,1)
      PS=P(2,2,1)
      CS=C(2,2,1)
      PMXS=PMX(2,2)
      GO TO 206
C**** FIRST NODE IN TEST PIPE
375 P(2,1,2)=P(1,NNOD(1),2)
      U(2,1,2)=U(1,NNOD(1),2)
      CALL WVSP(2,P(2,1,2),PMX(2,1),C(2,1,2))
      KITR(2,1)=0
      GO TO 270
C**** LAST NODE IN TEST PIPE
400 IF(NE.EQ.1) GO TO 425
      IF(NE.EQ.2) GO TO 450
      GO TO 34
C      FAR END IS CLOSED
425 CMI=CPL
      UR=-UA
      PR=PA
      GB=-GA
      GO TO 230
C      FAR END IS AT CONSTANT PRESSURE, PIN
450 CMI=CPL
      UR=UA
      PR=2.0*PIN-PA
      GB=GA
      GO TO 230
C      FAR END IS NON-REFLECTING
475 U(J,NN,2)=U(J,NN-1,2)
      P(J,NN,2)=P(J,NN-1,2)
      C(J,NN,2)=C(J,NN-1,2)
      KITR(J,NN)=0
      GO TO 270
C**** REDETERMINE DT FOR CONVERGENCE AND NUMERICAL STABILITY
600 DTM=DZ(2)/CEL(2)
      DO 601 J=1,2
          NN=NNOD(J)
          DO 601 I=1,NN
              DTI=DZ(J)/(CEL(J)+ABS(U(J,I,2)))
601 DTM=AMIN1(DTM,DTI)
      IF(DT.LE.DTM.AND.DT.GE.DTM/STBFAC) GO TO 100
      IF(JT.GT.25) GO TO 970
      TTT=T-DT
      DT=0.5*(DTM+DTM/STBFAC)
      T=TTT+DT
      JT=JT+1
      GO TO 200
C***** END OF PROBLEM

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C**** PRINT MAXIMUM STRESSES, MAXIMUM STRAINS, AND PLASTIC STRAINS
      DO WRITE(NOUT,1040) T
1040 FORMAT(1H1,20X,'END OF COMPUTATION, T =',E13.6,' SEC',
1      //,2X,'PIPE',2X,'NODE',4X,'SIGMAX, PSI',7X,'EPSMAX',
2      9X,'EPSPLAS')
      DO 805 J=1,2
      NN=NNOD(J)
      WRITE(NOUT,1041) J
1041 FORMAT(1H0,2X,14)
      DO 805 I=1,NN
      PXX=AMAX1(PMX(J,I),P(J,I,2))
      CALL STRAIN(J,PXX,SMAX,EPS,EPSPL)
      SXX=SMAX/PFAC
      WRITE(NOUT,1042) I,SXX,EPS,EPSPL
1042 FORMAT(9X,14,3E16.6)
      DO5 CONTINUE
C**** DO NEXT PROBLEM
      R10 NPRO=NPRO+1
      IF(NPRO.LE.NPROB) GO TO 5
      STOP
C***** CONVERGENCE MESSAGES
C**** NO CONVERGENCE IN ITERATION FOR CA
      920 WRITE(NOUT,1050) J,I,T,VPL1,VPL,CA,CPA,CPL,UA,UP,UPL
1050 FORMAT(1H1,5X,'NO CONVERGENCE FOR CA FOR J =',I3,', I =',I4,
1      ', T =',E12.5,'/,10X,'VPL1 =',E12.5,', VPL =',E12.5,
2      '/,10X,'CA =',E12.5,', CPA =',E12.5,', CPL =',E12.5,
3      '/,10X,'UA =',E12.5,', UP =',E12.5,', UPL =',E12.5)
      GO TO 810
C**** NO CONVERGENCE IN ITERATION FOR CB
      925 WRITE(NOUT,1051) J,I,T,VMI1,VMI,CB,CPB,CMI,UB,UP,UMI
1051 FORMAT(1H1,5X,'NO CONVERGENCE FOR CB FOR J =',I3,', I =',I4,
1      ', T =',E12.5,'/,10X,'VMI1 =',E12.5,', VMI =',E12.5,
2      '/,10X,'CB =',E12.5,', CPB =',E12.5,', CMI =',E12.5,
3      '/,10X,'UB =',E12.5,', UP =',E12.5,', UMI =',E12.5)
      WRITE(NOUT,1060) KN,JA,JB,NN,U(2,2,1),P(2,2,1)
1060 FORMAT(1H0,5X,'KN,JA,JB,NN,U(2,2,1),P(2,2,1)',/,
1      5X,4I4,2E15.5)
      GO TO 810
C**** CONVERGENCE RESULTS FOR NODE P
      950 WRITE(NOUT,1052) J,I,JT,T,DT
1052 FORMAT(1H1,5X,'J =',I3,', I =',I4,', JT =',I3,', T =',E12.5,
1      ', DT =',E12.5,/)
      PRW=PR/PFAC
      PQW=PQ/PFAC
      PSW=PS/PFAC
      PMXRW=PMXR/PFAC
      PMXQW=PMXC/PFAC
      PMXSW=PMXS/PFAC
      WRITE(NOUT,1053) PRW,PQW,PSW,UR,UQ,US,PMXRW,PMXQW,PMXSW,
1      CR,CQA,CQB,CS
1053 FORMAT(5X,'VALUES OF PR,PQ,PS,UR,UQ,US / PMXR,PMXQ,',
1      ' PMXS,CR,CQA,CQB,CS',//,8X,6E15.5,/,4X,7E15.5,/)
      WRITE(NOUT,1054)
1054 FORMAT(5X,'VALUES OF KIT,PP,PA,PB,UP / CPA,CPB,CA,CB,UA,UB',/)
      955 PPW=PP/PFAC

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      PAW=PA/PFAC
      PBW=PB/PFAC
      WRITE(NOUT,1055) KIT,PPW,PAW,PBW,UP,CPA,CPB,CA,CB,UA,UB
1055  FORMAT(2X,I4,4E15.5,/,10X,6E15.5,/)
      GO TO 252
C**** NO CONVERGENCE IN ITERATION FOR NODE P
      960 PPW=PP/PFAC
      PPIW=PP/PFAC
      WRITE(NOUT,1056) J,I,T,UP,UP1,PPW,PPIW
1056  FORMAT(1H0,5X,'NO CONVERGENCE IN ITERATION AT NODE J =',I3,
1      ', I =',I4,' WITH T =',E13.6,/,5X,'UP =',E13.6,
2      ', UP1 =',E13.6,', PP =',E13.6,', PPI =',E13.6)
      GO TO 810
C**** NO CONVERGENCE FOR DT
      970 WRITE(NOUT,1057) DT,DTM
1057  FORMAT(1H0,5X,'NO CONVERGENCE FOR DT',/,
1      5X,'DT =',E13.6,' SEC, DTM =',E13.6,' SEC')
      GO TO 810
      END

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SUBROUTINE WVSP(J,PP,PX,CC)
C**** SUBROUTINE WHICH COMPUTES WAVE SPEED AS FUNCTION OF PRESSURE
C**** FOR VARIOUS MATERIALS
C      MAT=1 - 304 STAINLESS STEEL
C      MAT=2 - 316 STAINLESS STEEL
C      MAT=3 - LINEAR ELASTIC AND PLASTIC REGIONS WITH ELLIPTIC
C              TRANSITION
C      MAT=4 - SAME AS 3 EXCEPT FOR DIFFERENT MATERIAL CONSTANTS
C      MAT=5 - BILINER STRESS-STRAIN RELATION
C      MAT=6 - RIGID PIPE WALL
COMMON /RT/TR
1  /PW/MAT(2),BB(2),BC(2),CC,CEL(2),KW
2  /RS/A(2,10),SY(2),SA(2),EE(2),EPSY(2),RM(2),EP(2),SGO(2)
DIMENSION PY(2),GMY(2),GPA(2),CPLAS(2)
DATA NOUT/6/,RRP/1.0001/,PFAC/144.0/,ELAR/998.0E+06/
SS=PP/BB(J)
SX=PX/BB(J)
M=MAT(J)
FRIG=ELAR*PFAC
GO TO (100,200,300,400,500,600),M
WRITE(NOUT,10) J,M
10 FORMAT(1H0,5X,'MATERIAL NUMBER IS INCORRECT, J =',I3,
1      ', MAT(J) =',I3)
STOP
C**** 304 STAINLESS STEEL PROPERTIES FROM LMFBR MATERIALS HANDBOOK
C      MAT(J)=1
DATA AS1,AS2,AS3,AS4/11.29E+03,1.23,27.5E+03,10.7/,
1  AF1,AF2,AF3/3.0E-06,5.03E-06,0.0143/,
2  AF4,AF5,AF6/9.3E-06,7.23E-05,0.1877/
100 IF(KW.GT.0) GO TO 120
C      COMPUTE MATERIAL PROPERTIES
SY(J)=(AS1-AS2*TR)*PFAC
SA(J)=(AS3-AS4*TR)*PFAC
A(J,1)=AF1/PFAC
A(J,2)=AF2*TR-AF3
A(J,3)=AF4/PFAC
A(J,4)=AF5*TR-AF6
KSS=304
105 EE(J)=SY(J)/(A(J,1)*SY(J)+A(J,2))*2
CEL(J)=CD/SQRT(1.0+BC(J)/EE(J))
EPSY(J)=SY(J)/EE(J)
PY(J)=SY(J)*BB(J)
EEF=EE(J)/PFAC
SYF=SY(J)/PFAC
SAF=SA(J)/PFAC
PYF=PY(J)/PFAC
WRITE(NOUT,110) KSS,TR,EEF,CEL(J),SYF,PYF,EPSY(J),SAF
110 FORMAT(5X,'TYPE ',I3,' STAINLESS STEEL AT',E13.5,' DEG. F',/,
1  5X,'ELASTIC MODULUS =',E12.5,' PSI',/,
2  5X,'ELASTIC WAVE SPEED =',E13.6,' FT/SEC',/,
3  5X,'YIELD STRESS =',E12.5,' PSI',/,
4  5X,'YIELD PRESSURE =',E12.5,' PSI',/,
5  5X,'YIELD STRAIN =',E12.5,/,

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      6      5X,'SIGA =',E12.5,' PSI',////)
C      COMPUTE WAVE SPEED
120 IF(SS.GT.SX/RRR.AND.SS.GT.SY(J)) GO TO 130
C      DEFORMATION IS ELASTIC
      CC=CEL(J)
      RETURN
C      DEFORMATION IS PLASTIC
130 IF(SS.GT.SA(J)) GO TO 140
C      STRESS IS BETWEEN SY AND SA
      EPL=0.5/A(J,1)/(A(J,1)*SS+A(J,2))
      GO TO 150
C      STRESS IS GREATER THAN SA
140 EPL=0.5/A(J,3)/(A(J,3)*SS+A(J,4))
150 CC=CC/SQRT(1.0+BC(J)/(EPL-2.0*SS))
      RETURN
C**** 316 STAINLESS STEEL PROPERTIES FROM LMFBR MATERIALS HANDBOOK
C      MAT(J)=2
      DATA BS1,BS2,BS3,BS4/10.27E+03,2.764E+06,13.373E+03,3.616E+06/,
1      EF1,BF2,BF3/1.8E-06,3.802,3.9E-03/,
2      BF4,BF5,BF6/8.2E-06,26.945,0.0817/
200 IF(KW.GT.0) GO TO 120
C      COMPUTE MATERIAL PROPERTIES
      SY(J)=(BS1+BS2/TR)*PFAC
      SA(J)=(BS3+BS4/TR)*PFAC
      A(J,1)=BF1/PFAC
      A(J,2)=BF3-BF2/TR
      A(J,3)=BF4/PFAC
      A(J,4)=-BF5/TR-BF6
      KSS=316
      GO TO 105
C**** LINEAR ELASTIC AND PLASTIC REGIONS WITH ELLIPTIC TRANSITION
C      MAT(J)=3
C      DATA FOR NICKEL 200
      DATA EE1/30.0E+06/,EP1/0.405E+06/,SGO1/29.0E+03/,
1      GMY1/0.75/,GMA1/1.25/
300 IF(KW.GT.0) GO TO 320
C      COMPUTE MATERIAL PROPERTIES
      EE(J)=EE1*PFAC
      EP(J)=EP1*PFAC
      SGO(J)=SGO1*PFAC
      GMY(J)=GMY1
      GMA(J)=GMA1
305 RM(J)=EP(J)/EE(J)
      CPLAS(J)=CO/SQRT(1.0+BC(J)/EP(J))
      SY(J)=SGO(J)*GMY(J)
      SA(J)=SGO(J)*GMA(J)
      PY(J)=SY(J)*RB(J)
      IF(EE(J).GT.ERIG) GO TO 306
      EPSY(J)=SY(J)/EE(J)
      CEL(J)=CO/SQRT(1.0+BC(J)/EE(J))
      GO TO 307
306 RM(J)=0.0
      EPSY(J)=0.0
      CEL(J)=CO
307 WRITE(NOUT,308)

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308 FORMAT(5X,'LINEAR ELASTIC AND PLASTIC REGIONS WITH ',
1      'ELLIPTIC TRANSITION')
      IF(EE(J).GT.ERIG) GO TO 312
      EEF=EE(J)/PFAC
      WRITE(NOUT,310) EEF
310 FORMAT(5X,'ELASTIC MODULUS =',E12.5,' PSI')
      GO TO 314
312 WRITE(NOUT,313)
313 FORMAT(5X,'ELASTIC MODULUS IS INFINITE')
314 SYF=SY(J)/PFAC
      SAF=SA(J)/PFAC
      PYF=PY(J)/PFAC
      EPF=EP(J)/PFAC
      SGOF=SGO(J)/PFAC
      WRITE(NOUT,317) EPF,RM(J),CEL(J),CPLAS(J),SYF,PYF,EPF(J)
317 FORMAT(5X,'PLASTIC MODULUS =',E12.5,' PSI',/,
1      5X,'MODULUS RATIO =',E12.5,/,
2      5X,'ELASTIC WAVE SPEED =',E13.6,' FT/SEC',/,
3      5X,'PLASTIC WAVE SPEED =',E13.6,' FT/SEC',/,
4      5X,'YIELD STRESS =',E12.5,' PSI',/,
5      5X,'YIELD PRESSURE =',E12.5,' PSI',/,
6      5X,'YIELD STRAIN =',E12.5)
      WRITE(NOUT,318) SGOF,SAF,GM(J),GMA(J)
318 FORMAT(5X,'SIGO =',E12.5,' PSI',/,
1      5X,'SIGA =',E12.5,' PSI',/,
2      5X,'GAMY =',E12.5,/,5X,'GAMA =',E12.5,////)
C      COMPUTE AUXILIARY CONSTANTS FOR ELLIPTIC TRANSITION
      DY=1.0-GMY(J)
      DA=GMA(J)-1.0
      R=RM(J)
      A(J,1)=DA+K*DY
      A(J,2)=DA-R*DY
      A(J,3)=DA*GMY(J)-R*DY*GMA(J)
      A(J,4)=GMA(J)-GMY(J)
      A(J,5)=DA*DY+R*(1.0-GMY(J)*GMA(J))
      A(J,6)=DY-DA
      A(J,7)=GMY(J)**2*(1.0-R**2)*DA**2
C      COMPUTE WAVE SPEED
320 IF(SS.GT.SX/RRR.AND.SS.GT.SY(J)) GO TO 330
C      DEFORMATION IS ELASTIC
      CC=CEL(J)
      RETURN
C      DEFORMATION IS PLASTIC
330 IF(SS.LT.SA(J)) GO TO 340
C      STRESS IS IN LINEAR PLASTIC REGION
      CC=CO/SQRT(1.0+BC(J)/(EP(J)-2.0*SS))
      RETURN
C      STRESS IS IN ELLIPTIC TRANSITION REGION
340 SRAT=SS/SGO(J)
      ARG=1.0-(A(J,7)+A(J,1)*(A(J,2)*SRAT**2-2.0*A(J,3)*SRAT))*
1      A(J,6)/A(J,4)/A(J,5)**2
      IF(ARG.LT.0.0) GO TO 350
      SLOPE=EP(J)*A(J,4)*A(J,5)*SQRT(ARG)/A(J,1)/(A(J,2)*SRAT-A(J,3))
      IF(SLOPE.LE.0.0) GO TO 355
      CC=CO/SQRT(1.0+BC(J)/(SLOPE-2.0*SS))

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      RETURN
350 WRITE(NOUT,351) J,MAT(J),ARG
351 FORMAT(1H0,5X,'ARGUMENT IN WVSP IS NEGATIVE',/,
1      5X,'J =',I3,', MAT =',I3,', ARG =',E12.5)
      GO TO 360
355 WRITE(NOUT,356) J,MAT(J),SLOPE
356 FORMAT(1H0,5X,'SLOPE IN WVSP IS NEGATIVE',/,
1      5X,'J =',I3,', MAT =',I3,', SLOPE =',E12.5)
360 WRITE(NOUT,361) PP,PX
361 FORMAT(1H0,5X,'PP =',E12.5,' LBS/FT2, PX =',E12.5,' LBS/FT2')
      STOP
C**** LINEAR ELASTIC AND PLASTIC REGIONS WITH ELLIPTIC TRANSITION
C      MAT(J)=4
      DATA EE2/30.0E+06/,EP2/4.05E+05/,SGQ2/29.0E+03/,
1      GMY2/1.0/,GMA2/1.0/
400 IF(KW.GT.0) GO TO 320
      COMPUTE MATERIAL PROPERTIES
      EE(J)=EE2*PFAC
      EP(J)=EP2*PFAC
      SGQ(J)=SGQ2*PFAC
      GMY(J)=GMY2
      GMA(J)=GMA2
      GO TO 305
C**** BILINEAR STRESS-STRAIN RELATION
C      MAT(J)=5
      DATA EEB/24.8E+06/,EPB/2.48E+06/,SYB/50.0E+03/
500 IF(KW.GT.0) GO TO 520
      COMPUTE MATERIAL PROPERTIES
      EE(J)=EEB*PFAC
      EP(J)=EPB*PFAC
      SY(J)=SYB*PFAC
      IF(EE(J).GT.ERIG) GO TO 505
      RM(J)=EP(J)/EE(J)
      EPSY(J)=SY(J)/EE(J)
      CEL(J)=CO/SQRT(1.0+BC(J)/EE(J))
      GO TO 507
505 RM(J)=0.0
      EPSY(J)=0.0
      CEL(J)=CO
507 CPLAS(J)=CO/SQRT(1.0+BC(J)/EP(J))
      PY(J)=SY(J)*BB(J)
      WRITE(NOUT,508)
508 FORMAT(5X,'BILINEAR STRESS-STRAIN RELATION')
      IF(EE(J).GT.ERIG) GO TO 512
      FEF=EE(J)/PFAC
      WRITE(NOUT,310) FEF
      GO TO 514
512 WRITE(NOUT,313)
514 SYF=SY(J)/PFAC
      PYF=PY(J)/PFAC
      EPF=EP(J)/PFAC
      WRITE(NOUT,317) EPF,RM(J),CEL(J),CPLAS(J),SYF,PYF,EPSY(J)
      WRITE(NOUT,516)
516 FORMAT(///)
C      COMPUTE WAVE SPEED

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      520 IF(SS.GT.SX/RRR.AND.SS.GT.SY(J)) GO TO 530
C      DEFORMATION IS ELASTIC
      CC=CEL(J)
      RETURN
C      DEFORMATION IS PLASTIC
      530 CC=CO/SQRT(1.0+BC(J)/(EP(J)-2.0*SS))
      RETURN
C**** RIGID PIPE WALL
C      MAT(J)=6
      600 IF(KW.GT.0) GO TO 620
      WRITE(NDOUT,602)
      602 FORMAT(5X,'RIGID PIPE WALL',////)
      CEL(J)=CO
      620 CC=CO
      RETURN
      END

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      SUBROUTINE FLPRP(SPWT,RHO,AMU,CC)
C**** SUBROUTINE DETERMINES FLUID PROPERTIES AT TEMPERATURE TR, DEG.F
C**** SPWT - SPECIFIC WEIGHT, LBS/FT3
C**** RHO - DENSITY, LBS-SEC2/FT4
C**** AMU - VISCOSITY, LBS-SEC/FT2
C**** CO - SOUND SPEED, FT/SEC
C
C**** SODIUM PROPERTIES
C
      COMMON /RT/TR
      DATA DL0/59.566/,DL1/-0.79504E-02/,DL2/-0.2872E-06/,
1      DL3/0.60350E-10/,CL1/8285./,CL2/-2187./,CL3/2290./,GC/32.173/
      SPWT=DL0+DL1*TR+DL2*TR**2+DL3*TR**3
      RHO=SPWT/GC
      AMU=EXP(2.303*(1.0203+397.17/(TR+460.)-0.4925*ALOG10(TR+460.)))/
1      3600./GC
      CO=CL1+CL2*(TR-210.)/CL3
      RETURN
      END

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      SUBROUTINE FRTRM(J,U,G)
C**** SUBROUTINE WHICH COMPUTES 'WEISBACH' FRICTION FACTOR F USING
C      'COLEBROOK-WHITE' CORRELATION AND RETURNS FRICTION TERM G,
C       $G = F * U * \text{ABS}(U) / (2 * D)$ 
      COMMON /RF/ D,DENS,VISC,KFR,RF(2)
      DATA NOUT/6/
      F=0.0
      G=0.0
      IF(KFR.EQ.1) GO TO 2
      RETURN
C**** IF KFR=1, FRICTION IS INCLUDED
2  N=0
      RR=RF(J)/D
      Q=ABS(U)
      RE=DENS*Q*D/VISC
      IF(RE.LE.0.1) RETURN
      IF(RE.GT.3000.) GO TO 5
C**** LAMINAR FLOW
      F=64./RE
      GO TO 70
5  IF(RR.GT.1.E-4) GO TO 10
C**** INITIAL ESTIMATE FOR SMOOTH PIPE BY 'BLASIUS' EQUATION
      F=0.316/RE**.25
      GO TO 25
C**** INITIAL ESTIMATE FOR ROUGH PIPE BY 'VON KARMAN' EQUATION
10  F=1./(-2.*ALOG10(RR)+1.14)**2
C**** F IS DETERMINED ITERATIVELY BY 'COLEBROOK-WHITE' CORRELATION
25  SF=SQRT(F)
      GO TO 35
30  SF=FF
35  DD=1.14-2.*ALOG10(RR+9.35/(RE*SF))
      FF=1./DD
      IF((ABS(FF-SF)/SF).LE.0.01) GO TO 50
      IF(N.GE.50) GO TO 45
      N=N+1
      GO TO 30
45  WRITE(NOUT,100) J,U
50  F=FF**2
70  G=F*U*Q/(2.*D)
      RETURN
100 FORMAT(5X,'NO CONVERGENCE FOR FRICTION FACTOR',/,
1       5X,'J =',I2,', U =',E12.5,/)
      END

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      SUBROUTINE STRAIN(J,P,S,EPS,EPSPL)
C**** SUBROUTINE WHICH COMPUTES STRESS, STRAIN, AND RESIDUAL PLASTIC
C**** STRAIN FOR A GIVEN PRESSURE AND SAME MATERIAL PROPERTIES AS USED
C**** IN SUBROUTINE WVSP
      COMMON /RW/MAT(2),BB(2),BC(2),CO,CEL(2),KW
1      /RS/A(2,10),SY(2),SA(2),EE(2),EPSY(2),RM(2),EP(2),SGD(2)
      S=P/BB(J)
      M=MAT(J)
      IF(M.EQ.6) GO TO 10
      IF(S.GT.SY(J)) GO TO 20
C**** DEFORMATION IS ELASTIC
      IF(RM(J).EQ.0.0) GO TO 10
      EPS=S/EE(J)
      EPSPL=0.0
      RETURN
10  EPS=0.0
      EPSPL=0.0
      RETURN
C**** DEFORMATION IS PLASTIC
20  GO TO (100,100,300,300,500),M
      STOP
C**** MATERIAL IS 304 OR 316 STAINLESS STEEL
100 IF(S.GT.SA(J)) GO TO 120
      EPS=(A(J,1)*S+A(J,2))**2
      GO TO 130
120 EPS=(A(J,3)*S+A(J,4))**2
130 EPSPL=EPS-S/EE(J)
      RETURN
C**** LINEAR ELASTIC AND PLASTIC REGIONS WITH ELLIPTIC TRANSITION
300 IF(S.LT.SA(J)) GO TO 320
C      LINEAR PLASTIC REGION
310 EPS=(S+SGD(J)*(RM(J)-1.0))/EP(J)
      EPSPL=(S-SGD(J))*(1.0-RM(J))/EP(J)
      RETURN
C      ELLIPTIC TRANSITION REGION
320 SRAT=S/SGD(J)
      X=A(J,7)+A(J,1)*(A(J,2)*SRAT**2-2.0*A(J,3)*SRAT)
      ARG=(A(J,4)*A(J,5))**2-A(J,4)*A(J,6)*X
      EPS=SGD(J)*X/(A(J,4)*A(J,5)+SQRT(ARG))/EP(J)
      EPSPL=EPS-S*RM(J)/EP(J)
      RETURN
C**** BILINEAR STRESS-STRAIN RELATION
500 GO TO 310
      END

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      SUBROUTINE PULSE(T,P)
C**** SUBROUTINE WHICH PROVIDES INPUT PRESSURE
C**** POLYGONAL PULSE ADDED TO UNIFORM INITIAL PRESSURE
C**** VERTEX PRESSURE AP(I) OCCURS AT AT(I), 1.LE.I.LE.IPUL
      COMMON /RP/ AP(99),AT(99),KPUL,PIN,IPUL
      DATA NOUT/6/
      IF(KPUL)1,1,10
C**** PRINT PULSE VERTICES
      1 WRITE(NOUT,100) PIN
      100 FORMAT(5X,'POLYGONAL PRESSURE PULSE',/,5X,
      1      ' INITIAL AND FINAL PRESSURE IS',E12.5,' PSI',/,
      2      13X,'P, PSI',11X,'T, SEC')
      DO 6 I=1,IPUL
      PP=AP(I)+PIN
      WRITE(NOUT,101) PP,AT(I)
      101 FORMAT(9X,E13.5,4X,E13.5)
      6 CONTINUE
      RETURN
C**** RETURN P(T)
      10 DO 20 I=2,IPUL
      IF(T.GE.AT(I-1).AND.T.LE.AT(I)) GO TO 25
      20 CONTINUE
      P=PIN
      RETURN
      25 IF(AT(I-1).EQ.AT(I)) GO TO 30
      
$$P = (AP(I-1) * (AT(I) - T) + AP(I) * (T - AT(I-1))) / (AT(I) - AT(I-1)) + PIN$$

      RETURN
      30 P=AP(I)+PIN
      RETURN
      END

```

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