MINIMAL REGGE MODEL FOR MESON-BARYON SCATTERING:
DUALITY, SU(3) AND PHASE-MODIFIED ABSORPTIVE CUTS

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Minimal Regge model for meson-baryon scattering:
Duality, SU(3) and phase-modified absorptive cuts

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Minimal Regge model for meson-baryon scattering:
Duality, SU(3) and phase-modified absorptive cuts

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A model is presented which incorporates economically all of the modifications to simple SU(3)-symmetric dual Regge pole theory which are required by existing data on $0^{-}\frac{1}{2}^{+} \rightarrow 0^{-}\frac{1}{2}^{+}$ processes. The basic assumptions are no-exotics duality, minimally broken SU(3) symmetry, and absorptive Regge cuts phase-modified by the Ringland prescription. We first describe qualitatively how these assumptions suffice for the description of all measured reactions, and then present the results of a detailed fit to 1987 data points for 18 different reactions.

I. INTRODUCTION

At the present time there is no complete theoretical description of the strong interactions of elementary particles. No basic set of principles has been deduced from which precise relationships between experimental observables may be calculated. Instead, the body of knowledge regarding high energy phenomena may loosely be classified under three divisions. First, there are certain hypotheses which have been given the status of axioms. Lorentz invariance and unitarity, for example, are universally accepted. Second are such precepts as maximal analyticity and crossing symmetry which are not so fundamentally based but are widely regarded as true. These propositions all serve to constrain the direction of theoretical investigations, but are insufficient to determine a solution. At the third level are various ideas, less specific in nature, which seem to give insight into the systematics of some limited class of phenomena. Although these are invariably incomplete, incalculable and lacking in resemblance to rigorous theory, they do serve to reduce the vast diversity of high energy data into a certain orderliness and support the belief that one may understand many features of the data in terms of a few physical ideas. The modest goal of much contemporary theoretical work is simply to seek and evaluate unifying relationships among the data in the hope of reducing the scope of an eventual synthesis into detailed theory. That is the spirit in which this paper is presented.

Much of this paper will report the results of an analysis of pseudoscalar meson - baryon ($0^{-\frac{1}{2}+}$) scattering data. This class of reactions was chosen because several elegant and powerful hypotheses relating diverse phenomena may be realized in model form for these multiplets and the plenitude of data permits a nontrivial test of their validity. The basic ingredients in this investigation were Regge poles, SU(3) symmetry and global duality, and Regge cuts. We discuss each of these topics briefly before describing the analysis in detail.

A. Regge Poles

In this section we introduce the Regge picture of high energy scattering which is basic to our analysis. Enough of the development will be sketched to demonstrate the intimate relationship between the high energy behavior of an amplitude and the location of poles in the complex angular momentum plane. The full details of standard Regge theory may be found in Reference 1.

We consider only the spin zero, equal mass case since it illustrates the important physical ideas without the great technical complexity present in the analysis of more general reactions. We use the Lorentz invariant scattering amplitude $T(s,t)$ which is related to observable quantities by the equations

$$\sigma_{\text{Total}} = \frac{2\pi^2}{q\sqrt{s}} \text{Im } T(s, t = 0),$$

$$\frac{d\sigma}{dt} = \frac{\pi^2}{2s q^2} |T(s, t)|^2,$$

where s is the center of mass energy squared, t is the invariant momentum transfer and q is the center of mass momentum. The independent variable t will often be replaced with z , the cosine of the center of mass scattering angle.

The scattering amplitude may be expanded into partial waves according to

$$T(s, z) = \sum_{J=0}^{\infty} (2J+1) f^J(s) P_J(z) \quad (1)$$

where the f^J are partial wave amplitudes and the P_J are Legendre polynomials. This equation may be inverted to give

$$f^J(s) = \frac{1}{2} \int_{-1}^{+1} dz P_J(z) T(s, z) \quad (2)$$

According to the principle of maximal analyticity, $T(s, t)$ at fixed t is assumed to have only those singularities in s which are demanded by unitarity. These are just branch cuts along the real axis from $\pm 4m^2$ to $\pm \infty$ with the possibility of simple poles between the cuts. Unitarity in the t channel implies the same cuts in t for fixed s . Assuming for the moment that the amplitude vanishes for $|z| \rightarrow \infty$ we use Cauchy's theorem and some minor manipulations of the kinematic variables to write a fixed s dispersion relation

$$T(s, z) = \frac{1}{\pi} \int_{z_0}^{\infty} \frac{dz' D_t(s, z')}{z' - z} + \frac{1}{\pi} \int_{z_0}^{\infty} \frac{dz' D_u(s, z')}{z' + z} \quad (3)$$

D_t is the t channel discontinuity and D_u is the u channel discontinuity.

We can write equation 3 as $T(s, z) = T_R(s, z) + T_L(s, z)$ where T_R and T_L

have singularities in the right and left z plane respectively. If we define signatured amplitudes $T^{\pm}(s, z)$ according to

$$T^{\pm}(s, z) = T_R(s, z) \pm T_L(s, -z). \quad (4)$$

we can write the simple dispersion relation

$$T^{\pm}(s, z) = \frac{1}{\pi} \int_{z_0}^{\infty} \frac{dz' D^{\pm}(s, z')}{z' - z} \quad (5)$$

where $D^{\pm} = D_t \pm D_u$. Then, from equations 2 and 3, we have for the signatured partial wave amplitudes:

$$f_{\pm}^J(s) = \frac{1}{2\pi} \int_{z_0}^{\infty} dz' D^{\pm}(s, z') \int_{-1}^1 dz \frac{P_J(z)}{z' - z},$$

or

$$f_{\pm}^J(s) = \frac{1}{\pi} \int_{z_0}^{\infty} dz' D^{\pm}(s, z') Q_J(z'). \quad (6)$$

Q_J is the Legendre polynomial of the second kind with well known mathematical properties. Equation 6 is known as the Froissart-Gribov projection for $f_{\pm}^J$.

In the general case $T^{\pm}(s, z)$ does not vanish as z goes to infinity, but rather $T^{\pm}(s, z) = O(z^{N(s) - \epsilon})$ where $N(s)$ is an integer and ϵ is some positive value. In this case we must write a subtracted dispersion relation and the projection of the partial wave amplitude becomes

$$f_J^+(s) = \frac{1}{2} \sum_{m=0}^{N(s)-1} \frac{1}{m!} C_m \int_{-1}^{+1} dz P_J(z) z^m + \frac{1}{2\pi} \int_{z_0}^{\infty} dz D^+(s, z) \int_{-1}^{+1} dZ \left(\frac{Z}{z}\right)^{N(s)} \frac{P_J(z)}{Z^1 - Z}$$

By using the orthogonality of the P_J we can easily show that this reduces to the Froissart-Gribov projection for $J \geq N(s)$. Thus we have the important result that $T^+(s, z) = O(z^{N(s)-\epsilon})$ implies $|z| \rightarrow \infty$

$$f_J^+(s) = \frac{1}{\pi} \int_{z_0}^{\infty} D^+(s, z) Q_J(z) dz \text{ for } J \geq N(s).$$

Using the known properties of the Q_J and the power boundedness of D^+ it can be shown that the integral converges and can be used for an analytical continuation in J of the $f_{\pm}^J(s)$. This statement is not true for the projection of unsignatured amplitudes. Writing the continued amplitude as $f_{\pm}(J, s)$, we note that $f_{\pm}(J, s)$ has no singularities in J for $J \geq N(s)$. This indicates the close relationship between asymptotic behavior and singularities in the J plane. Since $z \rightarrow \infty$ corresponds to $s \rightarrow \infty$ in the crossed channel, we see that the highest lying J plane singularities determine the high energy behavior of the crossed channel amplitude.

Maximal analyticity of the second kind asserts that equation 6 may be continued below $J = N(s)$ with only isolated singularities being encountered. Accepting this, we use the standard Sommerfeld-Watson transformation to write the partial wave sum in equation 1 as an integral over a contour encircling the positive real axis in the J plane. Moving the contour to the line $\text{Re } J = -\frac{1}{2}$, we obtain a background integral

which vanishes for large z plus contributions from all the singularities to the left of $J = N(s)$. A pole in $f_{\pm}(J, s)$ at $J = \alpha_{\pm}(s)$ with residue $\beta_{\pm}(\alpha, s)$ contributes

$$T_{\text{pole}}^{\pm} = -\pi(2\alpha_{\pm} + 1) \beta_{\pm}(\alpha_{\pm}, s) \frac{P_{\alpha_{\pm}}(-z)}{2 \sin \pi \alpha_{\pm}}$$

to the signed amplitude. When we continue this expression to the high energy, small t region of the crossed channel and use the asymptotic properties of the P_{α} , we find that a pole of even/odd signature contributes

$$T_{\text{pole}}^{\pm}(s, t) = -\pi(2\alpha_{\pm}(t) + 1) \beta_{\pm}(\alpha_{\pm}, t) \frac{1 \pm \exp(-i\pi\alpha_{\pm})}{2 \sin \pi \alpha_{\pm}} \left(\frac{s-u}{s_0}\right)^{\alpha_{\pm}}.$$

In the direct channel, the Regge amplitude has resonance-like poles at even/odd integer values of $\alpha_{\pm}$ for even/odd signature poles, so the J plane poles are commonly interpreted as being associated with the exchange of Regge trajectories bearing the quantum numbers (except for spin) of the particles lying on those trajectories.

B. SU(3) and Duality

The unadorned Regge model described above suffers from an excess of unknown quantities. Since the residue from each pole is an unknown function, model fits are often doomed to questionable ambiguity by the proliferation of parameters. By permitting the calculation of relationships between various residues, SU(3) and duality do much to alleviate this problem.

$SU(3)$ is the group of all unitary 3×3 matrices with zero trace.² It has been noticed that the observed particles seem to group themselves into multiplets of $SU(3)$. Groups of particles having identical non- $SU(3)$ quantum numbers and similar masses may often be placed in correspondence with states forming the basis for a representation of $SU(3)$. The conjecture which naturally follows, that $SU(3)$ is a symmetry of nature, has significant implications for high energy scattering. In the case we shall consider, the pseudoscalar mesons and the spin $\frac{1}{2}$ baryons both belong to $SU(3)$ octets. There are eight of the mesons and eight baryons. The use of $SU(3)$ alone reduces the number of independent amplitudes for all $0^{-}\frac{1}{2}^{+}$ reactions to eight for each helicity configuration. This results from the fact that two octets couple to a small number of other representations, and a scalar scattering operator does not permit a change of representation during the scattering process.

In reality it is apparent that $SU(3)$ is not a perfect symmetry. Particles within a multiplet do not have identical masses. Scattering models which incorporate perfect $SU(3)$ are generally unsuccessful. One of the important goals of particle phenomenology is to understand the role of $SU(3)$ symmetry in high energy reactions. Elucidation of a simple, natural breaking mechanism (e.g. by the particle masses themselves) which accommodated the data would be of great value as a unifying force in the analysis of particle data.

Before discussing duality, we introduce the concept of a finite energy sum rule (FESR).³ An amplitude may always be written as the sum of two terms, one of which is even under $s - u$ crossing and receives contributions only from even signature trajectories while the other is

odd and receives contributions only from odd signature trajectories. We define $v = (s-u)/s_0$ and consider an amplitude which is antisymmetric under $s \rightarrow u$ crossing: $T(v,t) = -T(-v,t)$. This amplitude satisfies the dispersion relation

$$T(v,t) = \frac{2v}{\pi} \int_0^{\infty} \frac{dv' \operatorname{Im} T(v',t)}{v'^2 - v^2} \quad (7)$$

where, by writing an unsubtracted dispersion relation, we have assumed that $T(v,t) \underset{|v| \rightarrow \infty}{=} O(v^{-1-\epsilon})$. If we apply this condition to the right hand side of equation 7,

$$\frac{2v}{\pi} \int_0^{\infty} \frac{dv' \operatorname{Im} T(v',t)}{v'^2 - v^2} \underset{|v| \rightarrow \infty}{=} O(v^{-1-\epsilon}),$$

we obtain the requirement

$$\int_0^{\infty} dv' \operatorname{Im} T(v',t) = 0. \quad (8)$$

In general we have $\operatorname{Im} T(v,t) \underset{|v| \rightarrow \infty}{\rightarrow} \sum_i C_i v^{\alpha_i(t)}$ and we must subtract the contributions of poles for which $\alpha \geq -1$ to get the convergence relation of equation 8,

$$\int_0^{\infty} dv' [\operatorname{Im} T(v',t) - \sum_{\alpha \geq -1} C_i v'^{\alpha_i}] = 0. \quad (9)$$

If v_R corresponds to a point where the Regge limit has been reached, we can write, for $v \geq v_R$, $T(v,t) = \sum_{\alpha} C_i v^{\alpha_i}$ and equation 9 can be

written as:

$$\int_0^{v_R} dv [\text{Im } T(v, t) - \sum_{\alpha \geq -1} C_i v^{\alpha_i}] + \int_{v_R}^{\infty} dv \sum_{\alpha \geq -1} C_i v^{\alpha_i} = 0. \quad (10)$$

Evaluation of the integrals gives:

$$\int_0^{v_R} dv \text{Im } T(v, t) = \sum_{\alpha} \frac{C_i}{\alpha_i} v_R^{\alpha_i+1} \quad (11)$$

This is a finite energy sum rule relating the low energy part of the amplitude to the residues which control high energy scattering.

The principal assumption of FESR duality is that the imaginary part of a low energy amplitude arises from direct channel resonances. When the direct channel quantum numbers are "exotic", i.e. correspond to no known particle states, the left hand side of equation 11 is zero, giving a constraint upon the Regge residues. The constraints of SU(3) and duality can be imposed upon the Regge amplitudes to give a model of much elegance and simplicity, with the couplings represented pictorially in terms of duality diagrams. It is this dual Regge model that we shall use as the basis for our analysis of the data.

C. Regge Cuts

In our discussion of Regge poles, we assumed that the continuation of the Froissart-Gribov projection below $J = N(s)$ encountered only isolated singularities. Nothing in our assumptions precludes the possibility of branch cuts being included among these singularities. In fact, the data seem to require their inclusion in any realistic Regge

model. The physical basis of Regge cuts is not as apparent as that of the poles which are associated with observable particles. For poles, we have direct information on the locations of the singularities while the residues are unknown. In the case of cuts, there is no model independent information on either the locations or the discontinuities. It is perhaps reassuring that all the successful cut models, e.g. the multiple scattering quark model,⁴ the absorption model⁵ and the eikonal model,⁶ generate cuts which are similar in form and have much common ground in their physical bases. In all cases, the cuts may be interpreted as arising from the successive exchanges of Regge trajectories, so each cut is physically associated with poles. All agree that the intercept of the cut trajectory is lower than that of its generating poles and that the trajectory slope is smaller.

A simple "derivation" of Regge cuts which reproduces the results of the absorption model and illustrates the presumed basis of the cuts in multiple exchange may be given as follows. Assume that high energy scattering takes place through some basic two body interaction which we will take to be given by the Regge amplitude and that an infinite number of rescatterings in the final state are possible. Take $S = I + iT$ to be the full scattering operator and $S_s = I + iT_s$ to be the operator for a single scatter. We then write

$$\langle f|S|n \rangle = \langle f|S_s|n_1 \rangle \langle n_1|S_s|n_2 \rangle \cdots \langle n_N|S_s|i \rangle$$

where the sums over n_i are sums over discrete quantum numbers and integrals over momenta. This gives rise to a multiple scattering sum of

the form

$$\begin{aligned}
 \langle f|T|i \rangle &= \langle f|NT_s|i \rangle + \frac{N(N-1)}{2N^2} i \langle f|NT_s|n \rangle \langle n|NT_s|i \rangle \\
 &+ \frac{N(N-1)(N-2)}{6N^3} i^2 \langle f|NT_s|n_1 \rangle \langle n_1|NT_s|n_2 \rangle \langle n_2|NT_s|i \rangle \\
 &+ \dots
 \end{aligned}$$

Letting $N \rightarrow \infty$ and taking NT_s to be the Regge pole amplitude, one can evaluate the terms if the residues are suitably parameterized.

D. Phenomenological Overview

The general class of reactions $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$, where 0^{-} is a pseudo-scalar meson and $\frac{1}{2}^{+}$ is a spin $\frac{1}{2}$ baryon, has undergone extensive experimental study.⁷⁻⁵⁴ In a general way, the data exhibit many features of Regge behavior, both s and t dependence being roughly exponential in nature.⁵⁵ However, as we have discussed previously, detailed agreement with the simple Regge pole model is difficult to confirm conclusively because of the great freedom in parameterization.⁵⁶ In addition, there are features of the data, such as charge exchange polarization, which seem beyond the ability of a poles-only model to explain. The dual Regge model is highly constrained, but makes several flat predictions, such as zero polarization for exotic reactions and invariance under line reversal, which are contradicted by the data.⁵⁷⁻⁶² Cut corrections are clearly required in this case and, given the simplicity of the pole amplitudes, several general conclusions regarding their nature can be drawn without a detailed fit.

Qualitatively, one can see that the data are consistent with a scheme in which the flip amplitude is not absorbed, the vector exchange non-flip amplitude receives a correction which is less destructive in the real part than the absorption model predicts, and the tensor exchange non-flip amplitude is only slightly absorbed. This picture is consistent not only with the sign of line reversal breaking in differential cross sections, but with the signs of inelastic polarizations, nearly all of which are entirely cut-induced in our model. For example, in reactions which are exotic according to duality diagrams, the flip amplitude is purely real and the polarization arises from the imaginary part of the cut. The data are all consistent with a featureless destructive cut in the imaginary part of the vector exchange amplitude. A destructive cut in the tensor exchange amplitude contributes to the polarization with the opposite sign. (Throughout this paper we shall use the word "destructive" to imply that an RP cut interferes destructively with the corresponding Regge pole term.)

Recently, a modification of the absorption prescription has been suggested by Ringland et al.⁵⁹ which has the effects just described: cuts in tensor exchange amplitudes are suppressed, and corrections to real parts of vector exchange amplitudes are small or constructive. Although we use Ringland's prescription in the work to be described, we obtain equally good fits by suppressing tensor exchange cuts with a strength parameter and rotating (in a crossing-symmetric way) the vector exchange cut toward the imaginary axis. Evidently, the details of the modifications of absorption model cuts are not of critical importance so long as: (a)

cuts in tensor exchange amplitudes are suppressed, and (b) real parts in vector exchange cuts are less destructive than in the absorption model.

Section II contains a description of the model used in our analysis. Section III presents some implications that can be drawn from the inelastic polarization data. This discussion does not depend on the specific numerical results of our fit. Section IV explains how the Ringland phase modification resolves the conflict between the experimental data and the traditional absorption model. Section V then contains the specific results of our fit to the complete set of data, detailing the mechanisms responsible for the generally good agreement and the disagreement, whenever it exists. Our final amplitudes are presented here. Section VI lists our conclusions.

II. DESCRIPTION OF THE MODEL

We have attempted in this analysis to use the constraints of $SU(3)$ and duality to the fullest extent permitted by the data. The Regge pole part of the amplitude is constructed with $SU(3)$ -symmetric residues satisfying all the constraints of FESR duality, including, through factorization, those obtained from the consideration of pseudoscalar-pseudoscalar scattering. Two meson trajectories are used: one for the non-strange mesons and one for the exchange degenerate K^*-K^{**} pair. The pomeron is treated as an ordinary Regge pole with a trajectory of nonzero slope, and is assumed to be a mixture of singlet and octet components. $SU(3)$ is broken only by the inequality of the strange and nonstrange trajectories and by the octet part of the pomeron. Absorptive corrections to the non-slip amplitudes are calculated by performing the usual convolution⁵⁵ of Regge poles with the pomeron, after the phase modification suggested by Ringland has been applied. Corrections to the pomeron amplitude are made by including the first two terms in the eikonal expansion.⁶

A. $SU(3)$ Details

Work with $SU(3)$ -symmetric residues can be greatly simplified by using a representation of Clebsch-Gordan coefficients in terms of quark wave functions for the particle states. We use the notation $|8,i\rangle$, $i = 1, 2, \dots, 8$ for single particle octet states and $|8_s, i\rangle$ and $|8_a, i\rangle$ for symmetric and antisymmetric two particle octet states. Defining eight 3×3 matrices according to $|8_i\rangle = (M_i)_{jk} |q_j \bar{q}_k\rangle$, one can derive the following representation for the Clebsch-Gordan coefficients:

$$\langle 8a; 8b | 8_s c \rangle = \sqrt{\frac{3}{10}} \langle \tilde{M}_c \{M_a, M_b\} \rangle \quad (12)$$

$$\langle 8a; 8b | 8_{ac} \rangle = -\frac{1}{\sqrt{6}} \langle \tilde{M}_c [M_a, M_b] \rangle \quad (13)$$

On the right hand side, brackets $\langle \rangle$ imply taking the trace and tilde implies transposition; $\{ \}$ and $[\]$ imply commutator and anti-commutator respectively. Coupling of two octets to the singlet states is given by:

$$\langle 8a; 8b | 1 \rangle = -\frac{1}{4\sqrt{2}} \langle \{M_a, M_b\} \rangle \quad (14)$$

One also derives the useful results:

$$\sum_{i=1}^8 \langle 8a; 8b | 8_i \rangle \langle 8c; 8d | 8_i \rangle = -\frac{1}{6} \langle [\tilde{M}_a, \tilde{M}_b] [M_c, M_d] \rangle \quad (15)$$

$$\begin{aligned} \sum_{i=1}^8 \langle 8a; 8b | 8_i \rangle \langle 8c; 8d | 8_i \rangle &= \frac{3}{10} \langle \{\tilde{M}_a, \tilde{M}_b\} \{M_c, M_d\} \rangle \\ &\quad - \frac{1}{10} \langle \{\tilde{M}_a, \tilde{M}_b\} \rangle \langle \{M_c, M_d\} \rangle \end{aligned} \quad (16)$$

$$\sum_{i=1}^8 \langle 8a; 8b | 8_i \rangle \langle 8c; 8d | 8_i \rangle = \frac{1}{2\sqrt{5}} \langle [\tilde{M}_a, \tilde{M}_b] \{M_c, M_d\} \rangle \quad (17)$$

For mesons, these traces have direct interpretations as quark diagrams.⁶³ For example, the term

$$\langle \tilde{M}_b \tilde{M}_a M_c M_d \rangle = (M_b)_{ji} (M_a)_{kj} (M_c)_{kl} (M_d)_{li}$$

may be diagrammed as shown in Fig. 1. For baryons, we write the quark wave functions as $|8_i\rangle = (B_i)_{jkl} |q_j q_k q_l\rangle$ and make the replacement

Fig. 1. Quark diagram for four mesons, corresponding to the trace $\langle \tilde{M}_b \tilde{M}_a \tilde{M}_c \tilde{M}_d \rangle$.

$(M_i)_{jk} = \frac{1}{\sqrt{2}}(B_i)_{jrs}\epsilon_{krs}$ in the traces. For calculational purposes, it is convenient to use the 3×3 matrices, even for baryons. A sample trace calculation is given in Appendix A.

Analogous results may readily be derived for couplings involving 10 and 10^* as well, as shown in Appendix B.

B. The Regge Pole Amplitude

The constraints imposed by duality upon the factorizable residues have been worked out previously and we do not repeat them here.⁶⁴ The process of obtaining a general expression for the amplitude given the residue constraints is quite easy using the trace methods, since the sum over t channel exchanges for a general set of external particles can be done with equations (15) - (17). Using the s channel helicity amplitudes of Cohen-Tannoudji et al.,⁶⁵ we obtain:

$$\begin{aligned}
 \langle cd|T|ab \rangle = & (-t)^{\frac{\lambda_d - \lambda_b}{2}} \{ [\langle \tilde{d}a \tilde{c}b \rangle (D+F)_{\lambda_d \lambda_b} \\
 & + (\langle \tilde{d}b \tilde{a}c \rangle - \langle \tilde{d}b \rangle \langle \tilde{c}a \rangle) (D-F)_{\lambda_d \lambda_b}] \\
 & + [\langle \tilde{d} \tilde{c} ab \rangle (D+F)_{\lambda_d \lambda_b} + (\langle \tilde{d}b \tilde{c}a \rangle \\
 & - \langle \tilde{d}b \rangle \langle \tilde{c}a \rangle) (D-F)_{\lambda_d \lambda_b}] e^{-i\pi\alpha} \} \left(\frac{s}{s_0}\right)^\alpha
 \end{aligned} \tag{18}$$

$D(t)$ and $F(t)$ are independent residues corresponding to symmetric and antisymmetric coupling respectively at the baryon vertex. "d", for

example, refers to the transpose of M_d , the 3×3 matrix previously defined. In terms of duality diagrams, the traces may be expressed as shown in Fig. 2.

The same methods can be used to derive duality diagrams for other reactions, including some involving decuplets, such as $PB \rightarrow PD$ which is calculated in Appendix B.

C. The Vacuum Exchange Amplitude

Since the πN and KN total cross sections differ substantially over the momentum range considered, the pomeron cannot be regarded as an $SU(3)$ singlet in our model. We take account of this by giving the pomeron an explicit $I=Y=0$ octet component. Our amplitude is:

$$\begin{aligned}
 \langle cd|T|ab\rangle = & \left\{ \frac{1}{8} \langle \tilde{d}b \rangle \langle \tilde{c}a \rangle P_s \right. \\
 & - \frac{1}{2\sqrt{5}} \langle [d,b] M_8 \rangle \langle [c,a] M_8 \rangle P_{8a} \\
 & \left. + \frac{3}{10} \langle [d,b] M_8 \rangle \langle [c,a] M_8 \rangle P_{8s} e^{\frac{-i\pi\alpha_P}{2}} \left(\frac{s}{s_0} \right)^{\alpha_P} \right\} \quad (19)
 \end{aligned}$$

M_8 is the matrix corresponding to the $I=Y=0$ octet state, P_s is the residue for the singlet part of the pomeron, and P_{8s} and P_{8a} are residues for the octet part with symmetric and antisymmetric coupling respectively to $\bar{B}B$. In the fit we present in Sec. V, P_{8a} has been omitted entirely; fits including it as a free parameter take it to a very small and unimportant contribution, so we believe that only P_s and P_{8s} are necessary in this amplitude. We have included a small helicity flip term, also of the structure given in equation 19.

$$\langle \tilde{d} b a \tilde{c} \rangle - \langle \tilde{d} b \rangle \langle \tilde{c} a \rangle =$$

$$\langle \tilde{d} a \tilde{c} b \rangle =$$

$$\langle \tilde{d} b \tilde{c} a \rangle - \langle \tilde{d} b \rangle \langle \tilde{c} a \rangle =$$

$$\langle \tilde{d} \tilde{c} a b \rangle =$$

Fig. 2. Quark diagrams for the four traces in Equation 18. The circles represent antisymmetrized quarks.

For the vacuum cut, we have kept the first two terms in the eikonal expansion, thus getting a contribution of either sign, and we have fitted an overall multiplicative cut strength. Since each of the pomeron terms can be written as Ae^{at} , the cut can be evaluated according to:⁵⁵

$$Ae^{at} \otimes Be^{bt} = \frac{1}{2} \left(\frac{iq}{4\sqrt{s}} \right) \frac{4\pi AB}{s(a+b)} e^{\frac{ab}{a+b} t} \quad (20)$$

$$Ae^{at} \otimes Be^{bt} \otimes Ce^{ct} = \frac{1}{6} \left(\frac{iq}{4\sqrt{s}} \right)^2 \left(\frac{4\pi}{s} \right)^2 \frac{ABC}{ab+bc+ac} e^{\frac{abc}{ab+bc+ac} t} \quad (21)$$

The pomeron flip amplitudes have been omitted from the cuts.

D. The Regge-Pomeron Cut Amplitude

In the high energy limit, the absorption prescription for RP cuts gives, for two exponentials, just the result of equation 20. Effectively, the Ringland modification amounts to multiplication of the non-rotating part of the pole by "i" before convolution with the pomeron. In crossing symmetric form, one makes the substitution:

$$(e^{-i\pi\alpha} + \tau) \left(\frac{s}{s_0} \right)^\alpha \rightarrow \left[\ln \left(\frac{s}{is_a} \right) \right]^{-1} \left(e^{-i\frac{\pi}{4}} e^{-i\pi\alpha} + \tau e^{i\frac{\pi}{4}} \right) \left(\frac{s}{s_0} \right)^\alpha \quad (22)$$

We choose s_a to give the multiplicative factor phase $\frac{\pi}{4}$ at a central energy. For even signature, this change can be written as:

$$e^{\frac{-i\pi\alpha}{2}} \cos \frac{\pi\alpha}{2} \rightarrow \left[\ln \left(\frac{s}{is_a} \right) \right]^{-1} \cos \frac{\pi}{2} \left(\alpha + \frac{1}{2} \right) e^{\frac{-i\pi\alpha}{2}} \quad (23)$$

For odd signature:

$$i \sin \frac{\pi\alpha}{\alpha} e^{\frac{-i\pi\alpha}{2}} \rightarrow [\ln(\frac{s}{s_a})]^{-1} i \sin \frac{\pi}{2} (\alpha + \frac{1}{2}) e^{\frac{-i\pi\alpha}{2}} \quad (24)$$

We discuss the implications of this modification in a later section.

E. SU(3) Breaking

We allow SU(3) breaking only by the octet component of the pomeron and by trajectory splitting between $K^* - K^{**}$ and the nonstrange mesons. Both of these mechanisms seem to be required by considerations other than detailed fitting of the data; i.e. inequality of the asymptotic πN and KN total cross sections and the positions of mesons on the Chew-Frautschi plot. We have chosen standard values for the trajectory parameters: $\alpha = .55 + .9t$ for the nonstrange mesons, and $\alpha = .35 + .8t$ for $K^* - K^{**}$. Our residues all satisfy perfect SU(3).

Of course, the octet part of the pomeron also contributes to SU(3) breaking in inelastic amplitudes through its appearance in the RP cut. However, the octet part is about 30% of the pomeron, and the RP cut is typically 25% of the pole, so the effect is not large.

III. QUALITATIVE IMPLICATIONS OF POLARIZATION DATA

Before discussing the model further, we attempt to draw from the inelastic polarization data some inferences which are more general than the results of an explicit fit. This analysis involves the following assumptions:

- (1) The phase of the cut varies slowly with t , so
neither the real nor imaginary part changes sign
in the region of interest.
- (2) The flip amplitude satisfies strong exchange degeneracy.
- (3) $(F/D)_{++} < -1$ and $(F/D)_{+-} \approx \frac{1}{3}$ for all $|t| \leq 1$, where the subscripts refer to s-channel helicities.

These conditions are all satisfied in our model.

We now examine several reactions individually.

In this reaction, only the ρ is exchanged, and the sign of the polarization is given by $P \sim \sin^2 \frac{\pi\alpha}{2} (1 - (Re/Im)_c \cot \frac{\pi\alpha}{2})$, where $(Re/Im)_c$ is the real to imaginary ratio for the ρP cut, and we have assumed that the imaginary part is destructive of the pole. If the polarization is not to change sign in the region $t > -0.6$, the condition $(Re/Im)_c < \tan \frac{\pi\alpha}{2}$ must be satisfied. Since $(Re/Im)_c$ is assumed to vary slowly with t while $\tan \frac{\pi\alpha}{2}$ decreases rapidly away from $t=0$, we conclude that $(Re/Im)_c$ must be small, even at $t=0$. The absorption model can accomplish this only with a pomeron trajectory of large slope which is difficult to reconcile with the elastic differential cross section data.

We conclude that the charge exchange polarization data is one bit of evidence for a ρP cut which is destructive and more imaginary than in the absorption model.

$$B. \quad K^- p \rightarrow \pi^- \Sigma^+$$

Reactions such as this one, being exotic according to duality diagrams, are useful to consider because the flip amplitude is purely real, and the polarization therefore isolates the imaginary part of the cut.

Given the assumed values for the F/D ratios, the polarization in this reaction takes the sign $P \sim \text{Im}(K^{**} \otimes P) - \text{Im}(K^* \otimes P)$, where the terms are intrinsically positive if destructive of the pole. The measured polarization is negative, which indicates that the imaginary part of K^* receives a greater destructive correction than does K^{**} . In fact, since a destructive K^{**} cut gives a positive contribution to the $K^- p \rightarrow \pi^- \Sigma^+$ polarization, the rather large magnitude of the polarization in this reaction may be taken as evidence for a very small destructive, or even constructive, correction to the imaginary part of K^{**} .

These conclusions also apply to the exotic reactions $K^- n \rightarrow \pi^- \Lambda$ and $K^- p \rightarrow \pi^0 \Lambda$. In both cases, the polarization is large and its sign indicates a more destructive cut in K^* than in K^{**} .

$$C. \quad \pi^- p \rightarrow \eta n$$

From the polarization in this reaction, we can probably conclude that the systematics of the $A_2 \otimes P$ cut are different from those of $\rho \otimes P$. A destructive, predominantly imaginary cut in the A_2 nonflip amplitude gives negative polarization, while the measured values are nearly all positive.

D. KN Charge Exchange

Although experimental data on these reactions are not yet available, they are of particular interest here because they provide rather stringent tests of our basic hypotheses. Our assumptions lead to a strong prediction in the case of K^+N charge exchange. The flip amplitude should be purely real, and a destructive $\rho \leftrightarrow P$ cut gives positive polarization over the entire region $t \gtrsim -1$.

For K^-p charge exchange, we expect the polarization to arise primarily from the destructive imaginary part of the $\rho \leftrightarrow P$ cut, in which case the polarization has the sign $P \sim -\cos \pi\alpha$. Thus, we predict negative polarization, at least for $t \lesssim -0.1$.

IV. QUALITATIVE FEATURES OF THE MODEL.

Having obtained from the data some fairly direct information on certain features of the cuts, we now discuss the qualitative nature of absorption as given by our model. In particular, we show how the Ringland phase modification makes the absorption model consistent with the conclusions reached in the previous section.

For the purpose of drawing qualitative conclusions, we replace the logarithmic factor $[\ln(\frac{s}{s_a})]^{-1}$ in equations (23) and (24) with $e^{\frac{i\pi}{4}}$, which is approximately justified over a wide energy range. The statements we make below are all verified by exact calculations.

When the above approximation is made, the cut in the vector exchange amplitude is obtained by convoluting:

$$e^{\frac{i\pi}{4}} \left[-i \sin \frac{\pi}{2} \left(\alpha + \frac{1}{2} \right) e^{-i \frac{\pi\alpha}{2}} \right] \left(\frac{s}{s_0} \right)^\alpha \quad (25)$$

with the pomeron. If the pomeron slope is small ($\alpha'_p \approx 3$), we can neglect the real part of the pomeron for our present purposes. The major contribution to the convolution integral comes from the small t region because of the exponential decrease of the amplitudes, so the phase of the cut is given roughly by:

$$V \otimes P \sim i e^{-i \frac{\pi}{2} (\alpha_0 - \frac{1}{2})} \quad (26)$$

For the nonstrange exchanges ($\alpha_0 = .55$), the cut is destructive and almost purely imaginary.

For K^* ($\alpha_0 = .35$) the situation is not quite as simple. The lower intercept causes a constructive real part to arise in addition to the destructive imaginary part. Were we to multiply the non-rotating part of the pole by $e^{i\pi\alpha_0}$ rather than "i", greater symmetry between the cases of strangeness exchange and non-strangeness exchange would result; the K^* cut would also be nearly purely imaginary. To investigate the importance of the constructive real part in the K^* cut, consider:

$$\left(\frac{d\sigma}{dt}\right)_R \sim |[K^{**} + (K^{**} \otimes P)] + [K^* + (K^* \otimes P)]|^2$$

$$\left(\frac{d\sigma}{dt}\right)_{N.R.} \sim |[K^{**} + (K^{**} \otimes P)] - [K^* + (K^* \otimes P)]|^2$$

Here N.R. refers to a reaction with a purely real ("non-rotating") pole amplitude, and R. refers to its line-reversed ("rotating") counterpart, with phase $e^{-i\pi\alpha}$. If we ignore terms quadratic in the cuts, we have:

$$\left(\frac{d\sigma}{dt}\right)_{N.R.} - \left(\frac{d\sigma}{dt}\right)_R \sim -2 \operatorname{Re}[K^{**}(K^* \otimes P)^* + K^*(K^{**} \otimes P)^*]$$

where "*" on the cut terms implies complex conjugation. The K^* and K^{**} pole terms have imaginary parts of the same sign and real parts of opposite signs. Thus, we see that for both cuts, positive contributions to the difference come from destructive imaginary parts and constructive real parts. (In the absorption model, the large, destructive real parts of the cuts give the wrong sign for line-reversal breaking; the real parts are over-absorbed.) Qualitatively, therefore, the polarizations we have considered and the signs of line-reversal breaking do not dis-

tinguish a vanishing real part in the K^* cut from a constructive one. We find on the basis of chi-squared that the earlier prescription (i rather than $e^{i\pi\alpha_0}$) is somewhat preferred. The constructive real part plays a role in "fine-tuning" the fit to differential cross section magnitudes, but has no strong qualitative effect. The important point is that the destructive real part, as given by the absorption model, has been modified. Whether it goes to zero or becomes constructive is more a matter of detail, and is probably not well decided by our parameterization-dependent fit.

For tensor exchange, we convolute:

$$e^{-i\frac{\pi}{2}(\alpha - \frac{1}{2})} \cos \frac{\pi}{2} (\alpha + \frac{1}{2}) \left(\frac{s}{s_0}\right)^\alpha \quad (27)$$

In the case of K^{**} , $\cos \frac{\pi}{2} (\alpha + \frac{1}{2})$ is not zero near $t=0$, so our previous arguments give the phase:

$$K^{**} \otimes P \sim -e^{-i\frac{\pi}{2}(\alpha_0 + \frac{1}{2})} \quad (28)$$

However, $\cos \frac{\pi}{2} (\alpha + \frac{1}{2})$ is sufficiently smaller than $\sin \frac{\pi}{2} (\alpha + \frac{1}{2})$ at small t , even for the $K^* - K^{**}$ trajectory, that the K^{**} cut is suppressed relative to K^* . We see from equation (28) that $K^{**} \otimes P$ has a constructive imaginary part and destructive real part. However, just as is the case for $\text{Re}(K^* \otimes P)$, the K^{**} cut plays no essential role in the fit. Its effect is to increase somewhat the polarization in exotic reactions and to decrease slightly the amount of line-reversal breaking. This is the effect one gets by increasing the imaginary part of $(K^* \otimes P)$ at the

expense of the real part, which may indicate the possibility of a vanishing K^{**} cut, with the K^* cut similar to that of the ρ .

For the A_2 amplitude, $\cos \frac{\pi}{2} (\alpha + \frac{1}{2})$ goes through zero at small t , and our simple arguments cannot be used. Actual calculation shows that the A_2 cut has a very small constructive imaginary part which gives the positive sign for the polarization in $\pi^- p \rightarrow \eta^0 n$ and plays no other role.

Thus, we conclude from this and the previous section that the cut terms of dominant importance are the destructive imaginary parts in vector exchange. In a qualitative description of a large amount of $0^{-\frac{1}{2}+}$ data, no other cuts at all are necessary, and in our explicit model fit, they are by far of the greatest importance. Ringland's prescription is successful because it modifies the destructive real part in vector exchange as given by the absorption model and decreases the magnitude of tensor exchange cuts.

Our results are in disagreement with the dual absorption model, since we do not find peripheral imaginary parts for tensor exchange amplitudes.⁶²

V. DETAILS OF THE FIT

We turn now to a more detailed examination of the model. First we give the explicit parameterization, and then discuss the comparison with the data.

A. Parameterization of the Model

Using the notation of equation 18, we have for the nonflip Regge residues:

$$D + F = (A_1 + A_2 t + A_3 t^2) e^{at}$$

$$D - F = B e^{bt}$$

For the helicity flip residues:

$$D + F = \Gamma(1 - \alpha) C e^{ct}$$

$$D - F = \Gamma(1 - \alpha) D e^{ct}$$

The pomeron parameters are defined in the notation of equation 19

$$P_s(t) = P_1 \exp(p_1 t) + P_2 \exp(p_2 t)$$

$$P_{8s}(t) = P_3 \exp(p_3 t)$$

for the non-flip residues, and the following for the flip residues,

$$P_s(t) = P_4 \exp(p_4 t)$$

$$P_{8s}(t) = \frac{P_3 P_4}{p_1 + p_2} \exp(p_4 t).$$

The pomeron trajectory function is written as $\alpha_p = 1 + \alpha'_p t$. Two multiplicative cut strengths were varied in the fit: C_p for the vacuum cut and C_r for the RP cut. Parameter values are given in Table I.

Observable quantities are related to our amplitudes as follows:

Total cross section

$$\sigma_T = \frac{2\pi^2(0.3893)}{q\sqrt{s}} \text{Im } A_{++} (t=0)$$

Differential cross section

$$\frac{d\sigma}{dt} = \frac{(0.3893)\pi^3}{4q^2s} (|A_{++}|^2 + |A_{+-}|^2)$$

Polarization

$$P = 2 \text{Im } (A_{++} A_{+-}^*) / (|A_{++}|^2 + |A_{+-}|^2)$$

$$R = \frac{-\cos \theta (|A_{++}|^2 - |A_{+-}|^2) + 2 \sin \theta \text{Re } (A_{++} A_{+-}^*)}{|A_{++}|^2 + |A_{+-}|^2}$$

$$A = \frac{\sin \theta (|A_{++}|^2 - |A_{+-}|^2) + 2 \cos \theta \text{Re } (A_{++} A_{+-}^*)}{|A_{++}|^2 + |A_{+-}|^2}$$

where q is the magnitude of the initial center of mass momentum, s is the square of the center of mass energy, and θ is the laboratory recoil angle of the final state baryon.

Table 1. Parameters determined by the
fit presented in Section V.

$A_1 = - .537$	$P_1 = -9.589$
$A_2 = -2.031$	$P_2 = -3.181$
$A_3 = -2.330$	$P_3 = -2.480$
$a = 1.154$	$P_4 = -1.078$
$B = 1.096$	$P_1 = 2.625$
$b = 2.220$	$P_2 = - .314$
$C = 1.342$	$P_3 = 3.306$
$D = .653$	$P_4 = 2.877$
$c = .941$	$\alpha'_p = .294$
$C_r = .325$	$C_p = 1.439$

B. Comparison with the Data

1. Total cross sections (TCS)

The π^-p TCS is well reproduced at all momenta between 5 and 65 GeV/c, never straying further than one third of a millibarn from the data. The π^+p TCS is, however, consistently about half a millibarn higher than the data between 20 and 40 GeV/c. Similarly, the K^-p TCS is fitted very well at all momenta, while the K^+p TCS is somewhat higher than our results at the high momenta. It is actually the TCS differences that are poorly fitted. Since the pomeron and vacuum cut amplitudes cannot contribute to either TCS difference, the difficulty may be in the energy dependence of the RP cut contribution. Since the inelastic data are available only over a relatively small energy range, the energy dependence of the RP cut is not well determined in the rest of the fit. We have checked that the discrepancy between the model and the data is of the same order of magnitude as the RP cut contribution, and we conclude that the energy dependence of RP cut models deserves further study when high energy inelastic data become available. The total cross sections are shown in Fig. 3.

2. Elastic differential cross sections (DCS)

All the elastic differential cross sections, with the possible exception of K^+p at the lowest energies, are fitted very well at lab momenta above 3.0 GeV/c. In particular, the new high energy DCS for π^-p and K^-p at 25 and 40 GeV/c are well described in both normalization and t -dependence. The model gives a slightly steeper slope to the K^+p

Fig. 3. Experimental measurements (Refs. 7-10, 19, 20, 25-27) and model results for the total cross sections.

DCS than is indicated by the data, but the fit is, in any case, quite satisfactory. The elastic differential cross sections are shown in Figs. 5 and 7.

The new Argonne data at 3.0, 3.65, 5.0, and 6.0 GeV/c provide precise determinations of the DCS differences. Under certain assumptions⁶⁶ one can obtain from these differences the approximate imaginary parts of the ρ and ω exchange nonslip amplitudes. In our model, the correct description of the crossover points in KN is due entirely to a very strong RP cut moving the ω signature zero toward $t=0$. This is direct evidence in favor of the Ringland phase modification for vector $l=0$ exchange. We also note at this point that the primary purpose of the quadratic factor in the non-flip ($D + F$) is apparently to adjust the relative strength of the RP cut in hypercharge and non-hypercharge exchange reactions. In fact, omitting the quadratic residue results in a πN DCS crossover point closer to $t=0$ than is required by the data, at the expense of a deterioration in the fit to hypercharge exchange data. The DCS differences are shown in Fig. 13.

3. Elastic polarizations

While the K^+p and K^-p polarizations are well fitted at all energies, there is some disagreement in $\pi^\pm p$ at the lower energies for values of t beyond the ρ signature zero. In particular, the model exhibits less energy dependence than is present in the data, and prefers the gentler behavior seen at the higher momenta.

Conventionally, the πp polarizations are explained in terms of a flat pomeron and a ρ -dominated flip amplitude, giving a strong quadratic

Fig. 4. Experimental measurements (Ref. 10) and model results for the real to imaginary ratio at $t=0$. (a) $\pi^+ p$ elastic; (b) $\pi^- p$ elastic.

Fig. 5. Experimental measurements and model results for differential cross sections and polarizations. (a) π^+p elastic (Refs. 11-15); (b) π^-p elastic (Refs. 11,12). Not all data fitted are shown.

Fig. 6. Experimental measurements and model results for the R parameter (Ref. 18). (a) π^+p elastic at 6.0 GeV/c; (b) π^-p elastic at 6.0 GeV/c; (c) π^-p elastic at 16.0 GeV/c.

Fig. 7. Experimental measurements and model results for differential cross sections and polarizations. (a) K^+p elastic (Refs. 11, 16, 22, 24, 28); (b) K^-p elastic (Refs. 11, 16, 21-24). Note all data fitted are shown.

zero at the wrong signature point. Although we find complete ρ dominance of the flip amplitude, the non-zero pomeron slope causes a splitting of the quadratic zero into two linear zeros. The vacuum cut only makes matters worse, since it gives a subtractive contribution with more closely spaced linear zeroes, causing an even greater separation in the net polarization. The energy dependence of the vacuum cut phase would require some modification to reproduce the marked double zero behavior seen at 6 GeV/c. The polarizations are shown in Figs. 5 and 7.

4. Real to imaginary ratios, R and A parameters

The real/imaginary ratio at $t=0$ for π^+p is fitted very well; however, the corresponding result for π^-p is about a factor two too small in magnitude. This result is very sensitive to the t -dependence of our residues at small t , and thus is not a major concern.

We have also included R and A for π^-p and R for π^+p in our model fit. Due to the relative scarcity and uncertainty in these data, they do not influence our amplitudes much in the χ^2 fit. The main conclusion to be drawn from these data is that each of our amplitudes is correct in sign and qualitatively correct in t -dependence for $|t| \leq 0.6$. These data are the main reason for including a flip term in the pomeron amplitude. These data are shown in Figs. 4 and 6.

5. K^-p , K^+p charge exchange

At lab momenta above 5 GeV/c, these reactions show little line reversal breaking. In terms of our model, this is explained by the dominance of the flip amplitude, in which there is no cut. The KN CEX differential cross sections are shown in Fig. 8.

Fig. 8. Experimental measurements and model results for differential cross sections. (a) K^+n charge exchange (Refs. 37, 38, 40); (b) K^-p charge exchange (Refs. 35, 36, 39).

6. $\pi^- p \rightarrow \pi^0 n$

The one remaining major discrepancy between our fit and the data is in the charge exchange differential cross section beyond the dip, where the model is systematically low. We find that if agreement is forced by alteration of the residues or strengthening of the RP cut, then agreement with the hypercharge exchange data deteriorates. The source of this problem might be anything from a simple inadequacy of the parameterization to a necessity for more extensive breaking of $SU(3)$ than we have allowed. Note, however, that agreement is good for $|t| \lesssim 0.6$.

As previously discussed, the polarization is explained in terms of a destructive, predominantly imaginary cut in the non-flip amplitude together with an unabsorbed flip amplitude. The DCS and polarization data are shown in Fig. 9.

7. $\pi^- p \rightarrow \eta n$

We find that mixing of the η must be taken into account in order to fit the magnitude of the differential cross sections. The theory is too large by several standard deviations when the η is treated as pure octet. After fitting the rest of the data, we used the singlet coupling strength given by duality⁶⁰ (and the quark model) and adjusted the mixing angle by inspection. The value we require is $\theta = 5^\circ$, which is opposite in sign from the result of Martin and Michael.⁶⁷ These data are shown

in Fig. 9.

Fig. 9. Experimental measurements and model results for differential cross sections and polarizations. (a) π^-p charge exchange (Refs. 30-34); (b) $\pi^-p \rightarrow \eta^0 n$ (Refs. 29-32). Not all the polarization data fitted are shown.

8. $\pi^- p \rightarrow K^0 \Lambda$, $K^- n \rightarrow \pi^- \Lambda$, $K^- p \rightarrow \pi^0 \Lambda$

The polarizations and differential cross sections are reasonably well fitted down to 3 GeV/c. The slopes of the differential cross sections are somewhat smaller for the two exotic reactions, and our model also reproduces this feature.

Interpretation of the $\pi^- p \rightarrow K^0 \Lambda$ polarization is rather complicated, since the flip amplitude is not purely real. The data for this reaction and $\pi^+ p \rightarrow K^+ \Sigma^+$, which is theoretically similar, both indicate a sign change at $|t| \approx .25$, after which point the polarization takes the sign contributed by the destructive imaginary part of the K^* cut. In terms of the model, this sign change arises from the real part of the cut, to which the constructive real part in $K^* \otimes P$ and the destructive real part in $K^{**} \otimes P$ add constructively. At small t , the imaginary part of the flip amplitude is dominant, so the real part of the cut dominates the polarization. However, the imaginary part of flip decreases rapidly and changes sign at $t \approx -.4$ while the real part does not change sign, so for $|t| \geq .25$ the imaginary part of the cut dominates the polarization, and for $|t| \geq .4$, the two contributions have the same sign.

This sign change seems definitely to be present in both the $\pi^- p \rightarrow K^0 \Lambda$ and $\pi^+ p \rightarrow K^+ \Sigma^+$ data, and might be regarded as evidence against the similarity of $\bar{p}$ and K^* cuts if the explanation above, requiring an appreciable real part in the K^* cut, is correct.

As previously discussed, the exotic polarizations can be explained as arising from the destructive imaginary part of the K^* cut. These data are shown in Fig. 10.

9. $\pi^+ p \rightarrow K^+ \Sigma^+$, $K^- p \rightarrow \pi^- \Sigma^+$

In addition to the line reversal breaking present in these reactions, a very noticeable feature of the differential cross sections is a distinct change of slope at about $|t| = 0.5$ for the reaction with rotating phase. The slope change is also present in $\pi^- p \rightarrow K^0 \Lambda$ and $\pi^- p \rightarrow K^0 \Sigma^0$. In our model, this break is the result of a sharp dip in the contribution to the DCS from the non-flip amplitude. The dip is not related to a signature zero, but instead is the result of a strong destructive real part of the RP cut causing a near coincidence of the zeros in the real and imaginary parts of the non-flip amplitude.

The polarization for $\pi^+ p \rightarrow K^+ \Sigma^+$ has been discussed in the previous section, and the polarization for the exotic reaction $K^- p \rightarrow \pi^- \Sigma^+$ has also been discussed at some length in Section III. Both are well reproduced by the model. In both reactions, however, we are unable to fit the differential cross section magnitudes below 5.0 GeV/c. We do fit $\pi^- p \rightarrow K^0 \Sigma^0$, which is related by t channel isospin to $\pi^+ p \rightarrow K^+ \Sigma^+$, down to 3.0 GeV/c. For both reactions, the model results are too large at the lower energies. These data are presented in Fig. 10.

10. $K^- p \rightarrow \eta \Lambda$, $K^- p \rightarrow \eta' \Lambda$

For $K^- p \rightarrow \eta \Lambda$, the data indicate a pronounced dip at $|t| \approx .4$. In the absence of mixing, the pole part of the amplitude is given by $A \sim K^{**} + 3K^*$, so the dip can be attributed to the signature zero in the dominant K^* exchange amplitude. As shown in Fig. 12, our model agrees only roughly with the data. Contributions from the cut and the K^{**}

Fig. 10. Experimental measurements and model results for differential cross sections and polarizations. (a) $\pi^+p \rightarrow K^+\Sigma^+$ (Refs. 51-53), $K^-p \rightarrow \pi^-\Sigma^+$ (Refs. 35, 39, 48-50); (b) $\pi^-p \rightarrow K^0\Lambda$ (Refs. 41, 42), $K^-n \rightarrow \pi^-\Lambda$ (Refs. 44-46); (c) $K^-p \rightarrow \pi^0\Lambda$ (Refs. 35, 39, 47, 48). Not all the polarization data fitted are shown.

Fig. 11. Experimental measurements and model results for differential cross sections. (a) $\pi^-p \rightarrow K^0\Sigma^0$ (Refs. 41, 42); (b) $\pi^-p \rightarrow K^0\Lambda/\Sigma^0$ (Ref. 43).

Fig. 12. Experimental measurements and model results for differential cross sections and polarizations in the reactions $K^-p \rightarrow \eta\Lambda$ and $K^-p \rightarrow \eta'\Lambda$ (Refs. 35, 54).

Fig. 13. Experimental (Ref. 11) and model results for elastic differential cross section differences at 6 GeV/c.

(a) $[\text{d}\sigma^-/\text{d}t - \text{d}\sigma^+/\text{d}t]/\sqrt{8(\text{d}\sigma^+/\text{d}t + \text{d}\sigma^-/\text{d}t)}$ for πN .

(b) the same quantity as in (a) for KN .

amplitude are sufficient to fill in the dip and give instead a very sharp break in slope. Agreement with the featureless shape of $K^-p \rightarrow \eta' \Lambda$ is reasonable.

It should be noted that the $\eta - \eta'$ mixing angle, which affects the differential cross section magnitudes for these reactions, was determined solely from the $\pi^-p \rightarrow \eta^0 n$ data.

C. Amplitudes

Since considerable polarization is observed in several reactions which are exotic according to duality diagrams, one obviously cannot say that the amplitudes corresponding to nonplanar duality diagrams are purely real. However, if our model results are taken seriously, one can make the following statements:

- (1) The flip amplitude corresponding to a nonplanar diagram is purely real, while the amplitude for a planar diagram has phase $e^{-i\pi\alpha}$.
- (2) The nonflip amplitude corresponding to a planar diagram has a peripheral imaginary part with the same zeros as given by the dual absorption model. For a nonplanar diagram, the non-flip amplitude has a structureless, nonperipheral imaginary part which does not change sign for $|t| \lesssim 1.5$, and the real to imaginary ratio at $t=0$ is a factor of 3-5 larger than is the case for planar amplitudes.

The structure of our exotic nonflip amplitudes is considerably different from that obtained in a recent amplitude analysis⁶² based

on the dual absorption model.⁶⁶ In that analysis, both vector and tensor exchange amplitudes were taken to be peripheral. While the K^+n charge exchange amplitude (which is quark-model exotic) was found to have a small imaginary part, amplitudes for processes which are exotic only by duality diagrams were found to be similar in structure to their line-reversed counterparts, with large, peripheral imaginary parts. Since both models reproduce the polarization data, measurements of the spin rotation parameters will be required to determine which, if either, picture is correct. Some of the amplitudes are shown in Figs. 14 and 15.

D. Some Technical Aspects of the Fit

In this section we wish to discuss briefly the statistical quality of our analysis. The χ^2 per data point for the fitted result presented is 2.6 for 1987 data points. We remark that this χ^2 reflects not only the deviation of our model from the parent distribution function, but also the deviation of the data from the parent distribution. When one analyzes 143 different experiments simultaneously, the total effect of systematic errors can become comparable to the statistical errors. By varying the normalization of each experiment within the systematic errors quoted by the authors, we determined that about one third of the χ^2 may come from systematic errors. However, since the average of all renormalizations was very nearly zero, and since none of the parameters was seriously affected, we chose to ignore these errors during our final analysis. We hope that experimentalists will soon consider it worth-

Fig. 14. Model results for individual particle exchange amplitudes at 6.0 GeV/c.

Fig. 15. Model results for the reactions (a) $\pi^+p \rightarrow K^+\Sigma^+$; (b) $K^-p \rightarrow \pi^-\Sigma^+$.

while to refine our knowledge by repeating many important experiments in the intermediate energy range (3-30 GeV/c) with higher statistics and smaller systematic errors.

VI. SUMMARY AND CONCLUSIONS

In this analysis, a model with 20 variable parameters was fit to most of the existing high energy data for the reactions $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$. A total of 1987 data points forming 143 separate s-dependent or t-dependent measurements on 18 reactions were included. In terms of the number of distinct reactions treated simultaneously, we believe this represents the most extensive high energy fit performed to date.

In high energy physics, a model fit seldom passes conclusive judgment upon the hypotheses being tested. Invariably there is insufficient knowledge of the dynamical details to support an unambiguous translation of ones hypotheses into a mathematical model. As a result, there is an arbitrariness of form that makes a rigorous statistical test of the propositions impossible. Instead a subjective interpretation of the results is required in which an attempt must be made to understand the nature and significance of those ad hoc assumptions made in formulation of the model. In our fit, chi squared per data point was 2.6 which is large when rigorously interpreted. However, the arbitrary choices for the residue parametrizations and the cut modifications are not dictated by the principles of the model, and their inadequacies must not be allowed to reflect unfairly upon the basic ideas. From the standpoint of reproducing the trends and important features of the data, we believe the fit to be successful. Most of the chi squared contributions come from disparities in detailed t dependence and not from failures to fit the structure of the data. It seems quite plausible to blame much of this on the t-dependent residues.

The picture of high energy scattering supported by this fit is attractive in many respects. The basic and dominant feature is the dual Regge amplitude with the full constraints of $SU(3)$ and duality intact except as broken by mass differences. $SU(3)$ is further broken only by the octet component of the pomeron which, at least in the context of our model, is dictated directly by total cross section differences. As contrasted with the pole amplitude, our cut formulation is highly phenomenological in nature. The cuts do have the $SU(3)$ properties appropriate to multiple exchange, but the Ringland modification has no known physical basis. It essentially reflects the fact that dual Regge amplitudes require appreciable cut corrections only to the imaginary part of vector exchange. It remains to be seen whether a fundamental difference between vector and tensor exchange can be understood in a deeper sense and the cut formulation placed upon a more physically rigorous foundation.

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VIII. APPENDIX A

If we number the states of the octet as shown in Fig. 16, the matrices appropriate to equations 1-3 are:

$$M_1 = \begin{vmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$M_2 = \begin{vmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$M_3 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{vmatrix}$$

$$M_4 = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$M_5 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$M_6 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$M_7 = \sqrt{1/2} \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$M_8 = \sqrt{1/6} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{vmatrix}$$

According to equation 12, the SU(3) Clebsch-Gordan coefficient for the symmetric coupling of $p\bar{K}^*$ to Σ^+ is given by

$$\langle p, \bar{K}^{*0} | \Sigma^+ \rangle = \sqrt{3/10} \langle M_1 \{ M_2, M_6 \} \rangle = \sqrt{3/10}$$

Fig. 16. Numbering of the octet states.

IX. APPENDIX B

In this section we illustrate the calculation of dual Regge amplitudes using techniques in which Clebsch-Gordan coefficients are expressed as quark diagrams. Duality diagrams emerge in natural and straightforward fashion from this approach. We choose the class of reactions in which a pseudoscalar meson and an octet baryon produce a pseudoscalar meson and a decuplet baryon ($PB \rightarrow PD$) since no derivation of duality diagrams for a reaction involving decuplets has appeared in the literature.

We write the decuplet states as $|\underline{10} i\rangle = (D_i)_{jkl} |q_j q_k q_l\rangle$ where D_i are fully symmetric in all indices. As before, we write the meson states as $|\underline{8} i\rangle = (M_i)_{jk} |q_j q_k\rangle$ and the octet baryon states as

$$|\underline{8} i\rangle = (B_i)_{jkl} |q_j q_k q_l\rangle.$$

Octet antibaryon states are written as $|\underline{8} i\rangle = (\bar{B}_i)_{jkl} |\bar{q}_j \bar{q}_k \bar{q}_l\rangle$. The B_i , $\bar{B}_i$ and M_i are related by

$$(B_i)_{jkl} = \frac{1}{\sqrt{2}} (M_i)_{jn} \epsilon_{nkl},$$

$$(\bar{B}_i)_{jkl} = \frac{1}{\sqrt{2}} (M_i)_{nl} \epsilon_{nkl},$$

and

$$B_a = (-1)^{Q_a} B_{\bar{a}}$$

where Q_a is the charge of state a of the octet. The Clebsch-Gordan coefficient which we wish to calculate is $\langle \underline{8} a; \underline{10} b | \underline{8} c \rangle$. Since we

want our quark diagram to represent $\overline{BD} \rightarrow M$, we proceed accordingly, using the fact that

$$\frac{1}{\sqrt{2}} \epsilon_{ijk} |q_j q_k\rangle$$

transforms as $|\overline{q}_i\rangle$ and

$$\frac{1}{\sqrt{2}} \epsilon_{ijk} |\overline{q}_j \overline{q}_k\rangle$$

transforms as $|q_i\rangle$. For reasons which will become clear momentarily, we write

$$|\underline{8}_a; \underline{10}_b\rangle = (\overline{B}_a)_{ijk} (D_b)_{lmn} |\overline{q}_i \overline{q}_j \overline{q}_k q_l q_m q_n\rangle$$

and use the above transformation properties to make the replacement

$$|\underline{8}_a; \underline{10}_b\rangle = (\overline{B}_a)_{ijk} (D_b)_{lmn} \epsilon_{kpq} \epsilon_{nrs} |\overline{q}_i \overline{q}_j q_p q_q q_l q_m \overline{q}_r \overline{q}_s\rangle.$$

We have ignored normalization for the moment. Similarly, we can write

$$|\underline{8}_c\rangle = (M_c)_{de} \epsilon_{fgh} \epsilon_{hij} \epsilon_{ikl} \epsilon_{lmn} \epsilon_{jpq} \epsilon_{qrs} |q_f q_g q_k q_p \overline{q}_m \overline{q}_n \overline{q}_r \overline{q}_s\rangle.$$

We have now reduced the two states in our Clebsch-Gordan coefficient to forms in which the quarks and antiquarks match and we can calculate the overlap. We use the properties of the ϵ_{ijk} to obtain

$$\langle \underline{8}_a; \underline{10}_b | \underline{8}_c \rangle = \left(\frac{2}{\sqrt{5}}\right) (\overline{B}_a)_{ijk} (D_b)_{lkj} (M_c)_{li} \quad (B-1)$$

where we obtain the normalization and sign by computing one coefficient

and comparing with the standard tables. Using the properties of the M_i we can also derive:

$$\sum_{m=1}^8 \langle \underline{8}_i; \underline{10}_j | \underline{8}_m \rangle \langle \underline{8}_a m | \underline{8}_k; \underline{8}_l \rangle = \frac{-2}{\sqrt{30}} (\overline{B}_i)_{fbc} (D_j)_{bcg} [M_k, M_l]_{gf}$$

$$\sum_{m=1}^8 \langle \underline{8}_i; \underline{10}_j | \underline{8}_m \rangle \langle \underline{8}_s m | \underline{8}_k; \underline{8}_l \rangle = \frac{\sqrt{6}}{5} (\overline{B}_i)_{fbc} (D_j)_{bcg} \{M_k, M_l\}_{gf}$$

Now consider the crossing relation

$$\langle c d | T^{(s)} | a b \rangle = \langle \overline{b} d | T^{(t)} | a \overline{c} \rangle \quad (B-2)$$

where a and c are mesons, d is a decuplet baryon, and b is an octet baryon. The only non-exotic t channel amplitudes are $T_{\underline{8}_a}$ and $T_{\underline{8}_s}$ where the "s" and the "a" indicate symmetric and antisymmetric coupling respectively at the PP vertex. $T_{\underline{8}_a}$ is odd signature (vector exchange) and $T_{\underline{8}_s}$ is even signature (tensor exchange). The Regge forms of these amplitudes are

$$T_{\underline{8}_a}^{(t)} = d_{BDV} f_{PPV} A^{(-)}$$

$$T_{\underline{8}_s}^{(t)} = d_{BDT} f_{PPT} A^{(+)}$$

where

$$A^{(\pm)} = [(\exp(-i\pi\alpha) \pm 1) / \sin \pi\alpha] ((s-u) | s_0)^\alpha.$$

The d's and f's are factorized Regge residues for symmetric and anti-symmetric coupling and the subscripts indicate the class of particle involved in the coupling. The no-exotics constraint is $\text{Im} T_{\underline{27}}^{(s)} = 0$ which through use of SU(3) crossing matrix gives

$$2/5 \text{Im} T_{\underline{8}_s}^{(t)} - 2\sqrt{5}/15 \text{Im} T_{\underline{8}_a}^{(t)} = 0.$$

In terms of residues, the constraint is

$$2/5 d_{\underline{8}DT} d_{\text{PPT}} - 2\sqrt{5}/15 d_{\underline{8}DV} f_{\text{PPV}} = 0. \quad (\text{B-3})$$

From a similar analysis of pseudoscalar-pseudoscalar scattering, we know that $f_{\text{PPV}} = \pm 3/\sqrt{5} d_{\text{PPT}}$. Using this relation in equation B-3 we obtain $d_{\underline{8}DT} = \pm d_{\underline{8}DV}$.

In terms of SU(3) amplitudes, equation B-2 may be written as

$$\begin{aligned} \langle c d | T^{(s)} | a b \rangle &= \sum_{m=1}^8 \langle \underline{8} b; \underline{10} d | \underline{8} m \rangle \langle \underline{8} a; \underline{8} \bar{c} | \underline{8}_a m \rangle T_{\underline{8}_a}^{(t)} \\ &+ \sum_{m=1}^8 \langle \underline{8} \bar{b}; \underline{10} d | \underline{8} m \rangle \langle \underline{8} a; \underline{8} \bar{c} | \underline{8} m \rangle T_{\underline{8}_s}^{(t)} \end{aligned}$$

Using the relations given above, we reduce this equation to

$$\langle c d | T^{(s)} | a b \rangle = 2\sqrt{6}/5 d_{\underline{8}DT} d_{\text{PPT}}$$

$$\{ [(B_{\bar{b}})_{ijk} (D)_{jkl} (M_{\bar{c}})_{lm} (M_a)_{MJ}] (A^{(+)} + A^{(-)}) \}$$

$$+ [(B_{\bar{b}})_{ijk} (D)_{jkl} (M_a)_{lm} (M_{\bar{c}})_{MJ}] A^{(+)} - A^{(-)} \}.$$

As in the case of other reactions, this takes the form of a nonplanar duality diagram multiplying a real term plus a planar diagram multiplying a term with rotating phase.

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