THE DYNAMIC APERTURE OF AN ELECTROSTATIC QUADRUPOLE
LATTICE*

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Abstract

In heavy-ion-driven inertial fusion accelerator concepts, dynamic aperture is important to the cost of the accelerator, most especially for designs which envision multibeam linacs, where extra clearance for each beam greatly enlarges the transverse scale of the machine. In many designs the low-energy end of such an accelerator uses electrostatic quadrupole focusing. The dynamic aperture of such a lattice has been investigated here for intense, space-charge-dominated ion beams using the 2-D transverse slice version of the 3-D particle-in-cell simulation code WARP. The representation of the focusing field used is a 3-D solution of the Laplace equation for the biased focusing elements, as opposed to previous calculations, which used a less-accurate multipole approximation. 80-85% radial filling of the aperture is found to be possible. Results from the simulations, as well as corroborating data from the High Current Experiment at LBNL, are presented.

INTRODUCTION

Electrostatic alternating-gradient focusing lattices have traditionally been used at the low-energy end in designs of the accelerator, or driver, for heavy-ion-driven inertial fusion. Electrostatic quadrupoles are attractive because of their low cost and compact longitudinal structure, which allows them to fit within the short lattice period at low energy. To reduce cost, especially in designs employing a multiple-beam linac, it is important to minimize the number of beams by filling as much of the transport system aperture as possible. In this paper the dynamic aperture of such a lattice is explored via computer simulation and experiment, and the results are compared.

The experimental measurements were done at the High Current Experiment (HCX) [1], an experiment of the Heavy Ion Fusion Virtual National Laboratory at LBNL. The HCX transports a coasting, driver-scale K⁺ beam through an alternating gradient FODO lattice consisting of 10 electrostatic quadrupoles, followed by 4 magnetic quadrupoles (not discussed here). Measurements described here were made for a 1 MeV beam, while the simulations used a 1.8 MeV beam, in order to maintain consistency with older results and to explore the parameter range expected in the driver. The current for the simulations has been scaled appropriately, so that for the same radial filling of the aperture in the simulation and experiment, particle orbits are the same for both, except for the effect of a small nonlinearity in the focusing field. This so-called “energy effect”—i.e., the effect of the increase in energy of the beam as it approaches the charged quadrupole electrodes—affects the simulation beam less than the experiment beam, since the change in energy is a smaller proportion of the total beam energy. This does not appear to change the results noticeably.

DYNAMIC APERTURE SIMULATIONS

Simulations were performed using the 2D (x-y) transverse version of the particle-in-cell code WARP [2]. A 3D solution of the Poisson equation done for the HCX quadrupoles (Fig. 1), including the supporting metal plates, was used to obtain the focusing fields. The quadrupole aperture is 2.3 cm. Unlike previous calculations [3,4], the full field (to a resolution of 0.49 mm in x and y and 1.7 mm in z) was applied to the particles, rather than a multipole approximation, which is inaccurate at large radius. Results were found to be very similar to the previous results using the multipole approximation. In the simulation the HCX lattice was extended to 50 lattice periods (and some longer runs of 100-300 periods were done) for increased sensitivity in detecting longer lengthscale effects. Image forces were calculated by the code using the structure of Fig. 1 and assuming all surfaces near the beam to be perfect conductors. Between quadrupoles the experimental aperture is larger than the simulation grid, and has no measurable effect on the beam. Therefore the simulation model used a square conducting box (9.86 cm wide) as a boundary condition there. A timestep corresponding to 80 steps per 0.4352 m lattice period was used in order to adequately resolve the focusing fringe fields.

The beam was initialized at the center of a drift space with a semigaussian distribution. The initial emittance was set to 5 times the thermal emittance of a 0.1 eV, 5 cm radius source for a beam with I=576 mA. It was then scaled with the square root of the current for other values.

of the current, simulating the effect of changing the source diameter for the same diode/injector. This scaling neglects differences in injector aberrations with beam size. Since ultimately the injector would be designed for the desired current, this seems to be a reasonable approximation. The emittance values used correspond to a space-charge-dominated beam. Care was taken to match the beam, and in all cases the radial oscillation was less than +/- 0.75%.

As in references [3] and [4], one can see from Fig. 2 that for $\sigma_0$ for above 70°, beam loss at a given filling factor rises with $\sigma_0$. This behavior appears to be related to the effect described in the paper at this conference by Lund et. al. [6], and to the experimental results of Tiefenback et. al. [7]. A series of runs was done for filling factor of approximately 80% and various values of $\sigma_0$ above 70°, and the results are shown by the filled diamond markers in Fig. 2. Emittance growth of a few percent in 50 lattice periods also occurs for these high values of $\sigma_0$. Though this growth is small, it is in notable contrast to the lack of growth at lower $\sigma_0$ (i.e., $\sigma_0$$\leq$70°). Because initial emittance was chosen to scale as the square root of the current, as filling factor is increased for a given $\sigma_0$, depressed phase advance decreases. The increase in the difference between the 70° and 80° curves in Fig. 2 as current increases could be correlated with this or simply with the increase in nonlinear forces.

Though all of these results pertain to a semigaussian initial distribution function, some runs have also been done with the multipole field approximation for a distribution reconstructed from experimental data, and the results are similar to those for the semigaussian[3].

**EXPERIMENTAL RESULTS**

The HCX, its layout and diagnostics, as well as detail about the experiments discussed here, are described in refs. [1] and [5]. We discuss here the comparison between the experimental data and the simulation results given above.

Two radial filling factors were investigated in the experiment, where the beam filled 60% and 80% of the radial aperture. For each fill factor the beam phase space was measured in detail both upstream and downstream of the electrostatic focusing section. Though the distribution function was not semigaussian as in the simulations, as mentioned above, simulations have shown similar results for a measured distribution and the semigaussian [3]. In the experiment the filling factor was increased by decreasing the quadrupole focusing strength instead of

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**Figure 1:** An HCX electrostatic quadrupole

A series of simulations was done for different beam currents for undepressed phase advance per lattice period, $\sigma_0$, of 50°, 60°, 70°, and 80°. For each $\sigma_0$ the beam current was increased until unacceptable beam behavior occurred, with emittance scaling as the beam radius. As in previous studies [3,4], which used the multipole approximation to the focusing field, the dynamic aperture was found to be determined by particle loss, rather than degradation of the transported beam. Emittance growth, except near $\sigma_0$=80° as discussed below, is negligible. Figure 2 shows the beam loss vs. radial filling factor (where “filling factor” is the ratio of beam major radius to quadrupole aperture and the aperture is defined as the radial distance to a quadrupole electrode surface) for various values of $\sigma_0$. The loss is essentially the same for all $\sigma_0$ less than or equal to 70°, with beam loss beginning at filling factors of about 80%. The rate of loss for high filling factor decreases with increasing $z$ until it becomes approximately constant, while the rate is approximately constant for filling factors $\leq$75%. The exact value of filling factor chosen for the aperture for an accelerator design would depend on the amount of allowable beam loss per meter and the length of the accelerator. But the results shown in this figure indicate that for a heavy ion fusion driver the choice of 80-85% fill factor and $\sigma_0$$\leq$70° for low energy transport would be reasonable. Given the practical considerations of a real accelerator with non-ideal beams, aperture would be added to this for beam halo, envelope mismatch, and centroid misalignment.

**Figure 2.** Beam loss in 50 focusing periods vs. radial filling factor for various values of $\sigma_0$. 

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adding current, but given the lack of dependence of the particle loss on \( \alpha \), in fig. 2, results at a given filling factor are directly comparable to the simulation results shown. \( \alpha \) for the two HCX cases was 69° and 48° for the 60% and 80% fill factor cases, respectively. The depressed phase advance per lattice period (\( \alpha \)) due to the self potential of the beam was 13° and 8° for the 60% and 80% fill factor cases. Within experimental uncertainty there is no evidence of a change in initial emittance of the two cases due to changes in the matching section used to produce the correct initial envelope for the focusing lattice.

Within the experimental sensitivity (10\% ± 10%), there was no evidence of emittance growth at the end of the electrostatic lattice for either the 60% or the 80% fill factor in either transverse dimension. The details of the beam phase-space distribution also remained practically unchanged, as shown in Fig. 4 for the horizontal dimension. In the length of the electrostatic transport section the beam loss was measured to be in the range of 0.2-1% (depending on the secondary electron coefficient assumed) at midpulse for both fill factor cases, as measured by the charge collected by the quadrupoles. This is consistent with the upstream vs downstream Faraday cup measurements, which gave beam loss of 1%±1%. Ionization of background gas, with the attendant expulsion of the ions from the gas by the space charge of the beam, can account for only 0.05% beam loss at the 2×10⁻⁶ Torr vacuum of the experiment.

The HCX is too short to determine the dynamic aperture, since the degree of beam loss that signals the limit, if the simulations are correct, can be measured only for a much longer lattice, of the order of at least 50 lattice periods. However one can look for consistency between the simulation and experimental results. In both, no beam degradation, including emittance growth, was seen for filling factors up to 80%, except for a small amount of beam loss. The 0.2-1% beam loss in the experiments is greater than that seen in the simulations (which would be negligible in this short lattice), but the fact that it was similar for the two fill factors indicates a likelihood that scrapeoff of an initial beam halo is responsible. This would not be seen in the simulations, which were done for an idealized distribution function. In the future, modeling could be initialized using a measured distribution function, in order to more closely examine beam loss, but this requires more upstream diagnostics, and would be of more benefit in a longer experiment.

### CONCLUSIONS

Simulations indicate that transport of an intense beam filling 80-85% of the radial aperture of an electrostatic quadrupole system is possible without significant beam degradation. The signature of the dynamic aperture limit is beam loss, which at 85% radial filling is a few percent in 50 lattice periods for undepressed phase advance less than 70°. Above 70°, particle loss increases with increasing \( \alpha \) (see Fig. 2), and near and above 80° some emittance growth is seen, which rises with increasing filling factor. These simulations were done for a focusing lattice occupancy of approximately 0.77, and more exploration will be done in the future to check the validity of the conclusions for a lower-occupancy lattice.

Experiments on the HCX were consistent with simulation in showing no emittance growth or significant phase space changes for filling factors up to 80%. However, 0.2-1% beam loss was seen for both 60 and 80% fill factors-- more than the simulation would predict, probably due to beam halo scrapeoff.

These results show that a relatively large filling factor is practical for space-charge-dominated beams in an electrostatic FODO lattice. This is important for heavy-ion-driven inertial fusion accelerators, especially those based on multiple-beam linacs, where increase in the filling factor has a large impact on cost.

### REFERENCES