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We report a differential synthetic aperture ladar (DSAL) concept that relaxes platform and laser requirements compared to conventional SAL. Line-of-sight translation/vibration constraints are reduced by several orders of magnitude, while laser frequency stability is typically relaxed by an order of magnitude. The technique is most advantageous for shorter laser wavelengths, ultraviolet to mid-infrared. Analytical and modeling results, including the effect of speckle and atmospheric turbulence, are presented.

Synthetic aperture ladars are of growing interest, and several theoretical [1,2] and experimental [3-5] papers have been published on the subject. Compared to RF synthetic aperture radar (SAR), platform/ladar motion and transmitter bandwidth constraints are especially demanding at optical wavelengths. For mid-IR and shorter wavelengths, deviations from a linear trajectory along the synthetic aperture length have to be sub-micron, or their magnitude must be measured to that precision for compensation. The laser coherence time has to be the synthetic aperture transit time, or transmitter phase has to be recorded and a correction applied on detection.

The DSAL concept* is depicted in Fig. 1 for a "Stripmap mode" [6] configuration. The transceiver aperture, of width d, is divided into two equal subapertures along the flight direction. The laser beam is transmitted through the full aperture, while return radiation is collected through both subapertures. Phase differentials between the subapertures are measured by heterodyning with a common local oscillator. These steps are repeated every distance d/4 of travel.

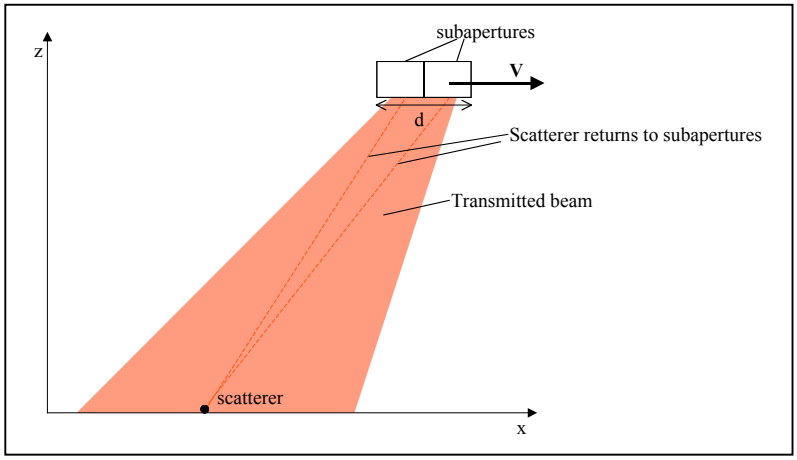


Fig.1. DSAL Concept

As will be shown below, the phase in either subaperture can be reconstructed by summing these differentials. In the process, common-mode phase errors between the two subapertures cancel. In particular, DSAL is insensitive to line-of-sight translation and vibration. As an example, for a single point source scatterer at location x_p, the path length difference to the center of the two subapertures is:

$$\Delta s(z) \sim \frac{\left(x - x_p + d/4\right)^2 - \left(x - x_p - d/4\right)^2}{2z} \sim \frac{\left(x - x_p\right)}{2z}$$

and, to first order in z:

$$\delta(\Delta\varphi) = \frac{\delta z}{z}\Delta\varphi$$

Since $\Delta\phi$ is on the order of 2π, the error is negligible for realistic δz values. Laser coherence is only required over a single roundtrip since differential phase measurements are independent. Phase distortions in the laser and local oscillator waveforms are common-mode as well, but such distortions must still need to be kept small, or recorded for maximum range resolution (not discussed in the present paper). Pointing stability requirements are similar to SAL.

The ideal, noise-free operation described above is degraded by detection noise, in particular inherent LO shot-noise. Since this noise is uncorrelated for the two differential measurements from which absolute phases are computed, an SNR-dependent stitching error results that accumulates along the synthetic aperture, in a random walk type manner. However, at optical wavelengths, unlike RF, the number of samples along a synthetic aperture length, $N_s \sim 4\lambda d/R^2$, is small, and the image distortion is modest.

We now present a more quantitative discussion of the DSAL concept. Equations 1 describe the received field in the two subapertures, for a Gaussian transmitter beam. Simulations have shown the conclusions derived from them to be generally correct.

Right subaperture:

$$Er_j = \sum_p a_p e^{\frac{-\left(x_j - x_p\right)^2}{{w_0}^2}} e^{\frac{-ik\left(x_j - x_p\right)^2}{2R}} e^{\frac{-ik\left(x_j + d/4 - x_p\right)^2}{2z}}$$

Left subaperture:

$$El_j = \sum_p a_p e^{\frac{-\left(x_j - x_p\right)^2}{{w_0}^2}} e^{\frac{-ik\left(x_j - x_p\right)^2}{2R}} e^{\frac{-ik\left(x_j - d/4 - x_p\right)^2}{2z}}$$

where $x_j = j*d/4$ and x_p are the positions of the aperture center and target point scatterers, respectively, z is the range, and w_0 and R(z) are the transmitter beam waist and radius of curvature at range z, respectively. It can be verified by inspection that, for R=z, $El_{j+1}=Er_j$, i.e. the field in the left subaperture for sample (j+1) equals the field in the right subaperture for sample j. Therefore, absolute phases can be obtained by adding these differentials, as illustrated in Fig. 2. The condition R=z means that the object must be in the far field of the transmitter, which is typically the case; simulations show that deviations up to approximately 10% are acceptable (intermediate field).

Platform Position	**Measured Phase Differential**	**Calculated Phase**
		$\phi 1=0$
I	$\phi 2-\phi 1$	$\phi 2$
II	$\phi 3-\phi 2$	$\phi 3$
III	$\phi 4-\phi 3$	$\phi 4$

Fig.2. Differential and absolute phases versus platform position

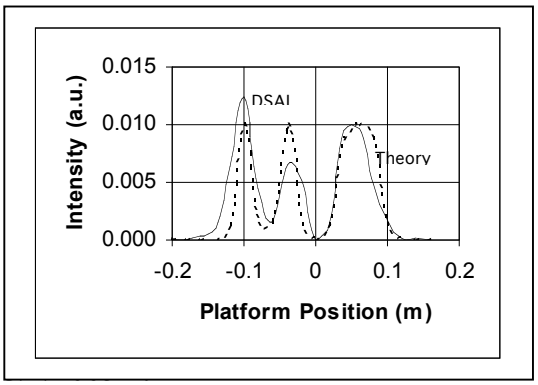
Images are constructed from the subaperture fields using Eq. 2, adapted from the conventional SAR theory [6]:

$$I_m = \left| \sum_j \left| Er_j \right| e^{-i\varphi r_j} G_{j+m} \right|^2$$

where:

$$G_j = e^{\frac{ikx_j^2}{2R}} e^{\frac{ik\left(x_j + d/4\right)^2}{2L}}$$

Figures 3-4 show representative wave-optics simulation results. Shot noise was added, for each phase measurement, using Poisson statistics for the received photon number. The associated phase error variance can be shown to be $\sigma^2(\delta\phi)=1/n_s$, where n_s is the number of photo-electrons. Figure 3 shows images for an ideal straight trajectory, for $n_s=8$; even for small photon numbers, SAL and DSAL provide similar performance. Figure 4 illustrates how DSAL operation is insensitive to LOS motion/vibration when SAL images are not feasible; a random phase according to a uniform probability density function was added at each platform position.



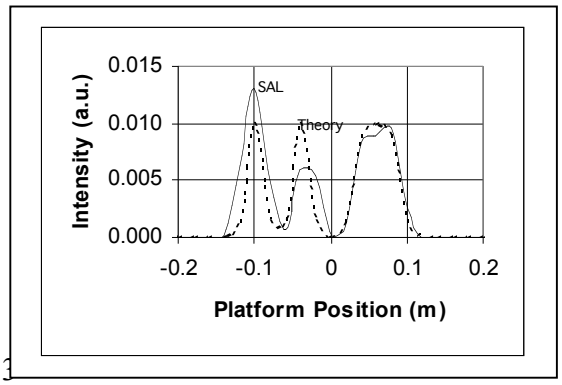


Fig.3. Reconstructed images with shot noise, n_s=8. The dotted lines are the ideal images. Left: DSAL, Right: SAL

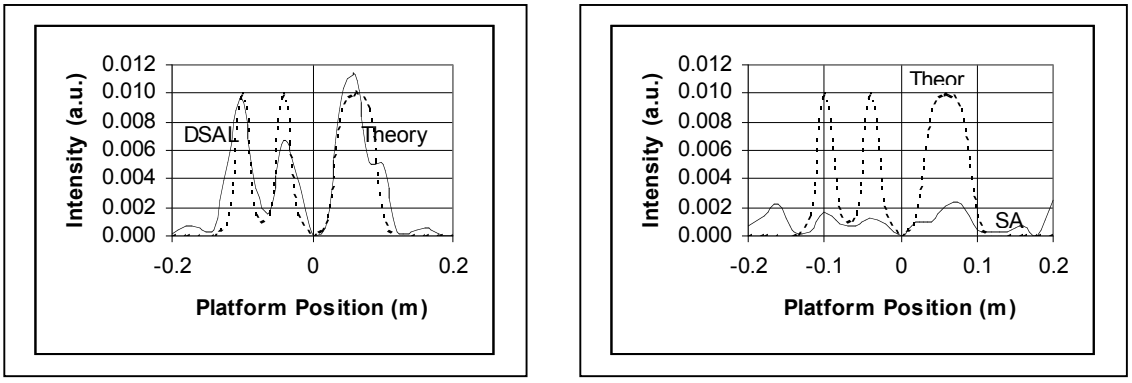


Fig.4. Reconstructed images with random vibration (λ/5) + shot noise, n_s=8. Left: DSAL, Right: SAL

Preliminary studies have been performed on the effect of speckle and atmospheric turbulence. They show that both effects are similar for DSAL and SAL. Figure 5 shows 3D simulation results for two targets, for two values of the Fried parameter, r_0 [7], for noise-free detection: SAL and DSAL results are indistinguishable. Significant image degradation is observed, for both configurations, for the smaller r_0 value.

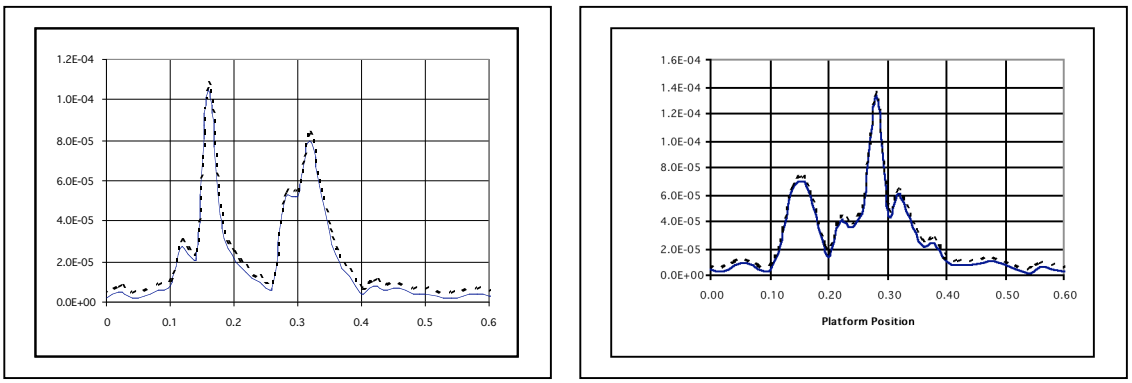


Fig.5. Imaging through turbulence: λ=2 μm, d = 8 cm, range = 13.3 km . Left: r_0(up) = 1 m, right: r_0(up) = 3.5 cm. A small vertical offset was added to the dashed SAL images.

In conclusion, analytical and modeling studies have been carried out on a differential synthetic aperture ladar concept. The concept greatly relaxes motion/vibration and laser spectral purity requirements, and is most beneficial for shorter wavelength, UV-to-MIR, imaging systems.

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