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LEVEL STRUCTURE IN ^{15}O FROM THE PROTON BOMBARDMENT OF ^{14}N
(Thesis)

Martin Luther West, II

Submitted as a dissertation to the Graduate Council of the University of Texas
in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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PHYSICS DIVISION

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ACKNOWLEDGMENTS

The work described in this thesis was performed in the Physics Division of the Oak Ridge National Laboratory, operated by Union Carbide Corporation for the U. S. Atomic Energy Commission. The research was supported by the Graduate Fellowship Program of the Oak Ridge Associated Universities and was under the direction of Dr. Harvey B. Willard and Dr. Charles M. Jones of the Physics Division of ORNL and Dr. Robert N. Little, Dr. Peter J. Riley, Dr. E. L. Hudspeth, and Dr. E. M. Bernstein of the Department of Physics of the University of Texas.

The author wishes to express his gratitude to Dr. Harvey B. Willard for his supervision of the theoretical analysis and interpretation and for his critical reading of the entire thesis, Dr. Charles M. Jones for his full time assistance with the experimental work and data reduction, and Dr. Joe K. Bair, who initially suggested the experiment, for his assistance with the data acquisition and reduction. Special thanks are due Dr. Robert N. Little who originally suggested the trip to ORNL and who offered encouragement throughout the stay in Oak Ridge.

The author is also deeply indebted to the entire staff of the High Voltage Laboratory of ORNL for their willing assistance and

many helpful suggestions, Steve Thornton for his assistance in data accumulation, Betty Hensley for her typing of the preliminary drafts of the thesis, and Mrs. Linda Moore for her competent typing of the final manuscript.

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CHAPTER I

INTRODUCTION

A vast amount of experimental and theoretical data has been compiled on the energy levels of light nuclei. Such a compilation and cataloging of the properties of nuclear levels is expected to yield information that will lead toward deeper understanding of the nucleus. Properties that experiments should seek to disclose include the existence of nuclear levels and their energies of excitation. More indirect properties include the total angular momentum, parity, total level width, and the partial level widths for the various decay modes of the state. A very fruitful technique for obtaining information about many of these properties is the elastic and inelastic scattering of charged particles. In elastic scattering the Coulomb and nuclear amplitudes interfere coherently which, in principle, allows the relative phase and magnitude of the unknown nuclear phases to be determined. It is of particular value to study the nuclear amplitudes in off-resonance regions in order to gain information about the background scattering. Inelastic scattering allows us to study nuclear levels without interference by the Coulomb and the large potential amplitudes present in the elastic scattering.

In the present experiment the elastic and inelastic scattering of protons from ^{14}N has been studied to obtain information concerning the excited states formed in the compound nucleus ^{15}O , as well as to investigate the nature of the "off-resonance" scattering. The nucleus ^{15}O is of particular interest because it consists of eight protons and

seven neutrons and hence needs only one nucleon to complete the 1p shell. Theoretical predictions about such a nucleus are expected to be in reasonable agreement with experiment and a comparison of experimental data with theory should provide a fair test of the shell model picture. The mirror nucleus ^{15}N has been extensively studied and it is of interest to compare the levels of these two nuclei. A detailed discussion of ^{15}N and ^{15}O level structure up to 10 MeV excitation has been given by Warburton, Olness, and Alburger¹⁾. Figure 1.1 shows the level structure of ^{15}O as reported by Ajzenberg-Selove and Lauretsen²⁾. Figure 2.2 shows the level structure of the mirror nucleus ^{15}N .

When ^{14}N is bombarded by protons with energies below about 6 MeV, the reaction can proceed through compound states in ^{15}O with excitation energies between 7.3 and 12.9 MeV by means of the following processes:

Previous knowledge of the level structure of ^{15}O in this energy region has come predominantly from studies of the radiative capture and the elastic scattering processes. Duncan and Perey³⁾ first studied the

Figure 1.1 Energy Levels of ^{15}O .

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Figure 1.2 Energy Levels of ^{15}N .

$^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction (1951) by measuring the yield of positrons from the decay of the ground state of ^{15}O for proton bombarding energies between 0.25 and 2.6 MeV. Absolute cross sections were obtained and all presently known levels in this energy region were observed.

The elastic scattering of protons by ^{14}N has been studied by Webb, et al.⁴⁾, Gove, et al.⁵⁾, Tautfest and Rubin⁶⁾, Bolmgren, et al.⁷⁾, Hagedorn, et al.⁸⁾, Olness, et al.⁹⁾, Ferguson, et al.¹⁰⁻¹²⁾, Bashkin, et al.¹³⁾, Val'tar, et al.¹⁴⁾, and Cohen-Ganouna, et al.¹⁵⁾. The experiments of Gove, et al.⁵⁾, and Ferguson, et al.¹⁰⁻¹²⁾ covered the proton bombarding energy range of 1.0 to 3.0 MeV. A phase shift analysis was made on a set of angular distributions as a means of understanding the non-resonant scattering in this energy region. Their work established the existence of a significant p-wave contribution to the off resonance scattering and also suggested that a simple potential well was an inadequate description of the interaction above 2.3 MeV bombarding energy. A total angular momentum and parity assignment of $1/2^+$ was made to the resonance at a bombarding energy of 1.557 MeV and an assignment of $1/2^+$ with a total width of 550 keV was made to the 2.32 MeV level. This latter result was in disagreement with the value of 1.2 MeV as obtained earlier by Duncan and Perey³⁾ from their capture data. It was suggested by Ferguson¹²⁾ that this disparity could be due to the presence of an appreciable non-resonant contribution to the (p,γ) cross section whose presence would increase the apparent width of the resonance if not explicitly included in the calculation.

Tautfest and Rubin⁶⁾ studied the bombarding energy region of 0.85 to 1.9 MeV and concluded that their work was in agreement with an

assignment of $1/2^+$ to the 1.060 and 1.557 MeV levels. Their data was not sufficient to decide between the possible assignments of $3/2^-$ or $5/2^-$ to the 1.74 and 1.803 MeV levels as suggested by Webb, et al.⁴⁾ and Gove, et al.⁵⁾.

Hagedorn, et al.⁸⁾ obtained excitation curves at four angles for proton bombarding energies between 0.620 and 1.820 MeV. They showed that the 1.060 and 1.55 MeV levels were formed predominantly by s-wave protons, thus limiting their spin and parity to $1/2^+$ or $3/2^+$. Assignments to the 1.75 and 1.81 MeV levels presented difficulties because of the large "off-resonance" scattering. This background was not of the pure s-wave type and part of it appeared to come from a broad level located near 2.4 MeV. Qualitative fits to the 1.75 MeV resonance indicated p-wave formation of either a $1/2^-$ or $3/2^-$ state.

The work of Bolmgren, et al.⁷⁾ covered the bombarding energy range of 1.5 to 3.5 MeV. Bashkin, et al.¹³⁾ and Olness, et al.⁹⁾ extended this range to 4.0 MeV. Additional resonances were reported at bombarding energies of 2.535, 3.2, 3.9, and 3.98 MeV. Olness, et al.⁹⁾ also measured the inelastic cross sections for the 3.9 and 3.98 MeV levels and obtained widths for the four resonances above 2.5 MeV. A probable assignment of $5/2^+$ was made to the 3.9 MeV level in ^{15}O and it was concluded that the 3.98 MeV resonance most probably involved $J \geq 3/2$, $\ell' \geq 1$ and that the inelastic width was less than the elastic width.

A. K. Val'tar, et al.¹⁴⁾ studied the energy region of 1.7 to 3.5 MeV, including both the elastic scattering cross section and the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction. They were able to assign resonance energies and widths to most of the levels in this region. Recently Cohen-Ganouna,

et al.¹⁵), made an analysis of the 1.75 and 1.81 MeV levels by studying the shapes of resonances associated with various spin and parity assignments. They concluded that the 1.75 MeV level was formed by d-wave protons with a probable assignment of $5/2^+$ and that the 1.81 MeV level was formed by p-wave protons with a probable assignment of $5/2^-$.

Above 4.0 MeV bombarding energy there is no experimental data on the elastic scattering of protons by ^{14}N . Bair, et al.¹⁶) studied the reaction $^{14}\text{N}(p,p'\gamma)^{14}\text{N}$ for proton bombarding energies between 2.5 and 5.1 MeV. Four levels were observed at excitation energies of 11.0, 11.08, 11.83, and 11.91 MeV corresponding to proton energies of 3.91, 4.0, 4.8, and 4.89 MeV. No spin and parity assignments could be made from their work.

The results of these investigations on the levels of ^{15}O are summarized in Table 1.1; in column 1 are the resonance energies, (laboratory system), in column 2 the total level widths (laboratory system) of the corresponding levels and in column 3 the spins and parities as determined by the various authors. It is evident from this table that the proton bombardment of ^{14}N below 4.0 MeV has been investigated rather extensively and the resonance energies and widths of most of the levels have been determined with a fair amount of accuracy. As regards to the spins and parities, however, unambiguous results have been obtained only for the 1.55 MeV level.

In the present experiment the proton bombarding energy has been extended to 5.7 MeV. Absolute differential cross sections for the elastic and inelastic scattering of protons from ^{15}O have been measured at laboratory angles of 67.5, 85.9, 105.8, 123.7, 135.0, and 157.5

TABLE 1.1

EXCITED STATES OF ^{15}O FROM $\text{P} + ^{14}\text{N}$

E_R (keV)	Γ_{Lab} (keV)	J^π
280 ^{a)}	2 ^{a)}	$[1/2^+]^a)$
1064 ^{a)} , 1050 ^{b)} , 1054 ^{c)} , 1060 ^{d,e)}	4.8 ^{a)} , 3 ^{c)}	$[3/2^+]^c)$, $[1/2^+]^e)$
1550 ^{a,d,f)} , 1540 ^{b)} , 1544 ^{c)} , 1554 ^{e)} , 1557 ^{g)} , 1530 ^{h)}	50 ^{a)} , 34 ^{d)}	$[1/2^+]^{c,e,h)}$
1730 ^{b)} , 1737 ^{c)} , 1740 ^{e,f)} , 1720 ^{h)} , 1743 ⁱ⁾ , 1744 ^{j)}	11 ^{a)} , 4 ^{c)} , $\leq 3.5^j)$	$[\leq 5/2^+]^c)$, $[3/2^-, 5/2^-]^e)$, $[3/2^-]^h)$, $[5/2^+]^i)$
1815 ^{a)} , 1799 ^{c)} , 1803 ^{e)} , 1805 ^{f)} , 1809 ⁱ⁾ , 1806 ^{j)}	7 ^{a)} , 4.8 ^{c)} , 3.7 ^{j)}	$[1/2^-]^c)$, $[3/2^-, 5/2^-]^e)$, $[5/2^-]^i)$
2356 ^{a)} , 2320 ^{b)} , 2360 ^{f)} , 2350 ^{j)}	14 ^{a)} , 11 ^{j)}	
2600 ^{a)} , 2300 ^{g)} , 2400 ^{j)}	1270 ^{a)} , 500 ^{g)} , 300 ^{j)}	$[1/2^+, 3/2^+]^a)$, $[1/2^+]^g)$
2489 ^{a)} , 2450 ^{b)} , 2490 ^{f)} , 2478 ^{j)}	11 ^{a)} , 9.3 ^{j)}	
3130 ^{b)} , 3200 ^{f,j)}	17 ^{j)}	
3878 ^{k)} , 3910 ^{l)}	85 ^{k)} , 90 ^{l)}	$[5/2^+]^k)$

TABLE 1.1 (continued)

E_R (keV)	Γ_{Lab} (keV)	J^π
$3980^k), 4000^1)$	$<30^k), 25^1)$	$[\leq 3/2^-]^k)$
$4800^1)$	$80^1)$	
$4890^1)$	$70^1)$	

a) Reference 3)

b) Reference 13)

c) Reference 8)

d) Reference 4)

e) Reference 6)

f) Reference 7)

g) Reference 10)

h) Reference 5)

i) Reference 15)

j) Reference 14)

k) Reference 9)

l) Reference 16)

degrees for bombarding energies between 3.8 and 5.7 MeV. In addition, angular distributions were measured at 21 selected energies. Thirteen resonances were observed in the elastic scattering cross section and five of these had appreciable partial widths for the $^{14}\text{N}(p,p_1)^{14}\text{N}^*$ reaction.

An attempt has been made to fit these data using a multi-level R-Matrix approximation and a computer program based on the R-matrix "two-level" approximation has been written for calculations of the elastic cross sections. It has been possible to deduce resonance parameters for some of these resonances by making semi-qualitative fits to the elastic scattering excitation functions.

CHAPTER II

EXPERIMENTAL APPARATUS AND PROCEDURE

2.1 General Discussion

The basic experimental apparatus consisted of the Oak Ridge National Laboratory 5.5 MV Van de Graaff accelerator, a general purpose gas scattering chamber, and solid state detectors with associated electronics. A complete description of this gas scattering chamber and its auxillary equipment has been given by C. M. Jones¹⁷). Only those features essential to this experiment are discussed here.

Protons from the Van de Graaff accelerator entered, and were deflected by, a 90 degree analyzing magnet. A horizontal slit system at the exit of this analyzing magnet served to regulate the generator voltage and, in conjunction with the entrance slits of the magnet, defined the proton beam energy to within about ± 0.05 percent. The beam entered the scattering chamber through a differentially pumped tube which also served to collimate the beam, defining it geometrically to a maximum angular divergence of about 0.1 degree.

The direct proton beam passed through the scattering chamber and then through an aluminum foil into a Faraday cup at the rear of the chamber. The entire chamber was filled with nitrogen gas and scattering occurred all along the beam path. However, only that scattering from a small region near the center of the chamber reached the detectors through their narrow analyzing slits. The scattered particles were incident upon surface barrier detectors located at various angles in the

scattering chamber. There was provision for one moveable counter that could view scattered particles at all angles between 20 degrees and 160 degrees with respect to the incident beam. Pulses from these detectors were fed to scalers and into a multichannel analyzer where they were sorted and stored for future analysis.

2.2 Scattering Chamber Assembly

The scattering chamber proper (Figure 2.1) consisted of a cylindrical cavity 14 inches in diameter and 2.25 inches deep enclosed by a removeable aluminum cover and a moveable lower section attached through a rotating vacuum seal. The moveable detector and associated slit system were mounted to this lower section and could be rotated to the desired position by a hand crank. The moveable counter axis was inclined at an angle of 20 degrees to the plane of the chamber and intersected the beam axis on the symmetry axis of the cylindrical chamber housing. The lower section of the chamber had an engraved angular scale which was read with an associated vernier to an accuracy of 0.5 minutes. Entrance portholes positioned at intervals around the cylindrical housing allowed fixed detectors to be placed at any of eleven pre-selected angles.

2.3 Analyzing System

The basic slit system, which served to define the volume from which a detector would accept scattered particles, is shown in Figure 2.2. Two conflicting considerations entered in the design of these slit systems. It was desirable to keep the angular aperture as small as possible for maximum angular resolution and, at the same time, a large

Figure 2.2 Basic Slit Geometry.

aperture was desirable for good counting statistics in a reasonable length of time. Physical dimensions of these slit systems were expressed in terms of a geometry factor G which depended on the solid angle and target length as seen by the detector. The moving detector slit system was rotated so that the vertical axis of the slits was always perpendicular to the plane defined by the beam axis and the axis of the analyzing system. The angular rotation of the vertical axis of the moving detector's analyzer slits was measured with an angular scale inscribed on the rotating counter assembly. To set this moving detector to a given laboratory angle, it was necessary to adjust both the chamber angle and the slit axis angle.

For the present experiment there were three basic slit systems. Two slit systems were fixed and the third was attached to the moving counter. Specific slit dependent quantities and their estimated uncertainties are listed in Table 2.1.

2.4 Differential Pumping System

Figure 2.1 displays the differential pumping system assembly. The scattering chamber was separated from high vacuum of the accelerator tube by several capillary impedances and two pumping stations. The impedance section which separated the chamber proper from the first stage pump consisted of 41 tantalum discs 30 mils thick with 0.125 inch separation between them. A mechanical pump with a pumping speed of 15 cubic feet per minute made up the first stage located nearest the scattering chamber. A diffusion pump with a pumping speed of 300 liters per second comprised the second stage pumping system. The first and second stage were separated by another capillary impedance section

TABLE 2.1

SPECIFIC SLIT DEPENDENT QUANTITIES

Slit System:	1m (Movable)	1 (Fixed)	2 (Fixed)
Nominal Dimensions (inches): (front and back)	0.125 x 0.625	0.150 x 0.750	0.085 x 0.425
Maximum Angular Opening: $(2(b_1 + b_2)/h)$	$\pm 1.4^\circ$	$\pm 1.6^\circ$	$\pm 0.9^\circ$
Path Length Along Beam (inches): (full width at half max.)	$\frac{0.225}{\sin \theta_{\text{Lab}}}$	$\frac{0.288}{\sin \theta_{\text{Lab}}}$	$\frac{0.16}{\sin \theta_{\text{Lab}}}$
Angular Uncertainty ^{a,b}):	± 3.0 (min)	± 2.0 (min)	± 2.0 (min)
Geometric Uncertainty ^b): (total uncertainty in "G" factor) (including slit measurement)	$\pm 0.5\%$	$\pm 0.4\%$	$\pm 0.7\%$

^a) Unless otherwise specified all uncertainties quoted are the standard deviation of an assumed normal distribution.

^b) These are initial estimates.

similar to the first, but consisted of 28 tantalum discs. A similar impedance section of 36 tantalum discs separated the second stage diffusion pump from high vacuum on the accelerator side. These capillary impedance sections served to retard gas flow between pumping stations and were preferred to smooth tubes because of reduced beam scattering and energy degradation by the walls. This pumping arrangement allowed the maintenance of a constant pressure of about 1.0 cm. of mercury in the scattering chamber. First stage pressure was dropped to 1.0 mm., second stage pressure was maintained at about 10^{-3} mm. and the third stage operated at about 10^{-6} mm. of mercury.

2.5 Gas Supply System

Gas had to be continuously pumped through the system in order to maintain constant pressure in the scattering chamber. Figure 2.3 shows a schematic diagram of this gas circulating system. Nitrogen entered the chamber through an automatic pressure control valve. Chamber pressure was regulated by a diaphragm type differential pressure transducer known as a capacitance manometer. First a reference volume of desired pressure was closed off from the circulating gas system and the capacitance manometer was placed between this reference volume and the chamber. Pressure differences were sensed by the manometer and signals from it were used to regulate the automatic pressure control valve at the gas supply entrance. If chamber pressure dropped below the reference pressure, the automatic pressure control temporarily increased the gas supply until equilibrium was maintained.

Gas pressure in the chamber determined the effective target thickness. This thickness could be easily varied and depended on the

ORNL-DWG 67-4200

Figure 2.3 Gas Flow and Pumping Systems.

energy resolution desired. Energy resolution increased with increasing pressure as a result of increased target thickness and as a result of increased straggling of the beam as it passed through the chamber to the effective target volume.

2.6 Beam Collection

Protons from the direct beam were collected in a Faraday cup. This tantalum lined cup was separated from the chamber assembly by an aluminum window 1.75 inches in diameter and 0.00025 inches thick and was also insulated electrically from it. To decrease gas ionization currents which could cause an appreciable error in the current measurement, a vacuum of 10^{-6} mm. of mercury was maintained on the cup side of the aluminum foil.

An important source of error in beam current measurements results from incomplete suppression of secondary electrons knocked from the aluminum entrance foil and walls of the collection cup. In the case of the aluminum foil, there will be a reduction in charge collected and hence an increase in apparent cross section. Electrons ejected from the cup walls will cause an increase in the ratio of collected charge to beam charge and a decrease in apparent cross section. Partial suppression of these stray electrons was accomplished by a ring of permanent magnets placed around the Faraday cup.

A reduction in the ratio of collected charge to true charge is also caused by particles lost through Coulomb scattering. These losses depend strongly on the type of gas used, foil thickness, and cup design. Errors introduced by this outscattering were estimated to be less than 0.1 percent under present conditions.

2.7 Beam Charge Measurement

Beam charge collected in the Faraday cup was fed through an ORNL Model 12 current integrator. Basic components of this system consisted of a series of precision capacitors that were precharged to a precisely known voltage. Charge from the Faraday cup served to discharge these capacitors. For a particular setting of the capacitance and applied voltage, the amount of charge necessary to cause a discharge remained fixed. Output pulses resulting from discharge of these capacitors gave an accurate measurement of integrated beam current. This instrument was calibrated internally and against a known current source consisting of precision resistors, standard dry cells, and a precision voltmeter. The error or uncertainty in charge integration was less than 0.1 percent.

2.8 Detection Equipment

Scattered particles were detected with solid state counters placed at selected angles in the chamber. Figure 2.4 shows a schematic diagram of one detector and its associated electronics. Pulses from the surface barrier detectors entered charge sensitive preamplifiers and then passed through low noise transistorized linear amplifiers. Output pulses from these amplifiers were fed into a multiplex routing and mixing unit^{*}. Here the pulses were sorted and stored in the pulse height analyzer according to their respective electronic input channel. This multiplex unit served to divide the 2048 channel analyzer into an

^{*}Designed at Oak Ridge National Laboratory.

Figure 2.4 Block Diagram of the Detector Electronic System.

equivalence of four separate 512 channel analyzers with the pulses from each detector stored separately. Output data were stored in the form of punched paper tape, magnetic tape, and visual display on the analyzer scope.

2.9 Pressure Measurement

An oil manometer connected to the scattering chamber gas system (Figure 2.3) was used to record target gas pressure. Readings of the oil levels were made through the telescope of a precision cathetometer located about 6 feet from the manometer. This instrument was capable of being read to the nearest 0.01 cm., so the minimum error expected in a difference reading of the oil columns was about 0.015 cm. Another error associated with rotation of the cathetometer about a non-level plane increased the total error in a measurement to approximately 0.3 percent.

2.10 Energy Calibration

The proton beam energy was determined by measuring the magnetic field required to bend the beam through preset slits at the entrance and exit of the 90 degree magnet. This field strength was determined by a nuclear moment resonance absorption device. The relativistically corrected energy of a charged particle beam from the Van de Graaff accelerator is given by

$$E_p \left(1 + \frac{E_p}{2m_0 c^2} \right) = k f^2 \quad (2.1)$$

where m_0 is the proton mass, c is the velocity of light, and f is the resonance frequency for either a hydrogen or lithium sample in the field of the analyzing magnet.

The absolute value of the constant k was determined by measuring the proton resonance frequency for the known resonance in the $C^{12}(p,p)C^{12}$ reaction at $4.808 \pm 0.008 \text{ MeV}^{18}$) and solving for k in equation 2.1. No measurements were made on the beam resolution or tail. From previous experience with this accelerator, the beam resolution was estimated to be about ± 0.05 percent fwhm. Based on later experimental measurements¹⁹), the integrated tail was thought to be less than 0.2 percent of the full energy peak.

CHAPTER III

DATA ACQUISITION AND REDUCTION

3.1 Data-Taking Procedure

The procedure for each run involved the recording of target gas pressure, chamber temperature, integrated beam current, nuclear magnetic resonance frequency, and total particle yield for each counter. Other auxiliary data were recorded when necessary. The energy spectrum of pulses from each counter was permanently recorded on magnetic tape for future analysis. The total yield from each counter, which was approximately proportional to the elastic scattering cross section, was plotted as a function of frequency as the data were being taken.

Two complete sweeps were made over the energy range of 3.71 to 5.67 MeV. Three counters were operated simultaneously during each sweep to give a total of six excitation curves. Five of the angles were chosen so as to lie near zeroes of Legendre polynomials for the elastic scattering and the sixth angle was the maximum possible back angle. Energy steps varied between 3 and 5 keV depending on the behavior of the cross section. In flat regions of the cross section, the larger energy intervals were used. After completing these excitation curves, a series of twenty-one angular distributions were taken at selected energies in this energy range. Angular distributions were taken with the moving counter in 10 degree steps from 30 degrees to 160 degrees. The energy spectrum of pulses from each counter was stored on punched

paper tape and magnetic tape. From time to time these spectra were printed out for visual inspection and preliminary analysis.

3.2 Data Reduction

The raw data for each energy and angle, consisting of frequency, yield spectrum, pressure, beam current, temperature, and other measurements had to be reduced to a form suitable for analysis. Thus the corrected energy and absolute cross section at each angle for all reactions had to be extracted. A general data reducing program for the CDC 1604 was written to handle these calculations²⁰). The essential parts of this program were:

1. INPUT - Two types of data were fed into this program.

Level one data consisted of information that was more or less constant over a period of time. This included calibration constants, energy loss tables, and type of reaction products involved. Level two data included total counts, beam current, frequency, chamber pressure, temperature, angles, run time, and run number.

2. Spectra Splitting - The experimental apparatus was set up to record data from several detectors simultaneously and produce spectra that were really a composite of spectra from several detectors. These individual spectra were split and labeled for proper identification.
3. Spectra Stripping - This sub-program²¹) performed a background subtraction, found peaks, and fit peaks with a skewed gaussian function

$$y_1 = \frac{A}{\sqrt{2\pi} \sigma} e^{-1/2\beta^2(1 - \alpha/2)} \quad (3.1)$$

where

$$\beta = \frac{x - x_0}{\sigma} \quad (3.2)$$

Here A is the area, σ is the width, x is the center of the peak, and α is the skew parameter (0.0005 in our case).

4. Energy Calculations - The proton energy at the scattering chamber center was calculated for each run. Calculation of the beam energy as it leaves the accelerator has been previously discussed (see page 22). Incident beam energy was corrected for energy losses in the chamber as the protons passed through the gaseous atmosphere. This energy loss was given by

$$\Delta E_{\text{gas}} = K_{\text{cham}} \cdot \frac{P_{\text{cham}}}{T(K)} \cdot \frac{dE}{dx_{\text{gas}}} \quad (3.3)$$

where

$T(K)$ = chamber temperature in degrees Kelvin

K_{cham} = an experimentally determined constant

$\frac{dE}{dx_{\text{gas}}}$ = energy loss of protons in ^{14}N

P_{cham} = pressure in chamber.

The constant K_{cham} was determined by observing energy shifts of the $^{12}\text{C}(p,p)^{12}\text{C}$ resonance at 4.808 MeV as a function of chamber pressure. The absolute uncertainty in this energy correction was estimated to be ± 0.3 keV/cm. or ± 3.3 keV for typical operating pressures. The relative data is, however, much better.

5. Dead Time Calculation - Analyzer dead time introduced an error in the cross section and had to be taken into account. The fractional dead time was taken as

$$DT = \frac{(\text{total scaler counts})_i - (\text{total analyzer counts})_i}{(\text{total scaler counts})_i}$$

where i refers to the appropriate electronic channel.

Typical dead times ranged between 5 and 20 percent.

6. Cross Section Calculation - The laboratory differential cross section $\sigma(\theta)$ was given by

$$\sigma(\theta) = \frac{K \cdot N \cdot T^0(K) \cdot \text{SIN } \theta}{P_{\text{cham}} \cdot CI \cdot C \cdot G \cdot \left(1 - \frac{DT}{100}\right) \cdot Na} \quad (3.4)$$

where

$K = \text{constant} = 2.11666 \times 10^{-14}$

$N = \text{number counts in each peak after background subtraction}$

$T(K) = \text{chamber temperature in degrees Kelvin}$

$\theta = \text{laboratory scattering angle}$

P_{cham} = chamber pressure in cm. of oil

CI = number of current integrator counts

C = number of Coulombs of charge per current integrator count

G = geometry factor of the detector slit system

DT = fractional dead time correction

N_a = number of atoms per molecule in the target gas

The percentage statistical error $\frac{\Delta\sigma}{\sigma}$ for each cross section measurement was also calculated.

7. Conversion to Center of Mass - For purpose of analysis, some of the calculated cross sections were converted to the center of mass system. This transformation of coordinates was given non-relativistically by

$$\sigma_{\text{cm}} = \frac{\sin^2 \theta_L}{\sin^2 \theta_{\text{cm}}} \cdot \cos(\theta_{\text{cm}} - \theta_L) \cdot \sigma_{\text{Lab}} \quad (3.5)$$

Finally, the run number, corrected center of mass cross section (or laboratory cross section), electronic channel number, beam energy at the chamber center, statistical error, and scattering angle for each measurement were punched on paper cards. The desired cross sections with their statistical errors were plotted on graph paper.

3.3 Corrections Not Made

Because of the large number of data points (~ 5000) and the insensitivity of the theoretical analysis to small errors in the absolute cross section a few of the minor corrections were omitted. These included:

1. slit edge scattering
2. multiple scattering
3. detector efficiency
4. loss of beam particles between target volume and Faraday cup
5. finite slit size effect.

Subsequent investigation¹⁹⁾ indicated that these effects were less than a few tenths percent.

3.4 Accuracy of the Cross Sections

No attempt was made to measure these cross sections to the ultimate accuracy attainable with the scattering chamber. In addition to the ignored corrections described above, it was later discovered¹⁹⁾ that the Faraday cup was not operating satisfactorily. In particular, it appeared that the magnetic secondary electron suppressor system was not completely effective. This effect was beam dependent and probably time dependent and it was difficult to estimate how much error was introduced into the present absolute and relative measurements. A safe estimate was to ascribe a normal distribution error with standard deviation ± 4 percent to all cross sections presented. This estimate was based primarily on the fact that a careful measurement of p-p scattering was in good agreement (0.5 percent) with previous

measurements²²). Also it should be noted that unexplained discrepancies in the present cross sections of as much as 1.5 percent appeared in some cases.

3.5 Accuracy of the Energy Measurement

Errors in the absolute energy measurement as a result of uncertainties in the calculated energy loss of protons as they passed through nitrogen gas, uncertainties in the magnet calibration constant, and finite beam resolution have been previously discussed. In addition, there was a spreading of the energy as a result of straggling in the chamber gas and as a result of finite target thickness. Straggling was calculated for typical energy and pressure (5.0 MeV and 11.0 cm.) using the accurate theory of Symon²³). The result was a moderately asymmetric distribution ($\lambda = 1.25$) and a standard deviation of $\lambda = \pm 3$ keV. Exact distributions may be seen in Rossi's book²⁴) (page 34). At 4.8 MeV, the energy loss for protons in ^{14}N (diatomic) at 0.01 atmosphere (typical of the present experiment) is 2.26 keV/inch. The actual target thickness may be computed by multiplying this number by the path length given in Table 2.1.

Absolute accuracy of the energy scale was estimated to be ± 15 keV with the relative accuracy being somewhat better. It was found necessary to shift excitation functions measured at different times by as much as 5 keV to produce a satisfactory match. Calculations of the energy included corrections for relativistic effects and magnet inhomogeneity.

3.6 Summary of the Experimental Data

The data which resulted from this experiment consisted of the following:

I. $^{14}\text{N}(\text{p},\text{p})^{14}\text{N}$ and $^{14}\text{N}(\text{p},\text{p}_1)^{14}\text{N}$

Excitation functions in the energy range 3.707 MeV to 5.665 MeV were measured at the angles shown below. This data is presented in Figures 3.1-3.4.

Laboratory Angle	Slit System	Electronic Channel
67.51°	2 (fixed)	3
85.91°	1M (movable)	1
105.81°	1M (movable)	1
123.71°	1 (fixed)	2
135.00°	1 (fixed)	2
157.48°	3 (fixed)	3

Angular distributions (see Figures 3.5-3.6) were measured at bombarding energies of 3.868, 3.900, 3.966, 3.988, 3.995, 4.021, 4.126, 4.186, 4.430, 4.548, 4.575, 4.770, 4.801, 4.870, 4.911, 4.986, 5.021, 5.193, 5.304, 5.506, and 5.550 MeV.

II. $^{14}\text{N}(\text{p},\text{p}_2)^{14}\text{N}$ and $^{14}\text{N}(\text{p},\alpha)^{11}\text{C}$

Data are presented for a $^{14}\text{N}(\text{p},\text{p}_2)^{14}\text{N}$ excitation function at a laboratory angle of 85.91 degrees in the bombarding energy range 5.274 MeV to 5.665 MeV. Limited angular distributions for $^{14}\text{N}(\text{p},\text{p}_2)^{14}\text{N}$ and $^{14}\text{N}(\text{p},\alpha)^{11}\text{C}$ were measured at bombarding energies of 4.986, 5.021, 5.193, 5.304, 5.506, and 5.550 MeV. Additional raw data exists for excitation functions in the following regions:

Reaction	Laboratory Angle	Energy Range (MeV)
$^{14}\text{N}(\text{p}, \text{p}_2)^{14}\text{N}$	123.71°	5.298-5.665
$^{14}\text{N}(\text{p}, \text{p}_2)^{14}\text{N}$	157.48°	5.370-5.665
$^{14}\text{N}(\text{p}, \text{p}_2)^{14}\text{N}$	105.81°	5.125-5.687
$^{14}\text{N}(\text{p}, \text{p}_2)^{14}\text{N}$	135.00°	5.327-5.687
$^{14}\text{N}(\text{p}, \alpha_0)^{11}\text{B}$	85.91°	5.316-5.665

The spectrum stripping code used to reduce the data of this experiment did not give reliable results for those peaks in the regions indicated above and it would have been necessary to analyze the spectra for these runs by hand. It was clear from an examination of the data that no additional sharp structure existed in the excitation functions not presented.

Figure 3.1 Experimental Differential Cross Section Curves.

Figure 3.2 Experimental Differential Cross Section Curves.

Figure 3.3 Experimental Differential Cross Section Curves.

Figure 3.4 Experimental Differential Cross Section Curves.

Figure 3.5 Experimental Angular Distributions.

Figure 3.6 Experimental Angular Distributions.

Figure 3.7 Experimental Angular Distributions and Cross Section Curve.

Figure 3.8 Experimental Angular Distributions.

Figure 3.9 Experimental Angular Distributions.

CHAPTER IV

THEORY

4.1 General Discussion

The scattering of charged particles from light nuclei at low energies often furnishes information about the properties of excited levels of the compound nucleus. The immediate aim of an analysis in such a scattering experiment is the assignment of level parameters to resonances observed in the excitation curves. These resonances may be thought of as being produced by virtual, or unbound, levels in the compound nucleus formed by the target and bombarding particle. The mathematical theory used in a treatment of these virtual levels was first developed by Breit and Wigner²⁵), and later extended by Wigner and Eisenbud²⁶). The most comprehensive development of this theory, based on the R-matrix formalism of Wigner and Eisenbud is given by Lane and Thomas²⁷). This treatment forms a suitable approach for the discussion of nuclear reactions and makes possible a reasonable physical interpretation of the parameters involved.

In this approach to the study of nuclear reactions the system of bombarding particles and nucleus is thought to be composed of a multi-dimensional configuration space divided into two parts. There exists an internal region where the separation of all nucleons of the systems is of the order of nuclear dimensions. This is the region in which all nuclear reactions are to take place. The external region forms the rest of configuration space and usually describes processes

outside the range of nuclear forces. These two regions are separated by a multi-dimensional surface S . In the internal region the system is composed of many channels, each channel C representing a method or mode of formation of the compound nucleus. For a given channel C there is defined an interaction radius a_c which is the minimum radial distance of separation of the pair at which neither nucleus experiences polarizing forces from the other.

The essence of the formal theory of scattering is the description of this internal region in terms of a set of physically interpretable parameters that can, in turn, be directly related to the scattering cross section. The development is as follows: The cross section is expanded in terms of a collision matrix U , the internal region is described by an R -matrix which is then expanded into a set of nuclear parameters, and the collision matrix is then evaluated in terms of the R -matrix. The result is an experimental cross section expressed in terms of a formal set of parameters that are amenable to theoretical calculations. Since the development of theoretical expressions for the cross section have been fully described in the literature^{27,28,29}, only the relations required for an understanding of the analysis will be discussed. In general, the notations and definitions follow those of Lane and Thomas.

4.2 Notation and General Formulae

Consider a reaction of the type

in which particle A collides with nucleus X. After the collision, particle B emerges at an angle θ to the direction of the beam and the recoil nucleus Y is projected at an angle ϕ . The incident particle and target nucleus are considered to be in a channel C represented by the indices (α, ℓ, s) . The channel index α gives the type of incoming particle and the state of the struck nucleus and ℓ gives their relative orbital angular momentum. The channel spin $\vec{s}$ is the total spin angular momentum obtained by vector addition of the intrinsic spins of the projectile and target nucleus, $\vec{I}_1$ and $\vec{I}_2$,

$$\vec{s} = \vec{I}_1 + \vec{I}_2 \quad (4.2)$$

The Z-component of $\vec{s}$ is denoted by m_s where the Z-axis is taken in the direction of the incident beam of particles. The channel spin $\vec{s}$ can take on values in integral steps from $|\vec{I}_1| + |\vec{I}_2|$ to $|\vec{I}_1| - |\vec{I}_2|$. The total angular momentum is obtained by coupling $\vec{\ell}$ and $\vec{s}$,

$$\vec{J} = \vec{\ell} + \vec{s} \quad (4.3)$$

The reaction products are considered to be in a channel C' represented by indices (α', ℓ', s') . Conservation of angular momentum and its Z-component give the following relations:

$$\vec{\ell} + \vec{s} = \vec{J} = \vec{\ell}' + \vec{s}' \quad (4.4)$$

$$m_s = M = m_{s'} + m_{\ell'}$$

where $m_{\ell} = 0$ because the Z-axis was chosen along the incident beam.

If the incident beam and target nuclei are unpolarized, the theoretical differential cross section for the reaction $\alpha \rightarrow \alpha'$ is

$$d\sigma_{\alpha, \alpha'}(\theta, \varphi) = \frac{1}{(2I_1 + 1)(2I_2 + 1)} \sum_{\substack{ss' \\ m_s m_{s'}}} |A_{\alpha' s' m_{s'}, \alpha s m_s}|^2 \quad (4.5a)$$

The scattering amplitude, $A_{\alpha' s' m_{s'}, \alpha s m_s}$ is given by

$$\begin{aligned} A_{\alpha' s' m_{s'}, \alpha s m_s} &= \frac{\sqrt{\pi}}{k_\alpha} \{ -C_{\alpha'}(\theta) \delta_{\alpha' s' m_{s'}, \alpha s m_s} \\ &+ i \sum_{J M l l'} (2l + 1)^{1/2} (s l m_s 0 | J M) (s' l' m_{s'} m_{l'} | J M) \cdot \\ &T_{\alpha' s' l', \alpha s l}^J \cdot Y_{l' m_{l'}}(\theta, \varphi) \end{aligned} \quad (4.5b)$$

The quantity $C(\theta)$ represents the Coulomb scattering amplitude and is given by

$$C_{\alpha'}(\theta) = \frac{1}{\sqrt{4\pi}} \cdot \eta_{\alpha'} \operatorname{cosec}^2(\theta/2) \exp \{ -2i\eta_{\alpha'} \cdot \ln(\sin \theta/2) \} \quad (4.6)$$

The Coulomb parameter η is defined by

$$\eta = \frac{Z_1 Z_2 e^2 \mu}{\hbar k} \quad (4.7)$$

The de Broglie wave number of relative motion is given by

$$k = \left(\frac{2\mu E}{\hbar^2} \right)^{1/2} \quad (4.8)$$

The quantities μ , E , and θ represent the reduced mass, the center of mass energy, and the center of mass scattering angle, respectively. The $Y_{\ell, m_{\ell}}$ are the usual spherical harmonics. The T matrix, as defined by Lane and Thomas, can be written in terms of the unitary and symmetric collision matrix U:

$$T_{\alpha' \ell' s', \alpha \ell s}^J = e^{2i\omega_{\alpha' \ell'}} \delta_{\alpha' \ell' s', \alpha \ell s} - U_{\alpha' \ell' s', \alpha \ell s}^J \quad (4.9)$$

where J is the total angular momentum as previously defined and $\delta_{\alpha' \ell' s', \alpha \ell s}$ is a Kroneker delta which is unity if $s' = s$, $\ell' = \ell$, $\alpha' = \alpha$ and zero otherwise. The Coulomb phase shift for orbital angular momentum ℓ relative to the $\ell = 0$ Coulomb phase shift is given by:

$$\omega_{\ell} = \sum_{m=1}^{\ell} \tan^{-1}(\eta/m), \quad \omega_0 = 0. \quad (4.10)$$

The differential cross section as given by equation 4.5 does not rely on any assumptions concerning a nuclear model. It is valid for any nonrelativistic reaction involving an initial and final projectile and target nucleus. The nature of the nuclear force is contained in the collision matrix U. This collision matrix can be expressed in terms of an R-matrix which in turn is directly related to the parameters involved in the analysis of data.

The collision matrix may be expressed, in matrix notation, in the following manner²⁷):

$$\begin{aligned} U^J &= \Omega W^J \Omega \\ &= \Omega [\underline{1} + P^{1/2} (\underline{1} - R^J L^0) R^J P^{1/2} W] \Omega \end{aligned} \quad (4.11)$$

Here Ω , P , L^0 , and w are diagonal matrices whose elements are

$$\begin{aligned}\Omega_{cc}' &= \delta_{cc}' e^{i(\omega_c - \varphi_c)} \\ P_{cc}' &= \delta_{cc}' \rho_c / (F_c^2 + G_c^2) \\ L_{cc}^0 &= \delta_{cc}' [S_c - B_c + iP_c] \\ w_{cc}' &= 2i\delta_{cc}'\end{aligned}\tag{4.12}$$

where ω_c is the previously defined Coulomb phase shift, φ_c is the hard sphere phase shift

$$\varphi_c = \tan^{-1} F_c / G_c\tag{4.13}$$

F_c and G_c are the regular and irregular solutions for channel C in the external region, P_c is the penetrability, S_c is the energy shift term defined by

$$S_c = \rho_c (F_c F_c' + G_c G_c') / (F_c^2 + G_c^2)\tag{4.14}$$

and B_c is the arbitrary boundary value parameter. The dimensionless quantity ρ_c is given by

$$\rho_c = k_c r_c\tag{4.15}$$

where k_c is the wave number and r_c is the radial coordinate for channel C . The primes indicate differentiation with respect to ρ_c .

R^J is the Wigner R-matrix given by

$$R^J = \sum \frac{(\vec{\gamma}_\lambda \times \vec{\gamma}_\lambda)}{E_\lambda - E} \quad (4.16)$$

where $\vec{\gamma}_\lambda$ is a vector whose elements, $\gamma_{\lambda c}$, are the reduced width amplitudes for the various channels. The E_λ are the characteristic energies of the nuclear states. $(\vec{\gamma}_\lambda \times \vec{\gamma}_\lambda)$ is the direct product of the vectors $\vec{\gamma}_\lambda$ and $\vec{\gamma}_\lambda$.

The expression for U^J , equation 4.11, involves inversion of the matrix $(\underline{1} - R^J L^O)$ whose dimensions equal the number of channels. It is convenient to replace the problem of inverting the channel matrix $(\underline{1} - R^J L^O)$, the components of which refer to channel C, to the one involving the inversion of a level matrix, the components of which refer to the levels of the system. The R^J matrix is split into two parts corresponding to a division of the levels into two groups:

$$R^J = R^O + R' \quad (4.17)$$

where R' are those levels of a given spin and parity which are treated explicitly in the analysis and R^O is the "background" matrix consisting of the tails from all far away resonances. The matrix product $(\underline{1} - R^J L^O) R^J$ (equation 4.11) now becomes

$$\begin{aligned} (\underline{1} - R^J L^O) R^J = \\ (\underline{1} - R^O L^O)^{-1} R^O + (\underline{1} - R^O L^O)^{-1} (\underline{1} - R' L')^{-1} R' (\underline{1} - L^O R^O)^{-1} \end{aligned} \quad (4.18)$$

where L' is given by

$$L' = L^O (\underline{1} - R^O L^O)^{-1} \quad (4.19)$$

The matrix $(\underline{1} - R'L')R'$ can be written in the form

$$(\underline{1} - R'L')^{-1}R' = \sum_{\lambda\mu} (\vec{\gamma}_\lambda \times \vec{\gamma}_\mu) A_{\lambda\mu} \quad (4.20)$$

Substitution of equation 4.18 and 4.20 into equation 4.11 leads to the desired expression for the collision matrix,

$$U^J = U^0 + \Omega \cdot P^{1/2} \left(\sum_{\lambda\mu} (\vec{\alpha}_\lambda \times \vec{\alpha}_\mu) A_{\lambda\mu} \right) P^{1/2} \cdot w\Omega \quad (4.21)$$

where

$$\vec{\alpha}_\lambda = (\underline{1} - R^0 L^0)^{-1} \vec{\gamma}_\lambda$$

with the background matrix U^0 given by

$$U^0 = \Omega (\underline{1} + P^{1/2} (\underline{1} - R^0 L^0)^{-1} R^0 P^{1/2} w) \Omega \quad (4.22)$$

The $A_{\lambda\mu}$ are elements of a level matrix A given by

$$A = (e - E - \xi)^{-1} \quad (4.23)$$

where e is a diagonal matrix with component E_λ , E is the energy E times the unit matrix and ξ is a symmetrical matrix given by

$$\xi = -\Delta + i \Gamma/2 \quad (4.24)$$

The matrix Δ of the real part is referred to as the Thomas level shift matrix whose elements are

$$\Delta_{\lambda\mu} = (\vec{\alpha}_\lambda^*, (L^{0*} R^0 L^0 - S^0) \vec{\alpha}_\mu), \quad S^0 = S - B \quad (4.25)$$

expressed as a scalar product with respect to channels. The matrix Γ of the imaginary part of ξ is the width matrix and is given by

$$\Gamma_{\lambda\mu} = \sum_C 2P_C \alpha_{\lambda C}^* \alpha_{\mu C} \quad (4.26)$$

where the sum C is over channels.

These are the general results of R-matrix theory. In practical application of this theory it will be necessary to make some simplifying approximations.

4.3 Approximations in the R-Matrix Formulism

I. Single Level Approximation

The most common approximation used to simplify the analysis of data is the neglect of all but one of the levels E_λ in the R-matrix expansion. It should be remembered that the R function refers to levels of a specified spin and parity. Hence in making this approximation we are simply neglecting all other levels of the same spin and parity as denoted by the R^J . The single level formula becomes an approximation only when there are nearby states which have the same spin and parity. A general criteria for validity of the single level formula is that the level width Γ be much smaller than the level spacing.

In this single level approximation we set $R^0 = 0$ and $\lambda = \mu = 1$ and equation 4.21 becomes

$$U = U^0 + \frac{\Omega P^{1/2} w(\vec{\gamma}_0 \times \vec{\gamma}_0) P^{1/2} \Omega}{E_0 - E + \Delta - i\Gamma_0/2} \quad (4.27)$$

In this case,

$$U_{cc}^0 = \delta_{cc} e^{2i(\omega_c - \varphi_c)}$$

$$\Omega_{cc} = \delta_{cc} e^{i(\omega_c - \varphi_c)}$$

$$\begin{aligned}
P_{cc}' &= \delta_{cc}' \rho_c / (F_c^2 + G_c^2) \\
L_{cc}^0 &= \delta_{cc}' [S_c - B_c + iP_c] \\
\Delta^0 &= \sum_c (S_c - B_c) \gamma_c^2 \\
\Gamma_0 &= 2 \sum_c P_c \gamma_c^2 \\
U_{cc}^J &= \delta_{cc}' e^{2i(\omega_c - \varphi_c)} + 2ie^{i(\omega_c + \omega_{c'} - \varphi_c - \varphi_{c'})} \\
&\quad \frac{x(P_c P_{c'})^{1/2} \gamma_c \gamma_{c'}}{E_0 + \Delta - E - i\Gamma_0/2}
\end{aligned} \tag{4.28}$$

II. Two Level Approximation

In some cases there will be two levels of the same spin and parity that do not satisfy the condition that the width of these resonances is less than their separation. Here the two level approximation must be used. In this case we set $R_0 = 0$ and

$$R = R_1 + R_2 = \frac{\vec{\gamma}_1 \times \vec{\gamma}}{E_1 - E} + \frac{\vec{\gamma}_2 \times \vec{\gamma}_2}{E_2 - E} \tag{4.29}$$

The collision matrix U from equation 4.11 reduces to

$$U = U_0 + \vec{\Omega} P^{1/2} w \left(\sum_{\substack{\lambda = 1,2 \\ \mu = 1,2}} (\vec{\alpha}_\lambda \times \vec{\alpha}_\mu) A_{\lambda\mu} \right) P^{1/2} \Omega \tag{4.30}$$

with elements

$$\begin{aligned}
U_{cc}' &= \delta_{cc}' e^{2i(\omega_c - \varphi_c)} + 2ie^{i(\omega_c + \omega_{c'} - \varphi_c - \varphi_{c'})} (P_c P_{c'})^{1/2} \\
&\quad \sum_{\lambda, \mu = 1,2} \gamma_{\lambda c} \gamma_{\mu c'} A_{\lambda\mu}
\end{aligned} \tag{4.31}$$

The elements of the level matrix A, are

$$\begin{aligned} A_{11} &= (E_2 - E - \xi_{22})/D \\ A_{22} &= (E_1 - E - \xi_{11})/D \\ A_{12} &= A_{21} = \xi_{12}/D \end{aligned} \quad (4.32)$$

Here D is the determinant of this level matrix,

$$D = (E_1 - E - \xi_{11})(E_2 - E - \xi_{22}) - \xi_{12}^2 \quad (4.33)$$

Also,

$$\xi_{\lambda\mu} = -\Delta_{\lambda\mu} + i/2 \Gamma_{\lambda\mu} \quad (4.34)$$

with

$$\Delta_{\lambda\mu} = \sum_c \gamma_{\lambda c} \gamma_{\mu c} (S_c - B_c) \quad (4.35)$$

and

$$\Gamma_{\lambda\mu} = \sum_c \gamma_{\lambda c} \gamma_{\mu c} P_c \quad (4.36)$$

U_{cc} may now be evaluated by substituting these elements of A into equation 4.31.

4.4 Discussion of Parameters

A large number of parameters are introduced in the R-matrix treatment of nuclear reactions. Before applying these formulae to an analysis of cross section data it is desirable to understand the role of each parameter and just what physical interpretation can be ascribed to it.

I. The Boundary Value B_c

The boundary condition used on the surface S separating the internal and external regions of space serves to define a set of arbitrary states χ_λ in the internal region. In theory its value is quite arbitrary but, in practice, we should like to choose a value that would make some states of this complete orthonormal set χ_λ coincide with states of the actual internal wave function. This can best be done by choosing B_c so that the E_0 of theory falls within the observed experimental resonance. A suitable choice is to make the shift function in the denominator of equation 4.27 equal to 0 at $E = E_0$. This places E_0 at the maximum of the resonant term of $U_{cc'}$.

II. The Interaction Radius

The radius "a" is also an adjustable parameter that is generally chosen as the minimum radial distance of separation of the interacting pair at which neither nucleus experiences any polarizing force from the other. A commonly used prescription has been

$$a = r_0 (A_1^{1/3} + A_2^{1/3}) \quad (4.37)$$

where r_0 is independent of A_1 and A_2 (mass numbers) and has a numerical value of about 1.45×10^{-13} cm. It should be pointed out that it is not necessary to place this restriction on "a". Instead, one need only insist that the radial wave equation in the external region be solvable. Having a non-vanishing potential in the external region would merely modify the form of the incoming and outgoing waves.

The potential phase shifts of equation 4.13 depend strongly on "a". The dependence of cross section between resonances which is

determined by this potential scattering is very artificial. It is compensated for in the exact theory by the distant levels which also depend on the radius.

III. The Potential Phase Shifts

The collision matrix of equation 4.9 may be written as a sum of two parts (as previously discussed)

$$U = U^O + U^R \quad (4.38)$$

where U^O is the background scattering associated with distant levels,

$$U^O = \Omega (\underline{1} + P^{1/2} (\underline{1} - R^O L^O)^{-1} R^O P^{1/2} w) \Omega \quad (4.39)$$

and U^R arises from the nearby resonances that are treated explicitly,

$$U_R = \Omega P^{1/2} \left(\sum_{\lambda\mu} (\bar{\alpha}_\lambda \times \bar{\alpha}_\mu) A_{\lambda\mu} \right) P^{1/2} w \Omega \quad (4.40)$$

For the case of a single isolated resonance where distant levels are neglected ($R^O = 0$) the background matrix is diagonal with elements

$$U_{cc}^O = e^{2i(\omega_c - \varphi_c)} \quad (4.41)$$

where ω_c and φ_c are the Coulomb and hard sphere phase shifts. For the case where there are contributions from distant levels but R^O is considered to be diagonal, the background matrix is again diagonal with elements

$$U_{cc} = e^{2i(\omega_c - \delta_c)} \quad (4.42)$$

where δ_c is the potential phase shift. In the limit that far off levels do not contribute to the scattering the δ_c reduce to hard sphere phase

shifts φ_c , where $\varphi_c = \arctan (F_c/G_c)$ the F_c and G_c being the regular and irregular Coulomb functions mentioned previously. The theory in which δ_c is replaced by φ_ℓ also requires that the potential or nonresonant scattering in other channels be insignificant and that the elastic scattering polarization be close to zero away from resonance levels (since φ_ℓ is independent of J).

In practice the potential phases are assumed to have only an ℓ dependence as is the case for hard sphere phases. They are then varied in order to give a reasonable fit to the data in off-resonance regions. If all resonances were known and could be treated explicitly then the δ_c would be given purely by the hard sphere contribution.

IV. Reduced Width and Energy Eigenvalues

The parameters E_λ and γ_λ^2 are real energy independent constants that depend on the radius "a" and the boundary value B_c . The order of magnitude of the reduced width is an indication of the single particle character of a level. Teichmann and Wigner³⁰⁾ have derived a sum rule which places a limit on the value of the reduced width. If the single particle description is applicable for the internal wave functions, i.e., if they can be expressed as the product of a wave function of a stationary state and a function of the distance of the extra particle from the residual nucleus, then

$$\gamma_\lambda^2 = \frac{3\hbar^2}{2\mu a} = \gamma_w^2 \quad (4.43)$$

If a multiple particle picture applies, the sum over the stationary states of the residual nucleus cannot be reduced to a single term as

in the single particle states. In this case γ_λ^2 is much smaller than the quantity given above.

If a level can be described by a single particle wave function, the reduced width is always of the order of magnitude of the Wigner limit although the converse is not true. A comparison of the reduced widths of a level with the Wigner single particle limit provides an indication of the applicability of the single particle picture to a description of the level.

CHAPTER V

ANALYSIS OF THE DATA

5.1 General Discussion

In general, two types of analyses are performed on elastic scattering data and these are classified by the number and type of parameters used to describe the scattering. One type of analysis is normally used when it is necessary to determine the energy dependence of several slowly varying phases. Such a situation occurs when the resonant levels are very broad and overlapping or when it is desired to determine the behavior of several non-resonant phases. In this type of analysis, angular distributions are measured over the energy region of interest and are subsequently analyzed to gain information on the magnitude and energy dependence of terms describing the scattering.

In another type of analysis used, for example, in the vicinity of narrow or isolated resonances; excitation curves are studied at several angles. Because the resonant scattering amplitude interferes with the Coulomb and nuclear potential scattering amplitudes, the qualitative shape and magnitude of a resonance depends strongly upon the particular values of l and J ; thus the total angular momentum and parity of a resonance are determined from an analysis of the excitation curves.

It is often necessary to apply both types of analyses in a scattering experiment. Angular distributions must be analyzed to obtain an adequate description of scattering in the off-resonance

regions. Then resonant scattering amplitudes are examined as a function of energy to determine parameters associated with the nucleus, such as resonant energies, spins, parities, and widths of nuclear states.

An inspection of Figures 3.1 - 3.4 shows that many resonances were observed in the experimental elastic scattering cross section of protons by ^{14}N and that several of these resonances had significant partial widths for the inelastic channel. Analysis of these data was carried out using the R-matrix formalism of Chapter IV. For resonances with narrow widths, the one level approximation for the R-matrix can be successfully applied. For broad resonances, however, the effects of interference from nearby levels must be considered. The differential cross section for elastic scattering of charged particles is found from equations 4.5 and 4.9.

These equations are exact; it is only in their application to a particular scattering problem that specific approximations must be made. The usual procedure is to assume a narrow level hypothesis which states that levels of the same spin and parity do not overlap appreciably. It is also usually assumed that only those levels explicitly considered contribute to the scattering, i.e., that no contribution results from distant levels.

Under these assumptions, elements of the collision matrix may be written as (equation 4.27)

$$U_{s'l',sl}^J = e^{i(\omega_l - \varphi_l)} e^{i(\omega_{l'} - \varphi_{l'})} \left\{ \delta_{s'l',sl} + \frac{1(\Gamma_{sl}^J \Gamma_{s'l'}^J)^{1/2}}{\Gamma_T^J} \right\} \\ E_\lambda + \Delta - E - i\frac{\Gamma_T}{2} \quad (5.1)$$

where the $\Gamma_{s\ell}^J$ are partial widths for formation (or decay) of a state with spin J , formed with orbital angular momentum ℓ through channel spin s . E_λ is the eigenenergy of the state, Δ_λ the Thomas level shift, E the incident particle energy, and Γ_T^J is the total width of the state. φ_ℓ is the hard sphere scattering phase shift for orbital angular momentum ℓ and is given by (equation 4.13)

$$\varphi_\ell = \tan^{-1}(F_\ell/G_\ell)_a \quad (5.2)$$

where F_ℓ and G_ℓ are Coulomb wave functions evaluated at radius "a" (usually taken as the nuclear radius).

This expression for elements of the collision matrix is also valid when there are background processes which contribute only to diagonal elements of the R-matrix. This assumption of a diagonal background matrix is not unreasonable because individual level contributions to the nondiagonal components can be positive as well as negative and tend to cancel, whereas the diagonal components are all proportional to the positive reduced widths. In this situation the definition of Δ_λ , φ_ℓ , and $\Gamma_{s\ell}^J$ must be slightly altered²⁷). However, this provides for the possibility that φ_ℓ need not be strictly interpreted as a hard sphere phase shift and allows considerable freedom in fitting experimental data without considering more complicated forms of the collision matrix.

The collision matrix U^J is separated into a diagonal part D^J , leaving a generally non-diagonal remainder X^J ,

$$U^J = D^J + X^J \quad (5.3)$$

The general scattering formulae become especially simple if D^J , like $C(\theta)$ is independent of J ,

$$D_{s'l',sl}^J = D_{sl} \quad (5.4)$$

Then

$$D_{s'l'm_s',m_l',slm_s} = \delta_{s'l'm_s',m_l',slm_s} D_{sl} \quad (5.5)$$

and by defining a potential scattering amplitude as

$$\begin{aligned} P_s &= -i \sum_l (2l+1)^{1/2} (e^{2i\omega_l} - D_{sl}) Y_l^0 \\ &= -i(4\pi)^{-1/2} \sum_l (2l+1) (e^{2i\omega_l} - D_{sl}) P_l(\cos \theta) \end{aligned} \quad (5.6)$$

the complete scattering amplitude of equation 4.5 reduces to

$$\begin{aligned} A_{s'm_s',sm_s} &= \frac{\sqrt{\pi}}{k} [(C(\theta) + P_s(\theta)) \delta_{s'm_s',sm_s} + \\ &\quad i \sum_{Jl} (2l+1)^{1/2} (slm_s | JM) (s'l'm_s',m_l' | JM) \\ &\quad \cdot X_{s'l',sl}^J \cdot Y_{l'm_l'}] \end{aligned} \quad (5.7)$$

By assuming that the off resonance scattering is given by Coulomb scattering, hard sphere scattering, and a contribution from the tails of far-away levels and that this diagonal contribution is the same for levels of all spins and parities, the diagonal matrix becomes

$$D_{sl} = e^{2i(\omega_l - \delta_{sl})} \quad (5.8)$$

where the δ_{sl} refer to potential phase shifts. If distant levels are neglected or do not contribute, the δ_{sl} become hard sphere phase shifts φ_l (see equation 5.2). The non-diagonal matrix X^J describes resonance scattering and in the one-level approximation is given by

$$X_{s'l'sl} = e^{i(\omega_l - \delta_{sl})} e^{i(\omega_{l'} - \delta_{s'l'})} \frac{(\Gamma_{sl}^J \Gamma_{s'l'}^J)^{1/2}}{E_0 + \Delta - E - i\Gamma_T^J/2} \quad (5.9)$$

In some cases the two level approximation discussed in Chapter IV must be used.

The partial width of a state Γ_{sl}^J can be written in terms of the channel spin ratio

$$\Gamma_{sl}^J = (\alpha_{sl}^J)^2 \Gamma_l^J \quad (5.10)$$

where the coefficients are normalized to unity:

$$\sum_0 (\alpha_{sl}^J)^2 = 1 \quad (5.11)$$

The partial width Γ_l^J of a level is related to the penetration factor P_l by,

$$\Gamma_l^J = 2 P_l (\gamma_l^J)^2 \quad (5.12)$$

where γ_l^J is the reduced width amplitude of the compound nuclear state.

The theoretical expression for the differential cross section (equation 4.5a) was programmed for the CDC 1604 computer. This code was made sufficiently general so that the elastic scattering of charged or uncharged particles of arbitrary spin could be analyzed. The total scattering amplitudes, $A_{s'm_s',sm_s}$, were taken from equation 5.7. The

two-level approximation for the collision matrix (equation 4.31) was used in order that two resonances of the same spin and parity in close proximity could be treated exactly. For the case of an isolated resonance, the resonant part of this two-level approximation reduces to equation 5.9. The potential scattering amplitudes were taken from equations 5.6 and 5.8.

As many as sixteen resonances of different spin and parity could be treated simultaneously. Present limitations on the code are that the number of scattering amplitudes cannot exceed one hundred and no more than two ℓ -values forming a given J^π value are allowed.

Input to the code was in the form of trial resonant parameters for a pre-selected number of resonances and an ℓ -dependent background that varied with energy. Options to the background included hard sphere phase shifts (equation 5.4), hard sphere phase shifts plus a constant term, phase shifts that were independent of energy, phase shifts that were linear in energy, and phase shifts that were functions of the energy squared. Other forms are allowed and can be easily included.

Output was in the form of computer plotted excitation functions. Both the theoretical (calculated) and experimental (input data) cross sections were plotted as an aid to analysis, at up to six scattering angles, over a specified energy range.

5.2 Application of the Theory to $^{14}\text{N}(p,p)$ Scattering

When protons are incident upon ^{14}N at bombarding energies between 3.7 and 5.7 MeV the total width of each resonance is composed of several particle widths:

$$\Gamma_T = \Gamma_p + \Gamma_{p'} + \Gamma_\gamma + \Gamma_\alpha \quad (5.13)$$

where p is the elastic channel, p' is the inelastic channel, γ is the capture gamma ray channel and α is the alpha particle channel. The ratio of the partial proton width for elastic scattering to the total width Γ_p/Γ_T appears as a parameter in the differential cross section for elastic scattering. In this analysis the α and γ channels were neglected. The capture gamma ray channel is much smaller than the corresponding particle channels and its neglect is justified. No significant cross section was observed for the alpha particle channel and for the channel corresponding to inelastic scattering from the second excited state of ^{14}N . Thus, the total width for each resonance was given by the partial widths for elastic scattering and for inelastic scattering from the first excited state of ^{14}N .

The non-zero spin of ^{14}N complicated the analysis by increasing considerably the number of parameters to be determined. One must combine a proton of spin and parity $1/2^+$ with a ^{14}N nucleus of spin and parity 1^+ to give channel spins ($\vec{s} = \vec{s}_p + \vec{s}_N$) of $1/2$ and $3/2$. Thus, there were as many as thirty-six channel-to-channel combinations $(s, m_s) \rightarrow (s', m_{s'})$ giving the number of scattering amplitudes, $A_{s', m_{s'}, s, m_s}$ to be determined. Each state of a given spin and parity could be formed by two partial waves; hence, the value of ℓ could change during the scattering process. All possible ℓ values for a given J^π value are shown in Table 5.1. The initial method of analysis was to study theoretical resonance shapes for these various possible ℓ, J, π , and channel spin configurations. However, before this could be done, it was necessary to have an adequate description of the background scattering.

TABLE 5.1

ALLOWED VALUES OF THE ORBITAL ANGULAR MOMENTUM

J^π	ℓ_{p_0}		$\ell_{p_1}^{a)}$
	$s = 1/2$	$s = 3/2$	$s = 1/2$
$1/2^+$	0	2	0
$1/2^-$	1	1	1
$3/2^+$	2	0,2	2
$3/2^-$	1	1,3	1
$5/2^+$	2	2,4	2
$5/2^-$	3	1,3	3
$7/2^+$	4	2,4	4
$7/2^-$	3	3,5	3

^{a)} Channel spin $s = 3/2$ is not allowed for the inelastically scattered protons.

5.3 Background Scattering

Background scattering processes were described by scattering amplitudes,

$$A_{s', m_s', sm_s} = \frac{\sqrt{\pi}}{k} \{C(\theta) + P_s(\theta)\} \delta_{s', m_s', sm_s} \quad (5.14)$$

where $C(\theta)$ and $p_s(\theta)$ are defined by equations 4.6 and 5.6, respectively. A fit to the off resonance scattering was attempted with these equations by varying the phase shifts $\delta_{s\ell}$. In some scattering problems these phase shifts are given entirely by the hard sphere values of equation 5.2 and no adjustment of the background is necessary. Note that the potential phases of equation 5.6 are dependent on s and ℓ , whereas the hard sphere phase shifts depend only on ℓ . The introduction of a set of phases that depend on both ℓ and s produced more parameters than could be conveniently handled in the analysis. For this reason the background was limited to a set of ℓ -dependent phases. This limitation is not serious provided the δ_ℓ are not allowed to deviate far from the tabulated hard sphere values. A large anomalous (deviating from the hard sphere value) potential scattering would be expected to be J dependent and could at best be treated by including arbitrary resonance terms with large total widths. The hard sphere phase shifts, φ_ℓ , are shown in Figure 5.1. Their contribution to the scattering amplitude is proportional to $\sin \varphi_\ell$ so that no large errors were introduced by neglecting values for $\ell > 4$.

First a theoretical background was calculated assuming only a hard sphere contribution. The radius chosen for these calculations was

Figure 5.1 ϕ_l vs. E_{LAB} for $l = 0$ to $l = 3$.

$$\begin{aligned}
 r &= 1.45(A_p^{1/3} + A_N^{1/3}) \\
 &= 4.77 \text{ fermis}
 \end{aligned}
 \tag{5.15}$$

Background curves were calculated at angles corresponding to the six excitation functions of Figures 3.1 and 3.2. These theoretical curves did not adequately fit the experimentally observed background. The background phases were then varied (assuming only an ℓ dependence) until a fit was found. Several problems were involved in this analysis. First, the fit to the six excitation curves was rather poor and the solution was not unique. Second, the energy variation of these phase shifts was not known and had to be chosen arbitrarily. In order to get a realistic set of phase shifts it would be necessary to have a complete set of off-resonance angular distributions taken throughout the energy region. Unfortunately, there was only one region, near 4.43 MeV, where the scattering was essentially off resonance.

A complete angular distribution was taken in this approximate off-resonance region and a least square fit was made to determine the background phases. This fit is shown in Figure 5.2 (also shown is the background predicted by hard sphere scattering). The shape of this hard sphere background at 4.43 MeV was characteristic of the hard sphere background at all energies; whereas, the experimental curve at 4.43 MeV was characteristic of all the experimental angular distributions taken in approximate off-resonance regions. This new set of phase shifts was used as a starting point in determining a set of phases that would reasonable approximate off-resonance behavior at all energies.

Figure 5.2 Fits to the Off-Resonance Angular Distribution at 4.43 MeV.

The resulting set of phase shifts was:

$$\varphi_0 = \varphi_{\text{H.S.}} + 10.0^\circ$$

$$\varphi_1 = 12.0^\circ$$

$$\varphi_2 = 1.75^\circ$$

$$\varphi_2 = 1.0^\circ$$

$$\varphi_4 = 0.0^\circ \tag{5.16}$$

5.4 Resonance Scattering

Next, theoretical resonance shapes were calculated for each allowed combination of ℓ, s, J , and π . Figures 5.3 and 5.4 show the calculated resonance shapes for a resonance energy of 4.0 MeV (indicated by E_0). Partial wave formations of $\ell = 0, 4$ are shown along with their corresponding channel spin and total spin assignment. The angles shown represent angles at which the experimental data were taken. Resonances showed distinct shapes for the various ℓ -wave formations and resonances formed by a particular ℓ -wave also showed a characteristic variation with angle. In particular, $\ell = 0$ resonances had a large interference dip at all angles, $\ell = 1$ resonances were peaked at the forward angles and showed a minimum at the backward angles, and $\ell = 2$ resonances were peaked at the forward and backward angles and exhibited interference dips at angles near 90 degrees. Resonance shapes were calculated for various values of the background phases and it was noted that the over-all characteristic shape for a given partial wave formation did

Figure 5.3 Theoretical Isolated Resonance Shapes for the Elastic Scattering of Protons by ^{14}N .

Figure 5.4 Theoretical Isolated Resonance Shapes for the Elastic Scattering of Protons by ^{14}N .

not change significantly. However, the resonance magnitude and detailed shape were sensitive to assumed values of the background phases.

These theoretical curves were then compared to the experimentally observed resonances to give an initial guess on resonance parameters. Resonance parameters were then adjusted to give a best fit to the data.

5.5 Inclusion of the Inelastic Process

Thirteen resonances were observed in the elastic scattering cross section and five of these had significant partial widths for inelastic scattering. The inelastic process manifested itself in the elastic scattering analysis through the shift factor, Δ_λ , and through the total width, Γ_T , (see equation 4.28). Therefore, a detailed fitting of the elastic excitation curves made it necessary to consider the inelastic scattering data.

Six excitation curves were taken for inelastic scattering as shown in Figures 3.3 and 3.4. Distinct resonance phenomena were observed at proton bombarding energies of 3.88, 3.99, 4.78, 4.86. and 5.55 MeV and behavior of the data between 4.9 and 5.7 MeV indicated additional resonance processes of unknown origin. This could have been partially the result of direct interaction processes. Angular distributions were measured at 21 selected energies and each angular distribution contained at least twelve data points spread over the angular range of 30 degrees - 160 degrees in the laboratory system. (Figures 3.5 and 3.6.)

Analysis of the inelastic data was hoped to play an essential role in the assignment of level parameters. An expression for the inelastic differential cross section is obtained from equations 4.4 and 4.5,

$$\begin{aligned}
 d\sigma_{p,p'}(\theta,\varphi) &= \frac{1}{(2I_1 + 1)(2I_2 + 1)} \sum_{\substack{s s' \\ m_s m_{s'}}} |A_{p's'm_s',psm_s}|^2 \\
 A_{p's'm_s',psm_s} &= \frac{\sqrt{\pi}}{k} [i \sum_{J M \ell \ell' m_\ell} (2\ell + 1)^{1/2} (s \ell m_s o | JM) (s' \ell' m_{s'} m_\ell | JM) \\
 &\quad \cdot X_{p's'\ell',ps\ell}^J Y_{\ell'm_\ell}^J(\theta, \varphi) \\
 X_{p's'\ell',ps\ell}^J &= \frac{e^{i(\omega_{p\ell} - \varphi_{p\ell})} e^{i(\omega_{p'\ell'} - \varphi_{p'\ell'})} (\Gamma_{ps\ell} \Gamma_{p's'\ell'})^{1/2}}{E_0 + \Delta - E - i\Gamma_T^J/2}
 \end{aligned}
 \tag{5.17}$$

An expression for the total inelastic cross section results from integration of the differential cross section over the full solid angle:

$$\begin{aligned}
 \sigma_{p,p'} &= \frac{\pi}{k^2} \sum_{J \ell \ell' s s'} \frac{(2J + 1)}{(2I_1 + 1)(2I_2 + 1)} |X_{p's'\ell',ps\ell}^J|^2 \\
 &= \sigma_1 + \sigma_2 + \sigma_3 + \dots
 \end{aligned}
 \tag{5.18}$$

where $\sigma_1, \sigma_2, \sigma_3, \dots$ designate contributions from levels of different spins and parities.

Features of the inelastic cross section formulae which were useful in analysis of this data are:

1. For an isolated resonance, the order of the Legendre polynomial must be even; hence, an angular distribution

over such a resonance is symmetric about 90 degrees in the center mass system.

2. The energy dependent shape of an isolated resonance is independent of angle.
3. Interference between levels of opposite parity produces Legendre polynomials of odd order.
4. Interference between levels of the same parity produces Legendre polynomials of even order.
5. Interference effects between levels of different spins and parities do not appear in the integral inelastic cross sections.
6. For levels of opposite parity, an excitation curve at 90 degrees center of mass will have the same shape as an excitation curve of the total inelastic cross section.
7. The total inelastic cross section for an isolated resonance is given by

$$G_i = \frac{\pi}{k^2} \frac{(2J+1)}{(2I_1+1)(2I_2+1)} \frac{\Gamma_e \Gamma_i}{(E_0 + \Delta - E)^2 + (\Gamma/2)^2} \quad (5.19)$$

Hence, if the total width and inelastic cross section of an isolated resonance are known, Γ_e and Γ_i can be determined.

CHAPTER VI

RESULTS AND CONCLUSIONS

6.1 Background Scattering

The background was assumed to be of an ℓ dependent type with the phase shifts given by equation 5.16. These phase shifts represent a best fit to the data and are not necessarily a true description of the actual scattering processes. This set of phases did not deviate drastically from the hard sphere values; in particular, the phase shift for $\ell = 0$ (which accounted for most of the scattering) was very near φ_0 (hard sphere). Since qualitative shapes of resonances did not depend strongly on variations in these phases it was possible to use them as a means of analyzing the data.

A least squares fit was also performed on the off-resonance angular distribution near 4.43 MeV by assuming a set of phases that depended on the incident particle spin. This gave rise to a set of phase shifts $\delta_{\ell + 1/2}$, $\delta_{\ell - 1/2}$ which described scattering by a potential which depends on coupling of the incident particle spin with the orbital angular momentum. Such a potential is usually assumed for the optical model. This fit to the 4.43 MeV angular distribution was better than the fit for an assumed set of simple ℓ -dependent phases. However, the inclusion of this new set of phases into the general analysis would have been somewhat involved and the fit to the off-resonance data was not sufficiently improved to justify this procedure.

If there had existed a more complete set of off-resonance angular distributions at several energies so that a set of phases could have been determined accurately, then such an analysis would have been in order.

An optical model search was also made on the off-resonance scattering but this proved to be unsuccessful. No realistic set of optical parameters could be found that would produce a satisfactory fit to the data. This resulted from insufficient off-resonance data and from complications introduced by the target spin.

6.2 The States in ^{15}O .

Differential cross sections for elastic scattering in the center of mass system are shown in Figures 6.1 and 6.2. The solid curves indicate the best theoretical fits to these data and a summary of the properties of the states in ^{15}O deduced from this analysis is given in Table 6.2. Assignments for each of these states are discussed separately.

(a) The 3.88 and 3.99 MeV states

There is no doubt that the 3.88 MeV state was formed by $\ell = 2$, as can be seen from the single level shapes of Figures 5.3 and 5.4. For this partial wave formation, assignments of $J^\pi = 1/2^+$, $3/2^+$, $5/2^+$, and $7/2^+$ were possible. The possible channel spin configurations are also indicated on the figures. The $1/2^+$ and $3/2^+$ assignments led to cross sections that were too small and the $7/2^+$ assignment produced too large of cross section at the backward angles. An assignment of $J^\pi = 5/2^+$, $\ell = 2$, $s = 1/2$, $\Gamma = 90$ keV gave the best fit and no improvement in fit resulted from an admixture of $\ell = 4$. This was not surprising since the penetrability of $\ell = 4$ was down by a factor of ten. A small admixture

Figure 6.1 Theoretical Fits to the Differential Cross Section Curves.

Figure 6.2. Theoretical Fits to the Differential Cross Section Curves.

of $s = 3/2$ did not greatly affect the results and since this would result in no improvements in fit, it was omitted. The general philosophy in analyzing this data was to assume the simplest possible configuration. If no improvement in fit resulted for more complicated configurations, they were excluded. Analysis of the 3.88 MeV state was complicated by the presence of a significant inelastic cross section; but, from Figures 3.1 - 3.4 it was apparent that $\Gamma_i \ll \Gamma_e$ so, as a first approximation, the analysis was performed by assuming $\Gamma = \Gamma_e$.

The 3.99 MeV state showed a characteristic $\ell = 1$ shape, i.e., it was peaked at the forward angles and showed interference minima at the backward angles. Not much could be said about this resonance from a study of the elastic scattering data. It appeared only weakly in the elastic excitation curves and from Figures 3.1 - 3.4 it was apparent that the inelastic width was at least of the same order of magnitude as the elastic width.

For a more complete analysis of these two resonances it was necessary to include the inelastic data. Seven complete inelastic angular distributions were taken over the energy region of these two resonances. The total inelastic cross sections were obtained from integration of these angular distributions. The experimental values of the peak inelastic cross sections as determined from these angular distributions were 61 mb for the 3.88 MeV resonance and 43 mb for the 3.99 MeV resonance. With these values of σ_i , equation 5.19 was used to calculate the inelastic and elastic partial widths for various assignments of J^π . The results are indicated in Table 6.1. There were two solutions to equation 5.19 but one of these was eliminated when a fit to the elastic data

TABLE 6.1

ELASTIC AND INELASTIC WIDTHS FOR VARIOUS ASSIGNMENTS OF J^π

J^π	ℓ	ℓ'	Γ_e (keV)	Γ_i (keV)	$(\Gamma_i)_w^a$ (keV)
<u>3.88 MeV Resonance</u>					
$1/2^+$	0,2	0	55	35	2170
$1/2^-$	1	1	55	35	790
$3/2^+$	0,2	2	77.5	12.5	96
$3/2^-$	1,3	1	77.5	12.5	790
$5/2^+$	2,4	2	82.2	7.8	96
$5/2^-$	1,3	3	82.2	7.8	4.6
$7/2^+$	2,4	4	84.5	5.5	< 1
$7/2^-$	3,5	3	84.5	5.5	4.6
<u>3.99 MeV Resonance</u>					
$1/2^+$	0,2	0	15.4	4.6	2490
$1/2^-$	1	1	15.4	4.6	970
$3/2^+$	0,2	2	18.0	2.0	100
$3/2^-$	1,3	1	18.0	2.0	970
$5/2^+$	2,4	2	18.7	1.3	128
$5/2^-$	1,3	3	18.7	1.3	6.8
$7/2^+$	2,4	4	19.0	1.0	1.9
$7/2^-$	3,5	3	19.0	1.0	6.8

^{a)} The maximum allowed widths, corresponding to a reduced width of the full Wigner limit $\gamma_w^2 = 2.93$ MeV.

was considered. Also shown for each of the possible J^π assignments are the allowed values of ℓ and ℓ' , the angular momenta of the incoming proton and the inelastically scattered proton, respectively. The maximum possible width, assuming a reduced inelastic width of the full Wigner limit (equation 4.43) is also shown.

For the 3.88 MeV resonance, the assignments involving $\ell' \geq 3$ could be partially ruled out since the calculated partial widths exceeded the Wigner limit. The more probable assignments were $J^\pi = 3/2^-$, $\ell' = 1$, $J^\pi = 3/2^+$, $\ell' = 2$, and $J^\pi = 5/2^+$, $\ell' = 2$. The only assignment consistent with the elastic analysis was $J^\pi = 5/2^+$.

With reference to Table 6.2, the only assignment for the 3.99 MeV level consistent with the observed elastic and inelastic cross sections was $J^\pi = 1/2^-$. Other assignments led to an elastic width that was either too large or too small. This assignment represented the best fit to the data.

The next logical step in this analysis was a study of the inelastic angular distributions. Angular distributions over the 3.88 MeV resonance were nearly isotropic and those near the 3.99 MeV resonance indicated interference with a level of opposite parity - a result consistent with the initial parity assignments. Unfortunately, shapes of the inelastic angular distributions near 3.88 MeV were inconsistent with the formation of an $\ell = 2$ resonance. Such a resonance predicted large P_2 and P_4 terms in a Legendre polynomial expansion. Isotropic angular distributions were predicted only by an $\ell = 0$ resonance, but this could be ruled out by the large cross section in the elastic excitation curves and by the shape of the resonance curves of Figures 5.4 and 5.5.

A possible explanation of this would be the existence of a third resonance with a large inelastic width. Interference of a $5/2^+$ resonance with a $3/2^+$ resonance produces large P_2 and P_4 terms which could conceivably diminish the P_2 and P_4 terms of an isolated $5/2^+$ resonance. A third resonance would also help to explain the unusual asymmetry of the peaks as observed in the inelastic excitation curves. A detailed fitting of these inelastic angular distributions is complicated by the large number of possible channel spin configurations and the variety of angular distributions that result. The interference of two such resonances leads to a variety of interference terms.

(b) The 4.13 MeV state

This resonance had no noticeable cross section for the inelastic process and was detectable only at certain angles in the elastic scattering. Resolution broadening effects prohibited a detailed analysis of very narrow resonances and the only parameters obtainable in this case were the resonant energy and an upper limit on the total width ($\Gamma \leq 10$ kev).

(c) The 4.20 state

No inelastic cross section was observed so the total width was assumed to be $\Gamma = \Gamma_e$. This resonance had the characteristic $\ell = 2$ shape and was limited to assignments of $J^\pi = 1/2^+$, $s = 3/2$, and $J^\pi = 3/2^+$, $s = 1/2$. A $5/2^+$ or $7/2^+$ assignment was ruled out because of the relative small cross section. An assignment of $J^\pi = 3/2^+$, $s = 3/2$ was not probable because, for this channel spin, the $\ell = 0$ process would be expected to dominate. An assignment of $J^\pi = 1/2^+$, $s = 3/2$ gave the best fit and was also more consistent with the assignment of $5/2^+$ to the 3.88 MeV resonance. A $3/2^+$ assignment should result in a resonance peak

with a magnitude somewhere between the values for the 3.88 and 4.13 MeV resonances. It was unfortunate that this relatively isolated resonance did not have a detectable inelastic width.

(d) The 4.575, 4.58, and 4.63 MeV states

The elastic scattering data revealed three closely spaced resonances with definite interference effects. There was no appreciable inelastic width for any of these resonances. The resonance at 4.58 MeV was found to be 25 kev wide and had a narrow resonance at 4.575 MeV situated near its peak. The narrow resonance was not sufficiently resolved to allow a complete analysis. Fortunately it was small enough so as not to completely destroy the characteristic shape of the 4.58 MeV resonance. The 4.58 MeV resonance did not show the characteristic shape of a single partial wave formation and could only be fit by making an assignment of $J^\pi = 5/2^-$, $\ell = 1, 3$, $s = 3/2, 1/2$. No other assignment would give the proper shape nor would produce the observed interference effects with the 4.63 MeV resonance. The 4.63 MeV resonance had the characteristic shape of $\ell = 1$ formation, but, as previously mentioned, it was distorted by the interference from nearby levels. This resonance could best be fit with an assignment of $J^\pi = 3/2^-$, $\ell = 1$, $s = 3/2$, $\Gamma = 25$ kev.

(e) The 4.74, 4.78, and 4.86 MeV states

The narrow 4.74 MeV level had no detectable inelastic width and appeared as a small bump in the elastic cross section. An upper limit on its width was $\Gamma \leq 10$ kev, so that it could not be properly analyzed. This resonance had no appreciable effect on the 4.78 level.

The resonances at 4.78 and 4.86 MeV were fairly broad and overlapping and interference effects were observed. Both levels had

significant partial widths for inelastic scattering which had to be considered in the detailed analysis. The 4.78 MeV resonance had the characteristic $\ell = 2$ shape which was somewhat distorted by interference and the 4.86 MeV resonance had the characteristic $\ell = 1$ shape.

Six inelastic angular distributions were taken over the energy region of these resonances (Figure 3.6). The total inelastic cross section obtained from integration of these angular distributions was used to deduce the inelastic widths. An inspection of Figure 3.6 shows that the inelastic cross sections for angles less than 30 degrees, where they were apparently a maximum, were not known. Estimated values of the total inelastic cross sections as determined from these angular distributions were ~ 50 mb for the 4.78 MeV level and ~ 20 mb for the 4.86 MeV level. Inelastic widths deduced from these measurements were not accurate, but since these inelastic widths were much smaller than the corresponding elastic widths, the approximate fits to the elastic data were not appreciably affected.

The inelastic angular distributions were complicated by terms which indicated interference between levels of opposite parities. If the inelastic differential cross sections were known at all angles and the total contribution from each resonance known accurately, then a detailed analysis would have been possible. Angular distributions taken near the 4.86 MeV resonance were consistent with those predicted by an $\ell = 2$ resonance having the assumed channel spin and J^π assignments.

(f) The 5.02 MeV state

The resonance at 5.02 MeV was peaked at all angles and did not exhibit the characteristic shape of a single partial wave formation.

When analyzing this resonance, the effects of nearby levels had to be considered. An $\ell = 4$ formation was peaked at all angles, but the magnitude and detailed shape of theoretical curves were not consistent with the experimental data. A combination of $\ell = 1$ and $\ell = 3$ produced the desired effects and restricted the J^π assignment to $5/2^-$. The resonance at 4.58 MeV had a similar shape and was also fit with this assignment. Differences in the shapes of these two resonances resulted from interference effects of nearby levels and from variations in background at the two energies. An assignment of $J^\pi = 5/2^-$, $\ell = 1, 3$, $s = 1/2, 3/2$, $\Gamma = 30$ kev gave the best fit.

(g) The 5.18 MeV state

This broad resonance had the characteristic $\ell = 2$ shape and interfered with many of the nearby levels. The inelastic cross section was not sufficiently large to be extracted from the background. Such a small inelastic cross section resulted in a negligible inelastic width for this $5/2^+$ resonance. A best fit to the data was obtained with an assignment of $J^\pi = 5/2^+$, $\ell = 2$, $s = 3/2$, $\Gamma = 160$ kev. An inelastic angular distribution taken in the vicinity of this resonance was consistent with this assignment but, because of the possible background interference, no detailed fit could be made.

(h) The 5.55 MeV state

Cross sections were detected for the $^{14}\text{N}(p, p_1)^{14}\text{N}^*$, $^{14}\text{N}(p, p_2)^{14}\text{N}^*$, and $^{14}\text{N}(p, \alpha)^{11}\text{C}$ reactions but their widths were much smaller than the corresponding elastic width. Inelastic angular distributions over this resonance were not symmetric about 90 degrees center of mass, indicating interference with a background or with levels

of opposite parity. Incomplete data and background interference prohibited the extraction of reliable reaction widths. This resonance exhibited the characteristic $\ell = 1$ shape and was best fit with an assignment of $J^\pi = 5/2^-$, $\ell = 1$, $s = 3/2$, $\Gamma = 60$ kev.

6.3 Comparison of Levels in ^{15}N and ^{15}O .

It is of interest to compare the results of the present analysis on the states of ^{15}O to the states in the mirror nucleus ^{15}N . Figure 6.3 shows a scale diagram for the energy levels of ^{15}N and ^{15}O in the energy region of interest. The lower energy levels (below 10.5 MeV excitation energy) have been compared quite successfully by a number of authors.^{1,31}) and are shown in Figure 6.4. In many cases there was an inversion of level ordering and the average level spacing was not constant. These lower levels of ^{15}N and ^{15}O have also been compared to theoretical calculations and there appeared to be a qualitative agreement regarding average level density and distribution of spins, but there was also considerable level shifting, at times causing reversal of level order. Theoretical calculations have not been made for the higher levels of ^{15}O . This is a result of not knowing the effective interaction for shell model calculations and of not having definite level parameters with which to make a comparison.

Thirteen states were observed in ^{15}O with excitation energies between 10.9 and 12.47 MeV. Levels in ^{15}N were shifted up by 200-500 kev, the shift varying with each level. There appeared to be a gap in the level structure for both nuclei and the region of interest began just above this gap. The level density in ^{15}N was similar to that of ^{15}O but ambiguous spin assignments for both nuclei made a one to one comparison

Figure 6.3 Energy Level Diagrams of ^{15}N and ^{15}O , Illustrating the Excitation Energies and Possible Spin-Parity Assignments.

Figure 6.4 Comparison of the Low-Lying Energy Levels of ^{15}N and ^{15}O (see reference 13).

between the levels impossible. A comparison of the present results with the possible assignments in ^{15}N showed that there was qualitative agreement with regard to density of states, density of spins and parities, and approximate level ordering. The broad state (~ 500 kev) at 11.62 MeV in ^{15}N had been assigned $T = 3/2$. If isospin is a good quantum number, this state and other $T = 3/2$ states should not show up in the elastic scattering of protons from ^{14}N . The total width of states in these nuclei were not in agreement, but this is not surprising, since it is the reduced widths and not the partial widths that need to be compared.

6.4 Concluding Remarks.

As a result of this experiment, seven previously unobserved resonances and six previously observed resonances were found and identified. Resonance shapes for each partial wave were quite distinct; hence, reliable ℓ - wave and parity assignments could be made for most of the levels. Resonance widths varied from ≤ 10 kev to a maximum of 160 kev and many of the resonances were fairly broad and overlapping. The resonant energies were taken from the best fit to the data. Errors associated with these energies arose from experimental uncertainties (Chapter III) and from insensitivity of the fit to small variations in resonant energy. Errors associated with the widths were also due to this insensitivity to fit.

Four resonances had the characteristic $\ell = 2$ shape and were located at energies of 3.88, 4.2, 4.78, and 5.18 MeV. Resonances at energies of 3.99, 4.63, 4.86, and 5.55 MeV had the characteristic $\ell = 1$ shape. Resonances at 4.58 and 5.02 MeV could only be fit by assuming

large contributions from two partial waves. This was not unreasonable as the contribution from $\ell = 3$ had channel spin $s = 1/2$ and this channel spin was not allowed for the $\ell = 1$ partial wave. If the level formation were channel spin dependent and favored $s = 1/2$, one would expect the $\ell = 3$ contribution to be significant.

The experimentally observed value of the peak inelastic cross sections for the 3.88 MeV level (61 mb) was in good agreement with previous measurements^{9,16} (58.8 and 60.1 mb). Relative magnitudes of the inelastic cross sections for the 4.78 and 4.86 MeV levels were also in agreement with previous measurements¹⁶).

It was unfortunate that after such an extensive analysis, many of the resonance parameters were still in doubt. For most resonances, there were several possible J assignments and the one chosen represented the best fit to the data. These uncertainties were the result of unknown background or off-resonance processes that were also present at lower bombarding energies. Similar problems arose in the analysis of the elastic scattering of neutrons from ¹⁴N³⁵). For this case, φ_0 was approximately the hard sphere value, φ_1 was less than hard sphere, and φ_2 was very small - a result consistent with the present analysis. The question is raised that perhaps an interaction was present which could be best described by a spin dependent potential, depending on the nuclear spin and the spin of the incident particle. This could hopefully be resolved by polarization studies in the off-resonant regions. It would also be of interest to study complete angular distributions at slightly higher energies as a means of determining the validity of the optical model in this energy region.

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Figure 6.5 Energy Level Diagram of ^{15}O , Illustrating the Previously Unreported Levels.

TABLE 6.2

RESONANCE PARAMETERS^{a)}

E_p (MeV)	E_x (MeV)	J^π	Γ_T (keV)	Γ_E (keV)	ℓ_0	S_0	$\gamma_{p_c \ell_0 s_0}^2$ (keV)	Γ_i (keV)	ℓ_1	$\gamma_{p_1 \ell_1 1/2}^2$ (keV)
3.88	10.91	$5/2^+$	90	82.2	2	1/2	0.109	7.8	2	0.238
3.99	11.01	$1/2^-$	20	15.4	1	1/2	0.008	4.6	1	0.014
4.13	11.14	-	< 10	< 10	-	-	-	-	-	-
4.20	11.21	$1/2^+$	35	35	2	3/2	0.039	-	-	-
4.575	11.56	-	< 10	< 10	-	-	-	-	-	-
4.58	11.565	$5/2^-$	25	25	1,3	3/2, 1/2	0.005, 0.054	-	-	-
4.63	11.61	$3/2^-$	25	25	1	3/2	0.011	-	-	-
4.74	11.71	-	< 10	< 10	-	-	-	-	-	-
4.78	11.75	$5/2^+$	80	77.5	2	3/2	0.066	2.5	2	0.014
4.86	11.83	$5/2^-$	50	45.6	1	3/2	0.018	4.4	3	0.235
5.02	11.97	$5/2^-$	30	30	1,3	3/2, 1/2	0.007, 0.055	-	-	-
5.18	12.12	$5/2^+$	160	160	2	3/2	0.116	-	-	-
5.55	12.47	$5/2^-$	60	58.3	1	3/2	0.020	1.7	3	0.067

^{a)} These parameters were used to obtain the theoretical fits of Figure 6.1 and 6.2. All widths are in the center of mass system.

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