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Recommendations Concerning the Computation and Reporting of Counting Statistics for the Nevada Applied Ecology Group

September 1975

**Prepared for the U.S. Energy
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Battelle
Pacific Northwest Laboratories

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RECOMMENDATIONS CONCERNING THE COMPUTATION AND
REPORTING OF COUNTING STATISTICS FOR THE
NEVADA APPLIED ECOLOGY GROUP

by

Richard O. Gilbert

Systems Department
Statistics Section

September 1975

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ABSTRACT

Recommendations based on work by Nicholson (1963) are made concerning the computation and reporting of count statistics for Group H8 at the Los Alamos Scientific Laboratory (LASL) relative to their analysis of environmental samples for plutonium, uranium, and other radionuclides for the Nevada Applied Ecology Group (NAEG). Computational formulas are given for (1) determining whether contamination above background level exists in the sample, (2) computing confidence intervals about the true net counting or disintegration rate of the sample, (3) determining the minimum true net disintegration rate that will be "detected" most of the time, and (4) reporting of net counts and their associated counting errors. Recommendations for answering these questions are made for four counting situations: (a) a sample (gross) count and a single background count, (b) a sample count with multiple background counts, (c) "base line" correction for background in gamma spectrometry, and (d) alpha spectrometry where a tracer is added to the sample. In addition, a "rule of thumb" (from Nicholson) is suggested for choosing between alternative methods of deciding whether contamination above background exists in the sample. An "exact" test is preferred when background counts are very low and/or the ratio of sample to background counting times is greater than 1. The concepts of "decision limits" and "detection limits" are discussed and mathematical derivations of the various computational formulas are given in an appendix. Tables are included for computing tests and confidence limits. Approximate counting error formulas for alpha spectrometry are given for various levels of generality relative to efficiencies of counting instruments and to equality of counting times for the sample, background, and tracer.

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1. INTRODUCTION

In the course of statistical consultations with scientists engaged in radio-ecological research for the Nevada Applied Ecology Group (NAEG)¹, certain questions have arisen concerning the appropriate methods of computing and reporting "counting errors" associated with net counts of radionuclides in soil, vegetation, and other environmental samples. As an initial response to this need this paper has been prepared which discusses the reporting of count data, the computation of "decision limits" to decide whether "something" is in the sample other than background, and the calculation of the "detection limit". Consideration is first given to the simplest case where a single background and sample count are available. Attention is then directed toward the more complicated situations of (a) multiple independent background readings, (b) "base line" correction for background in gamma spectrometry, and (c) alpha spectrometry where a tracer is added to the sample to be counted.

This report is based primarily on the work of Nicholson (1963 and 1966), Currie (1968), and Hartwell (1972). Currie (1968) points out the present confusion surrounding the "limit of detection". He defines 3 types of limits ("decision limit", "detection limit", and "determination limit") and shows how they are used in the specification of lower limits for a measurement process. Hartwell (1972) discusses and endorses Currie's work and illustrates the computation of "decision limits" and "detection limits". These two papers are directed at the radiochemist while the paper by

¹The NAEG was established by the Atomic Energy Commission, Nevada Operations Office in July 1970 (now the Environmental Research and Development Administration (ERDA)), for the purpose of determining and predicting the effects of radioactivity on biota, particularly with respect to food chains and environmental factors affecting man in the environs of the Nevada Test Site.

Nicholson (1963) discusses many of the same topics in a more classical statistical terminology. Nicholson (1963) studies several possible "decision limits" one of which is that suggested by Currie (1968). Nicholson's (1963) paper (summarized in Nicholson (1966)) is directed toward applications where background counts are appreciably large relative to the total (gross) counts. In this case, statistical procedures based on the normal distribution (appropriate when background counts are not a significant proportion of the total counts) may no longer be applicable.

Specific recommendations are made (summarized in Appendix A) concerning the computation and reporting of counting error information for the three cases given above. These recommendations (based largely on Nicholson's work) are directed toward counting procedures currently in use by Group H8 at the Los Alamos Scientific Laboratory (LASL). References of interest not covered in this report are those of Pasternack and Liuzzi (1968), who discuss the estimation of radionuclide concentrations in mixtures using gamma ray spectrometry, and Head (1972) on the "minimum detectable true area" of a photopeak in a Ge(Li) gamma ray spectrum.

2. SPECIFICATION OF THE PROBLEM

Consider the simplest situation of a single background count. Let

x = total (gross) number of counts in the sample obtained in time t ,
and y = background count obtained in time s ,
where x and y were obtained on count instruments with efficiencies e_s and e_g ,

respectively.² Then

$$\tilde{C} = x/e_s t - y/e_B s \quad (1)$$

= estimated total (sample) disintegrations per minute (dpm)
- estimated background dpm

= estimated net dpm.

$\tilde{C}$ is an estimate of C , the true (unknown) net disintegration rate, and $y/e_B s$ estimates B , the true (unknown) background disintegration rate.

In this paper the following questions are considered:

- Q1. How does one determine from $\tilde{C}$ whether or not $C > 0$? If it is decided that $C > 0$, then the conclusion is made that activity above background has been "detected".
- Q2. How does one compute confidence intervals about C , the true net disintegration rate?
- Q3. How are the $\tilde{C}$ and their counting errors to be reported to the scientific community for evaluation and statistical analysis for both the "detected" and "not detected" cases?
- Q4. How does one determine the minimum true net disintegration rate C_0 that we can be confident will be "detected" most of the time. (Nicholson terms C_0 the "threshold" rate. Currie and Hartwell denote C_0 by L_D .)

² In most, if not all, applications the gross and background counts will be determined on the same counting instrument in which case e_s will equal e_B . However, for the purpose of greater generality, most of the formulas given in this report are in terms of e_B and e_s . They are appropriate whether or not $e_B = e_s$.

In section 3 the answers to these questions are considered for the simple case described above, while later sections deal with the more complicated counting situations mentioned in the Introduction.

3. A SINGLE BACKGROUND READING

3.1 "Decision Limits"

In this section the various formulas presented are in terms of counts per minute (cpm) rather than dpm in order to simplify the introduction of some of the statistical concepts. Later sections are in terms of dpm and hence involve the efficiencies e_s and e_B .

The answer to Q1 involves computing $\hat{c}$ using Eq. (1) in addition to L_C , which Currie (1968) calls the "decision limit" and Hartwell (1972) refers to as the "critical level". In any case,

L_C = the net count rate which must be exceeded before a sample can be said to contain any material above background.

Hartwell (1972) suggests L_C be computed as follows:

$$\begin{aligned} L_C &= k_\alpha \sigma_B (1 + s/t)^{1/2} \\ &= k_\alpha \left[\frac{y}{s} \left(\frac{1}{t} + \frac{1}{s} \right) \right]^{1/2} \\ &= k_\alpha \sigma_U \end{aligned} \tag{2}$$

where

σ_B = standard deviation of the background count rate based on one background count determination y
 $= \sqrt{y}/s$,

σ_0 = standard deviation of $\tilde{C}$ when in fact $C = 0$ (see Section (1.1) of Appendix B for a derivation; also see Nicholson (1963: p. 17, 18)), and

k_α = 100 (1- α) percentile of the unit normal distribution.

Then, computing $\tilde{C}$ using Eq. (1) the decision whether the true net count rate C is greater than zero is made by comparing $\tilde{C}$ with L_C as follows:

	<u>Decision</u>	<u>Inference</u>	
If $\tilde{C} > L_C$	"detected"	$C > 0$	
If $\tilde{C} \leq L_C$	"not detected"	$C = 0$	(3)

Thus, the solution to Q1 suggested by Currie (1968) and Hartwell (1972) is simply to compute $\tilde{C}$ from Eq. (1) and L_C from Eq. (2) and see whether $\tilde{C} > L_C$. Note that the decision of "detected" or "not detected" can be made only after the gross and background count have been determined. Note too that the decision is not being made whether or not C is detectable, but rather, whether it has been "detected".

It is important to recognize that the above approach is actually a test of the null hypothesis $H_0 : C = 0$, where α is the "level of significance" or "Type I error" (α is discussed in the next paragraph; see Figure 1). The recognition of this fact may help avoid confusion with the "detection limit" discussed in Section (3.4) below. Now, there may be some differences of opinion concerning whether Eq. (2) is the best way to compute L_C (in fact, a different formula is recommended below), but, hopefully, the basic notion of using a test of the hypothesis $H_0 : C = 0$ to decide whether something has been "detected" can be accepted as a reasonable procedure.

**RELATIONSHIP BETWEEN THE "DECISION LIMIT"
 L_C AND THE "DETECTION LIMIT" L_D**

α = TYPE I ERROR (LEVEL OF SIGNIFICANCE)

β = TYPE II ERROR

$1 - \beta$ = "POWER" OF THE TEST

σ_0 = STANDARD DEVIATION OF $\tilde{c}$ WHEN $C = 0$

σ_D = STANDARD DEVIATION OF $\tilde{c}$ WHEN $C = L_D$

Figure 1

It will be useful for later discussions to review the role of α in hypothesis testing. The "Type I error" (α) is the probability (specified by the experimenter) of wrongly concluding that something above background exists in the sample. Hence, if in fact $C = 0$, we would wrongly conclude that $C > 0$ about 100α percent of the time if the procedure in Eq. (3) is used. The particular value of α used should be decided upon (before counts are made) on the basis of the practical consequences of making a Type I error. In general, we want α to be small, and values of 0.01, 0.05 and 0.10 are often used. However, α would ordinarily never be greater than 0.10 since the risk (α) of falsely concluding that $C > 0$ would then be too high. The values of k_α in Eq. (2) for $\alpha = 0.01, 0.05$ and 0.10 are $k_{0.01} = 2.326$, $k_{0.05} = 1.645$ and $k_{0.10} = 1.282$, respectively.

The above approach to testing $H_0 : C = 0$ is based on the assumption that $\tilde{C}$ is approximately normally distributed which allows us to use the particular values of k_α specified above. The study by Nicholson (1963:p. 3) indicates that even when the number of counts is quite low, the distribution of $\tilde{C}$ is still approximated quite well by the normal distribution.

Nicholson (1963) compared the method of computing L_C using Eq. (2) with three other methods, one of which (Eq. (4) below) he found to be preferable. He suggests the testing procedure (Eq. (3)) be followed when L_C is calculated as

$$L_C = k_\alpha \left[(x + y)/ts \right]^{1/2} , \quad (4)$$

which makes use of the gross sample count x in estimating σ_0 (see Nicholson (1963: p. 19) for a derivation of Eq. (4)). His argument for preferring Eq. (4) to Eq. (2) is based on the fact that, if in reality the true net count rate C is zero, then x as well as y is an estimate of the background count and hence a better estimate of σ_0 is obtained by using both x and y . Nicholson denotes the tests using Eqs. (2) and (4) as decision rules D_2 and D_3 , respectively. The remainder of this section is devoted to a discussion of the merits of D_2 , D_3 , and an "exact" test D_e , leading to a rule of thumb (Eq. (5)) for deciding which rule to use in a given situation.

Nicholson's work indicates that the exact significance levels of the decision rule D_3 are closer to the stated level α than is the case for rule D_2 . (The significance level of the tests D_2 and D_3 are not exactly α for any finite sample and background count since only when these counts are infinitely large does $\tilde{c}$ become exactly normally distributed.) In particular, he found that when Eq. (2) (rule D_2) was used, too many "detected" conclusions were reached. Table D.1 in Appendix D (an abbreviated version of Nicholson's Table 1) compares the exact significance levels of D_2 and D_3 with the stated level α . As an example of its use, consider the case where $\alpha = 0.05$ and the expected background count (tB) is 2 counts in time t . If $\rho = t/s$ (ratio of sample count time t to background count time s) is $1/3$, then when $C = 0$ we see that the decision "detected" is reached about 9.5% of the time when D_2 is used, which is considerably greater than the expected 5% rate. For rule D_3 (Eq. (4)) the exact level is elevated only slightly to 5.9%.

An inspection of Table D.1 shows that the difference between exact and stated significance levels of both tests varies depending on the count time ratio ρ and the expected background count tB . The exact levels of D_2

become extremely elevated for $\rho \geq 1$ when $tB \leq 5$. In general, the test D_3 is less affected than D_2 , but extreme underestimation of α can occur when $tB \leq 2$ when $\rho \geq 1$.

Now, an evaluation of D_2 and D_3 must also consider their performance when $C > 0$, i.e., when we would be making a "Type II error" (see Figure 1 and the discussion in Section 3.4) if we did not reject $H_0 : C = 0$. Nicholson's results (his figures 2, 3, 4) indicate that rule D_2 (Eq. (2)) is slightly more effective, i.e., is more powerful, at detecting $C > 0$ than is D_3 (Eq. (4)), but as pointed out above, at the expense of an elevated significance level. Hence, when C is greater than zero, Eq. (2) is somewhat better than Eq. (4) in detecting it, but when $C = 0$, Eq. (2) tends to wrongly conclude that $C > 0$ a greater proportion of the time than does Eq. (4). Thus, a decision as to whether Eq. (2) or Eq. (4) is to be used in testing $H_0 : C = 0$ depends on the magnitude of these effects (which depend on ρ and tB) and to the importance attached to these two types of errors.

Figures B.1 - B.18 in Nicholson's report give the "power" (probability of rejecting $H_0 : C = 0$ when in fact $C > 0$) of rule D_3 for various values of α , ρ , the expected background count tB , and the ratio of expected counts $k = tC/(tB)^{1/2}$. Five of these are reproduced here in Appendix D as Figures D.1 - D.4. These are for $\rho = 1$ and $1/2$ when $\alpha = 0.01$ and for $\rho = 1/3, 1/2$ and 1 when $\alpha = 0.05$. Each curve in these figures corresponds to an expected background count tB and shows how for given tB the ability or "power" of rule D_3 using Eq. (4) to detect a positive net count increases as the ratio k increases. These figures also show that for a given tB and k , the power of D_3 increases as ρ increases, i.e., as the sample is counted for a longer time relative to the background count time.

Nicholson (1963: p. 6) derives an exact test D_e for use when Eq. (4) does not perform well. He suggests the following rule of thumb:

$$\begin{aligned} & \text{"For stated } \alpha \text{ equal to } \begin{Bmatrix} 0.10 \\ 0.05 \\ 0.01 \end{Bmatrix}^3 \text{ use decision rule } D_3 \text{ when } \rho \leq 1 \\ & \text{and } y > \begin{Bmatrix} 5 \\ 10 \\ 15 \end{Bmatrix}; \text{ use exact decision rule } D_e \text{ when } \rho > 1 \text{ and/or} \\ & y \leq \begin{Bmatrix} 5 \\ 10 \\ 15 \end{Bmatrix} \text{"}. \end{aligned} \quad (5)$$

This appears to be a reasonable guideline and its adoption by the participating laboratories in the NAEG program is recommended.

The rule D_e (Nicholson (1963: p. 24)) is exact in the sense that its significance level is exactly α rather than being elevated or depressed for certain values of ρ and tB as happens with D_2 and D_3 . It is based on the binomial rather than the normal distribution and is an example of a randomized decision rule. The decision "detected" or "not detected" is made by comparing the gross count x with the number X , where tables of X and its associated probability P are given in Nicholson (1963; p. A.1-A.6). More specifically:

		Decision	Inference
Rule D_e :	If $x > X$	"detected"	$C > 0$
	If $x = X$	"detected" with probability P	$C > 0$ or $C = 0$ depending on outcome of random number draw (see text)
	If $x < X$	"not detected"	$C = 0$

³ "In the remainder of this statement, use the top quantity in each bracket with 0.10, the second with 0.05, etc."

Nicholson gives tables of the fixed numbers X and P for $\alpha = 0.10, 0.05$ and 0.01 and values of $x + y$ from 1 to 60 and ρ from $1/10$ to 10. These are reproduced here as Table D.2 in Appendix D.

When $x = X$, rule D_e specifies that the decision "detected" or "not detected" depends on the "randomization" probability P . If this should occur in practice, i.e., if $x = X$, the decision can be reached by choosing at random a 3 digit number, say P' , from a random number table (i.e., a table of randomly assorted digits from 0 to 9), dividing it by 1000 and then comparing it with the tabled P value for specified α , $x + y$ and ρ . Thus, when $x = X$

	<u>Decision</u>	<u>Inference</u>
If $\frac{P'}{1000} > P$	"detected"	$C > 0$
If $\frac{P'}{1000} \leq P$	"not detected"	$C = 0$

where P is tabled in Table D.2.

As an example, suppose it is desired to make an $\alpha = 0.05$ level test when $\rho = 2$ and $y = 4$ so that according to the rule of thumb (Eq. (5)) the exact test D_e should be used. Suppose $x = 20$ so that $x + y = 24$. Entering Table D.2 for $\alpha = 0.05$, $x + y = 24$ and $\rho = 2$ we find $X = 20$ and $P = 0.762$. Since $x = X$, a random 3 digit number P' from a random number table is chosen and divided by 1000, giving say, $P'/1000 = 0.037$. Since 0.037 is less than the tabled P value of 0.762, we decide "not detected" and infer $C = 0$. The validity of the procedure depends on the 3 digit number P' being selected at random from the random number table so that any number between 000 and 999 is equally likely to be chosen.

The test is very simple to make when $x > X$ or $x < X$, since there is then no need to take the additional step of using a random number table. Random number tables are often given in standard statistical methods textbooks (e.g., Snedecor and Cochran (1967)). More extensive tables are given in "A Million Random Digits With 100,000 Normal Deviates" published by The Rand Corporation (1955). A more sophisticated approach would be to use a pseudo-random number generator on a digital computer to obtain a 3 digit number for use when $x = X$, but some assurance must be given that the random number generator does in fact generate truly random digits.

3.2 Confidence Intervals

Concerning Q2 in Section 2 on the computation of confidence intervals, it is recommended that these be computed in units of dpm, i.e., after correcting for the efficiencies e_s and e_B of the sample and background counting instrument(s), respectively. When $t_B > 10$ and one background count determination is available, the recommended method for computing the upper and lower 100 $(1-\alpha)$ percent confidence limits on the true net disintegration rate C is

$$(x/e_s t - y/e_B s) \pm k_{\alpha/2} (x/e_s^2 t^2 + y/e_B^2 s^2)^{1/2} \quad (6)$$

The quantity under the square root sign is an estimate of the variance of the estimated net dpm ($\hat{C}$) when $C \geq 0$ (see Section (1.1) of Appendix B for a proof). Values of $k_{\alpha/2}$ for $\alpha = 0.10, 0.05$ and 0.01 are $k_{0.10/2} = 1.645$, $k_{0.05/2} = 1.960$ and $k_{0.01/2} = 2.576$, respectively. For example, a 95% confidence interval on C would be obtained using $k_{0.05/2} = 1.960$. Eq. (6)

(with $e_s = e_B = 1$)⁴ is the formula used by Hartwell (1972: p. 10) to compute confidence limits on the net count rate when $\tilde{c} > L_C$. However, there is no necessity to limit the computation of a confidence interval to the situation where $\tilde{c} > L_C$. It is an equally valid computation when $\tilde{c} \leq L_C$, i.e., when "not detected" is the conclusion reached from testing $H_0 : C = 0$. It is not recommended, however, that a confidence interval be reported on a routine basis by the laboratories (see Section (3.3) below).

Nicholson (1963: p. 5) considers an alternative method of computing confidence intervals which he recommends over Eq. (6) when y (the background count) lies between $2 + 3/2\rho$ and $6 + 12/\rho$. The form of the confidence interval on the true net count rate C is

$$\frac{L}{t} \leq C \leq \frac{U}{t} \quad (7a)$$

where L/t and U/t are the lower and upper $100(1-\alpha)$ percent confidence limits. His method may also be used to put confidence intervals on the true net disintegration rate C when $e_s = e_B$, in which case Eq. (7a) becomes

$$\frac{L}{te_s} \leq C \leq \frac{U}{te_s} \quad (7b)$$

In the unlikely event that $e_s \neq e_B$ then Eq. (6) must be used. The limits L and U depend on x , y , $\rho = t/s$ and α , and are given in Table D of Nicholson (1963: pp. D.1-D.99) for $1-\alpha = 0.90, 0.95$ and 0.99 , for ρ between 2 and $1/10$. Unfortunately, this table is too long to include in its entirety here, but in Table D.3 in Appendix D his tables for $1-\alpha = 0.95$ and $\rho = 1$ and $1/2$ are given.

⁴ No counting instrument has an efficiency of 1. However, by setting $e_s = e_B = 1$ in Eq. (6), confidence limits are obtained on the net count rate rather than the net disintegration rate.

From Nicholson (1963, p. 8) it appears that the differences between Eqs. (6) and (7a) become negligible for expected backgrounds >10 . While he does not specifically discuss what happens when $e_s = e_B \neq 1$ (the case in practice) it is suggested here that Eq. (7b) be used in place of Eq. (6) whenever $e_s = e_B$ and $t_B < 10$. If Table D.3 is not adequate or if Nicholson's Table D is not available, Eq. (6) may be used in any case. Nicholson (1963: p. 8) states that the use of either method is questionable if $y < 2 + 3/2p$.

3.3 Reporting of Data

Now, concerning question Q3 in Section (2) above on the reporting of data, we feel that "less than" numbers or the letters "ND", used in place of a numerical result to signify "not detected", should not be reported, regardless of the outcome of the recommended testing procedures in Section (3.1). Whether or not the decision "detected" has been reached, the net dpm and its counting error should be reported. All net disintegrations $\hat{C}$ obtained are important even if they are negative or very near zero since they provide information about the total variability associated with a given counting system. Concerning negative numbers, if $C = 0$ we would in fact expect to obtain negative as well as positive net results since x and y are both estimating background levels. If the decision "not detected" is reached, this could be indicated on the output sheet by the symbol, say $+$ (the symbol $*$ should not be used since this customarily indicates statistical significance, which is opposite to the intended meaning here).

To be specific, the recommended reporting system is

$$\hat{C}^+ \pm "1 \sigma \text{ counting error}" \quad , \quad (8)$$

which, when one background count determination is being used to estimate the background rate, is computed as

$$(x/e_s t - y/e_B s)^{\dagger} \pm (x/e_s^2 t^2 + y/e_B^2 s^2)^{1/2}, \quad (9)$$

where the symbol $\dagger$ is used only if the test of $H_0 : C = 0$ (see Section (3.1)) results in the decision "not detected". If desired, the absolute value of the percent counting error ($100 \sigma/\tilde{c}$) could also be reported.

This method of reporting data is somewhat different from that suggested by Hartwell (p. 10). He recommends that if a net count is "detected", the 95% confidence interval on the net count rate (computed as in Eq. (6) above) be reported, i.e.,

$$\tilde{c} \pm k_{.05/2} (x/t^2 + y/s^2)^{1/2}, \quad (10)$$

which is similar to the reporting system above except that he multiplies the right-hand side of Eq. (8) above by $k_{.05/2} = 1.96$. Now, when the decision "not detected" is reached, Hartwell and Currie recommend that the upper 95 percent one-sided confidence limit

$$U = (x/t - y/s) + 1.645 (x/t^2 + y/s^2)^{1/2} \quad (11)$$

be computed and that the sample be reported as having a net count rate of "less than" the computed U with 95% confidence. We have noted above our objection to reporting "less than" numbers since it throws away information. While there is nothing statistically wrong with reporting results as in Eq. (10), the recommended method (Eq. (9)) is more flexible since confidence intervals for any desired level of significance (not just $\alpha = 0.05$) can be quickly computed by multiplying the right-hand side of Eq. (9) by $k_{\alpha/2}$.

Note too that U in Eq. (11) can be easily computed from the information in Eq. (9) if the individual investigator so wishes. Also, Hartwell (p. 10) indicates that his reporting method can be confusing to those who do not adequately understand the method since it is likely that some positive values reported will be less than other "less than" reported values. The use of Eq. (8) above should avoid this kind of confusion.

3.4 "Detection Limit" or Threshold

Quoting from Hartwell (p. 7):

"Suppose someone calls you on the phone and wants to know what your 'lower detection limit' is. He is really asking, 'If I send you a sample, how much material must be in that sample for you to be sure (95 percent sure) that you will find it?'".

Currie (1968) denoted that amount of material by L_D , the "detection limit", where

L_D = the smallest net count rate for which we can be
100(1- β) percent sure of detecting the material
present in the sample using a given measurement
process.

The "95 percent sure" in the above quote is the "100(1- β) percent sure" used in this definition of L_D . Currie showed how to compute L_D and Hartwell extended his results to the case where the sample count time (t) is different from the background count time (s). In the present discussion L_D is defined in terms of dpm rather than cpm.

In Section (1.2) of Appendix B it is shown that when working with disintegration rates, L_D is computed as follows:

$$L_D = L_C + \frac{k_\beta^2}{2e_s} \left[1 + \left(1 + \frac{4L_C e_s}{k_\beta^2} + \frac{4L_C^2 e_s^2}{k_\alpha^2 k_\beta^2} \right)^{1/2} \right] \quad (12)$$

which reduces to

$$L_D = 2L_C + k^2/e_s, \quad (13)$$

when $k_\alpha = k_\beta = k$ (Hartwell (p. 9)). Figure 1 shows the relationship between L_C and L_D . The "decision limit" L_C in Eqs. (12) and (13) should be computed using Eq. (A.3) in Appendix A unless it is known without a doubt that the sample contains no activity above background. In the latter case L_C could be computed using the right hand side of Eq. (A.1) since both x and y will then estimate background. The point here is that σ_0^2 should estimate the variance of $\hat{C}$ when $C = 0$, and this will not be the case if x is used in the estimation of σ_0^2 when (unknown to us) $C > 0$. An examination of Figure 1 may help here (it is also useful in understanding the proof of Eq. (12) in Section (1.2) of Appendix B).

The quantity k_β is the $100(1-\beta)$ percentile of the unit normal distribution where β is the "Type II error" mentioned above in Section 3.1 and is defined as the probability of wrongly concluding that nothing above background exists in the sample. Conversely, $1-\beta$ refers to the "power" or confidence we desire in being able to detect the net rate L_D . Values of k_β for $\beta = 0.01, 0.02, 0.05$ and 0.10 are the same as those given for k_α in Section 3.1, for $\alpha = 0.01, 0.02, 0.05$ and 0.10 , respectively.

Notice from Eqs. (12) and (13) that the value of the "detection limit" L_D changes depending on the background count y , the counting times t and s , the efficiencies e_s and e_b , and the values of k_α and k_β . Hence, the answer to the question quoted from Hartwell at the beginning of this section can only

be given after these quantities are specified for a given measurement process. The purpose of computing L_D is to obtain some notion of the level of net disintegration rate that must be present in a sample before it will be "detected" a large proportion $(1-\beta)$ of the time by a given measurement technique. If $1-\beta$ is increased, we see that k increases which implies L_D also increases. Hence, if we want to be 99% sure ($1-\beta = 0.99$) of detecting material present in the sample, L_D will be a larger fixed net disintegration rate than will be the case if we need to be only 80% sure ($1-\beta = 0.80$).

We have seen from Nicholson's study that the significance level associated with L_C computed using Eq. (2) tends to be greater than the stated level α , particularly for $tB \leq 5$ and $\rho \geq 1$. Referring to Figure 1 this implies the area α under Curve I is too large, i.e., L_C is farther to the left than it should be. This in turn implies that L_D is somewhat smaller than it should be relative to the stated α and β requirements. Unfortunately, we have no ready solution to this problem other than to point it out and to urge caution in the interpretations and applications of L_D for small background counts and $\rho \geq 1$. The magnitude of this problem when $e_s \neq e_B$ is unknown.

4. MULTIPLE INDEPENDENT BACKGROUND READINGS

If n background counts $y_1, y_2, \dots, y_n$ counted for times $s_1, s_2, \dots, s_n$, respectively, have been made to estimate the true background disintegration rate B , these should be utilized in testing $H_0 : C = 0$ and in computing L_D . The generalizations of Eqs. (1) and (2) to the case $n > 1$ (assuming the n background readings all estimate the same true background level and all are counted with the same efficiency) are

$$\hat{C} = x/t - \bar{y}/\bar{s} \quad (14)$$

and

$$\begin{aligned} L_C &= k_\alpha \sigma_0 \\ &= k_\alpha [(\bar{y}/\bar{s}) (1/t + 1/n\bar{s})]^{1/2}, \end{aligned} \quad (15)$$

where

$$\bar{y} = \left(\sum_{i=1}^n y_i \right) / n$$

and

$$\bar{s} = \left(\sum_{i=1}^n s_i \right) / n$$

so that

$$\bar{y}/\bar{s} = \sum_{i=1}^n y_i / \sum_{i=1}^n s_i \quad .^\dagger$$

There are several comments we wish to make at this point. First of all, we note that Eq. (15) reduces to Eq. (2) when $n = 1$. Secondly, the proof that $\sigma_0 = [(\bar{y}/\bar{s}) (1/t + 1/n\bar{s})]^{1/2}$ when $C = 0$ (see Section (1.1) of Appendix B) depends on the assumption that $\bar{s}$, the average background count time, is a fixed constant rather than a random variable. $\bar{s}$ would be a random variable e.g., if the background counts were obtained by letting the counting continue until a fixed number of background counts were obtained. Eq. (15) is not appropriate for that type of procedure, nor for that matter are any of the formulas given in this paper. Nicholson (1963: p. 5) notes that Birnbaum (1954) discusses random time procedures.

The "rule of thumb" given in Eq. (5) for $n = 1$ indicates Eq. (4) be used to test $H_0 : C = 0$ unless the background count is very low and/or the ratio of count time $\rho = t/s$ is greater than 1. Nicholson (1963) did not consider the case $n > 1$, but using the method by which he derived Eq. (4)

[†]The estimate $\bar{y}/\bar{s}$, rather than the average rate $(1/n) \sum_{i=1}^n (y_i/s_i)$, is used to estimate B since $\text{Var}(y_i)$ (which equals $E(y_i)$) is proportional to count time s_i (see, e.g., Armitage (1971: p. 278)).

(weighting the $x, y_1, y_2, \dots, y_n$ by the inverse of their variances) it is shown in Section (1.3) of Appendix B that Eq. (4) can be generalized for $n > 1$ to

$$L_C = k_\alpha \sigma_0$$

$$= k_\alpha \left[\frac{\left(x + \sum_{i=1}^n y_i \right)}{\left(t + \sum_{i=1}^n s_i \right)} \left(\frac{1}{t} + \frac{1}{n\bar{s}} \right) \right]^{1/2} \quad (16)$$

Eqs. (2), (4), and (15) are all special cases of Eq. (16), since (16) reduces to (4) when $n = 1$ and to (15) when the gross count information x and t are not used. Eq. (2) is a special case of Eq. (15) when $n = 1$.

Analogous to the use of Eq. (4) when $n = 1$, a test of $H_0: C = 0$ is made by seeing whether $\tilde{c} = x/t - \bar{y}/\bar{s}$ is greater than L_C computed using Eq. (16): if so, we decide something has been "detected" and infer $C > 0$; otherwise we decide "not detected" and infer $C = 0$. We have not constructed an exact test of $H_0: C = 0$ analogous to D_e for $n = 1$ (although this could probably be done). Nor have we made any study of the exact significance levels of the test using Eq. (16) to see whether an exact test is even required. However, Nicholson (p. 26) proved when the gross and background counts are large for $n = 1$, the exact test D_e is equivalent to D_3 (the test using Eq. (4)). This suggests the significance levels of Eq. (16) may not be as biased as are those for D_3 since $n > 1$ background counts are pooled to make a larger total background event than when $n = 1$. In any case, based on Nicholson's study for $n = 1$, the significance level of

the testing using Eq. (16) should be uniformly closer to the stated level than that of the test using Eq. (15). Hence, it is recommended that Eq. (16) be used to compute L_C when $n > 1$, with the cautionary note that for $\rho \geq 1$ and very low background counts too few "detected" decisions may be obtained when in fact $C = 0$.

The above discussion has been in terms of cpm rather than dpm. In Appendix A, Eqs. (14) and (16) are rewritten as Eq. (A.1) for the more general case where the background determinations were all taken on instruments with the same efficiency e_B , but e_B need not equal e_s , nor be equal to one. Hence, a test of $H_0 : C = 0$ can be carried out in this more general case by using Eq. (A.1). As with the case where $n = 1$, if it should happen that $e_s = e_B$ then Eq. (16) may be used to make the test, i.e., the decision reached will be the same whether Eq. (16) or (A.1) is used. But the values of $\hat{C}$ and the decision limit L_C themselves are not the same using the two equations.

Concerning the computation of a $100(1-\alpha)$ percent confidence interval on C when $n > 1$, the straightforward generalization of Eq. (6) is recommended, i.e.,

$$\left[\frac{\frac{x}{e_s t} - \frac{\sum_{i=1}^n y_i}{e_B \sum_{i=1}^n s_i} \right] \pm k_{\alpha/2} \left[\frac{\frac{x}{e_s^2 t^2} + \frac{\sum_{i=1}^n y_i}{e_B^2 \left(\sum_{i=1}^n s_i \right)^2} \right]^{1/2}, \quad (17)$$

where the standard deviation on the right hand side is derived in Section (1.1) of Appendix B. The data reported by the laboratories should be $\hat{C} \pm 1 \sigma$, i.e.,

$$\left[\frac{x}{e_s t} - \frac{\sum_{i=1}^n y_i}{e_B \sum_{i=1}^n s_i} \right]^+ \pm \left[\frac{x}{e_s^2 t^2} + \frac{\sum_{i=1}^n y_i}{e_B^2 \left(\sum_{i=1}^n s_i \right)^2} \right]^{1/2} \quad (18)$$

where the + symbol is used only if $\tilde{c} \leq L_C$, where $\tilde{c}$ and L_C are computed as in Eq. (A.1) in Appendix A. The absolute value of the percent counting error $|100 \sigma/\tilde{c}|$ could also be reported.

The limit L_D can be computed using Eqs. (12) or (13) (whichever is appropriate depending on whether or not $k_\alpha = k_\beta$), where L_C in these equations is computed using Eq. (A.3) in Appendix A rather than the right hand side of Eq. (A.1) for reasons discussed in Section (3.4) for the case $n = 1$.

5. "BASE LINE" CORRECTION FOR BACKGROUND IN GAMMA SPECTROMETRY

In gamma spectrometry the determination of background is not simply a matter of counting a background sample. In this case the technique known as "base line" correction can be used to estimate background levels. The computational formulas for L_C , L_D , and confidence intervals are slightly different than those given above.

In "base line" correction the "peak" channel is first located and a certain number of channels on either side are used to estimate the total (gross) count. Then channels on either side of these "peak" channels are averaged to estimate the background count. In this situation the counting time for both gross and background counts are necessarily equal and denoted here by t . Similarly there is only one counting instrument and hence one efficiency e_B . Let

$\sum_{i=1}^{n_1} x_i$ = total gross counts over n_1 "peak" channels

$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$ = average background count over n_2 channels
bracketing the n_1 "peak" channels.

Then

$$\hat{c} = \frac{1}{te_B} \sum_{i=1}^{n_1} x_i - n_1 \left(\frac{1}{te_B n_2} \sum_{i=1}^{n_2} y_i \right) \quad (19)$$

= gross dpm - (n_1) (average background dpm)

= estimated net dpm.

Then for testing $H_0: C = 0$ we may compare $\hat{c}$ computed using Eq. (19) with (see Section 2.2 in Appendix B for the derivation of Eq. (20))

$$L_C = k_\alpha \sigma_0$$

$$= \frac{k_\alpha}{te_B} \left[\frac{n_1}{n_2} \left(\sum_{i=1}^{n_1} x_i + \sum_{i=1}^{n_2} y_i \right) \right]^{1/2}, \quad (20)$$

the decision "detected" being reached if $\hat{c} > L_C$; otherwise we conclude "not detected". The $100(1 - \alpha)$ percent confidence interval about C is given by the limits

$$\hat{c} \pm \frac{k_{\alpha/2}}{te_B} \left[\sum_{i=1}^{n_1} x_i + (n_1/n_2)^2 \sum_{i=1}^{n_2} y_i \right]^{1/2}, \quad (21)$$

where

$$\frac{1}{t^2 e_B^2} \left(\sum_{i=1}^{n_1} x_i + (n_1/n_2)^2 \sum_{i=1}^{n_2} y_i \right) \quad (22)$$

is an estimate of the variance of $\hat{c}$ when $C \geq 0$. The data reported by

the laboratories should be

$$\tilde{c}^{\dagger} \pm [\text{Var}(\tilde{c})]^{1/2},$$

where $\dagger$ is used only if $\tilde{c} \leq L_C$, where $\tilde{c}$ is given by Eq. (19), and $\text{Var}(\tilde{c})$ by Eq. (22). The computation of L_D is according to Eqs. (12) or (13) with L_C computed as

$$\frac{k_{\alpha}}{e_B t} \left[\frac{n_1 (n_1 + n_2)}{n_2^2} \sum_{i=1}^{n_2} y_i \right]^{1/2} \quad (23)$$

Note that L_D now depends on $\bar{y}$, e_B , t , n_1 , n_2 , k_{α} and k_{β} . The derivations of the various variance formula given here for "base line" correction are given in Sections 2.1 and 2.2 of Appendix B.

6. APPROXIMATE TESTS AND CONFIDENCE INTERVALS WITH ALPHA SPECTROMETRY

When ^{239}Pu is determined using alpha spectrometry where a tracer, say ^{242}Pu , is added to the sample, the estimation problems can become rather complicated. This complexity arises because of the need to (i) correct for contamination of the sample by ^{239}Pu introduced with the ^{242}Pu tracer, (ii) to take into account the yield of the ^{242}Pu obtained from the sample, and (iii) to allow for different counting times and efficiencies of the various counting instruments used. For example, using the notation defined in Appendix (C) it is stated there (Eq. (C.2)) that the estimated disintegration rate of ^{239}Pu is given by

$$\tilde{c} = \frac{1}{e_T} \left[\frac{\left(\frac{x}{t} - \frac{b_x}{s} \right) \left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)}{\left(\frac{w}{t} - \frac{b_w}{s} \right)} - \left(\frac{v}{t_T} - \frac{b_v}{s_T} \right) \right] \quad (24)$$

An approximate expression for $\text{Var}(\hat{c})$ can be obtained using a technique commonly known as "propagating the errors" or the "delta method", which uses the first few terms of a Taylor series expansion of $\hat{c}$. In Appendix C 3 levels of generality are considered relative to the equality of counting times for sample and background count determinations and to whether the ^{242}Pu tracer is contaminated with ^{239}Pu . For each case we derive by "propagating the errors" the appropriate formula for (i) computing the "1 σ counting error" for reporting with the net dpm, and for (ii) computing the variance of $\hat{c}$ used to calculate L_C for use in computing L_D and for testing whether the "true" net count is zero. Since these approximate expressions are rather lengthy and are given in Appendix C, we do not repeat them here. These results are also summarized in Appendix A.

The accuracy of the approximate variance formulae given in Appendix C will depend on the magnitude of the relative variances $[\text{Var}(\text{count})]/(\text{count})^2$ of the various counts, which will be large when counting times are short and/or the count rate is low. One approach that could be used to study their accuracy would be a computer simulation study. The approximate variance formulas would be evaluated for various sample, background, and tracer count levels, and for different counting times and yields. Another more direct approach would be to try and enlarge upon the work of Goodman (1962) who found an exact expression to replace the approximate (propagation of errors) formula for the variance of the product of two means. In the present context we would need to consider in addition the ratio of means. Until such studies have been made, care should be used in the interpretation of tests of significant or in the "detection limit" L_D computed when sample and background counts are few in number.

Concerning L_D , the approximate variance expressions in Appendix C indicate that the "detection limit" depends on not only the sample and background counts, but also on the yield, efficiency, counting time and the amount of ^{239}Pu contamination present in the tracer. Hence, the utility of computing such a limit would seem to be reduced if the computed L_D varies widely as these variables change. Some consideration should also be addressed to whether the formulae for L_D given in Eqs. (12) and (13) are the most appropriate when working with alpha spectrometry. We have not looked at the problem but suspect that a more complicated expression may be required.

7. DISCUSSION

In this paper certain recommendations have been made relative to the computation and reporting of count statistics for counting procedures currently practiced by Group H8 at LASL. While the topics discussed here are necessarily limited in scope, future work could be directed toward counting procedures currently in use at other NAEG participating laboratories. The estimation techniques discussed by Pasternack and Liuzzi (1968) might also be applied to gamma ray spectrometry estimation problems at the various laboratories.

Hopefully the discussion here of the "decision limit" L_C and the "detection limit" L_D will result in a better understanding of the different roles played by these two quantities. A mutual understanding of these concepts might contribute to more uniform methods of deciding whether something above background is in the sample and to more uniform reporting methods.

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APPENDIX A

SUMMARY OF RECOMMENDATIONS

APPENDIX A1. NOTATION

x = total (gross) sample counts in time t

y_i = i^{th} background count in time s_i , $i = 1, 2, \dots, n$ ($n \geq 1$)

C = true (unknown) net disintegration rate (dpm)

B = true (unknown) background disintegration rate (dpm)

$\hat{C}$ = estimated net disintegration rate (dpm)

$\rho = t/s$ = ratio of sample to background count time when $n = 1$

k_α = 100(1 - α) percentile of the unit normal distribution

n = number of independent background count determinations ($n \geq 1$)

n_1 = number of "peak" channels in "base line" correction for background

n_2 = number of "background" channels in "base line" correction for background

e_s = efficiency of the counter on which x is obtained

e_B = efficiency of the counter on which each of the n background counts were obtained.

e = efficiency of the counter in "base line" correction for background

2. RECOMMENDATIONS

Q1: How to test $H_0: C = 0$, i.e., whether $\hat{C}$ indicates something above background has been "detected".

A: One or more independent background readings ($n \geq 1$).⁵

Decide "detected" if

⁵ Use decision rule D_e rather than Eq. (A.1) when $\rho > 1$ and/or $\sum_{i=1}^n y_i \leq \begin{Bmatrix} 5 \\ 10 \\ 15 \end{Bmatrix}$.
See "rule of thumb" (Eq. (5)).

$$\left(\frac{x}{e_s t} - \frac{\sum_{i=1}^n y_i}{e_B \sum_{i=1}^n s_i} \right) > k_\alpha \left[\frac{\left(x + \sum_{i=1}^n y_i \right)}{\left(t + \sum_{i=1}^n s_i \right)} \left(\frac{1}{e_s^2 t} + \frac{1}{e_B^2 n \bar{s}} \right) \right]^{1/2} \quad (\text{A.1})$$

Otherwise decide "not detected".

B: "Base line" correction

"Detected" if

$$\frac{1}{te} \left[\sum_{i=1}^{n_1} x_i - (n_1/n_2) \sum_{i=1}^{n_2} y_i \right] > \frac{k_\alpha}{te} \left[\frac{n_1}{n_2} \left(\sum_{i=1}^{n_1} x_i + \sum_{i=1}^{n_2} y_i \right) \right]^{1/2} \quad (\text{A.2})$$

Otherwise decide "not detected".

C: Alpha spectrometry (see Appendix C for approximate formulas and notation).

Case	"Detected" if
1 (most general)	Eq. (C.2) > k_α [Eq. (C.12)]
2 ($t = s$, $t_T = s_T$)	Eq. (C.5) > k_α [Eq. (C.13)]
3 ($t = s$, $t_T = s_T$, $v = 0$, $b_v = 0$)	Eq. (C.7) > k_α [Eq. (C.14)]

Otherwise decide "not detected".

Q2: How to Compute a 100(1- α) percent confidence interval on C.

A: One or more independent background readings ($n \geq 1$)

A.1. $e_s = e_B$ and $tB < 10$

$$\frac{L}{te_s} \leq C \leq \frac{U}{te_s}$$

where L and U are tabled by Nicholson (1963: Tables D.1, D.2, D.3) for $1 - \alpha = 0.90, 0.95$ and 0.99 for various values of ρ , x and y . See Table D.3 in the present report for his tables when $1 - \alpha = 0.95$ and $\rho = 1$ and $1/2$.

A.2 $e_s \neq e_B$

$$\left[\frac{x}{e_s t} - \frac{\sum_{i=1}^n y_i}{e_B \sum_{i=1}^n s_i} \right] \pm k_{\alpha/2} \left[\frac{x}{e_s^2 t^2} + \frac{\sum_{i=1}^n y_i}{e_B^2 \left(\sum_{i=1}^n s_i \right)^2} \right]^{1/2}$$

B: "Base line" correction.

$$\frac{1}{et} \left[\sum_{i=1}^n x_i - (n_1/n_2) \sum_{i=1}^n y_i \right] \pm \frac{k_{\alpha/2}}{et} \left[\sum_{i=1}^n x_i + (n_1/n_2)^2 \sum_{i=1}^n y_i \right]^{1/2}$$

C: Alpha spectrometry (all results are approximate)

<u>Case</u>	<u>Approximate Confidence Interval</u>
1	Eq. (C.2) $\pm k_{\alpha/2}$ [Eq. (C.4)]
2	Eq. (C.5) $\pm k_{\alpha/2}$ [Eq. (C.6)]
3	Eq. (C.7) $\pm k_{\alpha/2}$ [Eq. (C.8)]

Q3: How are data to be reported?

1. Never report "less than" numbers or "ND".
2. Always report the estimated net disintegration rate $\tilde{c}$ along with its 1σ counting error as follows:

$$\tilde{c} \pm 1\sigma \text{ counting error,}$$

where the symbol $+$ is present only when the decision "not detected" has been reached using the methods summarized above under Q1. Percent counting error ($100 \sigma/\tilde{c}$) may be reported if desired.

A: Multiple background readings ($n_i \geq 1$)

$$\left[\frac{x}{e_s t} - \frac{\sum_{i=1}^n y_i}{e_B \sum_{i=1}^n s_i} \right] \pm \left[\frac{x}{e_s^2 t^2} + \frac{\sum_{i=1}^n y_i}{e_B^2 \left(\sum_{i=1}^n s_i \right)^2} \right]^{1/2}$$

B: "Base line" correction

$$\frac{1}{et} \left[\sum_{i=1}^{n_1} x_i - (n_1/n_2) \sum_{i=1}^{n_2} y_i \right] \pm \frac{1}{et} \left[\sum_{i=1}^{n_1} x_i + (n_1/n_2)^2 \sum_{i=1}^{n_2} y_i \right]^{1/2}$$

C: Alpha spectrometry

<u>Case</u>	<u>Data to be Reported</u>
1	Eq. (C.2) $\pm$ Eq. (C.4)
2	Eq. (C.5) $\pm$ Eq. (C.6)
3	Eq. (C.7) $\pm$ Eq. (C.8)

Q4: How do we determine the minimum true net disintegration rate L_D

that we can be sure will be "detected" 100(1- β) percent of the time for significance level α ?

General Formula

(1) $k_\alpha \neq k_\beta$

$$L_D = L_C + \frac{k_\beta^2}{2e_s} \left[1 + \left(1 + \frac{4L_C e_s}{k_\beta^2} + \frac{4L_C^2 e_s^2}{k_\alpha^2 k_\beta^2} \right)^{1/2} \right]$$

(2) $k_\alpha = k_\beta = k$

$$L_D = 2L_C + k^2/e_s$$

The quantity L_C in these expressions is computed as follows:

A: Multiple background counts ($n \geq 1$)

$$L_C = k_\alpha \left[\frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n s_i} \left(\frac{1}{e_s^2 t} + \frac{1}{e_B^2 ns} \right) \right]^{1/2} \quad (A.3)$$

B: "Base line" correction

$$L_C = \frac{k_\alpha}{et} \left[\frac{n_1(n_1 + n_2)}{n_2^2} \sum_{i=1}^{n_2} y_i \right]^{1/2}$$

C: Alpha spectrometry (all results are approximate)

Case	L_C used to compute L_D
1	k_α [Eq. (C.12)]
2	k_α [Eq. (C.13)]
3	k_α [Eq. (C.14)]

APPENDIX B

DERIVATIONS OF VARIANCE FORMULAS FOR $n \geq 1$ BACKGROUND READINGS

APPENDIX B1.1 One or More Independent Background Determinations ($n \geq 1$)1.1 Derivation of general variance formulae for σ_0^2 and σ_D^2 .

When the n background determinations have the same efficiency e_B , the estimated net dpm is

$$\hat{c} = x/e_s t - \sum_{i=1}^n y_i / e_B \sum_{i=1}^n s_i$$

Therefore,

$$\begin{aligned} \text{Var}(\hat{c}) &= E(x) / e_s^2 t^2 + \sum_{i=1}^n E(y_i) / e_B^2 \left(\sum_{i=1}^n s_i \right)^2 \\ &= (C + B)t / e_s^2 t^2 + B / e_B^2 n\bar{s} \\ &= C / e_s^2 t + B \left(1/e_s^2 t + 1/e_B^2 n\bar{s} \right) \end{aligned} \quad (B.1)$$

When $C = 0$ then $\text{Var}(\hat{c})$ is given by the second term in Eq. (B.1) which is estimated by

$$\hat{\sigma}_0^2 = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n s_i} \left[1/e_s^2 t + 1/e_B^2 n\bar{s} \right] \quad (B.2)$$

which is used in the computation of L_D (Eq. (12) or (13)) when $n \geq 1$.

When $C \geq 0$, an estimate of Eq. (B.1) is

$$\hat{\sigma}_D^2 = x/e_s^2 t^2 + \sum_{i=1}^n y_i / e_B^2 \left(\sum_{i=1}^n s_i \right)^2$$

which is used in computing confidence intervals on C (Eq. (6) for $n = 1$; Eq. (17) for $n > 1$).

1.2 Derivation of the "Detection Limit" L_D (Eqs. (12) and (13))

From Currie (1968: p. 589) and Figure 1 we have that

$$L_D = L_C + k_\beta \sigma_D$$

Using Eq. (B.1), L_D in units of dpm may be written as

$$L_D = L_C + k_\beta (L_D/e_s + L_C^2/k_\alpha^2)^{1/2} \quad (B.3)$$

or

$$(L_D - L_C)^2 = k_\beta^2 (L_D/e_s + L_C^2/k_\alpha^2)$$

Expanding the left hand side and combining terms, we find

$$L_D^2 - L_D \left(2L_C + \frac{k_\beta^2}{e_s} \right) - \left(\frac{k_\beta^2}{k_\alpha^2} L_C^2 - L_C^2 \right) = 0$$

Using the quadratic formula to solve for L_D gives

$$\begin{aligned} L_D &= L_C + \frac{1}{2} \left\{ \frac{k_\beta^2}{e_s} \pm \left[\left(4L_C^2 + \frac{4L_C k_\beta^2}{e_s} + \frac{k_\beta^4}{e_s^2} \right) + 4L_C^2 \left(\frac{k_\alpha^2}{k_\beta^2} - 1 \right) \right]^{1/2} \right\} \\ &= L_C + \frac{k_\beta^2}{2e_s} \left[1 + \left(\frac{4L_C e_s}{k_\beta^2} + \frac{4L_C^2 e_s^2}{k_\alpha^2 k_\beta^2} \right)^{1/2} \right] \end{aligned}$$

when the negative sign is ignored. When $k_\alpha = k_\beta = k$, this reduces to

$$\begin{aligned} L_D &= L_C + \frac{k^2}{2e_s} + \frac{k^2}{2e_s} \left(1 + \frac{4L_C e_s}{k^2} + \frac{4L_C^2 e_s^2}{k_\alpha^2 k_\beta^2} \right)^{1/2} \\ &= L_C + \frac{k^2}{2e_s} + \left[\left(\frac{k^2}{2e_s} + L_C \right) \left(\frac{k^2}{2e_s} + L_C \right) \right]^{1/2} \\ &= 2L_C + \frac{k^2}{e_s} \end{aligned}$$

1.3 Derivation of σ_0 on Eq. (16)

We want to test $H_0: C = 0$ using a variance estimate of $\tilde{C}$ which makes use of $x, y_1, \dots, y_n$ when $C = 0$, where $\tilde{C} = x/te_s - \sum_{i=1}^n y_i / e_B \sum_{i=1}^n s_i$. When $C = 0$, x/te_s and each $y_i/s_i e_B$ estimate the true background disintegration rate B , i.e.,

$$E(x/te_s) = E(y_i/s_i e_B) = B \text{ for all } i.$$

Also,

$$\text{Var}(x/te_s) = Bte_s/t^2 e_s^2 = B/te_s$$

$$\text{Var}(y_i/s_i e_B) = Bs_i e_B/s_i^2 e_B^2 = B/s_i e_B$$

Following Nicholson (1963: p. 19), the minimum variance, unbiased, linear estimate of B is given by the weighted mean

$$\hat{B} = \frac{\omega_x (x/te_s) + \sum_{i=1}^n \omega_i (y_i/s_i e_B)}{\omega_x + \sum_{i=1}^n \omega_i}, \quad (B.4)$$

where the weights ω are the inverse of the variances of x/te_s and the $y_i/s_i e_B$, $i = 1, 2, \dots, n$. Substituting into Eq. (B.4) we find

$$\begin{aligned} \hat{B} &= \frac{\frac{x/t}{B/t} + \sum_{i=1}^n \frac{y_i/s_i}{B/s_i}}{\frac{1}{B/t} + \sum_{i=1}^n \frac{1}{B/s_i}} \\ &= \left(x + \sum_{i=1}^n y_i \right) / \left(t + \sum_{i=1}^n s_i \right) \end{aligned}$$

Substituting $\hat{B}$ into the second term of Eq. (B.1), which is $\text{Var}(\tilde{C})$ when $C = 0$, gives

$$\hat{\sigma}_0 = \left[\frac{\left(x + \sum_{i=1}^n y_i \right)}{\left(t + \sum_{i=1}^n s_i \right)} \left(\frac{1}{e_s^2 t} + \frac{1}{e_B^2 n \bar{s}} \right) \right]^{1/2}$$

which is used in Eq. (16).

2.0 "Base Line" Correction for Background

2.1 Derivation of general variance formulae for σ_0^2 and σ_D^2

Eq. (19) is

$$\tilde{c} = \frac{1}{te} \left[\sum_{i=1}^{n_1} x_i - (n_1/n_2) \sum_{i=1}^{n_2} y_i \right] \quad (B.5)$$

Therefore,

$$\begin{aligned} \text{Var}(\tilde{c}) &= \frac{1}{(te)^2} \left[\sum_{i=1}^{n_1} \text{Var}(x_i) + \left(\frac{n_1}{n_2} \right)^2 \sum_{i=1}^{n_2} \text{Var}(y_i) \right] \\ &= \frac{1}{(te)^2} \left[E \left(\sum_{i=1}^{n_1} x_i \right) + \left(\frac{n_1}{n_2} \right)^2 E \left(\sum_{i=1}^{n_2} y_i \right) \right] \\ &= \frac{1}{(te)^2} \left[(C + n_1 B) te + \left(\frac{n_1}{n_2} \right)^2 (n_2 B) te \right] \\ &= \frac{1}{te} \left[C + n_1 B (1 + n_1/n_2) \right] \quad (B.6) \end{aligned}$$

When $C = 0$, then

$$\text{Var}(\tilde{c}) = \sigma_0^2 = (n_1 B / te) (1 + n_1/n_2) \quad (B.7)$$

which is estimated by

$$\hat{\sigma}_0^2 = \left(n_1 \sum_{i=1}^{n_2} y_i / t^2 e^2 n_2 \right) (1 + n_1/n_2) .$$

This $\hat{\sigma}_0^2$ is used to compute L_C (Eq. (23)) which is used in the computation of L_D (Eq. (12) or (13)).

When $C \geq 0$, Eq. (B.6) is estimated by substituting Eq. (B.5) for C and $\sum_{i=1}^{n_2} y_i / t n_2$ for B , resulting in

$$\hat{\sigma}_D^2 = \sum_{i=1}^{n_1} x_i / t^2 e^2 + (n_1 / n_2)^2 \sum_{i=1}^{n_2} y_i / t^2 e^2$$

which is used in computing confidence intervals on C (Eq. (21)).

2.2 Derivation of σ_0 in Eq. (20)

We wish to test $H_0: C = 0$ using a variance estimate of $\hat{C}$ that makes optimal use of the x_i and y_i in Eq. (B.5). When $C = 0$ we have

$$E \left(\sum_{i=1}^{n_1} x_i / t \right) = (1/t) \sum_{i=1}^{n_1} tB = n_1 B$$

$$E \left(\frac{n_1}{t n_2} \sum_{i=1}^{n_2} y_i \right) = (n_1 / t n_2) \sum_{i=1}^{n_2} tB = n_1 B$$

$$\text{Var} \left[\sum_{i=1}^{n_1} x_i / t \right] = (1/t^2) \sum_{i=1}^{n_1} E(x_i) = n_1 B / t$$

$$\text{Var} \left[\frac{n_1}{t n_2} \sum_{i=1}^{n_2} y_i \right] = (n_1 / t n_2)^2 \sum_{i=1}^{n_2} E(y_i) = n_1^2 B / t n_2$$

Following Nicholson (1963:p. 19) the minimum variance, unbiased, linear estimate of $n_1 B$ is

$$n_1 \hat{B} = \frac{\left(\frac{\sum_{i=1}^{n_1} x_i / t}{n_1 B / t} + \frac{\frac{n_1}{t n_2} \sum_{i=1}^{n_2} y_i}{n_1^2 B / t n_2} \right)}{\left(t / n_1 B + t n_2 / n_1^2 B \right)}$$

$$= \frac{n_1 \left(\sum_{i=1}^{n_1} x_i + \sum_{i=1}^{n_2} y_i \right)}{t (n_1 + n_2)}$$

Substituting $n_1 \hat{B}$ into Eq. (B.7) gives the $\hat{\sigma}_o^2$ used in Eq. (20).

APPENDIX C

PROPAGATION OF ERRORS IN ALPHA SPECTROMETRY

APPENDIX C1. Notation

$\tilde{c}$ = estimated disintegration rate (dpm) of ^{239}Pu from the sample
(after correction for tracer and yield)

x = total (gross) counts of ^{239}Pu in the sample (includes ^{239}Pu contamination from the tracer).

w = total (gross) counts of ^{242}Pu tracer recovered in the sample.

z = total (gross) counts of ^{242}Pu tracer added to the sample

v = total (gross) counts of ^{239}Pu contamination in the ^{242}Pu tracer.

b_x, b_y, b_z, b_v = background counts for x, y, z and v , respectively.

t = count time for x and w

s = count time for b_x and b_w

t_T = count time for z and v (T stands for "tracer")

s_T = count time for b_z and b_v

e_s = efficiency of counter on which x, w, b_x and b_w are determined

e_T = efficiency of counter on which z, v, b_z and b_v are determined.

2. Estimation of $\tilde{c}$ and its Approximate Counting Error for Calculating Approximate Confidence Intervals

Case 1: (most general)

$$\tilde{c}_1 = \frac{\left(\frac{x}{te_s} - \frac{b_x}{se_s} \right) - \left(\frac{\frac{v}{t_T e_T} - \frac{b_v}{s_T e_T}}{\frac{z}{t_T e_T} - \frac{b_z}{s_T e_T}} \right) \left(\frac{w}{te_s} - \frac{b_w}{se_s} \right)}{\left(\frac{w}{te_s} - \frac{b_w}{se_s} \right) - \left(\frac{\frac{v}{t_T e_T} - \frac{b_v}{s_T e_T}}{\frac{z}{t_T e_T} - \frac{b_z}{s_T e_T}} \right) \left(\frac{x}{te_s} - \frac{b_x}{se_s} \right)} \quad (\text{C.1})$$

$$= \frac{1}{e_T} \left[\frac{\left(\frac{x}{t} - \frac{b_x}{s} \right) \left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)}{\left(\frac{w}{t} - \frac{b_w}{s} \right)} - \left(\frac{v}{t_T} - \frac{b_v}{s_T} \right) \right], \quad (C.2)$$

where the second term in the numerator of Eq. (C.1) is the amount (expressed as a rate) of ^{239}Pu contamination in the ^{242}Pu tracer added to the sample, and the quantity in parentheses in the denominator of Eq. (C.1) is the "yield", i.e., the proportion of the total tracer added that is recovered.

Using the approximate "propagation of errors" formula (see, e.g., Hahn and Shapiro (1967:p. 231)):

$$\begin{aligned} \text{Var}(\tilde{c}_1) \approx & \left(\frac{\partial \tilde{c}_1}{\partial x} \right)^2 \text{Var}(x) + \left(\frac{\partial \tilde{c}_1}{\partial b_x} \right)^2 \text{Var}(b_x) + \left(\frac{\partial \tilde{c}_1}{\partial w} \right)^2 \text{Var}(w) \\ & + \left(\frac{\partial \tilde{c}_1}{\partial b_w} \right)^2 \text{Var}(b_w) + \left(\frac{\partial \tilde{c}_1}{\partial v} \right)^2 \text{Var}(v) + \left(\frac{\partial \tilde{c}_1}{\partial b_v} \right)^2 \text{Var}(b_v) \\ & + \left(\frac{\partial \tilde{c}_1}{\partial z} \right)^2 \text{Var}(z) + \left(\frac{\partial \tilde{c}_1}{\partial b_z} \right)^2 \text{Var}(b_z) \end{aligned} \quad (C.3)$$

we find that when the counts are Poisson distributed, i.e., when $\text{Var}(x) = x$, $\text{Var}(w) = w$, etc.,

$$[\text{Var}(\tilde{c}_1)]^{1/2} = "1\sigma \text{ counting error}"$$

$$\begin{aligned}
& \approx \frac{1}{e_T} \left[\frac{\left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)^2}{\left(\frac{w}{t} - \frac{b_w}{s} \right)^2} \left(\frac{x}{t^2} + \frac{b_x}{s^2} \right) \right. \\
& + \frac{\left(\frac{x}{t} - \frac{b_x}{s} \right)^2}{\left(\frac{w}{t} - \frac{b_w}{s} \right)^4} \left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)^2 \left(\frac{w}{t^2} + \frac{b_w}{s^2} \right) \\
& \left. + \left(\frac{v}{t_T^2} - \frac{b_v}{s_T^2} \right) + \frac{\left(\frac{x}{t} - \frac{b_x}{s} \right)^2}{\left(\frac{w}{t} - \frac{b_w}{s} \right)^2} \left(\frac{z}{t_T^2} + \frac{b_z}{s_T^2} \right) \right]^{1/2} \quad (C.4)
\end{aligned}$$

Case 2: ($t = s$, $t_T = s_T$)

$$\hat{c}_2 = \frac{1}{e_T t_T} \left[\frac{(x - b_x)(z - b_z)}{(w - b_w)} - (v - b_v) \right] \quad (C.5)$$

and

$$\begin{aligned}
[\text{Var}(\hat{c}_2)]^{1/2} & \approx \frac{1}{e_T t_T} \left[\frac{(z - b_z)^2}{(w - b_w)^2} (x + b_x) \right. \\
& + \frac{(x - b_x)^2 (z - b_z)^2}{(w - b_w)^4} (w + b_w) \\
& \left. + (v + b_v) + \frac{(x - b_x)^2}{(w - b_w)^2} (z + b_z) \right]^{1/2} \quad (C.6)
\end{aligned}$$

Case 3: ($t = s$, $t_T = s_T$, $v = 0$, $b_v = 0$)

$$\tilde{c}_3 = \frac{(x - b_x)(z - b_z)}{e_T t_T (w - b_w)}, \quad (C.7)$$

and

$$[\text{Var}(\tilde{c}_3)]^{1/2} \approx \tilde{c}_3 \left[\frac{(x + b_x)^2}{(x - b_x)^2} + \frac{(w + b_w)^2}{(w - b_w)^2} + \frac{(z + b_z)^2}{(z - b_z)^2} \right]^{1/2}. \quad (C.8)$$

We note that

- (1) The efficiency e_s does not enter into these approximate counting error formulas.
- (2) The formulas when $t = s = t_T = s_T$ can be obtained using Eqs. (C.5) and (C.6).
- (3) The formulas when $(t = s, t_T \neq s_T)$ or $(t \neq s, t_T = s_T)$ can be obtained by substitution of the approximate counting times into Eqs. (C.2) and (C.4).

3. Approximate 1 σ Counting Errors for Computing L_C to Make Approximate Tests and for Computing L_D .

We wish to determine whether the estimated dpm of ^{239}Pu (excluding any contamination of ^{239}Pu added with the tracer) is sufficiently large to indicate something above background is in the sample and to compute an approximate "detection limit" L_D using Eq. (12) or (13). To do this we need an estimate of the variance of this estimated disintegration rate when only background is present. This variance can be approximated using the "propagation

of error" approach used above. Turning to Eq. (C.3) we recall that the variances in that approximate expression can be replaced by expected values since we are dealing with Poisson random variables. Actually we need concern ourselves only with $E(x)$ since this is the only expected value in Eq. (C.3) containing the "true" net ^{239}Pu count rate (exclusive of ^{239}Pu contamination). Setting this "true" rate to zero will then yield an approximate estimate of $\text{Var}(\hat{C})$ for use in testing $H_0: C = 0$ or for computing L_D .

Let

C_x = "true" net count rate of ^{239}Pu including any ^{239}Pu contamination added with the ^{242}Pu tracer

B_x = "true" background count rate for ^{239}Pu

C_{x1} = "true" net count rate of ^{239}Pu that does not include ^{239}Pu contamination

C_v = "true" net count rate of ^{239}Pu added with the ^{242}Pu tracer

C_z = "true" net count rate of ^{242}Pu tracer added to the sample

C_w = "true" net count rate of ^{242}Pu tracer recovered in the sample.

Then

$$C_{x1} = C_x - \frac{C_v}{C_z} C_w \quad (C.9)$$

Now, since

$$E(x) = (C_x + B_x)t$$

we have

$$E(x) = (C_{x1} + \frac{C_v}{C_z} C_w + B_x)t \quad (C.10)$$

Setting $C_{x1} = 0$ we find

$$E(x) = \left(\frac{C_v}{C_z} C_w + B_x \right) t$$

which is estimated by

$$\left[\frac{\left(\frac{v}{t_T} - \frac{b_v}{s_T} \right)}{\left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)} \left(\frac{w}{t} - \frac{b_w}{s} \right) + \frac{b_x}{s} \right] t \quad (C.11)$$

Substituting this expression for $\text{Var}(x)$ in Eq. (C.3) gives the following approximate equations for the standard deviations of $\tilde{c}_1$, $\tilde{c}_2$ and $\tilde{c}_3$ for

Case 1, 2 and 3, respectively, when $C_{x1} = 0$:

Case 1: (most general case)

$$\sigma_0 = [\text{Var}(\tilde{c}_1)]^{1/2} \approx \frac{1}{e_T} \left\{ \frac{\left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)^2}{\left(\frac{w}{t} - \frac{b_w}{s} \right)^2} \left[\frac{\left(\frac{v}{t_T} - \frac{b_v}{s_T} \right)}{t \left(\frac{z}{t_T} - \frac{b_z}{s_T} \right)} \left(\frac{w}{t} - \frac{b_w}{s} \right) + \frac{b_x}{s} \left(\frac{1}{t} + \frac{1}{s} \right) \right] + T_2 + T_3 + T_4 \right\}^{1/2} \quad (C.12)$$

where T_2 , T_3 and T_4 are the 2nd, 3rd, and 4th terms, respectively, under the square root in Eq. (C.4).

Case 2: ($t = s$, $t_T = s_T$)

$$[\text{Var}(\tilde{c}_2)]^{1/2} \approx \frac{1}{e_T t_T} \left[\frac{(z - b_z)}{(w - b_w)} \frac{(v - b_v)}{t} + 2b_x \frac{(z - b_z)^2}{(w - b_w)^2} + T_2 + T_3 + T_4 \right]^{1/2}, \quad (\text{C.13})$$

where T_2 , T_3 and T_4 are the 2nd, 3rd and 4th terms, respectively, under the square root in Eq. (C.6).

Case 3: ($t = s$, $t_T = s_T$, $v = 0$, $b_v = 0$)

$$[\text{Var}(\tilde{c}_3)]^{1/2} \approx \tilde{c}_3 \left[\frac{2b_x}{(x - b_x)^2} + \frac{(w + b_w)}{(w - b_w)^2} + \frac{(z + b_z)}{(z - b_z)^2} \right]^{1/2}. \quad (\text{C.14})$$

Multiplying Eqs. (C.12), (C.13) or (C.14) (whichever is appropriate) by k_α will yield an approximate value of L_C which may be compared with $\tilde{c}_1$, $\tilde{c}_2$ or $\tilde{c}_3$ for an approximate test of $H_0: C_{x1} = 0$. (see Section (6) of the text for further discussion).

APPENDIX D.

TABLES OF DECISION CRITERIA

APPENDIX D:

TABLE D.1.	EXACT SIGNIFICANCE LEVELS FOR DECISION RULES D_2 AND D_3
TABLE D.2.	SIGNIFICANCE CRITICAL POINTS X AND RANDOMIZATION PROBABILITIES P FOR DECISION RULE D_e
TABLE D.3.	CONFIDENCE INTERVAL END POINTS L AND U ON C.
FIGURES D.1 - D.4.	POWER OF DECISION RULE D_3

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TABLE D.1

EXACT SIGNIFICANCE LEVELS FOR DECISION RULES D_2 AND D_3 Stated Level of Significance: 1% ($k_\alpha = 2.326$)Expected Total Background Contribution: (tB)

$\rho = t/s$		<u>1</u>	<u>2</u>	<u>3</u>	<u>5</u>	<u>10</u>	<u>15</u>	<u>20</u>	<u>30</u>	<u>40</u>	<u>50</u>
0	D_2	0.018	0.017	0.012	0.014	0.014	0.019	0.013	0.015	0.014	0.012
	D_3	0.018	0.017	0.012	0.014	0.014	0.019	0.013	0.015	0.014	0.012
1/10	D_2	0.042	0.029	0.025	0.022	0.018	0.017	0.016	0.014	0.014	0.013
	D_3	0.024	0.022	0.020	0.017	0.015	0.014	0.014	0.013	0.013	0.012
1/5	D_2	0.041	0.038	0.031	0.023	0.020	0.018	0.017	0.016	0.015	0.014
	D_3	0.019	0.017	0.018	0.017	0.014	0.013	0.013	0.012	0.012	0.012
1/3	D_2	0.094	0.051	0.036	0.029	0.021	0.020	0.018	0.017	0.016	0.015
	D_3	0.021	0.019	0.016	0.014	0.012	0.013	0.012	0.012	0.012	0.012
1/2	D_2	0.113	0.072	0.052	0.037	0.026	0.022	0.020	0.018	0.017	0.016
	D_3	0.012	0.013	0.013	0.013	0.012	0.012	0.011	0.011	0.011	0.011
1	D_2	0.234	0.133	0.083	0.058	0.036	0.031	0.027	0.022	0.020	0.019
	D_3	0.000	0.003	0.006	0.009	0.009	0.010	0.010	0.010	0.010	0.010
2	D_2	0.383	0.319	0.216	0.109	0.058	0.046	0.039	0.030	0.027	0.025
	D_3	0.000	0.000	0.000	0.001	0.005	0.006	0.006	0.007	0.008	0.008

TABLE D.1 (Continued)

EXACT SIGNIFICANCE LEVELS FOR DECISION RULES D_2 AND D_3 Stated Level of Significance: 5% ($k_\alpha = 1.645$)

Expected Total Background Contribution: (tB)

$\rho = t/s$		1	2	3	5	10	15	20	30	40	50
0	D_2	0.080	0.053	0.084	0.068	0.049	0.053	0.052	0.046	0.053	0.056
	D_3	0.080	0.053	0.084	0.068	0.049	0.053	0.052	0.046	0.053	0.056
1/10	D_2	0.086	0.076	0.071	0.066	0.061	0.060	0.059	0.057	0.056	0.055
	D_3	0.067	0.064	0.063	0.060	0.058	0.056	0.055	0.054	0.054	0.054
1/5	D_2	0.107	0.086	0.077	0.069	0.065	0.065	0.060	0.058	0.057	0.057
	D_3	0.069	0.062	0.059	0.057	0.054	0.055	0.055	0.054	0.053	0.053
1/3	D_2	0.111	0.095	0.084	0.078	0.069	0.064	0.062	0.060	0.059	0.058
	D_3	0.094	0.059	0.056	0.057	0.055	0.054	0.054	0.053	0.052	0.052
1/2	D_2	0.166	0.122	0.107	0.083	0.071	0.069	0.066	0.063	0.061	0.060
	D_3	0.063	0.070	0.062	0.053	0.052	0.053	0.052	0.053	0.052	0.052
1	D_2	0.240	0.160	0.122	0.098	0.087	0.079	0.073	0.070	0.068	0.065
	D_3	0.030	0.050	0.050	0.052	0.049	0.050	0.050	0.050	0.051	0.050
2	D_2	0.383	0.320	0.224	0.139	0.116	0.095	0.090	0.081	0.076	0.072
	D_3	0.000	0.006	0.019	0.038	0.040	0.042	0.046	0.047	0.046	0.047

TABLE D.1 (Continued)

EXACT SIGNIFICANCE LEVELS FOR DECISION RULES D_2 AND D_3 Stated Level of Significance: 10% ($k_\alpha = 1.282$)

Expected Total Background Contribution: (tB)

$\rho = t/s$		1	2	3	5	10	15	20	30	40	50
0	D_2	0.080	0.143	0.084	0.133	0.083	0.125	0.112	0.089	0.092	0.092
	D_3	0.080	0.143	0.084	0.133	0.083	0.125	0.112	0.089	0.092	0.092
1/10	D_2	0.117	0.120	0.113	0.111	0.108	0.107	0.106	0.105	0.105	0.104
	D_3	0.114	0.113	0.113	0.107	0.106	0.105	0.103	0.103	0.102	0.102
1/5	D_2	0.129	0.127	0.128	0.118	0.116	0.109	0.109	0.107	0.107	0.106
	D_3	0.116	0.112	0.106	0.106	0.104	0.104	0.103	0.102	0.102	0.102
1/3	D_2	0.154	0.127	0.118	0.124	0.115	0.115	0.111	0.110	0.108	0.108
	D_3	0.112	0.115	0.115	0.105	0.104	0.101	0.103	0.102	0.101	0.101
1/2	D_2	0.183	0.155	0.146	0.134	0.121	0.117	0.114	0.111	0.111	0.110
	D_3	0.116	0.102	0.108	0.108	0.104	0.101	0.102	0.101	0.101	0.101
1	D_2	0.263	0.220	0.185	0.148	0.133	0.125	0.123	0.119	0.116	0.114
	D_3	0.104	0.124	0.115	0.101	0.101	0.101	0.101	0.101	0.100	0.099
2	D_2	0.384	0.324	0.241	0.179	0.155	0.141	0.140	0.134	0.127	0.123
	D_3	0.012	0.053	0.083	0.091	0.093	0.097	0.097	0.097	0.098	0.099

TABLE D.2

1% LEVEL OF SIGNIFICANCE CRITICAL POINTS $X_{.01}$
AND RANDOMIZATION PROBABILITIES $P_{.01}$ FOR DECISION RULE D_e

ρ = Sample - Background Counting Time Ratio (t/s)

Total Count $x + y$	10		5		3		2		1		1/2		1/3		1/5		1/10	
	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P
1	1	.011	1	.012	1	.013	1	.015	1	.020	1	.030	1	.040	1	.060	1	.110
2	2	.012	2	.014	2	.017	2	.022	2	.040	2	.090	2	.160	2	.359	1	.010
3	3	.013	3	.017	3	.023	3	.033	3	.080	3	.270	3	.640	2	.077	2	.410
4	4	.014	4	.020	4	.031	4	.050	4	.160	4	.810	3	.130	3	.597	2	.175
5	5	.016	5	.024	5	.042	5	.075	5	.320	4	.143	4	.616	3	.207	2	.055
6	6	.017	6	.029	6	.056	6	.113	6	.640	5	.524	4	.162	3	.024	3	.807
7	7	.019	7	.035	7	.074	7	.170	6	.040	5	.081	5	.750	4	.511	3	.450
8	8	.021	8	.042	8	.099	8	.256	7	.194	6	.434	5	.250	4	.206	3	.247
9	9	.023	9	.051	9	.133	9	.384	8	.457	6	.050	5	.000	4	.026	3	.114
10	10	.025	10	.061	10	.177	10	.576	9	.924	7	.405	6	.400	5	.580	3	.018
11	11	.028	11	.074	11	.236	11	.864	9	.154	7	.039	6	.091	5	.270	4	.709
12	12	.031	12	.089	12	.315	11	.049	10	.423	8	.412	7	.629	5	.072	4	.453
13	13	.034	13	.106	13	.420	12	.145	11	.870	8	.045	7	.233	6	.740	4	.276
14	14	.037	14	.128	14	.561	13	.274	11	.158	9	.445	8	.962	6	.393	4	.145
15	15	.041	15	.154	15	.748	14	.450	12	.454	9	.067	8	.443	6	.163	4	.044
16	16	.045	16	.184	16	.997	15	.696	13	.925	10	.500	8	.128	7	.991	5	.839
17	17	.050	17	.221	16	.058	15	.010	13	.200	10	.103	9	.743	7	.582	5	.581
18	18	.055	18	.266	17	.128	16	.124	14	.533	11	.578	9	.328	7	.304	5	.389
19	19	.061	19	.319	18	.215	17	.272	14	.017	11	.154	9	.055	7	.104	5	.241
20	20	.067	20	.383	19	.323	18	.468	15	.276	12	.678	10	.610	8	.850	5	.124
21	21	.074	21	.460	20	.457	19	.731	16	.659	12	.221	10	.251	8	.508	5	.028
22	22	.081	22	.552	21	.628	19	.026	16	.087	13	.802	10	.001	8	.260	6	.811
23	23	.089	23	.662	22	.844	20	.164	17	.389	13	.305	11	.524	8	.073	6	.586
24	24	.098	24	.795	22	.031	21	.341	18	.834	14	.952	11	.198	9	.790	6	.408
25	25	.108	25	.954	23	.118	22	.570	18	.187	14	.408	12	.900	9	.479	6	.265
26	26	.119	25	.027	24	.222	23	.872	19	.543	14	.051	12	.466	9	.245	6	.147
27	27	.131	26	.069	25	.349	23	.091	19	.025	15	.531	12	.160	9	.062	6	.048
28	28	.144	27	.115	26	.503	24	.260	20	.322	15	.137	13	.834	10	.777	7	.883
29	29	.158	28	.168	27	.694	25	.474	21	.742	16	.676	13	.429	10	.480	7	.661
30	30	.174	29	.229	28	.930	26	.750	21	.145	16	.239	13	.135	10	.251	7	.481

TABLE D.2 (Continued)

1% LEVEL OF SIGNIFICANCE CRITICAL POINTS $X_{.01}$
AND RANDOMIZATION PROBABILITIES $P_{.01}$ FOR DECISION RULE D_e

ρ = Sample - Background Counting Time Ratio (t/s)

Total Count $x + y$	10		5		3		2		1		$1/2$		$1/3$		$1/5$		$1/10$	
	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P
31	31	.192	30	.298	28	.070	26	.040	22	.496	17	.846	14	.793	10	.068	7	.332
32	32	.211	31	.377	29	.178	27	.207	23	.995	17	.360	14	.407	11	.797	7	.207
33	33	.232	32	.470	30	.307	28	.415	23	.300	17	.019	14	.121	11	.504	7	.101
34	34	.255	33	.576	31	.461	29	.678	24	.717	18	.502	15	.772	11	.274	7	.008
35	35	.281	34	.701	32	.648	29	.006	24	.137	18	.123	15	.398	11	.088	8	.793
36	36	.309	35	.845	33	.875	30	.174	25	.495	19	.667	15	.115	12	.844	8	.601
37	37	.340	35	.004	33	.054	31	.381	26	.994	19	.243	16	.766	12	.549	8	.440
38	38	.374	36	.057	34	.170	32	.639	26	.312	20	.858	16	.398	12	.314	8	.303
39	39	.411	37	.116	35	.306	33	.965	27	.739	20	.383	16	.117	12	.122	8	.185
40	40	.452	38	.182	36	.467	33	.157	27	.155	20	.037	17	.774	13	.914	8	.083
41	41	.498	39	.257	37	.660	34	.366	28	.529	21	.545	17	.409	13	.612	9	.981
42	42	.548	40	.341	38	.891	35	.624	28	.019	21	.159	17	.126	13	.369	9	.770
43	43	.602	41	.437	38	.065	36	.947	29	.352	22	.732	18	.792	13	.169	9	.590
44	44	.663	42	.546	39	.191	36	.152	30	.798	22	.299	18	.427	13	.002	9	.437
45	45	.729	43	.671	40	.338	37	.366	30	.199	23	.948	18	.142	14	.693	9	.304
46	46	.802	44	.814	41	.510	38	.627	31	.594	23	.461	19	.821	14	.439	9	.189
47	47	.882	45	.978	42	.714	39	.951	31	.064	23	.098	19	.454	14	.230	9	.087
48	48	.971	45	.054	43	.957	39	.160	32	.419	24	.647	19	.164	14	.054	10	.992
49	48	.013	46	.122	43	.101	40	.379	33	.891	24	.240	20	.860	15	.793	10	.789
50	49	.035	47	.198	44	.239	41	.647	33	.266	25	.860	20	.488	15	.526	10	.614
51	50	.057	48	.282	45	.399	42	.976	34	.688	25	.402	20	.193	15	.304	10	.462
52	51	.081	49	.377	46	.586	42	.178	34	.130	25	.054	21	.908	15	.117	10	.330
53	52	.106	50	.483	47	.806	43	.406	35	.511	26	.589	21	.529	16	.911	10	.213
54	53	.133	51	.603	47	.029	44	.681	35	.008	26	.198	21	.227	16	.628	10	.110
55	54	.162	52	.739	48	.161	44	.008	36	.356	27	.802	22	.964	16	.393	10	.017
56	55	.193	53	.893	49	.313	45	.207	37	.810	27	.364	22	.578	16	.194	11	.841
57	56	.226	53	.025	50	.489	46	.443	37	.217	27	.024	22	.268	16	.023	11	.665
58	57	.262	54	.097	51	.694	47	.729	38	.629	28	.552	22	.015	17	.748	11	.512
59	58	.300	55	.176	52	.934	47	.038	38	.092	28	.172	23	.634	17	.497	11	.377
60	59	.341	56	.264	52	.098	48	.245	39	.470	29	.767	23	.316	17	.284	11	.257

TABLE D.2 (Continued)

5% LEVEL OF SIGNIFICANCE CRITICAL POINTS $X_{.05}$
AND RANDOMIZATION PROBABILITIES $P_{.05}$ FOR DECISION RULE D_e

ρ = Sample - Background Counting Time Ratio (t/s)

Total Count $x + y$	10		5		3		2		1		1/2		1/3		1/5		1/10	
	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P
1	1	.055	1	.060	1	.066	1	.075	1	.100	1	.150	1	.200	1	.300	1	.550
2	2	.060	2	.072	2	.088	2	.112	2	.200	2	.450	2	.800	1	.079	1	.252
3	3	.066	3	.086	3	.118	3	.168	3	.400	2	.058	2	.244	2	.653	1	.118
4	4	.073	4	.103	4	.158	4	.253	4	.800	3	.381	3	.983	2	.291	1	.022
5	5	.080	5	.124	5	.210	5	.379	4	.120	3	.028	3	.391	2	.090	2	.700
6	6	.088	6	.149	6	.280	6	.569	5	.366	4	.390	3	.094	3	.770	2	.446
7	7	.097	7	.179	7	.374	7	.854	6	.771	4	.036	4	.643	3	.414	2	.279
8	8	.107	8	.214	8	.499	7	.070	6	.135	5	.444	4	.262	3	.185	2	.155
9	9	.117	9	.257	9	.665	8	.204	7	.433	5	.073	4	.009	3	.015	2	.055
10	10	.129	10	.309	10	.887	9	.376	8	.893	6	.533	5	.518	4	.636	3	.882
11	11	.142	11	.371	10	.050	10	.604	8	.214	6	.136	5	.195	4	.358	3	.633
12	12	.156	12	.445	11	.144	11	.914	9	.571	7	.655	6	.890	4	.153	3	.447
13	13	.172	13	.534	12	.254	11	.114	9	.044	7	.222	6	.459	5	.969	3	.301
14	14	.189	14	.641	13	.387	12	.289	10	.348	8	.810	6	.159	5	.619	3	.182
15	15	.208	15	.770	14	.548	13	.510	11	.778	8	.333	7	.831	5	.362	3	.080
16	16	.229	16	.924	15	.747	14	.794	11	.173	8	.000	7	.436	5	.161	4	.971
17	17	.252	16	.032	16	.997	14	.067	12	.539	9	.471	7	.146	6	.989	4	.740
18	18	.278	17	.091	16	.109	15	.251	12	.026	9	.103	8	.815	6	.657	4	.555
19	19	.305	18	.157	17	.236	16	.475	13	.352	10	.637	8	.436	6	.403	4	.403
20	20	.336	19	.229	18	.383	17	.756	14	.792	10	.227	8	.149	6	.198	4	.275
21	21	.370	20	.309	19	.558	17	.051	14	.195	11	.834	9	.826	6	.027	4	.164
22	22	.407	21	.400	20	.767	18	.244	15	.584	11	.375	9	.454	7	.734	4	.065
23	23	.447	22	.502	20	.009	19	.477	15	.058	11	.033	9	.166	7	.473	5	.940
24	24	.492	23	.619	21	.135	20	.762	16	.411	12	.549	10	.860	7	.261	5	.739
25	25	.541	24	.753	22	.279	20	.057	17	.879	12	.168	10	.488	7	.081	5	.570
26	26	.595	25	.908	23	.444	21	.262	17	.262	13	.753	10	.195	8	.843	5	.426
27	27	.655	25	.033	24	.638	22	.505	18	.683	13	.326	11	.910	8	.571	5	.300
28	28	.721	26	.108	25	.866	23	.801	18	.131	14	.989	11	.534	8	.346	5	.189
29	29	.793	27	.190	25	.064	23	.083	19	.516	14	.509	11	.235	8	.156	5	.089
30	30	.872	28	.279	26	.207	24	.300	19	.012	14	.140	12	.977	9	.981	6	.997

TABLE D.2 (Continued)

5% LEVEL OF SIGNIFICANCE CRITICAL POINTS $X_{.05}$
AND RANDOMIZATION PROBABILITIES $P_{.05}$ FOR DECISION RULE D_e

ρ = Sample - Background Counting Time Ratio (t/s)

Total Count $x + y$	10		5		3		2		1		1/2		1/3		1/5		1/10	
	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P
31	31	.959	29	.378	27	.370	25	.557	20	.370	15	.720	12	.594	9	.694	6	.804
32	31	.017	30	.488	28	.556	26	.866	21	.830	15	.308	12	.286	9	.454	6	.637
33	32	.048	31	.611	29	.773	26	.127	21	.240	16	.963	12	.031	9	.251	6	.492
34	33	.081	32	.749	29	.014	27	.357	22	.664	16	.500	13	.665	9	.074	6	.363
35	34	.115	33	.904	30	.158	28	.628	22	.121	16	.136	13	.348	10	.842	6	.248
36	35	.151	33	.035	31	.321	29	.954	23	.516	17	.720	13	.083	10	.585	6	.144
37	36	.189	34	.120	32	.505	29	.186	23	.012	17	.314	14	.748	10	.367	6	.049
38	37	.229	35	.212	33	.717	30	.431	24	.384	18	.972	14	.419	10	.177	7	.915
39	38	.271	36	.311	34	.961	31	.719	25	.851	18	.516	14	.145	10	.009	7	.744
40	39	.315	37	.421	34	.127	31	.035	25	.263	18	.151	15	.842	11	.740	7	.592
41	40	.363	38	.541	35	.290	32	.261	26	.700	19	.746	15	.501	11	.503	7	.457
42	41	.413	39	.674	36	.475	33	.522	26	.152	19	.339	15	.215	11	.298	7	.336
43	42	.468	40	.823	37	.684	34	.828	27	.564	19	.005	16	.948	11	.117	7	.226
44	43	.525	41	.988	38	.925	34	.110	27	.049	20	.552	16	.592	12	.920	7	.125
45	44	.587	41	.082	38	.108	35	.351	28	.439	20	.182	16	.294	12	.663	7	.031
46	45	.654	42	.177	39	.275	36	.629	29	.923	21	.793	16	.038	12	.440	8	.886
47	46	.725	43	.280	40	.461	37	.956	29	.324	21	.382	17	.693	12	.244	8	.726
48	47	.802	44	.391	41	.671	37	.199	30	.780	21	.041	17	.382	12	.070	8	.582
49	48	.885	45	.513	42	.910	38	.456	30	.217	22	.606	17	.115	13	.846	8	.452
50	49	.974	46	.646	42	.102	39	.753	31	.649	22	.229	18	.805	13	.603	8	.334
51	49	.028	47	.793	43	.272	39	.061	31	.117	23	.859	18	.479	13	.390	8	.225
52	50	.068	48	.955	44	.460	40	.301	32	.529	23	.440	18	.200	13	.201	8	.125
53	51	.109	48	.069	45	.672	41	.576	32	.023	23	.090	19	.928	13	.031	8	.032
54	52	.153	49	.168	46	.912	42	.894	33	.417	24	.676	19	.586	14	.789	9	.892
55	53	.198	50	.274	46	.106	42	.161	34	.897	24	.289	19	.293	14	.557	9	.737
56	54	.246	51	.389	47	.279	43	.418	34	.312	25	.941	19	.038	14	.351	9	.596
57	55	.297	52	.513	48	.471	44	.712	35	.769	25	.512	20	.702	14	.167	9	.468
58	56	.350	53	.649	49	.686	44	.033	35	.213	25	.152	20	.395	14	.000	9	.350
59	57	.405	54	.798	50	.928	45	.275	36	.650	26	.761	20	.128	15	.746	9	.242
60	58	.464	55	.961	50	.118	46	.550	36	.120	26	.363	21	.829	15	.522	9	.141

TABLE D.2 (Continued)

10% LEVEL OF SIGNIFICANCE CRITICAL POINTS $X_{.10}$
AND RANDOMIZATION PROBABILITIES $P_{.10}$ FOR DECISION RULE D_e

ρ = Sample - Background Counting Time Ratio (t/s)

Total Count $x + y$	10		5		3		2		1		1/2		1/3		1/5		1/10	
	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P
1	1	.110	1	.120	1	.133	1	.150	1	.200	1	.300	1	.400	1	.599	0	.010
2	2	.121	2	.144	2	.177	2	.225	2	.400	2	.900	1	.100	1	.259	1	.554
3	3	.133	3	.172	3	.237	3	.337	3	.800	2	.283	2	.600	1	.074	1	.340
4	4	.146	4	.207	4	.316	4	.506	3	.150	3	.887	2	.233	2	.723	1	.205
5	5	.161	5	.248	5	.421	5	.759	4	.440	3	.332	3	.960	2	.401	1	.101
6	6	.177	6	.298	6	.561	5	.046	5	.900	4	.998	3	.473	2	.187	1	.009
7	7	.194	7	.358	7	.749	6	.202	5	.228	4	.427	3	.170	2	.018	2	.743
8	8	.214	8	.429	8	.998	7	.390	6	.592	4	.070	4	.840	3	.665	2	.538
9	9	.235	9	.515	8	.110	8	.632	6	.061	5	.562	4	.437	3	.399	2	.382
10	10	.259	10	.619	9	.232	9	.953	7	.386	5	.171	4	.149	3	.195	2	.256
11	11	.285	11	.743	10	.373	9	.156	8	.835	6	.735	5	.817	3	.024	2	.149
12	12	.313	12	.891	11	.539	10	.362	8	.223	6	.301	5	.441	4	.716	2	.053
13	13	.345	12	.026	12	.740	11	.613	9	.617	7	.948	5	.157	4	.456	3	.905
14	14	.379	13	.101	13	.988	12	.931	9	.083	7	.461	6	.841	4	.248	3	.703
15	15	.417	14	.180	13	.127	12	.158	10	.444	7	.102	6	.472	4	.072	3	.539
16	16	.459	15	.265	14	.273	13	.381	11	.924	8	.653	6	.185	5	.822	3	.400
17	17	.505	16	.358	15	.439	14	.647	11	.299	8	.254	7	.894	5	.555	3	.280
18	18	.555	17	.461	16	.631	15	.975	12	.732	9	.881	7	.524	5	.337	3	.174
19	19	.611	18	.577	17	.858	15	.195	12	.171	9	.432	7	.231	5	.150	3	.077
20	20	.672	19	.708	17	.065	16	.434	13	.572	9	.082	8	.970	6	.971	4	.969
21	21	.740	20	.857	18	.217	17	.717	13	.055	10	.641	8	.594	6	.689	4	.782
22	22	.814	20	.013	19	.386	17	.034	14	.434	10	.251	8	.292	6	.456	4	.622
23	23	.895	21	.101	20	.578	18	.258	15	.914	11	.883	8	.036	6	.257	4	.482
24	24	.984	22	.194	21	.800	19	.515	15	.310	11	.466	9	.680	6	.081	4	.359
25	24	.033	23	.294	21	.032	20	.818	16	.757	11	.095	9	.367	7	.856	4	.248
26	25	.073	24	.403	22	.190	20	.104	16	.198	12	.670	9	.102	7	.605	4	.146
27	26	.115	25	.522	23	.364	21	.344	17	.619	12	.280	10	.780	7	.389	4	.051
28	27	.157	26	.653	24	.559	22	.620	17	.094	13	.926	10	.454	7	.200	5	.923
29	28	.202	27	.799	25	.780	23	.945	18	.496	13	.491	10	.179	7	.031	5	.757
30	29	.248	28	.961	25	.020	23	.194	19	.995	13	.134	11	.895	8	.783	5	.611

TABLE D.2 (Continued)

10% LEVEL OF SIGNIFICANCE CRITICAL POINTS $X_{.10}$
AND RANDOMIZATION PROBABILITIES $P_{.10}$ FOR DECISION RULE D_e

ρ = Sample - Background Counting Time Ratio (t/s)

Total Count $x + y$	10		5		3		2		1		1/2		1/3		1/5		1/10	
	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P	X	P
31	30	.296	28	.074	26	.184	24	.452	19	.383	14	.730	11	.555	8	.548	5	.480
32	31	.347	29	.174	27	.363	25	.748	20	.854	14	.334	11	.267	8	.343	5	.361
33	32	.400	30	.281	28	.562	25	.059	20	.279	14	.001	11	.015	8	.161	5	.252
34	33	.457	31	.396	29	.786	26	.303	21	.727	15	.560	12	.668	9	.991	5	.151
35	34	.517	32	.520	29	.025	27	.579	21	.181	15	.194	12	.367	9	.735	5	.057
36	35	.580	33	.656	30	.194	28	.898	22	.610	16	.815	12	.104	9	.511	6	.937
37	36	.648	34	.804	31	.379	28	.169	22	.089	16	.410	13	.794	9	.313	6	.780
38	37	.721	35	.968	32	.584	29	.430	23	.503	16	.066	13	.478	9	.134	6	.638
39	38	.798	35	.083	33	.812	30	.727	23	.002	17	.651	13	.202	10	.950	6	.510
40	39	.881	36	.189	33	.044	30	.045	24	.403	17	.274	14	.933	10	.705	6	.391
41	40	.970	37	.303	34	.219	31	.295	25	.882	18	.922	14	.600	10	.488	6	.282
42	40	.032	38	.424	35	.410	32	.576	25	.309	18	.504	14	.311	10	.294	6	.180
43	41	.080	39	.554	36	.621	33	.895	26	.769	18	.149	14	.055	10	.117	6	.085
44	42	.129	40	.696	37	.855	33	.171	26	.220	19	.761	15	.734	11	.926	7	.989
45	43	.180	41	.849	37	.074	34	.439	27	.664	19	.371	15	.430	11	.688	7	.834
46	44	.233	41	.010	38	.256	35	.740	27	.136	19	.033	15	.162	11	.476	7	.693
47	45	.288	42	.118	39	.453	35	.056	28	.566	20	.617	16	.880	11	.284	7	.564
48	46	.345	43	.231	40	.671	35	.313	28	.055	20	.248	16	.560	11	.109	7	.444
49	47	.405	44	.350	41	.911	37	.600	29	.473	21	.890	16	.278	12	.915	7	.333
50	48	.468	45	.479	41	.116	33	.924	30	.968	21	.485	16	.026	12	.683	7	.229
51	49	.534	46	.616	42	.304	33	.196	30	.385	21	.133	17	.701	12	.474	7	.131
52	50	.603	47	.764	43	.508	39	.471	31	.863	22	.746	17	.405	12	.283	7	.038
53	51	.676	48	.925	44	.733	40	.779	31	.302	22	.362	17	.140	12	.108	8	.913
54	52	.753	48	.061	45	.981	40	.087	32	.765	22	.026	18	.854	13	.916	8	.770
55	53	.835	49	.174	45	.168	41	.352	32	.221	23	.614	18	.541	13	.686	8	.639
56	54	.922	50	.294	46	.362	42	.646	33	.672	23	.248	18	.264	13	.479	8	.516
57	54	.007	51	.421	47	.574	43	.976	33	.145	24	.893	18	.012	13	.289	8	.402
58	55	.059	52	.556	48	.805	43	.240	34	.583	24	.492	19	.689	13	.114	8	.295
59	56	.113	53	.701	48	.041	44	.523	34	.071	24	.141	19	.397	14	.925	8	.194
60	57	.168	54	.858	49	.228	45	.839	35	.499	25	.760	19	.133	14	.698	8	.099

TABLE D.3

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE COEFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1

TOTAL SAMPLE COUNT	TOTAL BACKGROUND COUNT -- Y									
	0		1		2		3		4	
X	L	U	L	U	L	U	L	U	L	U
0	0	0	0	3.01	0	2.18	0	1.36	0	0.53
1	0	5.67	0	4.83	0	4.00	0	3.17	0	2.34
2	0	7.29	0	6.46	0	5.62	0	4.78	0	3.94
3	0	8.82	0	7.98	0	7.14	0	6.29	0	5.45
4	0	10.29	0	9.44	0	8.59	0	7.74	0	6.90
5	0.30	11.71	0	10.86	0	10.00	0	9.15	0	8.30
6	1.13	13.09	0	12.24	0	11.38	0	10.52	0	9.67
7	1.96	14.45	0.16	13.59	0	12.74	0	11.87	0	11.01
8	2.79	15.79	1.00	14.93	0	14.07	0	13.20	0	12.34
9	3.61	17.11	1.83	16.24	0.26	15.38	0	14.51	0	13.65
10	4.44	18.41	2.67	17.54	1.10	16.68	0	15.81	0	14.94
12	6.10	20.98	4.34	20.11	2.78	19.23	1.32	18.36	0	17.49
14	7.76	23.50	6.01	22.63	4.47	21.75	3.02	20.87	1.63	20.00
16	9.41	25.99	7.68	25.12	6.16	24.23	4.72	23.35	3.34	22.47
18	11.07	28.46	9.35	27.57	7.85	26.69	6.42	25.81	5.05	24.92
20	12.73	30.89	11.01	30.01	9.55	29.12	8.13	28.24	6.77	27.35
22	14.38	33.31	12.60	32.43	11.24	31.54	9.84	30.65	8.49	29.76
24	16.04	35.71	14.20	34.82	12.94	33.93	11.55	33.04	10.21	32.15
26	17.70	38.10	15.81	37.21	14.62	36.31	13.27	35.42	11.93	34.52
28	19.35	40.47	17.44	39.57	16.26	38.68	14.99	37.78	13.66	36.88
30	21.01	42.83	19.08	41.93	17.91	41.03	16.71	40.13	15.39	39.24
34	24.32	47.51	22.40	46.61	21.24	45.71	20.07	44.81	18.86	43.91
38	27.64	52.15	25.76	51.25	24.60	50.35	23.45	49.44	22.29	48.54
42	30.95	56.77	29.14	55.86	27.99	54.95	26.85	54.05	25.70	53.14
46	34.27	61.35	32.56	60.44	31.42	59.54	30.28	58.63	29.14	57.71
50	37.58	65.91	36.00	65.00	34.86	64.09	33.73	63.18	32.59	62.27
54	40.89	70.45	39.46	69.54	38.33	68.62	37.20	67.71	36.07	66.80
58	44.21	74.97	42.94	74.06	41.81	73.14	40.69	72.23	39.56	71.31
62	47.52	79.47	46.44	78.56	45.32	77.64	44.19	76.72	43.07	75.80
66	50.83	83.96	49.95	83.04	48.83	82.12	47.72	81.20	46.60	80.28

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE CO-EFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1

TOTAL SAMPLE COUNT	TOTAL BACKGROUND COUNT -- Y									
	5		6		7		8		9	
X	L	U	L	U	L	U	L	U	L	U
0	0	0	0	0	0	0	0	0	0	0
1	0	1.50	0	0.67	0	0	0	0	0	0
2	0	3.10	0	2.26	0	1.42	0	0.58	0	0
3	0	4.60	0	3.76	0	2.91	0	2.07	0	1.22
4	0	6.04	0	5.19	0	4.34	0	3.49	0	2.64
5	0	7.44	0	6.59	0	5.73	0	4.88	0	4.02
6	0	8.81	0	7.95	0	7.09	0	6.24	0	5.38
7	0	10.15	0	9.29	0	8.43	0	7.57	0	6.70
8	0	11.47	0	10.61	0	9.74	0	8.88	0	8.01
9	0	12.78	0	11.91	0	11.04	0	10.18	0	9.30
10	0	14.07	0	13.20	0	12.33	0	11.46	0	10.59
12	0	16.61	0	15.74	0	14.86	0	13.99	0	13.11
14	0.27	19.12	0	18.24	0	17.36	0	16.48	0	15.60
16	1.99	21.59	0.66	20.71	0	19.83	0	18.94	0	18.06
18	3.71	24.04	2.39	23.15	1.09	22.26	0	21.38	0	20.49
20	5.43	26.46	4.12	25.57	2.83	24.68	1.55	23.80	0.29	22.91
22	7.16	28.87	5.86	27.98	4.57	27.09	3.30	26.19	2.04	25.30
24	8.89	31.25	7.59	30.36	6.31	29.47	5.05	28.58	3.79	27.68
26	10.62	33.63	9.33	32.73	8.06	31.84	6.80	30.94	5.55	30.04
28	12.36	35.99	11.07	35.09	9.81	34.19	8.55	33.30	7.30	32.40
30	14.10	38.34	12.82	37.44	11.55	36.54	10.30	35.64	9.06	34.74
34	17.58	43.00	16.31	42.10	15.06	41.20	13.82	40.30	12.59	39.39
38	21.07	47.63	19.82	46.73	18.58	45.82	17.35	44.91	16.13	44.01
42	24.56	52.23	23.34	51.32	22.11	50.42	20.89	49.51	19.67	48.60
46	28.00	56.81	26.86	55.89	25.64	54.98	24.43	54.07	23.22	53.16
50	31.46	61.35	30.33	60.44	29.19	59.53	27.98	58.61	26.78	57.70
54	34.94	65.88	33.81	64.97	32.69	64.05	31.54	63.14	30.35	62.22
58	38.44	70.39	37.32	69.47	36.19	68.56	35.07	67.64	33.93	66.72
62	41.95	74.88	40.83	73.97	39.72	73.05	38.60	72.13	37.48	71.21
66	45.48	79.36	44.36	78.44	43.25	77.52	42.14	76.60	41.02	75.68

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE CO-EFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1

TOTAL SAMPLE COUNT	10		12		14		16		18	
	L	U	L	U	L	U	L	U	L	U
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0.37	0	0	0	0	0	0	0	0
4	0	1.79	0	0.08	0	0	0	0	0	0
5	0	3.17	0	1.45	0	0	0	0	0	0
6	0	4.52	0	2.79	0	1.07	0	0	0	0
7	0	5.84	0	4.11	0	2.38	0	0.65	0	0
8	0	7.14	0	5.41	0	3.67	0	1.93	0	0.19
9	0	8.44	0	6.70	0	4.95	0	3.21	0	1.46
10	0	9.71	0	7.97	0	6.22	0	4.47	0	2.72
12	0	12.24	0	10.48	0	8.72	0	6.97	0	5.20
14	0	14.72	0	12.96	0	11.20	0	9.43	0	7.66
16	0	17.17	0	15.41	0	13.64	0	11.86	0	10.09
18	0	19.61	0	17.83	0	16.06	0	14.28	0	12.50
20	0	22.02	0	20.24	0	18.46	0	16.67	0	14.89
22	0.79	24.41	0	22.62	0	20.84	0	19.05	0	17.26
24	2.55	26.78	0.09	24.99	0	23.20	0	21.41	0	19.62
26	4.31	29.15	1.86	27.36	0	25.56	0	23.76	0	21.97
28	6.07	31.50	3.62	29.70	1.20	27.90	0	26.10	0	24.30
30	7.83	33.84	5.40	32.04	2.98	30.24	0.59	28.43	0	26.63
34	11.37	38.49	8.94	36.68	6.55	34.87	4.16	33.06	1.80	31.25
38	14.91	43.10	12.50	41.29	10.11	39.47	7.75	37.66	5.39	35.84
42	18.46	47.69	16.07	45.87	13.69	44.05	11.33	42.23	8.99	40.40
46	22.02	52.25	19.64	50.42	17.27	48.60	14.92	46.77	12.59	44.95
50	25.59	56.79	23.22	54.96	20.86	53.13	18.52	51.30	16.19	49.47
54	29.16	61.30	26.80	59.47	24.46	57.64	22.12	55.80	19.81	53.97
58	32.75	65.80	30.40	63.97	28.06	62.13	25.74	60.29	23.42	58.46
62	36.34	70.29	33.99	68.45	31.67	66.61	29.35	64.77	27.05	62.93
66	39.91	74.76	37.60	72.92	35.29	71.07	32.98	69.23	30.68	67.39

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE COEFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1

TOTAL SAMPLE COUNT X	20		22		24		26		28	
	L	U	L	U	L	U	L	U	L	U
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0.97	0	0	0	0	0	0	0	0
12	0	3.44	0	1.68	0	0	0	0	0	0
14	0	5.89	0	4.12	0	2.35	0	0.57	0	0
16	0	8.31	0	6.54	0	4.76	0	2.98	0	1.19
18	0	10.72	0	8.93	0	7.15	0	5.36	0	3.57
20	0	13.10	0	11.31	0	9.52	0	7.73	0	5.94
22	0	15.47	0	13.67	0	11.88	0	10.09	0	8.29
24	0	17.82	0	16.02	0	14.23	0	12.43	0	10.63
26	0	20.16	0	18.36	0	16.56	0	14.76	0	12.95
28	0	22.50	0	20.69	0	18.89	0	17.08	0	15.27
30	0	24.82	0	23.01	0	21.20	0	19.39	0	17.58
34	0	29.43	0	27.62	0	25.81	0	23.99	0	22.17
38	3.05	34.02	0.72	32.20	0	30.38	0	28.56	0	26.74
42	6.65	38.58	4.33	36.76	2.02	34.93	0	33.10	0	31.28
46	10.26	43.12	7.95	41.29	5.64	39.46	3.35	37.63	1.05	35.80
50	13.87	47.64	11.57	45.80	9.27	43.97	6.98	42.13	4.70	40.30
54	17.49	52.13	15.19	50.30	12.90	48.46	10.62	46.62	8.34	44.78
58	21.12	56.62	18.83	54.78	16.54	52.94	14.26	51.10	11.99	49.25
62	24.75	61.08	22.47	59.24	20.18	57.40	17.91	55.55	15.64	53.71
66	28.39	65.54	26.11	63.70	23.83	61.85	21.56	60.00	19.30	58.15

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE CO-EFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1/2

TOTAL SAMPLE COUNT X	0		1		2		3		4	
	L	U	L	U	L	U	L	U	L	U
0	0	0	0	3.39	0	2.94	0	2.49	0	2.04
1	0	5.67	0	5.21	0	4.76	0	4.32	0	3.87
2	0	7.29	0	6.84	0	6.39	0	5.94	0	5.48
3	0.28	8.82	0	8.37	0	7.91	0	7.46	0	7.01
4	1.01	10.29	0.12	9.83	0	9.38	0	8.92	0	8.46
5	1.74	11.71	0.86	11.25	0.09	10.79	0	10.34	0	9.88
6	2.47	13.09	1.60	12.63	0.84	12.18	0.12	11.72	0	11.26
7	3.20	14.45	2.22	13.99	1.59	13.53	0.88	13.07	0.20	12.61
8	3.94	15.79	2.87	15.33	2.28	14.87	1.65	14.41	0.97	13.95
9	4.67	17.11	3.53	16.65	2.95	16.19	2.38	15.73	1.74	15.26
10	5.40	18.41	4.22	17.95	3.64	17.49	3.07	17.02	2.50	16.56
12	6.86	20.98	5.64	20.52	5.07	20.05	4.50	19.59	3.93	19.12
14	8.33	23.50	7.10	23.04	6.53	22.57	5.97	22.11	5.41	21.64
16	9.79	25.99	8.60	25.53	8.03	25.06	7.47	24.60	6.91	24.13
18	11.26	28.46	10.12	27.99	9.57	27.52	9.01	27.05	8.45	26.59
20	12.72	30.89	11.68	30.43	11.12	29.96	10.57	29.49	10.01	29.02
22	14.18	33.31	13.25	32.85	12.70	32.38	12.15	31.91	11.60	31.44
24	15.65	35.71	14.84	35.25	14.29	34.78	13.75	34.31	13.20	33.84
26	17.11	38.10	16.45	37.63	15.91	37.16	15.36	36.69	14.81	36.22
28	18.58	40.47	18.08	40.00	17.53	39.53	16.99	39.06	16.44	38.59
30	20.04	42.83	19.72	42.36	19.17	41.89	18.63	41.41	18.08	40.94
34	22.97	47.51	23.03	47.04	22.48	46.57	21.94	46.09	21.40	45.62
38	25.90	52.15	26.37	51.68	25.83	51.21	25.30	50.74	24.76	50.26
42	28.83	56.77	29.76	56.29	29.22	55.82	28.68	55.35	28.14	54.87
46	31.75	61.35	33.17	60.88	32.63	60.40	32.09	59.93	31.56	59.45
50	34.68	65.91	36.60	65.44	36.07	64.96	35.53	64.49	35.00	64.01
54	37.61	70.45	40.06	69.98	39.53	69.50	38.99	69.02	38.46	68.55
58	40.54	74.97	43.54	74.49	43.00	74.02	42.47	73.54	41.94	73.06
62	43.47	79.47	47.03	79.00	46.50	78.52	45.97	78.04	45.44	77.56
66	46.39	83.96	50.54	83.48	50.01	83.01	49.48	82.53	48.95	82.05

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE COEFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1/2

TOTAL SAMPLE COUNT	TOTAL BACKGROUND COUNT -- Y									
	5		6		7		8		9	
X	L	U	L	U	L	U	L	U	L	U
0	0	1.59	0	1.14	0	0.70	0	0.25	0	0
1	0	3.41	0	2.96	0	2.51	0	2.06	0	1.61
2	0	5.03	0	4.58	0	4.13	0	3.67	0	3.22
3	0	6.55	0	6.10	0	5.64	0	5.19	0	4.73
4	0	8.01	0	7.55	0	7.09	0	6.64	0	6.18
5	0	9.42	0	8.96	0	8.51	0	8.05	0	7.59
6	0	10.80	0	10.34	0	9.88	0	9.42	0	8.97
7	0	12.15	0	11.69	0	11.24	0	10.77	0	10.31
8	0.30	13.48	0	13.03	0	12.57	0	12.10	0	11.64
9	1.09	14.80	0.44	14.34	0	13.88	0	13.41	0	12.95
10	1.87	16.10	1.23	15.63	0.59	15.18	0	14.71	0	14.25
12	3.36	18.66	2.79	18.19	2.19	17.73	1.56	17.27	0.95	16.80
14	4.84	21.18	4.28	20.71	3.72	20.25	3.16	19.78	2.56	19.32
16	6.36	23.66	5.80	23.20	5.24	22.73	4.68	22.26	4.12	21.79
18	7.90	26.12	7.34	25.65	6.79	25.19	6.23	24.72	5.68	24.25
20	9.46	28.55	8.91	28.09	8.36	27.62	7.80	27.15	7.25	26.68
22	11.05	30.97	10.49	30.50	9.94	30.03	9.39	29.56	8.84	29.09
24	12.65	33.36	12.10	32.90	11.55	32.43	11.00	31.96	10.45	31.49
26	14.26	35.75	13.72	35.28	13.17	34.81	12.62	34.34	12.08	33.86
28	15.90	38.12	15.35	37.64	14.81	37.17	14.26	36.70	13.72	36.23
30	17.54	40.47	17.00	40.00	16.45	39.52	15.91	39.06	15.37	38.58
34	20.86	45.15	20.32	44.67	19.78	44.20	19.24	43.73	18.70	43.26
38	24.22	49.79	23.68	49.31	23.14	48.84	22.60	48.36	22.06	47.89
42	27.61	54.40	27.07	53.92	26.53	53.45	25.99	52.97	25.46	52.50
46	31.02	58.98	30.49	58.50	29.95	58.03	29.42	57.55	28.88	57.08
50	34.46	63.54	33.93	63.06	33.40	62.58	32.86	62.11	32.33	61.63
54	37.93	68.07	37.39	67.59	36.86	67.11	36.33	66.64	35.80	66.16
58	41.41	72.59	40.88	72.11	40.35	71.63	39.81	71.15	39.28	70.68
62	44.91	77.09	44.38	76.61	43.85	76.13	43.32	75.65	42.79	75.18
66	48.42	81.57	47.89	81.09	47.36	80.61	46.83	80.14	46.30	79.66

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE CO-EFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1/2

TOTAL SAMPLE COUNT X	T O T A L B A C K G R O U N D C O U N T -- Y									
	10		12		14		16		18	
	L	U	L	U	L	U	L	U	L	U
0	0	0	0	0	0	0	0	0	0	0
1	0	1.16	0	0.26	0	0	0	0	0	0
2	0	2.77	0	1.86	0	0.96	0	0.05	0	0
3	0	4.28	0	3.37	0	2.46	0	1.55	0	0.64
4	0	5.73	0	4.81	0	3.90	0	2.99	0	2.07
5	0	7.13	0	6.22	0	5.30	0	4.38	0	3.47
6	0	8.51	0	7.59	0	6.67	0	5.75	0	4.83
7	0	9.85	0	8.93	0	8.02	0	7.09	0	6.17
8	0	11.18	0	10.26	0	9.34	0	8.41	0	7.49
9	0	12.49	0	11.57	0	10.64	0	9.72	0	8.80
10	0	13.79	0	12.86	0	11.94	0	11.01	0	10.08
12	0.34	16.34	0	15.41	0	14.48	0	13.55	0	12.62
14	1.96	18.85	0.75	17.92	0	16.99	0	16.06	0	15.13
16	3.57	21.33	2.39	20.40	1.21	19.46	0.04	18.53	0	17.60
18	5.12	23.78	4.01	22.85	2.87	21.91	1.70	20.97	0.54	20.04
20	6.70	26.21	5.60	25.27	4.49	24.34	3.37	23.40	2.21	22.46
22	8.29	28.62	7.20	27.68	6.10	26.75	5.00	25.81	3.89	24.87
24	9.91	31.02	8.81	30.08	7.72	29.13	6.62	28.20	5.53	27.25
26	11.53	33.39	10.44	32.45	9.35	31.51	8.26	30.57	7.17	29.63
28	13.17	35.76	12.08	34.82	11.00	33.87	9.91	32.93	8.82	31.99
30	14.82	38.11	13.74	37.17	12.65	36.22	11.57	35.28	10.48	34.33
34	18.15	42.78	17.07	41.84	15.99	40.89	14.91	39.95	13.83	39.00
38	21.52	47.42	20.45	46.47	19.37	45.52	18.29	44.57	17.22	43.63
42	24.92	52.02	23.85	51.07	22.78	50.12	21.70	49.17	20.63	48.22
46	28.35	56.60	27.28	55.65	26.21	54.70	25.14	53.75	24.07	52.80
50	31.79	61.15	30.73	60.20	29.66	59.25	28.59	58.30	27.53	57.34
54	35.26	65.69	34.20	64.73	33.13	63.78	32.07	62.83	31.00	61.87
58	38.75	70.20	37.69	69.25	36.63	68.29	35.56	67.34	34.50	66.38
62	42.25	74.70	41.19	73.74	40.13	72.78	39.07	71.83	38.01	70.87
66	45.77	79.18	44.72	78.22	43.66	77.27	42.60	76.31	41.54	75.35

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE CO-EFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1/2

TOTAL SAMPLE COUNT X	20		22		24		26		28	
	L	U	L	U	L	U	L	U	L	U
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	1.16	0	0.24	0	0	0	0	0	0
5	0	2.55	0	1.64	0	0.72	0	0	0	0
6	0	3.91	0	2.99	0	2.07	0	1.16	0	0.23
7	0	5.25	0	4.33	0	3.41	0	2.49	0	1.56
8	0	6.57	0	5.64	0	4.72	0	3.80	0	2.87
9	0	7.87	0	6.94	0	6.02	0	5.09	0	4.17
10	0	9.15	0	8.23	0	7.30	0	6.38	0	5.45
12	0	11.69	0	10.77	0	9.84	0	8.90	0	7.97
14	0	14.19	0	13.26	0	12.33	0	11.40	0	10.46
16	0	16.66	0	15.73	0	14.79	0	13.86	0	12.92
18	0	19.10	0	18.17	0	17.23	0	16.30	0	15.36
20	1.06	21.53	0	20.59	0	19.65	0	18.71	0	17.77
22	2.75	23.93	1.61	22.99	0.47	22.05	0	21.11	0	20.17
24	4.44	26.31	3.30	25.37	2.17	24.43	1.04	23.49	0	22.55
26	6.08	28.69	5.00	27.74	3.88	26.80	2.75	25.86	1.63	24.92
28	7.74	31.04	6.65	30.10	5.57	29.16	4.47	28.21	3.35	27.27
30	9.40	33.39	8.32	32.44	7.24	31.50	6.15	30.56	5.07	29.61
34	12.76	38.05	11.68	37.10	10.60	36.16	9.52	35.21	8.45	34.26
38	16.14	42.68	15.07	41.73	13.99	40.78	12.92	39.83	11.85	38.88
42	19.56	47.27	18.49	46.32	17.42	45.38	16.35	44.42	15.28	43.47
46	23.00	51.84	21.93	50.89	20.86	49.94	19.79	48.99	18.73	48.04
50	26.46	56.39	25.39	55.44	24.33	54.49	23.26	53.53	22.20	52.58
54	29.94	60.92	28.88	59.96	27.81	59.01	26.75	58.06	25.69	57.10
58	33.44	65.43	32.38	64.47	31.32	63.52	30.25	62.56	29.19	61.61
62	36.95	69.92	35.89	68.96	34.83	68.01	33.77	67.05	32.72	66.09
66	40.48	74.39	39.42	73.44	38.36	72.48	37.31	71.52	36.25	70.57

TABLE D.3 (Cont.)

CONFIDENCE INTERVAL END POINTS L AND U FOR CONFIDENCE COEFFICIENT .95 AND SAMPLE-BACKGROUND COUNTING TIME RATIO 1/2

TOTAL SAMPLE COUNT X	30		34		38		42		46	
	L	U	L	U	L	U	L	U	L	U
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0.64	0	0	0	0	0	0	0	0
8	0	1.95	0	0.10	0	0	0	0	0	0
9	0	3.24	0	1.39	0	0	0	0	0	0
10	0	4.52	0	2.66	0	0.81	0	0	0	0
12	0	7.04	0	5.18	0	3.32	0	1.46	0	0
14	0	9.53	0	7.66	0	5.80	0	3.93	0	2.06
16	0	11.99	0	10.12	0	8.25	0	6.37	0	4.50
18	0	14.42	0	12.55	0	10.67	0	8.80	0	6.92
20	0	16.83	0	14.96	0	13.08	0	11.20	0	9.32
22	0	19.23	0	17.35	0	15.47	0	13.59	0	11.70
24	0	21.61	0	19.72	0	17.84	0	15.96	0	14.07
26	0.51	23.97	0	22.09	0	20.20	0	18.31	0	16.43
28	2.23	26.33	0.00	24.44	0	22.55	0	20.66	0	18.77
30	3.95	28.67	1.73	26.77	0	24.88	0	22.99	0	21.10
34	7.37	33.32	5.20	31.42	3.00	29.53	0.80	27.63	0	25.74
38	10.78	37.94	8.63	36.04	6.49	34.14	4.30	32.24	2.11	30.34
42	14.21	42.52	12.07	40.62	9.93	38.72	7.79	36.82	5.64	34.92
46	17.66	47.09	15.53	45.18	13.39	43.28	11.26	41.37	9.13	39.47
50	21.13	51.63	19.00	49.72	16.88	47.81	14.75	45.91	12.63	44.00
54	24.63	56.15	22.50	54.24	20.38	52.33	18.25	50.42	16.13	48.51
58	28.13	60.65	26.01	58.74	23.89	56.83	21.77	54.92	19.66	53.01
62	31.66	65.14	29.54	63.23	27.42	61.31	25.31	59.40	23.19	57.48
66	35.19	69.61	33.08	67.69	30.97	65.78	28.85	63.87	26.74	61.95

FIGURE D.1

Power of Rule D₃ (Using Eq. (4)) to Detect

$C > 0$ When $\alpha = 0.01$ and $\rho = t/s = 1/2$

FIGURE D.2
 Power of Decision Rule D_3 (Using Eq. (4)) to Detect
 $C > 0$ when $\alpha = 0.01$ and $\rho = 1$.

FIGURE D.3

Power of Decision Rule D_3 (Using Eq. (4)) to Detect $C > 0$ When
 $\alpha = 0.05$ and $\rho = 1/3$ and $1/2$.

FIGURE D.4

Power of Decision Rule D_3 (Using Eq. (4)) to Detect $C > 0$
When $\alpha = 0.05$ and $\rho = 1$.

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