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Paper No. 2

The Design of Large Cryogenic Magnet Coils*

C. E. Taylor and R. F. Post

If some difficult material and mechanical problems can be solved, it should be possible to construct highly efficient air core cryogenic magnet coils for the production of intense continuous magnetic fields. It has been determined that there are at least two pure metals which should be satisfactory as conductors in such a use. These are sodium and aluminum. Some preliminary design and experimental work aimed toward the fabrication of a large magnet coil with encapsulated sodium conductors will be outlined. Questions of cooling cycles, heat transfer, thermal stability and the mechanical reinforcement of such coils will be discussed.

* This work performed under the auspices of the U. S. Atomic Energy Commission.
Design of Large Cryogenic Magnet Coils
by Clyde E. Taylor and Richard F. Post *

The previous paper discusses the need for cryogenic coils and briefly discusses the various metals which are considered for electrical conductors. This paper will briefly describe some of the engineering problems of building such coils. The main points of interest are as follows:

1. Theoretical design of coils.
2. Heat transfer problems.
3. Construction techniques and example design of a 20" dia., 100KG coil.
5. Heat leak along leads.
6. Future plans.

The above items are discussed below:

1. Theoretical Design of Coils

Air-core solenoids with symmetry in the circumferential direction are considered. Any desired variation of current density with radial and axial position can be used to achieve results such as:

1) Minimum power requirement for a given central field
2) Uniform central field
3) Minimum stress in the structural members
4) Uniform heat generation rate
5) Particular field shapes
6) Structural simplicity
7) Ease of heat removal

* Work done under the auspices of the U.S. Atomic Energy Commission
Current density and coil cross section can be varied at the expense of structural simplicity. These problems are not unique, of course, to cryogenic coils and many calculations have been made in past years by people concerned with small, high field coils for research. See, for example references 1 through 5. For the present we will consider only simple coils with rectangular cross section and uniform current density.

The current density required to produce a given magnetic field at the center of such a solenoid is:

\[ \mathcal{J} = \frac{B}{\mu_0 Y R_i C} \]

where \( \mathcal{J} \) = current density, amps/m\(^2\)
\( B \) = field strength at geometric center of solenoid-webers/m\(^2\)
\( \mu_0 \) = free space magnetic permeability
\( 4 \pi \times 10^{-7} \) Webers/amp-m
\( R_i \) = inside radius of solenoid - m
\( Y \) = packing fraction: ratio of conductor cross section to total cross section
\( C \) = geometric constant depending on coil proportions.

The coil proportions are given in ratios:

\[ \alpha = \frac{r_o}{r_i} \]

where \( r_i \) = inside coil radius
\( r_o \) = outside coil radius

and \( \beta = \frac{L}{R_i} \) where \( L \) = total length of coil.
In terms of $\alpha$ and $\beta$,
\[
C = \frac{B}{2} \ln \frac{\alpha + \sqrt{\alpha^2 + B^2}}{1 + \frac{1}{\alpha} + \frac{B^2}{\alpha^2}}
\]
.................................(2)

An approximate formula for $C$ is:
\[
C = \frac{B(\alpha - 1)}{[(1 + \alpha)^2 + \beta^2]}
\]
.................................(3)

For most cases of interest the approximate equation (3) is sufficient. For $\alpha = 3$ and $\beta = 4$, for instance, $C$ from equation (3) is 0.9% higher than the exact value.

The heat generated per unit total coil volume is
\[
q = \frac{I^2 R}{V_0} = \xi R Y
\]
.................................(4)

where $q = \frac{\text{watts}}{\text{m}^3}$
$R$: electrical resistivity (average) ohm - cm
$Y$: packing fraction

From (1) and (3), the total power required is
\[
P = \frac{B^2 \rho r_i}{\mu_0^2 Y} \left[ \frac{\pi B (\alpha^2 - 1)}{C^2} \right]
\]
where $P$: watts
$B$:韦伯/m²
$\rho$: ohm - m
$r_i$: m
It can be shown that the coil proportions resulting in a minimum total power for a given inside radius and central field are \( \alpha = 3 \) and \( \beta = 4 \), giving a coil of the proportions shown in Fig. 1. It is of interest to determine the additional power required to produce a given central field, for coils which have proportions different from \( \gamma = 3 \) and \( \beta = 4 \).

Considerations such as heat transfer, field configuration, ease of fabrication and cost may dictate a departure from the optimum proportions. We define

\[
\varepsilon = \frac{P_{\text{actual}} - P_{\text{min}}}{P_{\text{min}}}
\]

where \( \varepsilon \) = fraction of "excess" power.

It can be shown that:

\[
\varepsilon = \frac{\beta (\alpha + 1)}{16 (\alpha - 1)} \left[ \left( \frac{1 + \alpha}{\beta} \right)^2 + 1 \right] - 1
\]

\[\text{..................(5)}\]

\( \varepsilon \) is plotted as a function of \( \alpha \) for various values of \( \beta \) in Fig. 2. For example, suppose we require a coil of length \( L = 2.7 \times r_1 \) (\( \beta = 2.7 \)).

The optimum outside radius for this \( \beta \) value is, \( \alpha = 2.25 \) and, from equation \( \varepsilon = 0.055 \). Such a coil requires 5.5% more power than a coil of proportions \( \alpha = 3 \) and \( \beta = 4 \) but requires 2.64 times smaller total coil volume.

2. Heat Transfer

Most efforts to cool coils have involved liquid coolants to take advantage of the high heat capacity, low pumping power requirement, and sometimes to achieve high heat transfer coefficients associated with boiling;
however, in large diameter coils, gas cooling becomes practical.

An example of large cryogenic coils which might be used in a mirror machine experiment in present day thermonuclear research is presented in order to give some idea of the engineering problems involved. Consider, for example, a pair of mirror coils, 20" inside diameter, to give a central magnetic field of about 100,000 gauss. These coils would be spaced about 20 in. apart along a common axis and a suitable vacuum vessel would be placed between and inside the coils. Plasma experiments, using the high magnetic field as a container, would be performed inside the vacuum chamber. For the 20" coils under discussion, about 2 Kg/sec of helium gas at about 70 K and 5 atmospheres pressure would be required. To achieve structural simplicity and minimum pressure drop, the coolant would circulate axially thru the coil, rather than circumferentially or radially.

3. Construction Techniques

The possible details of construction we will only mention briefly. Aluminum and sodium are both attractive coil materials; however, only sodium will be used for the following example. The sodium metal would be cast into square thin-walled stainless steel tubing. The tubing is wound in layers on cylinders. Each layer of conductor is structurally supported by a steel cylinder, to resist the high magnetic forces. The coolant grooves are small axial slots running axially through the coil. Figure 3 is a schematic drawing showing a single layer of the construction just described. For a space factor of 65%, a temperature drop for heat transfer between conductor and coolant of less than 20 K is possible. The following parameters are for a single coil of the above size:

- current density = 5400 amps/cm²
- current = 14,000 amps
conductor - 5/8 square x .010 wall stainless steel tubing filled with sodium
inductance - .18 henry
resistance (under load) 1.26 x 10^{-4} ohm
time constant (under load) L/R = 51 min.
volts - 1.8 volts
energy in field - 1.7 x 10^7 joules
heat generation rate .04 watts/cc
total power - 25 Kw
coolant - 1 Kg/sec of He gas at 5 at. pressure and about 70K
(cooling SCFM)
coolant passage 1/8" wide x .050" deep grooves
number of layers - 15
number of turns 41/layer, 615 total
coolant makes 3 passes of 5 layers each
coolant pressure drop .5 psi
coolant velocity - 35 ft/sec at inlet and 130 ft/sec at outlet
average ΔT for heat transfer in coil 1.70K (1/3 in .010" stainless steel tube wall and 2/3 in gas)
sodium required - 1500 lbs.
maximum magnetic "bursting" pressure - 1500 psi
structural support - 15 stainless steel cylinders from .35 to .12 inches thick, each cylinder self-supporting.

The refrigeration to operate a pair of such coils is about 50 Kw. It is assumed that a helium gas refrigeration cycle would be an attractive possibility with the refrigerator working gas passing directly through the coils.
The cycle involved might resemble that of a helium liquifier except liquid temperature would never be reached.

A schematic diagram of a simple gas cycle is shown with 2 expansion engines, in Fig. 4. The temperatures and pressures shown are not necessarily optimum values. The ratio of compressor work to refrigeration is about 100. For the above pair of coils, a compressor capacity of about 5,000 Kw would be required. The capital cost of such a refrigeration plant might be about $1.5 million as compared to over $3 million for a 50,000 Kw power supply to operate water-cooled copper coils of the same size and the power cost would be about 1/10 that of the water-cooled copper coils.

4. Stability

The stability of such a coil, in which resistance increases as any local temperature rise occurs, must be examined. Figure 5 is an illustration of stable vs unstable operation. Local heat production, for a particular coil, is plotted against conductor temperature and three lines representing heat removal rate (at some fixed coolant temperature) are shown. Intersection of heat production and heat removal curves represent possible operating conditions.

A represents stable operation, B represents stability at one temperature, and instability at a higher temperature, and C represents impossible operation. It can be shown that at low temperatures stability requires

\[ \frac{5}{(1+R)} T < \frac{1}{T - T_0} \]

which is satisfied by all \( T \) for \( R \geq 4 \) and by \( T < \frac{5T_0}{4-R} \) for \( R < 4 \).
where \( T \) = conductor temperature
\( T_c \) = coolant temperature

\[
R = \frac{\rho_i + \rho_c}{\rho_i \cdot \rho_c}
\]

For the case of the sodium coil described previously this stability criterion is satisfied for all practical purposes.

5. Heat Leak Along Leads

Heat leak into the low temperature region along the two electrical leads of the coil is inevitable; however, the magnitude of this heat leak is relatively small. For the sodium coils, assuming one end of the electrical lead at \( 10^6 \)K and the other end at room temperature, about 1.1 Kw of heat leak occurs per coil or 4.5% of the total heat load. This is computed for optimum sized leads which, for 1 ft. length, are 2.2 inches in diameter. If the warm ends of the leads are maintained at liquid nitrogen temperature, heat leak is about 1/4 of the above values for the optimum lead size of 1.3 in. diameter. By using smaller conductors and more turns, this heat leak can be reduced.

6. Future Plans and Conclusions

At present we are experimenting with distillation of sodium and coil construction techniques. We are constructing an 8" diameter coil to test these ideas. Techniques for welding and wrapping of 5/8" square by .010" wall stainless steel tubing have been developed. The sodium must solidify in a controlled
manner, after being cast into place, in order to eliminate formation of voids. We will conduct a short-duration test by blowing helium gas thru the 8" coil, measuring electrical and thermal behavior.

Construction of a pair of larger, D.C. coils, complete with refrigerator, will be considered after the small-scale testing is finished.

To summarize:

1. Large, gas cooled coils for 100,000 gauss D.C. magnetic fields appear to be practical.

2. Conductor purity using sodium or aluminum, can be made high enough to achieve a net power saving of about 10 over conventional copper coils.

3. Large refrigeration plants, of such size and type as to be unique, are required.

* * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Richard Mallon of Astra, Inc., has contributed significantly to this paper, especially through calculations of stability and heat leak through the leads.

References:

(3) Hitter - RSI Vol 7, pages 479, 1936
COIL PROPORTIONS GIVING MINIMUM POWER FOR A GIVEN FIELD AT THE CENTER

MUL-7553

COIL PROPORTIONS GIVING 5% MORE POWER THAN THE OPTIMUM PROPORTIONS FOR THE SAME FIELD AT THE CENTER (REQUIRES 2.2 TIMES SMALLER VOLUME THAN THE OPTIMUM COIL)

Figure 1
\[ \epsilon = \frac{\text{Power Req'd by Coil Having Proportions } \alpha \text{ & } \beta}{\text{Power Req'd by Coil Having Optimum Proportions}} \]

OPTIMUM PROPORTIONS ARE: $\alpha = 3$ & $\beta = 4$

Figure 2
Figure 3. Conductor and coolant passage arrangement.
COOLING WATER

\[ P = 25 \text{ ATM} \quad 293^\circ K \]

\[ \eta_1 = 0.65 \quad W = 1 \text{ MASS FLOW RATE} \]

\[ P = 5 \text{ ATM} \quad 290^\circ K \]

\[ \text{HEAT EXCHANGER \# 1} \]

\[ 42^\circ K \]

\[ 37.5^\circ K \]

\[ \text{HEAT EXCHANGER \# 2} \]

\[ 25.3^\circ K \]

\[ 24.8^\circ K \]

\[ \text{HEAT EXCHANGER \# 3} \]

\[ 11.6^\circ K \]

\[ \text{ENGINE \# 2} \quad \eta_2 = 0.85 \quad W = 0.72 \]

\[ 7.2^\circ K \]

\[ \text{LOAD} \]

\[ P = 5 \text{ ATM} \quad 10^\circ K \]

SCHEMATIC DIAGRAM OF A TYPICAL HELIUM GAS REFRIGERATION CYCLE

\[ \frac{\text{COMPRESSOR WORK}}{\text{REFRIGERATION}} = 97.6 \]

THE PARAMETERS CHOSEN ARE NOT NECESSARILY THE OPTIMUM PARAMETERS

Figure 4
ASSUMING \( P \) (HEAT GENERATION RATE) 
\[ = K_1 T^5 + K_2 \]
AND \( Q \) (HEAT REMOVAL RATE) 
\[ = K_3 (T - T_0) \]
THEN IT CAN BE SHOWN THAT FOR STABILITY:

\[ \frac{5}{(1+R)T} < \frac{1}{T-T_0} \]

WHERE \( R = \frac{P_i + P_B}{P_0} \)
\( T_0 = \) COOLANT TEMP.
\( P_i = \) IMPURITY RESISTIVITY
\( P_B = \) MAGNETIC RESISTIVITY
\( P_0 = \) INTRINSIC RESISTIVITY

THIS IS SATISFIED BY:

1. ALL \( T \) FOR \( R \geq 4 \)
2. \( T < \frac{5T_0}{4-R} \) FOR \( R < 4 \)

CONDITION FOR THERMAL STABILITY OF A CRYOGENIC COIL

Figure 5
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