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FINAL REPORT

VOLUME 77-5

EY-76-S-07-1601

I. Introduction

The final report under contract EY-76-S-07-1601 is being submitted as a series of volumes as follows:

Volume 77-0	*October, 1977
Volume 77-1	*September, 1977
Volume 77-2	December, 1977
Volume 77-3	December, 1977
Volume 77-4	December, 1977
Volume 77-5	December, 1977
Volume 77-6	December, 1977
Volume 77-7	January, 1978
Volume 77-8	January, 1978

*Submitted

Other volumes will be submitted in accordance with the contractual reporting schedule.

II. Reports Delivered

Task 76.1.21 Color photos

Stereo color aerial photographs were made available to industry for copying at industry's expense on November 28, 1977 at 11:00 a.m. This final report on this task merely records that fact.

Task 77.1.14 Proposal Review (MOD A002)

All activities under this task have been completed. No tangible deliverables were required.

Task 76.1.14 Simultaneous modeling of multiple data sets

A technique for simultaneous inversion of MT and Schlumberger data was developed and tested on some available deep crustal data from

South Africa. The publication resulting and included herewith is:

Petrick, W. R., W. H. Pelton, and S. H. Ward, 1977, Ridge Regression Inversion Applied to Crustal Resistivity Sounding Data from South Africa, *Geophysics*, vol. 42, no. 5, p. 995-1005.

Joint inversion of Schlumberger and electromagnetic sounding data was developed and applied to geothermal data from Roosevelt Hot Springs KGRA.

The publication resulting and included herewith is:

Tripp, A. C., S. H. Ward, W. R. Sill, C. M. Swift, Jr., W. R. Petrick, 1978, Electromagnetic and Schlumberger Resistivity Sounding in the Roosevelt Hot Springs KGRA, *Geophysics*, in press.

To utilize multiple inversion schemes at a convective hydrothermal system in the Eastern Great Basin will require applications of three-dimensional forward algorithms now available for gravity, magnetics, and AMT/MT. Unfortunately the pertinent data sets available for Roosevelt Hot Springs KGRA (or Monroe Hot Springs KGRA, for that matter) are not compatible since the physical property distributions giving rise to the gravity field is not coincident with that giving rise to the magnetic field, and so on.

Thus, we have turned to interpreting each data set independently and then drawing a schematic model of the subsurface which accepts all data sets. An example of this procedure is contained in "Ward, S. H., J. M. Bodell, W. D. Brumbaugh, J. A. Carter, K. L. Cook, T. J. Crebs, T. L. Olsen, W. T. Parry, W. R. Sill, R. B. Smith, I. Thangsuphanich, and A. C. Tripp, 1978, *Geophysics of the Roosevelt Hot Springs Thermal Area, Utah*", submitted to *Geology* and included in Final Report Volume 77-2.

Task 76.1.11 Drill and Log 10 Heat Flow Holes

All thermal gradients and heat flows appear in Technical Report 77-3. The summary of drilling progress, costs, and lithologs is enclosed.

Cost Analysis and Drilling Data
Thermal Gradient and Heat Flow Holes, 1976

by

J. A. Whelan

Cost Analysis and Drilling Data

Thermal Gradient and Heat Flow Holes, 1976

1.0 Introduction and Summary

1.1 This report summarizes the summer 1976 drilling program in the Roosevelt KGRA near Milford. Results of geologic, hydrothermal alteration, thermal gradient, heat flow, and rock properties made with cores or cuttings from these holes will be published in other reports.

Ten holes were drilled, totaling 2732 feet. Total funds expended, not including University of Utah supervision, or supplies, chiefly black-iron pipe, were \$29,320. Cost per foot for NX-core drilling, on holes was \$15.08. Cost per foot for rotary drilling in alluvium was \$4.98 per foot.

Locations of the holes are shown in Figures 1 through 5.

Summary data on individual holes is given in Appendix (A). Detailed logs are available at the University of Utah to interested parties. Cores and cuttings may be examined there.

2.0 Administration

2.1 Permission to Drill. On Federal Lands permission to drill was requested on Notice of Intent to Conduct Geothermal Resource Exploration Operations (Form 3200-9, December, 1973). On state lands, permission to drill was obtained from the Division of State Lands, Department of Natural Resources. If state lands were leased, permission was also obtained from the leasees. On private land, permission was obtained from the owners.

2.2 Two drilling contracts were utilized. General features are described below:

Figure 1. Location of Holes TGI, TGS, and Section 16

Figure 2. Location of Thermal Gradient Holes 2, 3, and 6

Figure 1. Location of Deep Flow Hole 1

Figure 5. Location of the Opal Dome Hole

2.2.1 Jensen Construction and Drilling Company. The basic features of this contract were as follows:

(a) Mobilization and moving were at \$1.50 per mile per unit (drill, water truck, pickup truck).

(b) Rig time, including the drill, a 1966 Hyarth, driller and helper, water truck and pickup truck was \$45.00 per hour.

(c) Water truck driver, when needed was furnished at \$70.00 per day.

(d) Bits were furnished at cost. Credit was given for diamond salvage.

(e) Core boxes and casing lost in the hole were paid for at cost.

(f) Other supplies - cement, drilling mud, casting plaster were charged for at cost plus 10 percent delivery charge.

2.2.2 The Wortley Engineering contract had the following features:

(a) No mobilization charge.

(b) Moves were at hourly "rig time" rate.

(c) Rig time was charged at \$24 per hour which included a driller and helper, pickup truck, a Joy 12 drill, an air compressor, and a water pump tank combination.

(d) Supplies were furnished at cost.

3.0 Procedure

3.1 Site Preparation. No holes required site preparation.

3.2 Surveying. All holes were tape and compass surveyed to the nearest land survey monument. Surveys are provided in hole data, Appendix (A).

3.3 Geologist. A geologist was present on the drill coring.

3.4 Hole completion. Thermal gradient holes and the alteration hole were completed by setting bottom capped one-inch black iron pipe in the hole, back filling with cuttings or sand to within 10 feet of the surface and from 10 feet to the surface with concrete. The heat flow holes were completed by setting one-inch black iron pipe with a check valve on the bottom. Water was circulated until good returns were obtained, followed by grout until it returned. Then a swabber, consisting of three tight fitting rubber stoppers on a threaded rod was forced into the check valve. After the grout or cement set, the black iron pipe was filled with water. All sites were raked or tilled and were seeded with 83 percent by weight crested wheat grass and 17 percent bitter brush between 15 September and December 1.

4.0 Costs

4.1 Costs for individual holes for Wortley Engineering are given in Appendix A. Costs for individual holes are not available for Jensen Drilling and Construction. Average costs for Jensen Drilling and Construction are given in Table 1 and for Wortley Engineering in Table 2.

TABLE 1 Average Costs, NX Core Drilling

Year	1975	1976	1976
Contract Agency	NSF	ERDA	USGS
Company	Boyles Brothers	Jensen	Jensen
Company Location	Salt Lake City, UT	Springville UT	Springville UT
Number of Holes	7	4	6
Total Footage	1727	1255	1589

Cost Per Foot

Mobilization	2.81	0.57	1.04
Move in, move out			
Rig Time	11.68	11.05	9.58
Bits (Net)	1.59	3.16	3.07
Supplies & Materials	1.50	0.30	0.51
Casing and Shoes lost in hole	<u>0.07</u>	<u>--</u>	<u>0.42</u>
	17.65	15.08	14.62

TABLE 2   Average Costs, Rotary Drilling, Wortley
Company, Summer, 1976

	Cost Per Foot
Rig Time	\$3.67
Diamond Bits	0.08
Rotary Bits	0.78
Surface Pipe	0.14
Supplies & Materials	<u>0.31</u>
Average Cost Per Foot	\$4.98

APPENDIX A - Descriptions of Holes

Thermal Gradient Hole 1

LOCATION: NW 1/4, SW 1/4, Sec. 15, T26S, R9W. (SLB & M)
994.8 Ft. S85°E from quarter corner 16/15.

LAND STATUS: Public Domain (BLM).

DATES OF DRILLING: July 12, 1976 - July 15, 1976

DEPTH: 185 Feet

COST ANALYSIS

	Total Cost \$	Cost Per Foot \$
Rig Time	614.00	3.32
Rotary Bits (2)	152.00	0.82
Supplies and Materials	<u>68.40</u>	<u>0.37</u>
Totals	834.40	4.51

SUMMARY OF DRILLERS LOG: 0-20 drilled and cemented surface, pipe.
0-205 feet. Drilled ahead of coring hole. Final
effective depth 185 feet.

SUMMARY OF LITHOLOGIC LOG: Alluvium. Poorly sorted subangular granite wash,
sand to gravel size. Contains streaks of magne-
tite.

WATER: None encountered

COMPLETION DATA: Set black iron pipe to 185 feet. Backfilled with cuttings
to 10 feet from the surface. Cemented top 10 feet. Site
tilled and reseeded with crested wheat grass and bitter
brush, 10:2 by weight.

Thermal Gradient Hole 2

LOCATION: SW 1/4 Sec. 5, T6S, R9W (SLB & M)
2469 Ft. East and 1108 Ft. North of $\frac{615}{718}$

LAND STATUS: Public Domain (BLM).

DEPTH: 232 Feet

COST ANALYSIS

	Total Cost \$	Cost Per Foot \$
Rig Time	870.00	3.75
Rotary Bits (2)	152.00	0.66
30-ft. of 2.5-inch casing @ \$1.50/foot	45.00	0.19
Supplies and Materials	<u>68.00</u>	<u>0.29</u>
Totals	1135.00	4.89

DRILLERS LOG: 0-15 feet. Drilled and cemented.
0-50 feet. Set casing to 30 feet. Hole coring.
50-125 feet. Drilling.
125-220 feet. Soft with small boulders.
220-232 feet. Set black iron pipe to 212 feet because of coring.

SUMMARY OF LITHOLOGIC LOG: Granite wash (cuttings from boulders), coarse sand with magnetite streaks.

WATER: None encountered.

COMPLETION DATA: Set capped black iron pipe to 212 feet. Backfilled with sand and cuttings to 10 feet from the surface. Cemented to surface. Site tilled and reseeded with crested wheat grass and bitter brush, 10:2 by weight.

Thermal Gradient Hole 3

LOCATION: SE 1/4 Sec. 19, T26S, R9W (SLB & M).

2670 feet N54°W from Sec. Corner $\frac{19}{30} \frac{20}{29}$

LAND STATUS: Private

DATES OF DRILLING: 24 June 1976 to 11 July 1976

COST ANALYSIS

	Total Cost \$	Cost per Foot \$
Rig Time	1194.00	3.67
Rotary Bits (3)	228.00	0.70
Surface Pipe 10'-3" iron	36.61	0.11
Supplies and Materials	<u>77.00</u>	<u>0.24</u>
Totals	1535.61	4.72

SUMMARY OF DRILLERS LOG: 0-10 feet. Set Casing. Cemented.
10-45 feet. Drill and cement each day.
45-132 feet. Drilled ahead with casing.

SUMMARY OF LITHOLOGIC LOG: 0-20 feet. Granite wash, gravelly sand.
20-45 feet. Silt
45-85 feet. Granite wash. Gravelly sand
85-95 feet. Gravel
95-325 feet. Granite wash sand.
Magnetite present through entire hole.

WATER: None encountered

COMPLETION DATA: Set capped black iron pipe to 305 feet. Backfilled with cuttings and sand to within 10 feet of the surface. Cemented top 10 feet. Cleaned and reseeded site with crested wheat grass and bitter brush (10:2 by weight).

Thermal Gradient Hole 5

LOCATION: SE 1/4, Sec. 14, T26S, R9W (SLB & M).
3047.4 feet N53E° of $\frac{S14}{S22}$ 1/4

LAND STATUS: Public Domain (BLM)

DATES DRILLED: 2 September 1976 - 7 September 1976

DEPTH: 170 Feet

COST ANALYSIS

	Total Cost \$	Cost Per Foot \$ (overall)
Rig Time	1158.00	6.81
Rotary Bits (3)	228.00	1.34*
Diamond Bits (1) 20' - 3 inch	100.00	0.53**
Casing	30.00	0.18
Supplies and Materials	<u>129.10</u>	<u>0.76</u>
Totals	1645.10	9.68

*150 feet drilled. \$1.52 per foot
**20 feet drilled. \$5.00 per foot.

SUMMARY OF DRILLERS LOG: 0-20 feet. Drilled. Set 20-foot casing.
20-150 feet. Rotary drilling. Lost circulation.
150-170 feet. Changed to Diamond Bit. Lost circulation.

SUMMARY OF LITHOLOGIC LOG: 0-150 feet. Granite wash with magnetite streaks.
150-170 feet. Fractured granite. Biotite altered to chlorite. Abundant iron and manganese staining.

WATER: None encountered

COMPLETION DATA: Ran capped PVC pipe to 162 feet. Backfilled to 10 feet from the surface. Cemented 10 feet to surface. Cleaned and re-seeded site.

Thermal Gradient Hole 6

LOCATION: SE 1/4, Sec. 7 T26S, R9W (SLB & M)
3710 feet N 54° W of

$$\begin{array}{r|l} 7 & 8 \\ \hline 18 & 17 \end{array}$$

LAND STATUS: Private.

DATES DRILLED: 20 August 1976 to 24 August 1976

DEPTH: 315 feet

COST ANALYSIS: Details not available. Total cost \$1065.45, or \$3.38 per foot.

SUMMARY OF DRILLERS LOG: Not available.

SUMMARY OF LITHOLOGIC LOG: Granite wash with streaks of magnetite.

WATER: None encountered

COMPLETION DATA: Set 305 feet of capped, one-inch PVC pipe in the hole. Back filled to 10 feet of surface. Cemented top 10 feet. Raked and reseeded drill site.

Thermal Gradient Sec. 16

LOCATION: NW 1/4, Sec. 16, T26S, R9W (SLB & M)

4483 ft. S41W from $\frac{9}{16} \frac{10}{15}$

LAND STATUS: State

DATES DRILLED: 2 September 1976 through 7 September 1976

DEPTH: 250 feet

COST ANALYSIS:

	Total Cost \$	Cost Per Foot \$
Rig Time	816.00	3.26
Rotary Bits (3)	228.00	0.91
Surface pipe 15'-3-inch	37.00	0.15
Supplies and Materials	<u>52.60</u>	<u>0.21</u>
Totals	1133.60	4.53

SUMMARY OF DRILLERS LOG: 0-15 feet. Drilled, set 3-inch steel casing, cemented.
15-55 feet. Drilling, hole coring.
55-250 feet. Drilling.

SUMMARY OF LITHOLOGIC LOG: 0-25 feet. Gravel of granitic composition.
25-185 feet. Granite wash with magnetite streaks.
185-215 feet. Quartz sand.
215-250 feet. Granite wash with magnetite streaks.

WATER: None encountered.

COMPLETION DATA: Ran 235 feet of capped one-inch black iron pipe. Backfilled to ten feet from surface, cemented ten feet to surface. Cleaned and reseeded site with 10:2 by weight crested wheat grass and bitter brush.

Alteration Hole 1-76

LOCATION: NE 1/4, SW 1/4, Sec. 34, T26S, R9W (SLB & M).
East 1926 feet and south 2756 feet from

28	27
33	34

LAND STATUS: Private

DATES DRILLED: 21 June 1976 - 25 June 1976

DEPTH: 201.8 feet.

COST ANALYSIS: Not available.

DRILLERS LOG: 0-10.8 feet. Rotary drilling.
10.8-201.8 feet. Core drilling.
Lost circulation at 150.8 feet, added cement.
Lost circulation at 188.0 feet, changed from water to bentonite
drilling mud.

CORE RECOVERY: 74 percent

SUMMARY OF LITHOLOGIC LOG: 0-10.8 feet. Granite wash
10.8-61.4 feet. Silica cemented granite wash
alluvium.
61.4-201.8 feet. Altered and fractured granite
containing some pyrite, chlorite, and sphene.

WATER: None encountered

COMPLETION DATA: Set capped one-inch black iron pipe to 200 feet. Backfilled
with cuttings to 10 feet from the surface. Site cleaned
and reseeded with 10:2 by weight crested wheat grass and
bitter brush.

Heat Flow Hole No. 1

LOCATION: NW 1/4, Sec. 8, T27S, R8W (SLB & M)
1620 feet East and 162 feet south of $\frac{6}{8}$

LAND STATUS: Public domain (BLM).

DATES DRILLED: 28 June 1976 - 15 July 1976; 2 August 1976 - 9 August 1976

DEPTH: 508.4 feet.

COST ANALYSIS: Not available

DRILLERS LOG: 0-101.7 feet. Rotary drilling.
101.7-508.4 feet. Coring.
Continuous sanding in hole. Lost circulation at 432.0 feet.
Wide crevice at 450 feet. Tried regaining circulation from
top and bottom. Failed. Cemented. Cased hole to 467.0 feet
with NX drill rods. Commenced drilling with BX rods. Extreme
difficulty in recovering NX rods.

LITHOLOGIC LOG: 0-101.7 feet. Muddy top soil.
101.7-316.5 feet. Altered granite, chlorite prominent.
316.5-318.0 feet. Basic dike.
316.5-508.4 feet. Intermittent aplite dikes in granite
containing biotite and chlorite. Iron, manganese, and chlorite
staining on fractures.

PERCENT CORE RECOVERY: 65 percent.

COMPLETION DATA: Set one-inch black iron pipe to 504 feet. Pumped cement
into pipe until surface returns were obtained and pushed
it out with a swabber. Cleaned site and reseeded site
with crested wheat grass and bitter bush (10:2 by weight).

Hole Number 3 Heat Flow

LOCATION: SE 1/4, Sec. 25, T26S, R9W
3432 ft. N79E from

26	25
35	36

LAND STATUS: Public Domain (BLM).

DATES DRILLED: 9 August 1976 - 27 August 1976

DEPTH: 489.3 feet.

COST ANALYSIS: Not available

DRILLERS LOG: 0-29.0 feet. Rotary drilling.
29.0-489.3 feet. Core drilling.

LITHOLOGIC LOG: 0-29.0 feet. Granite wash.
29.0-106.6 feet. Fractured granite with biotite and chlorite.
74.0-81.0 feet. Basic dike.
81.2-82.7 feet. Basic dike.
106.6-489.3 feet. Intermittent aphyre dike in granite containing biotite and chlorite. Some sphene, hematitic, manganese, and chloritic staining in fractures.

PERCENT CORE RECOVERY: 96

COMPLETION DATA: Set one-inch black iron pipe to 487 feet. Pumped cement into pipe until surface returns were obtained and pushed cement out of pipe with a swifter. Cleaned site and reseeded it with crested wheat grass and bitter bush (10:2 by weight).

Opal Dome Hole

LOCATION: NE 1/4 Sec. 16, R9W, T26S (SLB & M)
2548 feet S77W of

$\frac{9}{16} \frac{10}{15}$

LAND STATUS: State opal lease to A. & L. McDonald

DATES DRILLED: 9 August 1976

DEPTH: 55.2 feet

COST ANALYSIS: Not available

DRILLERS LOG: 0-4.7 feet. Rotary drilling.
4.7-55.2 feet. Core drilling.

LITHOLOGIC LOG: Coring began just below 5 feet. The first 23 feet consists dominantly of massive or banded opal with some clay interbeds, which become more abundant near the bottom of this section of the core. From 23 to 33 feet the core consists mainly of silicified sediment with minor opal layers. Below this to the bottom the core is made of cemented alluvium, either brown or light green, and varying considerably in its coherence.

COMPLETION: Abandoned.

FOR DETAILS SEE: Brown, F. H. (1977). Attempt at Paleomagnetic Dating of Opal, Roosevelt Hot Springs KGRA. Technical Report, vol. 77-1, Contract EY-76-S-07-1601, ERDA, 13 p.

ELECTROMAGNETIC AND SCHLUMBERGER RESISTIVITY SOUNDING
IN THE ROOSEVELT HOT SPRINGS KGRA

by

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ABSTRACT

One- and two-dimensional modeling of the Schlumberger soundings at the Roosevelt Hot Springs KGRA have indicated a low-resistivity zone of approximately 5 Ω -m paralleling the Dome Fault. The low resistivity of this zone is probably due to intensely fractured and altered rock. A zone of resistivity 12 Ω -m extending to the west of the Dome Fault is probably due to leakage of brine away from the geothermal system. A resistive basement underlies the conductive zones and is believed to be unaltered rock.

⊗ A major problem in the application of one-dimensional modeling of Schlumberger data in the Roosevelt Hot Springs KGRA is poor resolution of the one-dimensional model parameters. The joint inversion of Schlumberger and electromagnetic sounding data gives a least-squares one-dimensional conductivity model in which parameters are much better resolved than the model parameters estimated by the inversion of Schlumberger data alone.

One-dimensional modeling of Schlumberger soundings along a traverse does indicate the presence of a two-dimensional inhomogeneity but it gives no hint of the possible complexity of that inhomogeneity even though the parameters of the models fitting each sounding have acceptable standard deviations when constrained by electromagnetic sounding data. The model parameter standard deviations are a function of the partial derivatives of the model

values of apparent resistivity with respect to the one-dimensional model parameters evaluated at the model parameter values. Thus, good resolution of one-dimensional model parameters does not indicate that the assumption of a one-dimensional model is valid. On the other hand, the possible complexity of structure is brought out by two-dimensional modeling of the same data, but with no estimates of parameter reliability whatsoever. Since the degrees of freedom for complex two-dimensional models is large, a thorough study of the resolution of such models is prohibitively costly at present. In these circumstances, the best we can do is to constrain the two-dimensional models with independent geological or geophysical data and then take the subsequent best-fit model with an intuitive grain of salt.

INTRODUCTION

Since 1974, the University of Utah has done extensive geological, geochemical, and geophysical investigations of the Roosevelt Hot Springs KGRA, near Milford, Utah. A part of this investigation was the collection and interpretation of approximately 50 oscillating magnetic dipole soundings and 21 Schlumberger resistivity soundings. We shall report on these soundings here.

Investigation of geothermal areas via Schlumberger sounding has been extensive and worldwide (Breusse and Mathiez, 1956; Cheng, 1970; Duprat, 1970; Zohdy et al., 1973; Stanley et al., 1976). These investigations have invariably utilized one-dimensional modeling of individual soundings, with subsequent synthesis, to predict relevant geologic structure. Previous investigations of geothermal regions via artificial source electromagnetic methods include time-domain investigations by Keller (1970) and Stanley et al., (1976) and frequency-domain measurements by Keller (1970), Lumb and MacDonald (1970), and MacDonald and Muffler (1972).

The Schlumberger method was used in the Roosevelt Hot Springs KGRA for three reasons. First, finer resolution of the vertical geoelectric section was desired along traverse lines where extensive reconnaissance dipole-dipole sounding-profiling had been done. In particular, accurate estimates of the depth to crystalline bedrock were desirable, both as a counter-check of the gravity and

magnetic interpretation of the area (Crebs, 1976) and as a complement to the dipole-dipole interpretation (Ward and Sill, 1976). Second, the Roosevelt Hot Springs environment provided a field test of the utility of joint electromagnetic-resistivity one-dimensional least-squares inversion. Third, a test of the utility of the Schlumberger method in two-dimensional laterally inhomogeneous regions was desired. By use of a two-dimensional transmission surface analogy modeling program, a judgment of the merits of lumping one-dimensional interpretations to deduce a two-dimensional structure of dimensions comparable to those of the soundings was possible. Conversely, the relative effects of lateral conductivity inhomogeneities of the scale encountered at Roosevelt Hot Springs on one-dimensional interpretation schemes could be judged.

The electromagnetic method was used to detect shallow (depths to 100 meters) conductivity layering and to complement the resistivity data, as mentioned above.

The location of both Schlumberger and electromagnetic sounding sites was based on geological considerations and previous dipole-dipole resistivity surveys. Actual location of the Schlumberger and electromagnetic sounding-profiling is illustrated in Figure 1. Figure 1 also illustrates fractures previously delineated by several other geophysical and geological methods.

GEOLOGICAL SETTING

The Roosevelt Hot Springs KGRA lies to the immediate west of the Mineral Range. The major unit of the Mineral Range is a granitic pluton 32 km long and 8 km wide. This pluton is flanked by sedimentary units, basalt flows and Precambrian (?) metamorphics to the west. Rhyolitic volcanics, between 500,000 and 700,000 years old (Nash, 1976), are exposed in the central part of the range. North-south faulting paralleling the Mineral Range is extensive (Petersen, 1975; Crebs, 1976).

The survey area is mostly covered with alluvium of Tertiary and Quaternary age. Hot spring deposits occur along the north-south-Dome fault which is upthrown on the west and is thought to be a major structural control for the geothermal activity (Petersen, 1975). Core from shallow drill hole DDH 1A (Figure 1) and analysis of aeromagnetic data indicates that the alluvium to the west of the Dome Fault is underlain by Precambrian bedrock while bedrock is granite east of the fault (Ward and Sill, 1976). Extensive clay alteration has been found in drill-holes DDH 1A (Parry et al., 1976) and DDH 1B.

The major conductivity changes sought in the survey region occur at bedrock, at the water table, and at alteration zones and brine-filled fractures.

Figure 1. Map of previously interpreted fractures, showing the location of Schlumberger and electromagnetic soundings.

ELECTROMAGNETIC MEASUREMENTS AND INVERSION

The electromagnetic measurements at Roosevelt Hot Springs were made with the University of Utah's 14 frequency two-loop electromagnetic system. The system is designed to measure the tilt angle and ellipticity of the polarization ellipse of the magnetic field scattered by a conductive earth. The soundings utilized a transmitting coil of 10 m diameter, vertically suspended from an aluminum mast. The receiver used two tunable orthogonal ferrite-cored coils. The coils were supported by a vertical yoke, which enabled the operator to make the axes of the coils coplanar with the oscillating dipole and then align the coils with the major and minor axes of the polarization ellipse. Figure 2 illustrates the general transmitter-receiver configuration. A detailed description of the field technique and the system design is contained in Ward et al. (1974).

At the beginning of the field measurements for each traverse, it was necessary to establish the optimal transmitter-receiver separations. The transmitter-receiver separation was varied until low and high frequency asymptotes appeared in the tilt angle curves. The low-frequency asymptote, combined with ellipticity values very close to 0, indicated that the scattered field, to the resolving power of the polarization ellipse parameters, was zero. The high frequency asymptote indicated that the field was entirely reflected

Figure 2. Illustration of electromagnetic transmitter and receiver geometry.

α_T = TRANSMITTER ANGLE

H_T = TRANSMITTER HEIGHT

α_R = RECEIVER ANGLE

H_R = RECEIVER HEIGHT

ρ = TRANSMITTER-RECEIVER SEPARATION

at the air-earth interface. Thus the full resolving capabilities of the system could be utilized. This empirical technique gave transmitter-receiver separations which were optimal in the sense of Glenn and Ward (1976). In some cases, multiple separation soundings were performed to accentuate the resolution at depth.

Accurate interpretations of the soundings required that the effects of terrain be removed. Figure 2 illustrates the geometric parameters which must be taken into account in the terrain correction. H_T and H_R are the vertical elevations of the transmitting magnetic dipole and the receiving coils respectively while ρ is the distance between the transmitter and the receiver. These three parameters are measured directly. α_T is the angle between the transmitting dipole and the horizontal direction with respect to the "datum" while α_R is the angle between the receiver vertical axis and the vertical direction with respect to the datum. α_T and α_R are determined from a knowledge of the relative elevation of the transmitter base with respect to the receiver base. Since low frequency magnetic field tilt angles are solely dependent on the relative geometry of the transmitter and the receiver, theoretical low-frequency field tilt angles, computed using the measured values of H_T , H_R and ρ and the computed values of α_T and α_R , should agree with the measured low-frequency tilt angles. In practice, the average variance of these quantities is approximately 1° , which is significantly larger than any expected measurement error. Misorientation of the transmitting dipole with respect to the true horizontal direction is responsible for this discrepancy, since H_T ,

H_R , and ρ are measured precisely and since any orientation error of the receiver axis with respect to the vertical is eliminated by the measurement technique (Ward et al., 1974). Thus the angle α_T is adjusted to correct any low frequency mismatch between theoretical and observed tilt angles. Appendix A contains a short discussion of the actual computation of tilt angle and ellipticity when the transmitting dipole and the receiver axis are inclined with respect to a one-dimensional earth, whose surface in this case is the "datum".

After the low frequency tilt angle asymptotes were matched, the soundings were inverted to a layered geoelectric section using a generalized linear inversion routine, developed by Smith and Glenn and described by Glenn et al. (1973). Of course, the model earth surface is taken to be the "datum".

We have not made an a priori estimate of the data variance for ellipticity and tilt angle measurements. An empirical estimate of data variance is possible however. We have found that the electromagnetic soundings over approximately layered earths may be inverted to models which match the observed data to the extent illustrated in Figure 4. Assuming that the misfit in the curves of Figure 4 are indicative of the data noise, we will consider an inversion successful if the observed-theoretical data match is comparable, in a square residual sense, to those illustrated in Figure 4.

SCHLUMBERGER MEASUREMENTS AND INVERSION

The Schlumberger measurements were made using a non-coherent detector and a transmitting frequency of 0.1 hz. The received rectified voltage was averaged over at least 120 seconds, or 12 current cycles. A reading was made when the percent standard deviation of the time-averaged measurement indicated by the receiver was less than 2%. The transmitted current was electronically controlled by the transmitter to 3% of the specified current. Assuming that the distance $AB/2$ from the center of the Schlumberger expander to either current electrode and the distance MN between the potential electrodes both have 3% standard deviations, the percent standard deviation for the geometric factor $K = \pi[(AB/2)^2 / MN - MN/4]$ was found to be approximately 7% by the standard error propagation approximation (Bevington, 1969). Assuming these percent standard deviations for voltage drop, transmitted current, and geometric factor, and assuming that the time-averaged voltage deviation and the current deviation are uncorrelated, error propagation analysis gives a percent standard deviation for the apparent resistivity measurement of 8%.

Three soundings (Figure 1) centered at 4000N, 2700W; 3500N, 1000W; and 5950N, 2500W had maximum $AB/2$ values of 2000 meters. Soundings centered at 3500N, 200E; and 3500N, 00E had maximum $AB/2$ values of 1500 meters. The fifteen remaining soundings had

maximum $AB/2$ values of 1000 meters.

All soundings were inverted to plane-layered conductivity models using ridge regression, least-squares inversion, developed and described by Rijo et al. (1977). The starting model for each inversion was determined using a forward problem routine by manually varying model parameters until a close theoretical - observed data match was obtained. In all cases, discontinuities in apparent resistivity versus $AB/2$ introduced by expansion of the MN distance were left in the data during the inversion. However, in most cases, these discontinuities were minor.

GENERAL INVERSION PROCEDURE

In the inversion of the electromagnetic and the Schlumberger data to one-dimensional conductivity models, a fixed procedure was followed. If a data point was suspect in any of the sounding curves, it was excluded before the inversion. All data points retained for an inversion were given the weight 1. This assignment of weights is equivalent to the assumption that the variance of each observation is the same (Glenn et al., 1973). The final variance for the data was estimated via the reduced chi-square measure (Rijo et al., 1977; Glenn et al., 1973).

As a test of the stability of the inversion process, in numerous instances the inversion was begun using as initial guess a random model that did not fit the observed data at all well. In all cases, the model obtained by inversion was the same as that obtained from an initial model fitting the data very well, as long as the number of layers for the two models was the same. Thus, for the classes of models encountered in this study, there does not seem to be a problem with multiple minima in least squares residual space for either the Schlumberger or the electromagnetic soundings. Such stability also indicates that the programs which were used successfully dealt with convergence problems arising in non-linearity in the forward problem.

In considering the results of an inversion, three quantities

are taken into account. The reduced chi-square measure indicates whether a particular model is consistent with the data, the parameter variances give a measure of the range of possible parameter values, while the parameter correlation matrix estimates the interdependence of the parameter variances. Thus the reduced chi-square measure and the parameter variances are enlisted in judging the reliability of a particular best-fit model, while the correlation coefficient of two parameters, say X_1 and X_2 , indicates whether an independent estimate of one parameter will constrain the value of the other parameter. Note that an estimated correlation coefficient which is close to 1 in absolute value does not by itself indicate that only a product or quotient of X_1 and X_2 may be resolved. It is simply a measure of the tilt relative to the X_1 and X_2 axes and the ellipticity of the ellipse of standard deviation in the X_1 - X_2 plane for the two estimated parameters (Hamilton, 1964). Only when the standard deviations for X_1 and X_2 are large will the ellipse of standard deviation enclose an important area of the X_1 - X_2 plane.

The statistical estimates of parameter variance and parameter correlation assume that the forward problem theory is linear. Since this is only true locally, in the mathematical sense, the statistical estimates must only be used in a qualitative manner.

It was necessary to determine the number of conductivity layers to be included in the models. We believe the optimal number of layers to be the smallest number necessary to fit the data within the predetermined data variance. If the model has fewer layers, information contained in the data will be ignored. Adding

more layers does not significantly reduce the least-square residual, as judged by the F-test (Davis, 1973), while the added parameters are often unresolvable.

TWO-DIMENSIONAL MODELING

One of the Schlumberger soundings had a curve with a positive slope greater than 45° . The thorough interpretation of this sounding required a consideration of two-dimensional effects. It will be shown later that consideration of two-dimensional effects is quite important for an adequate interpretation of all the Schlumberger soundings on lines 3500N and 5950N (see Figure 1 for locations). For this reason, we utilized a two-dimensional transmission surface forward algorithm to model the soundings along lines 3500N and 5950N. This algorithm gives the apparent resistivities for various values of $AB/2$ and MN due to a previously specified two-dimensional model. Each change in the conductivity structure must be user-specified. We began with a model "compatible" with the one-dimensional resistivity and electromagnetic best-fit models. The position of lateral contacts between blocks of different resistivities was constrained to be compatible with the faults mapped by a gravity survey (Crebs, 1976) and by reconnaissance dipole-dipole resistivity (Ward and Sill, 1976). A trade-off between resolution of conductivity features and the ability to model a large area was necessary. Since we were not interested in shallow or localized effects, we modeled the resistivity soundings only for values of $AB/2$ greater than or equal to 50 meters. We made the vertical magnitude of the individual conductivity grid boxes greater than or equal to 15 meters.

Recall that a measure of model non-uniqueness for one-dimensional modeling is provided by the model parameter statistics and correlations calculated by the inversion program. Since the two-dimensional model program used did not give the derivatives of the data with respect to the model parameters, two-dimensional model parameter statistics are not given. Although the models were made as simple as was conducive to a good data fit, it is likely that model equivalence problems are present. Since the degrees of freedom for such models is large, a thorough study of the resolution of such models by utilizing the forward problem is prohibitively costly. In these circumstances, the best we can do is to constrain the modeling with independent geological or geophysical data whenever possible, and then take the subsequent best-fit model with an intuitive grain of salt.

MODELING RESULTS

Line 3500N

Electromagnetic soundings. - Figure 3 illustrates the least-squares models and model parameter standard deviations for the one-dimensional inversion of the electromagnetic data along line 3500N. Figure 4 illustrates the match obtained between theoretical and observed values of tilt angle and ellipticity for the soundings 1.2, 2.2, and 3.3 of Figure 3. These theoretical-observed data matches are typical of the matches obtained throughout the study.

The models in Figure 3 indicate a simple conductivity structure. West of 150W, the upper layer has a resistivity of approximately 100 Ω -m; this upper layer is underlain by a unit with a resistivity of approximately 10 Ω -m, which seems to surface at 50W. The interface between these two units dips approximately 10° to the west. To the east of 50W as far as 200E there are two conductive units. The surface unit has a resistivity in the range 35 Ω -m to 107 Ω -m. The lower unit has a resistivity of approximately 5 Ω -m. The interface between the two units has a fairly constant depth of, say, 25m.

Schlumberger soundings. - Figure 5 depicts models and model parameter standard deviations for the one-dimensional inversion of the Schlumberger data along line 3500N. During the inversions, all model parameter values were unconstrained. The parameter standard deviations in Figure 5 reveal that the resistivities and thicknesses

Figure 3. Least-squares one-dimensional models and subsequent percent parameter standard deviations for electromagnetic soundings along line 3500N. The values in the layers are layer resistivities in ohm-meters and percent standard deviations. The value beneath each interface is the percent standard deviation for the thickness of the layer above. The vertical exaggeration is 4:1.

Figure 4. Fit for three electromagnetic soundings obtained from the one-dimensional models illustrated in Figure 3. Dotted circles or triangles are data points while the solid lines are the theoretical curves.

Figure 5. Least-squares one-dimensional models and subsequent parameter standard deviations for Schlumberger soundings along line 3500N. The vertical exaggeration is 4:1. Superficial top layers are not depicted.

of the layers overlying the most conductive layer are in general well-resolved. The resistivities and thicknesses of the most conductive layer range from well-resolved to unresolved. The parameters of the resistive basement are mostly unresolved. Figure 6 depicts the small residuals obtained between theoretical and observed values of apparent resistivity for the soundings 500W, 200W, and 200E, which are representative of the residuals obtained for all of the soundings throughout this study.

Figure 7 depicts the one-dimensional models obtained from the inversion of the Schlumberger soundings when the basement resistivity is constrained to the value $300 \Omega\text{-m}$ everywhere. The inversions with this constraint were done to facilitate comparison of one-dimensional and two-dimensional interpretations of the Schlumberger soundings. As is evident from Figures 5 and 7 the estimated parameters for the inversions with the basement resistivity constrained are comparable to the estimated parameters with the basement resistivity unconstrained for the four soundings east of 300W. For the sounding at 500W, the "constrained" basement depth is shallower than the "unconstrained" depth.

Two-Dimensional Modeling. - Heretofore, the interpretation of the Schlumberger soundings along line 3500N has assumed that the earth sampled by the soundings is locally one-dimensional. However, the marked lateral variation in the conductivity-thickness product of the conductive layer across the Dome Fault shown in Figure 5 is indicative of lateral conductivity inhomogeneity. Lateral inhomogeneity is also indicated by the sounding centered at 1000W, which

Figure 6. Fit for three Schlumberger soundings obtained from the one-dimensional models illustrated in Figure 5. Dotted circles are data points while the solid lines are the theoretical curves.

Figure 7. Least-squares one-dimensional models and subsequent parameter standard deviations for Schlumberger soundings along line 3500N, obtained when basement resistivity is constrained to 300 Ω -m everywhere. The vertical exaggeration is 4:1. Superficial top layers are not depicted.

has a greater than 45° ascending slope for $AB/2$ values greater than 1000 meters (Figure 8). Since reconnaissance dipole-dipole resistivity sounding-profiling in the area of line 3500N indicates a two-dimensional conductivity structure roughly paralleling the Dome Fault (Ward and Sill, 1976), the use of the two-dimensional resistivity forward algorithm to test the conjectured one-dimensional interpretation is justified. Note that the model parameter standard deviations are functions of the partial derivatives of the model values of apparent resistivity with respect to the one-dimensional model parameters evaluated at the model parameter values. Thus, good resolution of one-dimensional model parameters does not indicate that the assumption of a one-dimensional model is valid.

For the two-dimensional modeling, we fixed the Dome Fault at 0, a location compatible with the one-dimensional interpretations of both Schlumberger and electromagnetic data. East of 500E, conductivities of the blocks overlying the basement were chosen to be roughly compatible with a dipole-dipole profile which utilized 100 meter dipoles. This was necessary because of the absence of Schlumberger sounding centers east of 200E. The resistivity of the basement was constrained to $300 \Omega\text{-m}$. This resistivity value is compatible with deep dipole-dipole sounding and with laboratory measurements on Precambrian gneiss saturated with brine of the same salinity as brine sampled in the Roosevelt Hot Springs area.

The two-dimensional conductivity model for line 3500N is illustrated in Figure 9. As shown, in Figure 10, the match between the observed and the model apparent resistivity values is good for

Figure 8. Data curve for Schlumberger sounding centered at 3500N, 1000W. The break in the curve at $AB/2 = 1000$ corresponds with the crossing of the Dome Fault by the east current electrode.

Figure 9. Two-dimensional resistivity model for Schlumberger soundings along line 3500N. Block resistivities are in ohm-meters. The vertical exaggeration is 4:1.

Figure 10. Fit for Schlumberger soundings obtained from the two-dimensional model illustrated in Figure 9. Dotted circles and solid circles are data points for $MN=4$ meters and $MN=80$ meters respectively, when a sizable variance occurs, while the solid lines are the theoretical curves. The dotted line for the sounding curve at 500W is the theoretical curve obtained by removing the thin 5 Ω -m strip from the model of Figure 9.

each Schlumberger sounding.

As mentioned above, the two-dimensional modeling program used does not give an estimate of model parameter standard deviations. A poor alternative is varying the model parameters and comparing the various results. Since this is clearly a quixotic undertaking, we will confine ourselves to comments on a few of the more problematic aspects of the model presented.

The several small features of the model between 200E and 300W and shallower than 25 meters were necessary to model the data. However, since the two-dimensional modeling mesh was coarse and since } the data for AB/2 values smaller than 50 meters was not modeled, these small features represent only a rough approximation to the actual conductivity structure. The narrow strip of 5 Ω -m conductivity located at 500W and running from a depth of 50 meters to a depth of 350 meters seems to be necessary to fit the sounding curve centered at 500W. Figure 10 shows the observed-theoretical data fit obtained by removing the 5 Ω -m strip. As is evident, the observed-theoretical data fit is markedly worsened.

The two-dimensional estimates of depth to the highly conductive zones and the resistivities of these zones are in close agreement with the one-dimensional estimates, depicted in Figure 7. However, the two-dimensional estimate of the depth to the resistivity basement is at variance with the one-dimensional estimate by as much as 200 meters.

The north-south sounding centered at 4000N, 1700W supports the two-dimensional interpretation; it gave a depth to the conductive

zone of approximately 120 meters, a value extremely close to the two-dimensional estimate.

Figure 11 illustrates the two-dimensional modeling for 300m and 1 km dipole-dipole data taken along line 4000N (Ward and Sill, 1976). The observed-theoretical data match is shown in Figure 12. The dipole-dipole two-dimensional modeling indicates a shallow zone of high conductivity between 900E and 900W. This zone, of 5-20 Ω -m resistivity, corresponds to the zone of 5-12 Ω -m shown by the Schlumberger two-dimensional modeling (compare Figures 9 and 11). The depth to the 300 Ω -m zone indicated by the dipole-dipole modeling, which is not shown in the figure, is 600m, which is much greater than the depth indicated by the Schlumberger modeling. Although this difference in the two-dimensional models may be due in part to real variances in the conductivity structure between lines 4000N and 3500N, it is likely that lack of model uniqueness of the dipole-dipole or the Schlumberger models is also responsible.

Line 5950N

Electromagnetic soundings. - Figure 13 depicts the least-square residual models and model parameter standard deviations for the one-dimensional inversion of the electromagnetic data along line 5950N.

The soundings to the east of 600W were modeled exclusively by two-layer models. The two soundings west of 600W could not be fitted by two-layer models, while the three-layer models gave small theoretical-observed data residuals and small model parameter standard deviations.

Figure 11. Two-dimensional resistivity model for dipole-dipole sounding-profiling data along line 4000N (after Ward and Sill, 1976). The vertical exaggeration is 4:1.

Figure 12. Theoretical-observed data fit obtained for the model of Figure 11. Solid lines are observed data contours while the dotted lines are theoretical contours.

Figure 13. Best fit one-dimensional models and subsequent percent parameter standard deviations for electromagnetic soundings along line 5950N. The vertical exaggeration is 4:1.

On line 5950N, the electromagnetic method was able to resolve the depth and the resistivity of the lower conductive half-space even through 80 meters of fairly conductive overburden, as in the case of sounding 8.2. The estimated depth to the lower conductive half-space seems to have a definite trend.

Schlumberger soundings. - Figure 14 presents the least-squares models obtained by the one-dimensional inversion of Schlumberger resistivity data taken along line 5950N. As is apparent from the figure, the soundings centered at 800W and at 200W do not resolve the highly conductive layer. None of the soundings resolves the resistivity of the basement.

Two-Dimensional Modeling. - It may be seen from Figure 14 that the one-dimensional model parameters seem to be self-consistent in indicating a surface layer of 50-120 Ω -m, a conductive second layer of 5-10 Ω -m, and a resistive basement. The first two layers are totally consistent with the model derived from the electromagnetic soundings. The large variation in the basement resistivities could be an effect of noise in measurements. However, since dipole-dipole resistivity reconnaissance surveying (Ward and Sill, 1976) indicates a two-dimensional structure paralleling the mapped fault in the area (Figure 1), two-dimensional conductivity modeling is justified.

The final two-dimensional model is shown in Figure 15. Figure 16 illustrates the agreement obtained between the sounding data and the two-dimensional model data. Since the soundings exhibit large discontinuities in observed apparent resistivity when the potential

Figure 14. Best fit one-dimensional models and parameter standard deviations for Schlumberger soundings along line 5950N. The vertical exaggeration is 4:1. Superficial top layers are not depicted.

Figure 15. Two-dimensional resistivity model for Schlumberger soundings along line 5950N. Block resistivities are in ohm-meters. The vertical exaggeration is 4:1.

Figure 16. Fit for Schlumberger soundings obtained from the two-dimensional model illustrated in Figure 15. Change in ρ_a due to expansion of MN distance is modeled. Dotted circles and solid circles are data points for MN=4 meters and MN=80 meters respectively.

electrode distance MN is expanded, the theoretical discontinuities due to the two-dimensional model were calculated for comparison. In three of the five soundings the theoretical and the observed discontinuities match exactly, as is shown in Figure 16. This suggests that the "clutch" can contain useful information and should be retained during modeling.

A comparison of Figure 14 and 15 reveals that the one-dimensional models provide a good first approximation of the two-dimensional model. The estimated depth to basement and the estimated resistivity of the very conductive units are in reasonable agreement. The two-dimensional model retains the shallow 20 to 50 Ω -m unit introduced by the one-dimensional inversion of the sounding at 800W. This unit has a counterpart in the third layer introduced by the electromagnetic modeling west of 600W.

SIMULTANEOUS ELECTROMAGNETIC - SCHLUMBERGER INVERSION

As has been observed throughout the study, a number of the one-dimensional Schlumberger models have had large percent standard deviations for the thickness and resistivity of the most conductive layer. Finer resolution of these model parameters utilizing the Schlumberger method is limited by the accuracy of the measurements, the finite number of data points, and the limited $AB/2$ distance used. Thus an independent source of model information is necessary to achieve finer resolution of conductivity model parameters.

It was noted during the study that in locations having both electromagnetic and Schlumberger soundings, the conductive layer resistivities for the electromagnetic sounding models were comparable to those of the Schlumberger models, but had smaller standard deviations. Since the conductive layer resistivity is usually highly correlated with the conductive layer thickness in the Schlumberger inversion, simultaneous inversion of the electromagnetic and Schlumberger data should resolve both parameters. In order to test this hypothesis, an existing simultaneous one-dimensional electromagnetic dipole-dipole resistivity least-squares inversion program was modified for the Schlumberger configuration.

At this juncture, it would be well to discuss briefly the actual joint inversion procedure used. It was found in the Roosevelt Hot Springs KGRA that the one-dimensional models for the Schlumberger

and electromagnetic soundings were often dissimilar for the layers above the most conductive zone, even though the depth and conductivity of this zone were similar. This diversity between models is to be expected, because of the difference in the scale of exploration of the two methods. Since such diversity is not germane to the problem at hand, the high frequency electromagnetic data was not used in the inversion. The choice of cutoff frequency was subjectively determined by inspection of the derivative matrix, evaluated at the electromagnetic model parameters. Suppose that the partial derivatives of ellipticity and tilt angle, with respect to the model parameters of the deep or interesting layers, are small at one frequency relative to the partial derivatives at other frequencies. Then discarding the data points at the first frequency will not greatly affect the resolvability of the interesting layers. An alternative approach would be to discard resistivity measurements for small $AB/2$ values. Note that a trade-off between the amount of useful information and contradictory information included in the inversion process is inescapable.

Once the data to be included in the inversion is specified, it is necessary to choose an appropriate scale for the data. For example, we must decide whether we wish to minimize the square residual of apparent resistivity or the square residual of log apparent resistivity. It is also necessary to decide what data weighting matrix is appropriate for the problem. Finally, it is necessary to decide upon the model parameterization used during the inversion.

The weighted data difference vector, $\Delta \mathbf{G}$, may be written as:

$$\Delta \mathbf{G} = \mathbf{W} \begin{pmatrix} \mathbf{G}_s \\ \mathbf{G}_e \\ \mathbf{G}_t \end{pmatrix}$$

where $\Delta \mathbf{G}_s$, $\Delta \mathbf{G}_e$, and $\Delta \mathbf{G}_t$ are the observed-theoretical data difference vectors for the Schlumberger apparent resistivity data, the ellipticity data, and the tilt angle data respectively, and $\mathbf{W}$ is the data weighting matrix. Since the apparent resistivity data has a large range of values, $\Delta \mathbf{G}_s$ is taken to be the column vector of the log apparent resistivity residuals $(\ln(\rho_a)_T - \ln(\rho_a)_O)$ where $(\rho_a)_T$ is the theoretical apparent resistivity and $(\rho_a)_O$ is the observed apparent resistivity. The elements of the ellipticity and tilt angle residual vectors, $\Delta \mathbf{G}_e$ and $\Delta \mathbf{G}_t$, will be in decimal fractions and radians respectively.

We take the weighting matrix $\mathbf{W}$ to be the diagonal matrix whose terms are the values necessary to scale the standard deviations for the ellipticity and tilt angle data to the same magnitude as the standard deviation of the log apparent resistivity data. This weighting scheme enables the inversion algorithm to fit each data point to within one standard deviation. This weighting is appropriate in our case since care is taken to assure that the data included in each inversion is self-consistent. For simplicity, we assumed that the standard deviation of the ellipticity data was approximately 1%, while the standard deviation of the tilt angle data was taken to be .5°. We believe these values to be reasonable.

The plane-layered conductivity model was parameterized in terms of log conductivities and log layer thicknesses to avoid the occurrence of negative layer conductivities and thicknesses. The estimated model parameter covariance matrix $\text{cov}(\mathbf{P})$ is:

$$\text{cov}(\mathbf{P}) = \chi_v^2 (\mathbf{A}^T \mathbf{W}^T \mathbf{W} \mathbf{A})^{-1}$$

where $\mathbf{A}$ is the system derivative matrix, v is the arithmetic difference between the number of observations and the number of model parameters, and

$$\chi_v^2 = \frac{\Delta \mathbf{G}^T \Delta \mathbf{G}}{v}$$

As seen in Figure 5, the values of ρ and t of the conductive layer for the Schlumberger sounding centered at 3500N, 200E are unresolved. However, the conductive layer resistivity of the least-squares model for the magnetic dipole sounding 1.1 is well-resolved, as is shown in Figure 3. Thus, this electromagnetic sounding and Schlumberger sounding are well adapted to joint one-dimensional inversion.

The electromagnetic data values for frequencies greater than 2 KHz were discarded for the purposes of the joint inversion because the data points did not markedly contribute to the resolution of the conductive zone resistivity.

The least-squares model, the model parameter standard deviations, and the theoretical-observed data residual for the joint inversion are shown in Figure 17. The percent standard deviations for ρ_3 and t_3 are greatly reduced, as was predicted. This example demon-

Figure 17. A) Best fit one-dimensional model and parameter standard deviations for the joint inversion of the electromagnetic and Schlumberger soundings at 3500N, 200E. B) Fit between theoretical ellipticity and tilt angles (solid lines) and observed ellipticity (dotted circles) and observed tilt angles (dotted triangles). C) Fit between theoretical ρ_a (solid lines) and observed ρ_a (dotted circles).

A)

$\rho_1 = 151 \Omega\text{-m}(128,178)$	$t_1 = 7.4 \text{ m}(5.8, 9.4)$
$\rho_2 = 39 \Omega\text{-m}(32,47)$	$t_2 = 36 \text{ m}(32,41)$
$\rho_3 = 2.4 \Omega\text{-m}(1.8,3.2)$	$t_3 = 46 \text{ m}(35,61)$
$\rho_4 = 73 \Omega\text{-m}(52,103)$	

B)

C)

strates that electromagnetic data does contain useful information not contained in the Schlumberger data. In the present case, this information has been used to gain confidence in the model advanced by the Schlumberger method.

GEOLOGICAL INTERPRETATION

We will now present interpretations of the two-dimensional conductivity structures advanced for line 3500N, depicted in Figure 9, and the two-dimensional conductivity structure advanced for line 5950N, depicted in Figure 15.

Rocks which do not contain massive amounts of semi-conducting minerals owe their conductivity almost solely to interstitial water and to water-saturated clay minerals. Thus we would expect a radical decrease in model resistivity for moderate depths to correspond to the water table wherever the water is saline. Thus, on lines 3500N and 5950N the interface between the shallow units with resistivity of order 30 Ω -m to 250 Ω -m and the conductive units of resistivity between 1 Ω -m and 12 Ω -m corresponds to the water table. Unfortunately, the water table was not noted during the drilling of DDH 1B, nor does the core give geochemical evidence of the depth to water table (Parry, pers. comm.). However, extrapolation of the depth to water table noted at DDH 1A (Parry et al., 1976) gives a depth to water table on line 3500N of approximately 40 meters. This is in good agreement with the depth predicted by the modeling on line 3500N at the site of DDH 1B (see Figure 9).

The two-dimensional modeling resolved zones of anomalously low resistivity within the main water-saturated zone. On line 3500N, the zone of resistivity 5 Ω -m is fairly localized between the Dome

Fault and Fault 1 (Figure 9). On line 5950N, the anomalous zone with a resistivity of 1 Ω -m, situated at 0, is associated with a previously mapped fracture (Figure 15). The possible and by no means mutually exclusive causes of these conductivity anomalies are increased fracturing and increased clay alteration. Ward and Sill (1976) showed that the presence of clay in typical amounts at the Roosevelt Hot Springs KGRA can lower the resistivity by a factor of four. Since the clay is undoubtedly associated with the fracturing, discrimination between clay conduction effects and fracture conduction effects seems unlikely.

The 12 Ω -m zone west of 500W and the 5 Ω -m zone west of 200W on line 3500N (Figure 9) are certainly water-saturated units. These low resistivity zones are compatible with the conclusion of Ward and Sill (1976) that brine is leaking from the vicinity of the Dome Fault westward and is saturating alluvium or altered bedrock.

If the saturated units are wholly alluvium, we would expect the bottom of the units to correspond with the gravity basement. While this correspondence is true on line 5950N (Figure 15) it does not appear to be true on line 3500N (Figure 9). On line 3500N, west of 400E, the gravity basement, extrapolated from the traverse along line 4000N (Crebs, 1976), is much shallower than the resistivity basement. There are several possible explanations of such a variance. It is likely that the depth to basement estimated by the two-dimensional Schlumberger method has a large associated standard deviation. If the depth to basement estimated by gravity lies within a standard deviation of the resistivity estimate, the

variance between the two estimates would not be statistically significant. Unfortunately, this hypothesis cannot be tested at present. The gravity modeling for line 4000N assumed that the density contrast between the alluvium and the bedrock is .5 gm/cc. This contrast is substantiated by Crebs' density profile measurements on dry alluvium. If we assume that the alluvial constituent material has a density of 2.67 gm/cc, Crebs' estimate of dry alluvium density gives an alluvial porosity of 20%. Saturating the alluvium thus reduced the density contrast between alluvium and bedrock to .3 gm/cc. Talwani gravity modeling indicates that between 500W and 1500W, the estimated depth to bedrock for a gravity model containing a saturated alluvium unit will be 40m to 120m greater than the depth estimated by Crebs.

We believe that part of the discrepancy in the gravity and resistivity depth estimates may also be due to high density, low resistivity altered rock. Densities for dried samples of the low resistivity altered core from DDH #1A range from 1.80 gm/cc to 2.48 gm/cc. The estimated wet density of these samples ranges from 2.1 gm/cc to 2.63 gm/cc. These values suggest that high density, low resistivity altered rock is present at Roosevelt Hot Springs.

Available heat-flow and geochemical data at Roosevelt Hot Springs support the notion that the depth of the resistive basement corresponds with the maximum depth of alteration. The K-feldspar stable temperature for values of pH and K^+ activity measured at Roosevelt Hot Springs (Parry et al., 1976) is approximately 150°C. Assuming a thermal gradient of 460°C/km measured in DDH 1B (Sill,

pers. comm.), this temperature is reached at a depth of 325 meters. This is in good agreement with the model depth to resistive basement found for line 3500N west of 500W (Figure 9). It is also in good agreement with the depth to resistive basement for the zone between the Dome Fault and Fault 1 on line 3500N. Assuming a thermal gradient of $800^{\circ}\text{C}/\text{km}$ measured in drill hole DDH 1976, #1 (Sill, pers. comm.), the 150°C transition temperature is reached at a depth of 186 meters. This agrees well with the depth to resistive basement predicted due north from DDH 1976 #1, on line 5950N.

The final geological interpretations of the two-dimensional modeling along lines 3500N and 5950N are illustrated in Figures 18 and 19 respectively.

Figure 18. Geological interpretation for line 3500N.

Figure 19. Geological interpretation for line 5950N.

CONCLUSIONS

Schlumberger and electromagnetic soundings have detected a large conductive zone which parallels the Dome Fault, thought to be a major structural control for the geothermal system at the Roosevelt Hot Springs, KGRA. This zone consists, we believe, of extensively fractured and altered rock. An extensive conductive zone to the west of the Dome Fault seems to indicate that brine is leaking from the geothermal system. This zone bottoms in a high resistivity unit, believed to be Precambrian (?) basement.

We have found that two-dimensional Schlumberger modeling is essential for adequate modeling of the geothermal system at Roosevelt Hot Springs; one-dimensional inverse modeling only provides a starting point for the two-dimensional forward modeling. The two-dimensional models appear to satisfy all of the demands placed on them by geological, heat flow, and gravity data. However, a method of estimating the uniqueness of the two-dimensional models advanced is needed. We have also found that the joint one-dimensional inversion of Schlumberger and electromagnetic data is superior to the inversion of Schlumberger data alone in resolving the vertical geoelectric section at Roosevelt Hot Springs at any single sounding site.

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APPENDIX A

We have mentioned that in the presence of terrain the transmitting magnetic dipole is rarely perfectly horizontal or vertical with respect to the "datum", as illustrated in Figure 2. Likewise, the receiver axis is rarely vertical with respect to the datum. Thus an algorithm is needed for computing the ellipticity and tilt angle of the magnetic field polarization ellipse which would be measured when the transmitter and receiver are inclined with respect to the datum. The presentation follows Thomas (1975).

We assume that the angle α_T between the transmitting dipole and the X-axis, as shown in Figure 2, is measured anticlockwise from the Ox direction. When the receiver is uphill from the transmitter, the receiver angle α_R is measured clockwise from the Oz direction. When the receiver is downhill from the transmitter, α_R is measured anticlockwise from the Oz direction.

Suppose that the magnetic field response of a one-dimensional earth to a vertical magnetic dipole of unit dipole moment has the components H_Z^V and H_X^V , while the response to a horizontal magnetic dipole of unit moment has the components H_Z^h and H_X^h . Now the field components H_Z^i and H_X^i for a unit moment dipole inclined an angle α_T with respect to the x axis may be computed by superposition of the fields due to a vertical dipole of moment $\sin \alpha_T$ and a horizontal dipole of moment $\cos \alpha_T$. Assuming the angle sign conventions

advanced above, we find the equations:

$$H'_Z = (\cos \alpha_T) H_Z^h + (\sin \alpha_T) H_Z^v \quad (1)$$

$$H'_X = (\cos \alpha_T) H_X^h + (\sin \alpha_T) H_X^v \quad (2)$$

Since the receiver axis is inclined to the datum by an angle α_R , the z and x field components referred to the coordinate system of the receiver, H''_Z and H''_X , are related to the components H'_Z and H'_X by the equations

$$H''_Z = (\cos \alpha_R) H'_Z + (\sin \alpha_R) H'_X \quad (3)$$

$$H''_X = (\cos \alpha_R) H'_X - (\sin \alpha_R) H'_Z \quad (4)$$

The magnetic field produced by an inclined transmitter of unit moment and measured by a receiver whose axis is tilted with respect to the datum may be calculated by means of equations (1) through (4). Since the ellipticity and tilt angle of the scattered field are independent of the oscillating dipole moment, the theoretical ellipticity ϵ and tilt angle α may be determined by substituting the values of H''_Z and H''_X in polar form, $H''_Z = H_Z e^{i\phi_Z}$, $H''_X = H_X e^{i\phi_X}$, into the equations (Smith and Ward, 1974):

$$\tan 2\alpha = \frac{2(H_Z/H_X) \cos \Delta \phi}{1 - (H_Z/H_X)^2} \quad (5)$$

and

$$\epsilon = \frac{H_Z H_X \sin \Delta \phi}{[H_Z e^{i\Delta \phi} \sin \alpha + H_X \cos \alpha]^2} \quad (6)$$

where $\Delta\phi = \phi_z - \phi_x$. The values of ellipticity and tilt angle calculated from equations (5) and (6) are the theoretical values used in the inversion algorithm.

RIDGE REGRESSION INVERSION APPLIED TO CRUSTAL RESISTIVITY SOUNDING DATA FROM SOUTH AFRICA

W. R. PETRICK*, W. H. PELTON*, AND S. H. WARD*

Ridge regression inversion has been used to test the applicability of various one-dimensional crustal models to the interpretation of deep Schlumberger sounding data from southern Africa (Van Zijl and Joubert, 1975). Four main models were investigated: a simple three-layered earth, a layered earth with a transition zone exhibiting a linear decrease in log resistivity with depth, a similar earth with the transition zone determined by cubic splines, and a model having exponential resistivity behavior at depth. The last model corresponds to temperature-dependent semiconduction through solid mineral grains (Brace, 1971). It was found that all of these models are capable of fitting the sounding data from southwestern Africa, while all except the semiconduction model fit the data from southeastern Africa. One is, thereby, immediately alerted to the problem of lack of resolution in Schlumberger sounding data where geologic control is not available.

A major problem with the inversion of Schlumberger data alone is that accurate information is

obtainable only for the resistivity-thickness product of the resistive portion of the crust. On the other hand, magnetotelluric data, when available, tends to provide information on the thickness, but very little information on the true resistivity of the section. In order to resolve both resistivity and thickness it is possible to invert simultaneously Schlumberger and magnetotelluric (MT) data. Results obtained from the combined inversion of the African resistivity data and hypothetical MT data show that a considerable improvement in model resolution can be achieved using MT amplitude data even of poor accuracy from a relatively limited frequency range (0.1 to 100 Hz), whereas inclusion of MT phase information is of negligible additional benefit.

Unfortunately, no significant test can be made, from data available at the time of our analysis, of the applicability of one-dimensional inversion in a geologic circumstance which probably demands more dimension.

INTRODUCTION

Deep crustal sounding data from various areas of the world usually indicate three main zones of differing conductivity. The surface zone is composed of porous, weathered material or sedimentary rocks with resistivity usually less than 500 Ω -m. Below this layer, the basement rocks typically have a much higher resistivity, between 10^4 and 10^5 Ω -m, corresponding to much lower porosities. These high resistivities persist for several kilometers, usually followed by a decrease in resistivity of at least an order of magnitude (Keller, 1971). The exact depth

at which this decrease occurs is usually not well known, nor is the mechanism which causes it.

Recently some very deep Schlumberger soundings (VES for vertical electric sounding) were conducted in southern Africa (Van Zijl and Joubert, 1975). Although their paper suggests a relatively detailed model for the resistivity of the crust, little information is given concerning the uncertainty of the model or whether other models may equally well fit the data. Accordingly, we have attempted a further analysis of the soundings using ridge regression inversion. We are attempting only the next obvious

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step in improvement of data analysis, and we are not attempting to extract nor evaluate the maximum amount of information from the available data sets. Therefore, the objectives of our analysis are limited to the following. We wished to ascertain (1) if the linear decrease in log resistivity at depth, indicated by Van Zijl and Joubert, was truly well-determined by the data, (2) if other crustal models such as that proposed by Brace (1971) could be safely excluded from consideration, and (3) what additional benefit magnetotelluric (MT) data would provide in resolving the resistivity of the crust.

INVERSION THEORY

We shall present only a brief summary of the least-squares inversion method used in this study. More detailed discussions regarding geophysical applications have been given by Glenn et al (1973) and Inman (1975). A description of the statistical estimates may be found in Glenn and Ward (1976). We will follow their notation as closely as is practical.

Least-squares method

Our geophysical problem may be linearized as follows:

$$\Delta \mathbf{G} = \mathbf{A} \Delta \mathbf{P} + \boldsymbol{\epsilon}, \quad (1)$$

where the $n \times m$ matrix of derivatives $\mathbf{A}$ relates a small change in model parameters $\Delta \mathbf{P}$ to a small change in data $\Delta \mathbf{G}$. When the departure from linearity $\boldsymbol{\epsilon}$ is small, we may obtain $\Delta \mathbf{P}$ from

$$\Delta \mathbf{P} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \Delta \mathbf{G}, \quad (2)$$

where we have merely multiplied both sides of (1) by the generalized inverse operator,

$$\mathbf{H} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T. \quad (3)$$

By premultiplying $\mathbf{H}$ by $\mathbf{A}$ we may define $\mathbf{S}$, the information density matrix. The diagonal of $\mathbf{S}$ contains large values at positions corresponding to data points which are contributing significant information to the total model determination. The derivative matrix $\mathbf{A}$, on the other hand, gives an approximate breakdown as to which data points contribute to the determination of each parameter. Therefore, through the use of either the information density matrix or the derivative matrix, it is possible to deduce the range of data required to resolve a particular model.

Once a model is selected and the data obtained, the object of least-squares inversion is to minimize the square error between the calculated model and the observed data, given by

$$\xi^2 = \Delta \mathbf{G}^T \Delta \mathbf{G}. \quad (4)$$

If the problem is exactly linear, only one determination of $\Delta \mathbf{P}$ will be necessary. All geophysical sounding problems, however, are nonlinear and usually several iterations are required to obtain a satisfactory solution from a given initial guess.

Weighted least-squares

Thus far, our least-squares inversion technique assumes uniform data error. This may be a rather poor assumption when considering several data sets simultaneously. To overcome this difficulty, we must resort to weighted least-squares in which the various data sets (or data points) are appropriately weighted in such a way that very noisy data do not contribute the same degree of influence over the inversion as relatively precise data.

We accomplish this by multiplying (1) by a weight matrix $\mathbf{W}$, so that the expression for parameter change becomes,

$$\Delta \mathbf{P} = (\mathbf{A}^T \mathbf{W}^T \mathbf{W} \mathbf{A})^{-1} (\mathbf{A}^T \mathbf{W}^T \mathbf{W}) \Delta \mathbf{G}, \quad (5)$$

and the weighted square error, which we minimize is,

$$\xi_w^2 = \Delta \mathbf{G}^T \mathbf{W}^T \mathbf{W} \Delta \mathbf{G}. \quad (6)$$

When the usual assumption is made that the error at each data point is independent of the error at all other data points, $\mathbf{W}$ reduces to a diagonal matrix. Thus if we know the standard deviation σ_i of each of the data points, it is natural to determine the elements of the weight matrix as follows (Lawson and Hanson, 1974):

$$W_{ii} = \frac{1}{\sigma_i^2} \quad i = 1, n. \quad (7)$$

Often, however, we have a better idea of the relative error between data sets than of the true error at each point. In such situations, we may use weights equal to 1.0 for the first data set and relative weights for each subsequent set. An estimate for the true variance of the data in the first set may then be obtained from the reduced chi-square,

$$\chi_v^2 = \frac{\xi_w^2}{v}, \quad (8)$$

where ξ_w^2 is defined by (6) and v is the degrees of freedom $n - m$.

Ridge regression

There is still one other difficulty with which we must contend before obtaining an efficient inversion

routine. Should one or more of the parameters be very poorly determined, the matrix $A^T A$ will be nearly singular and an eigenvalue decomposition of this matrix would reveal one or more near-zero eigenvalues. To overcome this difficulty, we could reduce the rank of the matrix by reparameterizing the problem. A similar result may more easily be achieved, however, by simply weighting out the very small eigenvalues through the addition of a small positive constant λ to the diagonal elements of $A^T A$ before inversion. This technique is known as Marquardt's method or damped least-squares (Lawson and Hanson, 1974). The equation for determining parameter change then becomes,

$$\Delta P = (A^T W^T W A + \lambda I)^{-1} (A^T W^T W) \Delta G. \quad (9)$$

A large value of λ results in an algorithm which is slow to converge, but which is very stable. On the other hand, a value of λ which is smaller leads to an algorithm which is faster, but which may diverge. Routines which carefully select an appropriate value for λ at each step in the inversion process are called ridge regression algorithms, and it is this type of algorithm which we have used in our inversion approach (Levenberg, 1944; Foster, 1961; Marquardt, 1963; Bevington, 1969). Note that all parameter statistics must be calculated with $\lambda = 0$.

Parameter statistics

Once the inversion process produces a model which best fits the observed data, the question of fundamental importance is: how accurately are the model parameters known?

We may obtain some feeling for how well our model is determined from the data by examining the parameter covariance matrix, which for the weighted least-squares technique is given by,

$$\text{cov}(\mathbf{P}) = \chi^2 (A^T W^T W A)^{-1}. \quad (10)$$

An estimate for the standard deviation of each parameter may be derived from the square root of the appropriate diagonal element of $\text{cov}(\mathbf{P})$ and the parameter correlation coefficients may be obtained by normalizing the parameter covariance matrix in the following manner:

$$\text{cor } P_{ij} = \frac{\text{cov } P_{ij}}{(\text{cov } P_{ii} \cdot \text{cov } P_{jj})^{1/2}}. \quad (11)$$

A linear relationship between two parameters is implied by a correlation coefficient with an absolute value near 1.0. If the correlation coefficient is near -1.0, only the product between two parameters

(e.g., $T = \rho t$) is well determined. Conversely, if the correlation coefficient is near +1.0, only the ratio between two parameters (e.g., $S = t/\rho$) is resolved. In order to estimate how well S or T is determined, we effectively reparameterize the problem by holding either the resistivity or the thickness constant and obtain statistics on the remaining parameter. Another approach, aimed at individual parameter definition, is to include additional data containing more information concerning one of the parameters.

PARAMETERIZATION OF PROBLEMS

There are perhaps three main considerations involved in parameterizing problems. The first of these is the choice of dimensions for the data. Since Schlumberger apparent resistivity data usually have an error which is a fixed percentage of each measurement, it is very convenient to parameterize the VES problem in terms of log apparent resistivity. In this way we avoid the requirement of a weight matrix with elements inversely proportional to the magnitude of each measurement.

The next important consideration is to try to choose as parameters only those truly important variables which have some hope of being resolved by the data. Of course, there are situations where the relative importance of a variable is not at all well-known beforehand. In these situations it is necessary to include the variable as a parameter and then to analyze the resulting statistics. Usually, however, the interpreter will have some feeling, perhaps from prior curve-matching experience, as to which simple models might be determined from inversion, and will thus be able to avoid selection of grossly insignificant variables as parameters.

The last consideration involves the choice of dimensions for the parameters. It is important that we choose an appropriately dimensioned parameter space such that our inversion problem is reasonably linear. This requirement is met for the layered earth model, if we invert with respect to log resistivities and log thicknesses. The logarithmic transformation has the effect of completely excluding negative resistivities and negative thicknesses from consideration. As a result, parameter confidence regions are approximately ellipsoidal and are accurately described by the parameter correlation matrix and percentage parameter standard deviations.

CALCULATION OF THE FORWARD PROBLEM

Since first derivatives with respect to any parameter of any model may be readily approximated by

FIG. 1. South African deep resistivity sounding data and theoretical curves presented by Van Zijl and Joubert (1975). Associated models are shown in Figure 3b.

taking the first forward difference, the main requirement of the ridge regression inversion technique is that a fast algorithm be available for the calculation of each model. The approach we used in our study was to segment our crustal models into thin layers and then use the fast algorithms already available for the layered earth MT and resistivity sounding problems (Ward, 1967; Rijo et al, 1976).

RESISTIVITY INVERSION

Shown in Figure 1 are the data from the three deep crustal Schlumberger soundings conducted in southern Africa. The reader is referred to the article by Van Zijl and Joubert (1975) for a discussion of the data acquisition and accuracy. The sounding locations are shown in Figure 2. Prior to inversion, the sounding data corresponding to $AB/2$ less than 3 km were deleted. This was necessary since the shorter $AB/2$ portion of the sounding data exhibits considerable error due to lateral variation of the S_1 value associated with the surface weathered layer. This effect is particularly noticeable for VES 2 and VES 3 in Figure 1. Since a correct fit to the remainder of the data depends upon the choice of S_1 , we chose a nominal 100 Ω -m resistivity value for the thin overburden and allowed the inversion process to vary t_1 . In this way we were guaranteed an optimum S_1 value compatible with the larger spacing data. Subsequent examination of the derivative matrices for all models confirmed that as long as S_1 was determined, the deletion of the shorter spacing data had negligible effect on the determination of the main crustal parameters of interest in our study.

Layered earth model

Our first approach was to invert each sounding separately in order to determine the best three-layer model fitting each data set. Although the data fits were just as good as those obtained by Van Zijl and Joubert in Figure 1, the parameter statistics were not very satisfactory, as shown in column 1A of Table 1. Evidently VES 1 is the only sounding which provides information on both the resistivity ρ_2 and thickness t_2 of the resistive portion of the crust. This layer is apparently very thin under south-eastern Africa so that VES 2 and VES 3 merely indicated $\rho_2 t_2$ products of $4.85 \times 10^8 \Omega\text{-m}^2$ and $4.77 \times 10^8 \Omega\text{-m}^2$, respectively.

Since ρ_2 is not at all well-determined by the latter two soundings alone, we wished to incorporate information somehow from the eastern part of the continent in order to arrive at a best estimate for ρ_2 compatible with all three sets of sounding data. To do this, we inverted the three VES simultaneously, thus coupling together information on ρ_2 from all three soundings into one derivative matrix. The results of the inversion are shown in columns II and IIA of Table 1 and in Figure 3a. The best estimate for ρ_2 common to all three soundings is 67,600 Ω -m, and since the data fit for the simultaneous inversion is approximately the same as that for each separate inversion, it appears that the hypothesis of a common ρ_2 is not totally unreasonable, considering the VES data alone.

FIG. 2. Location of three Schlumberger sounding sites (Van Zijl and Joubert, 1975) and geothermal gradient values from Lee and Uyeda (1965).

Table 1. Summary of percent parameter standard deviations associated with the three-layered earth model for various data set combinations.

		I	IA	II	II A	II B	II C	II D	II E	
	MODEL	PARAMETER	PARAMETER VALUE	EACH VES. SEPARATELY	PARAMETER VALUE	ALL VES. COMBINED	VES. and MT (ρ_a)	VES. and MT ($\rho_a/a\phi$)	MT ALONE (ρ_a)	MT ALONE ($\rho_a/a\phi$)
VES 1	KAMESKROON	ρ_1	100 Ω -m	CONSTRAINED	100 Ω -m	CONSTRAINED	CONSTRAINED	CONSTRAINED	>1000%	>1000%
		ρ_2	41900 Ω -m	22%	67600 Ω -m	COUPLED 48%	18%	16%	>1000%	>1000%
		ρ_3	539 Ω -m	21%	588 Ω -m	18%	16%	15%	130%	70%
		t_1	25.0m	16%	30.8m	13%	7%	7%	>1000%	>1000%
		t_2	18073m	20%	11500m	46%	17%	15%	>1000%	55%
VES 2	MAVONDE	ρ_1	100 Ω -m	CONSTRAINED	100 Ω -m	CONSTRAINED	CONSTRAINED	CONSTRAINED	>1000%	>1000%
		ρ_2	128x10 ⁵ Ω -m	592%	67600 Ω -m	COUPLED 48%	19%	17%	>1000%	>1000%
		ρ_3	755 Ω -m	12%	743 Ω -m	8%	10%	10%	130%	50%
		t_1	13.7m	66%	11.3m	24%	13%	12%	>1000%	>1000%
		t_2	3840m	578%	7180m	46%	16%	15%	>1000%	430%
VES 3	PETERSBURG	ρ_1	100 Ω -m	CONSTRAINED	100 Ω -m	CONSTRAINED	CONSTRAINED	CONSTRAINED	>1000%	>1000%
		ρ_2	135x10 ⁵ Ω -m	>1000%	67600 Ω -m	COUPLED 48%	25%	21%	>1000%	>1000%
		ρ_3	1230 Ω -m	6%	1230 Ω -m	12%	6%	5%	90%	53%
		t_1	56.2m	24%	53.9m	7%	4%	3%	>1000%	>1000%
		t_2	3520m	>1000%	7050m	49%	25%	21%	>1000%	>1000%

If we assume, then, a relatively constant ρ_2 , it would appear that the thickness t_2 of the resistive portion of the crust may decrease across southern Africa from about 11.5 km in the west to 7 km in the east. This decrease in thickness, however, is de-

pendent upon the assumption of a relatively homogeneous crust. As the gravity data shown by Hales and Gough (1962) indicate, this assumption may not be a particularly good one. The obvious next step in analyzing deep resistivity data in South

FIG. 3. Crustal conductivity models derived from the South African resistivity sounding data. (a) Our three-layered models. (b) Models proposed by Van Zijl and Joubert (1975).

FIG. 4. Reparameterized models using linear and spline functions to approximate the transition zone. Error bars show parameter standard deviations associated with various components of the models.

Africa is modeling of two- or three-dimensional structures by either forward or inverse algorithms. Unfortunately, the available electrical data do not warrant or even permit this degree of sophistication.

Linear transition zone model

The next one-dimensional crustal model which we chose to investigate was the one suggested by Van Zijl and Joubert (1975). As illustrated in Figure 3b, they propose a linear decrease in log resistivity with depth between the resistive and underlying conductive region of the crust. Since the authors neglected to mention evidence substantiating this feature, we wished to find out if the transition zone was truly well-determined by their data.

To parameterize the model, we chose the normal parameters for a three-layered earth and added one additional parameter, slope, which was responsible for specifying the resistivity and thickness of six or

seven additional layers which defined the transition zone. As shown in Figure 4a, this slope parameter is extremely poorly determined by the data from VES 2, and yet this sounding provided the best estimate for slope among the three VES. The main difficulty in determining the slope is that it is extremely highly correlated with the thickness t_2 of the second layer. Thus, there are many parameter combinations ranging from small t_2 and near-zero slope to large t_2 and near perpendicular slope which equally well fit the data.

Spline transition zone model

In order to explore further the transition zone postulated by Van Zijl and Joubert, we investigated a model having a very flexible cubic spline dependence for log resistivity as a function of depth. Although the transition zone is characterized by only two parameters, corresponding to the base points of

FIG. 5. Mineral semiconduction models determined for sounding location 1. A: Model for which E_0 , and ρ_0 were empirically determined. B: Model for which E_0 , ρ_0 and dT/dz were variables.

the spline, the model is free to simulate virtually any smooth functional behavior in the depth range 5 to 20 km. Beyond this range, the model again has all the standard three-layer earth parameters except for t_2 .

The results of the inversions with respect to the spline transition zone are shown in Figure 4b, c, and d. It is evident that the two base points specifying the functional behavior of the transition zone are very poorly determined. It is not really possible to ascertain if the transition is indeed abrupt as suggested by VES 1 or is slowly gradational, as perhaps indicated by VES 2. The only parameters which are reasonably well determined are the resistivities of the resistive zone and of the underlying conductive layer.

Semiconduction model

The last model which we wished to investigate was one which includes semiconduction through mineral grains as the major conduction mechanism at depth in the crust. For this model the rock resistivity depends upon absolute temperature T , as

$$\rho = \rho_0 e^{E_0/kT},$$

where E_0 is the activation energy, k is Boltzmann's

FIG. 6. VES 1 data fit for the mineral semiconduction model B. The data fit obtained using the three-layered model is included for comparison.

constant, and ρ_0 is an empirically determined constant (Parkhomenko, 1967; Hermance, 1973).

Brace (1971) has presented bounds for rock resistivity assuming crack and pore-dominated conduction in the upper few kilometers of the earth's crust and semiconduction at greater depth. His bounds for observable dry semiconduction behavior correspond to $E_0 = 0.6\text{ eV}$, $\rho_0 = 30\ \Omega\text{-m}$ (upper bound), and $E_0 = 0.7\text{ eV}$ and $\rho_0 = 0.03\ \Omega\text{-m}$ (lower bound). Accordingly, our first attempt at inversion of the semiconduction model was to choose median semiconduction behavior corresponding to

$$\rho = 1.0 e^{E_0/kT},$$

where T was specified in terms of the geothermal gradient s and depth z by

$$T^\circ\text{K} = sz + 300.$$

Inversion of VES 1 to a two-layered model modified by semiconduction behavior at depth, gave an estimate of 23°C/km for the geothermal gradient in southeastern Africa. This model is plotted as curve A in Figure 5. Also shown in the figure are the upper and lower Brace bounds, assuming a geothermal gradient of 22°C/km .

Our next step was to invert with respect to activa-

FIG. 7. Uncertainty in the estimation of the thickness of the resistive zone for various percent noise in MT data over the frequency range from 10^{-4} to 10^4 Hz.

tion energy E_0 and ρ_0 , as well as geothermal gradient s , in an attempt to obtain the optimum set of parameters fitting the data from VES 1. The resulting model is shown as curve B in Figure 5. There was relatively minor change in E_0 and ρ_0 , indicating that the average semiconduction behavior was a reasonable assumption. The estimate for geothermal gradient decreased slightly from $23^\circ\text{C}/\text{km}$ to $21^\circ\text{C}/\text{km}$.

Shown in Figure 6 is the resulting data fit for model B compared with the best-fitting 3 layered earth model for VES 1. The fit is about the same for both models. The results concerning the data from VES 2 and VES 3 are considerably different, however. There is no combination of semiconduction parameters which can provide a good fit to the latter two soundings.

MT INVERSION

In order to determine what role MT data might play in the resolution of a crustal conductivity model, we generated hypothetical MT amplitude and phase data using the models obtained from the simultaneous resistivity inversion. The MT data were then contaminated with prescribed amounts of log normal error to simulate the effect of real field data.

MT data were generated over two frequency ranges. One set covers the frequency range from 0.1 to 100 Hz while the other covers the extended range

FIG. 8. VES 1. Experimental Schlumberger and hypothetical MT sounding data with best fitting theoretical three-layer sounding curves. Associated model is shown in Figure 3b.

from 10^{-4} to 10^4 Hz. The narrow range contains only information concerning those parameters found to be poorly determined by the inversion of individual resistivity data sets, whereas the extended range represents the complete spectrum of useful frequencies concerning all of the three-layered model parameters. The appropriate frequency ranges were determined from an analysis of all columns of the derivative matrix A.

As illustrated in columns IID and IIE of Table 1, inversion of the narrow range amplitude data containing 50 percent noise was incapable of defining any of the model parameters. The inclusion of phase information at the same error level improved only the estimate of ρ_3 and t_2 .

Somewhat similar results were obtained when the extended data set was inverted. As shown in Figure 7, the addition of phase information had relatively minor influence on the resolution of the most important crustal parameter t_2 . This was not the case for all parameters of the three-layered model, however. When only MT amplitude data were available, the addition of phase information was useful in the total model determination.

FIG. 9. VES 2. Experimental Schlumberger and hypothetical MT sounding data with best fitting theoretical three-layer sounding curves. Associated model is shown in Figure 3b.

SIMULTANEOUS RESISTIVITY AND MT INVERSION

Although it is possible to invert resistivity and MT data separately, thus obtaining independent estimates for $\rho_2 t_2$ and t_2 , one of the major objectives of our study was to investigate the *simultaneous inversion* of resistivity and MT data. We particularly wished to determine whether noisy MT amplitude data from the narrow frequency range 0.1 to 100 Hz could substantially improve resolution of ρ_2 and t_2 .

To carry out the simultaneous inversion, we combined derivatives with respect to both VES data and MT amplitude data into a single A matrix. However, since the error in the MT data was much greater than that for the VES data, it was necessary to use a weight matrix W in order to adjust appropriately the contribution to parameter determination from each data set. For the elements in W, we chose the value 1.0 corresponding to the VES data and the ratio of logarithmic VES error to logarithmic MT error for the elements of W corresponding to the MT data.

Layered earth model

The results of the simultaneous inversion of log

resistivity data and log MT amplitude data to a layered earth model are shown in column IIB of Table 1. The data fits are shown in Figures 8 through 10. It is evident that the addition of MT amplitude information results in increased resolution of all parameters, particularly ρ_2 and t_2 for VES 2 and VES 3 (compare columns IA and IIB). The addition of *both* MT amplitude and phase information to the VES data, however, results in only relatively minor improvement over the addition of MT amplitude data alone, as shown in column IIC of Table 1.

Linear transition zone model

Since the addition of MT data significantly aided the resolution of poorly determined layered earth parameters, we chose as our last study in this paper to investigate the benefit of simultaneous VES and MT inversion on the resolution of the linear transition zone model proposed by Van Zijl and Joubert. We specifically wished to determine what accuracy of data would be required in order to resolve independently the highly correlated slope and t_2 parameters.

To investigate the effect of noise on both the VES data as well as the MT data, we generated theoretical data sets for both, using the models given by Van Zijl and Joubert in Figure 3b. The range of

FIG. 10. VES 3. Experimental Schlumberger and hypothetical MT sounding data with best fitting theoretical three-layer sounding curves. Associated model is shown in Figure 3b.

frequencies and AB/2 was the same as that shown in Figures 8 through 10.

Table 2 presents the results of the simultaneous inversions for various combinations of data noise. It is apparent that a data accuracy better than 10 percent for VES and better than 20 percent for MT amplitude data is required in order to resolve the model. The addition of MT phase information results in only minor improvement in resolution.

DISCUSSION AND CONCLUSIONS

Our analysis of the African sounding data was mainly intended to illustrate the application of ridge regression inversion to a difficult problem in geophysical interpretation. We have by no means exhausted all possibilities for useful data combinations or for one-dimensional models that will fit the data. Of those models investigated, three provide reasonable, if admittedly simplistic, views of the crustal conductivity profile.

The first acceptable model is that of a simple three-layered earth. Assuming a relatively constant resistivity of about 68,000 Ω -m for the resistive section of the crust, this layer appears to decrease in thickness from approximately 11.5 km in southwestern Africa to 7 km in southeastern Africa. Below this resistive zone, there is perhaps a two order of magnitude increase in conductivity.

One geologic section which is compatible with this three-layered model is that of a granitic upper crust underlain by basalt (Keller, 1971). The boundary between these two rock types may account for the increase in conductivity and for the increase in shear wave velocity to 3.75 km/sec at a depth of 8 km obtained by Bloch et al (1969) from surface wave dispersion analysis.

The second acceptable model for the crust is one which has a linear decrease in log resistivity with depth. Although Van Zijl and Joubert appear to favor this model, our analysis of their data indicates that the linear transition feature is very poorly resolved.

This poor resolution for the transition zone became very evident when the third model, exhibiting cubic spline functional behavior, was used to fit the data. Since the spline base points are highly correlated with one another, there are a great number of possible functional dependencies for the transition region, all of which fit the data equally well.

The only *unsuccessful* attempt at fitting the data occurred when we tried to fit the results from VES 2 and VES 3 to the semiconduction model. While the last data points from these soundings are relatively

Table 2. Summary of percent parameter standard deviations for parameters associated with a linear transition zone for simultaneous resistivity-MT inversion with various combinations of data noise.

RES PCT DATA ERROR	5%	5%	10%	20%	10%
MT $ p_0 $ PCT DATA ERROR	5%	5%	20%	20%	50%
MT ρ PCT DATA ERROR	5%	—	—	—	—
PCT STD DEV SLOPE	11%	14%	50%	70%	UNDET
PCT ST DEV THICKNESS t_2	13%	17%	55%	79%	UNDET

noisy, they do appear to indicate a reasonably constant resistivity of perhaps 700 Ω -m below the resistive zone. Assuming that these data are reliable, it is difficult to attribute the resistivity decrease under southeastern Africa to purely temperature-dependent semiconduction behavior and a relatively constant geothermal gradient.

There is no similar difficulty, however, with the data from VES 1 in southwestern Africa. Using average rock semiconduction properties, we were able to obtain a good fit to the data by inverting to an unknown geothermal gradient. As shown in Figure 2, the values of 21°C/km and 23°C/km obtained from the inversion compare very favorably with the nearest published value of 22°C/km obtained by Lee and Uyeda (1965).

Carte and van Rooyen (1971) have published geothermal gradient values of 18°C/km very close to VES 1, but a slight change in ρ_0 to .527 Ω -m and E_0 to .608 eV is sufficient to counter the effect of a lower geothermal gradient, and give essentially the same resistivity section as that of model B shown in Figure 5.

One of the main deficiencies with inverting VES data alone is that usually only a resistivity-thickness product may be determined when a thin resistive layer is encountered. We partially overcame this problem by simultaneously inverting VES 1 with VES 2 and VES 3 in order to obtain a value for ρ_2 which was compatible with all three soundings. The result, however, is only as good as the assumption that ρ_2 is reasonably constant from VES 1 to VES 2 and VES 3.

Much better results may be obtained when MT data are simultaneously inverted with VES data. Our analysis indicates that a substantial increase in parameter resolution can be obtained from inclusion of MT amplitude data, with as much as 50 percent noise, over the relatively narrow frequency range 0.1 Hz to 100 Hz.

Models more complex than a layered earth, however, will require more precise data in order to be truly well-resolved. For example, the detection of the linear transition zone postulated by Van Zijl and Joubert will probably require simultaneous inversion of resistivity data with less than 10 percent noise and MT amplitude data with less than 20 percent noise. This "noise level" includes all effects of 2-D inhomogeneities as well as measurement inaccuracies.

One somewhat surprising result indicated by our simultaneous inversion studies was the relatively minor benefit achieved by the addition of MT phase information. Although phase data are a useful addition to MT amplitude data alone, the constraints provided by phase appear very similar to those provided by VES data. Thus, when deep VES measurements are available as they are in southern Africa, it appears necessary only to collect MT amplitude information in order to provide increased parameter resolution.

The reader should be cautioned that we have not attempted to find a total suite of one-dimensional models which fit the data. As far as we have studied the problem, a simple three-layered model fits the data as well as the more complex models. Of even more importance, we have not evaluated 2-D nor 3-D models. The data do tend to be dominated by one-dimensional effects; much more data are required before a significant attempt at 2-D and 3-D modeling can be made.

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