Dr-1458

30
15-75
6-20 4 413 34
LA-5986-MS, Vol. I

Informal Report

UC-20 and UC-34

Reporting Date: May 1975

Issued: June 1975

**Optical Design of a Reaction Chamber for
Weakly Absorbed Light
I. Canted and Parallel Mirrors**

by

J. J. Devaney

D. E. Jackson

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

↓ ↓
An Affirmative Action/Equal Opportunity Employer

UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
CONTRACT W-7405-ENG. 36

MASTER

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

In the interest of prompt distribution, this report was not edited by the Technical Information staff.

Printed in the United States of America. Available from
National Technical Information Service
U S Department of Commerce
5285 Port Royal Road
Springfield, VA 22151
Price: Printed Copy \$4.00 Microfiche \$2.25

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

OPTICAL DESIGN OF A REACTION CHAMBER FOR WEAKLY ABSORBED LIGHT

I. CANTED AND PARALLEL MIRRORS

(With an Appendix for Strongly Absorbed Light Waves)

by

J. J. Devaney and D. E. Jackson

ABSTRACT

The optimization of light absorption in a weakly absorbing medium is studied by repeatedly reflecting a light beam from either plane or cylindrical canted and parallel mirrors. Vertical control is exerted either by horizontal mirror reflection, by cylindrical end mirrors, or by well-collimated or focused beams. Useful absorption for a number of numerical parameters is calculated. The gross properties of lasers are characterized for geometrical optical purposes. Beam insertion optics are evaluated. Chamber and mirror energetics are considered. In the Appendix one of us (JJD) gives the shape and speed of a light beam eating its way into a very highly absorbing medium.

I. INTRODUCTION

This report, the first of a series, deals with the optimum illumination of a reaction chamber by weakly absorbed light which, as a specific example, we take to be $\sim 0.4\text{-}\mu\text{m}$ wavelength. Only plane (or cylindrical) mirrors without external beam return are considered in this report.

Our reaction chamber or active region has width w , length L , and height h , across which (in the w -direction) we pass a light beam of height h , reflecting it from mirrors at the $h \times L$ faces in Fig. 1. We suppose as a nominal, but high-quality value, a beam loss per pass of 0.5% for mirror loss and useless gas absorption, a number which we will subsequently vary.

For the moment we study a single beam in plane or nearly plane mirror geometry. We call for beams of uniform intensity or nearly so; the examination of significantly nonuniform initial beams is reserved for later studies. It is, of course, evident that uniform beams lead to ease in rendering uniform illumination of the reaction chamber.

We will later fix many extraneous parameters so that we can concentrate on the effects of varying

Fig. 1. Reaction chamber.

just the optical beam parameters to determine beam smoothness, efficiency, intensity, and time.

Because our light is only weakly absorbed in the cavity medium, efficient use of the light beam requires many passes through the reaction chamber. Beam losses due to divergence and diffraction must therefore be carefully controlled.

The order of topics is: II. Vertical Containment, III. Horizontal Containment, IV. Laser Characterization, V. Beam Insertion Optics, VI. Numerical Examples, VII. Chamber Energetics - Mirror Intensities, and Appendix, High Absorption of Light, Bleaching.

II. VERTICAL CONTAINMENT

We will discuss vertical containment by: (a) horizontal mirrors; (b) a well-collimated beam; and (c) cylindrical end mirrors.

A. Horizontal Mirrors

If the chamber could be fitted with reflecting surfaces above and below, then from Born and Wolf,¹ for a grazing angle δ , from a medium of refractive index 1 to a metal of real index n and imaginary part $n\kappa$, the second-order (in δ) approximation to reflectivity R is given by:

$$R_{TE} \approx \frac{(\delta - U_2)^2 + V_2^2}{(\delta + U_2)^2 + V_2^2} \quad (1)$$

(electric vector perpendicular to plane of incidence) and

$$R_{TM} \approx \frac{[n^2(1 - \kappa^2)\delta - U_2]^2 + [2n^2\kappa\delta - V_2]^2}{[n^2(1 - \kappa^2)\delta + U_2]^2 + [2n^2\kappa\delta + V_2]^2} \quad (2)$$

(magnetic vector perpendicular to plane of incidence) where U_2 and V_2 are to be obtained from:

$$U_2^2 - V_2^2 = n^2 - n^2\kappa^2 - 1 \quad (3a)$$

and

$$U_2 V_2 = n^2 \kappa. \quad (3b)$$

Taking values of n and $n\kappa$ from the APS Handbook,² we obtain the reflectivities summarized in Table I.

Clearly our reflection losses will be large unless grazing angles are very small or highly reflecting surfaces are used. Note that TE reflectivity is greatly superior to TM.

TABLE I
GRAZING REFLECTIVITIES

Material	Wave-length, μm	Grazing Angle (δ) , rad	Reflectivities	
			R_{TE}	R_{TM}
Evaporated silver	0.4	2×10^{-5}	0.999 999 957	0.999 999 754
		0.1	0.999 781	0.998 78
	8	4.3×10^{-4}	0.999 996 15	0.987 77
Bulk nickel	0.42	2×10^{-5}	0.999 988	0.999 884
		0.1	0.9423	0.5694
	8.7	0.1	0.99312	0.5715
Bulk iron	0.589	2×10^{-5}	0.999 989	0.999 81
		0.1	0.945	0.408

B. Collimated Beam

Should other considerations preclude use of horizontal mirrors, we can control beam loss by controlling the beam quality. A beam with a divergence of the order of 0.1-rad half-angle will lose $\sim 95\%$ of its intensity in a single 200-cm pass. However, a diffraction-limited beam of $\sim 0.4\text{-}\mu\text{m}$ wavelength with a minimum vertical dimension of 1 cm (called the waist) expands by a factor of $\sqrt{2}$ in a distance (called the Rayleigh range) of 20 000 cm. By taking twice the Rayleigh range as the length of the usefully controlled beam, we could get 200 passes of 200 cm each in which the beam changes from a height of 1.41 cm to 1 cm (waist) and back to 1.41 cm. Similarly, the Rayleigh range of a 2-cm vertical opening is 80 000 cm or 800 passes (200 cm). The corresponding values for a collimated $8.6\text{-}\mu\text{m}$ beam of 1-cm waist are 911 cm - 9 passes; and for a 2-cm waist, 3644 cm - 36 passes. Thus vertical optics are easier for a 2-cm-high reaction chamber than for a 1-cm chamber (see also discussion of cylindrical mirrors, below).

To hold a 1-cm vertical beam for the order of 100 passes without cylindrical optical elements, the beam must be formable into a diffraction-limited beam. In contrast, as discussed below

(Sec. III), we can tolerate a much greater horizontal divergence if need be. Vertical tradeoffs are as follows: For a focused beam with a 1-cm waist and for 200 passes of 200-cm length (with no loss), the diffraction-limited variation of the 90% beam limit is 1.41 to 1 to 1.41 cm giving an effective illumination within 1 cm of 83%. For 90 no-loss passes the beam changes from 1.096 to 1 to 1.096 cm giving an effective 1-cm illumination of 95.4%. Average diffraction losses are summarized in Table II. Note that these losses are initial losses of a uniform beam in one dimension. Repeated passes lead to Gaussian shapes, with eventually vanishingly small diffraction losses, especially for concave mirrors; e.g., for confocal, 1-cm-high by 200-cm-radius TEM₀₀, the diffraction loss is less than 0.001% per pass.³

TABLE II
AVERAGE ONE-DIMENSIONAL INITIAL
DIFFRACTION LOSSES
(200-cm path length, uniform beam)

Vertical aperture, (cm)	1	1.41	2	4
Relative loss, (%)	0.16	0.08	0.04	0.01

If beam divergence is 2x diffraction limited (DL) the effective illumination drops to 61.8% for 200 passes (200 cm) and to 85.3% for 90 passes. For a 3x DL the no-loss beam is 48% effective for 200 passes and 74.7% for 90 passes. For 5x DL the effectiveness of 200 passes has dropped to 32.8%, whereas for 10x DL and 90 passes the effectiveness is 35.7%. Note that wider divergence implies a need for larger end mirrors; for example, a 2x diffraction-limited beam requires a mirror depth of 1.35 cm to reflect the same proportion of beam at 90 passes and requires 2.24 cm at 200 passes. A 3x diffraction-limited beam requires 1.69 cm for 90 passes and 3.16 cm for 200 passes. To a first approximation the early, average diffraction loss may be taken as a function of mirror depth only as given in Table II (e.g., 1-cm mirror, 0.16%/pass).

C. Cylindrical End Mirrors

As mentioned, we can improve beam effectiveness by using a deeper reaction chamber, by fewer passes, or by going to a cylindrical mirror for one end

(only one end is needed). However, the last method may reduce beam effectiveness due to a loss of uniformity which is especially serious close to absorption saturation.

The cylindrical and the plane mirror together form a stable resonator, which we should design for nearly uniform volume illumination. By using one cylindrical mirror of 20-km radius, we may increase the effectiveness of a diffraction-limited beam to 0.999975, or a 2x DL beam to 0.999, and of a 10x DL beam to 0.9975; that is, make the beam essentially 100% effective because it is effectively 100% paraxial.

Starting at the curved cylindrical mirror with a focal length $f_m = R/2$ and traversing the chamber twice, beyond the plane mirror and to just beyond the curved mirror again, the ray transfer matrix for 2n passes is from Sylvester's theorem and some algebra:³

$$M^n \approx \theta^{-1} \begin{bmatrix} \theta \cos(\frac{2n-1}{2} \theta) & 2w \sin n \theta \\ -\frac{1}{f_m} \sin n \theta & \theta \cos(\frac{2n-1}{2} \theta) - \theta^2 \sin n \theta \end{bmatrix} \quad (4)$$

for

$$\theta \equiv \sqrt{2w/f_m} \ll 1,$$

so that a ray starting at x_0 with slope x'_0 has

$$x_{2n} = x_0 \cos(\frac{2n-1}{2} \theta) + 2wx'_0 \sin n\theta/\theta \quad \text{and}$$

$$x'_{2n} = -x_0 \sin n\theta/f_m \theta$$

$$+ x'_0 \cos \frac{2n-1}{2} \theta - \theta x'_0 \sin n \theta \quad (5)$$

from

$$\begin{bmatrix} x_{2n} \\ x'_{2n} \end{bmatrix} = M^n \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}.$$

Equation (5) shows that provided the initial slope x'_0 is not too large, the ray is confined. Which is, of course, to say that the system is a stable resonator vertically, as expected. Indeed, for small x'_0 , $x_{2n} \sim x_0$.

Cylindrical mirrors of shorter radii allow us to handle very poor beams, which we will discuss by changing the beam characterization from a multiple of the diffraction-limited divergence to an effective source size s , and a useful half-angle ϕ , for the entrance lens with a focal length f . Let the entrance-window aperture be a , we have a single curved mirror of radius R , focal length $f_m = R/2$, and depth a , and a flat mirror of depth b , spaced $L = 200$ cm apart (see Fig. 2).

In Fig. 2, for optimum illumination the source must be imaged on the first mirror (flat) of size equal to the depth, h , of the reaction chamber, thus the entrance lens must magnify by an amount m , given by:

$$h = -ms. \quad (6)$$

The image distance Z and object distance Z' are also related to m through:

$$Z' = -mZ \quad (7)$$

and to f by:

$$\frac{1}{Z} + \frac{1}{Z'} = \frac{1}{f}, \quad (8)$$

so that setting $Z' = L$ we have

$$f = \frac{LS}{S + h}, \quad (9)$$

giving the focal length of the entrance lens with the source located at

$$Z = \frac{SL}{h}. \quad (10)$$

The maximum vertical source angle at which light is accepted into the system is ϕ , given by:

$$\tan \phi = \frac{a}{2Z} = \frac{ah}{2SL}. \quad (11)$$

Rays at any greater angle are lost. Within the chamber we refocus to h so we must put

$$\frac{L}{2} = f_m = \frac{R}{2}$$

or

$$R = L, \quad (12)$$

where R is the radius of curvature of the cylindrical mirror. If the source obeys Lambert's Law in

Fig. 2. Vertical optics - cylindrical mirror (expanded).

object space, the image does in image space, and we can therefore calculate the effectiveness of illumination of the reaction chamber for large angles, ϕ . We exhibit the result for $2h \geq a \leq h$, $L \gg h$ to wit: the ineffective fraction is;

$$F = \frac{a - h}{4h}, \quad (13)$$

and for $L \gg a \leq 2h$;

$$F = 1 - \frac{h}{a - h} \left(\frac{3}{4} + \frac{1}{2} \ln \left| \frac{a - h}{h} \right| \right). \quad (14)$$

Note however, that for a given source a larger aperture a always increases the absolute effective illumination up to saturation in the gas, even though the relative effectiveness decreases with a . Equations (13) and (14) give the fractional ineffectiveness of wide-angle light because the light is outside the reacting volume. In all of the above, the average early diffractive losses are roughly those of Table II, and thus depend primarily on a and b . As the beam progresses, loss becomes negligible as uniformity degenerates to a Gaussian shape.

D. Insertion Lens Control

If uniformity is not essential we can use a more converging lens than given by Eq. (9) and thereby control vertical divergence, or use plane mirrors plus a different lens than that given in Eq. (9). We will discuss the matter after we introduce laser characterization below.

III. HORIZONTAL CONTAINMENT

We now turn to illumination in the horizontal dimensions where we will "walk" the beam down and up (in the L dimension) from a single entrance window thus illuminating the whole volume. Both to avoid hot spots on the mirrors and for optimum reactivity in all the medium, it is best to start with a beam of uniform intensity and examine the effects of diffraction (from the entrance aperture), transmitting losses, time, and chamber geometry. This we do below, leaving the special effects of sharply non-uniform beams for a later study.

By making many passes through the medium volume we utilize the light, with its weak absorption, most effectively. In general and until diffraction

Fig. 3. Horizontal reaction chamber, Version a, parallel end mirrors. Beam-reversal mirror canted at angle R' width a (strictly speaking should be width of waist of beam which started at a).

broadens the beam excessively one does better in number of passes, in total absorption, and in intensity multiplication with narrow beams (~ 0.25 cm for $w = 200$ cm) at the cost of some illumination uniformity which may either pose an intensity-tolerance limit on mirrors and/or cause the loss of effective illumination at the corresponding low-level portions of the chamber.

We propose two designs. The first, Type a (Fig. 3), using plane parallel mirrors (the term plane refers to the horizontal dimension; the vertical options have been discussed in II), plus a plane reversing mirror has the merit of controlling the average uniformity in the chamber to a few percent. The second, Type b (Fig. 4), using slightly canted

Fig. 4. Horizontal reaction chamber, version b, canted end mirrors.

plane mirrors can achieve more than twice the best Type-a absorption, but of course through trapping the beam for a higher number of passes for a longer time and permitting greater intensity variations.

A. Parallel End Mirrors, Type a

This design depends on obtaining a well-collimated beam in the sense that divergence may be neglected while the beam is in the cavity. The angle R' is given by $\tan R' = a/2W$. For an entrance aperture of $a = 1$ cm, the number of passes before loss at the top is 39 in a 200×10 cm² chamber, with an optimum beam-width variation of 1.037-1.0-1.037 cm and a useful beam absorption of 2.58% with an average uniformity of $\pm 0.3\%$ over 1-cm increments in the downstream direction except for the entrance where the total intensity is 20% lower. For $a = 1$ cm, $R' = 0.0025$ rad.

Stretching this concept of nondivergence a little, the values for an entrance aperture of $a = 0.5$ cm are: number of passes, 79; optimum beam-width variation, 0.95-0.50-0.93 cm (hardly non-divergent); but a good useful absorption of 4.68% with an average uniformity of $\pm 1\%$ except for the entrance increment where the integrated intensity is 10% lower. For $a = 0.5$ cm, $R' = 0.00125$ rad.

These examples demonstrate some pros and cons of this version. We will give integrated intensities relative to entrance beam intensities and power together with those of Type b in Sec. VI.

B. Canted End Mirrors, Type b

Consider the horizontal geometry of Fig. 4. The two end mirrors are canted so that a beam introduced at the upper left, aperture a , will reflect back so as to just miss its entrance aperture and will continue reflecting down the reaction chamber but at ever decreasing intervals of L between reflections until it ceases to progress downstream, reversing at the stagnation line to progress upward with greater and greater intervals until it is lost at the top of the reaction chamber. Such an optical geometry has a very simple representation by forming the successive image spaces as in Fig. 5. Let the initial (uniform) beam be of width a and angular divergence, θ , so that its one-dimensional cross-section at a distance, Z , from the aperture is

$$C = a + 2Z \tan \theta. \quad (15)$$

Fig. 5. Equivalent (to Fig. 4) periodic optical system (not to scale).

For diffraction-limited beams in one dimension, $\theta = \lambda/a$ and in the far field ($Z \gg a$) C includes 89.1% of the beam. For simplicity we will take C to include (uniformly) all of the beam. The subscript L therefore refers to the left or upper edge of such a beam travelling right, no subscript to the center, and the subscript R to the right. The radius R is determined by trigonometry to be

$$R = W/\tan \beta, \quad (16)$$

and the angle R'_L is determined so that the left beam edge just clears the entrance window after the first reflection:

$$\cos(R'_L - 2\beta) = (1 + \frac{a \tan \beta}{W}) \cos R'_L, \quad (17)$$

$$R'_L + \theta = R'; R' + \theta = R'_R; R'_L + 2\theta = R''_R. \quad (18)$$

For a given ray, stagnation occurs when that ray is perpendicular to a mirror, say the n^{th} mirror image at an angle $n\beta$. Let the distance the ray has traveled to stagnation be L_s , then:

$$L_s = \frac{W \sin n\beta}{\tan \beta} = \frac{W \sin R'}{\tan \beta}, \quad (19)$$

where n is the number of passes to stagnation given by:

$$n = R'/r. \quad (20)$$

The distance downstream to stagnation is:

$$D = \frac{w(1 - \cos R')}{\tan \beta}. \quad (21)$$

The number of passes, n'_L , until the left or upper edge of the beam just leaks out the top is given by:

$$n'_L = 2 \left(\frac{R'}{\beta} - 1 \right). \quad (22)$$

The number of passes, n' , for the beam center to return to the entrance center is:

$$n' = 2R'/\beta, \quad (23)$$

and the corresponding total path to escape of the beam center is:

$$L_{\text{total}} = \left(a + 2 \frac{w}{\tan \beta} \right) \sin R'. \quad (24)$$

Equations (15) to (24) permit calculation of the geometrical excursions of a beam of width a and divergence θ . For example, for a $200 \times 10 \text{ cm}^2$ chamber and for $a = 0.5 \text{ cm}$, $\lambda \sim 0.4 \times 10^{-4} \text{ cm}$ and a divergence of 2λ diffraction limited, we find that the mirrors must be canted at $\beta = 2.66 \times 10^{-5} \text{ rad}$, out of parallel, and that the beam must be shot in at an angle of 0.00143 rad in order that the right beam edge will stagnate exactly at the far (bottom) edge of the mirrors in Fig. 4, giving 60 passes to stagnation and $n' = 108$ passes to central leakage at top. But more relevant parameters are the total useful absorption and the uniformity thereof, which we consider next.

A program, Absorption Distribution, has been written and tested for the HP-9820A computer, which does some of the preceding trigonometry, plus intensity calculations; namely it keeps track of the beam center and width, determines R' and β , and gives the total absorption as well as the relative absorption per cell. A cell is defined as $1/20$ th of the downstream reaction chamber times the full width and depth. At the center of the chamber width the

program adds the fraction of each beam in the cell times its intensity to give the total intensity in the cell. The inputs are (Fig. 4):

1. Beam entrance diameter, a ;
2. Distance of beam center to stagnation, R_0 ;
3. Downstream length of chamber, L ;
4. Cross-stream width of chamber, w ;
5. Beam divergence, half-angle, θ ;
6. REFL, reflectivity of mirrors less any other losses, R ;
7. TRANS, transmissivity of the gas, $T = 1 - \text{useful absorption} = 1 - w\phi\sigma$; and
8. REPORT, the number of passes (plus one) at which the absorption and its distribution are desired; these are always given for the maximum number of passes also.

As above, the program requires the diverging beam to just miss the entrance aperture after the first reflection. It then calculates R' and β to satisfy stagnation of the center at R_0 , and thus can calculate the entire trajectory of the beam.

Having these parameters describing the position and width of the beam, absorption for a particular pass is calculated as follows.

1. Calculate any energy that may be lost out of the chamber at either end.
2. Calculate how much of the remaining energy is absorbed by using the transmissivity of the gas.
3. Calculate the energy absorbed in each cell as proportional to the area common to the beam and that cell.

Energy loss to the wall is then calculated from wall reflectivity, followed by calculations of the new beam position and width, and so on until the trailing edge of the beam leaves the chamber.

The program prints θ , T , R , a , R_0 , L , w , $\beta/2$ (called ALPHA), and R' . It lists the center of the entrance window and gives the beam center and edges as well as the beam energy before the pass and the absorption of the last pass at the beginning of the third pass and at every tenth pass, or for every pass if desired. It gives the total absorption as a fraction of the input beam intensity and the percent of that absorption in each of the twenty cells; all being given both at the beginning of the REPORT

designated number of passes and at the end of the total possible passes. It lists that total and gives the turning-point location plus a check on the sum of cellular absorption. The parameter study below was made with the foregoing program.

IV. LASER CHARACTERIZATION

For estimating the cavity performance of beams far from diffraction limited, we need a description of the gross features of a laser adequate to give the device in sufficient detail to enable us to compute the photometry of optical systems in both the near and far fields of the various image spaces and to at least a useful approximation. The purpose here is to characterize a partially incoherent laser beam well enough to determine reaction chamber properties leaving the fine details for more sophisticated (and time consuming) theory such as that of Fox and Li,⁴ of Terrell,⁵ or of Suydam,⁶ all based on Huygen's principle. Laser beams that are nearly diffraction limited should be treated by the well known coherent Gaussian-mode beam optics.

We suggest then that a laser be characterized by a convex lens of aperture $2c$ equal to the effective exit window of the laser together with a flat plate disk light source of diameter $2b$ located at the focal point, f , of lens c (see Fig. 6). The brightness of source b is a function of position on the disk b , and for that matter the transmission of the lens c can also be made a function of position. Such a laser model, or its image space equivalent below, is easily propagated through an optical system by successive imaging. Moreover the characterization fits the gross features of lasers, the limiting features of lasers, and varies smoothly and plausibly between limiting features. For example,

Fig. 6. Laser characterization.

a diffraction-limited coherent beam, when we set b equal to zero, the true limit of the model in the real laser, being given by the ratio $b/f = \lambda/2c$. At the other extreme when $b/f \gg \lambda/2c$ (or $\lambda/2c$) the model describes the divergence of a super-radiant or near super-radiant laser, the intensity on the disk b giving the variation of the intensity with angle.

Actually we may transform the disk b of Fig. 6 into the image space of the lens c to get the fully equivalent characterization of Fig. 7. This e is the image space transform of the disk b ; it is the exit pupil of the system and determines the diffractive spread of the laser beam. The transmission may as before be varied across e . e is a flat plate light source with the ratio e/d_0 giving the intrinsic nondiffractive spread of the laser beam. As before, the intensity of the source as a function of angle is given by the (photometric) brightness function over the disk whose radius is e .

As an example we work a first approximation that certainly does not exhaust the details of the model much less the complexity of a real beam. We thus describe a beam for which we have only two experimental numbers, namely the laser exit beam diameter of 0.5 cm and a distant angular spread of 1.3 mrad. These values determine $e = 0.25$ cm and $e/d_0 = 1.3 \times 10^{-3}$ radians, and we put d_0 at the appropriate distance to give the proper diffraction, to wit, $e/d_0 = \lambda/2c$ for a coherent Gaussian beam whence for say, $\lambda \approx 0.4 \mu\text{m}$ and $d_0 = 5020$ cm we obtain $e = 6.5$ cm. We now can image e through our system by geometrical optics and obtain thereby a first, and for most photometric purposes, a more than adequate analysis.

Fig. 7. Equivalent laser characterization.

Again we would caution the reader that our characterization and certainly the above oversimplified model is not intended to be all things to all people. Rather we submit the model as a way to use a paucity of experimental data to make first-order calculations that must be carefully examined at each succeeding step to make sure that the model does not oversimplify physical reality. With these caveats, the model proves highly useful.

V. BEAM INSERTION OPTICS

In addition to the parameters of the chamber itself, we may optimize useful absorption by modifying the laser beam before insertion. We consider two methods: beam size modification and beam focusing. In both there are tradeoffs. Compressing the beam (horizontally in one dimension) is advantageous, but concomitantly the beam diverges, which is disadvantageous. Focusing allows the beam to return from the first reflection at a smaller angle and still miss the entrance hole thus packing more passes into the chamber, but contrarily the beam may diverge greater further on; or focusing may be used just to control diffraction sufficiently for the full beam travel to be accomplished. In all these variations, mirror damage thresholds and the effects on illumination uniformity must be kept in mind.

A. Beam Compression (Expansion)

Both for "poor" beams and for diffraction-limited beams, compression of the beam diameter by a factor m expands the beam divergence by a factor $1/m$. We illustrate this effect by the action of a Galilean telescope on a laser beam as characterized in Fig. 7 (see Fig. 8). The telescope is made up of a convex lens, f_3 , and a concave lens, f_4 , obeying $f_3 + f_4 = d$, and compressing the beam by a factor $m = -f_4/f_3$, simultaneously expanding the beam divergence from θ to θ/m , as we will show. The transfer matrix taking a ray from the source e to

Fig. 8. Beam compression by Galilean telescope.

just beyond the lens f_4 is:

$$M = \begin{bmatrix} m & d_1 + d_o m \\ 0 & 1/m \end{bmatrix}. \quad (25)$$

The beam edge ray at the source, e , has position e and divergence half angle or slope θ , so that the position vector at f_4 , call it X_o , is given by:

$$X_o = MX_e = M \begin{bmatrix} e \\ \theta \end{bmatrix} = \begin{bmatrix} me + (d_1 + d_o m)\theta \\ \theta/m \end{bmatrix} = \begin{bmatrix} x_o \\ x'_o \end{bmatrix}. \quad (26)$$

We can make d_o and d_1 as small as we please; $d_o m \theta$ is just the beam spread through the telescope, and $d_1 \theta$ is the beam spread from the laser to the telescope. Thus the effect of the telescope, as asserted, is to compress the beam by a factor m and enlarge the divergence by a factor $1/m$.

We now imagine the beam from lens f_4 to be introduced into the reaction chamber or into an insertion lens of the reaction chamber. What are the apparent size e_o and distance d_{oo} of the source viewed from the reaction chamber? If we make $\theta = e/d_o$ in Eq. (26) then clearly $e_o/d_{oo} = (1/m)e/d_o$. Using

$$e_o = me, \quad (27a)$$

we have

$$d_{oo} = m^2 d_o, \quad (27b)$$

thus giving the apparent source size e_o and distance d_{oo} if a telescope of magnification m is inserted in the beam.

B. Insertion Optics

Down beam from the telescope (or combined into lens f_4) we may want to insert a converging lens at the chamber entrance (usually). What performance can we expect? Consider Fig. 9. Our laser beam emanates from e at a distance d_o from the insertion lens of diameter $2c$ and focal length f , which focuses

Fig. 9. Insertion lens - effective beam spread.

the beam to a slope θ_0 at Z' with image e' . The term θ_d denotes the effective diffraction angle from the aperture $2c$; for a uniform one-dimensional beam its value is $\theta_d = \lambda/2c$. $w(Z)$ is the net width and θ the net divergence from c of the beam at distance Z from c . Usually we will want to minimize w and hence θ for a total path Z through the chamber. Also a slightly closer beam return to the entrance may give slightly better total absorption even though the beam escape losses are greater for larger w at larger Z where the attenuated beam contributes little absorption, a refinement we do not consider at this time. We define w' by

$$w' \equiv w - Z\theta_d. \quad (28)$$

Minimizing w' minimizes w . Observe in Fig. 9 that

$$\theta = (w - c)/Z. \quad (29)$$

In general we do best (minimum w or θ) by choosing the insertion lens focal length f so that the image e' coincides with w' and thus $Z' = Z$. That is also true when $e' > c$ (not illustrated). Since in this case $e' = Ze/d_0$, we have:

$$w = Ze/d_0 + Z\theta_d, \quad (30)$$

and using Eq. (30),

$$\theta = \frac{e}{d_0} - \frac{c}{Z} + \theta_d \quad (\text{Case I}), \quad (31)$$

which also equals the insertion angle, and

$$f = \frac{Zd_0}{d_0 + Z}. \quad (32)$$

However, for a limited range of parameters, detailed below, a slightly smaller Z' , giving a smaller e' , will yet give a smaller w or w' .

That is, for $Z' \leq Z$ and $e' \leq c$ we minimize

$$\begin{aligned} w'(Z) &= e' + (Z - Z')(c - e')/Z' \\ &= (2e/F) + (ZcF/d_0) - (eZ/d_0) - c, \end{aligned} \quad (33)$$

where

$$F \equiv (d_0/f) - 1 \text{ or } f = d_0/(1 + F) \quad (34)$$

with respect to f . We obtain thereby

$$F = \sqrt{2ed_0/Zc} \quad (\text{assumes } d_0 > f), \quad (35)$$

$$w' = 2\sqrt{2Zc e/d_0} - eZ/d_0 - c, \quad (36)$$

and

$$\theta = -(\sqrt{2c/Z} - \sqrt{e/d_0})^2 + \theta_d \quad (\text{Case II}), \quad (37)$$

valid in the range

$$c/(2e/d_0) \leq Z \leq 2c/(e/d_0). \quad (38)$$

Note the θ of Eq. (37) is less than or equal to that of Eq. (31) where Eq. (37) is valid. The insertion angle in this case (Case II) is:

$$\theta_{\text{insert}} = -\sqrt{e/d_0} (\sqrt{2c/Z} - \sqrt{e/d_0}) + \theta_d. \quad (39)$$

Thus whenever the conditions of Eq. (38) are met, use Eq. (37) Case II, otherwise use Eq. (31), Case I. In our numerical studies the word focused means choosing f from either Eqs. (35) and (34) or (32) and thereby optimizing θ .

VI. NUMERICAL EXAMPLES

We will use a number of typical parameters to give the reader some perspective and then vary them to illustrate tradeoffs. Refer to Table III where we maximize useful absorption with respect to the independent variables and not to the number of

TABLE III
SELECTED NUMERICAL RESULTS

Run Number	Beam Divergence Half-Angle θ (rad)	Linear Absorption, $\rho\sigma$ (cm ⁻¹)	Width w (cm)	Reflectivity, R	Aperature, a (cm)	Length L (cm)	Cant Angle (rad)	Total Fraction of Beam Usefully Absorbed	Number of Passes	Absorption Cell 2 (%)	Maximum Cell Absorption (%)	Absorption Cell 19 (%)	Beam Energy Escaping (%)	Remark
1	1.57(-4)	3.68(-6)	200.	0.995	0.25	10.	9.06(-6)	7.93(-2)	209.	4.2	8.6	2.5	37.6	Reference
	DL1							[6.66(-2) absorbed before beginning leak out at entrance]						
2	1.5 (-4)	3.68(-6)	200.	0.995	0.25	10.	8.9 (-6)	8.04(-2)	210.	4.2	8.7	2.4	37.3	
	focused													
3	3.14(-4)	3.68(-6)	200.	0.995	0.25	10.	1.46(-5)	6.10(-2)	169.	4.9	8.3	2.4	52.4	
4	3. (-4)	3.68(-6)	200.	0.995	0.25	10.	1.42(-5)	6.23(-2)	171.	4.9	8.5	2.4	51.4	
	focused													
5	7.84(-4)	3.68(-6)	200.	0.995	0.25	10.	5.14(-5)	~ 3.6 (-2)	~ 98.	8.9	10.4	1.5	71.9	Code limiting
6	7.75(-4)	3.68(-6)	200.	0.995	0.25	10.	5. (-5)	~ 3.7 (-2)	~ 99.	8.7	10.4	1.6	71.1	Code limiting
	focused													
7	1.57(-4)	6.11(-7)	200.	0.995	0.25	10.	9.06(-6)	1.38(-2)	209.	4.1	8.6	2.5	42.2	
	DL1													
8	1.57(-4)	1. (-7)	200.	0.995	0.25	10.	9.06(-6)	0.23(-2)	209.	4.1	8.6	2.5	43.0	
	DL1													
9	1.57(-4)	1. (-5)	200.	0.995	0.25	10.	9.06(-6)	19.8 (-2)	209.	4.3	8.6	2.4	30.8	
10	1.57(-4)	1. (-4)	200.	0.995	0.25	10.	9.06(-6)	79. (-2)	209.	8.1	8.1	1.1	1.3	
11	1.57(-4)	3.68(-6)	100.	0.995	0.25	10.	1.28(-5)	4.76(-2)	248.	3.7	8.7	3.5	30.5	
12	1.5 (-4)	3.68(-6)	100.	0.995	0.25	10.	1.26(-5)	4.78(-2)	250.	3.6	8.9	3.6	30.2	
	focused													
13	7.84(-5)	3.68(-6)	700.	0.995	0.5	10.	1. (-5)	16.3 (-2)	105.	3.5	8.4	2.8	51.9	
	DL1													
14	7.1 (-5)	3.68(-6)	700.	0.995	0.5	10.	9.5 (-6)	16.8 (-2)	107.	3.4	8.8	2.8	50.5	
17	3.92(-4)	3.68(-6)	50.	0.995	0.1	10.	7.5 (-6)	2.9 (-2)	468	5.5	8.5	1.9	16.8	
	DL1													
18	3.92(-4)	3.68(-6)	20.	0.995	0.1	10.	1.12(-5)	1.34(-2)	587	5.2	8.2	1.9	7.5	
	DL1													

(Continued)

TABLE III (continued)

Run Number	Beam Divergence Half-Angle θ (rad)	Linear Absorption, $\rho\sigma$ (cm^{-1})	Width w (cm)	Reflectivity, R	Aperature, a (cm)	Length L (cm)	Cant Angle (rad)	Total Fraction of Beam Usefully Absorbed	Number of Passes	Absorption Cell 2 (%)	Maximum Cell Absorption (%)	Absorption Cell 19 (%)	Beam Energy Escaping (%)	Remark
19	1.57(-4)	3.68(-6)	200.	0.995	0.25	20.	4.2 (-6)	10.9 (-2)	437.	5.	8.1	2.3	14.9	
	DL1													
21	1.57(-5)	3.68(-6)	200.	0.995	0.25	5.	1.88(-5)	4.78(-2)	101.	3.	8.7	2.8	62.7	
	DL1													
22	1.57(-4)	3.68(-6)	200.	0.995	0.25	2.5	4. (-5)	2.48(-2)	49.	.3	8.7	3.4	80.7	
24	1.57(-4)	3.68(-6)	200.	0.9	0.25	10.	9.1 (-6)	.73(-2)	209.	25.7	25.7	.005	$<10^{-4}$	
25	1.57(-4)	3.68(-6)	200.	0.96	0.25	10.	9.1 (-6)	1.8 (-2)	209.	12.3	12.3	.4	0.13	
26	1.57(-4)	3.68(-6)	200.	0.98	0.25	10.	9.1 (-6)	3.43(-2)	209.	7.1	7.8	1.3	3.24	
27	1.57(-4)	3.68(-6)	200.	0.99	0.25	10.	9.1 (-6)	5.71(-2)	209.	4.9	8.5	2.1	16.6	
28	1.57(-4)	3.68(-6)	200.	0.998	0.25	10.	9.1 (-6)	9.94(-2)	209.	3.9	8.6	2.6	63.0	
29	1.57(-4)	3.68(-6)	200.	0.999	0.25	10.	9.1 (-6)	10.8 (-2)	209.	3.9	8.5	2.7	74.6	
30	1.57(-4)	3.68(-6)	200.	0.9999	0.25	10.	9.1 (-6)	11.6 (-2)	209.	3.8	8.5	2.7	86.8	
							[10.8(-2) absorbed before beginning leak out at entrance]							
31	3.92(-5)	3.68(-6)	200.	0.995	1.0	10.	7.8 (-5)	4.02(-2)	68.	2.9	12.8	12.8	69.4	Cell 3 and 18
	DL1													
32	7.84(-5)	3.68(-6)	200.	0.995	0.5	10.	2.1 (-5)	6.55(-2)	134.	3.3	10.5	5.8	48.9	Cell 3 and 18
	DL1													
34	2.18(-4)	3.68(-6)	200.	0.995	0.18	10.	7.6 (-6)	7.72(-2)	232.	5.2	8.5	2.1	39.8	
	DL1													
33	3.92(-4)	3.68(-6)	200.	0.995	0.1	10.	1.08(-5)	~ 5.8 (-2)	192.	11.6	11.6	1.3	55.	Code limited
	DL1													
35	0	3.68(-6)	200.	0.995	0.6	10.	0	3.9 (-2)	63.	5.2	5.2	5.1	69.7	Parallel mirrors
36	3.92(-4)	3.68(-6)	20.	0.995	0.1	3.5	3.2 (-5)	8.64(-3)	206.	3.4	8.7	3.2	40.4	
	DL1													
37	3.92(-4)	6.11(-7)	20.	0.995	0.1	3.5	3.2 (-5)	1.44(-3)	206.	3.4	8.7	3.2	40.8	
	DL1													

(Continued)

TABLE III (continued)

Run Num- ber	Beam Diver- gence Half- Angle θ (rad)	Linear Absorp- tion, $\mu\sigma$ (cm ⁻¹)	Width w (cm)	Reflec- tivity R	Aper- ture a (cm)	Length L (cm)	Cant Angle (rad)	Total Fraction of Beam Usefully Absorbed	Number of Passes	Ab- sorp- tion Cell 2 (%)	Maxi- mum Cell Ab- sorp- tion (%)	Ab- sorp- tion Cell 19 (%)	Beam Energy Escaping (%)	Remark
38B	7.84(-4)	6.11(-7)	20.	0.995	0.05	3.5	2.35(-5)	~ 1.4 (-3)	263.	5.7	8.9	2.0	42.6	
	DL1													
39	7.84(-5)	6.11(-7)	20.	0.995	0.5	3.5	5.8 (-4)	4.83(-4)	46.	3.1	12.6	9.8	80.2	Cell 4
	DL1													
40	-1.82(-4)	6.11(-7)	20.	0.995	0.5	3.5	5.4 (-4)	5.12(-4)	48.	3.1	15.9	13.9	79.0	Cell 4
	focused													
41	1.3 (-3)	6.11(-7)	20.	0.995	0.5	3.5	8.3 (-4)	3.77(-4)	29.	2.9	9.2	4.4	84.6	Cell 4
42	9.9 (-4)	6.11(-7)	20.	0.995	0.5	3.5	7.7 (-4)	4.01(-4)	40.	4.3	9.9	4.8	83.6	Cell 5
	focused													
43	3.92(-4)	3.68(-6)	20.	0.995	0.1	5.5	2.0 (-5)	1.10(-2)	323.	4.1	8.7	2.7	24.0	
	DL1													
44	1.3 (-3)	6.11(-7)	20.	0.995	0.5	5.5	4.9 (-4)	6.14(-4)	65.	3.9	9.3	4.2	74.8	Cell 4
45	1.1 (-3)	6.11(-7)	20.	0.995	0.5	5.5	4.6 (-4)	6.35(-4)	66.	3.8	9.5	4.6	74.0	Cell 4
	focused													
46	3.7 (-4)	1. (-5)	20.	0.9999	0.1	5.5	1.96(-5)	5.58(-2)	333.	3.4	9.0	3.6	91.6	
	focused													
48	3. (-3)	3. (-7)	20.	0.9	0.5	3.5	1.28(-3)	5.52(-5)	31.	3.1	8.9	2.0	8.0	Cell 3

passes. Because of leakout of beam edges these quantities do not maximize together. Because our chambers are designed to optimize absorption for low values of the product of absorber molecular density ρ , and cross-section σ , the product $\rho\sigma$ in Column 3 of Table III is chosen to be a low number. Column 2 gives the far-field beam divergence; DL1 means that the half-angle is the first minimum in the one-dimensional Fraunhofer pattern of an initially uniform beam, whereas focused means the half-angle given is that calculated from the theory of Sec. IV. Refer to Fig. 1 for the meaning of the dimensions w and L . All losses, save useful absorption, are lumped into the reflectivity loss, $1-R$, where R is given in Column 5. The aperture a of Column 6 is the beam entrance width along the chamber dimension L . Column 8 gives the angle between the two canted mirrors. Column 9 is the chief result and is the fraction of beam usefully absorbed or reacted in the chamber for negligible absorber depletion. Column 10 gives the number of passes of the beam before it wholly exits from the chamber. Edges of the beam do leak out before this pass number is reached. We could make the number of passes higher but thereby lose useful absorption. Columns 11-13 indicate the uniformity of illumination in the cavity. The code divides up the $L \times h$ face (Fig. 1) of the cavity into 20 cells along the dimension L and sums the light intensity in each cell, printing the result as a percent of the total integrated intensity on the whole face or mirror. We give the result for the second cell inward, namely cell 2 in Column 11 near the entrance and cell 19 in Column 13 near the narrow part of the canted mirrors. If the aperture is appreciable compared with the length L , the absorption of the 2d cell is abnormally low (see Fig. 3) so that for these cases (Column 11) we take the outermost cell at which the beam begins to be reflected on itself in place of cell 2 and note the cell used in the last column, Remark. Column 12 lists the maximum integrated cell absorption in percent, which usually occurs close to the center of the chamber except when the losses per pass are high as in runs 24 and 25 for example. The maximum cell mirror intensity may be calculated by the formula:

$$\bar{I} = \frac{E_0}{w\rho\sigma} \frac{\bar{A}_c}{100} \frac{20 A_t}{hL}, \quad (40)$$

good for $w\rho\sigma \ll 1$, where E_0 is the initial beam energy, $\bar{A}_c$ is the maximum percent cell absorption of Column 12, A_t is the total useful absorption of Column 9, and $T = (1 - w\rho\sigma)$ is the transmission through the cavity in the dimension w . Thus the maximum cell intensity in Case 1 of Table III is 18.5 J/cm^2 per joule of beam power (the average mirror loading is 10.8 J/cm^2 per pulse per joule). Note that maximum cell intensity is not the highest possible intensity, one must guard against high-intensity diffractive spikes in any optical system.⁵ The 14th column gives the beam energy escaping the chamber, mostly back out the entrance aperture. For high reflectivity, a weakly absorbing medium, and a small chamber, the escape loss is large. One may have to spatially or temporally isolate the laser, or best of all, use time-dependent reflecting devices to return the beam for multiple travels in the chamber. In the last column the remark "code limiting" refers to the fact that our code will not permit simultaneous beam loss at both ends of the cavity in a given pass. The values so given are satisfactory but not as accurate as they might otherwise be. This quirk prevents the study of very highly divergent beams with our code as presently written.

Run 1 is our reference; we will vary parameters about the values of this run. The chamber size is $200 \times 10 \times 1 \text{ cm}^3$ with parasitic losses per pass of 0.5% and a useful medium linear absorption ($\rho\sigma$) of $3.68(-6) \text{ cm}^{-1}$. The aperture is 0.25 cm wide and a diffraction-limited $0.25 \times 1 \text{ cm}^2$ beam with a horizontal divergence of $1.57(-4)$ radians half-angle issues into the cavity. For these parameters our total beam fractional useful absorption is 0.0793 in 209 passes before the beam is wholly out of the chamber. The average uniformity of illumination is good, varying by about a factor of 2 over the chamber. Thirty-seven and six-tenths percent of the original beam energy escapes the chamber. Optimal focusing can improve matters only slightly for these parameters to 0.0804 fractional absorption as run 2 shows. The sequence of runs 1, 3, and 5 shows the

effect of loss of beam quality (divergence half-angle) on useful absorption, and runs 2, 4, and 6 give the same when optimally focused. Beam quality is an important parameter for these cavities, but cavity performance diminishes less than proportional to beam divergence. For these parameters, focusing is of negligible help, but may be significant for other parameters as shown in Table III.

The sequence of runs 8, 7, 1, 9, and 10 shows the effect of increasing the absorption $\rho\sigma$ of the medium; it is obvious that high-medium absorptivity is desired.

The run sequence 18, 17, 11, 1, and 13 shows the gains to be made by using wider (w) chambers, with runs 12 and 14 giving a check on the help that optimum focusing can give. Again the gains are not quite proportional to the increase in w with even less gain going to 700 cm.

The run sequence 22, 21, 1, and 19 shows the effect of longer (L) chambers. Here too the absorption gain is not quite proportional to the increase in L with least gain for the longest.

The sequence of runs 24, 25, 26, 27, 1, 28, 29, and 30 shows the advantages of higher mirror reflectivity or lower parasitic loss. Note that low loss to the mirrors means high escape to the entrance, as it should. Note also that unless one uses a size chamber or a type design that allows very many passes before leakout, one gains only a little beyond reflectivities of about 0.998 in this size chamber ($200 \times 10 \text{ cm}^2$). The gains at first are roughly proportional to $(1-R)^{-1}$ but taper off to less than that at high R . An infinitely trapped beam has useful absorption proportional to $w\rho\sigma/(1-RT)$, which for $w\rho\sigma \ll 1$ is proportional to $(1-R)^{-1}$. These remarks represent the fact that high reflectivities are utilized to the fullest only in highly trapped beam chambers.

The run sequence 31, 32, 1, 34, 33 shows the effect of changing the aperture a . Upon narrowing the aperture, one at first gains useful absorption because the beam can be returned at smaller angles and so pack more passes into the chamber, but the higher diffractive angles at the lower apertures eventually cause such beam spreading as to allow the escape of light from the cavity ends.

Run 35 shows what can be done in a $200 \times 10 \times 1 \text{ cm}^3$ chamber with parallel mirrors (Type a of

Sec. II). The beam is taken to have negligible divergence or convergence over its path. In point of fact a coherent Gaussian beam with a waist of 0.6 cm will spread to 0.8 cm on either edge of 63 passes $\times$ 200-cm width. The fractional useful absorption, E/E_0 , of a beam making n passes wholly within a reaction chamber is:

$$E/E_0 = w\rho\sigma(1 - R^n T^n)/(1 - RT). \quad (41)$$

This formula or alternatively our machine calculation for $\theta = 0$ gives the useful absorption of Line 35 in Table III. Run 35 shows a very respectable absorption of 3.9% with a nearly perfect average uniform distribution. We do not at this time study diffraction spikes in the beam itself which must be allowed for in practical designs.⁵

In runs 36 to 48 we study in greater detail the properties of a small cavity, one with $w = 20 \text{ cm}$ and $L = 3.5$ or 5.5 cm . Runs 36 and 37 show the effect of lowering $\rho\sigma$ by a factor of 5.57 for $L = 3.5 \text{ cm}$. Then at the lower medium absorption the best absorption occurs for apertures of about 0.1 cm as runs 37 and 38b show. If we must use an aperture of 0.5 cm (not necessary, one can use beam compression - see IVA), run 39 shows what can be achieved. Here focusing gives some improvement as run 40 shows. A highly divergent 0.5-cm beam of run 41 can be focused to give the result of run 42. Runs 43 to 45 repeat these calculations for the longer cavity, $L = 5.5 \text{ cm}$.

Last, runs 46 and 48 give the extremes that might be expected, taking for run 46 the most favorable parameters to be expected near the ones we have chosen, and for run 48 the least favorable, obtaining thereby relative absorptions in a 20-cm chamber ranging from 5.6% to 0.0055%. Of course, widely different parameters will give yet wider results, thus a $\rho\sigma$ of 10 cm^{-1} gives 100% absorption in 20 cm (one pass).

VII. CHAMBER ENERGETICS - MIRROR INTENSITIES

If the absorbing molecular density is ρ , the light energy per unit volume needed to react or be absorbed on all the molecules is then

$$h\nu/c_q, \quad (42)$$

where $h\nu$ is the quantum energy and c_q the quantum

efficiency of the reaction or absorption desired. During the pulse length, Δt , a summed beam of intensity I dumps an energy density of

$$\frac{dI}{dz} \Delta t = I \rho \sigma \Delta t \quad (43)$$

into the medium, so that the summed energy density per pulse for complete reaction is [equating (42) and (43)]:

$$I \Delta t = \frac{hc}{\lambda \sigma \epsilon_q} \quad (44)$$

where λ is the wavelength. This energy density is that needed to react the molecules in the chamber; it does not depend on the density of molecules, the chamber width, nor any other macroscopic variable save only that the absorption be weak else one must compensate for the intensity diminution of the beam across the chamber. Weak absorption means a low cross-section σ , which in turn requires high-intensity beams which may exceed mirror or other tolerances of the reaction chamber. Note that the energy density in Eq. (44) is the summed energy density so that if the mirror tolerance is based on power and not energy (i.e., say avalanche limiting versus impurity heat limiting), the high intensity of Eq. (44) may be safely achieved by multiple passes of weaker intensity beams.

For example let $\lambda \sim 0.4 \mu\text{m}$, $\sigma = 1.8 \times 10^{-20} \text{ cm}^2$, $\epsilon_q = 1$, then $I \Delta t = 28.2 \text{ J/cm}^2$. For Δt shorter than about $1 \mu\text{s}$ this energy density greatly exceeds the tolerance of most mirrors.⁷

Both the canted, Type b, and the parallel, Type a, reaction chambers multiply the beam intensity at a point by overlapping the beam. The latter, Type a, in the limit of low total loss increases the beam intensity by a factor of 4 uniformly throughout the chamber save at the ends. The canted mirror chamber can achieve higher beam intensity multiplication (in excess of 6x) but at the expense of uniformity. The actual multiplications in Table III are somewhat less than the above because of the attenuation of the returning beam. For example in place of x4 in the uniform case, Run 35 achieves

x3.3 for the chosen parameters, and the reference canted case, run 1, achieves an actual maximum multiplication of x4.6 versus a true effective beam overlap of 6x or more. Conversely, such beam multiplication may permit higher absorptions without exceeding mirror tolerance if the damage mechanism is power dependent (such as electron avalanche).

VIII. SUMMARY

Plane or cylindrically concave mirrors provide in either canted or parallel configurations a simple, adaptable (to changing beam parameters and absorption parameters) reaction cavity giving high relative absorption together with a high or nearly perfect uniformity of illumination. Useful absorption or reaction increases approximately linearly with absorber density, cross section, chamber width, or chamber length, and approximately inversely as the beam divergence and as the parasitic losses $(1 - R)$. In every case above, the increase is somewhat less than linear, tapering to appreciably less than linear as the given parameter is carried to an extreme - a law of diminishing returns. The use of beam compressing (expansion) by telescope permits adjusting the aperture to optimum. Insertion focusing can improve performance only slightly when long beam paths in the chamber are possible, but appreciably for short paths.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge conversations with, and comments from, the following Los Alamos Scientific Laboratory colleagues: L. A. Booth, R. S. Cooper, J. M. Williams, C. M. Gillespie, F. T. Finch, J. R. Bobbitt, E. Heck, P. Robinson, T. Cotter, H. Flicker, T. G. Frank, and F. Harlow.

REFERENCES

1. M. Born and E. Wolf, Principles of Optics (Pergamon Press, New York, 1959), p. 625.
2. D. E. Gray, Editor, American Institute of Physics Handbook, 2nd Edition (McGraw-Hill, New York, 1963).
3. H. Kogelnik and T. Li, "Laser Beams and Resonators," Appl. Optics 5, 1550 (1966).
4. A. G. Fox and T. Li [as quoted in A. E. Siegman, An Introduction to Lasers and Masers] (McGraw-Hill, New York, 1971), p. 335.
5. James Terrell, Los Alamos Scientific Laboratory, private communication, 1974.

6. Bergen Suydam, Los Alamos Scientific Laboratory, private communication, 1974.

7. For example see the "Third Conference on High-Power Infrared Laser Window Materials," Air Force Cambridge Research Laboratory report (to be published).

APPENDIX

HIGH ABSORPTION OF LIGHT, BLEACHING

J. J. Devaney

Absorption formulae for light are given for the case that appreciable depletion of the absorber is possible.

Suppose we have an absorber whose absorption cross section is σ , whose absorption molecular density is ρ , upon which we shine light of intensity I_0 , of photon energy $h\nu$, for a time from $t = 0$ to $t = t$, in the x -direction, normally incident on a semi-infinite slab (one-dimensional beam geometry). Unless density fluctuations exist, the effect of scattering is only to change the velocity of light from the vacuum value of c to c/n , where n is the index of refraction of the absorber. For the calculation of absorption itself the velocity of light is not pertinent, save that it be sufficiently fast compared to other parameters, so we need take no further notice of scattering. The effect of density fluctuations, an interesting subject, must be reserved for a future study.

If we have an initial absorbing molecular density ρ_0 that is not depleted, the familiar exponential beam absorption obtains:

$$I = I_0 e^{-\rho_0 \sigma x} \quad (\text{no depletion for all } t). \quad (\text{A-1})$$

In the other limit, when absorption and consequent depletion are large, one can approximate the light wave envelope by a square wave eating into the material and obtain the velocity of such a bleaching wave by balancing energy losses. The energy lost by the beam per unit area during a time, dt , is $\Delta I dt$, which is taken up by $\Delta \rho dx$ molecules per unit area so that:

$$\Delta I dt = h\nu \Delta \rho dx \quad (\text{A-2a})$$

or

$$\frac{\partial I}{\partial x} = h\nu \frac{\partial \rho}{\partial t}, \quad (\text{A-2b})$$

where one molecule is removed for one photon, $h\nu$, absorbed.

For a square wave, ΔI is precisely I_0 , and $\Delta \rho$ is precisely ρ_0 . Equation (A-2a) then gives the velocity, v , with which the light eats into the material, bleaching it:

$$v = I_0 / (h\nu \rho_0) \quad (\text{A-3})$$

(large depletion and large $\rho_0 \sigma$).

Thus for distances less than $x_v = vt$, we have $\rho = 0$ and $I = I_0$ and for greater distances, $\rho = \rho_0$ and $I = 0$. See Fig. A-1.

For the general case our intensity differential equation becomes nonlinear:

$$dI = -I(x,t)\rho(x,t)\sigma dx, \quad (\text{A-4})$$

with the boundary and initial conditions: For $t = 0$,

$$\rho = \rho_0, \quad (\text{A-5a})$$

$$I = I_0 e^{-\rho_0 \sigma x} \quad (\frac{c}{n} \text{ large}); \quad (\text{A-5b})$$

Fig. A-1. Very high absorption depletion waves.

For $x = 0$,

$$\rho = \rho_0 e^{-\rho_0 \sigma v t}, \quad (\text{A-6a})$$

$$I = I_0; \quad (\text{A-6b})$$

For $t = \infty$, $x \neq \infty$,

$$\rho = 0, \quad (\text{A-7a})$$

$$I = I_0; \quad (\text{A-7b})$$

For $t \neq \infty$, $x = \infty$,

$$\rho = \rho_0, \quad (\text{A-8a})$$

$$I = 0. \quad (\text{A-8b})$$

Using Eq. (A-2b) we obtain the solution

$$I = I_0 [1 - (1 - e^{\rho_0 \sigma x}) e^{-\rho_0 \sigma v t}]^{-1}, \quad (\text{A-9})$$

with $v = I_0 / h\nu \rho_0$, which satisfies Eqs. (A-5b), (A-6b), (A-7b), (A-8b), and

$$\rho = \rho_0 (I/I_0) e^{-\rho_0 \sigma x(x-vt)}, \quad (\text{A-10a})$$

or

$$\rho = \rho_0 e^{\rho_0 \sigma x(x-vt)} [1 - (1 - e^{\rho_0 \sigma x}) e^{-\rho_0 \sigma v t}]^{-1}, \quad (\text{A-10b})$$

which satisfies Eqs. (A-5a), (A-6a), (A-7a), and (A-8a).

Equations (A-9) and (A-10) are our general results.

The half-light intensity, $I_0/2$, and the mean depletion position, $\rho_0/2$, become identical at large $\rho_0 \sigma x$ and travel at the speed v of Eq. (A-3). For large $\rho_0 \sigma x$, I and ρ are symmetric about such half-point and complementary to each other, see Fig. A-2.

Moreover, as $\rho_0 \sigma$ increases, the waves steepen, approaching square waves, Fig. A-1, and Eq. (A-3) in the limit. The other limit, Eq. (A-1) is achieved for $\rho_0 \sigma v t \rightarrow 0$, which is to say when no time exists that will deplete ρ_0 , the usual (i.e., nondepletion) assumption.

Equation (A-10) not only gives the molecular density as a function of t for light of intensity, I_0 , beginning at $t = 0$ and lasting indefinitely, but Eq. (A-10b) also gives the resulting density, ρ , from a square pulse of intensity, I_0 , of length t , incident on a medium of initial molecular density, ρ_0 , and cross section, σ .

Figure A-3 presents a numerical example for an absorber of initial molecular density $\rho_0 = 2.75 \times 10^{14} \text{ cm}^{-1}$ and light intensity $I_0 = 1.27 \times 10^{10} \text{ erg/cm}^2\text{-s} = 0.00127 \text{ J/cm}^2\text{-}\mu\text{s}$. All waves exhibited regardless of cross section, σ , have the same half-intensity velocity, $v = 2.007 \times 10^8 \text{ cm/s}$.

Fig. A-2. General depletion waves. (Left hand: waves at $t_1 < t_2$, right hand: waves at t_2 .)

Fig. A-3. Bleaching of absorber illustrated at $t = 0.2 \mu s$.