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GRAPHS OF THE HYPERGEOMETRIC
O.C. AND A.O.Q. FUNCTIONS FOR
LOT SIZES 10 TO 225

by

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and

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SEPTEMBER 1959

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ABSTRACT

Graphs of the hypergeometric operating characteristic (O.C.) and average outgoing quality (A.O.Q.) functions are given for lot sizes 10 to 225 and various sample sizes. Section I includes graphs for acceptance numbers 0(1)2 and Section II includes graphs for acceptance numbers 0(1)3.

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GRAPHS OF THE HYPERGEOMETRIC O.C. AND A.O.Q. FUNCTIONS FOR LOT SIZES 10 TO 225

Most of the existing collections of operating characteristic curves for sampling plans based on sampling without replacement have used binomial or Poisson approximations to the hypergeometric distribution. When the lot size is small, or the sample size is a significant fraction of the lot size, these curves may not be sufficiently accurate to provide the sampling economy necessary in many applications. For this reason, Wiesen (Ref. 1) has compiled a set of graphs of the operating characteristic (O.C.) and average outgoing quality (A.O.Q.) functions for single sampling plans based on the hypergeometric distribution which covers the range of large lot sizes N and sample sizes n (see Fig. 1, code $\times$). The present monograph extends this collection to include the additional lot sizes and sample sizes plotted in Figure 1 (codes $\bullet$, $\circ$, and $\blacktriangle$).

As noted in Figure 1, the graphs are classified into two sections according to lot and sample size. The acceptance numbers for the curves of Section I (pp.11-50) are $a = 0, 1$, and 2 , and those for Section II (pp.51-172) are $a = 0, 1, 2$, and 3 .

The functions plotted are:

a. The O.C. function, $100P(D; N, n, a)$ versus $100 \frac{D}{N}$,

where*

$$P(D; N, n, a) = \sum_{x=0}^a \frac{\binom{D}{x} \binom{N-D}{n-x}}{\binom{N}{n}} \quad (1)$$

*The binomial coefficient, $\binom{a}{b}$, is defined to be $\frac{a!}{b!(a-b)!}$ for integral values of b and a satisfying the inequalities $0 \leq b \leq a$ and is zero elsewhere.

Fig. 1 -- Values of lot size N and sample size n for which graphs of the hypergeometric O. C. and A. O. Q. functions exist

is the probability of accepting a lot of size N containing exactly D defective items based on a single sampling plan which specifies sample size n and acceptance number a ; and

b. The A.O.Q. function, $100A(D; N, n, a)$ versus $100 \frac{D}{N}$,

where

$$A(D; N, n, a) = \frac{D}{N} P(D; N, n, a) \quad (2)$$

is the average proportion defective in all outgoing lots initially containing D defective items when all defective items in rejected lots are replaced with good ones, and the proportion defective in accepted lots is unchanged by the sampling procedure. Although the O.C. and A.O.Q. functions are defined only for integral values of D , they are plotted as continuous curves to conform with past practice and to give a better visual representation of the functions.

Reference 2 contains derivations of Equations (1) and (2) along with examples illustrative of their applications. Additional theory and tables and graphs of the O.C. functions for several sampling plans based on infinite lot approximations are available in References 3, 4, and 5.

Some recent contributions to the literature on hypergeometric sampling plans are given in References 6 and 7. Reference 6 contains graphs for determining the sample size necessary to ensure high lot reliability (maximum allowable value of $D/N = 0.0001, 0.0005, 0.001, 0.005$) with various confidences (0.50, 0.90, 0.95, and 0.99). The results are graphed for acceptance numbers 0 and 1.

A convenient approximation to the sample size necessary to ensure a desired lot reliability with arbitrary confidence for acceptance number 0 is given in Reference 7.

Tables of point and summed hypergeometric probabilities for small lot sizes ($1 \leq N \leq 50$) correct to six decimal places may be obtained from Reference 8.

The subdivision of the present set of graphs into two sections is motivated by the separate uses for which they were originally intended. The first author of this paper used an LGP-30 computer to compute the numbers plotted on the

graphs contained in Section I. These graphs are to be used by the Statistics and Evaluation Division of the Reliability Department 1590, Sandia Corporation. The graphs of Section II were compiled by the second author at the request of the Quality Assurance Procedures Division of the Quality Assurance Department 5510. Figure 1 facilitates easy reference to the proper section for a given lot size and sample size.

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SECTION I

O.C. and A.O.Q. Curves for

<u>N</u>	<u>n</u>
10	2, 4, 5
20	2, 5, 8, 10
30	5(5)15
40	5(5)20
50	10(5)25
60	15(5)30
70	20(5)35
80	20(5)40
90	20(5)45

SECTION II

O. C. and A. O. Q. Curves for

N	n
40	3(1)12
50	4(1)13
60	5(1)14
70	6(1)15
80	7(1)16
90	8(1)17
100	9(1)18
125	10(1)19
150	10(1)19
175	11(1)20
200	14(1)23
225	15(1)24