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...RESEARCH COLLOQUIUM

SOLAR WINDS

by

Eugene N. Parker

University of Chicago

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The general problem that I am interested in is the one sometimes called solar-terrestrial relations. A lot of things happen on the sun -- I might add that we really don't understand what they are; we only know them superficially. There are solar flares and various sorts of explosions and eruptions, but we see these things only from a distance of a hundred million miles, so it is really pretty difficult to know exactly what is going on. Immediately following, and therefore presumably caused by these various things on the sun, some things happen on the earth. The cosmic ray intensity may go up or it may go down. Or there may be a fairly bright aurora or a magnetic storm. There is even fairly good evidence that solar activity has some subtle effect on over-all weather patterns, but that is another order of magnitude more nebulous than the things I want to talk about today.

The difficulty is that you cannot get to the sun to observe closely what is going on there, nor can you observe what is going on between here and the sun, nor can you observe what is going on in the vicinity of the earth. While this may be changed in a few years, nonetheless right now it is the case. So, in a sense, it is theology instead of science I'll be talking about; that is, in the absence of observations, it is always possible to theorize. Of course, while you can theorize and generate ideas and have lots of fun, if you want to get any place, what you must do is to try to construct a working model of what you think happens at the sun and then see to what extent you can explain the things that happen at the earth without making additional assumptions. This is what one does ideally. I am not going to be able to follow this plan as closely as I might like.

I would like to begin with the dynamics of the sun, particularly the solar atmosphere and, after making a minimum number of assumptions, see what terrestrial effects we might be able to explain -- I am choosing my words carefully here -- without making additional assumptions. If the explanations outnumber the asumptions when we are done, we will mark up a success. On the other hand, if the assumptions and special hypotheses outnumber the explanations, we will have to go back and start over again. You see, then, the framework in which we are working. I think that in the next year or so there will be some fairly direct experimental tests, and I'll talk about these later.

First, let's consider the sun and what we know about it, or at least those things which may be relevant to our conversation. The photosphere, which is what you see when you look at the sun, is about 5600-5800°K. Let's say 6000°K -- a nice round number -- for the surface of the sun. (It is a rather fuzzy layer with a temperature gradient, so it isn't easy to say exactly what its temperature is.) Outside the photosphere is the solar corona. The solar corona has a temperature of the order of 1,000,000°K, but it may sometimes be as high as 4,000,000°K. It is a very nebulous bit of gas. At a distance of 300,000 kilometers above the surface of the sun -- I am just giving you a standard number which I'll apply later on -- the density is 3×10^7 particles per cubic centimeter, and the temperature is of the order of millions of degrees. In the interior of the sun the temperature is some 15,000,000°K; it drops to 6000 degrees at the photosphere and to about 4300 degrees just above the photosphere; then it begins to climb again to the corona.

So far as magnetic fields are concerned, our only direct evidence is from the photosphere. There are probably magnetic fields in the corona, but there is no optical emission by which to detect them. Information on magnetic fields comes from looking at the Zeeman effect -- the Babcocks have been doing so for some years now and there are improved versions of their apparatus. We find that within 40 degrees of the poles there is something like a dipole field, that is, there is about +1 gauss coming out of one hemisphere -- I don't remember which one -- and -1 gauss going back in the other hemisphere. In between, the fields are generally about 1 gauss, but they are all mixed up and change day by day. Of course, the fields are much more concentrated at sun spots, but I am not going to concern myself with sun spots at the moment. The sun, as you probably know, is composed pretty largely of hydrogen. In fact, I will not talk about other things, even though there may be as much as 10 percent helium.

This then, is the animal we are working with; what sense can we make out of it?

Sidney Chapman was the first one, I think, to try to write down a consistent set of conditions for the solar corona. If the solar corona is in static equilibrium, as it may be, what temperature would you expect to find as you go outwards from the sun? The thermal conductivity of an ionized gas -- ionized because it is at a million degrees -- is proportional to temperature to the five halves power: $K \sim T^{5/2}$. Solving the steady-state heat flow equation -- $\nabla \cdot (K \nabla T) = 0$ -- you find that the temperature drops off like the distance to the two-sevenths power. That is, $T = T_0 (a/r)^{2/7}$, where a is the radius of the base of the corona -- the smallest radial distance at which you would find this million degrees -- and r is some general distance from the sun. The important thing about this formula is that it drops off very slowly. If the corona is at 2,000,000°K, Chapman calculated that the temperature at the orbit of earth is about 200,000-300,000°K, which is pretty hot. Of course, there is very little material there, so the temperature is of no importance in heating ships and things of that nature. Chapman thought this might be important for heating the ionosphere, which is itself a fairly tenuous gas, and there has been a lot of discussion on that particular problem. I mentioned Chapman's investigation for several reasons; one is that his discussing this with me one time is what got me interested in the problem of the corona.

One should go one step further. Instead of just discussing the temperature and, in a vague way, how the density drops off, one should try to write down quantitatively how the density does drop off. If you have an equation of equilibrium, the pressure has to drop off as the density times the gravitational acceleration of the sun:

$$\frac{dP}{dr} = -NM \frac{GM_{\odot}}{r^2},$$

where N is the number density of atoms, M their individual masses, and $M_{\odot}$ the mass of the sun. So you can write down an equation for the gravitational equilibrium of the sun's atmosphere; and what's more, you can even solve it. You know the temperature, so you multiply top and bottom of the right side of this equation by 2kT. Now 2kTN is the hydrostatic pressure of an ionized gas, so you call it P. That is,

$$\frac{dP}{dr} = -2kTN \frac{MGM_{\odot}}{2kTr^2} = -P \frac{MGM_{\odot}}{2kTr^2}$$

You know how T varies with the distance, so you put it in. Then you put the P on the other side, and you can integrate right away. You get a very simple little formula,

$$\frac{1}{P}\frac{dP}{dr} = -\left(\frac{MGM_{\odot}}{2kT_o a^{2/7}}\right)\frac{1}{r^{12/7}}$$

$$P = P_o \exp\left[-\frac{7}{10}\lambda\left(1-\left(\frac{a}{r}\right)^{2/7}\right)\right]$$

$$\lambda = \frac{MGM_{\odot}}{kT_o a}$$

Lambda is a constant, just a comparison of gravitational energy with thermal energy.

The important thing about this formula is that it shows what happens when r goes to infinity. In other words, what is the pressure infinitely far from the sun? Well, you put your hand over the last term and you get:

$$P_{\infty} = P_o \exp\left[-\frac{7}{10}\lambda\right]$$

Lambda turns out to be 4, 5, 6, or 8, depending on how hot the corona is that day, and what this formula tells you is that the pressure infinitely far from the sun is equal to the pressure at the sun times e to the minus something of the order of 3 or 4, say e^{-4}. The embarrassing thing about this is that e^{-4} is only about 1/50, so the pressure at infinity is awfully high. The formula says, in other words, that if you want to keep the solar corona in hydrostatic equilibrium you have to build a box around the sun to hold the thing in because of the very large pressure -- a pressure which is down by only about a factor of 50 or so from what it is right at the sun.

Physically this is all very simple. Let's look at the sun this way: It has a gravitational field which pulls in on all the stuff sitting around it, but the corona is so hot and the temperature drops off so slowly that the gas density and pressure, which drops off as you go out from the sun, is still rather high when you get so far from the sun that you can neglect the gravitational field. From there on it doesn't drop off any more, and so at infinity you must have pressure.

There is at least one way around this. We made an assumption that the sun was in hydrostatic equilibrium, and we got into trouble. Let's go back and reinvestigate that assumption. Instead of assuming that the sun is in hydrostatic equilibrium, this high pressure suggests it might be expanding. So we write down a different set of equations. The density times the acceleration is equal to minus the pressure gradient (this is an ionized gas so the pressure is 2NkT), and we have to put in the gravitational attraction of the sun. We have:

$$\rho a = \frac{dP}{dr} - \rho g_{\odot}$$

$$NM\, v\frac{dv}{dr} = -\frac{d(2NkT)}{dr} - \frac{MGM_{\odot}N}{r^2}$$

We can solve this equation for a number of cases. For instance, if the temperature is constant, we get:

$$\psi - \ln\psi = \psi_0 - \ln\psi_0 + 4\ln\frac{r}{a} - \lambda\left(1 - \frac{a}{r}\right),$$

where ψ is a measure of velocity, actually the ratio of the kinetic energy of an atom to the thermal energy, and λ is the same constant as before, the ratio of the gravitational energy at the base of the corona to the thermal energy. That is,

$$\psi = \frac{Mv^2}{2kT_0} \quad \text{and} \quad \lambda = \frac{MGM_{\odot}}{akT_0}$$

This is for an isothermal corona, with the temperature assumed constant. Obviously this cannot be true to infinity. The corona is heated by some mechanism out to a distance which we are free to specify in an arbitrary way since we lack any observations. If, beyond that, there is a relation such that the temperature varies according to some power of the density, particularly adiabatically, then we can again solve the equation. Let me just write the adiabatic formula down, although it is of no great importance:

$$\psi - \psi_b = 5\left[1 - \left(\frac{\psi_0}{\psi}\right)^{1/3}\left(\frac{b}{r}\right)^{4/3}\right] - \frac{a}{b}\lambda\left(1 - \frac{b}{r}\right)$$

You get formulas like this, and you can plug in the numbers and see what happens. Say you put in 2,500,000°K for the temperature of the corona, which is a fairly routine temperature during the years of solar activity, and assume that this temperature extends out for several solar radii. (It is observed to be that hot out to 3 solar radii, beyond which the corona is too faint to be seen very well, so you don't know the temperature.) Suppose that the corona were at 2,500,000°K and were maintained at that temperature by some mechanism out to about 4 solar radii, and that there were then just adiabatic expansion, that is, absolutely no heating. You would find that at infinity (which is any large distance from the sun) the outward velocity is 540 kilometers per second and the density at the orbit of earth is a little bit over 200 particles per cubic centimeter. In other words, you can get very high velocities -- in excess of the speed of sound in the gas, I might add -- at large distances from the sun.

Now, before we go any further, let's see if there is any reason to believe that this expansion might take place. I think there are three observations which are relevant. The first and most important is comet tails. You all know that comet tails pretty largely point away from the sun. There are a few exceptions, but we won't worry about them at the moment; they turn out to be a different kind of comet with a dusty tail. Now any elementary textbook will tell you that comet tails point away from the sun because of radiation pressure. Biermann and others have looked into this problem quantitatively. Comet tails are composed of such things as ionized carbon monoxide, ionized nitrogen, and things of that nature. You can calculate the absorption cross sections, and lo and behold radiation pressure proves to be disgustingly inadequate to accelerate the tail away from the sun. Not only does the tail point away from the sun, but if you watch the motion of individual bumps and knots in the tail, you will find, as Biermann did, that when they come out of the comet head they soon show tremendous accelerations outwards from the sun. Radiation pressure is grossly inadequate to explain it.

Then there is another problem with comet tails; namely, ionized carbon monoxide and nitrogen are observed in excited states. The textbooks tell us that this ionization and excitation is produced by X-rays from the hot solar corona. After all, the sun is radiating at a couple of million degrees and ought to produce a lot of ultraviolet and soft X-rays. Unfortunately, recent measurements made directly from rockets show that the sun does not radiate nearly as much X-ray and ultraviolet as had been speculated. So you now have two problems on your hands. As is usually the case, it is nice to have your problems come in even numbers because you can then kill two birds with one stone.

In fishing around for an explanation of the motion of the comet tail, Biermann hit upon the idea that there are corpuscular radiations from the sun, that is, protons coming outward from the sun. These would bang into the atoms from the comet tails and blow them out. He got very rough numbers, and it is hard to get any more quantitative than this, but he estimated that if you had an outward flow of gas of 500 kilometers per second with densities of about 100 particles per cubic centimeter at the orbit of earth, then you could account for the outward blowing of comet tails. They would be simply blown out in the wind, just like smoke coming out of a chimney. Then Biermann looked at the problem of the ionization of carbon monoxide in comet tails. A proton with a 500-kilometer per second velocity has a little over a kilovolt of energy and a density of $100/cm^3$ proves to be adequate for producing the ionization and excitation. I want to emphasize again that the numbers are very rough, but nonetheless they seem to be of the right order of magnitude. So Biermann, being apparently able to kill two birds with one stone, suggested that the sun produces corpuscular radiation of 500 km/sec and $10^2/cm^3$ and that this radiation explains the motion of comet tails. He was able to observe that when the sun is particularly active, the corpuscular velocity seems to increase, sometimes to as high as 1500 kilometers per second. The comet tails are much more active then, and the density may go as high as 10^4 or even 10^5. Now comets are not confined entirely to the plane of the ecliptic as planets are; they sometimes go nearly over the poles of the sun. And wherever the comet may go around the sun, it shows the effects of this corpuscular radiation. Perhaps somewhat more corpuscular radiation comes out near the plane of the equator of the sun, but nonetheless there appears to be some in all directions and at all times. We can say "at all times" because comets come in a random way and the effects seem to be always present.

There is evidence on the earth which is very suggestive. If you go up in the auroral zone, say 70 degrees north latitude, then sometime during every clear night you will see an aurora. Aurorae are generally ascribed to particles from the sun, and the fact that an aurora is seen every clear night implies that particles must be coming from the sun every day. Again this leads to the same sort of conclusion that Biermann reached. Furthermore, as you go north of the auroral zones near the pole, you find that the magnetic field is constantly being agitated.

It is never quiet as it sometimes is at low latitudes. The suggestion is again that corpuscular radiation is always flowing past the earth and producing auroral and magnetic agitation and also blowing out the tails of any comets present.

Those are the observations; what I would like to suggest at this point is that this corpuscular radiation is nothing more than the hydrodynamic expansion of the solar corona. Since I am suggesting that it is a hydrodynamic expansion, I would like to use a hydrodynamic term to describe it; I would like to call it solar wind instead of corpuscular radiation. Corpuscular radiation always suggests to me Van de Graaff generators, electromagnetic fields, and such fancy things, whereas solar wind is simple hydrodynamics. While I don't regard any of the arguments for the existence of the solar wind conclusive, I must say they are very attractive when you want to explain things which occur on the earth. The really acid test for this idea will come any month now. Professor Rossi at MIT is going to look for corpuscular radiation, or solar wind, or whatever you would like to call it, directly from a space vehicle. He is going to fly a Faraday cage, which is an apparatus fairly simple in principle, although somewhat complicated if you want to build it properly. You make a little cup (see Figure 1) and put a negative grid across it to stop the electrons (A). Electrons moving at 500 kilometers per second don't have more than a volt of energy and will be stopped, but protons will come in. You put in another grid (B) to suppress photoelectric emission and then a couple more grids to do some other things. What you want to do is to measure the electric current caused by a flux of protons. A thousand kilometers per second is 10^8 centimeters per second; the density of 100 per cubic centimeter means you are going to pick up 10^{10} protons per square centimeter per second, and these will produce measurable currents. This will be a direct test and should allow us to write down much better numbers than the vagaries I have put on the board. The experiment will probably allow us to correlate more directly variations in the coronal temperature of the sun with the wind that blows near the earth.

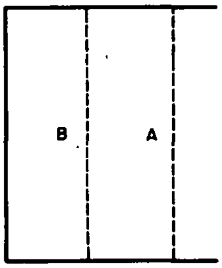

Figure 1

Speaking of variations, let me give you some reasonable extremes. The solar corona is not always at $2,500,000^{\circ}K$. When the sun is quiet for a year or so, that is, during sun-spot minimum, the temperature drops to something of the order of $1,000,000^{\circ}K$. The corona is then observed to extend rather farther into space, and if you use now 10^6 degrees out to 8 solar radii, the velocity that you get at the earth's orbit turns out to be 313 kilometers per second. On the other hand, when the solar corona is very hot, the temperature is sometimes observed to jump to $4,000,000^{\circ}K$. In that case you get out here an expansion velocity of 900 kilometers per second. Incidentally, the photosphere is about 10^8 times denser than any of the coronal gases, and so it is infinitely massive and its pressures are infinitely large compared to the pressures I have been talking about. The density in the photosphere is 10^{16} and the temperature is of the order of 10^4. The pressures are therefore much greater than those of the corona.

Question: Do your numbers correspond to adiabatic or isothermal expansion?

Both. The corona is taken to be isothermal out to some distance, say 4a. There I arbitrarily chop off my heating, and it expands adiabatically until finally at infinity you reach the supersonic velocities. You always get numbers of the order of hundreds of kilometers per second.

It seems, then, that one can fit the rather crude observational data now available. So the next step in our idealized attack on the problem of solar-terrestrial relations is to ask, "All right, if this is what comes from the sun, can I account for the effects that I observe at the earth?" This, you remember, was one of the goals I set for myself earlier. Let's see what happens if we assume as a working hypothesis that the sun cannot help expanding into space. Let's see where it leads us.

In Figure 2a, we are looking down on the north pole of the sun. Remember that the sun has a magnetic field which is roughly a dipole except for a mess around the equatorial regions. (Since this is a small-scale mess which does not extend very far into space, I suggest we forget it.) So for simplicity we have the sun without sun spots and with a dipole field. Lines of force come out of the north, make a circuit, and go back in the south. But what happens if the gas through which these lines thread should decide to expand outward into space? The gas is ionized; it is a conductor; and therefore it must carry its magnetic field with it. So the line of force which originally, in some primordial time, we may think of as having gone out and around back into the southern hemisphere, is now pulled outward. What we get are lines extended radially in all directions from the sun. Figure 2b shows this from the side. The original line of force is marked A, but the gas moved outward, so that the lines became radial, as shown. In a dipole field people expect lines of force to go out from one hemisphere and back in the other hemisphere. So they always say, "Well, where do they close?" I always give them the answer, "Far away." I don't know -- I am talking about things out as far as the orbit of the earth. If they close at all someplace beyond the orbit of earth, it is probably beyond the orbit of Jupiter -- but again I just don't know. I am, of course, aware that a line of force never ends any place, and so this is an interesting problem.

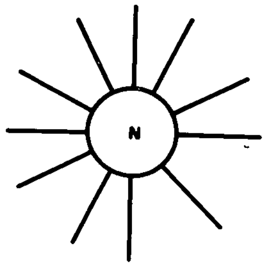

Figure 2a

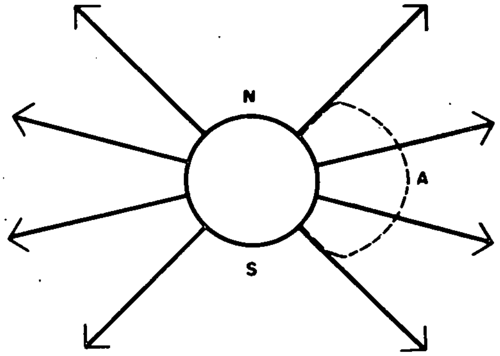

Figure 2b

We have a magnetic field sticking out from the sun. It is about 1 gauss at the sun, and for geometrical reasons it drops off as the distance squared. So at the orbit of earth, the interplanetary field is approximately radial and 2×10^{-5} gauss. To head off any coincidences, I would like to point out that this happens to be about the same as the galactic magnetic field but that this is purely coincidental and bears no relation to theories living or dead. The reason I say this is that some people have tried to make a great deal of the fact that the galactic field is just what you need to explain certain local cosmic ray effects.

So far I have spoken as though the sun did not rotate, but of course the sun does rotate once every four weeks (more or less, depending on what latitude you look at -- some parts rotate faster than others). Consequently the lines of force will not be straight. A particle of material (A in Figure 3) leaving the sun will always have the same line of force in it. The material moves exactly radially -- let me emphasize that point. Once it leaves the sun, the rotational velocity is completely negligible. By the time the material is at B, the sun would have rotated, so that the line of force which came out at A is now over at C. You can see that the lines of force will then have a spiral. The lines of force spiral, but the material flows exactly radially out of the sun. The degree of spiraling is not very great; with a 500-kilometer per second wind, the angle at the earth is about 40 degrees from the radial direction. For a 1000-kilometer per second wind, it is only about 20 degrees. I will often speak as though the fields were precisely radial, but I am perfectly aware that they are not. The inclination does not seem to be serious for most of our problems. So much, then, for the magnetic field of the

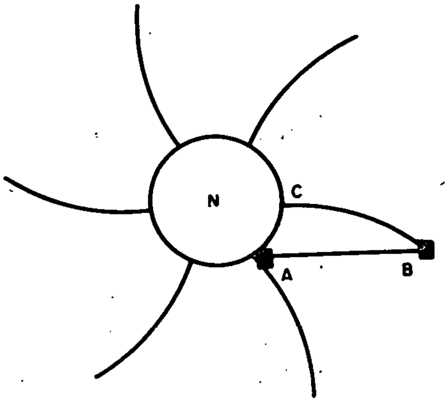

Figure 3

sun which extends into space. There is hope that it will soon be possible to measure this field with suitable magnetometers, probably using optical pumping schemes, and so forth. At least, people say it can be done.

Now for the effects that we observe on the earth. In what kind of position are we to explain the aurorae, magnetic storms, and things of that nature? Well, the picture that we have is an earth with a dipole field past which there is a wind blowing. (See Figure 4.) For the time being, let me speak of it as a perfectly steady wind. I know it isn't perfectly steady, because the coronal temperature jumps up and down pretty fast sometimes -- in a matter of an hour or so -- so there must be gusts. But for the moment, let's talk about a steady wind. The wind is an ionized gas; it is a good conductor of electricity; it cannot move across the lines of force of the geomagnetic field. It can push against them and it can push them to one side, but it cannot actually flow through them. So we can ask how far this gas will penetrate. We have a wind of density N, velocity U, and made up of hydrogen of mass M per ion. We would expect that the wind pressure would push apart the geomagnetic field down to the point where the wind pressure becomes comparable to the magnetic pressure,

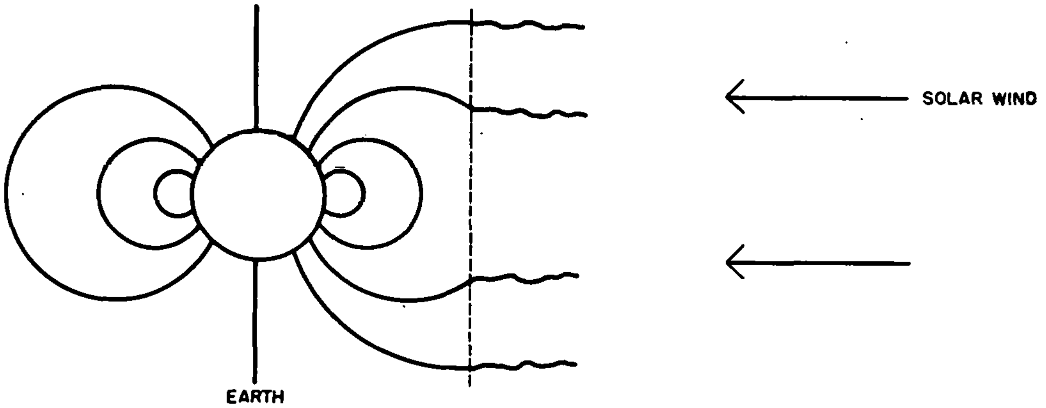

Figure 4

$$1/2\,NMU^2 = \frac{B^2}{8\pi}.$$

We know the geomagnetic field is a dipole, and we can calculate the field at any height. If we plug in 100 particles per cubic centimeter and 500 kilometers per second, we find that the wind can push down to about 5 earth radii from the center of earth. In other words, at 5 earth radii, the earth's field might flutter a little, but it certainly would not be seriously perturbed. Outside 5 earth radii, the field is messed up in a way about which we are totally unable to speculate. We would have to observe it to see what is happening. Beyond 5 earth radii, the wind pressure is greater than the magnetic pressure, and the field is at the mercy of the wind. It is easy to show theoretically that if the interface between the field and the wind were smooth, it would be unstable to perturbation. So the field must flutter. If you follow the line of force which just barely goes through 5 earth radii, you reach something like 65 degrees (north magnetic) latitude. This is very gratifying indeed, because that is the southern boundary of the auroral zone. We can say, therefore, that the auroral zone is produced by those lines of force which have access to the solar wind and can feed particles down into that region. If the solar wind blows at all times, probably there is an aurora at all times at approximately these latitudes. Obviously any line of force coming from nearer the pole goes farther out than 5 earth radii, and so it is constantly agitated by the solar wind. This is something which has been observed. A magnetometer in the polar regions never really quiets down; it always flutters a little bit. So in at least a rough qualitative way, we find that we are on the right track in explaining some of the observations made at the earth.

I think I will talk a little bit more about the aurora before going on to cosmic-ray effects. You had Joe Chamberlain* out here to talk to you about aurorae, so let me just refresh your minds with some of their salient features. I think that the most prominent feature of the aurora, from a theoretical point of view, is that we haven't the faintest idea of what is going on. On the other hand, to give the experimentalists their due, we do know some things that go on in an aurora, and those things are very important when it comes to constructing theories. Consider a homogeneous auroral arc, which occurs particularly when the sun is active. Chamberlain and Meinel have looked at H_α, one of the hydrogen lines, and have found it to be Doppler shifted due to its incoming motion. Chamberlain has put quite a bit of effort into trying to fit the observed luminosity as a function of height with the observed velocity spectrum of the incoming protons. I think he has had a measure of success; at least he has put together a picture that appears to be self-consistent. The striking thing is that you need a wide spectrum of primary protons to produce these homogeneous arcs; it is not sufficient to put in a single energy. This, of course, immediately rules out Størmer's theories of the aurora, which require not only single energies but the wrong energies to give the altitudes that are observed. Chamberlain and Meinel find that you need particles from about 500 kilometers per second up to about 10,000 kilometers per second issuing from space on to the top of the atmosphere if you want to produce the observed auroral arc.

There are a lot of other things in the arc besides these proton effects, and there are a lot of auroral forms besides arcs. That is why I started off by saying, "Let's face it, we don't know what an aurora is."

A couple of facts help us on our way. We need a variety of protons. These homogeneous arcs occur about 30 hours after there has been a big flare-up on the sun. As a result of the

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*Joseph W. Chamberlain, "Theories of Auroras," SCR-77(April 1959). (Ed. note) flare-up, the solar corona is suddenly heated to 4,000,000°K and practically explodes into space; and when this blast wave reaches the earth, the homogeneous auroral arcs are produced. It is not sufficient merely to assume that the auroral particles are solar wind particles that happen to get caught in some sort of magnetic funnel so that they ram down into the atmosphere; because if they were, you wouldn't find either the broad spread of particles or the high velocity end of the tail. Things come from the sun with maximum velocities of 2000 kilometers per second. So one does not account for 10,000-kilometer per second particles in that way. Hence, as Chamberlain has pointed out, local acceleration of particles is apparently necessary. We don't know with any certainty how this acceleration is produced, but let me suggest a possibility: the outer surface of the geomagnetic field flutters in the solar wind. Any charged particle in this fluttering magnetic field gets kicked first one way and then the other -- like a tennis ball if you can imagine a handball court with about 50 people swatting at a ball with tennis rackets. This is called Fermi acceleration, and it is the only hydromagnetic acceleration mechanism available in the presence of an ionized gas. So if you drop a particle into the fluttering medium, it is going to bounce around and then finally pop out of the fluttering layer. It turns out that we can fit the observed velocity spectrum with the observed solar wind velocity. So I offer this as a possible explanation for the protons in the auroral arcs. The thing that is important here -- at least it seems important to the over-all picture -- is that we don't have to make any additional assumptions to get this velocity spectrum. We have the solar wind, and we have shown that the geomagnetic field is unstable to fluttering at the outer surface; then the spectrum falls out automatically.

There is evidence that electrons are the dominant particles in auroral rays, that is, the diffused ray structures which follow arcs. There are plasma processes which seem able to produce the electrons, but I think that is a much more nebulous state of affairs.

Let's push on now and talk about some cosmic-ray effects that have been observed for some time and that seem to find an explanation in the solar wind model. The first thing is that about every five years there is a flare on the sun that looks like other flares except that for some reason it produces a great many cosmic rays. By cosmic rays I mean protons with energies of 1-10 Bev. The cosmic ray intensity normally doesn't fluctuate by more than ten percent, but when there is a flare of this kind on the sun, within a few minutes the cosmic-ray intensity may go up by a factor of 1000 and stay up, and only over periods of 20 hours or so decay back to normal. Even 30 to 40 hours later you can see that things are slightly above normal. (See Figure 5.) This is the observation. The cosmic rays must come straight from the sun; they can't come by a lengthy path because they arrive so promptly. They can't go any faster than light, and yet within a matter of a few minutes they are at the earth. This suggests that there is no hindrance between the sun and the earth, and that you must have a radial magnetic field from the sun if you have any magnetic field at all. Fortunately, our model has come up with a radial magnetic field and so fits with the free passage of cosmic rays from the sun to the earth.

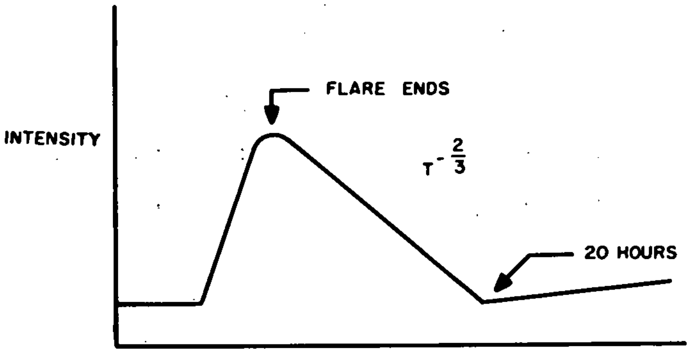

Figure 5

The question now is, "Why the long tail on Figure 5?" The flare lasts maybe half an hour. It is over by the time you reach the peak of the cosmic ray intensity, and yet cosmic rays come in for many, many hours afterwards. Of course you can always say that the sun continues to produce cosmic rays even after the visible flare has disappeared. Since we are completely ignorant of what a flare is, this is not an untenable hypothesis. On the other hand, it is not very attractive. It seems more appealing to assume that the cosmic rays were produced during the flare, that when the flare went away the sun ceased to produce particles, and that the particles were stored somehow. But how are they stored? Let me give you the analysis made before solar wind days.

Any child knows that the decay curve ought to be exponential; that is the way things behave. Unfortunately it is not exponential. The cosmic ray intensity falls off like $t^{-3/2}$. The curve gets a little steeper near the end as though it were trying to get to the exponential, but it is certainly not exponential over the bulk of the decay. There is only one kind of decay that anybody could think of that falls off like $t^{-3/2}$. Let me give you a thermal analogy. If I take a very large block of some medium, produce a burst of heat right in its center, and observe the temperature near the source of the heat, I find that it falls off like $t^{-3/2}$, and continues to do so until the heat begins to flow out the sides of the block sometime later. The decay of the temperature then goes over to exponential. You see that is similar to what we have here. So the model that was constructed was this: let us suppose that the particles are indeed produced by the flare only while it is visible on the surface of the sun and that they are stored. Look at Figure 6. There is free access between the sun and earth. There can't be any absorber or scatterer between them, because the particles come straight from the sun -- as you can see by their prompt arrival. Beyond the orbit of earth is a shell that must extend for a very large distance in order to get hours of $t^{-3/2}$ decay. If it were a thin shell, we would get exponential decay immediately, so it must extend well beyond the orbit of earth. By looking at the time when the decay ceases to be like $t^{-3/2}$ and tries to become exponential, you can guess that the outside of the shell is about five astronomical units from the sun, that is, in the vicinity of Jupiter. In other words, what I have is a block. I release a burst of heat, or in this case, cosmic rays. The rays diffuse out through the block, and when they begin to diffuse at a significant rate out of the outer surface you get a steeper decay. You can write down more properties of this block; you can say that this diffusing medium must be a magnetic field, for that is the only thing that affects cosmic rays in any interesting way. It must be a field of the order of 10^{-5} gauss, and it must be disordered. (If it were a smooth radial field, the particles would just go right out.) The scale of disordering must be about a million kilometers. That is a short distance, when you remember we are thinking of hundreds of millions of kilometers for the thickness of the shell. The idea simply is that when these cosmic-ray particles get out into this messed up field, they random walk, and eventually diffuse out the other side.

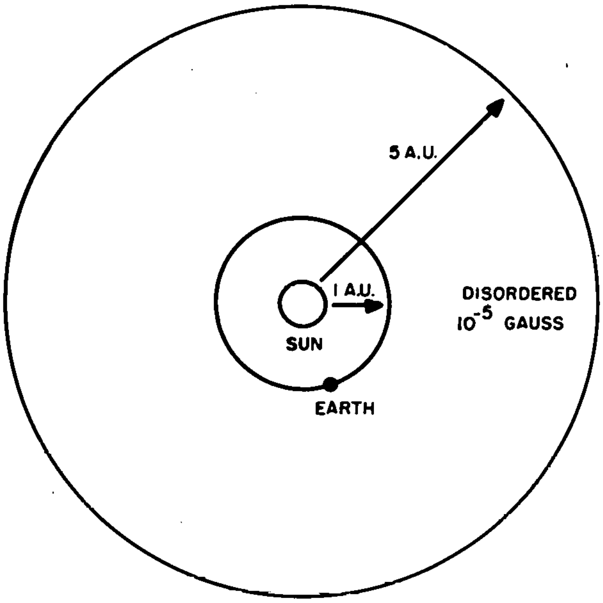

Figure 6

This was the picture constructed purely on the basis of looking at cosmic-ray decay. There was a lot of discussion as to what might produce the shell. Of course, you don't need the exact spherical symmetry I have drawn, but I couldn't solve the problem in anything but spherical symmetry, so that is why I draw everything nice and round.

Now let's go back and look at our solar wind picture on a different scale (see Figure 7). As lines of force go outward from the sun, the gas density drops to very low values (100 per cubic centimeter), collisions between atoms and electrons become rather infrequent, and in general the particles go freely on their way -- mainly moving radially outward. The temperature is probably quite low compared to the equivalent temperature of their velocity. Notice though how the volume expands. Suppose I enclose a bit of gas near the sun. When this

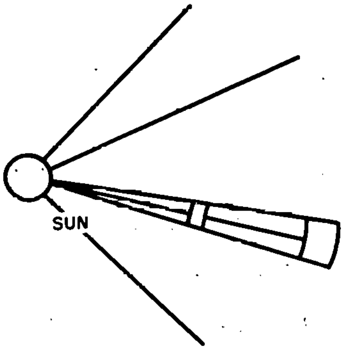

Figure 7

little enclosure gets farther out, as shown in the figure, it will have the same dimensions in the radial direction because everything is moving with constant velocity once you are away from the sun. But it will have expanded perpendicular to the radial direction, because everything is moving out radially. This is an anisotropic expansion. Remember that you have very low densities, so the atoms don't make many collisions; they are expanding one way but not the other. The thermal motions perpendicular to the radius, and hence perpendicular to the magnetic field, will be decreased by the expansion; there are no collisions, so they never equalize with the motions in the radial direction. You soon find that you have a gas which, if you look in the radial direction, is relatively hot and perpendicular to the radial direction is cold. In other words, the pressure perpendicular to the magnetic field must be rather less than the pressure parallel to the magnetic field.

Question: Are you assuming that the curvature of magnetic field is not significant?

In this particular problem, it is not a factor. Once I get beyond the orbit of earth, of course, the amount of inclination to the radius increases. At the moment I want to argue the simplest case, one with a radial field.

So I get an anisotropic pressure. Now one can show very formally with a lot of fancy equations that this leads to instability, but there is no need to go through them because the matter is very simple. We have a line of force in a conducting medium, and the medium and the line of force have to move together. What we are saying is that most of the thermal motions are along the lines of force and that there is very little perpendicular motion. Think of the lines of force as rubber bands. The particles are charged so they have to spiral along the line of force. What happens if I perturb the field as shown in Figure 8? A particle comes along and zooms around the curve. But as you all know there is an animal called centrifugal force, and as the particle goes around the curve, the centrifugal force pushes outwards and tends to pull out the curve further. You can imagine yourself driving on a rubber road and as you go around the curve the road actually pulls out into a bigger curve -- that is exactly what I am suggesting happens here. You can show that if the difference between these two pressures exceeds twice the magnetic pressure, that is, $P_{\parallel} - P_{\perp} > 2B^2/8\pi$, then the rubber road has less strength in it than the centrifugal force of the cars running around the curve and the road gets more and more curvy. The field becomes unstable whenever the pressure

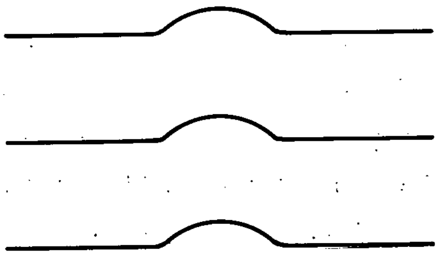

Figure 8

along the field becomes sufficiently greater than the pressure perpendicular to the field. That is an instability which we must watch out for.

Taking the very crude values we have for density, you can guess that the anisotropy in the pressure might begin out somewhere in the vicinity of the orbit of earth. I wouldn't argue whether it is Venus or Mars, but this instability should begin to occur somewhere in that general region. You can also calculate the scale of the instability. It is of the order of a million kilometers -- maybe a half a million kilometers, maybe a million kilometers. I have already written down the field strength. At earth it is 2 x 10^{-5} gauss; at Mars it is just 10^{-5} gauss, so you get the correct orders of magnitude. Somebody may ask, "What happens when the lines of force spiral closely?" Then, of course, the reverse is true because the directions are interchanged -- $P_{\perp}$ becomes bigger than $P_{\parallel}$. In that case, if you perturb the field in the same way -- if you make a weak spot -- the particles tend to congregate in it and inflate the weak spot, further weakening the field there. So we have another kind of instability. The point is that it starts off at a scale of 10^6 km and in a field of 10^{-5} gauss. This is a theoretically predicted instability. If you look back to see what we needed to explain the trapping of cosmic rays from solar flares, you see it is about this scale (10^6km) and field (10^{-5}gauss). And so I suggest that the radial field from the sun accounts for the free transit of particles from the sun to the earth and for their trapping in the inner solar system once they have gotten out into interplanetary space. I suggest this. I can dream up interplanetary temperatures which would not allow the instability and the trapping. If you put too low a temperature in the gas, then the coulomb cross section is too large, and you don't get large enough anisotropies. But since nobody knows the temperature anyway, I assume a reasonable value, namely, something a little less than 10^6 °K, but perhaps greater than 10^4 °K. So it looks as though we may have an explanation for cosmic-ray containment in the solar system.

Question:. What do you think the sun's magnetic field is like beyond five astronomical units? Does it become smooth again?

After five astronomical units, I don't know. All I am saying is that the observations show that something happens so that its trapping ability falls off. You suggested as good a possibility as any. I can give you a couple more possibilities, but I certainly wouldn't feel they were any more likely than yours. The field intensity is still dropping off, you see, as you go out. Even though the thing may be tightly spiralled out here, the intensity drops off like $1/r$, and maybe the field just becomes too weak after five astronomical units -- I really don't know.

Question: You say the solar wind continues after five astronomical units on the spiralling solar magnetic field?

I think it may continue because I can't think of any way of stopping it; but on the other hand, if somebody were to observe that it stops, I wouldn't argue with him. You see I am arguing from pure observational ignorance and theoretical arrogance, and I don't want to commit myself to anything beyond what produces some benefit. In other words, I commit myself on this disordered field because it seems to explain an observation; but we don't have any observations on what goes on beyond five astronomical units, and I just don't know. People have tried to guess how far out the solar wind blows; you can get anything from the orbit of Jupiter to 1/10 of a light year. You can think of all kinds of things which might happen. For instance, you know how at a drinking fountain water squirts up and forms a little cap of water on top that squashes the stream down. Well, you might have a big mass of stationary gas sitting out beyond Jupiter and being pulled in by the solar gravitational field, for it takes only 10^5 particles per cubic centimeter to do this. That might stop the solar wind, but I don't know -- there are just too many possibilities.

I have put down this instability in the field because I couldn't see any real convincing way of avoiding it, and fortunately it seemed to explain an observation. But there is one more thing I have to worry about. The gas in which these magnetic irregularities are embedded is moving outward from the sun. Now consider what happens when cosmic-ray particles wish to come in -- remember that we believe that cosmic rays are produced somewhere outside the solar system except when there is a solar flare. They have to get in past the outward sweeping fields. Well, the particle comes in and begins to random walk, and at the same time it is random walking it is being carried back out. As you can see, this will, at least to some degree, impede the progress of cosmic rays into the inner solar system. You can see that an equilibrium will be established in which the intensity close to the sun must be a little bit less than it is farther out, just because the particles coming in tend to be swept back out at the same time they are random walking. Having written down the characteristic scale for the random walk, and also the field strength so you can tell how it affects particles of different energies, you could calculate what the cosmic-ray spectrum at the earth's orbit should look like if you knew it beyond Jupiter. You suppose that during times of solar minimum the effects should be at a minimum. The corona is not as hot, and there is not as much solar wind. Maybe particles then come in fairly freely. If you make that assumption, the spectrum you see at that time becomes a representation of the galactic cosmic-ray spectrum (see Figure 9). Plotting the number of particles at a given energy against energy, you get a spectrum that is fairly

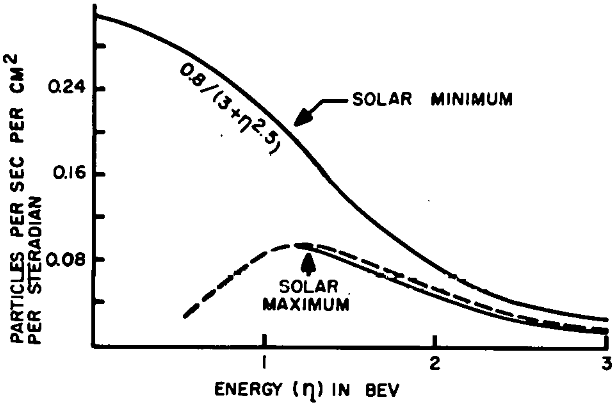

Figure 9

flat at the lower end; it doesn't show any signs of dropping off toward low energies. At higher energies, it drops off quite rapidly at something like $E^{-2.7}$ and goes out to extraordinarily high energies. Obviously at the high-energy end you aren't going to get any effects, because such particles are so rigid that they come right through the field. Then when the sun is active, four or five years later, you observe that the cosmic ray intensity, as you expected, is not affected at high energies; but as you come to lower energies, it seems to be greatly depressed and definitely turns over and drops off. Evidently the spectrum comes down to zero in some continuous manner.

If you make the assumption that the spectrum at solar minimum is the true galactic spectrum and then multiply by the sort of reduction factor given by the outward sweeping of the solar wind, lo and behold, you find a curve with just the right sort of form to fit the observed depressed cosmic ray spectrum at solar maximum. So a second result of disordering the field is that it may account for the 11 year cycle in cosmic ray intensity.

What I have done then, to recapitulate for a moment, is to look at the dynamics of the solar corona, which we found is probably expanding. From the expansion and resulting solar wind, we deduced the interplanetary and terrestrial magnetic field configurations. And we were able to explain, on a qualitative basis, some of the observed terrestrial effects. On the other hand, I want to emphasize that none of the arguments I have given are in any way conclusive; a large number of problems are still unexplained. Why are the aurorae observed principally on the night side, for instance? That is a beautiful little problem, and there are many other questions of like nature. We are better off with the solar wind than with most other theories, but there are still a lot of outstanding problems. I hope within the next four or five years there will be experimental tests of these things.

Questions

How long will the sun last at this rate of emission?

You mean how much mass does it lose? About one percent in five billion years. It does lose mass very fast sometimes, but that is only for a day or so at a time.

(Dealt with comets observed with tails going in the opposite or inward direction.)

There is no contradiction. There are comets which have tails, which instead of pointing away from the sun, just hang limply out of the comet. These tails seem to be composed largely of dust particles; that is, small pieces of stone and so forth, and you show that they would not be affected by the solar wind. One should be very cautious about this commentary, of course. As Biermann has pointed out, there are magnetic fields mixed up in it somehow, and we see some strange things. For example, a little L-shaped irregularity was seen on one side of a tail one day, and the next day it was pointing the other way but still had its shape. This might have been coincidence. But it means something strange is going on, and that is why I think we must wait until after Rossi gets through with his experiment to be absolutely sure that there is a solar wind. I am willing to put my money on this horse, but it is still a horse race, and we won't know until the race is over how it is coming out.

When you talked about the 20 hour falloff, the perturbations all seemed to be past the earth. I didn't understand the explanation.

This is the sun, this is the earth, and all the magnetic mess is out beyond. You produce particles at the sun and they move in perfectly straight lines with nearly the speed of light, and they come to the earth and apparently fill up the cavity inside the disordered field. Now to get out of the cavity, they must random walk through the disordered field and that takes time. So what you have is a sort of barrier through which things can diffuse slowly.

Does the solar wind theory predict aurorae in north and south hemispheres at the same time?

That is correct. And it is observed that if you see a lot of auroral activity in the north, you will see it in the south too.

Wouldn't you expect small-scale turbulence within the earth's orbit from such things as sunspots, flares, inconsistency of the corona, etc.? Wouldn't the wind be turbulent?

That is a real tough question. If you calculate a Reynolds number by writing down a characteristic scale, velocity, and viscosity, you get a pretty big number -- bigger than 2000. On the other hand, there is not much shear involved unless you happen to have a real hot spot on the sun which is poking stuff out faster in one than in neighboring directions. Then you expect some sort of a fast expansion and a shear along the boundary between the hot and cool corona. You might get turbulence there. The reason I didn't go into that is because this is about as far as I can go.

You used the word "gusts" of wind at one stage. . . .

What I meant there was something a little different. The 4,000,000° temperature apparently is achieved very quickly, in a matter of a few hours, in what is for all practical purposes an explosion, so that you get a blast wave from the sun. Among other things the fast moving blast wave straightens out the spiralling of the interplanetary field so that the lines of force jog across the front of the blast wave. The jog in the field sweeps back cosmic rays and appears to account quantitatively for the observed large and abrupt cosmic ray decreases. Thus, instead of gusts, perhaps I should have said blast waves.

You say that the earth is apparently about the right position to support the idea that the solar wind causes the aurora. How about Jupiter, do you expect it might not have an aurora?

Frankly I don't know. If you observe aurorae on Jupiter, it would not be inconsistent with any of the things I have said here.

Are you familiar with the concept of solar sailing* that has been proposed for interplanetary travel? How would the magnitude of the forces you spoke of compare with radiation forces on a satellite?

The solar wind, with its normal everyday value, has a pressure rather less than light pressure. On the other hand, the enhanced value, that is, corresponding to 1500, instead of 500, kilometers per second, would be many times light pressure, since the density goes up as well as the velocity. For solar sailing the solar wind is apparently more of a headache than anything else, not only because it produces transient loading, but also because it produces sputtering which evaporates the sail. You see, you need a metal-coated plastic. Now maybe the plastic dries up and blows away by getting brittle with ultraviolet; but the solar wind would evaporate the metal film itself. The present state of the matter is that the sputtering calculations which were made used an awfully high value of the solar wind -- a value not unreasonable for a blast wave when the sun is active, but certainly more than the everyday background solar wind. These calculations indicate that you lose your sail in a couple of days. On the other hand, if you put in the everyday values, which can probably be relied on for 11 months out of any year, (but of course you can't tell ahead of time which month it is going to be higher), you get a sail life of a year or more.

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*Theodore P. Cotter, "Solar Sailing," SCR-78 (April 1959)(Ed. note).

What about the satellite balloons?

They are well down inside the geomagnetic field, at least the ones I have heard talked about, and so are shielded from the solar wind. You do not have to worry about them.

They are now talking about something like four earth radii for balloons.

If you are inside five earth's radii you are probably all right most of the time. Once in a while when the solar wind is very high, it may butt down to within a couple of earth's radii.

What about a polar orbit?

You go through an auroral zone then. I wouldn't want to commit myself as to what happens. It would be the problem of how much bombardment a balloon can stand.

Does the solid matter in the solar system -- planets and other bodies -- act as any kind of sink?

Yes, though probably not as an important sink because they have such small cross sections. If you were to stand on the sun -- which is a preposterous state of affairs -- you would have a hard time seeing the earth. It would be pretty small, like one of the planets you see in the sky. And of course the earth is very well shielded by the magnetic field. You get a leakage which may produce the aurora, but it is actually a very small fraction of the total number of particles. I have been told that this is very good because there would be an anomaly with the abundance of argon in the atmosphere. Apparently there is more argon in the sun than in the atmosphere. If the solar wind really did get in, why shouldn't you see more argon in the atmosphere? The amount of argon, then, is a measure of the effectiveness of the shielding of the field. Of course, we can argue about whether Mercury, for example, has a field. I don't know.

APPENDIX

Editorial Note: Dr. Parker left a copy of his notes, from which comes the following expansion of the analysis of static and expanding coronas given in the text.

Static Corona

$$\frac{dP}{dr} = -\rho g_{\odot} = -NM\frac{GM_{\odot}}{r^2} \tag{1}$$

$$\nabla(K\nabla T) = 0 \tag{2}$$

Since $K \sim T^{5/2}$, Equation 2 yields

$$T = T_0\left(\frac{a}{r}\right)^{2/7} \tag{3}$$

Then, multiplying numerator and denominator of the right hand side of Equation 1 by 2kT and remembering that $P = 2kTN$, by substituting Equation 3 we get:

$$\frac{1}{P}\frac{dP}{dr} = -\left(\frac{MGM_{\odot}}{2kT_0 a^{2/7}}\right)\frac{1}{r^{12/7}} \tag{4}$$

Integrating

$$\begin{aligned} P &= P_0 \exp\left[-\frac{7}{10}\left(\frac{MGM_{\odot}}{kT_0 a}\right)\left(1 - \left|\frac{a}{r}\right|^{5/7}\right)\right] \\ &= P_0 \exp\left[-\frac{7}{10}\lambda\left(1 - \left|\frac{a}{r}\right|^{5/7}\right)\right] \end{aligned} \tag{5}$$

where

$$\lambda = \frac{MGM_{\odot}}{kT_0 a}$$

Thus if $\lambda = 6.28$, which corresponds to $T_0 = 2.5 \times 10^6$ K:

$$\begin{aligned} P_{\infty} &= P_0 \exp\left(-\frac{7}{10}\lambda\right) \qquad (6) \\ &= P_0 \exp(-4.4) \\ &= .012\, P_0 \end{aligned}$$

Expanding Corona

$$\rho a = -\frac{dP}{dr} - \rho g_{\odot} \tag{7}$$

$$NMv\,\frac{dv}{dr} = -\frac{d(2NkT)}{dr} - \frac{NMGM_\odot}{r^2} = 0 \qquad (8)$$

If $T = T_o$, which is to say isothermal expansion:

$$vdv = -\frac{2kT_o}{M}\frac{dN}{N} - GM_\odot\frac{dr}{r^2} \qquad (9)$$

Integrating

$$\frac{v^2}{2} = -\frac{2kT_o}{M}\ln N + \frac{GM_\odot}{r} + \text{const.}$$

or

$$\frac{Mv^2}{2kT_o} - \frac{Mv_o^{\,2}}{2kT_o} = 2\ln\frac{N}{N_o} - \frac{MGM_\odot}{2kT_o a}\left(1 - \frac{a}{r}\right) \qquad (10)$$

Thus if we have spherical expansion from the sun,

$$N = N_o\,\frac{v_o a^2}{vr^2} \qquad (11)$$

and with

$$\psi = \frac{Mv^2}{2kT_o} \quad\text{and}\quad \lambda = \frac{MGM_\odot}{kT_o a} \qquad (12)$$

$$\begin{aligned}\psi - \psi_o &= \ln\left(\frac{v}{v_o}\right)^2 + 4\ln\left(\frac{r}{a}\right) - \frac{MGM_\odot}{kT_o a}\left(1 - \frac{a}{r}\right)\\ &= \ln\frac{\psi}{\psi_o} + 4\ln\frac{r}{a} - \lambda\left(1 - \frac{a}{r}\right)\end{aligned}$$

or

$$\psi - \ln\psi = \psi_o - \ln\psi_o + 4\ln\frac{r}{a} - \lambda\left(1 - \frac{a}{r}\right) \qquad (13)$$

The right hand side and the left hand side must pass their minima simultaneously or else $\psi \to 0$ as $n \to \infty$. The left hand side is a minimum where $\psi = 1$, the right where $n = \frac{a\lambda}{4}$. They must be equal at their minimum, so that

$$\psi_o - \ln\psi_o = 1 - 4\ln\frac{\lambda}{4} + \lambda - 4 = -3 + \lambda - 4\ln\frac{\lambda}{4}$$

whence

$$\psi - \ln\psi = -3 - 4\ln\frac{\lambda}{4} + 4\ln\frac{r}{a} + \lambda\frac{a}{r} \qquad (14)$$

On the other hand, if we wish to treat adiabatic expansion, then:

$$T = T_o \left(\frac{N}{N_o}\right)^{\gamma-1}$$

Whence, instead of Equation 9 we get:

$$\begin{aligned} v\,dv &= -\frac{2kT_o}{MN_o^{\gamma-1}} \frac{dN^{\gamma}}{N} - GM_{\odot} \frac{dr}{r^2} \\ &= -\frac{2kT_o\gamma}{MN_o^{\gamma-1}} \frac{dN}{N^{2-\gamma}} - GM_{\odot} \frac{dr}{r^2} \end{aligned}$$

Integrating

$$\frac{v^2}{2} = -\frac{2kT_o\gamma}{MN_o^{\gamma-1}} \frac{N^{\gamma-1}}{\gamma-1} + \frac{GM_{\odot}}{r} + \text{const.}$$

$$\frac{Mv^2}{2kT_o} - \frac{Mv_o^2}{2kT_o} = \frac{2\gamma}{\gamma-1}\left[1 - \left(\frac{N}{N_o}\right)^{\gamma-1}\right] - \frac{MGM_{\odot}}{2kT_o b}\left(1 - \frac{b}{r}\right)$$

As before,

$$N = N_o \frac{v_o b^2}{vr^2},$$

where b is the radius at which the expansion becomes adiabatic (b > a), then:

$$\psi - \psi_o = \frac{2\gamma}{\gamma-1}\left[1 - \left(\frac{\psi_o}{\psi}\right)^{(\gamma-1/2)} \left(\frac{b}{r}\right)^{2(\gamma-1)}\right] - \frac{a}{b}\lambda\left(1 - \frac{b}{r}\right)$$

A reasonably well ionized gas much as we suppose the corona to be, will have $\gamma = 5/3$, whence,

$$\psi - \psi_o = 5\left[1 - \left(\frac{\psi_o}{\psi}\right)^{1/3}\left(\frac{b}{r}\right)^{4/3}\right] - \frac{a}{b}\lambda\left(1 - \frac{b}{r}\right)$$

Examples

1. If $T_o = 2.5 \times 10^6$ to $r = 4a$

   $\lambda = 6.28$

   Then $\psi_o = 0.31$ $v_o = 113$ km/sec

   $\psi_a = 3.59$ $v_a = 386$ km/sec

   $\psi_{\infty} = 7.02$ $v_{\infty} = 540$ km/sec

   At earth $N = 280\ \text{cm}^{-3}$

2. If $T_O = 10^6$ to $r = 8a$

Then $\psi_o = .74 \times 10^{-3}$ $v_o = 3.5$ km/sec

$\psi_a = 2.97$ $v_a = 218$ km/sec

$\psi_\infty = 5.91$ $v_\infty = 313$ km/sec

At earth $N = 15$ cm^{-3}

3. If $T_O = 4 \times 10^6$

Then $\psi_o = 1$ $v_o = 258$ km/sec

$\psi_a = 7.65$ $v_a = 712$ km/sec

$\psi_\infty = 12.17$ $v_\infty = 900$ km/sec