RECEIVED BY DTIC SEP 28 1967

MASTER

OAK RIDGE NATIONAL LABORATORY

operated by

UNION CARBIDE CORPORATION

NUCLEAR DIVISION

for the

U.S. ATOMIC ENERGY COMMISSION

ORNL - TM - 1914

TWO-PROTON FINAL-STATE INTERACTIONS IN THE REACTIONS

${}^3\text{He}(d, t)2p$, $d({}^3\text{He}, t)2p$, ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$, and $p({}^3\text{He}, d)2p$

(Thesis)

Bobby Joe Morton

Submitted as a dissertation to the Graduate Council of the University of Tennessee in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

— LEGAL NOTICE —

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

CR-11-11-11
H.C. \$ 3.00; MN 65

ORNL-TM-1914

Contract No. W-7405-eng-26

ELECTRONUCLEAR DIVISION

TWO-PROTON FINAL-STATE INTERACTIONS IN THE REACTIONS

$^3\text{He}(d, t)2p$, $d(^3\text{He}, t)2p$, $^3\text{He}(^3\text{He}, ^4\text{He})2p$, and $p(^3\text{He}, d)2p$

Bobby Joe Morton

Submitted as a dissertation to the Graduate Council of
The University of Tennessee in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

SEPTEMBER 1967

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee
Operated by
UNION CARBIDE CORPORATION
for the
U. S. ATOMIC ENERGY COMMISSION

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:
A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
14

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

ACKNOWLEDGEMENTS

This thesis describes research carried out in the Electronuclear Division of the Oak Ridge National Laboratory, operated by the Union Carbide Corporation under contract with the U. S. Atomic Energy Commission. The research was supported by a National Science Foundation Cooperative Fellowship administered through The University of Tennessee, and by the Oak Ridge Graduate Fellowship Program of the Oak Ridge Associated Universities. The research was directed by a committee appointed by Dr. Hilton A. Smith, Vice-President for Graduate Studies and Research at The University of Tennessee. This committee was composed of Dr. R. D. Present, Dr. W. M. Bugg, and Dr. C. R. Bingham of the Department of Physics and Dr. L. H. Turner of the Department of Mathematics at The University of Tennessee; and Dr. E. E. Gross and Dr. R. M. Drisko of the Electronuclear Division of the Oak Ridge National Laboratory.

The author expresses his sincere appreciation to Dr. E. E. Gross for his direction, advice, and assistance throughout the research; also to Dr. A. Zucker, Dr. M. P. Fricke, Dr. R. O. Ginaven, Dr. E. V. Hungerford, and Dr. J. J. Malanify for much helpful advice and assistance; and to Dr. R. M. Drisko, Dr. R. M. Haybron, and Dr. Y. E. Kim for many helpful discussions on the theoretical aspects of the problem.

The author is indebted to the many members of the Electronuclear Division for their excellent cooperation during the research, particularly to the operating staff of the Oak Ridge Isochronous Cyclotron; to the plate scanning group consisting of Polly Haydon, Vestel Jones, Jean Martin, and Ruby Shelton for their help in reducing the data; and to Jane Binkley for correcting and typing the manuscript.

ABSTRACT

The energy spectra of the observed particles for the reactions ${}^3\text{He}(d, t)2p$, $d({}^3\text{He}, t)2p$, ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$, and $p({}^3\text{He}, d)2p$ were measured at forward angles using the 36-MeV deuteron beam and the 53- and 74-MeV ${}^3\text{He}$ beams from the Oak Ridge Isochronous Cyclotron. Each of the spectra were characterized by a peak in the high-energy region corresponding to the two unobserved protons having a low relative energy. These peaks were attributed to the enhancement of the cross section in this energy region by the final-state interaction of the two protons. The data were analyzed using the final-state interaction theory of Watson and Migdal. This theory gave an adequate representation of the data for the reactions ${}^3\text{He}(d, t)2p$ and ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$ using the known proton-proton interaction. However, for the other reactions, $d({}^3\text{He}, t)2p$ and $p({}^3\text{He}, d)2p$, the observed peaks were much narrower than the theoretical predictions. For the reaction $d({}^3\text{He}, t)2p$ the narrower peak could be explained by assuming the primary reaction mechanism is a charge exchange process and calculating this effect using the impulse approximation.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION.	1
II. THEORY.	5
III. EXPERIMENTAL APPARATUS AND	
PROCEDURE.	18
Broad-Range Spectrograph	18
Targets.	21
Beam Preparation	24
Resolution	30
Measurement of Absolute Cross Sections	32
Absolute Energy Measurement	33
Spectrograph Calibration	34
Recording of Data	36
IV. EXPERIMENTAL RESULTS AND ANALYSIS. . . .	41
Experimental Results	41
Analysis Using the Watson-Migdal Theory . . .	46
Impulse Approximation	53
Lifetime of the Diproton.	56
V. DISCUSSION OF RESULTS	59
VI. CONCLUSIONS	67
REFERENCES	70
APPENDIX.	73

LIST OF FIGURES

FIGURE		PAGE
1.	Layout of Experimental Equipment.	19
2.	Details of the Broad-Range Spectrograph	20
3.	Experimental Results for the Reaction $d(^3\text{He}, t)2p$ at $\theta_L = 0^\circ$ and 3° and the Theoretical Predictions of the Watson- Migdal Theory for the Indicated Values of a_p	42
4.	Experimental Results for the Reaction $d(^3\text{He}, t)2p$ and the Theoretical Predictions of the Watson-Migdal Theory for the Indicated Values of a_p	44
5.	Experimental Results for the Reaction $^3\text{He}(d, t)2p$ and the Theoretical Predictions of the Watson-Migdal Theory for the Indicated Values of a_p	45
6.	Experimental Results for the Reaction $^3\text{He}(^3\text{He}, ^4\text{He})2p$ and the Theoretical Predictions of the Watson-Migdal Theory for the Indicated Values of a_p	47
7.	Triton Spectrum from the Reaction $^3\text{He}(^3\text{He}, t)3p$	48

FIGURE		PAGE
8.	Experimental Results for the Reaction $p(^3\text{He}, d)2p$ and the Theoretical Predictions of the Watson-Migdal Theory for the Indicated Values of a_p	49
9.	Triton Spectrum from the Reaction $d(^3\text{He}, t)2p$ at $\theta_L = 0^\circ$ and the Theoretical Predictions of the Impulse Approximation. . .	54
10.	Triton Spectrum from the Reaction $d(^3\text{He}, t)2p$ at $\theta_L = 3^\circ$ and the Theoretical Prediction of the Impulse Approximation . . .	55
11.	Triton Spectrum from the Reaction $d(^3\text{He}, t)2p$ for a Bombarding Energy of 74 MeV and the Theoretical Prediction of the Impulse Approximation.	57
12.	Possible Reaction Mechanisms for the Reaction $^3\text{He} + d \rightarrow t + 2p$	63
13.	Flow Diagram for the Program FSICAL	77
14.	Flow Diagram for the Least-Squares Fitting Subroutine LSQFIT	78
15.	Flow Diagram for the Subroutine CALC Which Calculates the Final-State Enhancement for Three-Particle Breakup Reactions.	79

CHAPTER I

INTRODUCTION

For nuclear reactions with three particles in the final state, it has long been recognized that interactions between two of the particles can influence the energy distribution of each of the particles. A well-known example is the influence of the Coulomb interaction on the beta decay energy spectrum. For β^- decay the Coulomb attraction of the nucleus enhances the number of low energy electrons and for β^+ decay the number of low energy positrons is reduced by the repulsive Coulomb potential.

A method of calculating this effect for short-range nuclear interactions was developed by Watson (1952). The method depends on being able to separate the reaction into two steps, the "primary mechanism" responsible for the three-body breakup and the "final-state interaction". The final-state interaction is assumed to be an interaction between a single pair of the particles in the three-body final state. If the primary mechanism does not affect the final energy distribution or if its effect can be unfolded, the final-state interaction provides an indirect means of measuring the interaction of two particles produced in a reaction. This can, in principle, provide useful information about the interaction of particles not otherwise available as targets or as projectiles for scattering experiments. This method was used by Brueckner and Watson (1951) and Brueckner (1951) to explain the results of early meson

production experiments, also experiments involving the π^- capture by deuterium leaving two neutrons and a γ -ray in the final state (Aamodt, Panofsky, and Phillips, 1951) were analyzed using this method by Watson and Stuart (1951). A similar theory was developed by Migdal (1955) intended for the application to nuclear reactions where two nucleons are produced with low relative energies.

Other calculations have been made for nuclear reactions including final-state interactions. In the analysis of the breakup of deuterons by protons and neutrons, Frank and Gammel (1954) treat the final-state interaction between the neutron and proton in the Born approximation using zero-range potentials. A method similar to this is used by Heckrotte and McGregor (1958) to analyze the experimental data of Nakada et al. (1958) on the breakup of deuterium by 8.9-MeV protons. Calculations using more realistic potentials have been made recently by Koehler and Mann (1964) and by Koehler (1965). Recently Henley, Richards, and Yu (1965) using various Born approximations found it necessary to include final-state interaction effects to explain the data of Jakobson, Manley, and Stokes (1965) on the $d(^3\text{He}, t)2p$ reaction at 21 MeV and the $^3\text{He}(d, t)2p$ reaction at 11.7 and 14.0 MeV.

Final-state interactions continue to be of interest, particularly from the point of view of determining the 1S_0 scattering length of two neutrons, a_n , in order to test the assumption of charge symmetry of nuclear forces. Recently (Baumgartner et al., 1966) the ^3He spectra at laboratory angles

of 6° and 8° from the reaction $t(d, {}^3\text{He})2n$ were analyzed using the final-state interaction theory of Watson (1952) and Migdal (1955) and gave a value of $a_n = -16.1 \pm 1.0$ F. The application of the simple Watson-Migdal theory was justified by the analysis of the triton spectrum at a laboratory angle of 8° from the reaction ${}^3\text{He}(d, t)2p$ for the same c.m. energy (Baumgartner et al., 1966). Using this theory a value of $a_p = -7.41 \begin{smallmatrix} + .39 \\ - .49 \end{smallmatrix}$ F was found, which agrees with the results of low energy scattering of free protons (Hulthén and Sagawara, 1957). The value of a_n from this experiment agrees with the recent experimental results from the π^- capture by deuterium by Haddock et al. (1965), where a value of $a_n = -16.4 \pm 1.3$ F was found. The value of a_n from both of these experiments is in agreement with the theoretical prediction of Heller, Signell, and Yoder (1964) based on the charge symmetry of nuclear forces and the low energy p-p scattering results. In this calculation they calculated the scattering length from various potentials found to reproduce the proton-proton scattering neglecting the Coulomb part of the potential. In disagreement with the above results, similar analyses (Cerineo et al., 1964 and Voitoveskii et al., 1964) of the proton spectra from the deuteron breakup reaction $d(n, p)2n$ for an incident neutron energy of 14 MeV gave $a_n = -21.2$ F and $-23.6 \begin{smallmatrix} + 2.0 \\ - 1.6 \end{smallmatrix}$ F. A difference between experiment and the Watson-Migdal theory has also been observed (Tombrello and Bacher, 1965) in the analysis of the final-state interaction of two protons for the

reaction ${}^3\text{He}(p, d)2p$ for 12-MeV incident protons. The high energy peak in the deuteron spectrum was found to be much narrower than the prediction of the Watson-Migdal theory.

For this thesis problem it was decided to further test the Watson-Migdal theory by studying several reactions involving two protons in the final state. By considering reactions involving two protons in the final state, the final-state interaction is known and it is possible to test the applicability of the theory. The reactions chosen were ${}^3\text{He}(d, t)2p$, $d({}^3\text{He}, t)2p$, $p({}^3\text{He}, d)2p$, and ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$ using the 36-MeV deuteron beam and the 53- and 74-MeV ${}^3\text{He}$ beams from the Oak Ridge Isochronous Cyclotron (ORIC). These reactions allow the study of the validity of the approximations used in the Watson-Migdal theory for a variety of reaction mechanisms, since the second reaction is probably dominated by a charge exchange process and the others probably by a pickup or stripping process. In fact the first two reactions involve the same particles in the final state and the beam energies were chosen so the three particles would have the same energy in the center-of-mass system; thus they only differ in the angle of detection of the observed particle.

CHAPTER II

THEORY

The final-state interaction theory has recently been summarized by Gillespie (1964). The treatment here follows closely the early development by Watson (1952). His approach was based on the assumption that the reaction process could be separated into two parts. The first step of the process is the breakup of the two colliding particles into three particles, the probability of this taking place being independent of the final-state interaction. The second step is then the interaction of a single pair of particles in the final state. For the final-state interaction effects to be observable and easily calculable, three conditions were specified by Watson. The first was that the three-particle breakup be due to a short-range interaction. It was suggested by Watson that any elementary particle interaction other than the electromagnetic interaction would probably satisfy this requirement. Second, the two particles for which the final-state interaction is being considered should have low relative energies. This condition can be achieved most easily by observing the third particle at small angles with respect to the incoming beam, then the region of the energy spectrum near the high energy kinematic limit corresponds to low relative energy of the other two particles. Finally, for the effect to be easily observable, the final-state interaction should be strong and attractive.

To gain an intuitive idea of the process, consider the time-reversed reaction starting from the three-particle final state. There the strong attractive force between two of the particles interacting in the final state causes them to come together and momentarily stick while the third particle approaches. This enhances the probability that the three particles will react to produce the original two, and by applying the principle of detailed balance, the three-particle breakup reaction should be correspondingly enhanced.

For a more formal approach, consider an initial state characterized by the state function χ_a , where χ_a describes the relative motion of the original particles and any internal coordinates necessary to describe the system, and satisfies the Schrödinger equation

$$H_0 \chi_a = E_a \chi_a . \quad (1)$$

H_0 is the Hamiltonian operator when the particles are too far apart to interact and is essentially the kinetic-energy operator. The χ_a are a complete set of eigenfunctions of H_0 and may be considered to be plane waves or the wave-packet extension of plane waves (Goldberger and Watson, 1964). We will be interested in transitions to the state described by χ_b , containing three particles, and satisfying the equation

$$H_0 \chi_b = E_b \chi_b . \quad (2)$$

The complete Hamiltonian will be $H_0 + U$, where U is the interaction which gives rise to the transition from state "a" to state "b". In order to study the final-state interaction, U will be decomposed into $U = V + v$, where V is defined as $U - v$, and v is the specific interaction between a pair of particles to be considered as a final-state interaction. The potential v may be thought of as vanishing except when operating on the state b containing the pair of interacting particles, i. e., it may be considered to be multiplied by a projection operator which picks out state b . This implies there will be no connection between states a and b through the interaction v or $(\chi_b, v \chi_a) = 0$. Only the potential V is assumed able to create state b from the original state a .

The state function for the system including the interaction is given by the solution to the Lippman-Schwinger equation (Lippman and Schwinger, 1950)

$$\psi_a^\pm = \chi_a + \frac{1}{E - H_0 \pm i\epsilon} (V + v) \psi_a^\pm . \quad (3)$$

A similar equation may be written for the particles interacting in the final state through the potential v :

$$\varphi_b^\pm = \chi_b + \frac{1}{E - H_0 \pm i\epsilon} v \varphi_b^\pm . \quad (4)$$

The transition matrix element giving the probability for the transition from state a to state b is given by

$$T_{ba} = \langle \chi_b, (V + v) \psi_a^+ \rangle . \quad (5)$$

Substituting from equation (4) using the φ_b^- form gives

$$T_{ba} = \langle \varphi_b^-, \frac{1}{E - H_0 - i\epsilon} v \varphi_b^-, (V + v) \psi_a^+ \rangle .$$

Using

$$(V + v) \psi_a^+ = (E - H_0 + i\epsilon) (\psi_a^+ - \chi_a)$$

from equation (3) and regrouping terms yields

$$T_{ba} = \langle \varphi_b^-, (V + v) \psi_a^+ \rangle - \langle \frac{1}{E - H_0 - i\epsilon} v \varphi_b^-, (E - H_0 + i\epsilon) \psi_a^+ - \chi_a \rangle .$$

Making use of the principle of superposition and Hermiticity of the operators, i.e., $(\hat{F} + \hat{G})\psi = \hat{F}\psi + \hat{G}\psi$ and $\langle F^+ \psi_a, \psi_b \rangle = \langle \psi_a, F \psi_b \rangle$ where the adjoint operation implies transposing and complex conjugation, we find

$$\begin{aligned} T_{ba} &= \langle \varphi_b^-, (V + v) \psi_a^+ \rangle - \langle \varphi_b^-, (\frac{1}{E - H_0 - i\epsilon} v)^+ (E - H_0 + i\epsilon) (\psi_a^+ - \chi_a) \rangle \\ &= \langle \varphi_b^-, (V + v) \psi_a^+ \rangle - \langle \varphi_b^-, (v \frac{1}{E - H_0 + i\epsilon}) (E - H_0 + i\epsilon) (\psi_a^+ - \chi_a) \rangle \\ &= \langle \varphi_b^-, (V + v) \psi_a^+ \rangle - \langle \varphi_b^-, v (\psi_a^+ - \chi_a) \rangle . \end{aligned}$$

The last step involves assuming the ϵ 's used in equations (3) and (4) are the same. This point is discussed further by Waston (1952). The complex conjugation above makes it necessary to use the φ^-

form of the final-state wave function. Now, using the superposition principle gives

$$\begin{aligned} T_{ba} &= \langle \varphi_b^-, V \psi_a^+ \rangle + \langle \varphi_b^-, v \psi_a^+ \rangle - \langle \varphi_b^-, v \psi_a^+ \rangle + \langle \varphi_b^-, v \chi_a \rangle \\ &= \langle \varphi_b^-, V \psi_a^+ \rangle + \langle \varphi_b^-, v \chi_a \rangle . \end{aligned}$$

Then for the assumption that v does not connect the initial state χ_a to the final state the second term is zero. Thus the final transition amplitude may be written

$$T_{ba} = \langle \varphi_b^-, V \psi_a^+ \rangle . \quad (6)$$

For a reaction of the type $A + B \rightarrow C + 2N$, the differential cross section for the production of C going off into solid angle Ω with energy E in the c.m. system is

$$\frac{d^2\sigma}{dE d\Omega} = \frac{2\pi}{\hbar} \frac{1}{v_a} \left| T_{ba} \right|^2 \rho(E) , \quad (7)$$

where v_a is the relative velocity of the two initial particles, $\rho(E)$ is the density of states or phase space factor given by $E^{1/2} (E_{\max} - E)^{1/2}$, where $E_{\max}$ is the kinematic maximum energy allowed for particle C . E is related to the relative momentum k of the two interacting nucleons through the conservation of energy.

When the two nucleons have a small relative momentum, i. e., satisfy Watson's third requirement, the scattering solution of the Lippman-Schwinger equation ψ_a^+ will be approximately the

state function for a two-body system consisting of particle C and a composite particle 2N. The dependence on the nucleon-nucleon interaction v will be weak, and most of the final-state interaction effects will be contained in the φ_b^- , which is essentially the continuum wave function for a two-nucleon system. The φ_b^- are given by the solution of the differential equation

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r}) - \frac{L^2}{r^2} + k^2 - v \right] \varphi_b^- = 0 ,$$

where L^2 is the angular momentum operator. Since only low relative energies of the two nucleons are to be considered, the partial wave expansion keeping only S-wave terms will be used. This should be valid up to relative energies of several MeV since the P-wave phase-shift has been shown to be negligible in nucleon-nucleon scattering up to about 10 MeV (Hulthén and Sugawara, 1957).

For n-n or n-p final-state interactions where the Coulomb effects may be neglected, the asymptotic form of the wave function may be written

$$\varphi_b^-(kr) = e^{-i\delta(k)} \sin [\delta(k) + kr]/kr . \quad (8)$$

$\delta(k)$ is the S-wave phase shift. Expanding $\sin (\delta + kr)$

$$\varphi_b^-(kr) = e^{-i\delta} (\sin \delta \cos kr + \cos \delta \sin kr)/kr .$$

For $kr \ll 1$, $\cos kr \simeq 1 - k^2 r^2/2$ and $\sin kr \simeq kr - k^3 r^3/6$, neglecting terms of order $k^2 r^2$, φ_b^- is given by

$$\varphi_b^-(kr) = e^{-i\delta} \sin \delta (1 + kr \cot \delta)/kr .$$

Now using the scattering length and effective range representation for δ discussed by Jackson and Blatt (1950), $k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \dots$

$$\varphi_b^- = e^{-i\delta} \sin \delta (1 - \frac{r}{a} + \frac{1}{2} r r_0 k^2 + \dots)/kr .$$

Neglecting the terms of order $rr_0 k^2$ and higher powers of k gives

$$\varphi_b^- = e^{-i\delta} \frac{\sin \delta}{kr} (1 - \frac{r}{a}) = e^{-i\delta} \frac{\sin \delta}{k} f(r) , \quad (9)$$

where the k and r dependence have been separated. As discussed by Watson (1952) and Bethe (1949), this separability is expected to also apply inside the range of nuclear forces. There the function $f(r)$ may not be the simple form given above, but will be the radial solution of the Schrödinger equation with the nuclear potential, and should be insensitive to k since the kinetic energy, k^2 , is small compared to the nuclear potential depth. Also, in the region of the interaction, kr will be less than one and the power series expansion of terms like $\cos kr$ and $\sin kr$ will be more accurate. Using the above form of φ_b^- and assuming ψ_a^+ is at most slowly varying with k , the transition amplitude may be written

$$T_{ba} = e^{-i\delta} \frac{\sin \delta}{k} \langle f(r), V \psi_a^+ \rangle = T_0 e^{-i\delta} \frac{\sin \delta}{k} , \quad (10)$$

where the T_0 gives the probability of the three-particle breakup and $e^{-i\delta} \sin \delta/k$ is an enhancement factor due to the final-state interaction which only depends on the relative momentum of the two nucleons and their interaction potential.

For final-state interactions involving two protons, the asymptotic form of φ_b^- including Coulomb effects may be written

$$\varphi_b^-(kr) = e^{-i\delta} (F_0 \cos \delta + G_0 \sin \delta)/kr . \quad (11)$$

F_0 and G_0 are the regular and irregular S-wave Coulomb functions.

For $kr \ll 1$

$$F_0 = C(\eta) kr \left(1 + \frac{r}{2R} + \dots\right)$$

and

$$G_0 = C^{-1}(\eta) \left(1 + \left[\frac{r}{R} \left(\ln \frac{r}{R} + h(\eta) + 2\gamma - 1\right) + \dots\right] + \dots\right) ,$$

where $C(\eta) = [2\pi\eta/(\exp(2\pi\eta) - 1)]^{1/2}$, $h(\eta) = \text{Re}[\Gamma'(-i\eta)/\Gamma(-i\eta)] - \ln(\eta)$, $R = \frac{\hbar^2}{2\mu e^2} = 2.88 \times 10^{-12}$ cm, and $\eta = \frac{e^2}{\hbar v}$ where v is the relative velocity of the two protons. Using the scattering length and effective range expansion

$$C(\eta) k \cot \delta + h(\eta)/R = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \dots \quad (12)$$

and neglecting terms of order $k^2 r^2$, $k^2 r r_0$, and r/R , we find

$$\varphi_b^-(kr) = e^{-i\delta} \frac{\sin \delta}{C(\eta) kr} \left(1 - \frac{r}{a}\right) = e^{-i\delta} \frac{\sin \delta}{C(\eta) k} f(r) . \quad (13)$$

The transition amplitude may then be written

$$T_{ba} = T_o e^{-i\delta} \sin \delta / C(\eta) k \quad (14)$$

similar to the non-Coulomb case, except now the phase shift and scattering length are related by equation (12). For the Coulomb case the cross section may be written

$$\frac{d^2\sigma}{dE d\Omega} = \frac{|g(\theta)|^2 C^2(\eta) \rho(E)}{C^4(\eta) E_{2p} + \frac{\hbar^2}{m_p} \left[-\frac{1}{a_p} - \frac{h(\eta)}{R} + \gamma_p E_{2p} \right]^2} \quad (15)$$

where E_{2p} is the relative energy of the two protons and proportional to k^2 , and $\gamma_p = r_o m_p / 2\hbar^2$. The $g(\theta)$, where θ is the c.m. angle of the observed particle, includes T_o and all factors independent of E_{2p} and in effect was an arbitrary normalization factor. This equation including the Coulomb effects is the same as the equation (6) of Migdal (1955).

As will be discussed more completely later, the theory as formulated above does not agree with the experimental results for the reaction $d(^3\text{He}, t)2p$. To improve the agreement it was believed that a more detailed treatment of the primary three-particle breakup was required. It was noted that the shape of the peak due to the final-state enhancement was very similar to that predicted for the reaction $d(p, n)2p$ by Phillips (1964) assuming a charge-exchange process. Clearly a similar process could be responsible for the $d(^3\text{He}, t)2p$ reaction. As discussed by Phillips, using the impulse approximation, the transition amplitude is given by the

overlap integral of the initial deuteron wave function φ_d and the two-proton final-state function φ_b^-

$$T_{ba} = T_o \int \varphi_b^-(kr) \varphi_d(r) \exp(i \underline{Q''} \cdot \underline{r}/2) d^3r . \quad (16)$$

Q'' is the momentum transferred to the two-proton system and T_o is assumed to contain all factors independent of k . A Hulthén form was used for the deuteron wave function:

$$\varphi_d = [\exp(-.232 r) - \exp(-1.202 r)]/r ,$$

and the form of φ_b^- of equation (11) was used (before the separation of the k and r dependence). Since Q'' , the momentum transferred to the two-nucleon system, is small and weakly dependent on the two-nucleon relative momentum k , the term $\exp(i Q'' \cdot r/2)$ is very nearly equal to unity and only slowly dependent on k . Therefore, this term will be neglected and the transition amplitude given by

$$T_{ba} = T_o \int \varphi_b^-(kr) \varphi_d(r) d^3r . \quad (17)$$

The impulse approximation calculation of Phillips can be seen to be equivalent to the Born approximation of Henley, Richards, and Yu (1965) if one neglects the k dependence of Q'' . The name "impulse approximation" comes from the fact that only one of the nucleons in the projectile is assumed to interact with one of the nucleons in the target nucleus. For the reaction $d(^3\text{He}, t)2p$ the charge-exchange process is assumed to take place by one of the protons in the ^3He undergoing a charge exchange with the neutron

in the deuteron. In the plane-wave Born approximation the transition amplitude for this process may be written

$$\begin{aligned}
 T_{ba} = & 2V_o N \int \exp(-\beta^2 Y^2) \exp(i \underline{Q}'' \cdot \underline{Y}) d^3 Y \quad X \\
 & \int \exp(-i \frac{1}{2} \underline{Q}'' \cdot \underline{r}) \varphi_b^- \varphi_d d^3 r \int \exp(-i \frac{1}{3} \underline{Q}'' \cdot \underline{Z}) v'^*(Z) v'(Z) d^3 Z \quad X \\
 & \int \exp(i \frac{1}{2} \underline{Q}'' \cdot \underline{\chi}) v^*(\chi) v(\chi) d^3 \chi \quad X \\
 & \langle \xi_{3_{\text{He}}} (1, 2, 3) \xi_{pp} (4, 5) \left| \frac{1 - \frac{1}{4} A (1 + \underline{\sigma}_1 \cdot \underline{\sigma}_4) (1 + \underline{\tau}_1 \cdot \underline{\tau}_4)}{2} \right| \\
 & \xi_{3_{\text{He}}} (1, 2, 3) \xi_d (4, 5) \rangle \quad (18)
 \end{aligned}$$

similar to the form given by Henley, Richards, and Yu (1965). In equation (18) σ and τ are the spin and isospin of the interacting nucleons, and the ξ 's are anti-symmetric spin-isospin functions.

A Gaussian form of the three-particle wave functions was used:

$$\begin{aligned}
 \psi(1, 2, 3) = & N \exp \left\{ -\gamma^2 \left[(\underline{r}_1 - \underline{r}_2)^2 / 2 + (\underline{r}_2 - \underline{r}_3)^2 + (\underline{r}_3 - \underline{r}_1)^2 \right] \right\} \\
 = & N v(\underline{r}_1 - \underline{r}_2) v' \left[\underline{r}_3 - \frac{1}{2} (\underline{r}_1 + \underline{r}_2) \right] .
 \end{aligned}$$

The variables used are defined as follows:

$$\begin{aligned}\underline{X} &= \underline{r}_1 - \underline{r}_2 \\ \underline{r} &= \underline{r}_4 - \underline{r}_5 \\ \underline{y} &= \underline{r}_3 - (\underline{r}_4 + \underline{r}_5)/2 \\ \underline{Z} &= \underline{r}_3 - (\underline{r}_1 + \underline{r}_2)/2 \\ \underline{Y} &= \underline{r}_1 - \underline{r}_4 = \underline{y} - \underline{Z} + (\underline{X} - \underline{r})/2 .\end{aligned}$$

The subscripts 1 and 2 refer to the two protons in the original ${}^3\text{He}$; and subscript 3, the neutron. Subscript 4 refers to the neutron in the deuteron; and 5, the proton. The reaction is assumed to take place by a charge-exchange spin-exchange process due to the interaction potential

$$V(1, 4) = V_0 \exp[-\beta^2(\underline{r}_1 - \underline{r}_4)^2] \left[1 - \frac{1}{4} A (1 + \underline{\sigma}_1 \cdot \underline{\sigma}_4) (1 + \underline{\tau}_1 \cdot \underline{\tau}_4)\right]/2$$

between particles 1 and 4. Now, by neglecting the k dependence of Q'' , all of the integrals from the Born approximation except the one involving the φ_b and φ_d may be included in the above-mentioned T_0 , and all that is left is the impulse approximation integral given by Phillips (1964). The form of the potential only enters the calculation of T_0 which was considered as an arbitrary normalization factor.

As pointed out by Phillips (1964), the charge-exchange process calculated in the impulse approximation predicts a much narrower peak than the Watson-Migdal theory. This difference for the impulse approximation arises from the approximation in the Watson-Migdal calculation that the k and r dependence can be separated over the

region of interaction. If one uses the form of φ_b^- given by equation (13), where the k and r dependence are separated, rather than that of equation (11) in the impulse approximation overlap integral, the transition amplitude becomes

$$T_{ba} = T_o \int e^{-i\delta} \frac{\sin \delta}{C(\eta)k} f(r) \varphi_d(r) \exp(i \underline{Q}'' \cdot \underline{r}/2) d^3r .$$

The $e^{-i\delta} \sin \delta$ factors out of the integral and the transition amplitude reduces to the form given by equation (10). This ability to separate k and r depends on the assumption that $kr \ll 1$ over the region of the interaction. This is apparently a good approximation if only small values of k are considered or the region in which the primary reaction takes place is small. The charge-exchange process is perhaps due to the long-range tail of nuclear forces arising from a single pion exchange and apparently violates the assumption of Watson of a small primary reaction volume. The larger reaction volume may also be in part due to the diffuse nature of the deuteron.

CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURE

The external beam from the Oak Ridge Isochronous Cyclotron (ORIC) was used in this experiment. Since energy resolution was of primary importance, it was decided to use the broad-range spectrograph recently installed at ORIC. The arrangement of the experimental equipment is shown in Figure 1. The extracted beam from the cyclotron was focused on the 4-jaw collimator at the entrance to the 153° analyzing magnet by two quadrupole doublets, Q-1 and Q-2. The energy analyzed beam emerging from the exit slit was then brought to a focus at the center of the spectrograph scattering chamber by two quadrupole triplets, Q-3 and Q-5. A horizontal focus was required between Q-3 and Q-5 for reasons to be discussed later. Data were taken for the reactions $d(^3\text{He}, t)2p$, $^3\text{He}(d, t)2p$, $p(^3\text{He}, d)2p$, and $^3\text{He}(^3\text{He}, ^4\text{He})2p$, using the 53- and 74-MeV $^3\text{He}^{++}$ and 36-MeV deuteron beams.

Broad-Range Spectrograph

The spectrograph is of the type described by Borggreen, Elbek, and Nielsen (1963). A more specific description of the system as installed at ORIC is given by J. B. Ball (1966). The spectrograph consists of a homogeneous field sector magnet with non-normal entrance and exit angles as shown in Figure 2. The

Figure 1. Layout of experimental equipment.

ORNL-DWG 67-6024

Figure 2. Details of the broad-range spectrograph.

particles enter the region of the magnetic field at an angle of 35° with respect to the normal to the magnetic field boundary and exit at an angle of -36.3° .¹ Also the point of entry into the field region lies on an extension of the exit boundary line. This assures that the exit angle of -36.3° and the total deflection angle of 107.4° will be independent of the radius of curvature of the particle path in the magnetic field. The fringing field due to the positive entrance angle and negative exit angle produce a focusing effect in the vertical or axial direction. However, the focal length is much larger than the focal length for the motion in the horizontal plane and a first order focus is not achieved at the focal plane, but some reduction in vertical spot size is achieved. The spectrograph was designed to cover a range of radii from 30 to 63 inches and to operate with fields as large as 13.5 kilogauss. This makes it capable of analyzing most particles from reactions involving the most energetic particles accelerated by ORIC.

Targets

In order to take full advantage of the resolution capabilities of the spectrograph, it is necessary to use thin targets. Since the incoming and outgoing particles have different rates of energy loss

¹The minus sign for the exit angle indicates the exit ray lies on the opposite side of the normal to the field boundary from the center of curvature for the path in the magnetic field, the same side is indicated by a plus sign.

in the target, a spread in the energy of the outgoing particles occurs depending on the depth in the target where the reaction takes place. In addition to the effect of "reaction depth", another small contribution to the energy spread results from statistical fluctuations in the energy lost in the target or "straggling".

For the deuterium target, a solid deuterated polyethylene foil was used. This was possible since all reaction products from the ^{12}C have a smaller momentum to charge ratio than the tritons from the $d(^3\text{He}, t)2p$ reaction and would not cause a background problem. The target was prepared by the Isotopes Division of the Oak Ridge National Laboratory. An isotopic analysis showed a hydrogen to carbon ratio of 1.98 to 1.99 and of the hydrogen, 98.31 percent was deuterium. From the material received two pieces were mounted in target holders. The thicknesses of the mounted targets were 0.3 and 0.5 mils or 0.53 and 0.95 mg/cm^2 . The thickness was measured by measuring the beam energy using the spectrograph set at 0° with and without the target in place. This method is probably only accurate to about 10 percent and agrees with a measurement using a micrometer. The error in the micrometer measurement is probably of the same order of magnitude for targets this thin.

For the proton target a solid polyethylene target could not be used due to the background of particles from reactions in ^{12}C . For this reason a gas target was designed and used for the reactions involving a proton or ^3He target. An important criteria in the design

was the desirability of taking data at small angles where the final-state enhancement is maximized. To take data at small angles and not have particles from reactions in the windows get into the spectrograph, it is necessary to have the target windows a large distance from the center of the chamber. This was achieved by using the spectrograph scattering chamber as a gas target and installing entrance and exit windows. The windows, of 0.00009 inch thick Havar foil,² were placed at the entrance to the chamber, 11.75 inches upstream from the center and 6 inches downstream from the center of the chamber at the exit port, which rotates with the magnet and is connected to the chamber with a sliding vacuum seal. In order to determine the effective volume of the gas target and the angular acceptance of the magnet, a collimating slit system was used. It consisted of a 4-jaw collimator located 32.8 inches from the center of the chamber and with micrometer read-out of the position of all four jaws together with a 1/16 inch x 1/4 inch slit milled from 0.060 inch thick tantalum positioned just in front of the exit window about 6 inches from the center of the chamber. In order to flush out the atmospheric gases in the target chamber, the chamber was first pumped out until a pressure of 1-10 microns of Hg was achieved. After a small amount of the filling gas

²Procured from the Hamilton Watch Company, Lancaster, Pennsylvania.

was allowed to flow in, the chamber was pumped out again. The chamber was then filled to the desired operating pressure with the target gas. For the normal operating pressure of 380 mm of Hg this procedure should result in less than one part in 10^6 of atmospheric contaminants. However, since the target is operated at less than atmospheric pressure, some N_2 and O_2 can leak in during the run. The pressure was recorded before and after each run and did not change enough to detect using a mercury manometer sensitive to ± 1.0 mm. This only limits the maximum contaminant to one part in 380. The contaminants leaking in from the atmosphere could have been reduced by operating the target at a pressure greater than atmospheric. However, this would have increased the effective target thickness and resulted in poorer resolution. The background from the contaminant was not found to be troublesome, and in fact proved helpful by providing a check of the energy calibration and resolution of the spectrograph.

Beam Preparation

Another consideration in achieving the best possible resolution is the beam preparation. It is also important from the point of view of maximizing the beam current in the scattering chamber in order to minimize the cyclotron time required to perform the experiment. The beam optics system is shown in Figure 1. To determine the best operating conditions for the beam optics, several calculations were made using the computer

program "OPTIK" (Devlin, 1961). The program OPTIK utilizes the linear matrix method for calculating the effect of a series of elements, such as a magnetic field, quadrupole field or a field free region, on a beam of particles. The beam may be represented by a six-element vector consisting of the horizontal and vertical displacement and angular divergence from the central ray and the deviation of momentum and mass from the central value. The effects of each optic element are represented by a transformation operator. A series of elements then may be combined using simple matrix multiplication. The program also can calculate the field parameters of a quadrupole doublet or triplet, necessary to bring about a specified focal condition. The conditions found to transmit the largest percent of the beam through the analyzing magnet with minimum energy spread, spot size, and angular divergence were: (1) to use Q-1 to make the beam coming out of the cyclotron parallel, and (2) to use Q-2 to produce a focus in both the horizontal and vertical planes at the entrance slit to the analyzing magnet.

The analyzing magnet is an inhomogeneous field ($n = 1/2$) magnet with a radius of curve of 72 inches and a deflection angle of 153° . Operated in this way it has equal entrance and exit focal lengths and a magnification of unity. The fractional spread in energy of particles exiting from the magnet is given by

$$\frac{\Delta E}{E} = \frac{1}{2r_0} (S_1 + mS_2) ,$$

where S_1 and S_2 are the entrance and exit slit widths, m the magnification which is unity, and r_0 the radius of curvature for the central ray and equal to 72 inches for this magnet. Thus $\Delta E/E$ may be written

$$\frac{\Delta E}{E} = \frac{1}{144} (S_1 + S_2) ,$$

and since $\Delta E/E = 2\Delta P/P$, the momentum spread is given by

$$\frac{\Delta P}{P} = \frac{1}{288} (S_1 + S_2) . \quad (19)$$

To determine the optimum conditions from this point on, it is necessary to consider the final beam desired in the spectrograph scattering chamber. As discussed by Ball (1966), some cancellation in the dispersions of the spectrograph and analyzing magnet can be brought about. This can be best understood by considering all factors which can cause the deviation of a particle from the central ray at the focal plane of the spectrograph. One of the equations resulting from the linear matrix method of treating the beam transport is

$$x = a_{11} x_0 + a_{12} x'_0 + a_{13} \Delta P/P \quad (20)$$

where x is the deviation in position at the focal plane and x_0 , x'_0 , and $\Delta P/P$ are the coordinates specifying the deviation in position, angle, and momentum from the central ray at some previous point.

This is assuming the motions in the horizontal and vertical planes are independent. If the initial position is taken at the center of the spectrograph scattering chamber, the object is to minimize x by the preparation of x_o , x_o' , and $\Delta P/P$. The position of the focal plane is chosen such that there is a first order focus in the horizontal plane, this means the matrix element a_{12} is equal to zero and the x position is independent of the angular spread from the target. The remaining two elements, x_o and $\Delta P/P$, are correlated since x_o is the beam particle position at the target which is the image produced by the beam optics of a point at the exit of the 153° analyzing magnet. At the exit of the analyzing magnet there is a correlation between momentum and position. In fact from equation (19) for the resolution of the analyzing magnet if $S_1 = 0$ the deviation in position at the exit slit and the deviation in momentum are related by

$$(\Delta P/P)_a = \frac{1}{288} x_a .$$

With S_1 not equal to zero the $\Delta P/P$ may be separated into a part correlated with position and an uncorrelated part as follows:

$$(\Delta P/P)_a = \frac{1}{288} x_a + \frac{1}{288} S_1 . \quad (21)$$

The position at the scattering chamber is related to the position at the exit slit by

$$x_o = Mx_a \quad (22)$$

where M is the magnification of the beam optics system which produces an image of the exit slit S_2 at the center of the spectrograph chamber and is adjustable to some degree by varying the currents in the quadrupoles $Q-3$ and $Q-5$. Finally the equation for the position at the focal plane may be written

$$x = a_{11} Mx_a + a_{13} \left[\frac{1}{288} x_a + \frac{1}{288} S_1 + (\Delta P/P)_T \right]$$

where $(\Delta P/P)_T$ is the additional spread in momentum of the reaction particles due to the depth in the target where the reaction takes place, straggling, and other effects produced by the target. For the ORIC broad-range spectrograph the matrix elements may be written (Devlin, 1961)

$$a_{11} = .36912 - 1.43478 \left(\frac{14.7648 + .95424 r}{57.3912 + r} \right)$$

and

$$a_{13} = 1.29904 r$$

Then the contribution to the final resolution introduced by S_2 may be eliminated by the proper choice of M . Setting

$$a_{11} Mx_a + \frac{a_{13}}{288} x_a = 0$$

and solving for M gives

$$M = - \frac{a_{13}}{288 a_{11}}$$

For a 60-inch radius of curvature in the spectrograph this cancellation is accomplished for $M = 0.5$. For a 30-inch radius a value of $M = 0.36$ is required. From a calculation using the above-mentioned beam optics program (OPTIK), values of M from 0.2 to 2.0 can be achieved easily by a variation in currents through the various elements of the quadrupoles. The above arguments require that the sign of M be positive, i.e., the energy gradient at the target must be the same as the energy gradient at the exit slit of the analyzing magnet. In light optics for a simple lens the object is inverted. If the beam optics were operated as a single thin lens, M would be negative and no cancellation would occur. This is averted by requiring the first quadrupole, Q-3, to form an intermediate image in the horizontal plane in front of Q-5; this gives one inversion. Then this image is used as an object for Q-5 and a second image is formed at the center of the chamber, with another inversion. The net result of the two inversions is an image with the desired energy gradient. The variation in magnification is accomplished by varying the position of this intermediate focus. The net result is that the momentum spread, or energy spread, introduced by the finite width of S_2 can be eliminated from the overall resolution. Nothing can be done to cancel the momentum spread which is uncorrelated with position and it just has to be minimized by using thin targets and a narrow slit at S_1 .

Resolution

Some of the factors which contribute to the energy resolution in the final spectrum have been discussed. Another effect is the kinematic spread due to the angular acceptance of the spectrograph and the fact that the energy of the reaction products varies with the angle of emission. This effect is more important where the target nucleus is lighter than the projectile because then the change of energy with angle is larger.

The various contributions to the resolution are summarized in Table I. The energy spread from straggling was negligible in most cases. Also when the incoming and outgoing particles have similar rates of energy loss in the target, as for the last two reactions listed in the table, the contribution from reaction depth is negligible. The largest contribution in most cases was the kinematic spread resulting from the dependence of the energy of the outgoing particle on the angle of scattering. Two factors contributed to the large energy spread from kinematics; one is the angular divergence in the beam which is partly due to small angle scattering in the windows, and the second is the angular acceptance of the collimating system.

The total energy spread for the various reactions indicated in Table I was calculated assuming all the individual contributions were uncorrelated. As mentioned in the previous section, part of the energy spread in the incident beam was cancelled by the dispersion of the spectrograph. This cancellation was observed

TABLE I
SUMMARY OF CONTRIBUTIONS TO OVERALL RESOLUTION
FOR THE REACTIONS INDICATED

Reaction	Beam Energy (MeV)	Laboratory Angle (Deg.)	Energy Spread (keV)				Total
			In Beam	Due to Straggling	Reaction Depth	Kinematics	
$d(^3\text{He}, t)2p$	53	0	40	8	25	25	62 ^a
$d(^3\text{He}, t)2p$	53	3	74	8	25	60	100 ^b
$d(^3\text{He}, t)2p$	74	0	102	11	60	28	122 ^c
$p(^3\text{He}, d)2p$	53	5	74	22	7	250	260
$^3\text{He}(^3\text{He}, \alpha)2p$	53	5	74	30	0	185	200
$^3\text{He}(d, t)2p$	36	5	50	17	0	95	110

^a Assuming the cancellation of dispersions of the analyzing magnet and spectrograph, the total energy spread would become 40 keV.

^b Assuming the cancellation, this would be 75 keV.

^c Assuming the cancellation, this would be 84 keV.

in the reaction $^{12}\text{C}(^3\text{He}, t)^{12}\text{N}$. In this reaction the various contributions were the same as those listed in the table for the reaction $d(^3\text{He}, t)2p$ except the kinematic contribution which was only 4 keV instead of 25. With the assumption of no cancellation this would predict a total energy spread of 48 keV. If the cancellation discussed in the previous section occurs, the total spread should be 33 keV. The observed width of the line was about 34 keV, indicating the cancellation does occur. Then assuming this cancellation takes place, the energy spread expected in the reaction $d(^3\text{He}, t)2p$ would be 40 keV for the 0° data and 75 keV for the 3° data for a bombarding energy of 53 MeV and 84 keV for the data taken using the 74-MeV beam. This cancellation does not occur for the data taken using a gas target.

Measurement of Absolute Cross Sections

As mentioned earlier, the final-state interaction theory does not predict the absolute value of the cross section. Therefore, no attempt was made to measure the absolute value accurately. However, it will be interesting to estimate the absolute value and compare the results for the various reactions. Some of the problems involved are the measurement of the beam current, the calibration of the solid angle accepted by the spectrograph, and the measurement of the target thickness.

For the 0° data a solid target was used and the target thickness and solid angle were fairly well determined. The beam current was also measured by a Faraday cup placed at the appropriate spot along the focal plane. However, it was not well determined what percent of the beam was lost between the target and the Faraday cup. By applying careful attention to these details, a more accurate determination of absolute cross section could be made. For data at angles other than 0° the incident beam has to be measured inside the scattering chamber. For large angles this is easy but for small angles one has to be careful that the beam monitor does not block some of the reaction particles and does not miss some of the beam particles. For the gas target there is the additional problem that the beam has to be stopped inside the gas target and part of the beam charge can be neutralized by the collection of electrons from the ionization of the target gas. Also for the gas target the determination of the solid angle and target thickness is more difficult. The effective volume of the gas and the solid angle accepted by the magnet are related to the beam spot size and the collimator slit system.

Absolute Energy Measurement

At these energies it becomes difficult to measure an absolute energy accurately. The energy of a beam from a cyclotron is determined by the radio frequency applied to the dee, the magnetic

field, and the extraction radius. In a spiral ridge cyclotron like ORIC, the particle orbits are not perfect circles and it is difficult to determine the radius of curvature at the extraction radius sufficiently well to precisely determine the beam energy and an external measurement is used. The 153° analyzing magnet can be used for this but there are some problems in calibrating it accurately. By surveying in the entrance and exit slits, the radius of curvature can be accurately determined but it is more difficult to measure the magnetic field accurately since it is an inhomogeneous field magnet. The current furnished to the coil windings can be measured to about five significant digits with a digital voltmeter and a calibrated shunt resistor but the effects of hysteresis in the magnet yoke prevents knowing the field this accurately. Also the field at a point off the central axis can be measured with an NMR probe but the relationship between the field at the position of the probe and the field at the central orbit is not well known. Also this relationship can change with the total excitation since hysteresis effects are more important in the region where the field is stronger. A better calibration and study of the effects of field strength is planned for the future.

Spectrograph Calibration

The broad-range spectrograph if calibrated would also provide an accurate absolute energy measurement. From field measurements its field was found to be homogeneous to a high

degree of accuracy over a large part of the field area. During normal operation this field can be measured with an NMR probe to about one Gauss or one part in 10^5 . However, the radius of curvature is not as accurately determined due to fringing field effects. In order to calibrate a point in the focal plane, a ^{212}Po source was used which provides a monoenergetic alpha beam of 8.78 MeV. Then for a fixed field the effective radius of curvature for this point was determined. To determine the corresponding radius of curvature for other points along the focal plane, the magnet was set at 0° and a 20-MeV proton beam put through it. The magnetic field was then changed in steps in order to record the beam spot in emulsions at approximately two-inch intervals along the focal plane. Using the known radius at one point, the radius of each of the other points was calculated. The results were summarized by an empirical equation for the radius as a function of position along the focal plane as follows:

$$r = 27.512 + 0.19389 x - 8.2777 \times 10^{-5} x^2 + 2.0628 \times 10^{-7} x^3,$$

where x is the position along the focal plane from an established index point and r is the radius of curvature in inches. For this calibration part of the field settings were made by approaching the final setting by increasing the field and part of them were made by decreasing the field to the desired setting. It has been suggested that this procedure could result in erroneous results and the calibration should be repeated using a standardized method of setting the field. This point is to be investigated at a later date.

Recording of Data

The energy spectrum of the observed outgoing particle was measured using nuclear emulsions along the focal plane of the broad-range magnetic spectrograph. For most of the plate exposures a beam of about 100 nA was used. For these beam currents, exposure times of 3 to 5 hours were required. For some earlier runs 25 micron thick 1 inch x 6 inch Kodak type NTB nuclear emulsions were used. They were loaded in lightproof aluminum holders having a 5-mil aluminum foil entrance window in a photographic darkroom. Then the holders were placed along the focal plane of the spectrograph for exposure. Later after testing plates exposed to higher levels of background light, we used 50 micron thick 2 inch x 10 inch Kodak type NTB emulsions which could be loaded directly in the spectrograph in subdued light consisting of the emergency light in the experimental room with a red lens and portable safelights with yellow filters. It was found that these emulsions can stand a considerable amount of red or yellow light without harmful effects.

During some of the earlier runs using the ^3He beam at 0° , an unexpected background was found. This was rather surprising since the tritons from the $d(^3\text{He}, t)^2\text{p}$ reaction were expected to have a larger radius of curvature in the spectrograph than any other particle. The background was found to be caused by the doubly charged beam particles picking up an electron in the target then losing it at some point in the magnetic field through an interaction

with the residual gas in the vacuum system, or picking up an electron in the residual gas. The background was identified by its variation in track length along the focal plane. All particles from the center of the scattering chamber following a normal path through the spectrograph cross the focal plane at an angle of 37° independent of position along the focal plane. The background was peaked at the position where the singly charged ^3He particles should appear and had the normal track length. However, away from the peak some of the tracks became shorter indicating the particles come through the emulsion at a different angle which would be true if they lost the extra electron in the region of the magnetic field. Also some of the tracks were longer than normal, indicating an increase of radius of curvature somewhere in the magnetic field as would be the case if one of the doubly charged beam particles picked up an electron in the residual gas. After recognizing the origin of the background, it was easily eliminated by placing enough aluminum absorber in front of the emulsions to stop the ^3He particles but to let the tritons through. The ^3He particles quickly lose the extra electron in the absorber, then have a much larger rate of energy loss since the energy loss of a charged particle of a given mass is proportional to the square of the charge. Using the absorbers we were able to take data with practically no background as demonstrated by very few tracks past the kinematic limit for tritons from the reaction being studied.

With the gas target a first attempt was made to take data at 3° . Due to the angular divergence in the beam and the small angle scattering in the windows, a large background was produced by the beam particles. By moving the magnet to 5° , satisfactory data could be taken.

After exposure the emulsions were developed according to the following procedure. The emulsions were developed for 30 minutes in a solution consisting of one part 100 percent D-19 Kodak developer and 6 parts of tap water, the solution was held at 68°F throughout development; then they were rinsed in running tap water for about 5 minutes; then fixed using full strength Kodak fixing solution for at least 30 minutes longer than required for clearing; the plates were then washed again in running tap water for about an hour and allowed to dry in a dust-free atmosphere. The developing process could be shortened for exploratory runs by using a stronger developing solution and shortening the fixing and washing times so the plates could be looked at under a microscope after about two hours.

The data was then retrieved by scanning the plates with a standard Bausch and Lomb microscope with a modified mechanical stage. The position of the mechanical stage was indicated by a dial indicator which reads from 0 to 7 cm in 0.01 mm steps to facilitate the accurate positioning of the plate. Most of the scanning was done using a 43 power objective lens and a 10 power eyepiece. This gives a circular field of view slightly larger than 250 microns

in diameter. The projection of the valid tracks on the surface of the 50 micron thick emulsion is about 67 microns long and thus easy to distinguish from miscellaneous background tracks and easy to count. The usual procedure was to use a reticle in the eyepiece of the microscope of a calibrated width and count all tracks starting at the top of the emulsion in the strip swept out by the reticle as the emulsion is moved under the microscope. Using this method the track counting was very reproducible. The number of counts in a strip of fixed width was converted to the differential cross section ($d^2\sigma/dEd\Omega$) by correcting for the fact that a fixed strip width corresponds to a different energy increment (dE) and different solid angle ($d\Omega$) at different points along the focal plane. The change in energy increment is due to the non-linearity of the energy scale in a magnetic spectrograph, and the change in solid angle occurs because the vertical magnification of the spectrograph is a function of the radius of curvature.

For the $^3\text{He}(^3\text{He}, ^4\text{He})2p$ reaction, there was a large background from the various hydrogen isotopes produced by other reactions in the gas target. This made the scanning difficult since nothing could be done experimentally to eliminate them while retaining the ^4He particles from the desired reaction. Although they could not be eliminated, they could be distinguished from the alpha tracks by the track density. This led to slower and much more difficult scanning since one had to make a decision based on track density concerning each track to be counted. This

additional difficulty added to the uncertainty in track identification and caused the fluctuations in this data to be larger than that expected from counting statistics alone. In spite of this the shape of the alpha spectrum was still well determined.

CHAPTER IV

EXPERIMENTAL RESULTS AND ANALYSIS

Experimental Results

The triton spectra for the reaction $d(^3\text{He}, t)2p$ for laboratory angles of 0° and 3° were measured using a 53-MeV ^3He beam. They are shown in Figure 3. The data points are shown as filled circles and typical errors from counting statistics are indicated by vertical lines with cross bars. To reduce the statistical error 500-600 counts per channel were accumulated in the vicinity of the peak. The smooth curves are calculated spectra which will be discussed later. The scale at the bottom is labeled to indicate the triton energy, E_t , and the relative energy of the two protons, E_{2p} . The vertical scale represents the differential cross section in arbitrary units. The data at 0° was taken during a 145-minute exposure with an average beam of 85 nA. This implies an absolute cross section of about 30 mb/sr-MeV in the vicinity of the peak. The uncertainty is hard to estimate and might be as large as 50 percent. The 3° data represents a six-hour exposure. For this data the beam current could not be measured because of the lack of room for a beam monitoring device in the scattering chamber while taking data at 3° . However, the beam current was estimated to be from 20 to 50 nA implying a cross section of 13 to 32 mb/sr-MeV in the region of the peak. The 0° data was taken with an overall resolution of 40 keV FWHM as

Figure 3. Experimental results for the reaction $d(^3\text{He}, t)2p$ at $\theta_L = 0^\circ$ and 3° and the theoretical predictions of the Watson-Migdal theory for the indicated values of a_p .

indicated by the line width of the tritons from the reaction $^{12}\text{C}(^3\text{He}, t)^{12}\text{N}$. The resolution for the 3° data was about 75 keV FWHM. The additional spread was due to kinematic effects which become important at angles other than 0° and the fact that wider slits were used in the 153° analyzing magnet. Similar data taken at 0° using a 74-MeV ^3He beam are shown in Figure 4. This data required a 240-minute exposure with a 20 nA beam, giving an absolute cross section of about 140 mb/sr-MeV, again with large uncertainties. The resolution was approximately 84 keV FWHM.

The triton energy spectrum from the reaction $^3\text{He}(d, t)^2\text{p}$ is shown in Figure 5. This spectrum was recorded at a laboratory angle of 5° using a ^3He gas target and a 36-MeV deuteron beam. This energy was chosen to give the same c.m. energy as for the 53-MeV ^3He beam on a deuterium target. The only difference between this reaction and the previous one is the c.m. angle of the observed tritons. It is expected that a different reaction mechanism may dominate at these two different angles. The cross section for this reaction in the vicinity of the peak was estimated to be 90 mb/sr-MeV with the large error limits discussed earlier for data taken with a gas target. The narrow line at a triton energy of 31.46 MeV may be attributed to the reaction $^{14}\text{N}(d, t)^{13}\text{N}$ on the nitrogen contaminant from the atmosphere. From the width of this line the overall resolution was about 100 keV FWHM.

Figure 4. Experimental results for the reaction $d(^3\text{He}, t)2p$ and the theoretical predictions of the Watson-Migdal theory for the indicated values of a_p .

ORNL-DWG 67-2382R

Figure 5. Experimental results for the reaction ${}^3\text{He}(d, t)2p$ and the theoretical predictions of the Watson-Migdal theory for the indicated values of a_p .

The alpha particle spectrum taken at 5° for the reaction ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2\text{p}$ is shown in Figure 6. The data were taken using the 53-MeV ${}^3\text{He}$ beam and the ${}^3\text{He}$ gas target with a resolution of approximately 200 keV FWHM. Due to the uncertainty in particle identification, the fluctuations of the data points were larger than expected from counting statistics. However, the shape of the final-state enhancement peak was well determined. The cross section was estimated to be about 100 mb/sr-MeV. Shown in Figure 7 is the triton spectrum near the upper kinematic limit for the reaction ${}^3\text{He}({}^3\text{He}, \text{t})3\text{p}$. This was taken simultaneously with the above alpha particle spectrum and does not show any peak due to the enhancement by the final-state interaction between three protons.

The deuteron spectrum taken at 5° for the reaction $\text{p}({}^3\text{He}, \text{d})2\text{p}$ using the 53-MeV ${}^3\text{He}$ beam is shown in Figure 8. The resolution for this experiment was about 260 keV FWHM. This large energy spread was due mainly to the kinematic effect which is large when the target particle is lighter than the projectile. The cross section was estimated to be about 2.5 mb/sr-MeV.

Analysis Using the Watson-Migdal Theory

The first attempt to explain the above reported experimental data utilized the final-state interaction theory of Watson and Migdal which was described in Chapter II. Equation (15) gives the relative cross section in the overall c. m. system for the production of two protons with a relative energy of $E_{2\text{p}}$ in the c. m. system of

Figure 6. Experimental results for the reaction ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$ and the theoretical predictions of the Watson-Migdal theory for the indicated values of a_p .

Figure 7. Triton spectrum from the reaction ${}^3\text{He}({}^3\text{He}, t){}_3\text{p}$.

Figure 8. Experimental results for the reaction $p(^3\text{He}, d)2p$ and the theoretical predictions of the Watson-Migdal theory for the indicated values of a_p .

the two protons. A computer program was written to convert the observed triton energy for each data point to the corresponding two-proton energy and to evaluate equation (15) numerically for this value of E_{2p} . The c.m. energy spectrum is then converted to the laboratory system by multiplying by the Jacobian for the transformation and each of the theoretical points is spread over a Gaussian distribution corresponding to the overall resolution. This theoretical cross section is then compared with the experimental data. Also included in the computer program is the ability to vary the proton-proton scattering length, a_p , and the effective range, r_o , to achieve the best fit to the data. The best fit is determined by minimizing the quantity χ^2 , where χ^2 is defined by

$$\chi^2 = \sum_j \left| \frac{\sigma_j(\text{Exp}) - \sigma_j(\text{theoretical})}{\Delta\sigma_j} \right|^2 .$$

The σ_j are the experimental or theoretical values of the cross section for each data point and the $\Delta\sigma_j$ is the uncertainty in the experimental value. The uncertainty used was just that arising from counting statistics alone which is just the square root of the number of counts. Also included in the program was the possibility of only utilizing a specified region of the data in the fitting procedure. This allows fitting only the region of the spectrum corresponding to small relative energies of the two protons where the approximations made in the theory are better.

In general the shape of the theoretical curve was not sensitive to the effective range parameter and throughout the calculations it was held fixed at the value determined from low energy proton-proton scattering ($r_0 = 2.65 \text{ F}$) (Hulthén and Sugawara, 1957). The sensitivity to changes in the scattering length is indicated in Figure 3. For this data the scattering length determined from low energy free p-p scattering ($a_p = -7.69 \text{ F}$) (Hulthén and Sugawara, 1957) clearly does not reproduce the shape of the peak. If this curve is normalized to go through the low points at the left, assuming this is close to the phase-space continuum, it would predict too little enhancement. To predict the magnitude of the enhancement from this assumption would require an $a_p = -16.0 \text{ F}$, the short dashed curve. However, this scattering length shifts the peak to smaller relative proton energies. A least squares fit to the data for the region $0.0 \leq E_{2p} \leq 0.5 \text{ MeV}$ requires an $a_p = -11.1 \pm 0.7 \text{ F}$ and to fit the region $0.0 \leq E_{2p} \leq 1.0 \text{ MeV}$ requires an $a_p = -12.6 \pm 0.6 \text{ F}$. All errors quoted are standard deviations determined by finding the value of a_p required to cause the χ^2 to change by one standard deviation ($\sqrt{2\chi^2}$). Using a similar procedure to fit the 3° data over the region $0.0 \leq E_{2p} \leq 0.5 \text{ MeV}$ gives $a_p = -11.0 \pm 0.7 \text{ F}$, and fitting the region $0.0 \leq E_{2p} \leq 1.0 \text{ MeV}$ gives $a_p = -12.0$. Figure 4 shows the results of similar calculations for the $d(^3\text{He}, t)2p$ reaction for a bombarding energy of 74 MeV. Fitting the region $0.0 \leq E_{2p} \leq 0.5 \text{ MeV}$ gave $a_p = -8.3 \pm 0.8$, and for the region $0.0 \leq E_{2p} \leq 1.0 \text{ MeV}$,

$a_p = -10.2 \pm 0.7$. These values are closer to the free p-p value. However, the overall character of the fits is very similar to that at the lower energy.

In contrast with the above results, as shown in Figure 5, the Watson-Migdal theory reproduces fairly well the general shape of the spectrum for the ${}^3\text{He}(d, t)2p$ reaction with the scattering length found from free p-p scattering. A least squares fit to the region $0.0 \leq E_{2p} \leq 1.5$ MeV gives a scattering length $a_p = -7.3 \pm 0.6$ F. This is in agreement with the experimental results of Baumgartner et al. (1966) for the same reaction using a 29.8-MeV deuteron beam.

As can be seen from Figure 6 the Watson-Migdal theory gives a reasonably good fit for the ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$ reaction with a scattering length between 6.5 and 9.0 F. The prediction still rises too fast and then does not fall off fast enough for large values of E_{2p} , but a reasonable fit is achieved over a wider range of E_{2p} than for the $d({}^3\text{He}, t)2p$, and the fit is achieved using a scattering length not differing greatly from the value found from free p-p scattering.

The theoretical predictions for the $p({}^3\text{He}, d)2p$ reaction are shown in Figure 8. Here, as in Figure 3, no value of the scattering length gives an accurate prediction of the shape of the deuteron spectrum. The width of the peak is much less than that predicted using the free p-p scattering parameters and the Watson-Migdal theory of final-state interactions.

Impulse Approximation

Considering the inability of the Watson-Migdal theory to reproduce the above discussed experimental results, a second method was tried. This consisted of an impulse approximation calculation for the charge exchange process as discussed by Phillips (1964) and in Chapter II. For this calculation a subroutine was incorporated in the computer program to calculate the transition amplitude from equation (17) rather than the Watson-Migdal form given by equation (14). The transformation from c.m. to the laboratory system and the procedure for folding in the instrumental resolution were carried out as in the previous calculations. The results of this calculation for the triton spectrum at 0° from the reaction $d(^3\text{He}, t)2p$ taken using the 53-MeV ^3He beam are shown in Figure 9. A scattering length of -7.7 F for the two-proton interactions now predicts a much narrower peak than the Watson-Migdal theory and is in fair agreement with the experimental results. The best fit determined by a least squares search gives a scattering length of $-9.5 \pm 0.4\text{ F}$. This is larger than the value determined from free p-p scattering by several standard deviations, but an excellent fit to the data is achieved. The 3° data were analyzed by the same method and the best fit is shown in Figure 10. Here the fit in the vicinity of the peak is very good for a scattering length of -8.15 ± 0.31 which is very close to the value from free p-p scattering. The data taken at 0° using a 74-MeV ^3He beam along with the results of the impulse approximation calculation are

ORNL-DWG 67-5202

Figure 9. Triton spectrum from the reaction $d(^3\text{He}, t)2p$ at $\theta_L = 0^\circ$ and the theoretical predictions of the impulse approximation.

Figure 10. Triton spectrum from the reaction $d(^3\text{He}, t)2p$ at $\theta_L = 3^\circ$ and the theoretical prediction of the impulse approximation.

shown in Figure 11. Here the fits are excellent and with a scattering length of -7.75 ± 0.44 F, in excellent agreement with the value from free p-p scattering. Perhaps this reflects an improvement in the validity of the impulse approximation as the bombarding energy is increased.

Lifetime of the Diproton.

Another interesting comparison of the results from the various reactions can be made. If, in the language of high energy physics, one considers the peak to be due to a resonance in the two-proton system, then the width of the peak is a measure of the mean lifetime of the unstable diproton. Assuming the width and lifetime are related by $\Delta E \Delta t = \hbar$, a summary of estimated widths and lifetimes are given in Table II. The widths ΔE are full width at half maximum measured from the relative energy of the two protons in their c.m. system. The estimated lifetime of 3.3×10^{-22} sec for the reaction ${}^3\text{He}(d, t)2p$ is in fair agreement with the estimate of Bilaniuk and Slobodrian (1963). Their estimate of the lifetime of the diproton was $2.0 \pm 1.0 \times 10^{-22}$ sec. From a calculation using low energy p-p phase shifts, they also arrived at a theoretical estimate of the mean lifetime to be 2×10^{-22} sec in good agreement with the experimental results. The longer lifetime from the other reactions could be attributed to a decrease in the width of the peak due to the preference for low relative energies for the two protons by the reaction mechanism. A similar result has been observed in reactions leaving two alpha particles in the final state (Berkowitz, 1964).

Figure 11. Triton spectrum from the reaction $d(^3\text{He}, t)2p$ for a bombarding energy of 74 MeV and the theoretical prediction of the impulse approximation.

TABLE II
SUMMARY OF PEAK WIDTHS AND CORRESPONDING LIFETIMES FOR THE
DIPROTON FROM THE REACTIONS INDICATED

Reaction	FWHM (MeV)	Lifetime (sec)
$d(^3\text{He}, t)2p$ (53 MeV)	0.9	7.33×10^{-22}
$d(^3\text{He}, t)2p$ (74 MeV)	1.0	6.6×10^{-22}
$p(^3\text{He}, d)2p$ (53 MeV)	1.0	6.6×10^{-22}
$^3\text{He}(d, t)2p$ (36 MeV)	2.0	3.3×10^{-22}
$^3\text{He}(^3\text{He}, ^4\text{He})2p$ (53 MeV)	2.5	2.6×10^{-22}

CHAPTER V

DISCUSSION OF RESULTS

All the data described in the previous chapter, with the exception of that shown in Figure 7 for the reaction ${}^3\text{He}({}^3\text{He}, t)3p$, are characterized by a peak near the high-energy kinematic limit. For all the reactions the position of the sharp rise as measured in the spectrograph agreed with the kinematic limit calculated from the beam energy within the accuracy of our absolute energy measurements (± 100 keV). The region where the peak occurs, therefore, corresponds to the production of two protons with low relative energy and the peak is attributed to the enhancement of the cross section for this region by the final-state interaction between the two protons. These peaks could also be attributed to a sequential process involving particle-unstable excited states in the residual nucleus after the first proton comes out. However, it is unlikely that the excited level would always be such as to produce the peak near the upper kinematic limit for the observed particle. Also for the three-nucleon system ($p + d$) from the reaction $p({}^3\text{He}, d)2p$, no excited level is known to exist. For the reaction $d({}^3\text{He}, t)2p$ the reported excited levels in ${}^4\text{He}$ at 19.94 and 21.25 MeV (Parker et al., 1965) would produce tritons having a maximum energy at least 12 MeV less than the kinematic maximum from the three-particle breakup. In the reaction ${}^3\text{He}({}^3\text{He}, \alpha)2p$ the sequential decay of the 16.65-MeV excited state

of ^5Li could contribute alpha particles near the kinematic limit. However, the peak observed was more similar to the other two-proton final-state interaction peaks than the broad symmetric peak attributed to the sequential decay of the ground state of ^5Li observed in the similar reaction for a bombarding energy of 12 MeV by Tombrello and Bacher (1965). For these reasons the peaks were attributed to the final-state interaction of two protons.

The calculated values for a_p using the Watson-Migdal theory are summarized in Table III. The lower limit on the scattering length is the value which best approximates the sharp rise beginning at $E_{2p} = 0$. The upper limit is the scattering length required to give the right magnitude for the peak if one normalizes the theoretical cross section at a point off the peak. These values are to be compared with the value of -7.69 ± 0.01 determined from low-energy free proton-proton scattering (Hulthén and Sugawara, 1957).

A most interesting fact is that the theory gives a reasonable fit to the results from the reaction $^3\text{He}(d, t)2p$ and not for the reaction $d(^3\text{He}, t)2p$. Since the beam energies were chosen so the relative motion of the deuteron and ^3He in the center-of-mass system is the same for both reactions, the only difference is the angle the outgoing triton makes to the incident deuteron. For the $^3\text{He}(d, t)2p$ reaction the triton was observed at an angle of 5° to the incident deuteron. For the reaction $d(^3\text{He}, t)2p$ the triton was observed at an angle of 180° relative to the incident deuteron.

TABLE III
SUMMARY OF SCATTERING LENGTHS WHICH GIVE
A BEST FIT TO THE EXPERIMENTAL DATA
USING THE WATSON-MIGDAL THEORY

Reaction	Beam Energy	Laboratory Angle	Scattering Length
$d(^3\text{He}, t)2p$	53 MeV	0°	-11.1 to -16.0 F
$d(^3\text{He}, t)2p$	53 MeV	3°	-11.0 to -16.0 F
$d(^3\text{He}, t)2p$	74 MeV	0°	-8.3 to -14.0 F
$p(^3\text{He}, d)2p$	53 MeV	5°	-8.6 to -14.0 F
$^3\text{He}(d, t)2p$	36 MeV	5°	-7.3 ± 0.6 F
$^3\text{He}(^3\text{He}, ^4\text{He})2p$	53 MeV	5°	-6.5 to -9.0 F

(or 0° relative to the ^3He). A possible difference in the two reactions is that a different reaction mechanism may dominate at the different c.m. angles. Diagrams indicating several possible reaction mechanisms are shown in Figure 12. Reaction mechanisms represented are: (a) charge exchange, (b) two-nucleon transfer or knock-out, (c) pickup or stripping with large-angle scattering, (d) pickup or stripping, (e) two-nucleon transfer or knock-out with large-angle scattering, and (f) charge exchange with large-angle scattering. One might expect that mechanism (a) dominates the reaction where the triton comes out parallel with the incident ^3He . However, for the second case it is probably true that mechanism (d) dominates. Mechanisms (b) and (e) are characteristic of higher order terms which should be negligible. Evidence to support the pickup mechanism is the diffraction pattern characteristic of direct interactions observed in the angular distribution for this reaction at lower energies by Jakobson, Manley, and Stokes (1965). The problem then is to try and understand how the different mechanisms affect the energy spectrum of the observed particle.

There are two differences in mechanisms (a) and (d). One difference is that in (a) the two protons come from the original deuteron and in (d) both protons come from the ^3He . The second difference is the momentum transferred to the two-proton system. When the triton continues in the direction of the incident ^3He , the incident and outgoing particles have nearly the same mass and approximately the same kinetic energy. Therefore, in order

Figure 12. Possible reaction mechanisms for the reaction ${}^3\text{He} + d \rightarrow t + 2p$.

to conserve energy and momentum, the momentum transferred to the two-proton system is very small. For the other case where the triton continues in the direction in the lab of the incident deuteron, the incoming and outgoing particles have different masses and to conserve momentum a large momentum has to be transferred to the two-proton system.

For the case where the final two protons come from the initial deuteron, and essentially zero momentum is transferred to the two-proton system, it is natural to assume the cross section for producing two protons with a given relative momentum depends on the relative momentum of the two nucleons in the initial deuteron. Then a measure of the yield would be the degree of overlap of the two-proton wave function and the deuteron wave function. This is just what appears in the charge exchange calculation in the impulse approximation, discussed in Chapter II. The excellent agreement of this calculation with the experimental results for a 74-MeV ^3He beam, as shown in Figure 11, might be taken as an indication that this is the dominant mechanism. The somewhat poorer agreement with the 53-MeV data, as shown in Figures 9 and 10, might be an indication that the impulse approximation is not as well justified at this energy. However, even for this data the impulse approximation gives much better agreement than the Watson-Migdal calculation.

For mechanism (c) and (d) the two protons come from the ${}^3\text{He}$ and the appropriate overlap is the overlap of the two-proton part of the ${}^3\text{He}$ wave function with the two-proton final-state wave function. Since the ${}^3\text{He}$ is much more tightly bound than the deuteron, this means the two protons from the ${}^3\text{He}$ have a smaller spatial distribution, or a larger momentum distribution, than two protons from the deuteron. This would indicate that the overlap with the two protons in the final state would be better over a wider range of two-proton relative momenta, producing a peak spread over a wider range of two-proton relative energies. This also gives an indication of why the Watson-Migdal calculation gives a better fit to the experimental data where the pickup mechanism dominates. At the c. m. angle where the charge-exchange mechanism is expected to dominate, the two protons come from the original deuteron, which has a mean radius of about 5 F; thus for $k = 0.16 \text{ F}^{-1}$ corresponding to $E_{2p} = 1 \text{ MeV}$, $kr \approx 0.8$. When the two protons come from the ${}^3\text{He}$ particle which has a mean radius of about 2 F, $kr \approx 0.3$ and the assumption that $kr \ll 1$ made in the Watson-Migdal theory is a better approximation.

For the other reactions the results are not as conclusive. The ${}^3\text{He}({}^3\text{He}, {}^4\text{He})2p$ reaction presumably takes place through a pickup process similar to (d), except the neutron from the ${}^3\text{He}$ is now picked up by the other ${}^3\text{He}$. To support the hypothesis of a pickup mechanism, a pronounced diffraction pattern in the angular distribution recently has been observed (Slobodrian et al., 1967) for

this reaction. For this reaction the results are fairly well reproduced by the Watson-Migdal theory. The fit is not as good as that for the previous reaction but the data are much poorer. For the $p(^3\text{He}, d)2p$ reaction, two reaction mechanisms are possible to produce a deuteron continuing in the direction of the incident ^3He particle, the most likely being the pickup (or knockout) of a proton from the ^3He by the incoming proton (in the c.m. system). An alternative method would be the pickup of a neutron from the ^3He by the incoming proton to form a deuteron, then the subsequent scattering at 180° . For the case where the two protons come from the ^3He particle, it was argued above that the Watson-Migdal theory should be applicable. However, for this reaction the experimental peak is much narrower than the prediction of the Watson-Migdal theory and may be due to an interference between the two mechanisms.

CHAPTER VI

CONCLUSIONS

This experimental program was started with the broad idea of achieving a better understanding of the interactions involved in few-nucleon systems. A more limited goal was to test the applicability of the final-state interaction theory for intermediate energy nuclear reactions and to lend confidence to neutron-neutron scattering parameters extracted in this way. Also it was hoped to get information on the importance of many-body forces and reaction mechanisms involved in these few-nucleon reactions.

From the results of these experiments it is clear one must be careful in extracting nucleon-nucleon scattering parameters from final-state interactions. For the reaction $d(^3\text{He}, t)2p$ the results were found to depend more on the correlation between the initial and final states than on the final-state interaction alone. However, for the reaction $^3\text{He}(d, t)2p$ the final-state interaction theory of Watson and Migdal adequately explains the experimental results. The validity of the theory for this case and not the previous one apparently depends on the fact that the two protons are created closer together when they come from the original ^3He . By assuming a different reaction mechanism we were able to fit the experimental data in both cases with the known proton-proton interaction. By understanding the reaction mechanism, one can hopefully extract some useful information on the nucleon-nucleon

scattering parameters. However, a more accurate treatment of the three-body breakup will have to be devised before one can hope to see the effects of many-body forces, or to determine the scattering parameters with a high degree of accuracy.

The effects of different reaction mechanisms are clearly demonstrated by the large difference observed for the two reactions discussed above which involve the same particles in the final state. Further information can probably be gained on the reaction mechanism from complete angular distributions and more accurate absolute cross sections. From the estimates of cross sections given in Chapter IV, there was an apparent increase in the cross section for the $d(^3\text{He}, t)2p$ reaction when the energy was increased from 53 to 74 MeV. This is in the opposite direction to what is normally expected for direct nuclear reactions and should be useful in testing reaction mechanisms.

In conclusion, it is clear that important and useful information can be extracted from reactions with three particles in the final state. The excellent agreement for the $d(^3\text{He}, t)2p$ reaction at 74 MeV seems to indicate that the impulse approximation is a fairly good approximation at this energy. However, for the lower energy and other reactions some refinements in the theoretical treatment may be necessary.

REFERENCES

REFERENCES

- Aamodt, R. L., W. K. H. Panofsky, and R. Phillips, Phys. Rev. 83, 1057 (1951).
- Ball, J. B., IEEE Trans. on Nucl. Sci., NS 13, 1340 (1966).
- Baumgartner, E., H. E. Conzett, E. Shield, and R. J. Slobodrian, Phys. Rev. Letters 16, 105 (1966).
- Berkowitz, E. H., Nucl. Phys. 60, 555 (1964).
- Bethe, H. A., Phys. Rev. 76, 38 (1949).
- Bilaniuk, O. M. and R. J. Slobodrian, Phys. Letters 7, 77 (1963).
- Borggreen, J., B. Elbek, and L. P. Nielsen, Nucl. Instr. and Methods 24, 1 (1963).
- Brueckner, K. A., Phys. Rev. 82, 598 (1951).
- Cerineo, M., K. Ilakovac, I. Šlaus, P. Tomaš, and V. Valković, Phys. Rev. 133, B948 (1964).
- Devlin, T. J., University of California Radiation Laboratory Report UCRL-9727, (1961) (unpublished).
- Frank, R. M. and J. L. Gammel, Phys. Rev. 93, 463 (1954).
- Gillespie, John, Final-State Interactions (Holden-Day, Inc., San Francisco, 1964).
- Goldberger, M. L. and K. M. Watson, Collision Theory (John Wiley and Sons, Inc., New York, 1964).
- Haddock, R. P., R. M. Slater, Jr., M. Zeller, J. B. Czirr, and D. R. Nygren, Phys. Rev. Letters 14, 318 (1965).
- Heckrotte, W. and M. MacGregor, Phys. Rev. 111, 593 (1958).
- Heller, L., P. Signell, and H. R. Yoder, Phys. Rev. Letters 13, 577 (1964).
- Henley, E. M., F. C. Richards, and D. U. L. Yu, Phys. Letters 15, 331 (1965).
- Hulthén, L. and M. Sugawara, in Handbuch der Physik, (Springer-Verlag, Berlin, 1959) Vol. 39, p. 1.

- Jackson, J. D. and J. M. Blatt, Rev. Mod. Phys. 22, 77 (1950).
- Jakobson, M., J. H. Manley, and R. H. Stokes, Nucl. Phys. 70, 97 (1965).
- Koehler, D. R. and R. A. Mann, Phys. Rev. 135, B91 (1964).
- Koehler, D. R., Phys. Rev. 138, B607 (1965).
- Lippmann, B. and J. Schwinger, Phys. Rev. 79, 469 (1950).
- Migdal, A. B., J. Exper. Theoret. Phys. USSR 28, 3 (1955)
[English translation: Soviet Physics JETP 1, 2(1955)].
- Nakada, M. P., J. D. Anderson, C. C. Gardner, J. McClure,
and C. Wang, Phys. Rev. 110, 594 (1958).
- Parker, P. D., P. F. Donovan, J. V. Kane, and J. F. Mollenauer,
Phys. Rev. Letters 14, 15 (1965).
- Phillips, R. J. N., Nucl. Phys. 53, 650 (1964).
- Slobodrian, R. J., D. J. Clark, J. S. C. McKee, W. F. Tivol,
and T. A. Tombrello, "Nuclear Chemistry Annual Report,"
University of California Lawrence Radiation Laboratory
Report UCRL-17299, (1966), p. 86 (unpublished).
- Tombrello, T. A. and A. D. Bacher, Phys. Letters 17, 37 (1965).
- Voitovetskii, V. K., I. L. Korsunskii, and Y. F. Pazhin,
Phys. Letters 10, 109 (1964).
- Watson, K. M., Phys. Rev. 88, 1163 (1952).
- Watson, K. M. and K. A. Brueckner, Phys. Rev. 83, 1 (1951).
- Watson, K. M. and R. N. Stuart, Phys. Rev. 82, 738 (1951).

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

APPENDIX

INSTRUCTIONS FOR USING FSICAL -- A PROGRAM TO CALCULATE THE EFFECTS OF THE NUCLEON- NUCLEON FINAL-STATE INTERACTION IN A THREE-PARTICLE BREAKUP REACTION

For a reaction of the type $A + B \rightarrow C + D + E$ the program FSICAL calculates the energy spectrum of particle "C" assuming a final-state interaction between "D" and "E". In the program it is assumed that the two interacting particles are nucleons with low relative energy so only the S-wave interaction is important. The final-state interaction is treated using the Watson-Migdal theory where the c.m. cross section is given by³

$$\frac{d^2\sigma}{dE d\Omega} = \frac{|g(\theta)|^2 C^2(\eta) \rho(E)}{C^4(\eta) E_{2p} + \frac{\hbar^2}{m_p} \left[-\frac{1}{a_p} - \frac{h(\eta)}{R} + \frac{1}{2} r_o k^2 \right]}.$$

Also, if experimental data are available, the program can perform a least squares fit to the data by varying any combination of the parameters a_p and r_o and by adjusting the energy scale and normalization factor.

³See Chapter II, equation (15) with $\gamma E_{2p} = \frac{1}{2} r_o k^2$.

FSICAL

A provision is also made for calculating the transition amplitude for a charge exchange process for the reaction $d(^3\text{He}, t)^2\text{p}$ using the impulse approximation discussed by Phillips. This calculation consists of a numerical integration of the overlap integral of the two-proton wave function and the deuteron wave function using Simpson's rule.

Instructions for Preparing Data Cards

Card 1. Columns 1-15 control the program options as follows; a 1 indicates the option is desired.

Column	1. Search on a_p
	2. Search on r_o
	3. Search on ΔE (change in energy scale)
	4. Search on normalization parameter
	5. Folds in instrumental resolution
	6. Utilizes damping factor in parameter changes to speed convergence (normally use this option)
	7. Performs Phillips' calculation (rather than Watson-Migdal)
	8. Neglects Coulomb effects to calculate n-n final-state interaction (this option is not available in the Phillips' calculation)
	9. Not used
	10. Does not read new data but uses that read-in for previous search

11. Plots spectrum after each calculation (normally only plots output for final calculation)
12. Prints out table of experimental and theoretical cross section values after each calculation
13. Prints out some of the intermediate results
14. Prints out matrix of normal equations and its inverse
15. Not used

Columns 16-70 contain additional information as noted.

- 16-18. Number of points used in fitting procedure (I3)
- 19-20. Number of search iterations (I2)
- 21-30. Convergence criteria (fractional change in parameters) (F 10.5)
- 31-40. a_p (in Fermis) (F 10.5)
- 41-50. r_o (in Fermis) (F 10.5)
- 51-60. Energy shift (MeV) (F 10.5)
- 61-70. Normalization factor (normally use 1.0) (F 10.5).

Card 2. Title card (may contain 72 alpha numeric characters) (format 12A6).

Card 3. Mass card.

- Column
- 1-10. Mass of particle A (in AMU) (F 10.5)
 - 11-20. Mass of particle B (in AMU) (F 10.5)
 - 21-30. Mass of particle C (in AMU) (F 10.5)
 - 31-40. Energy of particle A (MeV) (F 10.5)
 - 41-45. Lab angle of particle C (degrees) (F 5.3)

- 46-50. Resolution (FWHM in MeV) (F 5. 3)
51. Control integer which determines format for reading-in data (see 4th card below) (I1)
- 52-54. Number of data points (should not exceed 225) (I3)
- 55-62. Full scale value for plotting subroutine output (should be larger than largest value of experimental data or data is modified so it can be plotted by subtracting the value of full scale).

Card 4. If control integer = 1, this means calculation only is wanted and this card specifies the maximum energy and increments at which the calculation is desired (format 2F 10.5).

If control = 2, program reads maximum energy and increment as above, then reads additional cards containing data, 8 values per card (8F 9.0).

If control = 3, program reads energy and experimental cross section in pairs, 4 pairs per card at points where calculation is desired (arranged in order of descending energy). (Number of data points must be the same as specified on card 3.)

Figure 13. Flow diagram for the program FSICAL.

Figure 14. Flow diagram for the least-squares fitting subroutine LSQFIT.

Figure 15. Flow diagram for the subroutine CALC which calculates the final-state enhancement for three-particle breakup reactions.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

INTERNAL DISTRIBUTION

1. J. B. Ball
2. R. M. Drisko
3. R. O. Ginaven
4. C. D. Goodman
- 5-6. E. E. Gross
7. M. L. Halbert
8. R. M. Haybron
9. F. T. Howard
10. E. V. Hungerford, III
11. Y. E. Kim
12. R. S. Livingston
13. C. A. Ludemann
14. J. J. Malanify
- 15-34. B. J. Morton
- 35-36. L. A. Rayburn (ORAU)
37. R. W. Rutkowski
38. G. R. Satchler
- 39-40. A. Zucker
- 41-42. Central Research Library
- 43-44. Document Reference Section
- 45-54. Laboratory Records
55. Laboratory Records (ORNL-RC)
56. ORNL Patent Office
- 57-71. DTIE, AEC
72. Research and Development, ORO

EXTERNAL DISTRIBUTION

- 73. A. D. Bacher
California Institute of Technology
- 74. R. H. Bassel
Brookhaven National Laboratory
- 75. O. M. Bilaniuk
Swarthmore College
- 76. C. R. Bingham
University of Tennessee
- 77. L. N. Blumberg
Brookhaven National Laboratory
- 78. W. M. Bugg
University of Tennessee
- 79. M. P. Fricke
University of Michigan
- 80. Nelson Jarmie
Los Alamos Scientific Laboratory
- 81-90. R. D. Present
University of Tennessee
- 91. H. G. Pugh
University of Maryland
- 92. Louis Rosen
Los Alamos Scientific Laboratory
- 93. Ivo Slaus
Institute Ruder Boskovic
- 94. R. J. Slobodrian
Lawrence Radiation Laboratory, Berkeley
- 95. R. H. Stokes
Los Alamos Scientific Laboratory
- 96. T. A. Tombrello
California Institute of Technology