Sound Propagation in Normal and Superfluid $^3$He

J. B. Ketterson
Northwestern University, Evanston, Illinois 60201 and
Argonne National Laboratory, Argonne, Illinois 60439

and

Pat R. Roach, B. M. Abraham, and Paul D. Roach
Argonne National Laboratory, Argonne, Illinois 60439

ABSTRACT

We present measurements of the pressure dependence of the attenuation and velocity of sound in the hydrodynamic regime and in both the normal and superfluid zero sound regime of $^3$He. The velocity and attenuation were studied at a frequency of 20.24 MHz and at pressures of 17.00, 21.00, 21.50, 21.80, 22.00, 23.17, 26.00, and 28.00 bar in the zero sound regime. In the hydrodynamic region the temperature dependence of the attenuation was studied at 5.48 and 10.02 MHz and at pressures of 0.69, 1.38, and 2.76 bar. At a pressure of 29.3 bar and frequency of 20.24 MHz the transition from hydrodynamic to zero sound behavior was studied for both the velocity and attenuation. In addition, the anisotropy of the velocity and the attenuation as a function of the angle between the direction of an applied external magnetic field and the sound propagation direction was observed in the superfluid A phase at a pressure of 26.0 bar; no anisotropy was observed in the B phase at 21.0 bar. The observed behavior associated with a collective excitation of the order parameter in the B phase is shown to be qualitatively in

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agreement with theoretical predictions. At pressures slightly above the polycritical point some unexplained structure is observed in the velocity near the AB transition.
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INTRODUCTION

The propagation of ultrasound is a powerful probe for studying the liquid phases of $^3$He. The nature of the sound propagation depends on the magnitude of $\omega \tau$ where $\omega$ is the sound frequency and $\tau$ is a viscous relaxation time of the liquid. When $\omega \tau \ll 1$ (which is equivalent to the sound period being much longer than $\tau$) sound propagates as the usual density wave familiar in hydrodynamics (first sound); when $\omega \tau \gg 1$ density waves propagate as a collective mode known as collisionless or zero sound. 1-4

At high temperatures the attenuation is given by the hydrodynamic expression 5

$$\alpha = \frac{\omega^2}{2 \rho c^3_1} \left[ \frac{4}{3} \eta + \xi + \frac{\kappa}{C_p} \left( \frac{C_p}{C_v} - 1 \right) \right]$$

(1a)

where $\rho$, $c_1$, $\eta$, $\xi$, $\kappa$, $C_p$, and $C_v$ are the density, first sound velocity, first viscosity, second viscosity, thermal conductivity, and heat capacities at constant pressure and constant volume, respectively. For $^3$He $\xi$ is negligible and below 500 mK $C_p = C_v$; the attenuation is then given by

$$\alpha_1 = \frac{2 \omega^2 \eta}{3 c^3_1}$$

(1b)

and the attenuation becomes a direct measure of the viscosity. At much lower temperatures $^3$He behaves as a Fermi liquid. In this limit the first sound velocity can be derived from the relation 1

$$F_0 = \frac{\frac{3}{m} m^*}{p_F^2} - 1$$

(2)

where $m$ is the mass of a $^3$He atom, $p_F$ is the Fermi momentum ($p_F = (3\pi^2 n/m)^{1/3}$) and $F_0$ is a Landau parameter. The effective mass, $m^*$, is
related to $F_1$, another Landau parameter, through Eq. (3) and can be evaluated from the specific heat.

$$\frac{m^*}{m} = 1 + \frac{F_1}{3} = \frac{C_V(\text{REAL})}{C_V(\text{IDEAL})}$$

As the Fermi liquid regime is approached the viscous relaxation time and the viscosity take up an asymptotic $T^{-2}$ dependence; thus the attenuation behaves as $\alpha \propto \omega^2/T^2$. At sufficiently low temperatures the condition $\omega T > 1$ will be met and a transition to the zero sound mode will occur. The expressions for the zero sound velocity, $c_o$, and attenuation, $\alpha_o$, are lengthy and will not be given here. Qualitatively, upon passing into the zero sound mode the sound velocity changes quickly to a greater value in the vicinity of $\omega T = 1$, and the attenuation passes through a maximum. The attenuation of zero sound, $\alpha_o$, follows a $T^2$ law and is independent of frequency. At still lower temperatures direct excitation of quasiparticles can occur and the attenuation has the form $\alpha = \alpha_o \left(1 + \left(\frac{\hbar}{2\pi k_BT}\right)^2\right)$; this "quantum limit" effect has not been observed to date.

At temperatures below about $3 \times 10^{-3}$ K, $\kappa$ makes a transition into a superfluid phase; at still lower temperatures, depending on the pressure, another superfluid phase exists. The pressure, temperature, and magnetic field dependence of the transition temperatures has been studied in some detail. Figure 1 shows the phase diagram in the P-T plane as given by the group at Helsinki University of Technology. A line of second order phase transitions separates the normal liquid from the superfluid. A line of first order phase transitions, which intersects the line of second order transitions at a point known as the polycritical point (PCP), separates a superfluid region in the P-T plane at higher temperatures and pressures known as the A phase; the remainder of the superfluid region of the phase diagram is known as the B phase.
Both of the phases are believed to be Bardeen-Cooper-Schrieffer states in which quasiparticles of opposite momenta are paired; the unique feature of the $^3$He superfluids in that the pairing occurs in a triplet ($S = 1$) state which, by the Pauli principle, must have an odd angular momentum. Anderson and Morel$^{14}$ have proposed a state in which only the spin up and spin down components of the triplet are paired; a modification of this type of state proposed by Anderson and Brinkman,$^{15}$ known as the ABM, or axial, or anisotropic state, is believed to correspond to $^3$He A; the most convincing evidence for this identification comes from the interpretation of the NMR experiments.$^{8,16}$ For this phase the energy gap is anisotropic being zero along the angular momentum axis, $\ell$, and largest on the equator. Balian and Werthamer$^{16a}$ proposed a phase in which all three components of the triplet state take part in the pairing; this state, known as the BW or isotropic state, is believed to correspond to the B phase of superfluid $^3$He and here the energy gap is isotropic.

Theoretical aspects of sound propagation in superfluid $^3$He have been studied by Wölfle,$^{17}$ Ebisawa and Maki,$^{18}$ Maki,$^{19}$ and Serene.$^{20}$ All of the published work to date has neglected the effects of quasiparticle lifetime. Using methods developed by Wölfle$^{17}$ which incorporate pairing theory into Landau transport theory, Serene$^{20}$ has calculated the attenuation and velocity in both the A and B phases. Using thermal Green's function techniques the A phase and the B phase have been studied by Ebisawa and Maki$^{18}$ and by Maki,$^{19}$ respectively. The results of both methods are essentially identical. Wölfle$^{20a}$ and Maki and Ebisawa$^{20b}$ have recently extended the theory to include lifetime effects. They have made some detailed numerical calculations for the B phase.

The anisotropic nature of the A liquid leads to an anisotropy in the sound propagation. Serene has written the anisotropy in the following form
\[ a = \alpha_{\parallel}(T) \cos^4 \theta + 2\alpha_c(T) \cos^2 \theta \sin^2 \theta + \alpha_{\perp}(T) \sin^4 \theta \]  

\[ \Delta c = \Delta \alpha_{\parallel}(T) \cos^4 \theta + 2\Delta \alpha_c(T) \cos^2 \theta \sin^2 \theta + \Delta \alpha_{\perp}(T) \sin^4 \theta \]  

where \( \theta \) is the angle between the propagation direction, \( \hat{q} \), and the axial-state-energy-gap axis, \( \hat{\ell} \). The functions \( \alpha_{\parallel}(T) \), \( \alpha_c(T) \), \( \alpha_{\perp}(T) \), \( \Delta \alpha_{\parallel}(T) \), \( \Delta \alpha_c(T) \), and \( \Delta \alpha_{\perp}(T) \) have been computed by Serene (his Eq. (6.17); Fig. 13 (which will be discussed later) shows a plot of these quantities as a function of temperature for a pressure of 26.0 atm. Two physical processes produce structure in the attenuation and velocity. The first of these is pair breaking which occurs when \( \hbar \omega > 2\Delta \). The small sharp structure in the attenuation just below the transition temperature may result from pair breaking near the equator of the Fermi surface where there is a singularity in the density of states.

The second physical process is the excitation of a collective mode of the energy gap (or order parameter) by the sound wave; there are two such modes for the A phase and one mode in the B phase. The large sharp peak in \( \alpha_{\perp}(T) \) and the broad peak in \( \alpha_c(T) \) arise from the effects of these two modes.

The dipole-dipole interaction energy favors the angular momentum vector \( \hat{\ell} \) aligning in the plane perpendicular to the direction of an external magnetic field \( \hat{H} \) and may lie anywhere in this plane (in the absence of boundary effects which may be negligible in the bulk liquid). This situation is shown pictorially in Fig. 2. Let us assume that a bulk sample of A liquid in a magnetic field is made up of many domains or textures of liquid whose \( \hat{\ell} \) vectors lie randomly in the plane perpendicular to \( \hat{H} \). Let \( \psi \) be the angle between \( \hat{H} \) and \( \hat{q} \) and let \( \phi \) be the angle of \( \hat{\ell} \) relative to the intersection of the plane formed by \( \hat{H} \) and \( \hat{q} \), and the plane perpendicular to \( \hat{H} \). We then have 

\[ \cos \theta = \hat{\ell} \cdot \hat{q} = \cos \phi \sin \psi; \]  

inserting this expression in Eqs. (4a) and (4b) and averaging over \( \phi \) we obtain
\[ \alpha = \frac{1}{8} (3\alpha_\parallel + 2\alpha_c + 3\alpha_\perp) \sin^4 \psi + (\alpha_\parallel + \alpha_c) \cos^2 \psi \sin^2 \psi + \alpha_\perp \cos^4 \psi \]  

(5a)

\[ \Delta c = \frac{1}{8} (3\Delta c_\parallel + 2\Delta c_c + 3\Delta c_\perp) \sin^4 \psi + (\Delta c_\parallel + \Delta c_c) \cos^2 \psi \sin^2 \psi + \Delta c_\perp \cos^4 \psi \]  

(5b)

Note that Eqs. (4) and (5) have the same functional form for the angular dependence. By fitting the attenuation (or velocity) data to Eq. (5a) (or (5b)), which is a trigonometric polynomial with three coefficients, we can algebraically extract the three coefficients of Eq. (4a) (or (4b)). Thus, for sound propagation work, it may not be necessary to have a fully oriented sample. We note that the anisotropy predicted by Eq. (4) is identical to that observed in a smectic-A liquid crystal; Eq. (5b) has been used (in the high frequency limit) to represent a cholesteric liquid crystal as a "twisted" nematic.

The attenuation and velocity in the B phase were calculated by Serene (his Eqs. 7.90 and 7.91) and by Maki (his Eq. (34)). In the limit \( \hbar \omega \ll 4k_B T \) Serene and Maki found

\[ \alpha = \frac{3\pi \omega A^2}{5F_0 k_B T} \left\{ \frac{(\hbar \omega^2 - 4A^2)^{1/2}}{5\hbar^2 \omega^2 - 12A^2} \theta(\hbar \omega - 2A) + \frac{\pi}{5\sqrt{6}} \delta(\hbar \omega - 2\sqrt{3/5} A) \right\} \]  

(6a)

\[ \frac{\Delta c}{c} = \frac{3\pi \Delta^2}{5F_0 k_B T} \frac{(4A^2 - \hbar \omega^2)^{1/2}}{12A^2 - 5\hbar^2 \omega^2} \theta(2A - \hbar \omega) \]  

(6b)

where \( A \) is the (isotropic) energy gap, \( \theta \) and \( \delta \) are a unit step function and Dirac \( \delta \)-function, respectively, and \( F_0 \) is the zeroth order Landau parameter; the remaining symbols have their usual meanings. Qualitatively these formulas have the following features: (1) a broad attenuation band between
and the temperature at which \( T_C \) (this is associated with the pair breaking for \( \hbar \omega > 2\Delta \)), (2) a \( \delta \) function peak in the attenuation at \( \hbar \omega = 2\sqrt{3/5} \Delta(T) \) (arising from coupling to a collective mode of the energy gap), (3) no shift in the sound velocity for \( \hbar \omega > 2\Delta(T) \), and (4) a divergence with sign reversal in the sound velocity at \( \hbar \omega = 2\sqrt{3/5} \Delta(T) \).

EXPERIMENTAL TECHNIQUES

The measurements of the pressure dependence of the attenuation in the first sound regime were performed with a dilution refrigerator and a metallic sonic cell which has been described elsewhere. The very low temperature measurements were performed in an epoxy adiabatic-demagnetization cell; a schematic drawing is shown in Fig. 3. The cell was precooled through a specially designed mechanical heat switch which was an integral part of the epoxy mixing chamber of our \(^3\text{He}-^4\text{He}\) dilution refrigerator. The cell and heat switch are described in detail elsewhere. The heat switch was actuated by a bellows which could be expanded (switch closed) using pressurized liquid \(^4\text{He}\). The mechanical contact surfaces were 1 cm\(^2\) in area and were formed from a 1 cm high stack of 200 1 cm\(^2\) x 0.0025 cm Au foils which had been electron beam welded together along one edge, after which the welded edge was machined flat to form the contact surface. One of these foil brushes was sealed (around the edge of the welded surface) into the demagnetization cell and another into the mixing chamber. The resulting 400 cm\(^2\) surface area of foils of each switch section provided contact to the liquid in the demagnetization cell and in the mixing chamber. The demagnetization cell contained 23.2 grams of cerium magnesium nitrate (CMN) together with a measured volume of 2.4 cm\(^3\) of \(^3\text{He}\) (including the sonic chamber).
Figure 4 shows the sonic components which were contained in an epoxy plug which was screwed into the bottom of the demagnetization cell; the threads were sealed with Nonaq stopcock grease. The sound path was defined by a fused quartz annulus 0.6 cm long with an outside diameter of 1.27 cm and an inside diameter of 0.95 cm. An epoxy annulus slightly under 0.6 cm long was inserted into the quartz in order to reduce the volume of $^3$He in the cell. The ends of the quartz spacer were crenelated (to minimize spurious transmission through the spacer and to allow thermal contact to the liquid) and ground optically flat (to minimize phase cancellation of the wave front). Appropriately machined epoxy pieces were fitted over the spacer to reduce further the quantity of $^3$He; a hole together with various grooves allowed liquid thermal contact with the main reservoir. The ends of the spacer were capped by unloaded 20 MHz coaxially-plated x-cut quartz transducers; electrical contact and mechanical pressure between the transducers and the spacer were provided by gold-plated beryllium-copper spring contacts; the pairs of electrical leads to the input and output transducers were twisted to minimize cross talk.

The sound propagation axis, $\hat{q}$, was aligned perpendicular to the cryostat axis. A Helmholtz pair mounted on a rotatable base was located external to the cryostat such that the angle between $\hat{q}$ and $\hat{H}$ could be varied from 0-360°. Measurements of the temperature and field dependence (direction and magnitude) of the attenuation and velocity of 20.24 MHz sound were made using the phase comparison technique of Abraham, et al. The repetition rate was typically 3-10 Hz and the peak to peak output level of the 5 μsec transmitter pulse was $\sim 100 \text{ mV}$. A stainless steel toeppler pump system with an oil intensifier provided the hydrostatic pressures which were measured with a Texas Instruments precision quartz-Bourdon-tube gauge.

Temperatures were determined by monitoring the magnetic susceptibility of the CMN in the main body of the demagnetization cell using a standard mutual
inductance bridge. The thermometer was calibrated at high temperature against
the $^3$He vapor pressure scale. An additive correction to this T$^*$ scale was
arrived at by comparison with other measurements of the phase diagram;\textsuperscript{9,11}
a T$^2$ variation of the zero sound attenuation was observed in the normal liquid
at all pressures studied. It is important to take into account the effect of
the field of the Helmholtz pair on the susceptibility of the cerium magnesium
nitrate (CMN) used for thermometry. This was done by calibrating the CMN
against the temperature dependence of the attenuation of zero sound in both the
normal liquid and in the B phase. The attenuation peaks in the B phase were
identical at different fields except for a temperature shift due to the change
in the thermometer calibration. At pressures of 21.0 and 8.0 bar the attenuation
was measured at H = 0, 9, 18, and 27 gauss; the susceptibility, $\chi$, was
found to follow Curie's law in the form $\chi = C(H)/(T + \Delta)$ for all fields where
C(H) is a field dependent Curie constant and $\Delta$ is the additive correction to
the temperature scale. The most consistent calibrations were obtained with
the liquid in the B phase, presumably due to the better thermal equilibrium
in the superfluid state.

In most of the experiments the cell was precooled to a temperature of
17-20 mK in a field of 1.7 kG. Providing that the temperature at the end of
the previous demagnetization was below 3 or 4 mK, the time to cool back down
to 20 mK following remagnetization was on the order of 30 hrs; it is felt that
a better preparation of the contact flats of the heat switch could materially
reduce this time. The demagnetization schedule was determined from an optimized
computer simulation of the adiabatic demagnetization process\textsuperscript{26} as well as
experience with prior demagnetizations. The total demagnetization time was
approximately 4 hours.
The data reported here will be divided into two categories: (A) those for the normal fluid in both the hydrodynamic and zero sound regimes, and (B) those primarily concerned with the superfluid regime.

A. Sound Propagation in the Normal Fermi Liquid Regime

As discussed in the introduction the attenuation in the $\omega t \ll 1$ limit is, to a high degree of accuracy, directly related to the viscosity through Eq. (1b). The viscosity of $^3$He has been determined from measurements of capillary flow,\textsuperscript{27,28} damping of a torsionally oscillating cylinder,\textsuperscript{29-31,35} vibrating wire,\textsuperscript{32} and the attenuation of ultrasound.\textsuperscript{33,34}

Figure 5 shows our measurements of the attenuation of sound in liquid $^3$He at the vapor pressure. The squares show our most recent measurements at 10.02 MHz; shown also are our earlier measurements at 5.48 MHz.\textsuperscript{34} At very low temperatures, in the Fermi liquid regime, the attenuation, or equivalently the viscosity, should take up a $T^{-2}$ dependence; the solid straight line shows such a temperature dependence where the proportionality coefficient has been taken from the work of Abel, Anderson, and Wheatley.\textsuperscript{33} The remaining two lines are derived from the work of Betts, Osborne, Welber, and Wiks,\textsuperscript{30} and from the recent measurements of Black, Hall, and Thompson.\textsuperscript{32} Since in this regime the attenuation is proportional to the square of the frequency, $\omega$, we can present both sets of data on the same curve by plotting $\alpha/\omega^2$; the vapor pressure data are shown in this form in Fig. 6. The line represents a smooth curve through the data points. It will be observed that the data from the two frequencies are essentially identical with the exception of the three lowest temperature points at 10.02 MHz; here the discrepancy could be due to poor thermal contact between the metallic cell and the external CMN thermometer.
As is well known, it is difficult to determine absolute attenuations using the pulse echo technique because of spurious attenuation mechanisms such as nonparallel transducers, beam spreading, extraction of signal for detection, etc.; the pulse comparison or the pulse interference methods do yield accurate measurements of the relative attenuation. Our data were normalized by computing the attenuation near 500 mK (where it is quite small) from the viscosity measurements of Betts, et al. For 5.48 and 10.02 MHz this amounts to $\alpha = 0.4$ and 1.7 dB/cm, respectively.

Figure 7 shows our measurements of the temperature dependence of $\alpha/\nu^2$ under pressure; the pressures studied were 0.69, 1.38, and 2.76 bar, and the frequency was 10.02 MHz. The data under pressure were again normalized using the viscosity measurements under pressure; here we used the data of Betts, Keen, and Wilks. Near 500 mK this amounts to $\alpha = 1.3$, 1.1, and 0.8 dB/cm for pressures of 0.69, 1.38, and 2.76 bar.

In no case does our attenuation data strictly follow a $1/T^2$ law. The deviations of the viscosity from the asymptotic Landau-Fermi liquid regime have been studied by Emery; he has shown that the lowest order temperature dependent correction to the viscosity has the form

$$\left(\frac{1}{\eta T^2}\right)_0 - \left(\frac{1}{\eta T^2}\right) \propto T,$$

(7)

where $(1/\eta T^2)_0$ denotes the zero temperature limit.

In Fig. 8 we show a plot of $\eta T^2$ as a function of temperature for the vapor pressure and 0.2 and 0.69 bar; according to the above relation this product should be proportional to $T$ at low temperatures. We observe that, although the data are consistent with this behavior, the scatter is too large to extract the proportionality coefficient accurately; this plot does allow us to extract a value for $(\eta T^2)_0$. 
Figure 9 shows the temperature dependence of the attenuation and velocity at a pressure of 29.3 bar and a frequency of 20.24 MHz. All regimes discussed in the introduction, with the exception of the quantum limit, are clearly visible in this figure. The attenuation data are plotted on a log-log scale so that a power law behavior of the attenuation with temperature corresponds to a straight line. The lines through the high and low temperature points show a $T^{-2}$ and $T^2$ behavior; these two regions correspond to the hydrodynamic and zero sound regimes, respectively, and a transition between the two occurs in the vicinity of 9.0 mK. The sharp peak near the low temperature extreme of the data is associated with the superfluid transition. The upper part of Fig. 9 shows the temperature dependence of the velocity. Note that it is essentially temperature independent except in the vicinity of 9 mK where the velocity increases on going from the hydrodynamic to the zero sound regime; in addition the velocity falls rapidly at the superfluid transition and, at very low temperature, approaches the hydrodynamic velocity.

B. Sound Propagation in the Superfluid Regime

The attenuation of sound in superfluid $^3$He along the melting curve was first studied by Lawson, Gully, Goldstein, Richardson, and Lee. Measurements off the melting curve of both the attenuation and velocity in the A and B phases were performed by Paulson, Johnson, and Wheatley.

Figures 10a - 10h show our measurements of the attenuation and velocity of sound for 20.24 MHz sound at pressures of (a) 29.00, (b) 26.00, (c) 23.17, (d) 22.00, (e) 21.80, (f) 21.50, (g) 21.00, (h) 17.00 Bar. Some of these results have appeared previously. The velocity data were normalized relative to the first sound velocity (measured at 60 mK) in the normal liquid. At the lower pressures, where the signal became unobservable due to high attenuation, the difference in velocity before and after the signal loss is uncertain to within
multiples of the sound period (corresponding to an uncertainty in $\Delta c/c$ of multiples of $3 \times 10^{-3}$ at 21 bars). This uncertainty at the attenuation peak in the B phase below the PCP was unambiguously settled by requiring that the difference in velocity between the normal liquid and the B-phase liquid at $\sim 2.2$ mK be a smooth function of pressure from well above the PCP where we have continuous data to below the PCP. A similar uncertainty occurring in the normal liquid in the zero sound-to-first sound transition region was settled by comparison with the measurements of Paulson, et al. The normalization of the attenuation data was arrived at by observing the attenuator setting (at each pressure studied) at 450 mK and correcting for the attenuation in the liquid (which is quite low at this temperature) from the viscosity measurements of Betts, et al; fixing the normalization at one temperature yields a nearly $T^2$ dependence of the zero sound attenuation in the normal liquid. We have identified the appearance of superfluidity as the temperature where the attenuation departs from the normal liquid value; this is in agreement with theory for both phases. The data at higher pressures are similar to those observed by Paulson, Johnson, and Wheatley. Note that a discontinuity in the velocity and attenuation is observed at the B-A transition for most A phase pressures studied (at 28.00 bar the temperature did not extend low enough); this transition was not observed by Paulson, et al., for the pressures and temperatures studied by them. The data in Fig. 10e at 21.80 bar show the situation at a pressure only slightly above the PCP where the A and B transitions are now quite close to each other. Note, both the velocity and attenuation remain discontinuous at the B-A transition (it is still a first order transition) but at temperatures just above and just below the discontinuity the velocity turns up and down, respectively (appearing to anticipate the transition); the velocity still turns down just below $T_c$, however. The
signal was observed continuously through the maximum in the attenuation (\(\sim 18 \, \mu K\) below \(T_c\)) but at a level near the limit of our signal to noise which was \(\sim 73\, \text{dB/cm}\), and the peak in Fig. 10e is drawn through this value. In Fig. 10f we show the data at 21.50 bar. The discontinuity associated with the B-A transition is not observed here and presumably occurs near the attenuation maximum where the signal is lost. The velocity is still observed to turn down just below \(T_c\). Figure 10g shows the data at a pressure of 21.00 bar. The "A-like" behavior where the velocity turns down below \(T_c\) is now absent. Pressures at which "A-like" behavior is not observed are taken to be below the PCP. Note that the velocity turns down just below the attenuation peak and when it reappears it has the opposite temperature dependence; the behavior near the peak in the attenuation was not observable but the curves are presumably continuous since there is no phase transition at this point. The peculiar structure in the velocity was not observed in the data of Paulson, et al.,\(^{38}\) although a sharp drop in the velocity at the temperature of the attenuation maximum was seen in their data at 19.6 bar. The data at a pressure well below the PCP (17.00 bar) are shown in Fig. 10h. They are essentially similar to those at 21 bar.

Experiments in a magnetic field were performed in the A phase at 26.0 bar and in the B phase below the polycritical point at 21.0 bar. Data were collected during temperature drifts for specific fields and angles or during angular rotations at fixed fields and nearly constant temperature; the majority of the data were taken with the first technique. Demagnetizations were performed at the angles \(\psi = 0, 30, 60\) and \(90^\circ\). At each angle, the fields \(H = 0, 9, 18,\) and \(27\, \text{G}\) were studied.

Figure 11 shows the temperature dependence of the attenuation and velocity shift in the A phase at 26.0 bar for the angles \(\psi = 0, 30, 60,\) and \(90^\circ\) and for a field of \(27\, \text{G}\). To simplify comparison with theory, the data have been
normalized to be zero in the normal liquid. The anisotropy was quite apparent at 9 G and essentially saturated at 18 G. It is quite striking that such small magnetic fields will align the liquid and this observation would appear to open the way for a host of anisotropic transport measurements in the A liquid using CMN refrigeration techniques. Figure 12 shows the angular dependence of the attenuation and velocity shift in the A phase at 26.0 bar in a field of 18 G and at a temperature of 48 μK below T_c. These data have not been normalized to be zero in the normal liquid. The curves through the data are fits to Eq. (5a) and (5b); clearly, the data exhibit an angular dependence that is very close to the predicted form. Recent measurements by Lawson, et al., on the sound attenuation in the A phase at 20 MHz have also shown anisotropy.

DISCUSSION

Our sound propagation data in the normal fluid present no unusual or unexpected features. The attenuation data in the unoriented superfluid A phase are only in qualitative agreement with the angularly averaged theory of Wölfle and Serene. The width of the main peak is considerably broader than predicted and the small broad peak below the main one (arising from \( \alpha_c \)) is not observed. Lifetime effects might account for the widening of the main peak; however, one would not expect the elimination of the second peak. The small sharp feature just above the main peak should not exist according to Wölfle. Our data on the B phase are in qualitative agreement with features (2), (3), and (4) discussed in Sec. I in connection with Eq. (6), if the effect of lifetime broadening is included; feature (1) could easily be obscured by collision broadening of the \( \delta \) function peak. Since we were not able to follow the signal through the peak the strength of the \( \delta \) function (area under the peak) cannot be reliably estimated. However, if we assume that the B phase
theory is essentially correct we can use features (3) and (4) to evaluate $\Lambda(T)$ at two temperatures; i.e., when $\Lambda_0 = 2\Lambda(T)$ and $\Lambda_0 = 2\sqrt{3}/5 \Lambda(T)$. For the data at 21.00 bar we find $\Lambda(T) = 6.6 \times 10^{-20}$ erg and $8.5 \times 10^{-20}$ erg for $T_C - T = 10.5 \mu K$ and $16.5 \mu K$, respectively. Either of these numbers may be used to compute the specific heat discontinuity, $\Delta C$, at $T_C$ using Eq. (F-13) of Serene

$$\Delta(T)^2 = \frac{2}{3} \pi^2 (k_B T_C)^2 \frac{\Delta C}{C_N} \frac{(T_C - T)}{T_C}$$

where $C_N$ is the normal state heat capacity at the transition; we find $\Delta C/C_N = 1.5$ which is consistent with a low pressure extrapolation of the data of Webb, et al.,\(^{39}\) who found $\Delta C/C_N = 1.85$ at 34 bars and 1.55 at 23 bars.\(^{40}\) The agreement is impressive and lends support to the identification of the B phase as a Balian-Werthamer state.\(^{16a,15}\)

The effect of quasiparticle scattering has been studied recently by Wölfle\(^{20a}\) and Maki and Ebisawa.\(^{20b}\) Wölfle's calculations for the B phase are in excellent agreement with the data of Paulson, et al.,\(^{38}\) for both the velocity and attenuation at 15.15 MHz and 19.6 bar. However, his calculation does not show the structure in the velocity that we observe at 20.24 MHz. The calculation by Maki and Ebisawa,\(^{20b}\) on the other hand, shows striking structure in the velocity that is quite similar to the structure that is suggested by our data. Their calculation also predicts a low-temperature limit to the sound velocity that is slightly different from the hydrodynamic first sound velocity $c_1$.

We now discuss our attenuation and velocity measurements in the presence of a magnetic field. We have used the data of Fig. 11 to deduce the temperature dependence of each of the combination of coefficients entering Eq. (5). From these combinations we have solved for the temperature dependent coefficients $a_{\parallel}(T)$, $a_{\perp}(T)$, $a_{\parallel}(T)$, $\Delta c_{\parallel}(T)$, $\Delta c_{\perp}(T)$, and $\Delta c_{\parallel}(T)$. The temperature dependence of these coefficients is shown in Fig. 13. The dashed lines
show calculations using the programs of Serene for this pressure and frequency. Qualitatively the reduced data and the calculations for the attenuation are quite similar; \( \alpha_\perp \) has a sharp peak, \( \alpha_c \) has a broad peak at a lower temperature than that of \( \alpha_\perp \), and \( \alpha_\parallel \) rises quickly to a small value and then falls slowly. The theory does not include quasiparticle lifetime effects and the small sharp structure just below \( T_c \) predicted by the theory could easily be obscured or, as pointed out by Wülfler, be an artifact of the computer calculations. The fact that experimentally the peaks in \( \alpha_\perp \) and \( \alpha_c \) are occurring at higher temperatures suggests that the temperature dependence of the gap is larger than estimated by Serene from the heat capacity anomaly on entering the A phase; this would explain the problem raised in the beginning of this section. The agreement between theory and experiment for the velocity anisotropy is less satisfying; here lifetime effects may play a dominant role.

We have also made a search for anisotropy in the sound propagation in the B phase of superfluid \(^3\)He. Our measurements were made at a pressure of 21.0 bar which is slightly below the polycritical point. No anisotropy was observable within our resolution. This observation supports the conjecture that the B phase is a Balian-Werthamer state for which an isotropic gap (and thus no anisotropy) is predicted.

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20b. K. Maki and H. Ebisawa, to be published.
21. We wish to thank Dr. Serene for sending us his program for computing these quantities.
FIGURE CAPTIONS

Fig. 1  The phase diagram of superfluid $^3\text{He}$ in the P-T plane as given by Ahonen, et al.\textsuperscript{13}

Fig. 2  The "fan" of $\mathbf{k}$ vectors for a given field direction $\mathbf{H}$.

Fig. 3  Adiabatic demagnetization cell:
A) Mixing chamber input line; B) Bellows actuating capillary; 
C) Nylon nut; D) Epoxy support plate; E) $^4\text{He}$ bellows; F) Bellows end plates; G) Epoxy jacketed mixing chamber return line; H) Mixing chamber reservoir; I) Mixing chamber gold brush; J) Upper epoxy switch casting; K) Epoxy mixing chamber; L) Nylon support rods; M) Be-Cu epoxy to gold transition caps; N) Lower epoxy switch casting; O) Main epoxy cell body; P) Cell gold brush; Q) Paramagnetic powder and $^3\text{He}$; R) Removable epoxy cell bottom; S) Individual gold foil; T) Lens paper; U) Welded gold switch surface.

Fig. 4  A schematic drawing of the demagnetization cell and epoxy sound plug.

Fig. 5  The temperature dependence of the attenuation of sound in liquid $^3\text{He}$ at the vapor pressure at frequencies of 10.02 MHz, and 5.48 MHz; shown also are the results of Black, et al.,\textsuperscript{32} and Betts, et al.\textsuperscript{30} For reference, a line showing the attenuation which would result from a $T^{-2}$ dependence of $\eta$ is shown where the coefficient was extracted from the data of Abel, et al.\textsuperscript{33}

Fig. 6  The temperature dependence of $\alpha/v^2$ as a function of temperature. The squares, $\square$, and circles, $\bigcirc$, correspond to 5.48 and 10.02 MHz, respectively. The solid line is a smooth curve through the data.

Fig. 7  The temperature dependence of $\alpha/v^2$ at pressures of 0.69, 1.38, and 2.76 bar, respectively. The frequency was 10.02 MHz.
Fig. 8 A plot of $\eta^2$ vs temperature for the vapor pressure, 0.2 bar and 0.69 bar; the frequency was 10.02 MHz in all cases.

Fig. 9 The temperature dependence of the attenuation and velocity at a pressure of 29.31 bar and a frequency of 20.24 MHz.

Fig. 10 The attenuation and velocity of sound as a function of temperature at a frequency of 20.24 MHz: (a) 28.06 bar, (b) 26.00 bar, (c) 23.17 bar, (d) 22.00 bar, (e) 21.80 bar, (f) 21.50 bar, (g) 21.00 bar, (h) 17.00 bar.

Fig. 11 The temperature dependence of the attenuation and velocity shift of 20.24 MHz ultrasound in superfluid $^3$He A at a pressure of 26.0 bar and for angles, $\psi$, between the magnetic field and propagation direction of 0°, 30°, 60°, and 90°.

Fig. 12 The angular dependence of the attenuation and velocity shift of 20.24 MHz ultrasound in superfluid $^3$He A for a pressure of 26.0 bar and at a temperature of 48 $\mu$K below $T_c$. The curves are fits of the data to Eqs. (5a) and (5b).

Fig. 13 The temperature dependence of the quantities which determine the anisotropic attenuation and velocity shift for 20.24 MHz sound in $^3$He A at 26.0 bar; the dashed lines are the calculations of Serene (Ref. 20).
$\frac{(c - c_i)}{c_i} \times 10^3$

$P = 29.3$ BAR
$f = 20.24$ MHz
$c_i = 401.8$ m/sec

$\alpha = KT^2$
$\alpha = \frac{K}{T^2}$
\( P = 28.00 \text{ bars} \)
\( f = 20.24 \text{ MHz} \)
\( c_1 = 396.1 \text{ m/sec} \)

\( \Delta \rightarrow \text{Fermi Liquid} \)
$\frac{(c-c_1)}{c_1} \times 10^3$

$P = 23.17$ bars
$f = 20.24$ MHz
$c_1 = 368.5$ m/sec

$T^* - T_c^*$ (μK)

ATTENUATION (dB/cm)

$A \rightarrow$ Fermi Liquid

B$\rightarrow$A
$P = 21.80 \text{ bars}$

$f = 20.24 \text{ MHz}$

$c_l = 366.8 \text{ m/sec}$

$A \rightarrow $ Fermi Liquid

$B \rightarrow A$

$T^* - T_c^b (\mu K)$
$P = 21.5$ bars
$f = 20.24$ MHz
$c_1 = 365.3$ m/sec

$A \rightarrow$ Fermi Liquid
$\frac{(c-c_1)}{c_1} \times 10^3$

ATTENUATION (dB/cm)

$P = 17.00$ bars
$f = 20.24$ MHz
$c_1 = 340.7$ m/sec

$T^* - T_c^*$ (μK)

B → Fermi Liquid
The graph shows the attenuation coefficient $(\gamma - \gamma_0)/\gamma_0$ as a function of $T^\circ - T_c^\circ$ (in $\mu$K) for different values of $\psi$. The data points are represented by markers: $\psi = 0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$. The graph is labeled with a caption: $H = 27$ Gauss.
$c_l = 389 \text{ m/sec}$

$H = 18 \text{ Gauss}$

$T_c - T = 48 \mu K$
ATTENUATION (dB/cm) 

\[
\frac{(c - c_0)}{c_0} \times 10^3
\]