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    Strong Interactions

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Major progress in axiomatic field theory in recent years has been achieved in three different areas:

1. **Algebraic approach** A general study of particle statistics (including parastatistics) and field commutation relations based on the assumption of commutativity of observables at spacelike distance has been successfully carried through by Doplicher, Haag, and Roberts using an algebraic method. Their work was not reported on at this conference.

2. **Constructive field theory** This is an attempt to construct a quantum field theory for a given interaction such as $\phi^4$ and $\overline{\psi}\psi$ in a mathematically satisfactory manner, thereby establishing the existence of nontrivial models satisfying the basic axioms of quantum field theory and enabling a further mathematical study of the physical properties of these models such as broken symmetry. Recent remarkable progress was reported in the mini-rapporteur talks of Jaffe and Wightman.

3. **Properties of scattering amplitudes** There has been remarkable progress in the analysis of on-mass-shell n-point amplitudes by Bros, Epstein, and Glaser. This subject along with some recent work on the Pomeranchuk theorem was discussed in the mini-rapporteur talk of Martin (#788, 312, 313, 314). In addition, a parametric dispersion representation which contains only physical absorptive parts and follows from axiomatic analyticity for pion-pion scattering, was reported by Khuri (#787), and a connection between scaling light-cone singularities and the asymptotic behavior of the Jost-Lehmann-Dyson spectral function was discussed by Vladimirov (#917) and Stichel.

In addition to achievement in the above three areas, important progress in renormalization has been made by Epstein and Glaser. This work was not reported on.

In the area of mathematical aspects of quantum field theory, E. Mihul reported her work on the Bargman-Hall-Wightman theorem and on the extended tube, and Swieca (#637) discussed the unitary implementability of special conformal transformations for free fields.

In the following, areas (ii) and (iii) listed above are discussed in somewhat more detail.

## 1. Constructive Field Theory

One considers a Hamiltonian

$$H = H_0 + H_1 + H_c,$$

where $H_0$ is a free Hamiltonian, $H_1$ is an interaction Hamiltonian, and $H_c$ is an (infinite) counter-term. Typically, one starts out from a cutoff Hamiltonian, proves the self-adjointness and semi-boundedness of the cutoff Hamiltonian, defines a Heisenberg field $\phi(x) = e^{itH}\phi(0)e^{-itH}$ (in a cutoff theory) proves a finite propagation property, goes to the limit of no cutoff, and proves the Wightman axioms one by one. For super-renormalizable interactions, for those interactions...
which produce essentially only a finite number of divergent graphs such a program can be carried through by treating the divergences exactly and remarkable progress has been achieved over the past several years where outstanding contributions have been made by Glimm and Jaffe.

At the time of this conference all the Wightman axioms including the existence of a mass gap between the vacuum and the rest of the energy momentum spectrum had been established for $P(\phi)_2$ theory (a theory with an interaction $\int \mathcal{P}(\phi(x)) \, dx$ where $\mathcal{P}$ is a polynomial which is bounded below such as $\lambda \phi^4 - \alpha \phi^2 \lambda > 0$ and the space time dimension is 2) for small coupling constant. Jaffe predicted that the $(\text{Yukawa}_1)_2$ theory will be in similar shape within 12 months because of recent results on the Euclidean formulation for Fermions.\(^6\)\(^7\) (The Euclidean formulation in general will be discussed below.) Another recent breakthrough is a proof of the positivity of the Hamiltonian for $(\phi^4)_3$ theory. Rapid progress was also predicted for this interaction as well as the $(\text{Yukawa}_1)_2$ theory.

One of the most recent technical developments which has been of vital importance for the rapid progress in the past year and is responsible for the optimistic future predictions is concerned with the so-called Euclidean method. In particular, relations among Euclidean field theory classical statistical mechanics and quantum field theory were emphasized by Wightman and will be explained below in some detail.

It has long been known that the vacuum expectation values of products of fields in quantum field theory (VEV) can be continued analytically to Schwinger points i.e. points with pure imaginary times and real space coordinates. The VEV at Schwinger points is called a Schwinger function. Symanzik\(^8\) has developed a Euclidean field theory which yields Schwinger functions of a Minkowski quantum field theory as the expectation values of commuting fields with respect to a positive measure. Decisive progress has been achieved in the past year by Nelson\(^9\)\(^-\)\(^14\) who introduced the Markoff property to Euclidean field theory and showed that this property together with properties discussed by Symanzik permit the reconstruction of a Minkowski quantum field theory from a Euclidean field theory. Thus one can first construct the Euclidean field theory or Schwinger functions for a given interaction and then study the corresponding Minkowski quantum field theory. This is the so-called Euclidean method.

The Schwinger functions for a cutoff interaction are obtained by the Gell Mann Low formula of the infinite t limit of

$$\lim_{t \to \infty} \int \phi(x_1) \, \phi(x_n) \, d\mu(\phi) \int_{-\infty}^{\infty} d\phi \,$$

where $\phi(x)$ in this expression is a commuting Euclidean field

$$d\mu(\phi) = e^{-\frac{1}{2} \int_0^{\infty} d\phi_0 \int_{-\infty}^{\infty} d\phi_1 \int_{-\infty}^{\infty} V(\phi(\xi))} \, d\phi_0 \, d\phi_1$$

$V(\phi)$ is an interaction such as $\phi^4$ and $d\mu(\phi)$ is the Gaussian measure for free fields. The model without a spatial cutoff is obtained in the limit $t \to \infty$. Thus one is interested in the limit of a state given by a measure $\exp \left[ \int_{\Lambda} \mathcal{H}(\phi(\xi)) \, d\phi \right] d\mu(\phi)$ as the rectangle $\Lambda$ (in space time) becomes finite which makes the correspondence with classical statistical mechanics more than mere analogy. Such a limit is used to obtain an equilibrium state in classical statistical mechanics since abstract characterizations of equilibrium states by a variational principle and by other equivalent conditions are known for classical statistical mechanics in an infinite volume and
since new techniques for predicting the existence of phase transitions are being developed, one can expect similar developments for quantum field theory. A study along these lines is contained in Refs. 12 and 13.

Apart from the connection with statistical mechanics, the power of the Euclidean method can be seen in Nelson's symmetry:

\[
(\Phi_0, \exp[-tH_f]\Phi_0) = (\Phi_0, \exp[-tH]\Phi_0),
\]

where \(\Phi_0\) is the free vacuum and \(H_f\) denotes the cutoff full Hamiltonian, \(f\) being the space cutoff. Such a symmetry between the (time) parameter \(t\) and the space cutoff \(f\) is quite remarkable and is a simple consequence of the Feynman-Kac formula

\[
(\Phi_0, \exp[-tH_f]\Phi_0) = Z_{\perp f}
\]

Guerra was the first person to notice an important application of this symmetry, which started a fullscale use of the Euclidean method by many authors. Some of the advantages of the Euclidean method are the availability of the Feynman-Kac formula, Euclidean symmetry, the symmetry of Green's function due to unrestricted commutativity of fields, and the availability of a perturbation expansion using the Feynman propagator \((k^2 + m^2)^{-1}\) which greatly simplifies earlier estimates.

We include in the list of references those quoted by Jaffe and Wightman in connection with \(\wp_c\) theory, \(\phi^4\) theory, \(\wp_c\) and \(\phi^4\) theories, and (Yukawa) \(\wp_c\) theory.

II. Analyticity of n-Point Amplitudes on Mass-Shell

Analyticity of the \(A\) particle-\(B\) particle amplitude \(A + B \rightarrow C + D\) for particles satisfying stability conditions in a neighborhood of the physical region except for the energy cut was established some time ago by Bros, Epstein, and Glaser on an axiomatic basis, and the analyticity domain was improved by Martin on the basis of unitarity and positivity. The recent result of Bros, Epstein, and Glaser is concerned with \(n\)-point amplitudes on mass shell and shows that the physical amplitude is the sum of a finite number of boundary values of analytic functions.

The most spectacular progress has been achieved for the \(5\)-point amplitude \(4 + 5 \rightarrow 1 + 2 + 3\). It is proved in this case that above a certain incident center-of-mass energy \(4 \times 8\) times the common mass in the equal mass case, the calculation of \(4 \times 8\) being due to Martin, the amplitude at any physical point is the boundary value of a single analytic function, holomorphic in a "local tube" in all \((5\) complex) variables. To describe the result, let \(E_1, E_2, E_3\), be the center-of-mass energies of the final particles \(1, 2, 3\) and \(S_{12}, S_{23}, S_{31}\), be the (squared) two-particle subenergies of the final particles

\[S_{12} = (E_1 + E_2 + E_3)^2 - 2(E_1 E_2 + E_2 E_3 + E_3 E_1) + m_3^2, \text{ etc}\]

In addition, two angular variables \(\theta\) and \(\phi\), are necessary to describe completely the scattering process. For fixed \(E_1, E_2, E_3\), analyticity in the angular variables \(\theta\) and \(\phi\) has been known for some time. (In the quoted literature, an integration of the cross sections over all possible \(E_1, E_2, E_3\) at a given total incident energy \(E\) is obtained to obtain the analyticity of the production.
amplitude from the size of the ellipse of analyticity of the elastic amplitude $4 + 5 - 4 + 5$. However, Omnes and Martin noticed that analyticity of the amplitude in the angular variables as a distribution in $E_1$, $E_2$, $E_3$ is obtained by integrating over an arbitrarily small cell in $E_1$, $E_2$, $E_3$ and using the Schwarz inequality.) Bros, Glaser, and Epstein obtained analyticity in all variables $E_1$, $E_2$, $E_3$, $\theta$, $\phi$ in a region which is described by the following inequalities in the neighborhood of physical points:

$$\text{Im} S_{kl} > 0,$$

$$\left\{(E_j^2 - m_j^2)^{1/2} - \epsilon E_j \right\} \text{Im} S_{kl} + \left\{(E_j^2 - m_j^2)^{1/2} + \epsilon \left(E_j - m_j^2 (E_1 + E_2 + E_3)^{-1}\right) \right\} \text{Im} (S_{12} + S_{23} + S_{31}) < 0,$$

where $(j,k,l)$ is any cyclic permutation of $(1,2,3)$ and $\epsilon = \pm 1$. Here $E_j$ is the real part of the particle energy. As all $E_1$, $E_2$, $E_3$ tend to infinity, the second set of constraints on the relative magnitude of $\text{Im} S_{12}$, $\text{Im} S_{23}$, $\text{Im} S_{31}$ becomes weaker and weaker.

### III. Generalizations of the Pomeranchuk Theorem

The problem is to compare differential cross sections (and integrated cross sections) for line reversed processes

$$A + B \rightarrow C + D \text{ and } \bar{C} + B \rightarrow \bar{A} + D$$

At fixed momentum transfer, Cornille and Martin proved that if the phases of amplitudes for both processes (defined by continuity assuming no physical region zeros) grow separately less fast than $\log s$, then

$$\lim_{t \rightarrow 0} \left[ \frac{d\sigma}{dt}(s,t)\right]_{AB \rightarrow CD} = \frac{d\sigma}{dt}(s,t)_{CB \rightarrow AD} = 1$$

if in addition, this limit exists. (The case of a slowly varying momentum transfer can also be treated, but with great complications.) If the phases are bounded by $(\text{const}) \log s$ (a fact which can be proved from first principles for elastic scattering if the amplitude has no zeros at physical points), then Cornille and Martin prove that the limit is finite.

In the case of elastic scattering ($A = C, B = D$), it is shown that (1) widths of the diffraction peaks are asymptotically equal if the width of one amplitude has a nonzero limit and (2) if one amplitude exhibits persistent shrinking ($\geq$, monotonously tends to zero) and ($d\sigma/dt(t = 0) \geq \text{const}$) $d\sigma/dt(s,t)$, then the ratio of widths tends to 1. Here the width $\Delta(s)$ of an amplitude $F(s,t)$ is defined by $F(s,\Delta(s)) = aF(s,0)$ where $a$ is any fixed constant such as $1/2$.

### IV. Parametric Dispersion Representations

Starting from an analyticity domain in the two Mandelstam variables, Auberson and Khun derive a parametric dispersion representation for equal mass elastic scattering amplitudes which is symmetric in the three Mandelstam variables and contains only physical absorptive parts.

### V. Scaling, Light-Cone Singularities and Asymptotic Behavior of the Jost-Lehmann-Dyson Spectral Function

The fact that scaling follows from certain light-cone singularities has been discussed by many authors. Vladimirov presented some work done in collaboration with N. N. Bogolubov A. N. Tavkhelidze showing that a certain asymptotic behavior of the Jost-Lehmann-Dyson...
spectral function implies scaling and light-cone singularities. Stichel presented his work showing that scaling implies a certain asymptotic behavior of the Jost-Lehmann-Dyson spectral function.

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21. J. Glimm and T. Spencer, Wightman Axioms and the Mass Gap for the $\phi^4$ Quantum Field Theory, NYU preprint
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Currents I: Electron Positron Interactions

Organizer: G. Salvini
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Experimental results on electron-positron collisions at various energies and theoretical results connected with the interpretation and perspectives of the experiments have been presented in this parallel session.

We summarize very briefly on the experimental results, for they are largely covered in the invited paper on $e^+e^-$ interactions (V Silvestrini). The theoretical interpretations and contributions to this conference have been reported by S Drell and S Brodsky of SLAC in Section II of this summary.

I Experimental Results

(Reported by I Miroshnichenko, C Menciucchi, A Zichichi, R Little and H Lynch)

These results deal mostly with the following reactions:

1. $e^+e^- \rightarrow p + \bar{p}$
2. $e^+e^- \rightarrow$ pairs of $\pi$'s or $K$'s
3. $e^+e^- \rightarrow$ at least three hadrons (multihadronic production)
4. $e^+e^- \rightarrow$ other particles

At center-of-mass energies between 1 and 4 GeV, the results have been obtained with the storage rings at Novosibirsk, Frascati, and CEA. Other $e^+e^-$ rings are ready or in preparation and in particular, we mention the electron-positron storage ring SPEAR of SLAC which has reached a luminosity by now of $1.8 \times 10^{30}$ cm$^{-2}$ sec$^{-1}$ and is ready to start its experimental program. Among the experimental results reported at this session we mention:

- The cross section for reaction (1) $e^+e^- \rightarrow p + \bar{p}$ has been measured at a center-of-mass energy of 2100 MeV with result $\sigma = (9 \pm 2) \times 10^{-34}$ cm$^2$ (Naples group, ADONE).
- The pion form factor goes down with energy but its value is above the "p tail" (Bologna, CERN, Frascati). It is still difficult to distinguish among pairs of pions and kaons at the highest energies. At the lowest energies (1 18 and 1 34 GeV) the K form factor is within the expected tail of the $\phi$ resonance plus possible $\rho\omega$ interferences (Novosibirsk contribution).
- The multihadronic cross section has a "bump" around 1 6 GeV and the kinematical analysis of the $e^+e^- \rightarrow 4$ charged pion events is strongly indicative of a $\rho^+$ resonance with the same quantum numbers of the $\rho$ (contributions 560 and 561).
- The reaction (3) has been recently measured at 3 and 4 GeV. The values of the total cross section are very high (factor 2 to 5) when compared to the cross section for the process $e^+e^- \rightarrow \mu^+\mu^-$, which is an obvious point of reference for the asymptotic behavior of multihadronic production (contributions 456 and 562).

All electromagnetic processes ($e^+e^-$ scattering, radiative corrections $e^+e^- \rightarrow e^+e^-e^+e^-$) are in agreement with QED when all the proper diagrams are taken into account in the comparison with the experimental data.

- No evidence of heavy leptons is observed up to a possible mass of 900 MeV (Bologna, CERN, Frascati collaboration).
II. Theoretical Results

II.1 Recent Theoretical Work on $e^+e^-\,$ Annihilation and Continuation
From Inelastic Electron Scattering* (#619)

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Two lines of theoretical developments have emerged as a result of the observed scaling behavior in DIES (deep inelastic electron scattering). One is the parton model and the other is the light-cone algebra; and we ask what are their implications for the $e^+e^-\,$ colliding beam cross sections that we've been learning about.

The parton model, with its point-like constituents of the proton which scatter incoherently in very inelastic electromagnetic or weak interactions at high energies, leads to Bjorken scaling for energy and momentum transfers exceeding $\approx 1\,$GeV. This result defines the mass scale for partons as well as a dimension of roughly $10^{-14}\,$cm for point-like behavior. It remains for the future—hopefully, the very near future—to tell whether a further refinement of scale to dimensions $\approx 10^{-15}\,$cm will reveal deviations from scaling, perhaps due to a parton structure resulting from its gluon cloud—i.e., the radiation and self-reaction effects associated with the exchange of the quanta or gluons binding the partons in the proton. This is the way all other scaling laws have broken when probed on higher resolution scales, and such deviations should be kept in mind no matter what formal approach is adopted. If we ignore this possibility, the

*(Invited talk at the Parallel Session on Currents I: Electron Positron Interactions, at the XVI International Conference on High Energy Physics, September 6, 1972)

*Work supported by the U. S. Atomic Energy Commission.
parton model predicts

\[ \sigma_{e\bar{e} \rightarrow \text{hadrons}} = N \left( \frac{4\pi \alpha'^2}{3s} \right) \text{ for } s > M_p^2 \]  

with

\[ N = \sum_{Q_j = 1/2}^{1} + \frac{1}{4} \sum_{Q_j = 0}^{2} \]

We must always face, in this approach, the embarrassing question of where are these constituents if they really exist and if they are light enough to be produced so that we have precocious scaling as in DIES.

To avoid this embarrassment or paradox and construct a more general basis for understanding scaling, it is desirable to cast off the strict literal interpretation of a parton model and abstract just those general features being probed. This is the idea of the light-cone approach.

What we learned from DIES and the observation of scaling is that the singularities of a current commutator near the light cone are canonical—i.e., the same as in free field theory, since what is measured is the Fourier transform of this singularity.

Turning to e\bar{e} annihilation, the object under study is

\[ \sigma_{e\bar{e} \rightarrow \text{hadrons}} = \frac{4\pi \alpha'^2}{3s} \rho(s) \]

with

\[ \rho(s) = \frac{12\pi^2}{s} \int d^4x \, e^{i\sqrt{s}t} \left\langle 0 \left| \left[ J_1(x), J_1(0) \right] \right| 0 \right\rangle \]

and

\[ 0 \left( \frac{1}{\sqrt{s}} \right) \sim t > \left| \vec{x} \right| , \text{ i.e., the tip of the light cone.} \]
Again one is near the light cone at high energies and is measuring a Fourier transform of a singularity in the commutator which, if it is canonical, → scaling—i.e., 

\[ \rho(s) \sim \text{const} \]

and 

\[ \sigma_{e^+e^- \rightarrow h} = \left( \frac{4\pi\alpha^2}{3s} \right) N \]

This result was first given by Bjorken \(^1\) in 1966 from a sum rule in terms of equal time commutators and by Gribov, Ioffe, and Pomeranchuk \(^2\) in terms of canonical dimensions of the Schwinger term for hadrons. Presumably, this behavior sets in when no large masses are around to impede the approach to the light cone, and it is important to keep in mind that it is not yet clear when that will be. In fact, it could be that the large numbers observed for (1) reflect individual \(\rho', \rho''\), resonances, and that scaling is yet to be found at higher energies.

To go further and get the number \(N\) in (1), we must postulate the algebraic structure of the current operators in detail. It is necessary to make use not only of the symmetry properties of the algebra, but to treat the current operator as actually factorizable into products of quark operators. Technically, this is necessary because we only know how to get a value for \(N\) by using the propagator sum rule for a quark propagator—i.e., the result that asymptotically for small distances or high momenta, the quark propagator approaches a free propagator. It is clear that with these literal manipulations, we also open the possibility of creating single quarks as indicated schematically when we use closure to insert a complete set of states, i.e.,

\[ \langle 0|q^+(x)q(0)|0 \rangle = \sum_{\text{quark states } S} \langle 0|q^+(x)|S \rangle \langle S|q(0)|0 \rangle \]  

(3)
Their propagator vanishes and so does the annihilation cross section below the quark production threshold. This is a murky business of how to avoid creating the quarks. If the model nucleon consists of 3 Fermi-Dirac quarks, \( N = 2/3 \).

But then one needs some very high potential wall to confine the individual quarks and avoid single quark production. This requires an inquiry into possible dynamical origins of such a wall, as have been discussed by K. Johnson recently.

Another way to prevent quarks from emerging is to describe them by parastatistics of rank 3, with a physical restriction that all physical particles are bosons or fermions so individual quarks never appear; or equivalently to assume, as Gell-Mann has recently proposed, \(^3\) that there are three colors of quarks, and to restrict all physical states to be singlets in the SU3 of color. Then a current operator with the structure

\[
J \sim \bar{q}^R_R q^R_R + \bar{q}^W_W q^W_W + \bar{q}^B_B q^B_B, \quad \text{i.e., a color singlet,} \quad (4)
\]

forms only singlet states from the vacuum. In this case, \( N = 2 \), and the problem of incorporating into a quark-gluon field theory model the restriction that all physical states are color singlets has not been solved. The full current operator \(^5\) contributes in (2), and so \( N = 2 \), even though we rule out the possibility of creating higher states of color away from the light cone, as illustrated in Fig. 1.

Although it is without any internal inconsistencies, the light cone approach is murky with regard to quark production, and we would like to avoid literal treatment of currents as a factored operator when abstracting algebraic relations from field theory models. It is still a challenge to fix \( N \) without introducing quark states. Recently Crewther \(^6\) has shown how to construct a relation of \( N \) to the decay rate for \( \pi^0 \to 2\gamma \) without explicitly introducing the quark states. Using PCAC in the soft pion limit \( \mu_\pi \to 0 \), one needs for calculating \( \pi^0 \to 2\gamma \)

\[
\langle 0|J_{\alpha}(x)J_{\beta}(y)\partial_{\lambda}\bar{J}^5_{\lambda}(z)|0\rangle . \quad (5)
\]
In the zero frequency limit of PCAC, it is the light-cone behavior of the current products in (5) that determines the decay rate as shown by K. Wilson. Since we can go to the light cone in several independent ways among the three space-time variables, but must get consistent results no matter which directions we choose, there are constraints on the form of singularities. These constraints force a relation of the connected parts of operator products in (5) with the disconnected one needed in (2) for \( e \bar{e} \rightarrow \text{hadrons} \); and their constant of proportionality is given by the symmetry properties of the current algebra. This is as shown by Bardeen, Fritzsch, and Gell-Mann. In his work, Crewther first derived this connection on the basis of the short-distance algebra, plus an assumption of the conformal symmetry of the world.

In concluding this discussion, I have two comments:

1. Accepting the measured decay rate for \( \pi^0 \rightarrow 2\gamma \), we obtain from the Crewther relation \( N = 2 \) if the currents satisfy an underlying SU3 quark algebra. If we enlarge the symmetry group to SU4 to accommodate "charm," \( N = 3 \frac{1}{3} \). The added "charmed" quark has 2/3 charge, and 0 isotopic spin and strangeness. It is desired, for example, in the Weinberg theory with neutral currents to suppress \( K^0 \rightarrow \mu \bar{\mu} \) to observed levels.

2. The application of PCAC to singular products of local currents as in (5) for \( \pi^0 \rightarrow 2\gamma \) may not be as accurate as in its demonstrated successes for soft pion theorems in which the divergence of the axial current interacts with an extended composite hadron. The PCAC extrapolation for \( \pi^0 \rightarrow 2\gamma \) decay has been questioned in a version of weak PCAC that preserves all other successes. In this case, one cannot compute \( N \) from Crewther's relation but must return to the factorized form in (3).
We turn next to inclusive cross sections. The question discussed in contributions by Gatto, Menotti, and Vendramin, and Gatto and Preparata is what, if any, is the general connection between the structure functions in DIES

\[ e^- p \rightarrow e^- + \text{ anything: } W_1, \nu W_2 \quad \omega = \frac{2 \not{P} \cdot q}{Q^2} > 1 \]

\[ e^- e^+ \rightarrow p + \text{ anything: } \overline{W}_1, \nu \overline{W}_2 \quad \omega < 1 \]

In several models, a cut-off Yukawa field theory and a multiperipheral ladder model in which stable particles propagate, it has been shown that the scale and can be determined by a simple analytic continuation

\[ \overline{W}_1 (\omega) = -W_1 (\omega) \]

\[ \nu \overline{W}_2 (\omega) = +\nu W_2 (\omega) \quad (6) \]

This has also been shown to be true for the leading term as \((\omega - 1) \rightarrow 0\) in a Bethe-Salpeter ladder model for the bound state. But it is not a general result. The problem is that no longer are we dealing with a simple commutator of currents. Added terms appear in the commutator, which contains all four contributions shown in Fig. 2, and their sum actually vanishes for \(\omega < 1\) when we have canonical scaling.

With canonical scaling (6) follows from the formal substitution rule interchanging ingoing and outgoing fermions, but the usefulness of these equations lies in the possibility of giving them the character of an analytic continuation—i.e., continuing \(W\) from \(\omega > 1\) to \(\omega < 1\) to determine \(\overline{W}\). For practical purposes, the continuation is important near \(\omega \sim 1\).

It is not in general easy to accomplish this continuation because the \(W\) are squares of moduli of amplitudes and, as such, have not in general the good analytic properties for making such a continuation. So one has to prove under
what assumptions the $W(\omega)$ are continuable to $\omega < 1$ and that the $\bar{W}$ are actually the continuations of $W$.

By studies of single and double box diagrams with stable and unstable particle exchanges, Menotti\textsuperscript{15} has reviewed various theories and observed that in general the continuation breaks down for a class of box graphs with unstable internal particle lines as illustrated in Fig. 3.

These produce cuts for $Re \omega \leq 1$. The question of immediate practical importance is what effect these cuts have on the behavior of the structure functions near $\omega = 1$ so that one can make predictions for the annihilation cross sections near $\omega = 1^-$ by continuing the experimentally observed behavior of DISes near $\omega = 1^+$. Menotti showed that, in the Bethe-Salpeter ladder model, there is a cut along the real axis for $\omega \leq 1$ which, in general, interferes with the continuation. However, its contribution is proportional to $(\omega - 1)^5$ near $\omega = 1^-$ and, therefore, is a higher order correction to the leading threshold behavior $(\omega - 1)^3$. He also analyzed the dependence of the cut near $\omega = 1$ on the mass spectrum of the unstable particle propagator in Fig. 3, showing the conditions which allow the continuation to be made for the leading threshold behavior.

A general light-cone analysis (reported in another session in some detail) by Gatto and Preparata\textsuperscript{11} has shown that, with canonical dimensions, the scaling of the total cross section (1) implies scaling for the inclusive cross section $e^- e^- \rightarrow p +$ anything, but does not, in general, lead to scaling as in (6).

Finally, Gribov and Lipatov\textsuperscript{17} have completed a study of the behavior of electroproduction structure functions in two field-theory models, summed to all orders of $g^2 \log(Q^2/m^2)$; $g^2 \ll 1$. The theories are neutral pseudoscalar (bare $p$, $\pi^0$) and neutral vector (bare $p$, $\omega^0$). The important diagrams turn out to be $t$-channel ladder graphs, in which propagators and vertices are "exact," i.e., computed to all orders of $g^2 \log(Q^2/m^2)$.
The very same results were confirmed recently by Christ, Hasslacher, and Mueller\textsuperscript{18} in a less laborious way by studying the Callen-Symanzik equations. For the neutral vector model, they are also contained in studies by Fishbane and Sullivan. Among the conclusions are:

1. \( \sigma_S / \sigma_T \approx 0 \).

2. \( W_1 = W_1(\omega, \xi) \) where

\[
\xi = \frac{3}{4} \log \left[ 1 - \frac{g^2}{12 \pi^2} \log \frac{Q^2}{m^2} \right]^{-1}
\]

in the neutral vector model, i.e., it does not scale but grows slowly with \( Q^2 \) at fixed \( \omega \).

3. A crossing relation

\[
\nu \bar{W}_2(\omega, \xi) = -\left( \frac{1}{\omega^3} \right) \nu W_2 \left( \frac{1}{\omega}, \xi \right)
\]

This last has as its consequence that the multiplicity of protons grows as \( \ln Q^2 \) if \( \nu W_2(\omega) \longrightarrow \) constant for large \( \omega \), i.e.,

\[
\bar{n}_p = \frac{1}{\sigma} \int_{m/\sqrt{Q^2}}^{1} \frac{d\sigma}{d\omega} \sim \ln Q^2
\]
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5. W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, Contribution to the Topical
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8. For a discussion and review of this problem, see B. W. Lee, J. R. Primack,
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10. R. Gatto, P. Menotti, and I. Vendramin, Papers 87 and 89 contributed to
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    Cimento 7A, 118 (1972).

11. R. Gatto and G. Preparata, Paper 194 contributed to this Conference (to be
    published).


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15. For an excellent general discussion and review, see the talk by P. Menotti
    presented at the Informal Meeting on Electromagnetic Interactions, Frascati,


Fig. 1. Forbidden production of final states that are not color singlets.

Fig. 2. Amplitudes contributing to the absorptive part of forward Compton scattering for time-like virtual photons.

Fig. 3. Example of contribution to $\bar{W}_{\mu\nu}$ that violates crossing relation (6).
II.2. HADRON PRODUCTION IN $e^+e^−$ COLLISIONS—ONE- AND TWO-PHOTON PROCESSES*

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Hadron production in $e^+e^−$ collisions plays a fundamental role in particle physics since one studies the synthesis of hadronic matter from pure electromagnetic energy. In fact, as has been emphasized recently, states of both even and odd charge conjugation are involved, since higher order electromagnetic production processes are important in the energy range of the Frascati, SLAC, CEA, and DESY storage rings. In particular, the two-photon processes of Fig. 1(d) and 1(e) lead to logarithmically increasing cross sections, which, although order $\alpha^4$, eventually dominate a decreasing $e^+e^−$ annihilation cross section at high energy. In this short talk, I will briefly review the various components of hadron production in $e^+e^−$ collisions and report on some of the relevant contributed papers to this conference, including two contributions which evaluate the possibility of detecting neutral weak currents in purely leptonic amplitudes.

One-Photon Annihilation: $e^+e^−→γ^s→\text{hadrons} (C = −)$

The cross section of most current and critical interest is of course the order $\alpha^2$ one-photon annihilation cross section [Fig. 1(a)]. The exciting implication for hadronic physics if the annihilation cross section scales (i.e., $σ(e^+e^−→\text{hadrons})→C$ at large $s$), the significance of the value of $C$ for the light-cone algebra and constituent models of the hadrons, and the question of the continuation of scaling laws from deep inelastic electron-proton scattering are discussed in Professor Drell’s review. Predictions for specific exclusive channels are of interest and have been the focus of an abundant literature. In a contribution to this conference, G. Kramer and T. Walsh develop a systematic treatment of quasi-two-body production, using a helicity amplitude formalism, and they catalog the implications of polarization-correlation measurements. They also present a vector dominance model for various two-body resonance cross sections: cross sections near threshold in the range $1 < \sigma < 100$ nb for processes like

$$e^+e^−→π^0ω, πΛ_2, πΛ_1, \rho^+, ρ^−$$

are possible depending on sizes of the various form factors. Other estimates and discussions of quasi-two-body processes are given in Ref. 2.

Two-Photon Annihilation: $e^+e^−→γ^s + γ^s→\text{hadrons} (C = +)$

There has been very little discussion of this interesting amplitude [Fig. 1(b)] in the literature. Constituent (parton) models predict the cross section to fall as $s^{−1}$ or $\ln s/s$, i.e., the same as the corresponding lepton-production cross sections. The calculation of the $C = +$ hadronic amplitudes for specific channels involves the same uncertainties as the Cottingham formula for mass differences, the nucleon polarization correction to the hyperfine splitting of hydrogen, and the two-photon correction to electron-proton elastic scattering. The interference of this amplitude with the one-photon annihilation amplitude causes an asymmetry of charged hadron production relative to the incident electron direction and contributes a cross section of order $\alpha^3$.

Hadronic Photon Emission: $e^+e^−→γ^s→\text{hadrons} (C = +)$

This interesting process, which permits the study of two photon (one real plus one time-like) couplings of various $C = +$ hadronic systems, has been discussed in considerable detail by Einhorn and Creutz. Unlike the previously discussed processes, the hadronic system is not produced at rest in the $e^+e^−$ CM system.

*Work supported by the U.S. Atomic Energy Commission
The cross section is of order $\alpha^2$, decreases with $s$, and leads to asymmetries with the incident electron direction due to the interference with the amplitude in which the real photon is emitted from an incident lepton line.

**Hard Photon Emission:** $e^+e^-\rightarrow\gamma+\gamma^*\rightarrow\gamma+\text{hadrons} \quad (C = -)$

This ordinary radiative correction process—where the incident lepton bremsstrahlungs a photon of large momentum [Fig. 1(c)]—can play an insidious role in multiparticle hadron production experiments in which the energy of the final hadrons is not determined. The bremsstrahlung allows the production process to occur at a much lower energy—at the expense of one power of $\alpha$. In fact, if the actual annihilation cross section has asymptotic behavior $\sigma(s)\sim s^{-N}$, then the order $\alpha^3$ radiatively-corrected cross section only falls as $s^{-1}$, for any $N > 1$. If $N = 1$, then

$$\sigma(e^+e^-\rightarrow\gamma+\gamma^*\rightarrow\gamma+\text{hadrons}) \sim \frac{\alpha^3}{s} \log \frac{s}{s_{\text{min}}}$$

where $s_{\text{min}}$ is a constant determined by the threshold dependence of $\sigma_0$. The total momentum of the produced hadronic system ("Fireball") moves dominantly along the beam direction, leading to a forward-backward cross section. Discussion of the effects of this type of radiative correction, especially for $\rho^0$ production, is given in A. Litke's thesis. Other calculations of radiative corrections are given in Ref. 7. Clearly, much more work is needed in this important area.

**The Double Fireball Process:** $e^+e^-\rightarrow\gamma^*+\gamma^*\rightarrow\text{hadrons} \quad (C = -1) + \text{hadrons} \quad (C = -1)$

In an interesting contribution to this Conference, Cheng and Wu emphasize that the "double fireball" process (see Fig. 1(d)) could be a dominant cross section for specific hadronic channels at high energies at fixed momentum transfer (small production angles). The form of the cross section (which follows from $j = \frac{3}{2}$ lepton exchange in the $t$ channel) for the production of two fireballs, each of mass of order $m$, is

$$\frac{d\sigma}{d^2s_{cm}} \sim \frac{\alpha^4}{s} \left(\frac{1}{|t|} + m^2\right)^{\frac{1}{2}}$$

where $|t| \sim s^{1/2}$ is the momentum transfer. The sharp peaking at $t = 0$ (zero production angle relative to the beam direction) is of the same origin as that of the single fireball radiative correction. Using the above form, one sees that the two-fireball process becomes comparable with an isotropic cross section of order $\alpha^2/s$ if $\sqrt{s} > m/\alpha$, $\theta^2 \sim m^2/s^2 \sim \alpha$, i.e., very small production angles. Thus the total one-photon annihilation cross section will generally overwhelm the two-fireball cross section at present energies and normal angles. On the other hand, for the case of a specific channel suppressed in one-photon annihilation, especially forward-backward $p + p$, the two-fireball process can be the dominant production mechanism at high energies and fixed momentum transfer ($t = 0$). Cheng and Wu also demonstrate that final state interactions are negligible between the two fireballs at high $s$.

**The Two-Photon Process:**

- $e^+e^-\rightarrow\gamma^*\gamma^*\rightarrow e^+e^-\text{hadrons} \quad (C = \pm)$
- $e^+e^-\rightarrow\gamma\gamma\rightarrow e^+e^-\text{hadrons} \quad (C = -)$

In contrast to the $e^+e^-$ annihilation cross sections, the cross section for the two-photon process (in which the leptons survive in the final state—see Fig. 1(e) and 1(f)) is logarithmically increasing. For $s \gg s_{\text{min}} \gg 4m_e^2$, the cross sections are of order

$$\sigma \sim \frac{\alpha^4}{s_{\text{min}}} \log \frac{s}{m_e^2} \log \frac{s}{s_{\text{min}}} \quad (C = \pm)$$

$$\sim \frac{\alpha^4}{s_{\text{min}}} \log \frac{s}{m_e^2} \log \frac{s}{s_{\text{min}}} \quad (C = -)$$

In fact, the total cross section for $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ is larger than the annihilation cross section $e^+e^\rightarrow\mu^+\mu^-$. 

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for $E > 1 \text{ GeV}$ ($s > 4 \text{ GeV}^2$). The incident leptons are scattered predominantly in the forward direction, with approximately half of the leptons falling within a forward cone of opening angle $\sqrt{m_e/E}$. A simple practical estimate for the $C = +$ cross section is obtained from the equivalent photon approximation:

$$\sigma_{\gamma\gamma \rightarrow X} = \int_0^E \frac{d\omega_1}{\omega_1} N(\omega_1) \int_0^E \frac{d\omega_2}{\omega_2} N(\omega_2) \sigma_{\gamma\gamma \rightarrow X} (s \rightarrow \omega_1 \omega_2)$$

corresponding to the annihilation of two virtual bremsstrahlung beams each along their respective incident lepton direction. $\sigma_{\gamma\gamma \rightarrow X}$ is the cross section for real photon annihilation at $s = 4\omega_1 \omega_2$.

The simple Weisacker-Williams form

$$N_{WW}(\omega) = \frac{2\pi}{\pi} \left[ \frac{E^2 + (E - \omega)^2}{2E^2} \right] \log E/m_e$$

does give the correct total cross section behavior for $m_e/E \rightarrow 0$, but often is inaccurate to $\pm 30\%$ in applications. However, forms for $N(\omega)$ have been presented$^{12,14}$ which are, in fact, very accurate for most applications and which, in fact, reduce to the exact answer if the leptons in the final state are confined to a small forward angle ($\theta_e \ll 1$).

The two-photon process is, of course, separable from the various annihilation processes by tagging (either one or both of the final leptons), or, most dramatically, by the use of $e^+e^-$ collisions, as will be possible at DESY. We thus have the exciting potential to measure the (crossed Compton) processes $\gamma + \gamma \rightarrow \text{hadrons}$ for both real and virtual (spacelike) photons. Over the past two years, many comprehensive calculations of various two-photon processes have been exactly calculated and extensive work has been done on development reliable approximation methods.$^{12-18}$ A detailed review and further references are given in my Cornell talk.$^{16}$ Some typical two-photon cross sections taken from the work of Ref. 14 are shown in Fig. 2 and 3.

In a detailed contributed paper to this Conference, C. J. Brown and D. M. Lyth$^{18}$ give general formulas for $e^+e^- \rightarrow e^+e^-$ hadrons based on a helicity amplitude representation of the virtual $\gamma + \gamma \rightarrow \text{hadrons}$ cross section. They also derive an equivalent photon approximation for $N(E)$ identical to that of Ref. 14. Their numerical checks on the equivalent photon approximation are particularly instructive: for $e^+e^- \rightarrow e^+e^-$ (Born amplitudes) at $E = 2 \text{ GeV}$, the approximate form is essentially indistinguishable from the exact calculation if both leptons are detected within $\theta^{\text{max}}_e = 0.1$ (which gives, in fact, 90% of the total cross section), there is a $+6\%$ error at $\theta^{\text{max}}_e = 0.35$ (where $|\log E/m_e| \leq 0.1 \text{ GeV}^2$), and a $+10\%$ error in the case of the total cross section. The equivalent photon approximation used in these works (closely related to the work of Dalitz and Yennie) is much more accurate than the simple "Weisacker-Williams" approximation which retains only the leading $s/m_e^2$ factor and gives errors of 30% or more. Brown and Lyth$^{18}$ also verify that the kinematic approximation of ignoring the correlation angle $\phi_e$ between the lepton-scattering planes in these calculations is generally justified. On the other hand, measurements of the lepton coplanarity angle can provide parity information on the production of the hadron state. In some kinematical situations, e.g., noncoplanar pion pairs, the equivalent photon approximation is useless, and exact calculations are required.

One of the simplest two-photon processes, but probably of the most critical current interest, is the process $\gamma + \gamma \rightarrow \pi^+ + \pi^-$ for (almost) real photons. Measurements of this process provide a determination of $\pi\pi$ phase shifts (via a Watson theorem in the elastic region) exclusive of hadronic-target complications, as well as a check of the low-energy and soft-pion theorems for the crossed $(s \rightarrow t)$ Compton amplitude. More specifically, the $\sigma(\infty)$ and all $C = +$, $l$-even, positive-parity resonances are accessible from $\gamma\gamma$ annihilation. The magnitude of the resonance couplings to two photons is very much model dependent. In a contribution to this Conference, Lyth$^{19}$ argues from dispersion theory and specific assumptions on the asymptotic behavior of the $\gamma\gamma \rightarrow \pi\pi$ amplitude, that the $\sigma$ coupling will be weak, and Born approximation
should be reliable at small $\pi\pi$ invariant mass. Chanowitz and Ellis reach a similar conclusion on the basis of assumptions on the dimensionality of current and a low-energy theorem. On the other hand, Goble and Roesler (using current algebra arguments), Brodsky, Kinoshita, and Terazawa and Sarker have given models in which the $\sigma$ plays a dominant role in the $J = 0^+ \gamma\gamma \rightarrow \pi\pi$ channel. Other predictions have been given by F. Yndurain, B. Schremp-Otto et al., G. Scheirholz and K. Sundermeyer. The analytic procedure for extracting the $\gamma\gamma \rightarrow \pi\pi$ amplitudes from the $ee \rightarrow ee\pi\pi$ measurements is discussed by C. Carlson and Wu-Ki Tung. Estimates for other hadronic channels are reviewed in my Cornell talk.

There is also the potential for measuring extremely interesting virtual photon-photon annihilation cross sections. These include processes such as (a) $\gamma^+ + \gamma \rightarrow X$, "deep-inelastic scattering on a photon target" (where one lepton is detected forward, the other at large angles) discussed by Brodsky, Kinoshita, and Terazawa, Walsh, and Fujikawa (electromagnetic contributions). (b) $\gamma^+ + \gamma^+ \rightarrow \pi^+ \pi^-$ - the connection to the pion mass difference discussed by T. M. Yan. (c) $\gamma^+ + \gamma^+ \rightarrow X$ (scaling, Regge limits) discussed by Kunszt and Ter-Antonyan, Walsh and Zerwas, Kingsley, Carlson, and Terazawa. I would also like to emphasize the importance of measurements of $\gamma^+(q^2) + \gamma \rightarrow \pi^+ \pi^-$ as shown by Close, Gumon, and myself. The local behavior of the electromagnetic current evident in the SLAC-MIT measurements of scaling implies the existence of a component of this amplitude which is independent of $q^2$ at fixed $\pi\pi$ invariant mass. Similarly, the consequences of the Adler-Bell anomaly contribution in the virtual $\gamma\gamma \rightarrow \pi^0$ amplitude can be studied.

The two-photon processes are clearly of great theoretical interest and will broadly extend the physics capabilities of the high-luminosity storage rings. The first results on hadronic production from this channel are eagerly awaited.

Weak Interactions

The possible detection of neutral weak currents by electron-positron annihilation is discussed in contributions to this Conference by V. K. Cung, A. H. Mann, and E. A. Paschos, and by G. V. Grigoryan and V. A. Khoze. The asymmetry of the spin-averaged cross section for $e^+ e^- \rightarrow \mu^+ \mu^-$ due to the interference of a postulated $W^0 (Z^0)$ annihilation amplitude with the one-photon annihilation amplitude is given by Cung et al. as

$$A = \frac{d\sigma(\theta, \phi) - d\sigma(\pi - \theta, \phi)}{d\sigma(\theta, \phi) + d\sigma(\pi - \theta, \phi)} = \frac{\sqrt{2}}{4\pi} \frac{G_0 M_W^2}{S - M_W^2} \frac{S}{2 - \sin^2 \theta (1 + |P_1|^2)} 2\cos \theta$$

(The muon charge must be detected.) The asymmetry is enhanced if the circulating $e^+$ and $e^-$ beams are transversely polarized, which is expected to happen theoretically $|P_1 - P_2| < 0.927$ due to the effects of synchrotron radiation. Taking the maximum polarization, and $G_0 - G_F$, one obtains an asymmetry $A(s = 64 \text{ GeV}^2, \phi = 0, \theta = 65^\circ) = -2.6\%$. Already, however, neutrino and meson decay measurements indicate limits on the neutral current below $G_0 - G_F$. (See the reviews of B. W. Lee and D. H. Perkins, this Conference.) Thus, measurements of weak-electromagnetic interference will inevitably require very high energy and high-luminosity colliding-beam facilities.
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8. M. Cheng and T. T. Wu, Paper No. 139, contributed to this Conference.


18. C. J. Brown and D. M. Lyth, paper 107, contributed to this Conference.

19. D. M. Lyth, paper 108, contributed to this Conference.


37. V. K. Cung, A. H. Mann, E. A. Paschos, paper 296 contributed to this Conference and NAL preprint.
38. G. V. Grigoryan and V. A. Khoze, paper 894 contributed to this Conference.
Hadron Production in $e^+e^-$ Collisions

Lepton Annihilation

(a) $e^+ e^- \rightarrow \text{hadrons}$

$\sigma \sim \frac{\alpha^2}{s} \left[ 1 + O(\alpha) \right]$ (roughly isotropic)

(b) $e^+ e^- \rightarrow \text{hadrons}$

(c) $e^+ e^- \rightarrow \text{hadrons}$

$\sigma \sim \frac{\alpha^3}{s}$ (peaked)

(d) $e^+ e^- \rightarrow \text{hadrons}$

$\sigma \sim \frac{\alpha^4}{s}$ (peaked)

Two Photon Non-Annihilation

(Leptons Forward Peaked)

(e) $e^+ e^- \rightarrow \text{hadrons}$

$\sigma \sim \frac{\alpha^4}{s_{\text{min}}^2} \left( \log \frac{s}{m_e^2} \right)^2 \log \frac{s}{s_{\text{min}}}$

(f) $e^+ e^- \rightarrow \text{hadrons}$

$\sigma \sim \frac{\alpha^4}{s_{\text{min}}^2} \log \frac{s}{m_e^2} \log \frac{s}{s_{\text{min}}}$

Fig 1 The various components to hadron production in $e^+e^-$ collisions
Fig. 2. The total cross sections for $ee \rightarrow ee$ hadrons ($C = \pm$). Here $E = \sqrt{s}/2$ is the colliding-beam energy. The cross sections for $\pi^\pm$ and $\eta$ are exact and calculated without form factors. The cross sections for $\pi^+\pi^-$ and $\mu^+\mu^-$ are calculated in the equivalent photon approximation. From Ref. 14.
Fig 3. Representative estimated cross sections for hadron production via the two-photon process $E = \sqrt{s}$/2. See Refs. 14 and 16 for discussion.
Currents II: Deep Inelastic Scattering

(Electrons, Muons, and Neutrinos)

Organizer: J. I. Friedman
Scientific Secretaries: T. M. Knasel
E. A. Paschos
P. A. Schreiner
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This is a summary of papers submitted to this conference from DESY on electro and photo-production. The data will be discussed in terms of $Q^2, \nu$ mass squared and lab energy of the virtual photon.

\[ W \] effective mass of the hadron system produced

\[ e \]
\[ \gamma \]
\[ q^2 \]
\[ W \]
\[ p \]

I. Single-Arm Experiments

Alder et al.\(^1\) have separated the total inelastic cross section into its contributions $\sigma_T$ and $\sigma_L$ from transverse and longitudinal photons for $1.5 < W < 1.7$ GeV and $Q^2$ near 1 GeV\(^2\). The ratio $R = \sigma_L / \sigma_T$ is found to be less than 0.2 (Fig. 1), showing that $\sigma_L << \sigma_T$ also in the low-$W$, low-$Q^2$ region. A Karlsruhe-DESY collaboration\(^2\) has measured electroproduction on protons and deuterons in the region $0.1 < Q^2 < 1.5$ GeV\(^2\), $1.3 < W < 1.9$ GeV. From the data the ratio of the neutron to proton cross sections, $(D/H - 1)$, has been extracted by correcting for deuteron binding effects. The result is shown in Fig. 2 as a function of $1/\omega'$ where $\omega'$ is the scaling variable, $\omega' = (2m_v \nu + m^2) / Q^2$. The neutron cross section is smaller than the proton cross section, with the difference increasing with increasing values of $1/\omega'$. The straight line in Fig. 2 shows a fit to the MIT-SLAC data\(^3\) in the deep-inelastic region ($Q^2 > 1$ GeV\(^2\), $W > 2$ GeV). It is amazing how close the data from the resonance and the deep-inelastic region come when analyzed with respect to $\omega'$.

Several groups have made detailed studies of momentum spectra and angular distributions for inclusive $\pi$, $K$, and $p$ photo and electroproduction. The motivation was of course to look for qualitative changes when $Q^2$ increases. Since a number of the prominent variations with $Q^2$ that have been observed can be understood in terms of a few two-body processes we shall first present results on specific reactions and then turn to the inclusive spectra.

II. $\Delta$ and $\rho$ Electroproduction

\[ a. \ e^+p \to e\Delta^{++} + \pi^- \]
\[ e\Delta^{++} \to e\Delta^0 + \pi^- \]

Electroproduction of $\Delta^{++}$ and $\Delta^0$ via

\[ e\Delta^{++} \to e\Delta^0 + \pi^- \] (1)

\[ e\Delta^{++} \to e\Delta^0 + \pi^- \] (2)

has been measured by Dammann et al.\(^4\) for $0.2 < Q^2 < 0.8$ GeV\(^2\) and $2 < W < 2.4$ GeV in a two-arm spectrometer setup. In order to reduce systematic effects, reaction (1) was measured with
incident positrons, reaction (2) was measured with incident electrons. Figure 3(a) shows the $Q^2$ dependence of the differential cross section

$$\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

($t = \text{momentum transfer between incoming proton and } \Delta, \epsilon = \text{the polarization parameter}, \epsilon \approx 0.75$). The photoproduction points stem from Boyarski et al. scaled to $W = 2.23 \text{ GeV}$. The $\Delta^0$ cross section increases relative to that for $\Delta^{++}$ with larger $Q^2$

$$r \equiv \frac{(\Delta^0 \pi^+)/(\Delta^{++} \pi^-)}{0.37 \pm 0.04 \text{ at } Q^2 = 0}$$

$$0.67 \pm 0.09 \quad 0.2 < Q^2 < 0.7 \text{ GeV}$$

In photoproduction a similar increase was found with increasing $t$. Figure 3(b) shows $(\Delta^0 \pi^+)/ (\Delta^{++} \pi^-)$. The data are consistent with the assumption that the ratio changes with $t$ but is constant with $Q^2$ for fixed $t$. The observed $\Delta^0/\Delta^{++}$ ratio can be understood either in terms of isoscalar-isovector photon interference or by assuming a contribution from $T = 3/2$ in the $s$ channel of $\gamma p \rightarrow e^+ e^- p^0$

These are first results from a streamer-chamber experiment in which all charged particles produced in an $ep$ collision are detected. A 7.2-GeV electron beam strikes a liquid-hydrogen target inside the chamber (see Fig. 4). A hodoscope of scintillation and shower counters detect the scattered electron. Scattered electrons are accepted in the range $0.3 < Q^2 < 2 \text{ GeV}$. So far approximately 200,000 pictures have been taken, about one third of the data have been analyzed. The results presented come from 5000 events of which 2300 events have 3 or more charged outgoing hadrons.

Figure 5 shows the $W$ dependence of the cross section for $ep \rightarrow e\pi^+\pi^-$. The general characteristics are the same as in photoproduction of $\gamma p \rightarrow p\pi^+\pi^-$. The $\pi^+\pi^-$ mass distributions at low and high values of $W$ are shown in Fig. 6. The low-$W$ region ($W < 1.7 \text{ GeV}$) is dominated by $\Delta^{++}$ production, the region $W > 2 \text{ GeV}$ by $p^0$ production. From fits to the mass distributions the $\Delta^0$ and $p^0$ cross sections have been deduced.

The $Q^2$ dependence of $\sigma(ep^0)$ is shown in Fig. 7. For $W > 2 \text{ GeV}$ the $p^0$ cross section drops much faster with $Q^2$ than does the total inelastic cross section (dashed-dotted curve). For $2 < W < 2.7 \text{ GeV}$ the ratio

$$\frac{\sigma(p)/\sigma_T + \epsilon\sigma_L}{\sigma_T + \epsilon\sigma_L} = 16 \pm 0.5\% \text{ at } Q^2 = 0^9$$

$$9 \pm 2\% \quad 0.4 \text{ GeV}^2$$

$$7 \pm 2\% \quad 0.65 \text{ GeV}^2$$

$$4 \pm 2\% \quad 1.1 \text{ GeV}^2$$

Approximately $30\%$ of this decrease is due to the $t_{\text{min}}$ cutoff (see Fig. 7). With increasing $Q^2$ the minimum momentum transfer becomes larger.

The $t$ distributions for all events in the $p$ region ($0.60 < M_{\pi^+\pi^-} < 0.85 \text{ GeV}$) normalized to the total $p$ cross section are shown in Fig. 8 together with the photoproduction data which were obtained in the same manner. The $t$ distributions flatten as $Q^2$ increases. Fitting with an exponential, $d\sigma/dt = d\sigma^0/dt \exp(At)$ led to the following results for $2 < W < 2.7 \text{ GeV}$.
The slope of the t distribution definitely decreases with $Q^2$. Possible reasons are the following:
- More background under the $\rho$ at higher t and $Q^2$
- Large longitudinal $\rho$ cross sections with different t behavior
- Large real part at large t, $Q^2$ values
- The interaction radius of the photon shrinks with increasing $Q^2$ (Ref. 10)

The decay distribution for events in the $\rho$ region has been analyzed in the helicity system. The distribution of the cosine of the polar angle $\theta_H$ (see Fig. 9 for definition of the angles) is essentially flat (Fig. 10) in contrast to photoproduction where $W(\cos \theta) \sim \sin^2 \theta_H$. Barring any gross background effects this means that both transversely and longitudinally polarized $\rho^0$ mesons are produced. The distribution of the polarization angle $\psi$ peaks near 0°, 180°, and 360°; i.e., the decay pions emerge preferentially in the plane of polarization of transverse photons. This anisotropy can only come from production by transverse photons.

The density matrix elements in the helicity frame are listed in Table I together with the values expected if $\rho$ electroproduction conserves the s-channel helicity in the total cms (SCHC) for both transverse and longitudinal photons. The data was consistent with SCHC. It is somewhat

<table>
<thead>
<tr>
<th>Matrix Element</th>
<th>Experiment</th>
<th>Prediction from SCHC With $R = \sigma_T/\sigma_L = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0^0$</td>
<td>0.35±0.07</td>
<td>0.47</td>
</tr>
<tr>
<td>Re $\rho_0^0$</td>
<td>0.08±0.05</td>
<td>0</td>
</tr>
<tr>
<td>Im $\rho_0^0$</td>
<td>-0.09±0.07</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{1-1}^1$</td>
<td>-0.04±0.11</td>
<td>0</td>
</tr>
<tr>
<td>Re $\rho_{1-1}^1$</td>
<td>-0.08±0.07</td>
<td>0</td>
</tr>
<tr>
<td>Im $\rho_{1-1}^1$</td>
<td>0.03±0.07</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{10}^2$</td>
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<td>0.26</td>
</tr>
<tr>
<td>Im $\rho_{1-1}^2$</td>
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<td>0</td>
</tr>
<tr>
<td>Im $\rho_{1-1}^2$</td>
<td>-0.27±0.11</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

Table I. $\rho$ Density Matrix Elements (for Definition See Ref. 6) in the Helicity System from Events of the Reaction $e^+p \rightarrow e^+\pi^+\pi^-$ in the $\rho$ Region (0.65 < $M_{\pi^+\pi^-}$ < 0.84 GeV) Data from Ref. 7 for 2 < $W$ < 2.7 GeV, 0.3 < $Q^2$ < 1.5 GeV^2 with $<Q^2> = 0.6$ GeV^2.
surprising that the elements Re $r_{10}^5$ and Im $r_{10}^6$ which measure transverse-longitudinal interference are zero. If we assume SCHC then we can deduce the ratio $R = \sigma_L / \sigma_T$, from the value of $r_{10}^{04}$. The result is $R = 0.6 \pm 0.2$. Background corrections due to $\Delta^++$ production may reduce the value of $R$ somewhat.

III Multiplicities and Inclusive Particle Spectra in Photo and Electroproduction

a. Multiplicities

In the electroproduction experiment with the streamer chamber\(^7\) the average number $<n>$ of charged hadrons produced has been measured as a function of $Q^2$ and $W$ (Table II)\(^\text{a}\). A slight decrease (10-15%) between $Q^2 = 0$ and $W = 4$ GeV\(^2\) is observed for $2.2 < W < 2.7$ GeV.

Table II. Average Number of Charged Hadrons Produced in $\text{ep} \rightarrow \text{e}^+ \text{+} \text{Hadrons}$. Table from Ref. 7.

Photoproduction Data Have Been Taken from Ref. 11.

<table>
<thead>
<tr>
<th>$Q^2$(GeV$^2$)</th>
<th>1.5-1.8</th>
<th>1.8-2.2</th>
<th>2.2-2.7</th>
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</thead>
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<tr>
<td>0</td>
<td>1.93 ±0.25</td>
<td>2.2 ±0.1</td>
<td>2.6 ±0.1</td>
</tr>
<tr>
<td>0.3-0.5</td>
<td>1.7 ±0.1</td>
<td>2.1 ±0.1</td>
<td>2.4 ±0.1</td>
</tr>
<tr>
<td>0.5-0.8</td>
<td>1.9 ±0.2</td>
<td>2.1 ±0.2</td>
<td>2.3 ±0.2</td>
</tr>
<tr>
<td>0.8-1.5</td>
<td>2.1 ±0.2</td>
<td>2.1 ±0.2</td>
<td>2.4 ±0.2</td>
</tr>
</tbody>
</table>

b. Inclusive Spectra

Table III lists the experiments which have studied inclusive particle spectra. We start with a photoproduction result of Burfeindt et al.\(^1\). Figure 11 shows the $\pi^+/\pi^-$ ratio as a function of the Feynman variable $x$, $x = p_{\parallel}^w / p_{\parallel}^\text{max}$ ($p_{\parallel}^w$, $p_{\parallel}^\text{max}$ cms longitudinal and maximum allowed momentum of the particle in question) for different transverse momenta $p_T$. In the photon fragmentation region ($x \gtrsim 0.5$) the $\pi^+$ over the $\pi^-$ yield increases with increasing $p_T$. A similar observation is made for the $K^+/K^-$ ratio. A comparison with the data at 18 GeV by Boyarski et al.\(^1\) shows that the particle/antiparticle yield becomes more equal with increasing energy. According to Mueller\(^1\) in the asymptotic limit the $\pi^+/\pi^-$ and $K^+/K^-$ ratios should become equal. The data
suggest that asymptotia is reached by making $E_T$ large (of course) and $p_T$ small.

In Fig. 12 the $p_T^\pi$ transverse momentum distributions are shown for photo- and electro-production in the photon fragmentation region, $f$ is the invariant cross section, $f \equiv E d^2\sigma/dp^2$. The figure gives an impression of the statistical accuracy achieved. The $p_T^\pi$ distributions become wider with increasing $Q^2$, notably for $\pi^+$ and $\pi^-$ with $0.4 < x < 0.7$. In Fig. 13 the $\pi^-$ yield integrated over $p_T^\pi$ is shown for different $Q^2$ intervals and compared with photoproduction while there is no change for $x < 0.2$ (target fragmentation and central region) the $\pi^-$ yield drops by a factor of 2-4 for large $x$. The dashed curve shows the $\pi^-$ yield for $Q^2 = 0$ after $\rho^0$ events ($\gamma p \rightarrow pp\pi^-$) have been removed. The dashed curve seems to follow rather closely the electro-production points. At $Q^2 = 0$ most of the $\pi^-$ at large $x$ are seen to come from $\rho^0$ production. Since the $\rho^0$ cross section drops by a factor of 4 between $Q^2 = 0$ and 1.1 GeV$^2$, the reduction in $\pi^-$ yield can be understood mainly in terms of the diminishing $\rho^0$ cross section.

Particle spectra in the forward direction ($p_T^\pi = 0$)

In Figs. 14 and 15, the $\pi^+$ and $\pi^-$ yields in the forward direction ($p_T^\pi < 0.02$ GeV$^2$) at $Q^2 = 0$ (Ref. 13) and $Q^2 = 1.15$ GeV$^2$ (Ref. 14) are compared. Both the $\pi^+$ and the $\pi^-$ yield drop considerably in the $Q^2$ for $x \geq 0.2$ whereas at $Q^2 = 0$ the $\pi^+$ and $\pi^-$ yields are approximately equal. At $Q^2 = 1.15$ GeV$^2$ a factor of two or more $\pi^-$ are produced for $x > 0.4$. The dashed curves in Figs. 14 and 15 show the $\pi^+, \pi^-$ yield at $Q^2 = 0$ after a removal of the $\rho^0$ contribution. Again part of the observed $Q^2$ variation can be ascribed to the $\rho^0$. Another clue comes from the mass spectrum of the particle system opposite to $\pi^\pm$,

$$\gamma p \rightarrow \pi^\pm \text{missing mass},$$

which is shown in Fig. 16(a) and (b) for $Q^2 = 1.16$ GeV$^2$. Low missing masses correspond to large values of $x$. The $\pi^+$ production at large $x$ comes mainly from two-body channels, such as

$$\gamma p \rightarrow \pi^+ n, \pi^+ \Delta^0, \pi^+ N(1520)$$

In the case of $\pi^-$ production the only channel accessible at low missing masses is

$$\gamma p \rightarrow \pi^- \Delta^{++}.$$
The major features of the $Q^2$ dependence of the $\pi^+$ and $\pi^-$ yields can be understood as a consequence of a) diminishing $\rho$ production, and b) increasing importance of the two-body reactions $\gamma p \rightarrow \pi^+$ baryon. A summary of the various measurements of the $Q^2$ dependence of $\pi^+$ and $\pi^-$ yields is given in Fig 17. The $\pi^+/\pi^-$ ratio increases with $Q^2$ in the $x$ regions shown.

**Proton Spectra**

Figure 18 shows the proton yield at $Q^2 = 0$ and 1.16 GeV$^2$ for $p_T = 0$. The electroproduction points in the large $x$ region are lower by $-20-30\%$. Hence no excess of fast forward-going protons is found, contrary to the prediction of the parton model of Drell, Levy and Yan.

**K Spectra**

A comparison of the $K^+$ yield in the forward direction ($p_T = 0$) is given in Fig 19. A striking increase in the relative number of $K^+$ mesons by a factor of $2.5 \pm 0.4$ from $Q^2 = 0$ to 1.16 GeV$^2$ is observed. Besides, at $Q^2 = 1.16$ GeV$^2$ the $K^+$ and $\pi^-$ yields are approximately equal for lab momenta near 2 GeV (Ref 14) whereas at $Q^2 = 0$ the $\pi^-$ are more copious by a factor of 10-15. The distribution of the missing mass to the $K^+$, $\gamma_p \rightarrow K^+$ + missing mass suggests (Fig 20) that most of the high momenta $K^+$ mesons at $Q^2 = 1.16$ GeV$^2$ come from two-body processes such as $\gamma_p \rightarrow K^+ \left(\frac{1}{2}^+\right)$, $K^+ Y^* (1385)$ etc. The observed increase with $Q^2$ in $K^+$ yield (Fig 19) appears to be in line with a prediction by K Wilson, however the fact that the $K^+$ stem mainly from the two-body reactions seems to indicate that the production process is different from the mechanism Wilson has considered.

**Acknowledgments**

I am grateful to Dr. Schildknecht for numerous discussions.

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W. Struczninski et al., Inclusive Photoproduction of $\pi^\pm$ and p at Energies Up to 6.3 GeV and Comparison to Electroproduction. #668

H. Burfeindt et al., Inclusive Photoproduction of Protons, Kaons, and Protons at 6 GeV. #660

H. Burfeindt et al., Measurement of Inclusive Photoproduction at 3.2 GeV and Comparison with Electroproduction. #661


I. Dammann et al., Inclusive $\pi^+$ and $\pi^-$ Distributions in Electroproduction on Protons. #754


The $p^0$ contribution to the $\pi^\pm$ yield for $p_T^2 < 0.04 \text{ GeV}^2$ has been determined by K. C. Moffeit from the data of the SLAC-Berkeley-Tufts collaboration (Ref. 11).


Fig. 1. The ratio $R = \sigma_L/\sigma_T$ as a function of $W$. The errors include statistical and systematic errors. Figure taken from Ref. 1.
Fig. 2. The ratio $D/H - 1$ as a function of $1/\omega'$. The full line is a fit to the MIT-SLAC deep-inelastic data (Ref. 3). Figure taken from Ref. 2.
Fig. 3(a). Dependence of the differential cross sections for $e^+e^-\to \Delta^{++}$ and $e^+e^-\to \Delta^0$ on $Q^2$ for $t - t_{\text{min}} = 0.04 \text{GeV}^2$ and $W = 2.23 \text{GeV}$. The dashed and dotted curves show the predictions of a VDM calculation. Figure taken from Ref. 4.

Fig. 3(b). The ratio of the cross sections $\pi^+\Delta^0/\pi^-\Delta^{++}$ as a function of $t$. The electroproduction data are those of Fig. 3(a) plotted at their respective $t$ values. The photoproduction points are those of Ref. 5.
Fig. 4. Layout of the streamer-chamber experiment to study electroproduction. Figure taken from Ref. 7.
Fig. 5. Total cross section for the reaction $ep \to ep\pi^+\pi^-$ as a function of the total hadronic effective mass $W$ for different $Q^2$ intervals. Also shown is the total cross section for $yr \to yr\pi^+$ from the ABBHHM collaboration. Figure taken from Ref. 7.
Fig 6 Reaction $e^+p \rightarrow e^+\pi^-\pi^-$ (a) $\pi^+$ mass distribution for $1.3 < W < 1.7$ GeV for different $Q^2$ intervals

Fig 6(b) $\pi^+$ and $\pi^-\pi^+$ mass distributions for $2 < W < 2.7$ GeV for different $Q^2$ intervals
Fig 7 Reaction $ep \rightarrow e\rho^0$. Total cross section as a function of $Q^2$ for different $W$ intervals ($\bullet$).

The open points (o) show the cross-section data when multiplied by $e^{A|t|_{\min}}$ to account for the $|t|_{\min}$ cutoff. The values at $Q^2 = 0$ have been measured by the ABBHHM collaboration.

The dashed-dotted curves show the $Q^2$ dependence of the total inelastic $ep$ cross section normalized to the cross-section point at $Q^2 = 0$. Figure taken from Ref. 7.
Fig. 8. Reaction $ep \rightarrow ep\pi^+\pi^-$. Differential cross section, $d\sigma/dt$, for events in the $\rho$ region (●). The cross sections have been normalized to the total $\rho$ cross section. The open points (○) show the photoproduction data$^9$ treated in the same manner. Figure taken from Ref. 7.
Fig. 9. Definition of the polar and polarization decay angles $\theta_H, \psi$ in the helicity system for forward produced $\rho^0$s.
Fig. 10. Reaction $ep \rightarrow ep\rho^0$. Decay angular distributions in the helicity system for all events in the $\rho$ region. Figure taken from Ref. 7.
Fig. 11. The ratio of the invariant cross sections for $\gamma p \rightarrow \pi^+ X$ and $\gamma p \rightarrow \pi^- X$ versus $x$ for different transverse momenta $p_T$. Figure taken from Ref. 13.
Fig. 12. Invariant cross section $f = E d^3 \sigma / dp^3$ as a function of $p_T^2$ for different $x$ and $Q^2$ intervals. Figure taken from Ref 15.
Fig. 13. The normalized $\pi^-$ yield, $F(x) = \frac{1}{\sigma_{\text{tot}}} \int_0^{p_{\text{max}}^2} \frac{E^*}{p_{\text{max}}^*} \frac{d^2\sigma}{dp_{\perp}^2} dx$, for $ep \rightarrow e\pi^- + \cdots$ at $W = 2.6$ GeV and $Q^2 = 0$ (Ref. 12), and at $2.0 < W < 2.7$ GeV for different $Q^2$ intervals (Ref. 7). The curves show the $\pi^-$ yield at $Q^2 = 0$ when $p_0^0$ events from $\gamma p \rightarrow p p^0$ are removed. Figure taken from Ref. 7.
Fig. 14. The normalized invariant cross section for $\gamma p \rightarrow \pi^- X$ and $ep \rightarrow e\pi^- X$ for $p_T^2 < 0.02$ GeV$^2$ at $W = 2.48$ GeV, $Q^2 = 0$ (Ref. 11) and at $W = 2.63$ GeV, $Q^2 = 1.16$ GeV$^2$ (Ref. 14). The curve shows the $\pi^-$ cross section at $Q^2 = 0$ after $p^0$ events from $\gamma p \rightarrow pp^0$ have been removed.
Fig. 15. The normalized invariant cross section for $\gamma p \rightarrow \pi^+ X$ and $ep \rightarrow e\pi^+ X$ for $p_{\pi^+}^2 < 0.02$ GeV$^2$ at $W = 2.63$ GeV, $Q^2 = 0$ (Ref. 13) and $Q^2 = 1.16$ GeV$^2$ (Ref. 14). The curve shows the $\pi^+$ cross section at $Q^2 = 0$ after $\rho^0$ events from $\gamma p \rightarrow p\rho^0$ have been removed.\textsuperscript{18}
Fig. 16. Missing-mass spectrum for inclusive $\pi^\pm$ electroproduction at $p_T^2 < 0.02$ GeV$^2$, $W = 2.63$ GeV and $Q^2 = 1.16$ GeV$^2$. Figure taken from Ref. 14.
Fig. 17. Normalized invariant cross sections for inclusive $\pi^\pm$ production as a function of $Q^2$ for different $x$ intervals. Data from Refs 13-15. Figure taken from Ref. 15.
Fig. 18. The normalized invariant cross sections for inclusive proton production at $p_T^2 < 0.02$ GeV$^2$, $W = 2.63$ GeV, $Q^2 = 0$ (Ref. 13) and $Q^2 = 1.16$ GeV$^2$ (Ref. 14). Figure taken from Ref. 13.
Fig. 19. The normalized invariant cross sections for inclusive $K^+$ production at $p_T^2 < 0.02$ GeV$^2$, $W = 2.63$ GeV, $Q^2 = 0$ (Ref. 13) and $Q^2 = 1.16$ GeV$^2$ (Ref. 14). Figure taken from Ref. 13.
Fig. 20. Missing-mass spectrum for inclusive K± production at $p_\perp^2 < 0.02$ GeV$^2$, $W = 2.63$ GeV and $Q^2 = 1.16$ GeV. Figure taken from Ref. 14.
I have been asked to report on five coincidence electroproduction papers submitted by groups working at the Wilson Electron Synchrotron at Cornell University.

The Cornell Electron Scattering group has submitted three papers to the conference. They have measured the cross section for the detection of an inelastically-scattered electron in coincidence with a hadron emerging at large angles to the virtual photon direction. This coincident hadron, either a pion or a proton, was emitted backwards in the center-of-mass of the virtual photon-proton system.

Figure 1 shows the distribution in transverse momentum squared of the pion events. The slopes are all about 9.5 GeV$^{-2}$ with no immediately obvious trend in either $Q^2$ or $x$. For all events, $W$, the total virtual photon-proton center-of-mass energy was 3.0 GeV. These distributions can be integrated over all angles and normalized by $\sigma^{tot} (\gamma_p p)$ to give an average multiplicity of pions in the $x$ range specified. This average multiplicity shows little or no increase with $Q^2$ as some models might predict.

Figure 2 shows the missing-mass distribution obtained when the proton is the coincident hadron. Notice that the $p$ is acting quite differently as a function of $Q^2$ from the background. Let us sidestep the problem of separating the $p$ from the background and look instead at the distributions of proton events which have a missing mass greater than 1 GeV.

Figure 3 shows the $t$-distribution of the proton events for four bins in missing-mass squared and for three different values of $Q^2$. The change in slopes of the fits is gradual, essentially independent of $m^2$ and decreasing somewhat with $Q^2$.

Figure 4 indicates the slopes obtained for all the high mass recoil protons plotted versus $Q^2$. The slope is probably decreasing slowly with $Q^2$.

Now I would like to move on to the data of the Harvard-Cornell collaboration. This group detected an inelastically-scattered electron in coincidence with a fast hadron emerging forward along the direction of the virtual photon. The group has analyzed data taken at two points in the deep-inelastic scaling region. Notice

Point 6 $W = 2.15 \quad Q^2 = 1.2 \quad \omega = 4.1$
Point 7 $W = 2.67 \quad Q^2 = 2.0 \quad \omega = 4.1$

the points lie on a line of constant $\omega$.

Figure 5 shows the missing-mass spectrum obtained from electron-pion coincidences at the higher energy point. The shape of the spectrum at the lower energy point, scaled down by a factor 2.34, is indicated by the curve.

There is a strong signal at both points from the two-body pion electro-production channel.
The data in this peak have been analyzed using a Born-type theory in order to extract the value of the pion form factor.\(^2\)

The shape of the inclusive part of the \(\pi^+\) spectrum is displayed in Fig. 6 plotted as a function of the Feynman scaling variable \(x\). Note the structure functions are normalized by the total virtual photon-proton cross section. The conclusion is at constant \(\omega\), the structure function scales in \(x\) and has roughly a form \(1/x\).\(^3\) In the region \(0.5 < x < 0.9\) the pions have an exponential distribution in transverse momentum with a slope of about 6.5 GeV\(^{-2}\) at both data points.

The group also has data on \(\pi^-\) at the higher data point only. Figure 7 shows the ratio of the structure functions of \(\pi^+\)'s to \(\pi^-\)'s as a function of \(x\). The ratio is close to 1 for \(x < 0.3\) but is consistently greater than 1 for large \(x\).

Turning now to Fig. 8 we see the missing-mass spectrum for kaon electron coincidences. There is a slight proton contamination in this spectrum which accounts for the events above the physical end of the spectrum at the \(\Lambda\) mass. The two-body channels \(K^+\Lambda^0\), \(K^+\Sigma^+\) (1385), \(K^+\Sigma^+\) (1520) are quite prominent at both data points but the continuum of kaons does not appear large or unusual at the two data points.

The group also presented data on the proton spectrum which indicated that the identifiable two-body channels, \(180^\circ p\pi^0\) and \(180^\circ p\eta\) were dropping rapidly with energy and that the continuum of protons was being emitted in the backward hemisphere in the center-of-mass. Some theories would predict that protons should become more copious in the forward hemisphere as \(q^2\) increases; this was not seen in the data.

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Fig. 1. $p_T^2$ distributions of pion events for different $Q^2$ and $x$ bins, $W = 3.0$ GeV.
Fig. 2. Missing-mass squared distribution of proton events for two different angular bins; \( Q^2 = 1.2 \text{ GeV}, W = 3.0 \text{ GeV}. \)
Fig. 3. Semilogarithmic plot of distributions in $m_x^2$ and $t$ for three $Q^2$ values, $W = 3.0$ GeV.
Fig. 4. Slope $B(\text{GeV}^{-2})$ as a function of $Q^2$ for an \(\exp(Bt)\) fit of proton events with \(1.125 < M_{\text{MSQ}} < 1.875\).
$\theta^* \leq 9.6^\circ$
$W = 2.66 \text{ GeV}$
$k^2 = -2.02 \text{ GeV}^2$
$\omega = 4.07$
$\gamma_\nu + p \rightarrow \pi^+ + \text{MM}$

Fig. 5. Missing-mass distribution of forward positive pion events.
Fig. 6. Feynman scaling of forward positive pion events for two data points each having \( W = 4.1 \).
Fig. 7. $\pi^+ / \pi^-$ ratio for higher-energy forward pion events.
Fig. 8. Missing-mass spectrum for forward $K^+$ events.
I. Introduction

In this summary, I report on a preliminary analysis of a hybrid bubble-chamber experiment examining the process $\mu^- + p \rightarrow \pi^- + X$ where $X$ is detected with $4\pi$ geometry.

The general kinematics of the process is as follows:

\[ W = \sqrt{(P+Q)^2} \]

where conventionally we define,

\[ Q^2 = -(\mu - \mu')^2 = 2m^2 + 2(EE' - |\vec{p}| |\vec{p}'| \cos \theta) \]

\[ = Q_{\text{min}}^2 + 4|\vec{p}| |\vec{p}'| \sin^2 \theta/2 > 0 \]

\[ Q_{\text{min}}^2 = 2(EE' - |\vec{p}| |\vec{p}'| - m^2) \]

\[ \nu = \frac{P \cdot Q}{M} = E - E', \quad M = \text{mass of proton} \]

\[ s = W^2 = 2M\nu + M^2 - Q^2. \]

Note that $Q^2$, $\nu$, and $s$ are Lorentz invariants and $(E,p)$, $(E',p')$ are the incoming and scattered muon energy and momentum in the laboratory frame while $\theta$ is the laboratory scattering angle of the muon.

The double differential cross section for muon detection is usually expressed in two ways:

\[ \frac{d^2\sigma}{dQ^2ds} = \left( \frac{\pi}{2M\nu} \right)^2 \sigma_M \left[ W_2(s,Q^2) + 2 \tan^2 \theta/2 W_1(s,Q^2) \right], \]

where

\[ \sigma_M = \frac{4\alpha^2 E^2 \cos^2 \theta/2}{Q^4} \]

or

\[ \frac{d^2\sigma}{dQ^2ds} = \left( \frac{\pi}{2M\nu} \right)^2 \Gamma \left[ \sigma_T(s,Q^2) + (\epsilon + \delta) \sigma_S(s,Q^2) \right] \]

where

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\[ \Gamma_t = \frac{\alpha}{4\pi} \frac{W^2 - M^2}{M^2} \frac{E}{E'} (1 - \varepsilon) \]  
\[ \varepsilon = \frac{1}{1 + \frac{2(Q^2 + \nu^2)}{Q^2 \tan^2 \theta/2} \frac{\nu^2}{Q^2 (1 - Q_{min}^2/Q^2)^2}} \]  
\[ \frac{2m^2}{Q^2} (1 - \varepsilon) \]

and the factor \((\pi/2M_E')\) converts \(d\sigma/dQdE'\) to \(d\sigma/dQ^2 ds\). In our kinematical range \(Q_{min}^2\) and \(\delta\) are negligible. \(W_1(s,Q^2)\) and \(W_2(s,Q^2)\) are the structure functions of the proton, in general functions of both \(s\) and \(Q^2\). The single arm inclusive ep inelastic-scattering measurements done at SLAC over the past five years have shown that \(W_2\) scales, i.e., \(W_2 = F(s/Q^2)\) if \(Q^2 \gtrsim 1\) (GeV/c)^2 and \(s \gtrsim 4\) GeV^2, and so the use of the structure function representation has become particularly common. However, the second representation, where \(\sigma_T\) and \(\sigma_L\) are the total cross sections for transversely and longitudinally polarized photons respectively, is more relevant to our presentation here, since comparisons with photoproduction are quite instructive, and \(\sigma_T(s,Q^2)\) \(\to\) \(\sigma_P(s)\) as \(Q^2 \to 0\). (From its definition \(W_2 \to 0\) as \(Q^2 \to 0\).) Also, much of our data lies in the transition region between photoproduction and the scaling region and so expressing our results in terms of \(W_2\) does not seem particularly revealing at this time.

Figure 1 shows the inclusive ep inelastic cross section at \(W = 4\) GeV, as a function of \(Q^2\), from photoproduction to \(Q^2 = 1.5\) (GeV/c)^2, approximately the \(Q^2\) range of this experiment. The decrease of the \(\sigma_T^{tot}\) with \(Q^2\) may represent a rapidly changing transition region between photoproduction and the scaling region, with most of the transition completed by a \(Q^2\) of \(\gtrsim 0.6\) (GeV/c)^2. This feature of the ep inclusive data was a strong motivation for this experiment which attempts to investigate largely qualitative features of the inelastic final states with limited statistics using the unbiased 4\(\pi\) geometry of the bubble chamber.

When considering a bubble-chamber experiment where the total cross section to be investigated is less than 1 \(\mu\)b, new techniques are strongly recommended. Conventional bubble-chamber techniques would require \(\sim100\) million pictures for a few thousand events, clearly an unacceptable situation. Happily, the SLAC bubble-chamber operations group has been working on rapid cycling techniques with the 40-in. hydrogen bubble chamber over the past few years, and so this experiment was run with the bubble chamber pulsing at 10 pps for over 90% of our data taking time. Additionally, we were able to run with about 100 \(\mu\)s/pulse through the chamber, and with a muon trigger behind the bubble chamber which flashed the lights and took a picture only when there was a good chance an event had occurred. This procedure yielded about 5000 events in 90,000 pictures, with about \(30 \times 10^6\) expansions of the bubble chamber.

II. Results

A. Total Cross Section and Topological Partial Cross Sections

The total cross section, \(\Gamma_t + \epsilon\sigma_s\), is related to the experimental cross section by the flux factor \(\Gamma_t\) according to Eq. (7).
The resulting values for $a_T + a_S$ are shown in Figs. 2(a)-2(c). We estimate an overall systematic uncertainty on the order of ±10% for these values. No attempt has been made to separate $a_T$ from $a_S$. For the range of data considered (with the exception of photoproduction points), $k$ varies from 0.8 to 1.0.

Three $W$ bins are considered: $W = 1.4$ to 1.8 GeV [Fig. 2(a)], $W = 1.8$ to 2.8 GeV [Fig. 2(b)], and $W = 2.8$ to 3.8 GeV [Fig. 2(c)]. Each cross section point is broken down into its charged multiplicity. Only charged hadrons contribute to the prong count. The ratio of one-prong to $a_{tot}$ (observed value) and the ratio of three-prongs to $a_{tot}$ are given at each $Q^2$ for the points in Fig. 2(a). In Fig. 2(b) five-prongs also contribute to the total cross section and in Fig. 2(c), also seven-prongs.

The general features show agreement, within systematic errors, between $\mu$-beam total cross sections, and SLAC single arm electroproduction values. At $W = 1.4$-1.8 GeV, the single-prong contribution to $a_{tot}$ is consistent with a constant value of about 0.7, and the three-prongs contribute the rest of $a_{tot}$. The photoproduction points are obtained from Ref. 3. In Fig. 2(b), at $W = 1.8$ to 2.8 GeV, we see an increasing fraction of the total cross section in the one-prong topology, as $Q^2$ increases, and this increase is reflected in a corresponding decrease in the three-prong topology, while five-prongs contribute a small amount. In Fig. 2(c), for $W = 2.8$-3.8 GeV, the total cross section now also includes seven-prong events. The photoproduction values in Figs. 2(b) and 2(c) are obtained from data of Ref. 4.

B. Multiplicities

One may summarize the prong distributions shown in Fig. 2 by computing $<N>$, the mean charged hadronic prong multiplicity for various bins in $Q^2$ and $W$.

$$<N> = \frac{\sum N \cdot WT}{\sum WT},$$

where $N$ is the number of prongs in an event and $WT$ is the inverse of the acceptance probability for the event derived from a Monte-Carlo calculation. Figure 3(a) shows $<N>$ vs $Q^2$ for three $W$ ranges. There is a tendency for $<N>$ to decrease as $Q^2$ increases, particularly for $W = 1.8$-2.8 GeV where the effect clearly arises from the relative decrease in three-prong events and the corresponding increase in one-prong events.

The multiplicity was also studied as a function of $Q^2$ in the $W$ range 2-4 GeV in the photon fragmentation region, the target fragmentation region, and the central region. To do this we binned the charged hadrons from each event into their proper $Q^2$ and $x$ regions as shown in Fig. 3(b). In the case of events in which the positive track identification was ambiguous the pion fit was selected rather than the proton fit. This gives a different $x$ value for the track since $P_{max}$ evaluated in the center of mass is greater for the proton than for the pion and $x = (P_{max}/P_{max}^{CM})$. The photoproduction data were handled in the same way. The central region multiplicity is consistent with flat while the target and projectile region multiplicities seem to be falling slightly with $Q^2$.

Although the purely statistical evidence is fairly strong in favor of $<N>$ decreasing as $Q^2$ increases, we want to emphasize that our whole analysis is preliminary, that no effects of possible systematic errors in our experiment are included, and that these conclusions depend to a large extent on the comparison of the present experiment with the photoproduction experiment where there might be additional relative systematic errors.
C. \( \rho^0 \) Production

Events of the reaction

\[ \mu^- p \rightarrow \mu^- \rho^0 n^0 \]

could be clearly isolated by requiring a good 4C fit. The invariant mass of dipions in this sample, for \( 2.0 < W < 4.0 \) GeV, is shown in Fig. 4(a) where a clear \( \rho^0 \) signal is evident. For \( W < 2.0 \) GeV the signal is indistinct and so only the higher energy data were used in this analysis.

Superimposed on the mass spectrum in Fig. 4(a) is the best fit obtained by the SLAC-LBL-Tufts collaboration to the data of 2.8 GeV photoproduction (the mean \( W \) is approximately the same in the two cases). It appears that in inelastic \( \mu \) scattering the background beneath the \( \rho^0 \) has decreased in relative importance and the \( \rho^0 \) itself may be less skewed (a standard Breit-Wigner resonance shape provides a satisfactory description).

We estimate that the incoherent background is about 10% of all dipions with \( 0.65 < M_{2\pi} < 0.90 \) GeV, and that \( \rho^0 \) events lost outside this mass cut are also about 10%. Hence we refer to the cross section of dipions in the above mass cut as \( \sigma(\rho^0) \). The total \( \rho^0 \) cross section normalized to \( \sigma_{\text{tot}}(W,Q^2) \) is shown in Fig. 4(b) for our data. The point at \( Q^2 = 0 \) was obtained by averaging the \( \rho^0 \) and total photoproduction cross sections over our virtual photon flux, using the data of the SWT and SBT collaborations, which were deduced using the Sodging Model. The relative importance of \( \rho^0 \) photoproduction apparently decreases with \( Q^2 \) by about a factor of 2 over our \( Q^2 \) range.

At a fixed low value of \( W \), the minimum momentum transfer to the proton, \( t_{\text{min}} \), increases over that of photoproduction at the same \( W \). If we assume that \( d\sigma/dt(W,Q^2) \) is equal to \( d\sigma/dt(W,0) \) at all \( Q^2 \), then the low \( t \) region will be kinematically excluded. We find that this effect would give reduction in the total \( \rho^0 \) cross section of 10 and 30%, respectively, in the two highest \( Q^2 \) bins in Fig. 4, assuming a dependence in \( d\sigma/dt \) of \( Q^2 \).

D. \( \omega \) Production

In Fig. 5(a) we plot \( M(\pi^+\pi^-\pi^0) \) from the reaction \( \mu^- p \rightarrow \mu^- \omega n^0 \), selecting only unambiguous protons (\( |t| < 1 \) GeV²). The events shown have a 4C fit confidence level greater than 10%, \( Q^2 \geq 0.15 \) (GeV/c)² and no cut on \( W \). We observe a clear \( \omega \) peak. The shaded events have \( Q^2 \geq 0.5 \) (GeV/c)² and also show a \( \omega \) peak. We have estimated the \( \omega \) cross section by selecting events with \( 0.74 < M(3\pi) < 0.82 \) (GeV/c)², making a small background correction by hand, and correcting for our 10% \( x^2 \) probability cut and non \( 3\pi \) decay modes. In Fig. 5(b) we plot the ratio \( \sigma(\gamma p \rightarrow \omega n)/\sigma_{\text{tot}} \) vs \( Q^2 \) for \( 1.8 < W < 3.8 \) GeV. The average value of \( W \) for all events in this plot, i.e., all \( Q^2 \) bins, is 2.26 GeV. Using photoproduction data we have calculated the ratio \( \sigma(\omega)/\sigma_{\text{tot}} \) at \( Q^2 = 0 \) for the same \( W \) interval (correcting for the photon energy spectrum). The photoproduction value agrees well with our lowest \( Q^2 \) bin. Our highest \( Q^2 \) bin contains one \( \omega \) event. Fig. 5(b) indicates that the \( \omega \) remains a relatively small fraction of the cross section as \( Q^2 \) increases.

E. Inclusive \( \gamma_V + p + p + X \)

In Fig. 6(a) are shown the inclusive proton distributions for various \( W \), and \( Q^2 \) bins, as well as \( Q^2 = 0 \) photoproduction results. The data plotted are \( F(x) \) vs \( x \) where \( x = (P_\parallel /P_{\text{max}})_{\text{CM}} \). The proton and \( n^0 \) ambiguity was resolved by ionization up
to laboratory momenta of \( \approx 1 \ \text{GeV}/c \). In addition, for those events obtaining a 4C kinematic fit with a \( \chi^2 < 25 \), or a 1C fit with confidence level >10%, the mass interpretation of the fit was used. For ambiguous events the proton hypothesis was used. Thus Fig. 6(a) represents an upper limit on the proton inclusive distributions. The photoproduction was treated in a similar way. As is seen from Fig.6(a), as \( Q^2 \) increases the peak near \( x = -1.0 \) drops compared to photoproduction, in both \( W \) bins. One can see this feature more clearly in Fig. 6(b) where \( F(x) \) is plotted on a linear scale. In the \( x \)-region shown proton identification is unambiguous by ionization.

Also, in Fig. 6(b) \( \langle x' \rangle = \langle 1 + s/Q^2 \rangle = 10.6 \). The peak at \( x = -1 \) corresponds to slow protons in the laboratory frame.

F. \( p_{\perp}^2 \) Distributions

To study further the \( Q^2 \) dependence of the inclusive reaction \( \gamma_p \rightarrow p + \text{anything} \), we discuss next the inclusive variable \( p_{\perp}^2 \). We limit ourselves to the backward region, \(-1 \leq x \leq -0.5\), a region where the protons are well identified by ionization.

Figure 7 shows \( p_{\perp}^2 \) vs \( Q^2 \) for the \( W \) range 1.8-4 GeV (\( \langle W \rangle = 2.6 \ \text{GeV} \)), where \( \langle p_{\perp}^2 \rangle \) means the mean value of \( p_{\perp}^2 \) averaged over the experimental distribution; more precisely:

\[
\langle p_{\perp}^2 \rangle = \int_{-1}^{-0.5} \ dx \int_{0}^{\infty} \frac{\hat{E}_p}{p_{\max}} \ dx_{\perp} \ dx_{\parallel} \ dx_{\perp} \ dx_{\parallel} \ dx_{\perp} \ dx_{\parallel} \ dx_{\perp} \ dx_{\parallel}.
\]

For an exponential distribution in \( p_{\perp}^2 \) with slope parameter \( B \), \( B = \langle p_{\perp}^2 \rangle^{-1} \). The point at \( Q^2 = 0 \) has been calculated, using the same procedure, from the photoproduction data of Ref. 4 taken at a fixed energy \( W = 2.5 \ \text{GeV} \) (\( E = 2.8 \ \text{GeV} \)).

We see a tendency for the slope parameter to decrease with \( Q^2 \). This effect is even more striking if we remove the contribution from the diffractive processes \( \gamma_p \rightarrow (p^0 \text{ or } \omega^0) p \). For this latter process the slope parameter seems to fluctuate around the photoproduction value, although it is not inconsistent with being flat in \( Q^2 \). (Pearson probability = 3%).

III. Conclusions

Some major conclusions one can draw from the above are, as \( Q^2 \) increases:

a) The multiplicity seems to decrease.

b) The \( p^0 \) and \( \omega \) elastic contribution becomes smaller relative to the total cross section.

c) Slow protons in the laboratory are becoming rare.

d) The \( p_{\perp}^2 \) distribution for protons with \( x < -0.5 \), with elastic \( p^0 \) and \( \omega \) removed, is flattening.

References


2The radiatively corrected values for \( q_T + \epsilon q_S \) and the corresponding errors were obtained from SEARCH, a program which interpolates between measured values of the SLAC-MIT inelastic ep scattering data. The photoproduction point is an average from E. D. Bloom et al., SLAC-PUB-653, unpublished and D. O. Caldwell et al., Phys. Rev. Letters 25, 609 (1970).

3Preliminary results from Group A, LBL; private communication from Dr. H. Oberlack (August 1972)
A correction of 1.3% at 2.8 GeV and 2.9% at 4.7 GeV has been applied to the normalized distributions given in this paper to account for the π particles in the strange particle topologies. For γp + p + (anything) the one-prong events were excluded as were the strange particle topologies. The x distributions were normalized to the total cross section remaining after these subtractions. We point out that the proton distributions from these events may be different from those of the topologies shown.

The elastic ρ^0 and ω contributions have been removed by the simplest cuts: All events of γp + π^+π^-p with M_{π^+π^-} < 1.0 GeV are referred to as "elastic" ρ^0 events and all events of γp + π^+π^-p^0p with |M_{π^+π^-p^0} - M_ω| < 20 MeV are referred to as "elastic" ω events. In addition, the calculated photon energy was required to be within the energy peak of the photon spectrum for the ω events.
Fig. 1. $\sigma_T + \varepsilon \sigma_s$ vs $Q^2$ obtained by interpolating the single arm inelastic ep scattering data. The data are interpolated at a fixed $W = 4$ GeV. The photoproduction values were obtained as indicated in Ref. 2. The straight dashed lines are drawn to guide the eye, and do not represent significant fits.
Fig. 2(a). Total cross section values $\sigma_T + \varepsilon\sigma_S$ obtained from the data by extracting the virtual photon flux factor from the measured cross sections. The polarization parameter $\varepsilon$ is the ratio of longitudinal to transverse polarization of the virtual photons, and varies from 0.8 to 1.0 in these data. No attempt has been made to separate the longitudinal and transverse parts of the cross section. The solid line is the single arm electroproduction values for $\sigma_T + \varepsilon\sigma_S$ averaged over the same $W$-values as the data. Beneath the total cross sections, each point is broken into fractional contributions from different topologies. Only out-going charged hadrons are counted as prongs. The photoproduction points shown are from Ref. 3 for the $W = 1.4-1.8$ GeV data, and from Ref. 4 for the $W = 1.8-2.8$ and 2.8-3.8 GeV data.
Fig. 2(b). Total cross section values $\sigma_T + \epsilon \sigma_S$ obtained from the data by extracting the virtual photon flux factor from the measured cross sections. The polarization parameter $\epsilon$ is the ratio of longitudinal to transverse polarization of the virtual photons, and varies from 0.8 to 1.0 in these data. No attempt has been made to separate the longitudinal and transverse parts of the cross section. The solid line is the single arm electroproduction values for $\sigma_T + \epsilon \sigma_S$ averaged over the same $W$-values as the data. Beneath the total cross sections, each point is broken into fractional contributions from different topologies. Only out-going charged hadrons are counted as prongs. The photoproduction points shown are from Ref. 3 for the $W = 1.4-1.8$ GeV data, and from Ref. 4 for the $W = 1.8-2.8$ and $2.8-3.8$ GeV data.
Fig. 2(c). Total cross section values $\sigma_T + \epsilon \sigma_s$ obtained from the data by extracting the virtual photon flux factor from the measured cross sections. The polarization parameter $\epsilon$ is the ratio of longitudinal to transverse polarization of the virtual photons, and varies from 0.8 to 1.0 in these data. No attempt has been made to separate the longitudinal and transverse parts of the cross section. The solid line is the single arm electroproduction values for $\sigma_T + \epsilon \sigma_s$ averaged over the same W-values as the data. Beneath the total cross sections, each point is broken into fractional contributions from different topologies. Only out-going charged hadrons are counted as prongs. The photoproduction points shown are from Ref. 3 for the $W = 1.4-1.8$ GeV data, and from Ref. 4 for the $W = 1.8-2.8$ and $2.8-3.8$ GeV data.
Fig. 3(a). The charged hadronic multiplicity as a function of $Q^2$. The $Q^2$ intervals used were $0.15 < Q^2 < 0.4$ (GeV/c)$^2$, $0.4 < Q^2 < 0.8$ (GeV/c)$^2$, $0.8 < Q^2 < 2.5$ (GeV/c)$^2$. Three $W$ regions are shown: Bottom, $<W> = 1.6$ GeV for the range $1.4 < W < 1.8$ GeV; Middle, $<W> = 2.25$ GeV for the range $1.8 < W < 2.8$ GeV; Top, $<W> = 3.25$ GeV for the range $2.8 < W < 3.8$ GeV. The photoproduction points were obtained from Refs. 3 and 4.
Fig. 3(b). Average numbers of charged hadronic tracks in three regions of $x$ as a function of $Q^2$ for the $W$ range 2-4 GeV. The photoproduction values ($Q^2 = 0$) were obtained from the $W = 2.5$ GeV data, and scaled up so that $\langle N \rangle$ summed over all $x$ equalled the photoproduction multiplicity interpolated to $\langle W \rangle = 2.85$ GeV.
Fig. 4(a). The dipion mass spectrum from which the ratios below were derived. The superimposed curve is taken from Ref. 4 and represents the spectrum observed in \( \gamma + p \rightarrow \rho^0 p \) photoproduction.

Fig. 4(b). Cross sections for observed \( \rho^0 \)'s in the reaction \( \mu^- p + \mu^- p \rightarrow \rho^0 \) for \( W \) between 2 and 4 GeV, normalized to the total hadronic cross section. The triangle represents the photoproduction value averaged over our photon spectrum for the same interval in \( W \).
Fig. 5(a). $M(\pi^+\pi^-\pi^0)$ from fits to $\mu^-p \rightarrow \mu^-p\pi^+\pi^-\pi^0$ with $Q^2 > 0.15 \text{ (GeV/c)}^2$. For 1C fits we plot fits with a confidence level greater than 10%. The shaded events have $Q^2 > 0.5 \text{ (GeV/c)}^2$. The distribution is not weighted by our acceptance and there is no cut on $W$.

Fig. 5(b). Ratio of the cross section $\sigma(\gamma \nu p \rightarrow p\omega)/\sigma_{\text{TOT}}$ for $1.8 < W < 3.8 \text{ GeV}$ plotted vs $Q^2$. For comparison we plot the photoproduction point at $Q^2 = 0$. 

-79-
Fig. 6(a). Reaction $\gamma p \rightarrow p + (\text{anything})$: Normalized structure function $F(x)$ vs $x$ for the indicated $W$ and $Q^2$ intervals. To the right of the symbol $\pi^+$ some $\pi^0$ contamination occurs. Photoproduction data at $Q^2 = 0$ are taken from Ref. 4. The dotted curves are approximations to the photoproduction data for comparison to the $Q^2 = 0$ distributions. Elastic $p^0$ and $\omega$ events have been excluded from some points as noted.
\[ \gamma p \rightarrow p \, (\text{anything}) \]

\[ 2.5 < W < 4.0 \text{ GeV} \quad \langle W \rangle = 3.1 \text{ GeV} \]

- $x$ elastic $p^0$ and $\omega$ excluded

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Fig. 6(b). Reaction $\gamma p \rightarrow p + (\text{anything})$: Normalized structure function $F(x)$ vs $x$ for the indicated $Q^2$ intervals for $2.5 < W < 4.0$ GeV. The photoproduction data at $Q^2 = 0$ are taken from Ref. 4. Elastic $p^0$ and $\omega$ events have been excluded from some points as noted.
Fig. 7. $\langle p^2 \rangle$ as a function of $Q^2$ for backward protons in the inclusive reaction $\gamma N \rightarrow p + (\text{anything})$ for $-1 < x < -0.5$ in the $W$ range $1.8 - 4$ GeV. The upper graph is for all events. The middle graph is for events with a $4\pi$ elastic $p^0$ or $\omega$. The lower graph is for all events that do not have an elastic $p^0$ or $\omega$. 
INCLUSIVE ELECTROPRODUCTION OF HADRONS AT 19.5 GeV** (#440)

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I. Experimental Method

This is a report of inclusive measurements, in the forward direction, of the electroproduction of hadrons using a hydrogen target. The incident electron energy was 19.5 GeV; and the data reported extends over the range $-0.5 \leq q^2 \leq -2.5 \, (\text{GeV/c})^2$ and $4 < \nu < 14 \, \text{GeV}$. $q^2$ is the square of the four-momentum transferred at the electron vertex and $\nu$ is the energy loss of the electron in the laboratory system. The experiment also included measurements over a larger $q^2$ range, measurements of the electroproduction of $p^0$ mesons and measurements using a deuterium target. A brief description of these aspects of the experiment is given at the end of the report.
The experimental apparatus, Fig. 1, consisted of (1) a 4 cm long liquid hydrogen or deuterium target followed by, (2) a large 10 kGauss analyzing magnet with a 17 kGauss-meters field, (3) two large optical spark chambers separated by 1.7 m., (4) a double bank of scintillation counters, and finally (5) a bank of 11 lead-lucite shower counters. The angular acceptance at the target was vertically ± 17° and horizontally ± 7° for particles with laboratory momentum greater than 1 GeV/c.

The electron beam contained 1 to 2 x 10^4 e^- per 1.5 μsec long SLAC pulse. At the experimental target, the beam had an rms width of 0.5 mm x 0.5 mm, and an rms divergence less than 0.2 mrad x 0.2 mrad. There the beam was very well collimated, with fewer than 1 e^- in 10^5 outside a 0.5 cm diameter circle. The unscattered beam and the forward electromagnetic backgrounds passed through the magnet in a field-free region created by a cylindrical superconducting tube.\[1\]

The apparatus was triggered on the detection of a scattered electron by the scintillation counters and shower counters. The shower counter thresholds were set to ~ 5 GeV. Photon triggers were eliminated by the requirement that a shower counter fire coincident with the scintillators in front of it.

During the experiment we recorded 250,000 pictures with the H_2 target and 110,000 with D_2. These data samples contained 2.6 x 10^{12} and 0.7 x 10^{12} incident electrons respectively. Here we report the H_2 inclusive data.

If p and E are the three-momentum and energy of a hadron, then the
quantity \( \frac{\partial \sigma(q^2, v)}{\partial p} \) defines a Lorentz invariant differential cross section. Here \( \sigma(q^2, v) \) denotes \( \frac{\partial \sigma(e^- + p \rightarrow e^- + \text{anything})}{\partial q^2 \partial v} \). For various ranges of \( q^2 \) and \( v \), the inclusive data is presented in terms of the variables \( x, p_\perp^2 \) and \( \phi \). \( x \), the Feynman variable, is \( p_\parallel^*/p^* \) max. Here, in the center of mass system of the virtual photon and the proton, \( p_\parallel^* \) is the longitudinal momentum of the hadron along the direction of the virtual photon and \( p^*_\text{max} \) is its maximum value. \( p_\perp \) is the momentum of the hadron perpendicular to the direction of the virtual photon, and \( \phi \) is the azimuthal angle of the hadron measured from the plane of the electron scatter. We note that

\[
E \frac{\partial \sigma(q^2, v)}{\partial p} = 2 \frac{E^*}{p^*_\text{max}} \frac{\partial \sigma(q^2, v)}{\partial x \partial p_\perp^2 \partial \phi}
\]

We have no means of distinguishing pions, kaons, and protons from each other. Hence we will refer only to positively and negatively charged hadrons and use the symbol "h" to describe them. In computing \( x \), each \( h \) is assumed to be a \( \pi \).

We studied the zero order radiative effects by assuming that, in addition to the material that was actually present, there was \( (3/4) \cdot (\alpha/\pi) \cdot [\ln(-q^2/m^2) - 1] \) radiation lengths of material before and after the interaction. Data were simulated in which the incident and scattered electrons underwent random bremsstrahlung. This radiative degradation produced no noticeable effect in the shapes of the \( p_\perp^2 \) and \( x \) distributions at the statistical level of this experiment. No corrections for radiative effects have been made in the data presented here. The simulation indicated that spatial resolution effects are also negligible.
II. Distribution in Azimuthal Angle

With the assumption of one-photon exchange, the azimuthal angle (\(\phi\)) dependence must be of the form

\[
\frac{d\sigma}{d\phi} = A + \epsilon B \cos 2\phi + \sqrt{\epsilon(1 + \epsilon)} C \cos \phi
\]  

where \(\epsilon\) is the transverse polarization of the virtual photon. For the data presented here the average polarization was 83\%. The term proportional to \(\cos (2\phi)\) is due to transverse polarization of the photon and is present in \(\pi^-\) photoproduction at high \(x\) at the 30\% level.\(^2\) The term proportional to \(\cos \phi\) is due to interference between transverse and longitudinal photons and is unique to electroproduction.

The hadron azimuthal angle dependence of the data was studied for 2 different \(x\) ranges, and for positive and negative hadrons (see Fig. 2). We see no significant \(\phi\) dependence in the data, although the negative hadron data are consistent with the photoproduction results. As an aid in extracting \(p_T^2\) and \(x\) distributions, discussed later, we assume that the \(\phi\) distribution is flat.

III. Distributions in \(p_T^2\)

For this and later distributions we use the normalized differential cross section

\[
\left[ \frac{1}{\sigma(q^2,\nu)} \frac{E^*}{\pi p_{\text{max}}^*} \frac{d^2 \sigma(q^2,\nu)}{dp^2 dx} \right] ;
\]

where \(\sigma(q^2,\nu)\) is also taken from our data. The \(p_T^2\) dependence of the cross section is shown for negative hadrons (\(h^-\)) in Fig. 3. The positive hadron spectra is similar in shape. The exponential slope parameters (\(b\)) in the formula \(A e^{-b p_T^2}\) are presented for both signs in Fig. 4.
The $p_1^2$ dependence is seen to be roughly independent of $q^2$, and independent of hadron charge, at the statistical level of this experiment. All the data are fit well with the exponential, and with a slope parameter of typically $b=4.7 \ (\text{GeV/c})^{-2}$. This slope is, however, less steep than the slope observed in inclusive photoproduction. We have extracted a value of $b = 5.9 \pm 0.2 \ (\text{GeV/c})^{-2}$ for the same $x$ and $\nu (E_\gamma)$ range from the $\pi^-$ photoproduction data of Moffeit et al.\textsuperscript{2} The slightly less steep slopes observed here are not believed due to radiation or radiative effects; however, we do not think that this difference should be taken to be a significant effect until we have had an opportunity to perform a more exhaustive analysis of the data.

The $p_1^2$ dependence of the cross section is similarly independent of $\nu$, as is true approximately in photoproduction.\textsuperscript{2} No statistically significant $x$ dependence of the $p_1^2$ distribution was evident either.

IV. Distribution in $x$

The dependence of the invariant cross section on $x$ is shown in Figs. 5 and 6. The curve for negative hadrons agrees with the analogous photoproduction\textsuperscript{2} curve at low $x$, but falls a factor of 2 to 3 below it at higher $x$. The slow fall of the photoproduction distribution at moderate $x$ values is due largely to the production of rho mesons. If proportionally fewer rho mesons were electroproduced than photoproduced, one would expect the distribution to fall more sharply. The relative cross section for positive hadron production is observed to increase with $|q^2|$ while the negative hadrons stay fixed. This effect will be discussed in greater detail in the following section.
V. Charge Ratios

Figure 7 shows the ratio of the invariant cross sections for positive hadrons to those for negative hadrons as a function of $x$. There is a striking increase in the ratio as $x$ increases.

There are no published photoproduction data for this ratio. However, from available information we estimate the photoproduction ratio to be $1.20 \pm 0.10$ throughout this $x$ range. The SIAC-Berkeley-Tufts bubble chamber data\(^3\) give a ratio between 1.00 and 1.10 in this range. These data exclude two classes of events, one-prong events and visible strange particle production, which together compose ~ 15% of the cross section and which predominately yield positive hadrons. The ratio of $1.20 \pm 0.10$ is also consistent with low $p_T$ SIAC spectrometer data.\(^4\) Our data appear to approach the photoproduction value as $x \to 0$.

In Fig. 8 we show the charge ratio as a function of $q^2$ for two ranges of $x$. The ratio increases markedly as $|q^2|$ increases, and appears to approach the photoproduction value as $q^2 \to 0$.

It is not clear whether the effect shown in Fig. 6 is a function of $q^2$ or $\omega = 2Mv/q^2$ (or both). There is some evidence in the data that at fixed $q^2$ the charge ratio is largest at small $\omega$, but we have been unable to establish this on a statistically significant level for all of our data.

We consider these data to be significant and surprising. As $|q^2|$ increases or as $\omega$ decreases, by some mechanism, part of the charge of the proton is being projected forward. For example, consider the most naive proton model of electron-proton inelastic scattering in which the inelastic process is assumed to entirely consist of electromagnetic...
elastic scattering on the two quarks, each having charge $+2/3$ e, and the
one antiquark, which has charge $-1/3$ e. The amount of quark charge
which is scattered will have the positive to negative charge ratio of
$8/1$. Suppose we assume that the forward hadrons are produced by these
quarks (or antiquarks) picking up an antiquark (or quark) from a "sea" of
quark-antiquark pairs, and then leaving the proton as mesons. These
mesons will have a positive to negative charge ratio, $R_p$, of $8/1$ to $16/1$
depending on the proportion of strange quark-antiquark pairs in the "sea."
Of course these extreme values of $R_p$ can be greatly reduced if one also
allows elastic scattering on the quarks or antiquarks in the "sea."

VI. Partial Multiplicities

We define the partial multiplicity to be the intégral

$$\frac{1}{\sigma(q^2, \nu)} \int_0^1 dx \int_0^\infty dp_1 \frac{d^2 g(q^2, \nu)}{dp_1^2} \ dx$$

Table I gives partial multiplicities for $x_0 = 0.2$ as a function of $q^2$
summed over $\nu$ and as a function of $\nu$ summed over $q^2$.

VII. Other Aspects of the Experiment

1. The $q^2$ range of the experiment was $-1.0$ to $-4.5$ (GeV/c)$^2$. But
the analysis of the data outside of the $0.5 > q^2 > 2.5$ (GeV/c)$^2$ range
awaits the completion of more detailed geometric acceptance calculations
required in the extremes of the $q^2$ range.

2. Measurement of events in which two hadrons were detected allows
the study of the reaction $e + p \rightarrow e + p + \rho^0, \rho^0 \rightarrow \pi^+ + \pi^-$. While the
analysis of this reaction is not yet complete, we find that the $\rho^0$ mass
peak is well resolved and the background under the peak is relatively
small. These desirable properties come from the direct measurement of
the two pions; and from the high energy which separates the kinematic
region of $\rho^0$ production from the kinematic region of $n^*$ production.

3. From the deuterium data we will obtain inclusive data and in
particular $R_n$, the charge ratio from the neutron. We will also analyze
$\rho^0$ production on deuterium and neutron targets.

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1. F. Martin, S. J. St. Lorant and W. T. Toner, Nucl. Instr. and
Methods (to be published).
3. K. C. Moffeit (private communication).
4. A. M. Boyarski et al., Contribution to the International Symposium
On Electron and Photon Interactions at High Energy, Ithaca, New
York (1971) and private communication.
### TABLE I

Partial multiplicities for $x_0 = 0.2$: \( \frac{1}{\sigma(q^2, \nu)} \int \frac{dx}{x_0} \int_0^{1/d_p^2} \frac{d^2\sigma(q^2, \nu)}{dp^2} dx \)

#### A) $4 < \nu < 14 \text{ GeV}$

<table>
<thead>
<tr>
<th>$q^2 [(\text{GeV/c})^2]$</th>
<th>$h^-$</th>
<th>$h^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5 to -1.0</td>
<td>0.197±0.007</td>
<td>0.312±0.007</td>
</tr>
<tr>
<td>-1.0 to -1.5</td>
<td>0.211±0.011</td>
<td>0.361±0.015</td>
</tr>
<tr>
<td>-1.5 to -2.0</td>
<td>0.194±0.016</td>
<td>0.384±0.022</td>
</tr>
<tr>
<td>-2.0 to -2.5</td>
<td>0.221±0.027</td>
<td>0.436±0.035</td>
</tr>
</tbody>
</table>

#### B) $-0.5 > q^2 > -2.5 \ (\text{GeV/c})^2$

<table>
<thead>
<tr>
<th>$\nu$ (GeV)</th>
<th>$h^-$</th>
<th>$h^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0 to 6.5</td>
<td>0.191±0.011</td>
<td>0.398±0.015</td>
</tr>
<tr>
<td>6.5 to 9.0</td>
<td>0.203±0.010</td>
<td>0.353±0.013</td>
</tr>
<tr>
<td>9.0 to 11.5</td>
<td>0.218±0.011</td>
<td>0.334±0.012</td>
</tr>
<tr>
<td>11.5 to 14.0</td>
<td>0.193±0.011</td>
<td>0.301±0.013</td>
</tr>
</tbody>
</table>
Fig. 1. Schematic view of the apparatus.
Fig. 2. Dependence of the invariant cross section on $\phi$.

Fig. 3. Dependence of the invariant cross section for negative hadrons on $p_T^2$ for various $q^2$ ranges and $4 < \nu < 14$ GeV. The lines are best fits of the form $A e^{-b p_T^2}$. 
Fig 4. The slope parameters (b) taken from fits to the data in Figs. 3 and similar data for positive hadrons. A point from \( \pi^+ \) photoproduction (Ref. 2) is included.
Fig 5  Dependence of the invariant cross section on $x$ for $-0.5 > q^2 > -2.5 \text{(GeV/c)}^2$, $4 < \nu < 14 \text{GeV}$.

A line representing the data in $\pi^-$ photoproduction (Ref. 2) is included.
Fig 6  Dependence on $x$ for negative hadrons in four $q^2$ regions
The same line is drawn on all curves to aid in visual comparison

Fig 7  The ratio of the invariant cross section for positive to negative hadrons at each $x$ for $-0.5 > q^2 > -2.5$ (GeV/c)$^2$ and $4 < \nu < 14$ GeV
Fig. 8. The ratios of the invariant cross section for positive to negative hadrons at each $q^2$ for two different $x$ ranges. A point at $q^2 = 0$ from photoproduction (Refs. 3 and 4) is included.
An experiment performed at the Stanford Linear Accelerator Center in which the cross sections for inelastic electron-proton and electron-deuteron scattering were measured at angles of 18°, 26°, and 34° has yielded preliminary results for the proton and deuteron structure functions. From these results, information about the neutron has been extracted. Earlier results from this experiment and prior small angles measurements (1) were reported at the 1971 Cornell Conference (2). This report therefore will be brief and will concern itself with some additional features of the large angle data revealed by further analysis.

The proton and deuteron structure functions $W_1$ and $W_2$, obtained from this experiment are defined in the usual manner (2):

$$\frac{d^2 \sigma}{d \Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[ W_2(\nu, q^2) + 2W_1(\nu, q^2) \tan^2 \frac{\theta}{2} \right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{e^2}{2E}\right)^2 \cdot \frac{\cos^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}}$$

$$\nu = E - E'$$

$$q^2 = 4EE' \sin \frac{\theta}{2}$$

where $E$ and $E'$ are the incident and scattered electron energies and $\theta$ is the scattering angle, all measured in the laboratory system. $W_1$ and $W_2$ are related to $\sigma_s$ and $\sigma_t$, the total photoabsorption cross sections for longitudinal and transverse virtual photons, by the expression

$$R \equiv \frac{\sigma_s}{\sigma_t} = \left(\frac{1}{q^2} + \frac{\nu^2}{q^2}\right) \cdot \left(\frac{W_2}{W_1}\right) - 1$$

-98-
R for the proton had previously been determined (4) over a deep inelastic kinematic region extending approximately from $q^2 = 1.5$ to 11.0 (GeV/c)$^2$ and from $W = 2.0$ to about 4.0 GeV. The quantity $W$ is the mass of the final state hadronic system and is given by

$$W = m^2 + 2mv - q^2$$

where $m$ is the mass of the target particle. Data from earlier inelastic $e$-$p$ scattering experiments performed at 6° and 10° (5) and at 18°, 26°, and 34° (4) were employed for this determination. The measured values were in the range 0.0 to 0.5 and had no apparent kinematic dependence. Assuming $R$ to be a constant in this kinematic region, $R_p$ was found to be

$$R_p = 0.18 \pm 0.10$$

$R_p$ now has been determined using the present large angle data over a kinematic range from $q^2 = 2.5$ to 13.0 (GeV/c)$^2$ and from $W = 2.0$ to approximately 4.0 GeV. The results are in good agreement with the earlier ones. $R$ for the deuteron has also been determined over this same kinematic region, and we can now say that $R_p$ and $R_d$ are consistent with being equal to within the experimental errors.

Information about the neutron is extracted from our measurements on deuterium using the impulse approximation. The deuteron structure functions are expressed in terms of free neutron and proton structure functions averaged over the Fermi motion of the nucleons within the deuteron. The method used is that of West (7), with small modifications employed to study possible
off-mass-shell corrections (8).

With the exception of Glauber corrections which are known to be quite small, other corrections to the above picture cannot be estimated accurately and are assumed to be small. The results are quite insensitive to the choice of wave function used to determine the momentum distribution of the bound nucleons, as long as it is in satisfactory agreement with other experimentally determined properties of the deuteron and the n-p interaction. Our procedure was to apply to the experimentally determined proton structure functions the West method of averaging over the Fermi momentum thereby yielding a "smeared" proton cross section. Subtracting the smeared proton cross section from the deuteron cross section yielded by definition, the smeared neutron cross section:

\[ \frac{d^2 \sigma_n}{d \Omega dE} \mid_{\text{sm}} = \sigma_{n_{\text{sm}}} = \sigma_d - \sigma_{p_{\text{sm}}} \]

The quantity \( \frac{\sigma_{n_{\text{sm}}}}{\sigma_{p_{\text{sm}}}} = \frac{\sigma_d}{\sigma_{p_{\text{sm}}}} - 1 \) is plotted against \( x = 1/\omega \) in Figure 1, where \( \omega \) is the Bjorken scaling variable, \( \omega = 2mv/q^2 \). The points were obtained by calculating this ratio for all experimentally determined points outside the resonance region and then forming weighted averages of these points over small intervals in \( x \). The approximate equality of \( R_p \) and \( R_d \) implies that \( R_p \sim R_n \) as well, which permits us to equate \( \frac{\sigma_{n_{\text{sm}}}}{\sigma_{p_{\text{sm}}}} \) with \( \frac{W_{2n_{\text{sm}}}}{W_{2p_{\text{sm}}}} \)

It is meaningful to plot this ratio against a scaling variable because the deuteron is observed to scale approximately, as well as the proton (2).
For values of $q^2$, $v$ in the scaling region, the smearing corrections are also found to scale, as they must in order to be consistent with the above results. The Bjorken variable was chosen for the figures shown purely as a matter of convenience. Our preliminary results are insensitive to the choice of scaling variable.

During the analysis performed in preparation for the Cornell Conference, a preliminary version of the smearing procedure was employed. An improved version has since been developed and applied to the data. The corrected results from the large angle measurements shown in Figure 1 are largely unchanged from the previous results, save for the last two points at the largest values of $x$ which are raised by approximately one and one half standard deviations on the average.

The effect of smearing for the points shown can be significant. Figure 2 illustrates the error that would be made, were no smearing correction applied. The dashed line represents a curve drawn through the data shown in Figure 1, while the solid line represents the quantity $\sigma_d/\sigma_p - 1$ (i.e. smearing is ignored). The difference between the two curves is about 40 per cent at $x = 0.79$.

We are developing a nearly model independent program for removing the effects of smearing on the neutron structure functions. The neutron to proton ratio with the effects of smearing removed can then be obtained. Preliminary results show that this ratio is essentially unchanged from that shown in Figure 1 except at large values of $x$. At $x = 0.79$, the unsmeared ratio is lower than the smeared ratio by about 0.02.
Figure 3 shows $v(W_{2p_{sm}} - W_{2n_{sm}})$ versus $x$. Again the points shown represent averages of data points over small intervals of $x$. Non-diffractive contributions to the cross sections are evident.

References


   G. B. West, this conference (paper #517).

Fig. 1. Rate of smeared cross sections $\frac{\sigma_{n_{sm}}}{\sigma_{p_{sm}}}$ vs scaling variable $x$. 

$W > 2.0$
Fig. 2. Effect of smearing correction on data of Fig. 1.
Fig. 3. Plot of $\nu(W_{2P_{sm}} - W_{2n_{sm}})$ vs scaling variable $x$
I have been asked to summarize an experiment performed by J. F. Davis, S. Hayes, R. Imlay, P. C. Stein and P. J. Wanderer at the Cornell 12 GeV Electron Synchrotron. The authors were interested in studying the photoproduction of \( \mu \) pairs via the compton diagram, Figure 1(a), in order to test some two photon predictions of a parton model of deep inelastic electron scattering. Bjorken and Paschos have shown that the cross section for \( \mu \) pairs from this diagram can be written in the form:

\[
\sigma = g \frac{W_2(q^2, \nu)}{\Sigma Q^2}
\]

In this expression, \( q^2 = (k-p_+ - p_-)^2 \), \( \nu = k-E_+ - E_- \), \( g \) is a known function of the kinematic variables, \( W_2(q^2, \nu) \) is the ordinary
The structure function of deep inelastic electron scattering, and $<Q_{j}^{n}>$ is the average over the parton distribution in the nucleon of the n'th power of the parton charge. In comparing the experimental results with the model, the parton charges $Q_{j}$ were taken to be 1.

The Bethe-Heitler diagram, Figure 1(b) is a background in the experiment. However, the single-arm apparatus used by the authors (Figure 2) tended to suppress the Bethe-Heitler contribution, and the authors feel that they can accurately estimate its magnitude since this diagram has been extensively studied. In addition, the single-arm geometry biases the $\mu$ pairs toward low effective masses, the region of interest for the theoretical model.

The primary problems in this experiment are to establish (1) that two particles travelled through the range telescope, $R$, and (2) that these particles were muons originating in the target. The authors have demanded two particles in each of the hodoscopes $H_F$ and $H_R$, and have augmented this requirement with a careful study of the pulse heights in the $R$ counters. To prove that their particles are muons originating in the target, the authors have changed the length of the flight path to the absorber $A$, and the composition of the absorber.

Data were taken with: $10^\circ < \Theta_\mu < 15^\circ$, $2.58 \text{ GeV} < E_\mu < 3.49 \text{ GeV}$, and $210 \text{ MeV} < M_{\mu\mu} < 550 \text{ MeV}$. 
They ran with 3 different values of the maximum photon energy and obtained the following yields:

<table>
<thead>
<tr>
<th>$k_{\text{max}}$(GeV)</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>70</td>
</tr>
<tr>
<td>10.0</td>
<td>257</td>
</tr>
<tr>
<td>11.7</td>
<td>275</td>
</tr>
</tbody>
</table>

The cross sections obtained are presented in Table I, along with the theoretical predictions and calculated yields from the Bethe-Heitler diagram.

The authors have made an extensive study of possible backgrounds in order to try to account for the yields that are an order of magnitude larger than the theoretical prediction. From their measurements of rates with different absorber geometry and composition, they conclude that at most 10% of their yield comes from decays of $\pi$ mesons outside of the target. In addition, they have made generous estimates of the contributions to be expected from the following processes:

- $\gamma N \rightarrow \rho X; \rho \rightarrow \mu^+\mu^-$
- $\gamma N \rightarrow \rho X; \rho \rightarrow \pi^+\pi^- + \mu^+\mu^-$
- $\gamma N \rightarrow \eta X; \eta \rightarrow \gamma \mu^+\mu^-$
- $\gamma N \rightarrow \gamma X; \gamma \rightarrow K^+K^- + \mu^+\mu^-$
- $\gamma N \rightarrow K^0\bar{X}; K_i^0 \rightarrow \pi^+\pi^- + \mu^+\mu^-$
- $\gamma N \rightarrow K_L^0\bar{X}; K_i^0 \rightarrow \pi^+\pi^- + \mu^+\mu^-$

$\mu$ Pairs produced in the Absorber A

From their calculations, the largest single channel contributes less than 2% and the total contribution of these channels is less that 6%.

The authors have presented a strong case for their
conclusion that the observed rate is due to $\mu$ pairs produced in the target. Furthermore the authors cannot account for the yield as due to the decay of known short-lived particles into two muons. We are left with the difficult job of trying to explain these results.

REFERENCES

1. J. F. Davis, S. Hayes, R. Imlay, P. C. Stein, and P. J. Wanderer. Paper #773 submitted to this Conference and to be published

FIGURES

1. (a) Compton Diagram for $\mu$ pair photoproduction
   (b) Bethe-Heitler Diagram for $\mu$ pair photoproduction
2. Schematic diagram of the detector. Each gap in the iron contains a single R (range) or V (veto) counter, as indicated. Lead shielding between the detector and the photon beam is not shown. The beam enters from the left.
TABLE I

The calculated and measured cross sections for muon-pair photoproduction, integrated over the experimental apparatus. Units are picobarns per photon per beryllium nucleus. The kinematic parameters are defined in the text. The errors shown are statistical; normalization uncertainty is ~ 10%.

<table>
<thead>
<tr>
<th>Photon Energy Bin</th>
<th>Kinematic Region</th>
<th>Resonance BH</th>
<th>Continuum BH</th>
<th>Inelastic Compton</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0 GeV - 8.5 GeV</td>
<td>$1.7(\text{GeV}/c)^2 &lt;</td>
<td>q^2</td>
<td>&lt; 4.1(\text{GeV}/c)^2$</td>
<td>0.0006</td>
<td>0.0128</td>
</tr>
<tr>
<td></td>
<td>$1.5 \text{ GeV} &lt; \nu &lt; 4.8 \text{ GeV}$</td>
<td>0.0001</td>
<td>0.0085</td>
<td>0.0403</td>
<td>0.75±0.11</td>
</tr>
<tr>
<td></td>
<td>$1.1 \text{ GeV} &lt; W &lt; 2.8 \text{ GeV}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2.0(\text{GeV}/c)^2 &lt;</td>
<td>q^2</td>
<td>&lt; 5.0(\text{GeV}/c)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.7 GeV - 10.0 GeV</td>
<td>$3.0 \text{ GeV} &lt; \nu &lt; 6.5 \text{ GeV}$</td>
<td>0.0001</td>
<td>0.0085</td>
<td>0.0403</td>
<td>0.75±0.11</td>
</tr>
<tr>
<td></td>
<td>$1.5 \text{ GeV} &lt; W &lt; 3.3 \text{ GeV}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1
THE MEASUREMENTS OF THE PION AND NUCLEON FORM FACTORS IN A TIMELIKE REGION OF 1.72 fm⁻², 2.23 fm⁻², AND 2.88 fm⁻² MOMENTUM TRANSFERS (#863)

Presented by S. F. Berezhnev
Joint Institute for Nuclear Research
Dubna, USSR

In the study 63±18 events of the reaction of inverse pion electroproduction (IPE) \( \pi^- p \to e^+ e^- n \) were observed at the total energy in the \( \pi^- p \) center-of-mass system \( W = 1295 \) MeV. The analysis of the data obtained in the experiment has been performed with the help of the dispersion model which contains, as free parameters, the form factors of the pion \( F_\pi \), the isovector Dirac form factor of the nucleon \( F^V_1 \) and the form factor \( G^M_i \) entering the amplitude of magnetic dipole transition \( M^+ \). It was shown that the Born terms give the main contribution to the differential cross section and the contribution of the transition \( M^+ \) is practically completely canceled by its interference with the Born terms. Therefore, the cross section of the reaction is found to be dependent only on the form factors \( F_\pi \) and \( F^V_1 \). Assuming \( F_\pi = F^V_1 \) it was obtained that \( F_\pi = F^V_1 = 1.07 \pm 0.14 \) at the average value of the square of four-momentum transfer \( k^2 = 2 \) fm⁻². The accuracy of the theoretical description of the IPE increases with the approach of the effective mass of photon \( k^2 \) to its maximum possible value at the given \( W \). In this quasi-threshold case the magnetic transitions are found to be suppressed, in particular, this is true for the amplitude \( M^+ \) responsible for the basic corrections to the Born multipoles, which is of interest to us.

This study is an analysis of new experimental data obtained at the same value of \( W \) with the help of the recently used model. The events are divided in three groups by \( k^2 \). In each interval of \( k^2 \) the event distributions were plotted by cosine of the angle between the momentum of pion and photon in the \( \pi^- p \) center-of-mass system \( \cos \theta^V \). The differential cross-section values were compared with the theoretical ones which were simulated by a Monte-Carlo program for the set of form factors within the limits, \( 0 \leq F_\pi, F^V_1 \leq 2 \). When assuming point-like particles \( F_\pi = F^V_1 = 1 \) good agreement is obtained only for the events of the first group (\( \chi^2 = 5.6 \) at 5 degrees of freedom). With increasing of \( k^2 \) the fit is worse \( \chi^2 = 8.9 \) for the second group of events, and \( \chi^2 = 18.3 \) for the third one at the same number of degrees of freedom.

It was assumed further that the empirical relation \( F^V_1(k^2) = F^V_1(k_0^2) \) observed at the investigation of pion electroproduction holds for a timelike transfer as well. Besides, both form factors were considered to be real. With these assumptions the experimental data are well described by the model \( \chi^2 / \chi^2 = 1 \) with the following values of parameters: \( F_\pi = F^V_1 = 1.15 \pm 0.09 \) at \( k^2 = 0.067 \) (GeV/c)², \( F_\pi = F^V_1 = 1.14 \pm 0.06 \) at \( k^2 = 0.087 \) (GeV/c)², \( F_\pi = F^V_1 = 1.30 \pm 0.07 \) at \( k^2 = 0.112 \) (GeV/c)². The obtained values of form factors are shown in Fig. 1 together with results of the pion form factor measurement in a spacelike region of momentum transfer.

The errors in the form factor values do not contain the uncertainties in the total cross section which can give a systematical displacement to all the form factor values simultaneously.
within ±7%. From these data the electromagnetic radius was determined, the error of ±7% being assigned to the value $F_{1}^\pi(0) = 1$. The four obtained values of the form factor are well approximated by the function $F_{1}^\pi(k^2) = 1 + \frac{1}{6} r_{0}^2 k^2$ with $r_{0} = r_{F_{1}^\pi} = (0.73±0.13)\text{fm}$. The value of $r_{F_{1}^\pi}$ measured in ep-scattering experiments is $r_{F_{1}^\pi} = 0.72\text{ fm}$. The experimental data allow one to determine the form factors $F_{\pi}$ and $F_{4}$ without assumptions about any connection between them. However, at present there are only estimations of the form factors at the 68% confidence level.

<table>
<thead>
<tr>
<th>$k^2$ (fm$^{-2}$)</th>
<th>$F_{1}^\pi$</th>
<th>$F_{4}$</th>
</tr>
</thead>
<tbody>
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References

6. S. F. Berezhnev et al., #863.
Currents III: Scaling Phenomena
(And Other Theoretical Matters)

Organizer: D. J. Gross
Scientific Secretaries: E. A. Paschos
A. I. Sanda
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I wish to review recent work on the short-distance behavior of quantum electrodynamics and in particular to discuss the question of whether the physical fine structure constant $\alpha$ can be calculated within the framework of quantum electrodynamics. The basic requirement which we impose, in an attempt to get an eigenvalue condition for $\alpha$, is that the renormalization constants of quantum electrodynamics should all be finite. These constants are

- $m_0$ - electron bare mass
- $Z_2$ - electron wave function renormalization
- $Z_3$ - photon wave function renormalization

we require that as the cutoff $\Lambda$ used to calculate them becomes infinite, $m_0$, $Z_2$, and $Z_3$ should have finite limits. The condition on $Z_3$ can be stated in the alternative form that the renormalized photon propagator $d_{\alpha}(-q^2/m^2,\alpha)$ (which is normalized to unity at $q^2 = 0$) should approach the finite constant $Z_3^{-1} - \alpha_0/\alpha$ as $-q^2/m^2 \to \infty$.

A systematic, nonperturbative attack on the problem of whether $Z_3$ can be finite was made by Gell-Mann and Low in their classic 1954 paper on the renormalization group. They showed that there is indeed an eigenvalue condition imposed by requiring that $Z_3$ be finite, but that the condition takes the form

$$\psi(\alpha_0) = 0$$

and determines the asymptotic coupling $\alpha_0$ rather than the physical coupling $\alpha$. Their analysis leaves $\alpha$ a free parameter of the theory, restricted only by the condition $\alpha < \alpha_0$ coming from spectral-function positivity. This essential conclusion was retained in the subsequent work of Johnson, Baker, and Willey (JEW), who made two important advances over the work of Gell-Mann and Low. First, they showed that if $Z_3$ is finite then the renormalization constants $Z_2$ and $m_0$ can also be finite. The electron wave function renormalization $Z_2$, which is gauge-dependent, can be made finite by an appropriate choice of gauge (the Landau gauge) while the electron bare mass $m_0$ takes the simple scaling form

$$m_0 = \text{const} \times m \left( \frac{\Lambda^2}{m^2} \right)^{-\epsilon} \left( \frac{\alpha_0}{2\pi} + \frac{3}{8\pi^2} \right)^2$$
and therefore vanishes in the limit of infinite $A$ provided that $\epsilon > 0$ (A vanishing bare mass means that the physical mass of the electron arises entirely from its self-interaction) Second, Baker and Johnson\(^3\) showed that the Gell-Mann Low eigenvalue condition $\phi(\sigma_0) = 0$ implies the much simpler condition $F^{[1]}(\sigma_0) = 0$, where $F^{[1]}(y)$ is a function of coupling $y$ defined as follows

Let us define the photon renormalized proper self-energy $\pi_c(-q^2/m^2, y)$ by

$$d_c(-q^2/m^2, y) = [1 + yw_c(-q^2/m^2, y)]^{-1},$$

and let $\pi_c^{[1]}(-q^2/m^2, y)$ denote its single-fermion-loop part.

In the limit of asymptotic $-q^2/m^2$ it can be shown that $\pi_c^{[1]}(-q^2/m^2, y)$ grows at worst as a single power of $\ln(-q^2/m^2)$ [higher powers of $\ln(-q^2/m^2)$ can only come from multiple-fermion-loop diagrams where vacuum polarization insertions appear inside fermion loops],

$$\pi_c^{[1]}(-q^2/m^2, y) - C^{[1]}(y) + F^{[1]}(y) \ln(-q^2/m^2) + \text{vanishing terms}$$

The coefficient of the logarithm in Eq (6) is the function which gives the simplified eigenvalue condition, unlike the Gell-Mann Low function $\phi$, which involves all vacuum polarization diagrams the function $F^{[1]}$ involves only a very special subclass of these diagrams.

In addition to showing that $\phi(\sigma_0) = 0$ implies $F^{[1]}(\sigma_0) = 0$, the Baker-Johnson analysis also shows that $\phi(\sigma_0) = 0$ implies $T^{[1]}_{2n}(m=0, y=\sigma_0) = 0$ for $n \geq 2$, where $T^{[1]}_{2n}(m, y)$ is the sum of single-fermion-loop 2n-point functions.

Let us now use\(^4\) this powerful result in the following way. We take a single-fermion-loop 2n-point function and contract $n-1$ pairs of external photon lines with $n-1$ photon propagators, leaving only
two free external photons. The resulting object has the same Lorentz structure as the single-fermion-loop proper self energy \( \pi_c^{[1]} \), but some simple combinatorics shows that it is not \( \pi_c^{[1]} \) itself, but rather the coupling-constant derivative \( (d/dy)^{n-1} \pi_c^{[1]} \). That is, we have

\[
(d/dy)^{n-1} \pi_c^{[1]} \propto C
\]  

But since \( T_{2n}^{[1]}(m=0,y=\alpha_0) \) vanishes, we learn from Eqs. (6) and (8)

\[
\frac{d^{n-1}}{dy^{n-1}} F_2^{[1]}(y) \bigg|_{y=\alpha_0} = 0 \quad n \geq 2,
\]

that is, \( F_2^{[1]} \) vanishes with an infinite order zero at \( y = \alpha_0 \). A similar argument shows that \( T_{2n}^{[1]}(m=0,y) \) also vanishes with an infinite order zero at \( y = \alpha_0 \), and this in turn implies that the Gell-Mann Low function has a zero of infinite order. Hence, if the Gell-Mann Low function \( \psi \) has a zero for nonvanishing coupling, it must be a zero of infinite order—we see that electrodynamics must satisfy an extraordinarily strong condition in order for \( Z_\lambda \) to be finite.

Whether \( F_2^{[1]}(y) \) and \( T_{2n}^{[1]}(m=0,y) \) have the required infinite order zero is an open calculational question. There are two possibilities:

(A) \( F_2^{[1]}(y) \) and \( T_{2n}^{[1]}(m=0,y) \) do not have the required infinite order zero. Then the renormalization constants of electrodynamics cannot all be finite. (The only way to avoid this conclusion would be if a key technical assumption needed for the renormalization group analysis breaks down. The assumption states that terms which vanish asymptotically in each order of perturbation theory do not sum to give an asymptotically dominant result.)

(B) \( F_2^{[1]}(y) \) and \( T_{2n}^{[1]}(m=0,y) \) have an infinite order zero at \( y - y_0 > 0 \). As we have seen, this allows a class of solutions with finite \( Z_\lambda \), in which \( \alpha_0 \) is fixed to be \( y_0 \) and \( \alpha < y_0 \) is undetermined. We will now show that the presence of an infinite order zero allows one additional solution, in which the physical fine structure constant \( \alpha \) is fixed to be \( y_0 \).

The possibility of an additional solution arises because when an infinite order zero (an essential singularity) is present, different orders of summing perturbation theory lead to inequivalent theories. One natural way of summing perturbation theory is to sum "vacuum-polarization-insertion-wise." One first sums all internal photon self-energy parts, and then inserts the resulting full photon propagators in the vacuum polarization skeleton graphs. This order of summation is the one used by JBW, and leads to their form of the eigenvalue condition \( F_2^{[1]}(\alpha_0^*) = 0 \). To see this we apply "vacuum-polarization-insertion-wise" summation to the single-fermion loop skeleton graphs for the photon proper self-energy giving
where each shaded blob denotes a full renormalized propagator insertion \( \sigma d_c / q^2 \). Let us now assume that on letting \( -q^2 / m^2 \to \infty \), we can take the limit inside the infinite sum over insertions represented by Eq. (10), and therefore we replace each blob by its asymptotic limit \( \sigma d_c (\infty, a) / q^2 = \alpha \gamma / q^2 = \sigma_0 / q^2 \). This gives

\[
\pi_c^{[1]} [-q^2 / m^2, \sigma d_c] \sim \pi_c^{[1]} [-q^2 / m^2, \sigma_0] \sim G^{[1]}(\alpha_0) + F^{[1]}(\alpha_0) \ln(-q^2 / m^2) + \text{vanishing terms},
\]

so asymptotic finiteness requires the condition \( F^{[1]}(\alpha_0) = 0 \), i.e., \( \sigma_0 = \gamma_0 \). Similar arguments apply to the multiloop skeleton diagrams, when summed "vacuum-polarization-insertion-wise," and again give the condition \( \sigma_0 = \gamma_0 \).

There is, however, another natural summation order, which is to proceed "loopwise." One first sums all single-fermion-loop vacuum polarization graphs, then one sums all two-fermion-loop vacuum polarization graphs, and so forth. The sum of all single-fermion-loop graphs is just

\[
\pi_c^{[1]} [-q^2 / m^2, \sigma] \sim G^{[1]}(\alpha) + F^{[1]}(\alpha) \ln(-q^2 / m^2) + \text{vanishing terms},
\]

so asymptotic finiteness now requires \( F^{[1]}(\alpha) = 0 \), i.e., \( \gamma = \gamma_0 \). It is easily seen that the same condition \( \alpha = \gamma_0 \) guarantees asymptotic finiteness of the multiloop vacuum polarization graphs.

So we have found an additional, discrete solution in which \( \alpha \) is fixed to have the value \( \gamma_0 \). Since \( \sigma_0 > \gamma_0 \), for this solution \( \gamma_0 \) will be outside the region of analyticity of \( F^{[1]} \) and so the interchange of limit with sum used in the "vacuum-polarization-insertion-wise" summation procedure is invalid. Hence the condition \( F^{[1]}(\sigma_0) = 0 \) derived by JBW does not apply to the discrete solution in which \( \alpha \) is fixed. (If it did, one would have the contradictory equations \( \alpha = \sigma_0 = \gamma_0 \), \( \alpha < \sigma_0 \).

We conclude, then, that requiring the renormalization constants of electrodynamics to be finite, combined with "loopwise" summation, leads to an eigenvalue condition for \( \alpha \). We conjecture that this is the mechanism which fixes the value of the fine structure constant \( \alpha \). The eigenvalue condition has the appealing property that it is independent of the number of elementary charged fermion species which are present. To see this, we note that when \( j \) species are present, the coefficient of the logarithmic divergence in the single-fermion-loop photon proper self-energy is

\[
\sum_{f=1}^{j} F^{[1]}(\alpha_f),
\]

which vanishes if all \( \alpha_f = \gamma_0 \). The same condition guarantees vanishing of the multiloop vacuum polarization diagrams. So the value of \( \alpha \) which is determined is the same as in the one species case, and the \( j \) species are all required to have the same basic electromagnetic coupling \( \alpha / \sqrt{\gamma_0} \). Hence charge quantization appears in a natural way.
Let us now give a possible argument for the neglect of the strong interactions. Suppose that elementary charged fermions are present which have strong interactions mediated by neutral boson exchange (the gluon model). Although the bosons do not themselves contribute vacuum polarization loops, they could modify the fermion vacuum polarization loops when they appear as internal radiative corrections, e.g.

\[
\text{Gluon}
\]

(14)

However, let us now invoke the experimental observation of scaling in deep inelastic electron scattering, one explanation for which is that the exchanges which mediate the strong interactions are actually much more strongly damped at high four-momentum transfer than is the free boson propagator \((q^2 + \mu^2)^{-1}\). If this explanation proves correct, then vacuum polarization diagrams with gluon radiative corrections will by themselves be asymptotically finite, and therefore will not contribute to \(\Pi[1]\). This means, in turn, that the presence of strong interactions will not alter the eigenvalue condition for \(\sigma\).

What can be said about the prospects of calculating \(\Pi[1](y)\)? All that is known at present is the expansion through 6th order in perturbation theory,

\[
-\Pi[1](y) = \frac{2}{3} \left( \frac{y}{2\pi} \right)^2 + \frac{1}{3} \left( \frac{y}{2\pi} \right)^3 + \ldots
\]

(15)

Even though the perturbation theory calculations leading to Eq (15) are quite horrendous, the resulting coefficients are remarkably simple. A possible clue to the origin of this simplicity may be the fact that \(\Pi[1]\) is a property of electrodynamics in the zero fermion mass limit, in which limit the invariance group is the full conformal group, a much larger group than the usual inhomogeneous Lorentz group. Perhaps this fact can be used to develop means for calculating \(\Pi[1]\), or at least for approximating it well enough to determine the location of its singularities.

For example, we have shown that conformal invariance allows one to write the Feynman rules for massless electrodynamics (after rotation to a Euclidean metric) in terms of equivalent Feynman rules on the surface of the 5-dimensional unit hypersphere. In these rules, propagation of a virtual photon is described by

\[
\frac{1}{4\pi^2} \frac{\delta^{ab}}{(\eta_1 - \eta_2)^2}
\]

(16)

where the coordinates \(\eta_1\) and \(\eta_2\) are 5-dimensional unit vectors and where the indices \(a, b\) range from 1 to 5. Letting \(L_4^2 = (\eta_{1a} \partial / \partial \eta_{1b} - \eta_{1b} \partial / \partial \eta_{1a})^2\) denote the hyperspherical angular momentum-squared and letting \(\delta_S(\eta_1 - \eta_2)\) denote the hyperspherical surface delta-function, the wave equation for this propagator takes the form

\[
(L_4^2 - 4) \frac{1}{(\eta_1 - \eta_2)^2} = -8\pi^2 \delta_S(\eta_1 - \eta_2)
\]

(17)

What is significant about Eq (17) is that the wave operator \(L_4^2 - 4\) has the discrete spectrum \(-2(n+1)(n+2), n = 0, 1, 2, \ldots\), suggesting that the semiclassical region of large quantum numbers.
n >> 1, is a natural domain for making approximations. Conceivably, the development of techniques for making such semiclassical approximations on the hypersphere would permit the study of the singularity structure of $F^{(1)}(a)$.

An alternative idea for calculating $F^{(1)}(a)$ has been discussed by Baker and Johnson in a contribution submitted to this conference. They point out that in the absorptive part of the vacuum polarization in zero-fermion-mass electrodynamics, an infinite number of electron-positron pair production thresholds are coincident at $q^2 = 0$. Conceivably, an approximation which takes into account this piling up of singularities, but which ignores details of the pair-production matrix elements, would still permit one to study the analyticity properties of $F^{(1)}$. Specifically, Baker and Johnson write $F^{(1)}$ as a sum over $n$-pair contributions, with the leading coupling-constant dependence factored out,

$$F^{(1)}(y) = \sum_{n=1}^{\infty} y^{2n-2} b_n(y)$$

They then drop the radiative corrections to the pair-production amplitudes, giving

$$F^{(1)}(y) = \sum_{n=1}^{\infty} y^{2n-2} b_n(0)$$

The hope is that one could learn the location of the singularities of $F^{(1)}$ from the large-$n$ behavior of the coefficients $b_n(0)$ appearing in Eq (19). In order to determine these coefficients, one must learn to carry out a three-step calculation consisting of (i) calculating the tree graph contribution to $\langle 0 | j_{\mu} | n \text{ pairs} \rangle$, (ii) projecting the single-fermion-loop part from the product $\langle 0 | j_{\mu} | n \text{ pairs} \rangle \times \langle n \text{ pairs} | j_{\mu} | 0 \rangle$, and (iii) integrating over $2n$-particle phase space.

In criticism of both of the above-described ideas for approximating $F^{(1)}$, we note that neither exploits gauge invariance in a profound way, and neither gives an explanation for the astonishing and puzzling simplicity of the coefficients in Eq (15). I think it is likely that a key piece to the puzzle is still missing.

Throughout our discussion of the asymptotic behavior of the photon propagator we have not yet dealt with the question of "Where does asymptopia begin?" There turn out to be two different answers to this question, depending on whether one adopts the usual asymptotically finite solution $\alpha_0 = y_0$ or the new solution $\alpha = y_0$. Let us consider first the former case, we assume, as has been traditional, that $\alpha_0 = y_0$ — unity, and study the asymptotic photon propagator

$$d_c(q^2/m^2, a) = q(a) + p(a) \ln(-q^2/m^2) + r(a)\{\ln(-q^2/m^2)\}^2 +$$

$$+ 0\left(\frac{m^2}{q^2} \times \log s\right)$$

It is readily shown that the logarithmic series in Eq (20) is asymptotic to $\alpha_0$ only for momentum transfers satisfying $|q^2| \gg m^2 e^{1.37}$, corresponding to distances $\alpha \sim |q^2|^{-1/2} \sim 10^{-40}$ cm. Unfortunately, such supershort distances are already much smaller than the characteristic gravitational length $G^{1/2} \sim 10^{-33}$ cm, making it unlikely that pure quantum electrodynamics will actually describe the domain where Eq (20) becomes asymptotic. We next consider the new solution, in which the physical coupling constant $\alpha$ is determined to be the eigenvalue $y_0$. In this case it can
be shown that the coefficients $p(\sigma)$, $r(\sigma)$ of the logarithmic terms in Eq (20) all vanish, giving the simple asymptotic formula

$$d_c \left( -\frac{q^2}{m^2}, \sigma \right) = \sigma_0 + \frac{\left( -\frac{m^2}{2} \times \log s \right)}{q} \tag{21}$$

Asymptopia is now attained when the remainder term in Eq (21) is negligible relative to $\sigma_0$. The pair production argument of Eqs (18)-(19) suggests that this might occur when the virtual energy is large enough to produce $n \gg 137$ pairs, giving the estimate $|q^2| > (137 \times 2m)^2$ for the location of the asymptotic region. Thus, in the solution in which $\sigma$ is fixed asymptopia may well occur at experimentally attainable energies, and the neglect of gravitational effects is most likely justified.

Finally, let us comment briefly on the impact of the weak interactions on our argument. What should be said here of course depends on which type of weak-interaction theory ultimately turns out to be correct. If (as in the unified gauge theories of weak and electromagnetic interactions) the weak interactions are really basically electromagnetic in strength, then the arguments which we have given will have to be modified to include weak interaction effects, whether this would still lead to an eigenvalue condition for the coupling constants remains an open (and so far uninvestigated) question. If the weak interactions turn out to be really weak in strength relative to electromagnetism, then the arguments which we have given might survive without fundamental modification.

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1. M. Gell-Mann and F. E. Low, Phys Rev 95, 1300 (1954)
5. See e.g., D. J. Gross and S. B. Treiman, Phys Rev D4, 1059 (1971)
8. For a review of conformal invariance in field theory, see e.g., G. Mack and A. Salam, Ann Phys (N.Y.) 53, 174 (1969)
9. S. L. Adler, Massless, Euclidean Quantum Electrodynamics on the 5-Dimensional Unit Hyperbolic Plane, Phys Rev, to be published
The simple scaling behavior observed for the structure functions $W_1$ and $\nu W_2$ in the SLAC deep inelastic electron-scattering experiments has focused considerable interest on the large $q^2$ and $\nu = -p\cdot q/m$ limit of the matrix element

$$
\frac{1}{4\pi m} \sum_{s=\pm \frac{1}{2}} \int e^{-i|q|^2} d^4 x <p, s|J^\mu(x)J^\nu(0)|p, s> = (\delta_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) W_1(q^2, \omega) + \frac{1}{m^2} \left( p^\mu - q^\mu \frac{p\cdot q}{q^2} \right) \left( p^\nu - q^\nu \frac{p\cdot q}{q^2} \right) W_2(q^2, \omega),
$$

where $|p, s>$ is a single nucleon state with four-momentum $p$ and $z$-component of spin $s$, $J^\mu(x)$ is the usual electromagnetic current. In this talk, I would like to discuss the high $q^2$ and $\nu$ behavior of this matrix element when computed in renormalized perturbation theory.

As is well-known, the high $q^2$, fixed $\omega = -2p\cdot q/q^2$ limit of $\nu W_2(q^2, \omega)$ when evaluated in low-order perturbation theory is not independent of $q^2$ as is found experimentally but instead grows logarithmically with $q^2$. Consider, for example, a field theory of charged spin $1/2$ particles $\psi$ and neutral pseudoscalar particles $\phi$ described by the interaction Lagrangian

$$
L_I = ig_5 \bar{\psi} \gamma_5 \psi.
$$

For large $q^2$ the leading term in $\nu W_2$, computed to lowest order in $g$, comes from the square of the amplitude represented in Fig. 1 and yields

$$
\nu W_2(q^2, \omega) \sim \frac{G^2}{4\pi} \frac{\omega - 1}{\omega} \frac{1}{16\pi} \ln(q^2/m^2),
$$

a logarithmic violation of Bjorken scaling. It is interesting to note that the logarithm in Eq. (3) does not come from any of the infinite renormalizations required by the theory because the amplitude being computed is a completely finite "tree graph." Since the Bjorken limit does not allow all external four momenta to approach infinity ($p^2$ is kept fixed), this limit is not governed by the renormalization group results of Gell-Mann and Low.\footnote{Alfred P. Sloan Foundation Fellow}

Let us now consider the Bjorken limit for amplitudes of arbitrary order in perturbation theory. This problem has been attacked from two different directions. First, extensive explicit perturbation theory calculations have been performed over the last three years determining the Bjorken limit in leading logarithmic approximation for a variety of Feynman amplitudes.\footnote{Alfred P. Sloan Foundation Fellow}~5 The culmination of this first approach is the work of Gribov and Lipatov~6 in which the Bjorken limit is computed for all graphs in the pseudoscalar theory described above and a vector theory in which a neutral vector field $V_\mu$ is coupled to the charged spinor field $\psi$ through the interaction term

$$
L_I = igV_\mu \gamma_5 \gamma_\mu \psi.
$$
All of these calculations show quite complicated behavior for the Bjorken limit—not the simple canonical scaling seen experimentally.

A second approach\(^7\) extends the work of Callan and Symanzik\(^8,9\) to determine a series of equations obeyed by \(W_4(q^2, \omega)\) and \(\nu W_2(q^2, \omega)\) for large \(q^2\) to arbitrary order in perturbation theory. These equations, of a form similar to the renormalization group equations of Gell-Mann and Low,\(^3\) easily reproduce the results of the explicit calculations done earlier and permit speculation about the exact large \(q^2\) and \(\nu\) behavior of \(W_4\) and \(\nu W_2\). In this talk I will outline the derivation of these equations, apply them to determine and interpret the leading logarithmic results obtained previously and finally examine a possible asymptotic scaling behavior of their general solution.

We first consider the Callan-Symanzik equation in the pseudoscalar theory obeyed by the amplitudes for forward Compton scattering of virtual photons from spinor or pseudoscalar particles\(^10\)

\[
D_1 \langle 0 | T(\tilde{\phi}(-p)J_{\mu}(x)J_{\nu}(0)\tilde{\phi}(p)) | 0 \rangle = \langle 0 | T(\tilde{\phi}(-p)J_{\mu}(x)J_{\nu}(0)\tilde{\phi}(p)) | 0 \rangle,
\]

\[
D_2 \langle 0 | T(\tilde{\phi}(-p)J_{\mu}(x)J_{\nu}(0)\tilde{\phi}(p)) | 0 \rangle = \langle 0 | T(\tilde{\phi}(-p)J_{\mu}(x)J_{\nu}(0)\tilde{\phi}(p)) | 0 \rangle,
\]

where

\[
u = \frac{1}{2} \int d^4 x \{ \delta_4 m N[\psi(x)] + 2\mu^2 N[\delta(x)] \}
\]

is a mass insertion operator, the symbol \(N\) indicating the presence of subtraction terms making \(\nu\) a finite operator in the sense of Zimmerman. The differentiation operators \(D\) are given by

\[
D_\mu = m \frac{\partial}{\partial m} + \beta \frac{\partial}{\partial g} + \frac{\beta}{\partial h} - 2\gamma - \frac{\partial}{\partial \mu}.
\]

\(g\) and \(h\) are the renormalized spinor-spinor-pseudoscalar and pseudoscalar-four-point coupling constants respectively. The constants \(\beta, \beta', \gamma, \gamma', \delta, \delta'\) depend on \(g, h\) and the ratio \(\mu/m\) of physical pseudoscalar and Fermion masses. The Eqs. (5) and (6) simply state that the derivative of an amplitude with respect to the scale of the physical masses can be obtained by insertion of the mass operator \(\nu\) in the original amplitude, provided the mass dependence of the coupling constant \((-\beta, -\beta')\) and wave function \((\gamma)\) renormalizations has been subtracted. These equations are obeyed exactly order-by-order in perturbation theory.

If we Fourier transform the amplitudes on the left-hand side of Eq. (5) with respect to \(x\), calling the Fourier transform variable \(q\), then the imaginary part for space-like \(q^2\) and \(-2p q\) \(\geq q^2 \geq 0\) is directly related to \(\nu W_2\). In fact if we let

\[
\sum_{S = 2} \int e^{-i q \cdot x} \langle p, s | T\{J_{\mu}(x)J_{\nu}(0)\}| p, s \rangle
\]

\[
= -\left\{ \frac{\delta m}{q^2} - \frac{q^2}{q^2} \right\} T_L + \frac{2}{q^2} \left( p^\mu \nu q^2 + \delta^{\mu\nu}(p \cdot q)^2 - p^\mu q^\nu(p \cdot q) - p^\nu q^\mu(p \cdot q) \right) T_2,
\]

then

\[
W_4 = \frac{2}{m} \text{Im} \left[ -T_L + 2 \left( \frac{p \cdot q}{q^2} \right)^2 T_2 \right],
\]

\[
\nu W_2 = -\frac{4}{\pi} \text{Im} \left[ \frac{p \cdot q}{q^2} T_2 \right].
\]
The Callen-Symanzik Eqs. (5), (6) would be useful as they stand if we were interested in the general limit \( q_\mu, p_\mu \to \infty \). Then the unknown right-hand side, containing the extra mass operator \( u \), would vanish and we would obtain an equation for \( T_1 \) and \( T_2 \) equivalent to the renormalization group results of Gell-Mann and Low.\(^3\) However, we are interested in a different limit, in which \( p_\mu \) stays on the mass shell, so that further work is needed. We adopt a technique of Symanzik,\(^8\) inserting the Wilson expansion\(^12,13\)

\[
J\left(\frac{x^\mu + x^\nu}{2}\right) J\left(\frac{x^\rho + x^\sigma}{2}\right) = \delta^{\mu\nu} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} - \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} \frac{1}{\epsilon^2} \left\{ \sum_{n=0}^{\infty} \sum_{i=0}^{u_n} F^{(i)}_n (x^2 + i\epsilon x_0) \cdot \frac{(\mu_1, \ldots, \mu_n)}{\mu_{i+1} \ldots \mu_n} \right\}
\]

\[
+ \delta^\mu_\nu \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} - \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} \frac{1}{\epsilon^2} \left\{ \sum_{n=0}^{\infty} \sum_{i=0}^{u_n} E^{(i)}_n (x^2 + i\epsilon x_0) \cdot \frac{(\mu_1, \ldots, \mu_n)}{\mu_{i+1} \ldots \mu_n} \right\}
\]

into Eqs. (5) and (6). Here the operators \( O^{(i)}_{\alpha_1 \ldots \alpha_n} (y) \) are traceless and symmetric with respect to each pair of Lorentz indices,\(^14\) while the coefficients \( E^{(i)}_n (x^2), F^{(i)}_n (x^2) \) are c-number functions.

Following Brandt and Preparata we include only those terms for each \( n \) in the expansion (11) which are most singular on the light cone. For the pseudoscalar theory at hand there are two such terms for each \( n \)

\[
O^{(1)}_{\alpha_1 \ldots \alpha_n} (y) = \frac{1}{2n} \sum_{j=1}^{n} N(\psi(y)\delta_{\alpha_j \alpha_{j+1}} + \delta_{\alpha_{j-1} \alpha_j} \delta_{\alpha_j \alpha_{j+1}} + \delta_{\alpha_j \alpha_{j+1}}\psi(y)) + \text{terms containing } \delta_{\alpha_i \alpha_j}
\]

\[
O^{(2)}_{\alpha_1 \ldots \alpha_n} (y) = \frac{1}{2n} N(\psi(y)\delta_{\alpha_j \alpha_{j+1}} + \delta_{\alpha_{j-1} \alpha_j} \delta_{\alpha_j \alpha_{j+1}} + \delta_{\alpha_j \alpha_{j+1}}\psi(y)) + \text{terms containing } \delta_{\alpha_i \alpha_j}
\]

with coefficients of the form

\[
F^{(i)}_n (x^2) = \sum_{f=0}^{\infty} \sum_{r=0}^{\infty} f^{(i)}_n (f, r) g^{2f} f_n (x^2)
\]

\[
E^{(i)}_n (x^2) = \sum_{f=0}^{\infty} \sum_{r=0}^{\infty} e^{(i)}_n (f, r) g^{2f} f_n (x^2).
\]

The coefficients \( E^{(i)}_n (x^2) \) and \( F^{(i)}_n (x^2) \) can be related to \( vW_2(q^2, \omega) \) and \( W_4 \) by the formulae\(^16\)

\[
T_2^{AF} (q^2, \omega) = \sum_{n=0}^{\infty} v^{2n} \sum_{i=0}^{u_n} \frac{E^{(i)}_n (q^2)}{\epsilon (q^2)} c_n^{(i)}
\]

\[
T_L^{AF} (q^2, \omega) = \sum_{n=0}^{\infty} v^{2n} \sum_{i=0}^{u_n} \frac{E^{(i)}_n (q^2)}{\epsilon (q^2)} c_n^{(i)}
\]

or

\[
\sum_{n=2}^{\infty} c_n^{(1)} E^{(i)}_n (q^2) = \int_0^{\infty} \int_0^{\infty} vW_2(q^2, \omega) d\omega
\]

\[
= \int_0^1 \int_0^{\infty} vW_2(q^2, \omega) d\omega d\omega.
\]
\[
\sum_n (1) \overline{E}_n(q^2) = \int_1^{\infty} \omega^{-n-1} \left( \frac{1}{2} \nu W_2 (q^2, \omega) - m W_1 (q^2, \omega) \right) d\omega.
\]

The quantity \( \overline{E}_n(q^2) \) is a slightly generalized Fourier transform of \( \overline{E}_n(x^2) \)

\[
\overline{E}_n(x^2) = \frac{1}{8} \frac{\delta}{\delta (q^2)} \int d^4 x e^{-iq \cdot x} \overline{E}_n(x^2 + \epsilon)
\]

and

\[
\overline{E}_n(x^2) = \frac{1}{4} \frac{\delta}{\delta (q^2)} \int d^4 x e^{-iq \cdot x} x \overline{E}_n(x^2 + \epsilon)
\]

The constants \( c_n^{(1)} \) are determined by the matrix elements of the operators \( O^{(1)}_{\alpha_1 \ldots \alpha_n} \):

\[
\frac{1}{2} \sum_{s=\pm \frac{1}{2}} \langle p, s | O_{\alpha_1 \ldots \alpha_n} | p, s \rangle = c_n^{(1)} \alpha_1 \ldots \alpha_n \eta^{(1)} + \{ \text{terms containing } \delta_{\alpha_1 \alpha_2} \}.
\]

If the Wilson expansion (4) is substituted into the Callan-Symanzik Eqs. (5) and (6), the coefficients \( E_n(x^2) \) and \( F_n(x^2) \) determine the \( x^2 \)-dependence of both sides of those equations, the result is equations for \( E_n(x^2) \) and \( F_n(x^2) \). We take only the leading light cone singularity on both sides of Eqs. (5), (6) and therefore consider only the terms in the matrix elements of the operators \( O^{(1)}_{\alpha_1 \ldots \alpha_n} \) which contain the tensor \( \eta_{\alpha_1 \ldots \alpha_n} \). Equating the coefficients of equal powers of \( x \cdot p \), we obtain a series of equations for \( E_n(x^2) \) diagonal in \( n \)

\[
D \left\{ \sum_j E_{n-2}^{(1)}(x^2) b_{n,j}^{1,1} \right\} - \sum_j E_{n-2}^{(1)}(x^2) b_{n,j}^{1,1} = 0,
\]

\[
2 \leq n \leq \omega, \quad 1 \leq j \leq u_n, \quad \text{for even } n.
\]

Equation (18) is also obeyed by \( F_n^{(1)}(x^2) \) for \( 0 \leq n \leq \omega \). The constants \( a_n^{1,J} \) and \( b_n^{1,J} \) are determined by matrix elements of the operators \( O^{(1)}_{\alpha_1 \ldots \alpha_n} \). For simplicity we will normalize these operators so that \( b_{n,j}^{1,1} = \delta_{1J} \).

These equations can now be used to obtain the leading logarithmic results for \( \nu W_2 \) obtained previously by Chang, Fishbane, Gribov, and Lipatov. Consider first the set of ladder graphs in Fig. 2 studied by Chang and Fishbane. Since these graphs contain no renormalizations and no two pseudoscalar intermediate states, their asymptotic behavior follows from our Eq. (18) if \( \beta, \beta' > 1, Y_1, Y_2 > 0 \) are set equal to zero and the operators \( O^{(2)}_{\alpha_1 \ldots \alpha_n} \) dropped. Equation (18) then becomes

\[
x^2 \frac{\delta}{\delta x^2} E_n^{(1)}(x^2) = a_n^{4,4} E_n^{(1)}(x^2)
\]

and

\[
E_n^{(1)}(x^2) - \nu_n^{(1)}(x^2)^2 a_n^{4,4}.
\]

Recalling that \( E_n^{(1)}(x^2) \) when Fourier transformed is just the Mellin transform of \( \nu W_2(q^2, \omega) \), we find

\[
\int_0^1 \left( \frac{1}{\omega} \right)^n d(\frac{1}{\omega}) \nu W_2(q^2, \omega) = V_n(q^2/m^2)^{-a_{4,4}} n+2.
\]
The leading logarithmic approximation of Chang and Fishbane results if $a_n^{4,1}$ is replaced by its lowest-order perturbation-theory value

$$a_n^{4,1} = \frac{-2}{16\pi} \frac{1}{n(n+1)}$$

(22)

Thus, for the graphs of Fig. 2, the coefficients $E_n^{(2)}(x^2)$ behave like powers of $x^2$, so that the operator $O_{a_1 \ldots a_n}^{(1)}$ has anomalous dimension

$$d_n = n + 2 + 2a_n^{4,1}$$

(23)

in the sense of Wilson.

A similar result follows if we consider all graphs without self-energy or vertex corrections. In that case $\gamma_1 = \beta = \beta' = 0$ and Eq. (18) becomes

$$+ x^2 \frac{\partial}{\partial x^2} E_n^{(1)} = a_n^{4,1} E_n^{(1)} + a_n^{4,2} E_n^{(2)}$$

(24)

With the solution

$$E_{n+2}^{(1)}(x^2) = \frac{1}{n+2} \frac{1}{x^2} + \frac{1}{n+2} \frac{1}{x^2}$$

(25)

where the $\gamma_n^{(1)}$ are integration constants and

$$\gamma_n^{(1)} = \frac{1}{n} + \frac{2}{n^2} + (21 - 3) \left( \frac{1}{4} a_n^{4,1} - a_n^{4,2} \right)^{1/2}$$

(26)

for $i = 1, 2$. Now the combination of operators

$$O_{a_1 \ldots a_n}^{(1)} + \frac{\gamma_n^{(j)} - a_n^{4,1}}{a_n^{4,2}} O_{a_1 \ldots a_n}^{(2)}$$

(27)

has anomalous dimension

$$d_n^{(j)} = n + 2 + 2a_n^{(j)}$$

(28)

for $j = 1, 2$. Thus, if all renormalization effects are neglected we find a scaling behavior for the coefficients $E_n^{(1)}(x^2)$ in the Wilson expansion, but a more general scaling than that seen at SLAC. Here the Mellin transform of $vW_2(q^2, \omega)$ is found to grow as a power of $q^2$ with an exponent which depends on the Mellin transform variable. The series of operators $O_{a_1 \ldots a_n}$ in the Wilson expansion have anomalous dimensions which do not increase in simple integral steps with $n$. Hence these operators do not contribute equal powers of $q^2$ in the Bjorken limit.
Finally, let us find the complete solution to Eq. (18) in the leading logarithmic approximation. This can be done by replacing all the constants \( \beta, \beta', \gamma_1, \gamma_2 \) by their lowest-order perturbation-theory values and solving the resulting two coupled first-order differential equations. The solution,

\[
\frac{\bar{b}_{n}(q^2)}{v_n} = \frac{1}{v_n - v_n^0} \left[ \frac{1}{n+2(n+3)} - \frac{1}{4} \right] \left[ \frac{1}{n+2(n+3)} - \frac{1}{2} \right] v_n^0 + \frac{1}{v_n - v_n^0} \left[ \frac{1}{n+2(n+3)} - \frac{1}{2} \right] v_n^0 \tag{29}
\]

is identical to that obtained by Gribov and Lipatov. Here

\[
\xi = \frac{1}{5} \ln \left[ 1 - \frac{5}{16 \pi^2} \ln(q^2) \right],
\]

and

\[
\begin{align*}
\nu_n &= -\frac{5}{4} + \frac{1}{2(n+2)(n+3)} + \left[ \frac{3}{4} + \frac{1}{2(n+2)(n+3)} \right]^2 + \frac{4}{(n+2)(n+3)} \right]^{1/2} \\
\nu_n' &= -\frac{5}{4} + \frac{1}{2(n+2)(n+3)} - \left[ \frac{3}{4} + \frac{1}{2(n+2)(n+3)} \right]^2 + \frac{4}{(n+2)(n+3)} \right]^{1/2} \\
\end{align*}
\]

Perhaps the most significant aspect of this result is its complexity. There seems to be no sign of the simple Bjorken scaling behavior found at SLAC.

If we assume that the asymptotic form of \( \bar{b}_{n}(q^2, \omega) \) in the Bjorken limit can be obtained by summing the asymptotic form of the individual terms in the perturbation expansion, then we can use Eq. (18) to discuss the exact behavior of \( \bar{b}_{n}(q^2, \omega) \) in the Bjorken limit. For example, in the vector theory we could assume the existence of a zero at \( g = g_{\infty} \) for the function \( \beta(g) \) appearing in the analogue of Eq. (8). If we also assume that \( g_{\infty} \) is the actual value of the renormalized coupling constant \( g \), then all renormalization effects essentially drop out of the vector theory. The vector particle propagator is asymptotically free and the coefficients \( E_n^{(1)} \) are given by an equation of the form (25). Again the result is not the simple canonical scaling seen at SLAC unless all the exponents \( \nu_n^{(1)}(g) \) happen to be very near zero for \( g = g_{\infty} \).

Thus these perturbation theory results which I have outlined do not suggest a simple Bjorken limiting behavior for \( \bar{b}_{n}(q^2, \omega) \). If taken seriously, they instead support the view that the Bjorken scaling observed at SLAC is not an asymptotic phenomena and should be violated at energies significantly larger than typical hadronic rest masses. This deviation from scaling at higher energies would not result, necessarily, from the existence of new particles (quarks, intermediate weak vector bosons, etc.) but would come instead from the logarithms present in local field theory.

References

The momentum-dependent amplitudes appearing below are the Fourier transforms of the corresponding, position-dependent, time-ordered products. The subscript $A$ means that the propagators corresponding to external lines have been removed.


The validity of this expansion in perturbation theory has been established by R. Brandt, Ann. Phys. 44, 224 (1967) and W. Zimmerman in *Lectures on Elementary Particles and Quantum Field Theory*, op. cit.

We have omitted from Eq. (2) a third type of term containing operators of the form $O_{\mu_1 \ldots \mu_n \nu}$, antisymmetric under interchange of $\mu$ or $\nu$ with $\nu_j$. Such terms do not contribute to the spin-averaged matrix elements under consideration.


Following Symanzik (Ref. 11) we shall use the superscript $AF$ on a function, $f(q^2, \omega)$, to indicate that only those terms containing the highest power of $q^2$ are retained in each order of perturbation theory. That is, if

$$ f(q^2, \omega) = (q^2)^n \sum_{k=0}^{\infty} \sum_{\ell=0}^{\infty} f_{k, \ell}(q^2) (\ln q^2)^{-k} \ln(q^2)^{\ell}, $$

where $f_{k, \ell}(\omega)$ is a power series in $g$, then $f(q^2, \omega)^{AF}$ contains only the $k=0$ terms

$$ f(q^2, \omega)^{AF} = (q^2)^n \sum_{\ell=0}^{\infty} f_{0, \ell}(\omega) (\ln q^2)^{\ell}. $$
Fig. 1. The Feynman diagram representing the leading contribution to $\nu W_2$ to order $g^2$ in the pseudoscalar theory. The solid lines represent Fermion propagators, the dashed lines pseudoscalar propagators, and the wavy lines virtual photons.

Fig. 2. Ladder graphs representing the "outer rainbow" amplitudes considered in the Chang-Fishbane calculation.
The problem we are interested in is what relation, if any, exists between deep inelastic scattering (DIS) and deep inelastic annihilation (DIA). (See Fig. 1, p is any stable hadron like π, K, p, f, . . . )

In 1969 Drell and Yan in the context of their field theoretical parton model obtained a very interesting result, namely that the appropriate correlation function of DIA \( \nu W(q^2, \nu) \) scaled according to Bjorken (\( \omega = q^2/2 \nu \))

\[
\nu W(q^2, \nu) \rightarrow F(\omega) \quad (\omega > 1)
\]

and that \( F(\omega) \) was given by the analytic continuation for \( \omega > 1 \) of the analogous \( F(\omega) \) of DIS (see Fig. 2).

Our aim is then to see how general is this result. One would like to try a light cone (LC) treatment of the problem; this, however, seems very difficult. In fact, whereas DIS requires the knowledge at \( x^2 = 0 \) of the current correlation function \( <p|J(x)J(0)|p> \); DIA on the other hand leads us, by applying the LSZ formalism, to consider the correlation function

\[
<0|T[J_p(x)J(y)]T[J_p(z)J(0)]|0> \quad (y^2 = 0),
\]

which requires knowledge of the short distance behavior of the hadronic source \( j_p(x) \). This appears to be beyond present theoretical understanding.

Gatto, Menotti, and Vendramin (\#87, 88, 89) look at the problem in a perturbative framework and reach results which are in agreement with a general LC analysis by Gatto and myself (\#194).
In this work one looks at the "unequal mass" Compton amplitude (Fig. 3) as given by the LC operator product expansion. The reason for this is explained in Fig. 4, which implies

\[ \nu \bar{W}(q^2, \nu) = T_0(s+\epsilon, q_1^2+\epsilon, q_2^2-\epsilon) - T_0(s-\epsilon, q_1^2+\epsilon, q_2^2-\epsilon), \]

where \( T_0(s, q_1^2, q_2^2) \) is the t=0 piece of the Compton amplitude which has only s-channel normal thresholds.

Thus, in general we expect from (2) contributions which cannot be expressed in terms of a suitable continuation of \( F(\omega) \) alone. In fact \( F(\omega) \) only involves the "equal mass" Compton amplitude.

The results of this analysis are the following:

Under the assumption that \( \sigma_{ee}(e^+ e^-) \) has free-field behavior, then

1. either \( \nu \bar{W}(q^2, \nu) = F(\omega) = -F(\omega) (\omega > 1) \), like in Drell and Yan,

2. or \( \nu \bar{W}(q^2, \nu) = -\text{Re} F(\omega) + \gamma(\omega) > 0 \) scales nontrivially, and \( \gamma(\omega) \) is given in terms of a suitable limit of the unequal mass Compton amplitude. This latter possibility occurs in the paper by Gatto et al. (x89) due to diagrams of the type in Fig. 5, which are however ruled out in the Drell-Yan calculation due to the stability of the parton.
While field theory has been a proven success in describing phenomena involving only leptons, the question remains whether it is also relevant for phenomena involving hadrons. The question is a difficult one of course; to begin with we do not know which field theory, if any, is the chosen one. Furthermore, the renormalized perturbation series may or may not converge. (Perhaps it converges for some processes and diverges for others). Let us assume that the series converges for a given process and ask if we can calculate some hadronic quantities to any finite order in coupling constants and hence confront the series with experiment. The search for such quantities was initiated by Adler, Bell and Jackiw. The quantities found so far appear in the processes $\pi \to 2\gamma$, $\gamma \to \pi^+ \pi^-$, and $2\gamma \to 3\pi$.

Let us state the result as a theorem. Define

$$R_{\mu\nu}(k, q) \equiv \int d^4x d^4y e^{ikx+qy} \langle 0 | T\slashed{A}(0) \slashed{V}_\mu(x) \slashed{V}_\nu^\dagger(y) | 0 \rangle$$

Theorem: $f(0, 0, 0)$ may be calculated to any finite order in the coupling constant (assuming that the neutral axial charge commutes with the electromagnetic current).

To prove this theorem we consider the Callan-Symanzik equation for $R_{\mu\nu}$ which summarizes conveniently the renormalizability of the theory. In asymptotic applications one appeals to Weinberg's theorem and drops the scalar insertion term that appears in the C-S equation, thus losing part of the informational content of the equation. Instead, we note that this term may be evaluated by using a chiral Ward identity. The result stated in the theorem follows immediately. This technique of combining two types of Ward identities, chiral and dilatation, may prove to be a useful one.

Our discussion does not imply that the existence of the axial anomaly depends on perturbation theory, of course. A nonperturbative treatment has been given by Wilson, who argued that $f(0, 0, 0)$ is determined by the short distance singularity of $\langle 0 | T\slashed{A}(x) \slashed{V}_\mu(0) \slashed{V}_\nu(0) | 0 \rangle$. This singularity may be explored in a step-wise manner, by first letting $y \to 0$ and then letting $x \to 0$. Carrying out this analysis, Crewther found the beautiful theorem that $-i(3\pi^2)f(0, 0, 0) = KR'$. Here $K$ and $R'$ are constants that appear in the operator product expansions (assumed to be scale invariant at short distances) of $\slashed{V}_\mu(0)\slashed{V}_\nu(0)$ and of $\slashed{A}_\lambda(x)\slashed{A}_\sigma(0)$ and are in principle measurable.

It may be emphasized that the domains of validity of the two theorems we discussed do not overlap in general. It has been asserted, however, that they both hold in QED in the one-fermion-loop approximation. A number of interesting deductions then follow. The conclusion appears to be that the naive short distance expansion fails at the eigenvalue, even though it may be valid order by order in perturbation theory.
Perhaps the Sutherland-Veltman theorem is in fact correct and the $\pi^0 \rightarrow 2\gamma$ amplitude is actually order $m^2_\pi$ but with a large numerical coefficient. Can we exclude this possibility? There exist low-energy theorems relating $\gamma \rightarrow 3\pi$ and $2\gamma \rightarrow 3\pi$ to $\pi^0 \rightarrow 2\gamma$. A simple way to prove these theorems would be to show by the same argument outlined here that $\gamma \rightarrow 3\pi$ and $2\gamma \rightarrow 3\pi$ are not affected by radiative corrections. Then the relations valid with the interaction turned off are true to any finite order. When we compare these relations with experiment we necessarily make a PCAC error of order $m^2_\pi$. So, if the amplitudes involved are themselves of order $m^2_\pi$ the relations would most likely be contradicted by experiment. The question posed here may also be answered heuristically by studying the off-shell dependence of $R_{\Delta\mu\nu}$ in such processes as $\pi^0 \rightarrow e^+ e^-$. One may also adopt the famous attitude that while certain algebraic relations may be abstracted from free field theory the conclusions of renormalized perturbation theory to any finite order are to be rejected. Recently, interest has been focused on the question whether one may abstract the relation $[\sigma(e^+ e^- \rightarrow \text{hadrons})/\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)] \rightarrow \Sigma Q_i^2$ at asymptotic energies [Here $Q_i$ denotes the charge of the fundamental quarks (if any)] It should be emphasized that this relation is not on the same footing as the $\pi^0 \rightarrow 2\gamma$ result. In renormalized perturbation theory this ratio may approach a constant not necessarily equal to $\Sigma Q_i^2$, may vanish, or may oscillate indefinitely.

If we accept this relation we may be able to distinguish various quark charge schemes. As is well-known, the Zweig model appears to be incompatible with the $\pi^0 \rightarrow 2\gamma$ rate. One possible way to save the model involves invoking some version of "weak PCAC" [17]. Examples of models that can accommodate the $\pi^0 \rightarrow 2\gamma$ rate include the Han-Nambu model and the recently proposed "colored" model [18] of Gell-Mann.

If SU(3) is exact and if the electromagnetic current transforms like $V^3 + (1/\sqrt{3})V^8$, then the $\gamma \rightarrow 3\pi$ anomaly determines the vector current contribution to $K^0 \rightarrow \pi^0$. Recent $K^0 \rightarrow \pi^0$ data appear to support the Han-Nambu model and the "colored" model. There is no reason, however, to suppose that SU(3) symmetry breaking effect is small. This question may be settled in principle by measuring $e^+ e^- \rightarrow K^+ K^- \pi^0$, $K^0 \bar{K}^0 \pi^\pm$, and $K^+ K^- \pi^0$.

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Current Algebra: Quarks and What Else?

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I. INTRODUCTION

For more than a decade, we particle theorists have been squeezing predictions out of a mathematical field theory model of the hadrons that we don't fully believe - a model containing a triplet of spin 1/2 fields coupled universally to a neutral spin 1 field, that of the "gluon." In recent years, the triplet is usually taken to be the quark triplet, and it is supposed that there is a transformation, presumably unitary, that effectively converts the current quarks of the relativistic model into the constituent quarks of the naive quark model of baryon and meson spectrum and couplings.

We abstract results that are true in the model to all orders of the "gluon" coupling and postulate that they are really true of the electromagnetic and weak currents of hadrons to all orders of the strong interaction. In this way we build up a system of algebraic relations, so-called current algebra, and this algebraic system gets larger and larger as we abstract more and more properties of the model.

In Section III, we review briefly the various stages in the history of current algebra. The older abstractions are correct to each order of renormalized perturbation theory in the model, while the more recent ones, those of light cone current algebra, are true to all orders only formally. We describe the results of current algebra in terms of commutators on or near a null plane, say $x + x' = 0$.

In Section IV, we attempt to describe, in a little more detail, using null plane language, the system of commutation relations valid in a free quark model that are known to remain unchanged (at least formally) when the coupling to a vector "gluon" is turned on. These equations give us a formidable body of information about the hadrons and their currents, supposedly exact as far as the strong interaction is concerned, for comparison with experiment. However, they by no means exhaust the degrees of freedom present in the model; they do not
yield an algebraic system large enough to contain a complete description of the hadrons. In an Appendix, the equations of Section IV are related to form factor algebra.

In Section V, we discuss how further commutation of the physical quantities arising from light cone algebra leads, in the model field theory, to results dependent on the coupling constant, to formulae in which gluon field strength operators occur and bilocal current operators proliferate. Only when these relations are included do we finally get an algebraic system that contains nearly all the degrees of freedom of the model. We may well ask, however, whether it is the right algebraic system. We discuss briefly how the complete description of the hadrons involves the specification and slight enlargement of this algebraic system, the choice of representation of the algebra that corresponds to the complete set of hadron states, and the form of the mass or the energy operator, which must be expressible in terms of the algebra when it is complete. The choice of representation may be dictated by the algebra, and if so that would justify the use of a quark and gluon Fock space by some "parton" theorists.

Finally, in Section VI, it is suggested that perhaps there are alternatives to the vector gluon model as sources of information or as clues for the construction of the true hadron theory. Assuming we have described the quark part of the model correctly, can we replace the gluons by something else? The "string" or "rubber band" formulation, in ordinary coordinate space, of the zeroth approximation to the dual resonance model, is suggested as an interesting example.

Before embarking on our discussion of current algebra, we discuss in Section II the crucial point that quarks are probably not real particles and probably obey special statistics, along with related matters concerning the gluons of the field theory model.
II. FICTITIOUS QUARKS AND "GLUONS" AND THEIR STATISTICS

We assume here that quarks do not have real counterparts that are detectable in isolation in the laboratory – they are supposed to be permanently bound inside the mesons and baryons. In particular, we assume that they obey the special quark statistics, equivalent to "para-Fermi statistics of rank three" plus the requirement that mesons always be bosons and baryons fermions. The simplest description of quark statistics involves starting with three triplets of quarks, called red, white, and blue, distinguished only by the parameter referred to as color. These nine mathematical entities all obey Fermi-Dirac statistics, but real particles are required to be singlets with respect to the SU_3 of color, that is to say combinations acting like $\bar{q}_R^q q_R^q + \bar{q}_B^q q_B^q + \bar{q}_W^q q_W^q$ or $q_R^q q_B^q q_W^q - q_R^q q_B^q q_W^q - q_R^q q_B^q q_W^q + q_R^q q_B^q q_W^q$.

The assumption of quark statistics has been common for many years, although not necessarily described in quite this way, and it has always had the following advantage: the constituent quarks as well as current quarks would obey quark statistics, since the transformation between them would not affect statistics, and the constituent quark model would then assign the lowest-lying baryon states ($\mathbf{3}$ representation) to a symmetrical spatial configuration, as befits a very simple model.

Nowadays there is a further advantage. Using the algebraic relations abstracted formally from the quark-gluon model, one obtains a formula for the $\pi^0$ decay amplitude in the PCAC approximation, one that works beautifully for quark statistics but would fail by a factor 3 for a single Fermi-Dirac triplet.

We have the option, no matter how far we go in abstracting results from a field theory model, of treating only color singlet operators. All the currents, as well as the stress-energy-momentum tensor $\Theta_{\mu\nu}$ that couples to gravity and
defines the theory, are color singlets. We may, if we like, go further and
abstract operators with three quark fields, or four quark fields and an
antiquark field, and so forth, in order to connect the vacuum with baryon
states, but we still need select only those that are color singlets in order
to connect all physical hadron states with one another.

It might be a convenience to abstract quark operators themselves, or
other non-singlets with respect to color, along with fictitious sectors of
Hilbert space with triality non-zero, but it is not a necessity. It may not even
be much of a convenience, since we would then, in describing the spatial and
temporal variation of these fields, be discussing a fictitious spectrum for
each fictitious sector of Hilbert space, and we probably don't want to load
ourselves with so much spurious information.

We might eventually abstract from the quark-vector-gluon field theory
model enough algebraic information about the color singlet operators in the
model to describe all the degrees of freedom that are present.

For the real world of baryons and mesons, there must be a similar algebraic
system, which may differ in some respects from that of the model, but which is
in principle knowable. The operator $\Theta$ could then be expressed in terms of
this system, and the complete Hilbert space of baryons and mesons would be
a representation of it. We would have a complete theory of the hadrons and
their currents, and we need never mention any operators other than color singlets.

Now the interesting question has been raised lately whether we should
regard the gluons as well as the quarks as being non-singlets with respect
to color. For example, they could form a color octet of neutral vector fields
obeying the Yang-Mills equations. (We must, of course, consider whether it
is practical to add a common mass term for the gluon in that case—such a
mass term would show up physically as a term in $\Theta$ other than the quark bare
mass term. In the past, we have referred to such an additional term that
violates scale invariance but does not violate $SU_3 \times SU_3$ as $\delta$ and its
dimension as $\lambda_\delta$. Nowadays, ways of detecting expected values of $\delta$ are
emerging.)

If the gluons of the model are to be turned into color octets, then
an annoying asymmetry between quarks and gluons is removed, namely that there
is no physical channel with quark quantum numbers, while gluons communicate
freely with the channel containing the $\omega$ and $\phi$ mesons. (In fact, this communi-
cation of an elementary gluon potential with the real current of baryon
number makes it very difficult to believe that all the formal relations of
light cone current algebra could be true even in a "finite" version of singlet
neutral vector gluon field theory.)

If the gluons become a color octet, then we do not have to deal with
a gluon field strength standing alone, only with its square, summed over the
octet, and with quantities like $\bar{q}((3 - ig_\rho B_\mu)q$, where the $\rho$'s are the eight
$3 \times 3$ color matrices for the quark and the $B$'s are the eight gluon potentials.

Now, suppose we look at such a model field theory, with colored quarks and
colored gluons, including the stress-energy-momentum tensor. Basically the
questions we are asking are the following:

1) Up to what point does the algebraic system of the color singlet
operators for the real hadrons resemble that in the model? What
is it in fact?

2) Up to what point does the representation of the algebraic system by
the Hilbert space of physical hadron states resemble that in the
model? What is it in fact?

3) Up to what point does $\theta_{\mu\nu}$, expressed in terms of the algebraic
system, resemble that in the model? What is it in fact?
The measure of our ignorance is that for all we know, the algebra of color singlet operators, the representation, and even the form of $\theta_{\mu\nu}$ could be exactly as in the model. We don't yet know how to extract enough consequences of the model to have a decisive confrontation with experiment, nor can we solve the formal equations for large $g$.

If we were solving the equations of a model, the first question we would ask is: Are the quarks really kept inside or do they escape to infinity? By restricting physical states and interesting operators to color singlets only, we have to some extent begged that question. But it re-emerges in the following form:

With a given algebraic system for the color singlet operators, can we find a locally causal $\theta_{\mu\nu}$ that yields a spectrum corresponding to mesons and baryons and antibaryons and combinations thereof, or do we find a spectrum (in the color singlet states) that looks like combinations of free quarks and antiquarks and gluons?

In the next three Sections we shall usually treat the vector gluon, for convenience, as a color singlet.
III. REVIEW OF CURRENT ALGEBRA

In this section we sketch the gradual extension of algebraic results abstracted from free quark theory that remain true, either in renormalized perturbation theory or else only formally, when the coupling to a neutral vector gluon field is turned on.

The earlier abstractions were of equal-time commutation relations of current components. It was soon found that useful sum rules could best be derived from these by taking matrix elements between hadron states of equal \( P_3 \) as \( P_3 \to \infty \), selecting the "good" components of the currents (those with matrix elements finite in this limit rather than tending to zero), and adding the postulate that, in the sum over intermediate states in the commutator, only states of finite mass need be considered. Thus formulae like the Adler-Weisberger and Cabibbo-Radicati sum rules were obtained and roughly verified by experiment.

Nowadays, the same procedure is usually accomplished in a slightly different way that is a bit cleaner - the hadron momenta are left finite instead of being boosted by a limit of Lorentz transformations, and the equal time surface is transformed by a corresponding limit of Lorentz transformations into a null plane, with \( x_3 + x_0 = \) constant, say zero. The hypothesis of saturation by finite mass intermediate states is replaced by the hypothesis that the commutation rules of good components can be abstracted from the model not only on an equal time plane, but on a null plane as well.\(^7,8\)

In the last few years, the process of abstraction has been extended to a large class of algebraic relations (those of "light cone current algebra") that are true only formally in the model, but fail to each order of renormalized perturbation theory - they would be true to each order if the model were super-renormalizable. The motivation has been supplied by the compatibility of the deep inelastic electron scattering experiments performed at

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SLAC with the scaling predictions of Bjorken, which is the most basic feature of "light cone current algebra." The Bjorken scaling limit \((q^2 \to \infty, 2p \cdot q \to \infty, \xi \equiv q^2 / (-2p \cdot q) \text{ finite})\) corresponds in coordinate space to the singularity on the light cone \((x-y)^2 = 0\) of the current commutator \([j(x), j(y)]\), and the relations of light cone current algebra are obtained by abstracting the leading singularity on the light cone from the field theory model. The singular function of \(x-y\) is multiplied by a bilocal current operator \(\mathcal{O}(x,y)\) that reduces to a familiar local current as \(x-y \to 0\). The Bjorken scaling functions \(F(\xi)\) are Fourier transforms of the expected values of the bilocal operators. Numerous predictions emerge from the relations abstracted from the quark-gluon model for deep inelastic electron and neutrino cross-sections. For example, the spin 1/2 character of the quanta bearing the charge in the model is reflected in the prediction \(\sigma_L^{\text{bj}} / \sigma_T^{\text{bj}} \to 0\), while the charges of the quarks are reflected in the inequalities \(1/4 \leq F^{\text{en}}(\xi) / F^{\text{ep}}(\xi) \leq 4\). So far there is no clear sign of any contradiction between the formulae and the experimental results.

We may go further and abstract from the model also the light-cone commutators of bilocal currents, in the limit in which all the intervals among the four points approach zero, that is to say, when all four points tend to lie on a light-like line. The same bilocal operators then recur as coefficients of the singularity, and the algebraic system closes.

The light cone results can be reformulated in terms of the null plane. We consider a commutator of local currents at two points \(x\) and \(y\) and allow the two points to approach the same null plane, say \(x_+ = x_3 + x_0 = 0\), \(y_+ = y_3 + y_0 = 0\). As mentioned above, when both current components are "good," we obtain a local commutation relation on the null plane, yielding another good component, or else zero. But when neither component is good, there is a singularity of the form \(\delta(x_+ - y_+)\) and the coefficient is a bilocal current on the null plane.
It is this singularity, arising from the light-cone singularity, that gives the Bjorken scaling.

On the null plane, with $x_+ = 0$, the three coordinates are the transverse spacelike coordinates $x_1$ and $x_2$ (called $\bar{x}_i$) and the lightlike coordinate $x_- \equiv x_3 - x_0$. Our bilocal currents $\Theta(x,y)$ on the null plane are functions of four coordinates: $x_-, y_-$ and $\bar{x}_i = \bar{y}_i$, since the interval between $x$ and $y$ is lightlike.

We may now consider the commutator of two bilocal currents defined on neighboring null planes (in each case with a lightlike interval between the two arguments of the bilocal current). Again, when neither current component is good, there is a $\delta$-function singularity of the spacing between the two null planes and the coefficient is a bilocal current defined on the common limiting null plane. In this language, as before in the light cone language, the system of bilocal currents closes.

We may commute two good components of bilocal currents on the same null plane, and, as for local currents, we obtain a good component on the right-hand side, without any $\delta$-function singularity at coincidence of the two null planes. Thus the good components of the bilocal currents $\Theta(x,y)$ form a Lie algebra on the null plane, a generalization of the old Lie algebra of local good components on the null plane (recovered by putting $x_- = y_-$).

Now, how far can we generalize this new Lie algebra on the null plane and still obtain exact formulae, formally true to all orders in the gluon constant, but independent of it, so that free quark formulae apply?

In the next section, we take up that question, but first we summarize the situation of current algebra on and near the null plane.
IV. SUMMARY OF LIGHT CONE AND NULL PLANE RESULTS

Let us now be a little more explicit. We are dealing with 144 bilocal quantities: $\tilde{F}_{1a}$, $\tilde{F}_{1a}^5$, $\tilde{F}_{1a}^j$, $\tilde{F}_{1a}$, and $\tilde{F}_{1a\beta}$, all functions of $x-y$ with $(x-y)^2 \to 0$.

Let us select the 3-direction for our null planes. Then in the model we can set $B_+ \equiv B_3 + B_0 = 0$ for the gluon potential by a choice of gauge. The gauge-invariance factor $\exp ig \int B \cdot d\ell$ for a straight line path on a null plane is just $\exp \left[ i\frac{g}{2} B_+ (x-y) \right] = 1$. Thus we have simple correspondences between our quantities and operators in the model:

$$\tilde{F}_{1a}^5 (x,y) \sim \frac{1}{2} \bar{q}(x) \lambda_j \gamma_a q(y), \text{ etc.}$$

and we have introduced the notation $\mathcal{D}(x,y, \frac{1}{2} \lambda_j \gamma_a)$, etc., where

$$\mathcal{D}(x,y,G) \sim \bar{q}(x)Gq(y) \sim q^+(x)(\delta G)q(y).$$

We are dealing with $\mathcal{D}(x,y,G)$ for every $12 \times 12$ matrix $G$, with

$$\tilde{F}_{1a}^5 (x,y) = \mathcal{D}(x,y, \frac{1}{2} \lambda_j \gamma_a \gamma_5), \quad \tilde{F}_{1a}^j (x,y) = \mathcal{D}(x,y, \frac{1}{2} \lambda_j),$$

$$\mathcal{D}(x,y, \frac{1}{2} \lambda_j \gamma_5), \text{ and } \mathcal{D}_{1a\beta} (x,y) = \mathcal{D}(x,y, \frac{1}{2} \lambda_j \gamma_a \gamma_5).$$

The good components, in the old equal-time $P_3 \to \infty$ language, were those with finite matrix elements between states of finite mass and $P_3 \to \infty$. By contrast, bad components were those with matrix elements going like $P_3^{-1}$ and terrible components those with matrix elements going like $P_3^{-2}$.

In the null plane language, good components are those for which $\delta G$ is proportional to $1 + \alpha_3$; thus the 36 good components are $\tilde{F}_{1a}^+, \tilde{F}_{1a}^5, \tilde{F}_{1a}^{j+}, \tilde{F}_{1a}^{j2+}$ for $j = 0 \ldots 8$. The terrible components are those for which $\delta G$ is proportional to $1 - \alpha_3$, hence $\tilde{F}_{1a}^-, \tilde{F}_{1a}^5, \tilde{F}_{1a}^{j1-}, \text{ and } \tilde{F}_{1a}^{j2-}$. The rest are bad; they have $\delta G$ anti-commuting with $\alpha_3$ so that $\alpha_3$ is $-1$ on the left and $+1$ on the right or vice versa.
Now the leading light cone singularity in the commutator of two bilocals is just given by the formula
\[
[S(x_+,y,G), S(u,v,G')] = iG(x_+,y,G)\partial A(v-u) - iG(u,v,G)\partial A(v-x),
\]
with \( \Delta(z) = (2\pi)^{-1} \epsilon(z)\delta(z^2) \).

When we commute two operators with coordinates lying on neighboring null planes with separation \( \Delta x^+ \), a singularity of the type \( \delta(\Delta x^+) \) appears (as we have mentioned in Section III) multiplied by a bilocal operator, with coordinates lying in the common null plane as \( \Delta x^+ \to 0 \), and it is this term that gives rise to Bjorken scaling. The term in question comes from the component \( \frac{\partial}{\partial z^+} \Delta(z) \) in \( \partial A(z) \), and is thus multiplied by \( S(x_+,v,IG\gamma, G') \) and \( S(u,v,IG\gamma, G) \). Now \( \beta(IG\gamma, G') = (1-\alpha)(\beta G') \), so it is clear that the singular Bjorken scaling term vanishes for good-good and good-bad commutators. In the case of the other components, we have, schematically, \( [\text{bad, bad}] \to \text{good}, [\text{bad, terrible}] \to \text{bad}, \) and \( [\text{terrible, terrible}] \to \text{terrible} \) for the Bjorken singularity.

The vector and axial vector local currents \( j^V(x,x) \) and \( j^A(x,x) \) occur, of course, in the electromagnetic and weak interactions. The local scalar and pseudoscalar currents occur in the divergences of the non-conserved vector and the axial vector currents, with coefficients that are linear combinations of the bare quark masses, \( m_u, m_d, \) and \( m_s \), treated as a diagonal matrix \( \mathcal{M} \). (Here \( m_u \) would equal \( m_d \) if isotopic spin conservation were perfect, while the departure of \( m_s \) from the common value of \( m_u \) and \( m_d \) is what gives rise to \( SU_3 \) splitting; the non-vanishing of \( \mathcal{M} \) is what breaks \( SU_3 \times SU_3 \).)

We see that all the 144 bilocals are physically interesting, including the tensor currents, because they all occur in the commutators of these local \( V, A, S, \) and \( P \) densities as coefficients of the \( \delta(\Delta x^+) \) singularity. Commuting a
local scalar with itself or a local pseudoscalar with itself leads to the
same bilocal as commuting a transverse component of a vector with itself,
and thus the light cone commutator of current divergences is predicted to
lead to Bjorken scaling functions that are proportional to those observed in
the light cone commutation of currents, while the coefficients permit the
experimental determination of the squares of the quark bare masses. Unfor­
tunately, the relevant experiments are difficult. (The finiteness of the
bare masses, as compared with the divergences encountered term by term in
renormalized perturbation theory in a gluon model, presumably has the same
origin as the scaling, which also fails term by term in renormalized
perturbation theory.)

As we have outlined in Section III, we begin the construction of the
algebraic system on the null plane by commuting the good bilocals with one
another. The leading singularity on the light cone (Eq.(4.1) gives rise to
the simple closed algebra we have mentioned, but we need also the additional
assumption that lower singularities on the light cone give no contribution to
the good-good commutators on the null plane. This additional assumption can
be squeezed out of the model in various ways. The simplest, however, is to
use canonical quantization of the quark-gluon model on the null plane.

In the model, the quark field \( q \) is written as \( q_+ q_- \), where \( q_\pm = \frac{1}{2} (1 \pm \alpha_3) q \).
Then \( q_+ \) obeys the canonical rules:

\[
\{ q_+^\alpha (x), q_+^\beta (y) \} = 0, \quad \{ q_+^\alpha (x), q_+^\beta (y) \} =
\]

\[
\delta^{(3)} (x-y) \frac{1}{2} (1+\alpha_3) \alpha_\beta
\]
on the null plane, where \( \delta^{(3)} (x-y) = \delta (x_+ - y_+) \delta (x_- - y_-) \).

Thus for any good matrices \( \beta A_{++} \) and \( \beta B_{++} \), we have on the null plane

\[
[\mathcal{O}(x,y,\beta A_{++}),\mathcal{O}(u,v,\beta B_{++})] = \mathcal{O}(x,v,\beta A_{++} B_{++}) \delta^{(3)} (y-u) - \mathcal{O}(u,y,\beta B_{++} A_{++}) \delta^{(3)} (v-x),
\]

which is just what we would get from (4.1) with no additional contribution
from lower light cone singularities.
The good-good commutation relations (4.2) on the null plane, together with the equations (4.1) for the leading light cone singularity in the commutator of two bilocal currents, illustrate how far we can go with abstracting free quark formulae that are formally unchanged in the model when the gluon coupling is turned on.

One may go further in certain directions. For example, the formulae for the leading light cone singularity presumably apply to disconnected as well as connected parts of matrix elements, and thus the question of the vacuum expected value of a bilocal operator arises. In the model, the coefficient of the leading singularity as \((x-y)^2 \to 0\) of such an expected value is formally independent of the coupling constant, and we abstract that as well - the answer here is dependent on statistics, however, and we assume the validity of quark statistics. Thus we obtain predictions like the following:

\[
\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) \to 2
\]

at high energy to lowest order in the fine structure constant.

The leading light cone singularity of an operator product, or of a physical order \((T^2)\) product, may also be abstracted from the model, except for certain subtraction terms (often calculable and/or unimportant) that behave like four-dimensional \(\delta\)-functions in coordinate space. To go from a commutator formula to a product formula to a physical ordered product formula, we simply perform the substitutions

\[
(2\pi)^{-\frac{1}{2}} e(z)\delta(z^2) \to (4\pi^2)^{-\frac{1}{2}}(z^2-i\epsilon)^{-1} + (4\pi^2)^{-\frac{1}{2}}(z^2-i\epsilon)^{-1}.
\]

With the aid of the product formulae and the vacuum expected values, we obtain the PCAC value of the \(\pi^0 \to 2\gamma\) decay amplitude.

Other exact abstractions from the vector gluon model that do not contain divergences and curl relations for local \(V\) and \(A\) currents:
\[ \frac{\partial}{\partial x_\mu} \mathcal{D}(x,x, \frac{i}{2} \gamma^\mu) = \mathcal{D}(x,x, \frac{i}{2} \{\gamma, \gamma\}), \]  
(4.3)

\[ \frac{\partial}{\partial x_\mu} \mathcal{D}(x,x, \frac{i}{2} \gamma^\mu_5) = \mathcal{D}(x,x, \frac{i}{2} \{\gamma, \gamma\}_5), \]  
(4.4)

but we also have, as presented elsewhere \(^2\),

\[ \frac{\partial}{\partial x_\nu} \mathcal{D}(x,x, \frac{i}{2} \gamma_\nu) = -\mathcal{D}(x,x, \frac{i}{2} \{\gamma, \gamma\}_\nu) + \left[ \left( \frac{\partial}{\partial x_\nu} - \frac{\partial}{\partial y_\nu} \right) \mathcal{D}(x,y, \frac{i}{2} \gamma_\nu) \right]_{x=y}, \]  
(4.5)

\[ \frac{\partial}{\partial x_\nu} \mathcal{D}(x,x, \frac{i}{2} \gamma_\nu_5) = -\mathcal{D}(x,x, \frac{i}{2} \{\gamma, \gamma\}_\nu_5) + \left[ \left( \frac{\partial}{\partial x_\nu} - \frac{\partial}{\partial y_\nu} \right) \mathcal{D}(x,y, \frac{i}{2} \gamma_\nu_5) \right]_{x=y}, \]  
(4.6)

and a number of other formulae, including the following:

\[ \left[ \left( \frac{\partial}{\partial x_\nu} - \frac{\partial}{\partial y_\nu} \right) \mathcal{D}(x,y, \frac{i}{2} \gamma_\nu) \right]_{x=y} = \mathcal{D}(x,x, \frac{i}{2} \{\gamma, \gamma\}). \]  
(4.7)

In the last three formulae, it must be pointed out that for a general direction of \(x-y\) we have the gauge-invariant correspondence

\[ \mathcal{D}(x,y,G) \sim \bar{q}(x)Gq(y) \exp ig \int B \cdot dl, \]  
which is independent of the path from \(y\) to \(x\) when the coordinate difference and the path are taken as first order infinitesimals. The first internal derivative

\[ \left[ \left( \frac{\partial}{\partial x_\mu} - \frac{\partial}{\partial y_\mu} \right) \mathcal{D}(x,y,G) \right]_{x=y} \]

is physically interesting for all directions \(\mu\) (and not just the \(-\) direction), as a result of Lorentz covariance.

In Eqs. (4.5–4.7), we have for the moment thrown off the restriction to a single null plane.
In the next Section, we return to the consideration of the algebra on the null plane, and we see how further extensions give a much wider algebra, in which departures from free quark relations begin to appear.
V. THE FURTHER EXTENSION OF NULL PLANE ALGEBRA

We now look beyond the commutation relations of good bilocals on the null plane. In the model, then, we have to examine operators containing $q_-$ or $q_+^+$ or both. The Dirac equation in the gauge we are using ($B_+ = 0$ on the null plane) tells us that we have

$$-2i \frac{\partial q}{\partial x} = [q_1, (-i\psi_1 - g\psi_1) + \beta\eta] q_+.$$

(5.1)

In terms of Eq. (5.1), we can review the various anticommutators on the null plane. We have already discussed the trivial one,

$$\{q_+(x), q_+^+(y)\} = \delta(x_+ - y_+) \cdot \frac{1}{2} (1 + \alpha_3) \delta(x_+ - y_+).$$

(5.2)

Using (5.1), (5.2), the fact that $B_\perp$ commutes with $q_+$ on the null plane, and the equal-time anticommutator $\{q_-, q_+^+\} = 0$, we obtain the well-known result

$$\{q_-(x), q_+^+(y)\} = \frac{i}{4} \epsilon(x_+ - y_+) [q_1, (-i\psi_1 - g\psi_1) + \beta\eta] \frac{1}{2} (1 + \alpha_3) \delta(x_+ - y_+).$$

(5.3)

Using the same method a second time, one finds, for $y_+ > x_+$,

$$\{q_-(x), q_+^+(y)\} = \frac{1}{8} \int_{x_+}^{y_+} dr_+ [q_1, (-i\psi_1 - g\psi_1) + \beta\eta] \left(\frac{1 - \alpha_3}{2}\right) \delta(x_+ - y_+).$$

(5.4)

where the singularity at the coincidence of the two null planes appears as an unpleasant integration constant. This singularity is, of course, responsible in the model for the Bjorken singularity in the commutator of two bad or terrible operators.

Because of the singularity, it is clumsy to construct the wider algebra by commuting all our bilocals with one another. Instead, we adopt the following procedure. Whenever a bilocal operator corresponds to one in the model containing $q_+^+(x)$, we differentiate with respect to $x_-$; whenever it corresponds to one in the model containing $q_-(y)$, we differentiate with respect to $y_-$. Thus
we "promote" all our bilocals to good operators. We construct the wider algebra by starting with the original good bilocals and these promoted bad and terrible bilocals. We commute all of these, commute their commutators, and so forth, until the algebra closes. Then, later on, if we want to commute an unpromoted operator, we use the information contained in equations of the model like (5.1) - (5.3) to integrate over $x_-$ or $y_-$ or both and undo the promotion. (A similar situation obtains for operators corresponding to those in the model containing the longitudinal gluon potential $B_-$.)

Now let us classify the matrices $\beta G$ into four categories:

- The good ones, $\beta G = A_{++}$, with $\alpha_3 = 1$ on both sides;
- The bad ones $\beta G = A_{+-}$ that have $\alpha_3 = 1$ on the left and $-1$ on the right;
- The bad ones $\beta G = A_{-+}$ that have $\alpha_3 = -1$ on the left and $+1$ on the right;
- And the terrible ones $\beta G = A_{--}$, with $\alpha_3 = -1$ on both sides.

Then, wherever $q_-$ or $q_+^+$ appears, we promote the operator by differentiating $q_-$ or $q_+^+$ with respect to its argument in the $-$ direction. We obtain, then:

- $\mathcal{O}(x, y, \beta A_{++})$, the good operators, unchanged;
- $\frac{\partial}{\partial x_-} \mathcal{O}(x, y, \beta A_{+-})$ and $\frac{\partial}{\partial y_-} (x, y, \beta A_{-+})$, promoted bad operators;
- $\frac{\partial}{\partial x_-} \frac{\partial}{\partial y_-} \mathcal{O}(x, y, \beta A_{--})$, promoted terrible operators.

All 144 of these operators now are given, in the model, in terms of $q_+$ and $q_+^+$, but the promoted bad and terrible operators involve the expressions $(\gamma_\parallel - ig B_\parallel)q_+$ and $(\gamma_\perp + ig B_\perp)q_+^+$. In fact, substituting the Dirac equation for $\frac{\partial q_-}{\partial x_-}$ into the definitions for the promoted bad and terrible operators, we see that we obtain good operators (with coefficients depending on bare quark
masses) and also good matrices sandwiched between \((v_1^+ + ig B_1)q_+^+\) and \(q_+\) or
between \(q_+^+\) and \((v_1^- - ig B_1)q_+\) or between \((v_1^+ + ig B_1)q_+^+\) and \((v_1^- - ig B_1)q_+\).

The null plane commutators of all these operators with one another are
finite, well-defined, and physically meaningful, but they lead to an enormous
Lie algebra that is not identical with the one for free quarks, but instead
contains nearly all the degrees of freedom of the model.

Let us first ignore any lack of commutation of the \(B\)'s with one another.
We keep commuting the operators in question with one another. When \(v_1^+ \pm ig B_1\)
appears acting on a \(\delta^{(3)}\) function, we can always perform an integration and
fold it over onto an operator. Thus the number of applications of \(v_1^+ \pm ig B_1\)
grows without limit. Since these gauge derivatives do not commute with one
another, but give field strengths as commutators, it can easily be seen that
we end up with all possible operators corresponding to \(\bar{q}_+(x)Gq_+(y)\) acted on by
any gauge invariant combination of transverse gradients and potentials. We
have, to put it differently, the operators corresponding to \(\bar{q}_+(x)Gq_+(y)\exp ig \int B \cdot d\xi\)
for any pair of points \(x\) and \(y\) on the null plane connected by any path \(P\) lying
in the null plane. We could think of these as operators \(\mathcal{O}(x,y,G,P)\) depending
on the path \(P\), with \(\delta G = A_++\).

In fact the \(B\)'s do not commute with one another in the model, and so we
get an even more complicated result. We have \([B_{li}(x), B_{lj}(y)] = \epsilon(x-y)\delta(x_i-y_i)\delta_{ij}\)
on the null plane, and the commutation of promoted bad and terrible bilocals
with one another leads to operators corresponding to \(\bar{q}_+(x)Gq_+(y)\bar{q}_+(a)G'q_+(b)\).
Further commutation then introduces an unlimited number of sideways gradients,
gluon field strengths, and additional quark pairs, until we end up with all
possible operators of the model that can be constructed from equal numbers of
\(\bar{q}_+\)'s and \(q_+\)'s at any points on the null plane and from exponentials of \(ig \int B \cdot d\xi\)
for any paths connecting these points.
If we keep track of color, we note that only color singlets are generated. If the gluons are a color octet Yang-Mills field, we must make suitable changes in the formalism, but again we find that only color singlets are generated.

The coupling constant $g$ that occurs is, of course, the bare coupling constant. It may not be intrinsic to the algebraic system (equivalent to that of quarks and gluons) on the null plane, but it certainly enters importantly into the way we reach the system starting from well-known operators.

A troublesome feature of the extended null plane algebra is the apparent absence of operators corresponding to those in the model that contain only gluon field strengths and no quark operators; for a color singlet gluon, the field strength itself would be such an operator, while for a color octet gluon we would begin with bilinear forms in the field strength in order to obtain color singlet operators. Can we obtain these quark-free operators by investigating discontinuities at the coincidence of coordinates characterizing quark and antiquark fields in the model? At any rate, we certainly want these quark-free operators included in the extended algebra.

Now when our algebra has been extended to include the analogs of all relevant operators of the model on the null plane that are color singlets and have baryon number $A = 0$, then the Hilbert space of all physical hadron states with $A = 0$ is an irreducible representation of the algebra.

If we wish, we might as well extend the algebra further by including the analogs of color singlet operators of the model (on the null plane) that would change the number of baryons. In that case, the entire Hilbert space of all hadron states is an irreducible representation of the complete algebra. From now on, let us suppose that we are always dealing with the complete color singlet algebra (whether the one abstracted from the quark-gluon model or some other) and with the complete Hilbert space, which is an irreducible representation of it.
The representation may be determined by the algebra and the uniqueness of the physical vacuum. We note that we are dealing with arbitrarily multilocal operators, functions of any number of points on the null plane. We can Fourier transform with respect to all these variables and obtain Fourier variables \((k_+, k_L)\) in place of the space coordinates. Since \(B_+ = 0\), there is no formal obstacle to thinking of each \(k_+\) as being like the contribution of the individual quark, antiquark or gluon to the total \(P_+ = \sum k_+\). Now \(P_+ = 0\) for the vacuum, and for any other state we can get \(P_+ = 0\) only by taking \(P_z \rightarrow -\infty\). The same kind of smoothness assumption that allows scaling can allow us to forget about matrix elements to such infinite momentum states. In that case, we have the unique vacuum state of hadrons as the only state of \(P_+ = 0\), while all others have \(P_+ > 0\). All Fourier components of multilocal operators for which \(\sum k_+ < 0\) annihilate the physical vacuum. (Note in the null plane formalism we do not have to deal with a fictitious "free vacuum" as in the equal-time formalism.) The Fourier components of multilocal operators with \(\sum k_+ > 0\) act on the vacuum to create physical states, and the orthogonality properties of these states and the matrix elements of our operators sandwiched between them are determined largely or wholly by the algebra. The details have to be studied further to see to what extent the representation is really determined. (The vacuum expected values contain one adjustable parameter in the case of free quarks, namely the number of colors.)

Once we have the representation of the complete color singlet algebra on the null plane, as well as the algebra itself, then the physical states of hadrons can all be written as linear combinations of the normalized basis states of the representation. These coefficients represent a normalized set of Fock space wave functions for each physical hadron state, with orthogonality relations for orthogonal physical states. Since the matrix elements of all null
plane operators between basis states are known, the matrix elements between physical states of bilocal currents or other operators of interest are all calculable in terms of the Fock space wave functions. 9)

This situation is evidently the one contemplated by "parton" theorists such as Feynman and Bjorken; they suppose that we know the complete algebra, that it comes out to be a quark-gluon algebra, and that the representation is the familiar one, so that there is a simple Fock space of quark, antiquark, and gluon coordinates. In the Fourier transform, negative values of each k correspond to destruction and positive values to creation.

Now the listing of hadron states by quark and gluon momenta is a long way from listing by meson and baryon momenta. However, as long as we stick to color singlets, there is not necessarily any obstacle to getting one from the other by taking linear combinations. The operator $M^2 = -p_+^2 - P_+ P_-$ has to be such that its eigenvalues correspond to meson and baryon configurations, and not to a continuum of quarks, antiquarks, and gluons.

The important physical questions are whether we have the correct complete algebra and representation, and what the correct form of $\Theta_{\mu\nu}$ or $p_\mu$ or $M^2$ is, expressed in terms of that algebra.

In the quark-gluon model we have $\Theta_{\mu\nu} = \Theta_{\nu\mu}^{\text{quark}} + \Theta_{\nu\mu}^{\text{glue}}$, where

$$\Theta_{\mu\nu}^{\text{quark}} = \frac{1}{4} q_{\nu}^{\prime} (\partial_{\nu} - ig B_{\nu}) q + \frac{1}{4} q_{\mu}^{\prime} (\partial_{\mu} - ig B_{\mu}) q - \frac{1}{4} (\partial_{\mu} + ig B_{\mu}) q_{\nu}^{\prime} q - \frac{1}{4} (\partial_{\nu} + ig B_{\nu}) q_{\mu}^{\prime} q$$

and $\Theta_{\mu\nu}^{\text{glue}}$ does not involve the quark variables at all. The term $\Theta_{\mu\nu}^{\text{quark}}$, by itself, has the wrong commutation rules to be a true $\Theta_{\mu\nu}$ (unless $g = 0$). For example, $(P_1, P_2, P_3) \neq 0$. The correct commutation rules are restored when we add the contribution from $\Theta_{\mu\nu}^{\text{glue}}$.

We can abstract from the quark-gluon model some or all of the properties of $\Theta_{\mu\nu}$, in terms of the null plane algebra. We see that in the model we have
and, as is well-known, the expected value of the right-hand side in the proton state can be measured by deep inelastic experiments with electrons and neutrinos. All indications are that it is not equal to the expected value of $\theta^+_\perp$, but rather around half of that, so that half is attributable to gluons, or whatever replaces them in the real theory.

In general, using the gauge-invariant definition of $\mathcal{D}$, we have in the model

$$\theta^{\text{quark}}_{\perp\perp} = \left[ \left( \frac{\partial}{\partial y_-} - \frac{\partial}{\partial x_-} \right) \mathcal{O}(x,y,\frac{1}{2} \gamma_+) \right] \bigg|_{x=y} \tag{5.5}$$

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In general, using the gauge-invariant definition of $\mathcal{D}$, we have in the model

$$\theta^{\text{quark}}_{\mu\nu} = \left[ \left( \frac{\partial}{\partial y_\nu} - \frac{\partial}{\partial x_\nu} \right) \mathcal{O}(x,y,\frac{1}{4} \gamma_\mu) + \left( \frac{\partial}{\partial y_\mu} - \frac{\partial}{\partial x_\mu} \right) \mathcal{O}(x,y,\frac{1}{4} \gamma_\nu) \right] \bigg|_{x=y} \tag{5.6}$$

and Eq. (4.7) then gives us the obvious result

$$\theta^{\text{quark}}_{\mu\mu} = \mathcal{D}(x,x,\gamma_\mu). \tag{5.7}$$

Whereas in (5.5) we are dealing with an operator that belongs to the null plane algebra generated by good, promoted bad, and promoted terrible bilocal currents, other components of $\theta^{\text{quark}}_{\mu\nu}$ are not directly contained in the algebra; neither are the bad and terrible local currents, nor their internal derivatives in directions other than $\perp$. In order to obtain the commutation properties of all these operators with those actually in the algebra, we must, as we mentioned above, undo the promotions by abstracting the sort of information contained in (5.3) and (5.4). Thus we are really dealing with a wider mathematical system than the closed Lie algebra abstracted from that of operators in the model containing $q^+, q_-^+$ and $\bar{B}^\perp_L$ only.

We shall assume that the true algebraic system of hadrons resembles that of the quark-gluon model at least to the following extent:
1) The null plane algebra of good components (4.2) and the leading light cone singularities (4.1) are unchanged.

2) The system acts as if the quarks had vectorial coupling in the sense that the divergence equations (4.3) and (4.4) are unchanged.

3) There is a gauge derivative of some kind, with path-dependent bilocals that for an infinitesimal interval become path-independent. Eqs. (4.5) - (4.7) are then defined and we assume they also are unchanged.

4) The expression (5.6) for $\theta^{\text{quark}}_{\mu\nu}$ is also defined and we assume it, too, is unchanged, along with its corollary (5.7).

About the details of the form of the path-dependent null plane algebra, arising from the successive application of gauge derivatives, we are much less confident, and correspondingly we are also less confident of the nature of the gluons, even assuming that we can decide whether to use a color singlet or a color octet. What we do assert is that there is some algebraic structure analogous to that in quark-gluon theory and that it is in principle knowable.

One fascinating problem, of course, is to understand the conditions under which we can have an algebra resembling that for quarks and gluons and yet escape having real quarks and gluons. Under what conditions do the bilocals act as if they were the products of local operators without, in fact, being so? We seek answers to this and other questions by asking, "Are there models other than the quark-gluon field theory from which we can abstract results? Can we replace $\theta^{\text{glue}}_{\mu\nu}$ by something different and the gauge-derivative by a different gauge-derivative?"
VI. ARE THERE ALTERNATIVE MODELS?

In the search for alternatives to gluons, one case worth investigating is that of the simple dual resonance model. It can be considered in three stages: first, the theory of a huge infinity of free mesons of all spins; next, tree diagrams involving the interaction of these mesons; and finally loop diagrams. The theory is always treated as though referring to real mesons, and an S-matrix formulation is employed in which each meson is always on the mass shell.

Now the free meson stage of the model can easily be reformulated as a field theory in ordinary coordinate space, based on a field operator $\phi$ that is a function not of one point in space, but of a whole path - it is infinitely multilocal. The free approximation to the dual resonance model is then essentially the quantum theory of a relativistic string or linear rubber band in ordinary space.

The coupling that leads, on the mass shell, to the tree diagrams of the dual resonance model has not so far been successfully reformulated as a field theory coupling but we shall assume that this can be done. Then the whole model theory, including the loops, would be a theory of a large infinity of local meson fields, all described simultaneously by a grand infinitely multilocal field $\phi$, coupled to themselves and one another. The mesons, in the free approximation, lie on straight parallel Regge trajectories with a universal slope $\alpha'$.

In the simplest form of such a theory, the grand field $\phi$(path) can be resolved into local fields $\phi(R), \phi_{\mu}(R), \phi_{\mu,\nu}(R), \ldots$. There is a single scalar, a single infinity of vectors, a double infinity of tensors and scalars, and so forth. The matrices $a_{\mu\nu}$ and $a^+_{\mu\nu}$ of the dual theory connect these components of $\phi$ with one another.
Perhaps the model theory of a gluon field can be replaced by a field theory version of a dual resonance model; the properties of operators, including $\theta_{\mu\nu}$, would be abstracted from the new model instead of the old one. With $\alpha' \neq 0$, a term $\delta$ would naturally appear that violates scale invariance and is not related to the bare quark masses. (Probably $\delta = 0$ here rather than -2 as in the case of a gluon mass.) The gauge derivative in the other portion of $\theta_{\mu\nu}$, referring to the quarks, would then involve a special linear combination of the $\phi_{\mu
u}(R)$ instead of the gluon potential $B_{\mu}(R)$.

An amusing point is that in the limit of a dual resonance theory as $\alpha' \rightarrow 0$ (so that the trajectories become flat), with attention concentrated on the value $\alpha = 1$, if the mathematics of a Lie group is built into the model, then the mass shell predictions become those of the corresponding massless Yang-Mills theory. That suggests that one might even try a dual resonance model as a replacement of a color octet Yang-Mills gluon model, with abstraction of the properties of color singlet operators.

We are not at all sure that what we are discussing here is a practical scheme, and if it is, we do not know how the resulting algebraic system differs from that of gluons. We put it forward merely in order to stimulate thinking about whether or not there are candidates for the algebra, the representation, and the form of $\theta_{\mu\nu}$ other than those suggested by the gluon model.

Our attempt to use the dual model to construct a field theory has no bearing on whether the mass-shell dual model can lead to a complete S-matrix theory of hadrons; our suggestion resembles the use of limits of dual theories to obtain unified theories of weak and electromagnetic interactions or the theory of gravity.

One interesting speculation that is independent of what model we use for the stuff to which quarks are coupled is that perhaps when we perform the mathematical transformation from current quarks to constituent quarks and
obtain the crude naive quark model of meson and baryon spectra and couplings, the gluons or whatever they are will also be approximately transformed into fictitious constituents, so that meson states would appear that act as if they were made of gluons rather than $q\bar{q}$ pairs. If there are indeed ten low-lying scalar mesons rather than nine, then we might interpret the tenth one (roughly speaking, the $\pi^0$ meson) as the beginning of such a sequence of extra $SU_3$ singlet meson states. (A related question, much debated by specialists in the usual, mass-shell dual models, is whether the infinite sequence of meson and baryon Regge trajectories, all rising indefinitely and straight and parallel in zeroth approximation, should be extended to exotic channels, i.e., those with quantum numbers characteristic of $qqqq\bar{q}$, $q\bar{q}qq$, etc.)

Let us end by emphasizing our main point, that it may well be possible to construct an explicit theory of hadrons, based on quarks and some kind of glue, treated as fictitious, but with enough physical properties abstracted and applied to real hadrons to constitute a complete theory. Since the entities we start with are fictitious, there is no need for any conflict with the bootstrap or conventional dual model point of view.
We have described in Sections III and IV a Lie algebra of good components of bilocal operators on a null plane. The generators are 36 functions of $x_-, y_-$ and $x_0 = x_1$, namely $\mathcal{J}_1^+, \mathcal{J}_0^+, \mathcal{J}_1^+, \mathcal{J}_2^+$, and $\mathcal{J}_2^+$. We define $R \equiv 1/2 (x+y)$ and $z \equiv x-y$; then we have functions of $R_1, R_-, z_-.$

With $z_-$ set equal to zero, we have just the usual good local operators on the null plane, related to the corresponding good local operators at equal times with $P_3 \to \infty$. We recall that in the early work using $P_3 \to \infty$ the most useful applications (fixed virtual mass sum rules) involved matrix elements with no change of longitudinal momentum, i.e., transverse Fourier components of the operators. Dashen and Gell-Mann studied these operators and found that between finite mass states their matrix elements do not depend separately on the transverse momenta of the initial and final states, but only on the difference, which is the Fourier variable $k_1$. Thus they obtained a "form factor algebra" generated by operators $F_1(k_1)$ and $F_2(k_1)$, to which, of course, one may adjoin $T_{++}(k_1^2)$ and $T_{+2}(k_1^2)$.

We may consider the analogous quantities using the null plane method and generating to bilocals:

$$F_1(k_1, z_-) \equiv \int d^4R_+(R_1^2) \mathcal{J}_1^+(R, z_-) \exp i \frac{k_1}{R_1^2} (R_1^2 + R_2^2 - (A_1 + J_2)) \exp i \frac{k_2}{R_2^2} (R_2^2 + R_2^2 - (A_2 - J_1))$$

and so forth. Here the integration over $R_-$ assures us that $P_+ \equiv P_o + P_3$ is conserved by the operator. (We note that Minkowski and others have studied the interesting problem of extracting useful sum rules from operators unintegrated over $R_-$, but we do not discuss that here.) The quantities $P_+^{-1}(A_1 + J_2)$ and $P_+^{-1}(A_2 - J_1)$ act like negatives of center-of-mass coordinates,
-R₁ and -R₂, since on the null plane x⁺ = 0 we have Λ₁⁺J₂ = -∫ R⁺δ(x⁺) dσ(R⁺) and Λ₂⁺J₁ = -∫ R⁺δ(x⁺) dσ(R⁺), while F₂⁺ = ∫ R⁺δ(R⁺).

Our bilocal form factor algebra has the commutation rules

\[ [F_i(k_1', z_-), F_j(k_2', z_-')] = i \sum_{k_1} F_k(k_1 + k_1', z_- + z_-'), \]  

etc., where the structure constants in general are those of \[U_{\omega}^{\omega}.\] Putting \(z_- = z_-' = 0\), we obtain exactly the form factor algebra of Dashen and Gell-Mann. If we specialize further to \(k_1 = k_1' = 0\), we obtain the algebra \[U_{\omega}^{\omega}\], currents of vector, axial vector, and tensor charges. It is not, of course, identical to the approximate symmetry algebra \[U_{\omega}^{\omega}\] for baryon and meson spectra and vertices, but is related to it by a transformation, probably unitary. That is the transformation which we have described crudely as connecting current quarks and constituent quarks.

The behavior of the operators \(F_i(k_1), etc.,\) with respect to angular momentum in the s-channel is complicated and spectrum-dependent; it was described by Dashen and Gell-Mann in their angular condition. There is a similar angular condition for the bilocal generalizations \(F_i(k_1, z_-), etc.\)

The behavior of \(F_i(k_1', z_-)\) and the other bilocals with respect to angular momentum in the cross-channel is, in contrast, extremely simple. If we expand \(F_i(k_1', z_-)\) or \(F_i^5(k_1', z_-)\) in powers of \(z_-\), each power \(z_-\) corresponds to a single angular momentum, namely \(J = n + 1\).

As we expand \(F_i(k_1', z_-), etc.,\) in power series in \(z_-\), we note that each term, in \(z_-^{-1}\), has a pole in \(k_1^2\) at \(k_1^2 + M^2 = 0\), where \(M\) is the mass of any meson of spin \(J\). By an extension of the Regge procedure, we can keep \(k_1^2\) fixed and let the angular momentum vary by looking at the asymptotic behavior of matrix elements of \(F_i(k_1', z_-), etc.,\) at large \(z_-\). A Regge pole in the cross channel gives a contribution \(\frac{\alpha(-k_1^2)}{\beta(-k_1^2)}[\sin \alpha(-k_1^2)]^{-1}\) and a cut gives...
a corresponding integral over \( \alpha \). Thus the bilocal form factor \( F_i(k_1, z) \) couples to each Reggeon in the non-exotic meson system in the same way that \( F_i(k_1) \) couples to each vector meson. The contribution of each Regge pole to the asymptotic matrix element of \( F_i(k_1, z) \) between hadron states A and B is given by the coupling of \( F_i(k_1, z) \) to that Reggeon multiplied by the strong coupling constant of the Reggeon to A and B.

It would be nice to substitute the Regge asymptotic behavior of \( F_i(k_1, z) \) etc., into the commutation rules and obtain algebraic relations among the Regge residues. Unfortunately, the asymptotic limit is not approached uniformly in the different matrix elements, and the asymptotic Regge formulae cannot, therefore, be used for the operators everywhere in the equations (A.2); only partial results can be extracted.
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A COMMENT ON SATURATION OF THE ADLER SUM RULE

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One of the most rigorous theoretical results to be tested in the upcoming experiments is the Adler sum rule. In the past months we have witnessed renewed interest on the saturation of the sum rule, with several of the authors arriving at the conclusion that the sum rule will not be saturated even in the multi-GeV region. It is the purpose of this note to make a critical analysis of the situation. In the standard notation the sum rule reads

\[ \int_0^\infty [W_2^\nu(Q^2, \nu) - W_2^{\nu p}(Q^2, \nu)] d\nu = 2 \cos^2 \theta_c \approx 1.86 \]  

or in the Bjorken scaling region (keeping only the vector contribution)

\[ \int_0^1 \left[ F_2^{\nu}(x) - F_2^{\nu p}(x) \right] \frac{dx}{x} = \cos^2 \theta_c = 0.93. \]  

The sum rule has been tested (via PCAC) in the \( Q^2 \ll M^2 \) region. It was found that it saturates to more than 95% of its value for \( \nu \sim 5 \text{ GeV} \).

Under zero dynamical assumptions, the sum rule for the vector contribution alone can be bounded from above by using the Bjorken inequality

\[ 2.8 \pm 0.2 \geq 2 \int_0^1 \left[ F_2^{\nu p} + F_2^{\nu n} \right] \frac{dx}{x} \geq 2 \int_0^{0.05} \left[ F_2^{\nu p} + F_2^{\nu n} \right] \frac{dx}{x} \geq 2 \int_0^{0.05} \left[ F_2^{\nu n} - F_2^{\nu p} \right] \frac{dx}{x} \approx 0.93. \]  

The numerical value was evaluated in terms of the SLAC-MIT data. The bound is obviously very crude. However, it does not indicate anything very alarming. In fact, it does not exclude the possibility that the sum rule could be saturated (to within 90%) at \( x \sim 0.05 \), provided that the structure functions are quite different. Table I illustrates the situation. Define

\[ z = \frac{F_2^{\nu n}}{F_2^{\nu p}}. \]  

Then the sum rule reads

\[ z \frac{z-1}{z+1} \int_0^1 \left[ F_2^{\nu p} + F_2^{\nu n} \right] \frac{dx}{x} \geq \int_0^{0.05} \left[ F_2^{\nu n} + F_2^{\nu p} \right] \frac{dx}{x}. \]  

Assuming that the isoscalar contribution to the sum of the electroproduction functions is small (~10%) we obtain the average value of \( z \) required in the non-Regge and the intermediate region.

Table I gives the average \( z \) and the corresponding contribution to the sum rule is denoted by \( \Sigma \). The sum rule can be saturated easily provided that the ratio \( F_2^{\nu n}/F_2^{\nu p} \approx 2.5-4 \) in the non-Regge region.

*Operated by Universities Research Association, Inc. under contract with the United States Atomic Energy Commission.
Table I. Three Examples Which Provide Early Saturation of the Sum Rule.

<table>
<thead>
<tr>
<th>Non-Regge Region</th>
<th>Intermediate Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.2 \leq x \leq 1.0$</td>
<td>$0.1 \leq x \leq 0.2$</td>
</tr>
<tr>
<td>$\bar{z} = 2.5$</td>
<td>$\bar{z} = 2.4$</td>
</tr>
<tr>
<td>$\Sigma = 0.58$</td>
<td>$\Sigma = 0.32$</td>
</tr>
<tr>
<td>$\bar{z} = 3.0$</td>
<td>$\bar{z} = 1.8$</td>
</tr>
<tr>
<td>$\Sigma = 0.68$</td>
<td>$\Sigma = 0.22$</td>
</tr>
<tr>
<td>$\bar{z} = 4.0$</td>
<td>$\bar{z} = 1.25$</td>
</tr>
<tr>
<td>$\Sigma = 0.82$</td>
<td>$\Sigma = 0.08$</td>
</tr>
</tbody>
</table>

Such a value is not unreasonably large. Nachtmann's\(^3\) positivity relations give the bound

$$F_2^{\nu n} / F_2^{\nu p} \geq \frac{1}{2}. \quad (6)$$

Whenever the ratio $y(x) = F_2^{\nu n} / F_2^{\nu p}$ is known, as is the case with the MIT experiment,\(^4\) we can improve the bound on the neutrino structure functions

$$z(x) = \frac{F_2^{\nu n}}{F_2^{\nu p}} \geq \frac{1}{2} + \frac{4}{3} \frac{1}{4y - 1}. \quad (7)$$

This bound is better than Eq. (6) for $y < 2/3$. The results shown in Fig. 1 indicate that for small values of $y$, $z$ should be quite large.

The conclusion is that there is still enough freedom for the sum rule to be saturated for a value of $x > 0.050 - 0.025$. This, however, leads to other observable results at larger values of $x$.\(^5\) The early saturation requires a contribution to the cross section which is large at intermediate values of $x$ but goes away faster than $\sqrt{x}$ as $x \to 0$. Such a contribution can arise from a $J \leq 1/2$ singularity.

References

1. Slow convergence of the integral has been discussed by:
   b. J. D. Bjorken and S. F. Tuan, Comments on Nuclear and Particle Physics 5, No. 3, 70 (1972), #79.

In these papers the rate of convergence is determined essentially by the $\rho$-trajectory, whose residue is adjusted to fulfill the sum rule. Lower trajectories could in general be present and change the rate of saturation drastically.


Fig. 1. A lower bound (open circles) for the ratio $F_{2n}^{u}/F_{2}^{v}$ implied by the electroproduction data (solid dots with errors). The electroproduction data are taken from Ref. 4.
FIELD THEORY IN LESS THAN FOUR DIMENSIONS; THE RENORMALIZATION GROUP

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Two topics are summarized here. The first is field theory in space-time dimension \( d \) between 2 and 4. The second is a reformulation of the renormalization group. Until now the principal model field theory exhibiting anomalous dimensions has been the Thirring model in one-space, one-time dimension. This is a very special model and is insufficient to give a general understanding of the phenomenon.

In the past year a vast number of models exhibiting anomalous dimensions have become solvable. The study of these models has just begun; when it is complete one should have a much better understanding of the physics of anomalous dimensions.

The new models are field theories defined for space-time dimensions \( d \) in the range \( 2 < d < 4 \). They unfortunately all reduce to trivial free field theories for \( d = 4 \). The models include the standard theories which are renormalizable in four dimensions (\( \phi^4 \) theory, etc.) but with an unconventional renormalization procedure to prevent these theories from being super-renormalizable in less than 4 dimensions. For example, for \( \lambda_0 \phi^4 \) theory one requires \( \lambda_0 \) to behave as \( \Lambda^{4-d} \) as the cutoff \( \Lambda \to \infty \).

Figure 1 is a map showing the various types of model field theories studied to date. Mack’s work on the \( \phi^3 \) theory in \( 6 + \epsilon \) dimensions is related to the theories discussed here.

Two new expansion techniques have been developed to solve these theories. Both techniques are stolen from previous work on statistical mechanics. The simplest expansion is the \( 1/N \) expansion: \( N \) is the number of internal components of a scalar field \( \phi \) or a spinor field \( \psi \). The interactions \( \lambda_0 (\phi^2)^2 \) and \( G_0 (\bar{\psi} \psi)^2 \) (with \( \phi^2 = \sum_{i=1}^{N} \phi_i^2 \), etc.) have been studied for large \( N \). This is possible for the whole range \( 2 < d < 4 \). The idea that the \( N \to \infty \) limit is soluble is due to Stanley. The essential idea is that the easily calculable bubble graphs are the dominant graphs for large \( N \) (Fig. 2). The other technique, discovered by Wilson and Fisher, is an expansion in powers of \( \epsilon = 4 - d \). This can be done for any \( N \). Studies in statistical mechanics show that this expansion gives good results for \( d = 3 \). The continuation to nonintegral dimension \( d \) has been most extensively discussed by ’t Hooft and Veltman.

The most interesting two results obtained to date from these models are the following. Anomalous dimensions have been calculated in powers of \( \epsilon \) (or \( 1/N \)) for a number of operators. In particular the anomalous dimensions of the \( n \)th rank tensor operators governing deep inelastic scattering in the \( \phi^2 \) theory have been computed to order \( \epsilon^2 \). The result for all \( N \) is that the anomaly (departure from canonical dimension) is positive, different for different \( N \) and smaller than \( \epsilon^2/96 \). For \( N \to \infty \) the anomaly goes to zero. This is a remarkably small anomaly since the only reasonable values of \( \epsilon \) are 1 or 2. The second result is that for large \( N \) the Fermi interaction \( G_0 (\bar{\psi} \psi)^2 \) is equivalent

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to the Yukawa interaction $g \phi(\Psi)$ where $\phi$ is a single scalar field, provided the coupling constants are renormalized so that $G_0 \Lambda^{-\epsilon}$ and $g_0 \Lambda^{-\epsilon}$ are held fixed as the cutoff $\Lambda \to \infty$. This is a similar result to Bjorken's discussion of quantum electrodynamics arising from a vector Fermi interaction.\(^9\)

A deeper development of the last few years is a complete reformulation\(^10\) of the renormalization group ideas of Gell-Mann and Low.\(^11\) The reformulation borrows heavily from previous ideas of Kadanoff\(^12\) and others in statistical mechanics. It is a complex of ideas designed to meet head-on the problems which in the past have seemed hopeless both in field theory and statistical mechanics. It goes well beyond the (present) Glimm-Jaffe program\(^13\) in that it deals with theories requiring coupling-constant renormalization. The modern renormalization group is already important in classical statistical mechanics;\(^14\) considerable work remains before one will know whether it can solve the presently insoluble problems of strong interactions.

Two results will be quoted from the renormalization group work. The first result is an indication that the $\lambda \phi^4$ theory (with $N = 1$) is trivial after renormalization for any value of $\lambda_0$ in the range $0 \leq \lambda_0 \leq \infty$ (infinity is included).\(^15\) In other words, the only hope to obtain a nonzero renormalized coupling constant $\lambda_R$ is to allow negative or complex $\lambda_0$ in either case leading to unpleasant consequences. (This is similar to what happens in the Lee model.\(^16\)) This result is obtained by two completely unrelated methods. One method is to calculate a form of the Gell-Mann-Low eigenvalue function $\psi(\lambda_0)$ (see Adler's talk) using the Schiff expansion.\(^17\) In Schiff's approach one uses a lattice as a cutoff and expands in the part of the kinetic term $(\partial \phi^2)$ which couples different lattice sites. Using modern statistical mechanical techniques\(^18\) this expansion was calculated to 9 nontrivial orders. The other method is a direct approximate solution of the $\phi^4$ theory using an approximate formulation of the renormalization group.\(^19\) Either method by itself is unreliable, but both methods agree that $\lambda_R$ is zero in the limit of infinite cutoff for all $\lambda_0$ in the range $0 \leq \lambda_0 \leq \infty$.

The second result is the idea of "domains". The idea is this. Consider the interaction

$$L_I = u_0 \Lambda^\epsilon \left\{ \phi_1^4 + \phi_2^4 \right\} + v_0 \Lambda^\epsilon \phi_1^2 \phi_2^2.$$  

Here $\epsilon$ is $4 - d$; $\phi_1$ and $\phi_2$ are two scalar fields, and $u_0$ and $v_0$ are coupling constants to be held fixed as $\Lambda \to \infty$. Then there are domains in $u_0, v_0$ space corresponding to a unique renormalized theory. These domains are shown in Fig. 3. Any values $(u_0, v_0)$ in the region A, for example, give the same renormalized theory (apart from mass terms) with the same anomalous dimensions in the limit $\Lambda \to \infty$. The line B is a separate domain. Any point on the line B corresponds to another renormalized theory with another unique set of anomalous dimensions. The point D $(u_0 = v_0 = 0)$ defines a free-field theory with canonical dimensions. Associated with each domain there are a definite number of free parameters (masses or coupling constants) which affect low-energy behavior only (corresponding to the "generalized mass terms" of Ref. 4). In the example there are the standard mass terms (involving $\phi_1^2$ and $\phi_2^2$); in addition there are free coupling constants whenever any point in a domain can be approached from outside. The theory associated with the domain A has no free-coupling constants; the domains B and C each give one free-coupling constant, while the domain D gives two free-coupling constants (the domain D corresponds to the standard super-renormalizable theory with $\phi_1^4 + \phi_2^4$ and $\phi_1^2 \phi_2^2$ as interactions).
For a general discussion of domains see Ref. 10, Lecture XII; the domains for this example (for small $u_0$, $v_0$, and $\epsilon$) were derived in Ref. 6.

Reference 10 provides an extensive introduction to the $\epsilon$ expansion as well as the renormalization group.

References

15. See Ref. 10, Lecture XIII.
Fig. 1. Map showing various models of short distance behavior (the Thirring model, not shown, covers the entire \( d = 2 \) line). The equivalence of field theory to models of 2nd order phase transitions is reviewed in Ref. 10.
\[ X = x + \cdots \]

Fig. 2. Dominant bubble graphs for the four-point function in the $N \to \infty$ limit.

Fig. 3. Domains within which the renormalized theory is independent of the unrenormalized constants $u_0$ and $v_0$. Domains B and C are lines while D is a single point (the origin).
<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
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<tbody>
<tr>
<td>$\Lambda^0 \rightarrow \text{pev Decay}$</td>
<td>V. Soergel (Heidelberg)</td>
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<tr>
<td>1.1 Recent Experiments</td>
<td>R. Winston (EPINS, Chicago)</td>
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<td>1.2 Form Factors Evaluation</td>
<td>M. Roos (Helsinki)</td>
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<td>2. Cabibbo Fit to Hyperon Decays</td>
<td>H. Pietschmann (Vienna) and</td>
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<td>3. Second Class Currents</td>
<td>P. L. Pritchett (Northwestern)</td>
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<td>4. Measurement of the $\Xi^-$ Magnetic Moment</td>
<td>R. L. Cool (Rockefeller)</td>
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<td>J. Sandweiss (Yale)</td>
</tr>
</tbody>
</table>
WEAK INTERACTIONS I - HYPERON DECAYS

Summary by J.-M. Gaillard
Orsay/CERN

Hyperon leptonic decays are generally well described by the Cabibbo theory which assumes perfect $SU_3$. The availability of higher statistics data should sooner or later reveal deviations from that theory as we know that $SU_3$ is not an exact symmetry. In particular terms induced by $SU_3$ breaking and of the same structure as second-class currents are expected at some level which is difficult to evaluate theoretically. The possible occurrence of genuine second-class currents at a substantial level has been raised by the experiment of Wilkinson on the decays of mirror nuclei.

The recent development of high-energy charged hyperon beams at Brookhaven National Laboratory and at CERN will provide high statistics on $\Sigma$ leptonic decays, and their extrapolation to higher energy machines will permit the detailed study of $\Sigma^-$, $\Omega^-$ and $\Sigma^+$ decays. A large number of $\Lambda^0$ leptonic decays have also been collected in the CERN - Heidelberg experiment in a short neutral beam. However in these new experiments the hyperon polarization will always be small. In two recent experiments, highly polarized $\Lambda^0$ have been produced in a more conventional way yielding more than 1500 $\Lambda^0 \to p e^-$ events. The polarization provides observable asymmetries for the lepton, proton, and neutrino which are rather sensitive to the form factors.

1. $\Lambda^0 \to p e^-$ Decay (V. Soergel, Heidelberg, and R. Winston, Chicago)

In the Chicago-ANL experiment the $\Lambda^0$s were produced in a hydrogen target just below the $\Sigma - K$ threshold by the reaction:

$$\pi^- + p \to \Lambda^0 + K^0.$$  

The $\Lambda^0$ polarization was $0.85 \pm 0.06$. The charged products of the $\Lambda^0$ were magnetically analyzed, thus permitting a complete reconstruction in the $\Lambda^0$ center of mass. The analysis leads to an estimate of 6% for the only significant background due to $\Lambda^0 \to p e^-$ decays and 504 events have been collected; of these 409 events have been fully reconstructed. In the case of the CERN - Heidelberg experiment the production reaction was:

$$\pi^+ + Be \to \Lambda^0 + K^+.$$  

For a beam momentum of 4130 MeV/c, the $\Lambda^0$ polarization was $0.75 \pm 0.03$. Out of 1078 events, 817 have been completely reconstructed. It is estimated that this subsample contains a background of 48 $\Lambda^0 \to p e^-$ decays and less than 1% for all other background sources. The $K^+$ and the decay proton momenta were measured by time of flight. To perform the kinematical fit, additional constraints were imposed: the three components of the Fermi momentum of the target neutron should have the distribution obtained from events with $\Lambda^0 \to p e^-$ decay and the lepton momenta should be within their physical ranges.

Table I summarizes the results obtained in the two experiments for the electron-neutrino angular-correlation coefficient $a_{ev}$ and the updown asymmetry parameters with respect to the
production plane $\alpha_e$, $\alpha_\nu$, and $\alpha_p$. The coefficient $\alpha_T$ measures the correlation $\sigma_\Lambda \cdot \langle \vec{p}_e \times \vec{p}_e \rangle$; a non-zero value of $\alpha_T$ would indicate time-reversal violation. Clearly the experimental data does not show any violation of time-reversal invariance.

The determination of the asymmetry parameters and of the neutrino-electron correlation coefficient from the two experiments are in very good agreement. Figure 1 shows the expected curves for $\alpha_e$, $\alpha_\nu$, and $\alpha_p$ as a function of $g_4/f_4$ ($g_4$ and $f_4$ are the axial and the vector form factors) in the conventional Cabibbo theory. The results of the two experiments are indicated on these curves. To avoid confusion on the figure the error bars, which are the same for all asymmetry parameters, are given separately. The vertical line drawn corresponds to $g_4/f_4 = 0.71$, the value obtained from the Cabibbo fit to hyperon data. From the asymmetry parameters alone, the following values are obtained

$$g_4/f_4 = 0.49^{+0.16}_{-0.11} \quad \text{CERN - Heidelberg}$$

$$g_4/f_4 = 0.34^{+0.14}_{-0.11} \quad \text{Chicago - ANL}$$

The Chicago - ANL result differs substantially from $g_4/f_4 = 0.71$, the Cabibbo value. For the CERN - Heidelberg, the difference is less than 1.5 standard deviation.

The electron-neutrino correlation gives the following results

$$g_4/f_4 = 0.85^{+0.19}_{-0.13} \quad \text{Chicago - ANL}$$

$$g_4/f_4 = 0.66^{+0.09}_{-0.08} \quad \text{CERN - Heidelberg}$$

Is it necessary to include new couplings besides the vector ($f_4$), axial ($g_4$) and weak magnetism ($f_2$) terms? From a general fit to all data on $\Lambda^0$ leptonic decays (90% of the world data is contained in the two experiments under discussion), the Chicago - ANL Group finds a low probability of a few percent for a fit without additional terms. But the CERN - Heidelberg Group gets a good fit to the totality of their data using only $f_4$, $g_4$, and $f_2$

$$g_4/f_4 = 0.63 \pm 0.06 \quad f_2/f_4 = 0.80 \pm 0.90$$

with a $\chi^2$ probability of 76%

Figure 2 shows the results obtained in the CERN - Heidelberg experiment for the maximum-likelihood fits, obtained from $\alpha_e$, $\alpha_\nu$, and $\alpha_p$ and from $\alpha_e$, $\alpha_\nu$, $\alpha_p$, and $\alpha_{e\nu}$. The one and two standard deviation contours are drawn.

In conclusion the necessity for additional terms is not proven. The value obtained from all experiments

$$g_4/f_4 = 0.69 \pm 0.04$$

is in good agreement with the Cabibbo value.

---

*Private communication from A. Wagner. This value takes into account the correlation between the various asymmetries.*
II. Cabibbo Fit to Hyperon Decays (M. Roos, Helsinki)

There are only minute changes with respect to the fit published by M. Roos in Physics Letters 36B, 130 (1971). The one-angle fit from this article gave

\[ \sin \theta = 0.237 \pm 0.003 \]
\[ \alpha = 0.638 \pm 0.009 \]

with a \( \chi^2 \) of 5.4 for 6 degrees of freedom.

III. Second-Class Currents

III.1 ΔS ≠ 0 Second-Class Currents (H. Pietschmann, Vienna)

The experiment of Wilkinson \(^4\) suggested the possible existence of second-class currents. As no pseudotensor effect has been found by Wilkinson and Alburger \(^4\) in the decay of the mirror nuclei \(^8\)Li and \(^8\)B, a second-class current operator of purely mesonic composition was proposed by Lipkin. \(^5\) Pietschmann and Rupertsberger \(^6\) extend that model to the full SU\(_3\) to evaluate possible effects in \( \Sigma^- \to \Lambda^0 e^- \nu_e \) decays. They introduce additional second-class terms to the normal weak Hamiltonian

\[
\mathcal{H} = G_D V^2 (D_1 + i D_2) \overline{D} \gamma_\mu D^\mu + G_f \omega^\mu \pi^- \gamma_\mu + G_f (\rho^-)^\mu \eta^\mu f + \text{h.c.}
\]

Contributions from this Hamiltonian to \( \Sigma^- \to \Lambda^0 e^- \nu_e \) decays are shown by the graphs of Fig 3.

The experimental ratio

\[
R \frac{\Gamma(\Sigma^- \to \Lambda^0 e^- \nu_e)}{\Gamma(\Sigma^+ \to \Lambda^0 e^+ \nu_e)} = 0.63 \pm 0.22
\]

is used to set an upper limit on \( G_d \)

\[
G_d / G_V \leq 0.1,
\]

where \( G_V \) is the weak vector coupling constant. In the absence of second-class currents, \( R \) should be equal to 0.60.

III.2 Induced Pseudotensor Term in \( \Lambda^0 \) Beta Decay (P. L. Pritchett, Northwestern)

Deviations from the Cabibbo theory can be due either to SU\(_3\)-breaking effects or to second-class currents. Pritchett and Deshpande \(^7\) have evaluated the size of the induced pseudotensor term which could be produced by SU\(_3\)-breaking effects, in \( \Lambda^0 \) beta decay. This work had its origin in the earlier indications that pseudotensor currents could be necessary to explain the experimental results on \( \Lambda^0 \) beta decay. If an effect were observed at a level considerably larger than the pseudotensor term induced by SU\(_3\) breaking, it would require the introduction of second-class currents.

The induced pseudotensor form factor \( g_2 (q^2) \) is assumed to satisfy an unsubtracted dispersion relation.
It is also assumed that the dispersion integral is dominated by the lowest mass intermediate states. Allowing for mass breaking and a variation of ±30% from the SU$_3$ value for all unknown coupling constants, it is found that

$$-0.43 < g_2/f_1 < 0.23$$

with the matrix element of the hadronic current defined as

$$M_{\mu} = f_1(q^2)\gamma_{\mu} + f_2(q^2)\sigma_{\mu\nu}q_{\nu}/(M_1 + M_2) + f_3(q^2)q_{\mu}/(M_1 + M_2) + g_4(q^2)\gamma_5 + g_5(q^2)\sigma_{\mu\nu}q_{\nu}(M_1 + M_2) + g_6(q^2)q_{\mu}/(M_1 + M_2)$$

$M_1$ and $M_2$ are the masses of the two baryons involved in the decay.

IV Measurement of the $\Xi^-$ Magnetic Moment (R. L. Cool, Rockefeller)

In a magnetic field $\vec{B}$, the equation of motion of the polarization vector $\vec{\sigma}_{\Xi}$ in the $\Xi^-$ rest frame is

$$d\vec{\sigma}_{\Xi}/dt = (\mu_{\Xi}/s_{\Xi}) \cdot (\vec{\sigma}_{\Xi} \times \vec{B})$$

where $\mu_{\Xi}$ and $s_{\Xi}$ are respectively the magnetic moment and the spin angular momentum of the $\Xi^-$. In the experiment reported, transversely polarized $\Xi^-$'s were produced by $K^-$'s in a 14-cm long solid hydrogen target. The set of reactions and decays observed were

$$K^- + p \rightarrow \Xi^- + K^+$$

$$L_\Lambda^0 + \pi^-$$

$$L_p + \pi^-$$

A longitudinal magnetic field of average value 110 kG was produced by a superconducting solenoid 15-cm long, placed after the production target.

The data were taken at 1.74, 1.80, and 1.87 GeV/c for zero-magnetic field and at 1.83 GeV/c with the field both parallel and antiparallel to the $\Xi^-$ trajectory. The beam definition $\Delta p/p$ was ±2%.

The results are based on 1302 events with zero field and 1134 events with magnetic field. The $K^+$, $\Xi^-$ and $\Lambda^0$ lifetimes obtained from this data sample are in good agreement with published values.

With zero magnetic field all angular distributions in the rest frame of the decaying hyperon have form

$$1(\theta_1) \sim (1 + A_1 \cos \theta_1)$$

with
\[
A_1 = a_x \tilde{\beta} \quad \cos \theta_1 = (\tilde{\delta}_x \cdot \tilde{p}_\Lambda)
\]
\[
A_2 = a_y a_x \quad \cos \theta_2 = (\tilde{p}_p \cdot \tilde{p}_\Lambda)
\]
\[
A_3 = \frac{\pi}{4} \beta \tilde{a}_x \quad \cos \theta_3 = (\tilde{p}_p \cdot (\tilde{\delta}_x \times \tilde{p}_\Lambda))
\]
\[
A_4 = (1 + 2 \gamma) a_x \tilde{p} / 3 \quad \cos \theta_4 = (\tilde{\delta}_x \cdot \tilde{p}_p)
\]

where \(\tilde{\delta}_x = (\tilde{p}_x \times \tilde{p}_x)\), \(\tilde{\beta}\) are the unit vectors of the subscripted particle momenta, \(\tilde{P}\) is the average \(\Xi^-\) polarization and \(a, \beta, \gamma\) are the \(\Xi^-\) and \(\Lambda^0\) decay parameters.

In the presence of a magnetic field the angular distributions take the form

\[
I'(\phi) \sim 1 + A_1 (\cos \phi \cos \theta + \sin \phi \sin \theta)
\]

where \(\epsilon\) is the precession angle and \(\phi\) the azimuthal angle.

A maximum-likelihood fit was made with two parameters \(\mu_\Xi\) and \(\tilde{F}\), assuming \(a_\Lambda = 0.645\), \(a_\Xi = -0.40\) and \(\gamma_\Xi = 0.91\) without errors, giving

\[
\mu_\Xi = -2.2 \pm 0.8 \text{ nuclear magnetons}
\]
\[
\tilde{F} = 0.30 \pm 0.05.
\]

The value of \(\mu_\Xi\) corresponds to an angle \(\epsilon = 44.5^\circ \pm 14^\circ\).

The value predicted by SU$_3$ symmetry is

\[
\mu_\Xi = \mu_n - \mu_p = -0.9 \text{ nuclear magnetons}
\]

The difference of 1.7 standard deviations between this value and the experimental result is not a significant disagreement. However, mass-breaking effects should modify the latter value and probably lower it, but there is no reliable way to compute the mass-breaking effects at present.

V. Hyperon Beams and Experiments at BNL and at CERN (J. Sandweiss, Yale)

Two high energy negative hyperon beams have been constructed at the AGS and at the CERN PS. These beams are based on two simple ideas

a) As a function of the incident proton energy \(E_p\), the shielding length against hadrons goes as the logarithm of \(E_p\), while the hyperon decay length is proportional to \(E_p\).

b) At high momentum and in the forward direction the leading baryon should dominate the production.

Both beams are short (~4 meters), with high-density shielding (tungsten, uranium, lead, or brass). The particles are collected into a small-area (~6 cm$^2$) tapered-beam channel and are deflected by a strong magnetic field (~30 kG). The magnetic channel selects negative particles with a \(\Delta p/p\) ±10%.

The CERN beam is run with an external proton beam of \(2 \times 10^{11}\) protons and 24 GeV/c. Two superconducting quadrupoles give an outgoing parallel beam. The negative hyperons are identified by a special DISC Cerenkov counter. Figure 4 shows the mass distribution obtained by varying the SF$_6$ gas pressure in the DISC. This curve has been obtained with a resolution \(\Delta \beta/\beta = 1.3 \times 10^{-4}\).

To get decent fluxes, one has to work with a resolution about 4 times wider. This beam is now used to study electronic decays of \(\Sigma^-\)s and \(\Xi^-\)s. The decays are observed in two streamer
chambers, the second is placed in a magnet to measure momenta. The direction of the incoming hyperon is measured with two proportional chambers giving a resolution of 0.5 mrad in space.

The BNL beam is run also in an external proton beam, but at 29 GeV/c. A threshold Cerenkov counter in the magnetic channel vetoes pions and kaons. The hyperon direction is measured in a high pressure spark chamber 20-cm long with an accuracy of 0.5 mrad in space. Runs to study \( \Xi^- \) electronic decays have been completed and should give about 9000 events. The setup is shown on Fig. 5, it uses magnetostrictive wire chambers and proportional chambers.

Typical conditions for \( \Xi^- \) and \( \Sigma^- \) runs for a proton momentum of 29 GeV/c are:

<table>
<thead>
<tr>
<th>( \Xi^- ) Runs</th>
<th>( \Sigma^- ) Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel momentum</td>
<td>20.5 GeV/c</td>
</tr>
<tr>
<td>Proton intensity</td>
<td>( 10^{11} )/burst</td>
</tr>
<tr>
<td>8 triggers/burst</td>
<td></td>
</tr>
<tr>
<td>0.4 reconstructed</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- \rightarrow \Lambda^0 + \pi^- ) decay/burst</td>
<td>detected</td>
</tr>
<tr>
<td>( \Sigma^- \rightarrow n \pi^- ) detected by scintillation and neutron counters/burst</td>
<td></td>
</tr>
<tr>
<td>0.05 ( \Sigma^- \rightarrow n \bar{\nu} )/burst (usefully reconstructed events)</td>
<td></td>
</tr>
</tbody>
</table>

Run 11

<table>
<thead>
<tr>
<th>( \Xi^- \rightarrow \Lambda^0 + \pi^- ) detected</th>
<th>( \Omega^- \rightarrow \Lambda^0 + K^- ) detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt; 1.2 \times 10^{-3} ) at 90% confidence level</td>
<td></td>
</tr>
</tbody>
</table>

In the CERN and BNL experiments on leptonic decays, one will also get data on \( \Sigma^- \rightarrow \Lambda^0 \bar{\nu} \) and \( \Xi^- \rightarrow \Lambda^0 \bar{\nu} \) decays.

A search has been made in both beams for \( \Omega^- \), and limits have been obtained. Because of the large error on the \( \Omega^- \) lifetime it is more meaningful to give the result for the detector position rather than at the production target. The BNL experiment gives for a channel momentum of 20.5 GeV/c:

\[
\left( \frac{\Omega^- \rightarrow \Lambda^0 + K^-}{\Xi^- \rightarrow \Lambda^0 + \pi^-} \right)_{\text{detected}} < 1.2 \times 10^{-3} \text{ at 90\% confidence level}
\]

In the CERN experiment, for \( 10^{11} \) incident protons and a channel momentum of 18 GeV/c, the limit at 90\% confidence level is \( 0.9 \times 10^{-3} \) \( \Omega^- \) per burst. These limits are very similar, and show that \( \Omega^- \) beam experiments are not feasible at AGS or PS energies.

In the two beams the \( \Xi^-/\Sigma^- \) production ratio has been measured to be 1-2\% depending upon the channel momentum. At BNL about \( 10^6 \) triggers have been taken with a scintillation counter target to study peripheral production of \( Y^+ \)'s which have a decay channel involving a \( \Lambda^0 \rightarrow p \pi^- \) decay. At CERN the total cross sections of \( \Sigma^- \) on hydrogen and deuterium have been measured.

References

Table I.

<table>
<thead>
<tr>
<th>Branching Ratio</th>
<th>$\alpha_e$</th>
<th>$\alpha_v$</th>
<th>$\alpha_p$</th>
<th>$\alpha_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN - Heidelberg</td>
<td>$(8.4 \pm 0.4) \times 10^{-4}$</td>
<td>$0.07 \pm 0.09$</td>
<td>$0.15 \pm 0.09$</td>
<td>$0.89 \pm 0.08$</td>
</tr>
<tr>
<td>Chicago - ANL</td>
<td>$(8.3 \pm 1.0) \times 10^{-4}$</td>
<td>$-0.08 \pm 0.10$</td>
<td>$0.09 \pm 0.11$</td>
<td>$0.75 \pm 0.11$</td>
</tr>
</tbody>
</table>
Fig. 1. Updown asymmetry parameters $\alpha_e$, $\alpha_\nu$, and $\alpha_p$ as a function of $g_1/f_1$ and for $f_2/f_1 = 1$. The experimental results of CERN-Heidelberg and of Chicago-ANL experiments are given. To avoid too much confusion the error bars, which are the same for all asymmetry parameters, are given separately for each experiment.
Fig. 2. One- and two-standard deviation contours of the likelihood function from a combined fit of \( g_1 / f_1 \) and \( f_2 / f_1 \) with \( g_2 = 0 \) for the CERN - Heidelberg experiment. Full line: \( \sigma_e, \sigma_\nu, \sigma_p, \sigma_{ev} \) used; dotted line: only \( \sigma_e, \sigma_\nu, \) and \( \sigma_p \) used.
Fig. 3. Second-class currents contributions to $\Sigma - \Lambda$ decay.
Fig. 4. Velocity curve at 17 GeV/c obtained in the CERN hyperon beam with the DISC counter. It gives the counting rate versus $1 - \beta$. 
HIGH ENERGY NEGATIVE HYPERON BEAM

PROTON BEAM
PROTON STEERING MAGNET
TARGET
SPARK CHAMBER
CERENKOV COUNTER
UP-DOWN COUNTERS
HODOSCOPE COUNTERS
PION BEAM VETO

Fig 5 Hyperon experiment setup at Brookhaven National Laboratory
<table>
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<th>Topic</th>
<th>Speaker</th>
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</thead>
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<tr>
<td>1. Lepton Conservation</td>
<td>E. Pioro (Milano)</td>
</tr>
<tr>
<td>2. Neutral Currents</td>
<td></td>
</tr>
<tr>
<td>2.1 Theoretical Considerations</td>
<td>B. W. Lee (SUNY, Stony Brook)</td>
</tr>
<tr>
<td>2.2 Search for the Processes</td>
<td>V. Brisson (CERN)</td>
</tr>
<tr>
<td>$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$</td>
<td></td>
</tr>
<tr>
<td>$\bar{\nu}<em>\mu + e^- \rightarrow \bar{\nu}</em>\mu + e^-$</td>
<td></td>
</tr>
<tr>
<td>2.3 Limits on Neutral Currents</td>
<td>Y. Cho (ANL)</td>
</tr>
<tr>
<td>3. Neutrino-Nucleon Interactions</td>
<td></td>
</tr>
<tr>
<td>3.1 Quasi-Elastic Scattering</td>
<td>P. Schreiner (ANL)</td>
</tr>
<tr>
<td>3.2 Total Cross Sections for $\nu$-Nucleon and $\bar{\nu}$-Nucleon Interactions</td>
<td>Ph. Heusse (CERN)</td>
</tr>
</tbody>
</table>
WEAK INTERACTIONS II - NEUTRINO INTERACTIONS

Summary prepared by C. Franzinetti
Torino University
Torino, Italy

I. LOW-ENERGY NEUTRINOS

I.1 Neutrinoless Double $\beta$ Decay of $^{76}$Ge ($\Delta I = 2$)

Presented by E. Fiorini
Milano University
Milano, Italy

An experiment is described, based on the use of a Ge(Li) crystal, both as a source and as a detector of neutrinoless double $\beta$ decay

$$^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^-.$$  \hspace{1cm} (I.1)

Single $\beta$ decay ($^{76}\text{Ge} \rightarrow ^{76}\text{As} + e^- + \bar{\nu}$) is forbidden by energy conservation.

The experiment was carried out in the Mont Blanc Tunnel under 4,200 m(w.e.) which provides an attenuation factor of about $10^6$ of the cosmic-ray component. The spectrum did not contain any peak corresponding to reaction I.1 ($E = 2.045 \pm 0.003$ MeV). The background in the same region was $(2.0 \pm 0.2) \times 10^{-3}$ counts keV$^{-1}$ h$^{-1}$. After 4300 hrs of counting, the lower limit for the lifetime of the above process obtained was

$$T > 5 \times 10^{21} \text{ yrs}$$

with 68% confidence. Table I compares the results obtained on other triplets. In Fig. 1 the spectral distribution is shown from the Ge(Li) crystal in the region of neutrinoless double $\beta$ decay.

Discussion

P. Gollon (NAL): Is your lower limit on the neutrinoless decay lifetime proportional to the background counting rate in the region of the expected peak, or did you try to fit a small peak to your data and take as a limit the largest peak that could remain hidden in the statistical fluctuations of the data?

E. Fiorini: Our limit was obtained with a maximum likelihood method, by trying to fit a Poisson-distributed peak on our experimental spectrum.

G. Shapiro (LBL): Did you look at transitions to excited states of the daughter nucleus?

E. Fiorini: Yes, we looked without finding anything. In these regions, however, the background was higher, and neutrinoless $\beta\beta$ decay, if it exists, should occur mostly to the $^{68}$ ground state of $^{76}$Se.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Transition</th>
<th>Method</th>
<th>Experimental Limit on $\tau_{0\nu}(\text{yr})$</th>
<th>Corresponding Limit on the Lepton Nonconserving Parameter $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bardin et al.</td>
<td>$^{48}\text{Ca} - ^{48}\text{Tl}$</td>
<td>Discharge chamber</td>
<td>$2 \times 10^{21}$ ($80% X^2$)</td>
<td>$10^{-3.6\pm1}$</td>
</tr>
<tr>
<td>Kirsten et al.</td>
<td>$^{82}\text{Se} - ^{82}\text{Kr}$</td>
<td>Geological</td>
<td>$6 \times 10^{19\pm0.2}$</td>
<td>$10^{-2.4\pm1}$</td>
</tr>
<tr>
<td>Kirsten et al.</td>
<td>$^{130}\text{Te} - ^{130}\text{Xe}$</td>
<td>Geological</td>
<td>$2.2 \times 10^{21}$</td>
<td>$10^{-3\pm1}$</td>
</tr>
<tr>
<td>Present experiment</td>
<td>$^{76}\text{Ge} - ^{76}\text{Se}$</td>
<td>Ge(Li) detector</td>
<td>$5 \times 10^{21}$</td>
<td>$10^{-3\pm1}$</td>
</tr>
</tbody>
</table>
Expected Location of Neutrinoless Double $\beta$ Decay

Fig. 1
II. NEUTRAL CURRENTS

II.1 Theoretical Considerations on Neutral Currents

B. W. Lee
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A. Introduction

I would like to address myself to the question of neutral currents in neutrino-induced reactions. The recent resurgence of theoretical interest in this subject is due to the possibility of constructing a renormalizable theory of weak (and electromagnetic) interactions based on gauge symmetry. It can be shown on very general grounds that such a model must contain either a neutral current interaction, or heavy leptons, or both.

A prototype of models which feature a neutral current interaction is the Weinberg model, where the massive neutral vector boson $Z$ couples to the neutral current

$$Z = \alpha \gamma^5 \frac{1 - \gamma_5}{2} \nu_\mu - \frac{1}{2} \gamma^5 \left( 1 - 2 \sin^2 \theta_w - \frac{1}{2} \gamma_5 \right) \mu + (\mu \to \nu) + \left( J^{\alpha}_3 \gamma^5 - 2 \sin^2 \theta_w J^{\alpha}_em \right) \nu_{\mu},$$

where $x$ is a parameter of the model, $f \sin \theta_w \cos \theta_w = e$, and $J^{\alpha}_3$ is the third component of the isospin $V$-$A$ current and $J^{\alpha}_em$ is the hadronic electromagnetic current. The mass of $Z$ is given by the formula

$$m_Z = \frac{e^2}{\sqrt{2} \cos^2 \theta_w \sin^2 \theta_w}.$$  

In the following we shall discuss the effects of the above neutral-current interactions on purely leptonic and semileptonic processes, and give bounds on the parameter $x = \sin^2 \theta_w$ implied by the existing data, and present theoretical predictions on possible future experiments. I shall not quote experimental data to be presented at this Conference.

B. Leptonic Interactions

Consider, first, the process

$$\bar{\nu}_e + e \to \nu_e + e.$$  

There are two Feynman diagrams which contribute to this process.

$$\begin{array}{c}
\text{Fig. 2}
\end{array}$$

The effective interaction for this process can be written as

$$\frac{G_F}{\sqrt{2}} \left[ \bar{\nu}_e \gamma^\alpha (1 - \gamma_5) \nu_e \right] \left[ \delta \gamma_\alpha (C_V - C_A \gamma_5) e \right].$$

where

$$C_V = \frac{1}{2} + 2 \sin^2 \theta_w,$$

and

$$C_A = \frac{1}{2}.$$  

The expected rate of events for this process is plotted against $x = \sin^2 \theta_w$ in...
Fig. 3. The expected rates for $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$, normalized to the Feynman-Gell-Mann theory prediction are given as a function of $x (= \sin^2 \theta_W)$. The ratio $C_V/C_A$ is related to $x$ by $C_V/C_A = 1 + 4x$.

The most recent results of Gurr, Reines, and Sobel gives

$$\frac{\sigma_{\text{exp}}}{\sigma_{\text{PG}}} = 1.0 \pm 0.9,$$

where $\sigma_{\text{PG}}$ is the prediction of the Feynman-Gell-Mann theory. From the figure above, we see that

$$\sin^2 \theta_W < 0.4 \text{ (about 90% confidence level).} \quad \text{(II.6)}$$

Next, consider the process

$$\nu_{\mu} + e + \nu_{\mu} + e. \quad \text{(II.7)}$$

This process is of interest since, in the conventional Feynman-Gell-Mann theory, it proceeds only in higher orders. In the Weinberg model, the effective matrix element for this process is given by

$$\frac{G}{\sqrt{2}} \left[ \bar{\nu}_\mu \gamma_a (1 - \gamma_5) \nu_\mu \right] \left[ \bar{e} \gamma^a (C_V' - C_A' \gamma_5) e \right], \quad \text{(II.8)}$$

where

$$C_A' = \frac{1}{2}, \text{ and } C_V' = \frac{1}{2} - 2 \sin^2 \theta_W. \quad \text{(II.9)}$$
Figure 4 is a plot of the expected rate of events versus x, for a typical experimental condition at CERN. We await the latest result of CERN to be discussed at this session (see Section II.2).

Fig. 4. The expected rates for $\nu_\mu + e^{-} \rightarrow \nu_\mu + e$ (CERN), normalized to the Feynman-Gell-Mann theory prediction for $\nu_\mu + e^{-} \rightarrow \nu_\mu + e$, are given as a function of $x (= \sin^2 \theta_W)$. The ratio $C_Y/C_A'$ is related to $x$ by $C_Y/C_A' = 1 - 4x$. The $\nu_\mu$ spectrum of Holder et al., Nuovo Cimento 57A, 338 (1970) is used. Minimum energy of the recoil electron is taken to be 1 GeV, following Sterner, Phys. Rev. Letters 24, 1330 (1970).

C. Semileptonic Processes

In the following we shall set the Cabibbo angle equal to zero. The effective interaction may be written as

$$\frac{G_F}{\sqrt{2}} \left[ \bar{\nu}_Y (1 - \gamma_5) \nu_\mu \left\{ J_1^{\alpha} + \frac{J_2^{\alpha}}{\sqrt{2}} \right\} + \bar{\nu}_Y (1 - \gamma_5) \nu_\mu \left\{ J_3^{\alpha} - 2 \sin^2 \theta_W J_{em}^{\alpha} \right\} \right]. \quad (11.10)$$

In making estimates of cross sections for pion production at currently available neutrino energies, it is customary to make the assumption of $\Delta(1236)$ dominance. Alternatively, we may assume the $I = 3/2$ and 1/2 amplitudes, $X_3$ and $X_1$, to be incoherent and assume

$$|X_1|^2 / \left[ |X_3|^2 + |X_1|^2 \right] \leq 0.3 \quad (11.11)$$

in the relevant energy range. In the following discussion we shall use the values of Argonne;

$$\sigma (\nu + p + \mu^- + p + \pi^+) = (0.78 + 0.16) \times 10^{-38} \text{ cm}^2 \quad (11.12)$$
rather than the bigger value
\[(1.13 \pm 0.28) \times 10^{-38} \text{ cm}^2 \quad \text{(II.12a)}\]
based on the 1967 CERN experiment.\(^7\) The lower value [see Eq. (II.12)] results in less stringent lower bounds quoted below.

1. \(\nu + p \to \nu + p\)
   The magnitude of this cross section can be bounded on the basis of our knowledge of the electromagnetic form factors of the proton. Pais and Treiman\(^7\) deduce the lower and upper bounds in this way, assuming \(\sin^2 \theta_w \leq 0.35\):
   \[0.15 \leq \frac{\sigma(\nu + p \to \nu + p)}{\sigma(\nu + n \to \nu^- + p)} \leq 0.25 \quad \text{(II.13)}\]

2. \(\nu + p \to \nu + n + \pi^+\)
   This process is an analogue of \(\nu + p \to \nu^- + p + \pi^+\). Unfortunately, the Clebsch-Gordan coefficient is unfavorable for the neutral current process:
   \[R_0 = \frac{\sigma(\nu + p \to \nu + n + \pi^+)}{\sigma(\nu + p \to \nu^- + p + \pi^+)} = \frac{1}{9} \quad \text{(II.14)}\]
in the \(\Lambda\) dominance approximation and in the limit \(x = 0\) (Weinberg).\(^4\) Without these approximations, but merely assuming Eqs. (II.6) and (II.11) and incoherence of \(X_1\) and \(X_3\), Albright et al. deduce\(^5\)
   \[R_0 \geq 0.03. \quad \text{(II.15)}\]

Let us recall the result of Cundy et al.,\(^8\) \(R_0 = 0.08 \pm 0.04\). The bound [Eq. (II.15)] applies also to the related ratio \(\sigma(\nu + n \to \nu + p + \pi^-)/\sigma(\nu + p \to \nu^- + p + \pi^+)\).

3. \(\nu + p \to \nu + p + \nu^0\) and \(\nu + n \to \nu + n + \nu^0\)
   The ratio we shall consider is
   \[R_1 = \frac{\sigma(\nu + p \to \nu + p + \nu^0) + \sigma(\nu + n \to \nu + n + \nu^0)}{2\sigma(\nu + n \to \nu^- + p + \pi^0)} \quad \text{(II.16)}\]
The Clebsch-Gordan coefficient is favorable for this process:
   \[R_1 = 1 \quad \text{(II.17)}\]
in the \(\Delta\)-dominance approximation and in the limit \(x = 0\). B. Lee\(^9\) estimated this ratio in the static model of the \(\Delta\) production, assuming \(x < 0.35\):
   \[R_1 \geq 0.4 \approx 0.6. \quad \text{(II.18)}\]
Assuming \(\Delta\) dominance and \(x < 0.35\), but otherwise making no dynamical assumption, Paschos and Wolfenstein\(^10\) obtain
   \[R_1 \geq 0.4. \quad \text{(II.18)}\]
   [This number is based on the cross section (II.12a)] With the smaller value (II.12) and \(x \leq 0.4\), one gets \(R_1 \geq 0.3.\) If one considers the effect of \(X_1\) with the bound given by Eq. (II.11), assuming \(X_1\) and \(X_3\) to be incoherent, and using Eq. (II.12) and \(x \leq 0.4\), one gets
   \[R_1 \geq 0.19. \quad \text{(II.18)}\]
The above result is to be compared with the experimental bound given by W. Lee\(^11\)
   \[R_1 \leq 0.14 \text{ (90\% confidence).} \quad \text{(II.19)}\]

(Ed. Note: For relevant data presented to this Conference, see Section II.3.)
D. Inclusive Processes

The presently available data on the neutral current ($\Delta S = 0$) interaction is inconclusive as to its existence, even though the result (II.19) presents serious trouble for Weinberg-type models.

More decisive experimental tests are clearly called for. A promising approach may be to look at inclusive processes

$$\nu + N \rightarrow \nu + \text{anything}$$  \hspace{1cm} (II.20)

Pais and Treiman,\textsuperscript{7} and Paschos and Wolfenstein\textsuperscript{10} have considered the bounds on process (II.20) at NAL energies.

Without any dynamical assumptions, Paschos and Wolfenstein\textsuperscript{10} show that

$$R_{\text{inc}} = \frac{\sigma(\nu + p \rightarrow \nu + \text{anything}) + \sigma(\nu + n \rightarrow \nu + \text{anything})}{\sigma(\nu + p \rightarrow \mu + \text{anything}) + \sigma(\nu + n \rightarrow \mu + \text{anything})} \geq 0.18$$  \hspace{1cm} (II.21)

If one makes further assumptions that the neutrino-induced production scales as does the electroproduction, and that the main contribution to the total cross-section comes from the scaling region, the bound can be tightened:

$$R_{\text{inc}} \geq 0.23.$$  \hspace{1cm} (II.22)

E. Concluding Remarks

It must be emphasized that the failure to detect a neutral current effect in any of the processes discussed above does not rule out the correctness of a unified gauge theory of weak and electromagnetic interactions. It may be that the nature makes use of a different scheme\textsuperscript{12} than Weinberg's, which calls for the existence of heavy leptons but dispenses with neutral currents.

In such models, neutral current effects such as $\nu_\mu + e^- \rightarrow \nu_\mu + e$, $\nu_\mu + p \rightarrow \nu_\mu + \text{anything}$ will be induced in higher orders, and their magnitudes we expected to be of order of $G_F^2$.

References


Discussion

S. P. Rosen (Purdue Univ., Ind.): Does the result for $\bar{\nu}_e e$ scattering quoted by Dr. Lee, namely
\[ \sigma_{\text{exp}}/\sigma_{\text{FG}} = 1.0 \pm 0.9 \]

mean that Gurr, Reines, and Sobel believe that they have actually seen neutrino-electron scattering?

B. Lee: No.

II.2 Search for the Processes \((\nu_\mu + e^- \rightarrow \nu_\mu + e^-)\) and \((\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-)\) (#785)

Presented by V. Brisson
Ecole Polytechnique
Paris, France

The leptonic processes

\[ \nu_\mu + e^- \rightarrow \nu_\mu + e^- \]
\[ \bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- \]

are forbidden in a charged current-current weak interaction theory. Evidence for such reaction was searched in 167,000 muon pictures of Gargamelle exposed to the CERN \(\nu\) beam and 223,000 pictures of the same chamber to the \(\bar{\nu}\) beam. Events consisting of single electrons were searched for, where the electrons had a lab energy larger than 300 MeV and formed an angle smaller than 5° with the beam direction. These cuts ensured a good scanning efficiency, removed the background due to low-energy \(\gamma\) rays, without reducing the number of the genuine \((\nu e)\) events if they existed.

No "candidate" both for \(\nu\) or \(\bar{\nu}\) neutral interactions, was observed. To estimate upper limits for the cross sections contributions have been made for - scanning efficiency (found to be 80%).
- detection efficiency due to the geometric and kinematical cuts (found to be 87%).

Limits on the cross-sections were derived from the value \(\sigma_{\text{total}} = 0.8 \pm 0.2 \times 10^{-38} \text{ cm}^2\) for charged current events, using as a normalization the effective number of events with a \(\mu^- (\mu^+)\) for \(\nu \) observed in the experiment.

With 90% confidence, the limits are:

\[ \sigma(\nu_\mu + e^- \rightarrow \nu_\mu + e^-) < 0.7 \times 10^{-41} \text{ E}_\nu \text{ cm}^2 \]
\[ \sigma(\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-) < 1.0 \times 10^{-41} \text{ E}_\bar{\nu} \text{ cm}^2 \]

A comparison with the Weinberg model is given in Fig. 5. It can be seen that the result does not rule out the model, but restricts the value of \(\theta\), the "Weinberg angle" to \(\sin^2 \theta < \nu 0.6 \) or \(\theta < \sim 50^\circ\). The forthcoming experiment with Gargamelle will probably increase the available statistics by a factor of 3. Then (see Fig. 5) it should be possible to reach a more definite conclusion on this question.

II.3 Searches for Neutral Currents in
\(\nu + \text{Nucleon} \rightarrow \nu + \text{Nucleon} + \gamma\) (#239, 473, 785)

Presented by Y. Cho
Argonne National Laboratory
Argonne, Illinois

The results of a Columbia optical spark-chamber experiment (#239) yielded

\[ R_1 = \frac{\sigma(\nu_\mu n + \nu_\mu n^0) + \sigma(\nu_\mu p + \nu_\mu p^0)}{2\sigma(\nu_\mu n + \mu^- p^0)} < 0.14 \text{ (90% CL)} \]
This experiment saw no events which were candidates for the numerator reaction and had 12 candidates for the denominator reaction. Of these 12 events, 3 were estimated to be background.

The 12-foot bubble-chamber group at Argonne reported on the basis of 361,000 pictures in $\text{H}_2$ and 145,000 pictures in $\text{D}_2$. They find

$$R_2 = \frac{\sigma(\mu^- p \rightarrow \nu^- p n^0) + \sigma(\nu^- p \rightarrow \mu^- n^0)}{\sigma(\mu^- p \rightarrow \mu^- p n^0)} < 0.31 \text{ (90\% CL)},$$

where there are "about 121" candidates for the denominator reaction.

These experimental limits are compared with the theoretical predictions of the Weinberg model in Fig. 6.

In addition, the Gargamelle collaboration presented a very preliminary result based on an analysis of 90,000 pictures, $R_4 \equiv 0.11 \text{ (90\% CL)}$ where $R_4$ is defined above. This result was presented with the note of caution that the same data yields

$$R_4 = \frac{\sigma(\mu^- n \rightarrow \mu^- p n^0)}{\sigma(\mu^- n \rightarrow \mu^- n p^0) + \sigma(\mu^- p \rightarrow \mu^- p n^0)} = 0.5$$

whereas $R_4 = 0.2$ is predicted by $\Delta$ dominance. Either the reactions do not proceed via the $\Delta$ or reinteractions inside the nucleus are so important that all channels are finally mixed. Before doing any comparison these reinteractions have to be known and corrected for.

Table II summarizes the present experimental limits.

Discussion

Comments by C. Baltay (Columbia): Because of the uncertainties about the extent of $I = 3/2$ dominance and the serious reabsorption problems in heavy nuclei, I think that it is dangerous to draw any conclusions about the validity of the Weinberg model from the present experimental limits on the neutral current single $n^0$ production. The $n^+ \rightarrow n^0$ ratio, which is around 2 in freon, and is expected to be around 5 if $I = 3/2$ dominance were complete, indicates that either $I = 3/2$ dominance is not complete or reabsorption effects are very important (or both).
Results of Searches for Neutral Currents in $\nu_\mu N \rightarrow \nu_\mu N\pi$

\[
R_1 = \frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0) + \sigma(\nu_\mu + n \rightarrow \nu_\mu + n + \pi^0)}{2\sigma(\nu_\mu + n \rightarrow \mu^- + p + \pi^0)}
\]

\[
R_2 = \frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0) + \sigma(\nu_\mu + p \rightarrow \nu_\mu + n + \pi^+)}{\sigma(\nu_\mu + p \rightarrow \mu^- + p + \pi^+)}
\]

References to Papers Presented to this Conference:

$\bar{\nu}_e e \rightarrow \bar{\nu}_e e$: Reference given in Section 2.1.

ANL: Paper #473

Columbia-BNL: Paper #239

Paschos and Wolfenstein: Paper #540

Paschos and Lee: Paper #786

Fig. 6
<table>
<thead>
<tr>
<th>Cross-Section Ratio</th>
<th>Approximate Upper Limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sigma(\nu_\mu + e^- \rightarrow \nu_\mu + e^-)}{V-A \text{ theory for } \sigma(\nu_\mu + e^- \rightarrow \nu_\mu + e^-)}$</td>
<td>0.44 (90% CL)</td>
<td>Paper #785 this Conference</td>
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<tr>
<td>$\frac{\sigma(\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-)}{V-A \text{ theory for } \sigma(\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-)}$</td>
<td>2.1 (90% CL)</td>
<td>Paper #785 this Conference (CERN-Gargamelle Coll.)</td>
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<tr>
<td>$\frac{\sigma(\nu_e + e^- \rightarrow \nu_e + e^-)}{V-A \text{ theory for } \sigma(\nu_e + e^- \rightarrow \nu_e + e^-)}$</td>
<td>3.0 (90% CL)</td>
<td>Phys. Rev. Letters 28, 1406 (1972) (Gurr, Reines, and Sobal)</td>
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<tr>
<td>$R_1 = \frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + n + \pi^0) + \sigma(\bar{\nu}<em>\mu + n + \mu^- + p + \pi^0) \times 2}{\sigma(\nu</em>\mu + n + \mu^- + p + \pi^0)}$</td>
<td>0.14 (90% CL)$^3$</td>
<td>Paper #239 this Conference (W. Lee)</td>
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<td>$R_2 = \frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + n + \pi^0) + \sigma(\bar{\nu}<em>\mu + p \rightarrow \nu</em>\mu + p + \pi^0)}{\sigma(\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0)}$</td>
<td>0.31 (90% CL)</td>
<td>Paper #473 this Conference (M. Derrick et al.)</td>
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<td>$R_0 = \frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + n + \pi^0)}{\sigma(\nu_\mu + p \rightarrow \nu_\mu + n + \pi^0)}$</td>
<td>0.16 (90% CL)</td>
<td>Phys. Letters 31B, 478 (1970) (D. C. Cundy et al.)</td>
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<tr>
<td>$\frac{\sigma(\nu_\mu + n \rightarrow \nu_\mu + p)}{\sigma(\nu_\mu + n \rightarrow \mu^- + p + \pi^0)}$</td>
<td>0.24 (90% CL)</td>
<td>Phys. Letters 31B, 478 (1970) (D. C. Cundy et al.)</td>
</tr>
</tbody>
</table>

$^1$Nonconference Data from C. Baltay—Review talk at 1972 Neutrino Conference, Balaton.

$^2$Note that (1236) dominance yields $R_0 = 1/3$ $R_2 = 1/9$ $R_1$.

$^3$Very preliminary results were also presented by the CERN-Gargamelle collaboration (see Section II.3).
Three two-body neutrino reactions have been studied during the last year. They all have non-zero cross sections:

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Laboratory</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\nu}_p \rightarrow \mu^+ A^0 )</td>
<td>CERN</td>
<td>First observation</td>
</tr>
<tr>
<td>( \nu_n \rightarrow \mu^- p )</td>
<td>ANL</td>
<td>First time studied in D_2</td>
</tr>
<tr>
<td>( \nu_p \rightarrow \mu^+ A^{++} )</td>
<td>ANL</td>
<td>First time studied in H_2</td>
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</table>

This reaction was first observed this year\(^1\) at CERN, in the Gargamelle freon bubble chamber. From 250,000 pictures, the following event samples were obtained:

- **Events**
  - 10 \( \mu^+ A^0 \)
  - 2 \( \mu^+ \)
  - 8 \( \mu^+ A + \pi, p, n \) etc.
  - 8 \( \mu^+ YK \)

The experimenters estimate that the number of actual \( \mu^+ A^0 \) events is 10, plus 3 \( \mu^+ LN \) events which have a very slow nucleon (<30 MeV/c). They subtract as background 1.5 events coming from associated production of \( A K^0 \) with an unobserved \( K^0 \) decay. The resulting cross section on freon is \( \sigma = \left( 1.3 \pm 0.6 \right) \times 10^{-40} \text{ cm}^2/\text{nucleon.} \)

To determine the cross section on protons, a difficult calculation must be performed to account for nuclear effects. For example, it is estimated that the \( A^0 \) absorption rate is 15% and the \( \Sigma + A \) conversion rate is 20%. The resulting proton cross section is \( \sigma = \left( 1.3 \pm 0.9 \right) \times 10^{-40} \text{ cm}^2/\text{proton.} \) For comparison with theory, one assumes exact SU_3, Cabibbo = 0.24 independent of \( Q^2 \), \( P/D = 0.45/0.78 \), and an axial form factor of \( (1 + Q^2/0.71)^{-2} \). After averaging over the CERN neutrino flux, one has \( \sigma = 2.4 \times 10^{-40} \text{ cm}^2. \) These data and theory are in good agreement.

\( \nu_n \rightarrow \mu^- p \)

Argonne has studied this reaction in 145,000 pictures from the 12-foot chamber filled with deuterium.\(^2\) After fiducial volume cuts, there are 95 \( v_d \rightarrow \mu^- nn \) events, with a background of less than 5%. The only model one needs to analyze the data is the deuterium wave function.

Previous experiments on elastic scattering\(^3-6\) have been done in \( CP_3 Br, Al, Fe, \) and \( C_3 H_6 \). Great difficulties were encountered in extracting the free nucleon cross section from such complex nuclei targets. We believe this experiment in D_2 has fewer systematic problems than these previous studies.

The comparison of data with theory has been carried out using the standard theoretical assumptions: \( T \) invariance, no second class currents, CVC, small induced pseudoscalar term, axial form factor \( \left( 1 + Q^2/M_A^2 \right)^{-2} \), and a Hulthen wave function.

Figure 7(a) shows the neutrino energy distribution for the 95 events; this is clearly a quite low energy experiment. Figure 7(b) shows the differential cross...
Fig. 7. For the reaction $\nu d \rightarrow \mu^{-} p p_{s}$: (a) neutrino energy distribution in laboratory, (b) differential cross section, (c) maximum likelihood function versus $M_{A}$. 
section. The dip in the forward direction is mainly caused by the Pauli exclusion principle. To determine the best value of \( M_A \), a maximum likelihood fit was performed to the shape of the differential cross section and the shape and magnitude of the total cross section. The preliminary result, displayed in Fig. 7(c), is

\[ M_A = 0.92^{+0.14}_{-0.13} \text{ GeV/c}^2. \]

To test the consistency of the data, separate fits were performed to the shape of the \( Q^2 \) and \( P_\nu \) distributions, and to the \( \sigma_T \) distribution. The results are:

1. **SHAPE \( Q^2 \), \( P_\nu \)**
   \[ M_A = 0.97^{+0.22}_{-0.19} \text{ GeV/c}^2 \]

2. **\( \sigma_T \)**
   \[ M_A = 0.88^{+0.17}_{-0.14} \text{ GeV/c}^2 \]

An average of the results from the five experiments on \( \nu n \to \mu^- p \) gives

\[ <M_A> = 0.87^{+0.09}_{-0.11} \text{ GeV/c}^2. \]

The chi-squared probability for the five experiments being consistent is 74%.

\[ \nu p \to \mu^- A^{++} \to \mu^- \bar{p} \]

Argonne has also studied this reaction in the 12-foot chamber. From 361,000 pictures in \( H_2 \) there are 105 events with a background of \( \gamma \mu \). From 145,000 pictures in \( D_2 \), there are 48 events with a background of \( \gamma \mu \). This combined sample is about three times larger than the CERN data sample in propane.

Figure 8 shows the invariant mass distributions for the 153 events. The final state is clearly dominated by \( \Lambda(1236) \) production. A fit to the mass spectrum gives the resonant fraction as 95%. One notes that there is no evidence for lepton-hadron mass enhancements at 420 MeV in the \( \pi \mu \) system or at 1900 MeV in the \( \nu \bar{p} \) system. The curves on the \( \pi \mu \) and \( \nu \bar{p} \) plots are the expected reflections of the \( \Lambda(1236) \).

Figure 9 shows the cross section measurements. The solid dots are the preliminary Argonne data and the open dots are the CERN results from propane. The error bars include all systematic errors. Below 1.5 GeV, the two experiments agree very well. Above 1.5 GeV, the Argonne data are lower than that of CERN, but the two experiments are consistent.

Comparison of the data with theory is difficult. One, of course, assumes \( T \) invariance and CVC; but there are still four unknown axial form factors to be determined. Therefore, one is forced to perform a model dependent analysis of the data. Theorists have stressed comparing their models with cross section measurements, but cross sections are certainly difficult to measure. In fact, all models can be made to agree with the data, so no decisive tests can be made. F. von Hippel and P. Schreiner have pointed out that the density matrix elements of the \( \Lambda \) can provide sensitive tests of the models, because they do not depend upon the overall normalization of the neutrino flux. One can write the differential cross section for \( \Lambda \) production as follows:

\[
\frac{d^4\sigma}{dWdk^2d\Omega} = \frac{d^2\sigma}{dWdk^2} \left\{ \frac{1}{\sqrt{4\pi}} \left[ Y_0^0 - \frac{2}{\sqrt{5}} \left( \frac{3}{2} \right) Y_2^0 - \frac{4}{\sqrt{10}} \gamma_3 \Gamma_2 + \frac{4}{\sqrt{10}} \gamma_3 \Gamma_2 \right] \right\}
\]

where \( W \) is the \( \tau n \) mass, \( \Omega \) are the \( \tau^+ \) spherical angles in the \( \tau n \) rest frame, and \( Y_L^M(\Omega) \) are the usual spherical harmonics in a coordinate system for the \( \Lambda \) decay which is right-handed, with the \( z \) axis along the momentum transfer direction \( \hat{P}_\nu - \hat{P}_\mu \) and the \( y \)-axis along the production plane normal \( \hat{P}_\nu \times \hat{P}_\mu \). Maximum likelihood
Fig. 8. Invariant mass distributions for the reaction $\nu p + \mu^- \pi^+ p$. 
Fig. 9. Total cross sections for the reaction $\nu p \rightarrow \mu^- \pi^+ p$ as a function of beam momentum.
fits to the Argonne data have been performed (using this differential cross section) for different models, with $M_A$ as a free parameter. Table III displays the results. Salin's model has a quite low value for $M_A$ and a much too large $\hat{\beta}_{31}$ matrix element. Bijtebier also has a too large $\hat{\beta}_{31}$. The predictions of Zucker are not bad, but the Adler model fits the data best.

| TABLE III. Experimental Density Matrix Elements and, for Each Model: Fitted Values of $M_A$ and Density Matrix Elements |
|---------------------------------|---------------|---------------|---------------|
|                                | $M_A$ GeV/c $^2$ | $\hat{\beta}_{33}$ | $\hat{\beta}_{3-1}$ | $\hat{\beta}_{31}$ |
| Data                           |               |               |               |               |
| Salin                          | $0.53 \pm 0.10$ | $0.62$         | $0.08$         | $0.17$         |
| Adler                          | $1.13 \pm 0.18$ | $0.69$         | $-0.02$        | $-0.11$        |
| Bijtebier                      | $0.71 \pm 0.08$ | $0.66$         | $-0.02$        | $0.14$         |
| Zucker                         | $0.80 \pm 0.09$ | $0.77$         | $-0.02$        | $-0.12$        |

References

9. P. Schreiner and F. von Hippel, #805.

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III.2 Preliminary Result on the Ratio of Antineutrino to Neutrino Total Cross Sections (#941)

Presented by Ph. Heusse
Orsay

We present here the preliminary results of the CERN 1971 neutrino experiment. This experiment uses the large heavy liquid bubble chamber "Gargamelle" filled with freon (CF$_3$Br), and the CERN neutrino beam with an ejected proton beam of 26 GeV on a beryllium target.

We have taken a total of 500,000 photographs, equally divided between neutrino and antineutrino exposures for each running period—so the conditions of fluxes are the same for both neutrino and antineutrino data.

The results presented here concern only a part of our sample of neutrino, but the total for antineutrino.

Analysis

The analysis was carried out in the same way for neutrinos and antineutrinos. For each event, we have required that:

1) the total visible energy be greater than 1 GeV, because the flux below this energy is not precisely known.

2) the longitudinal momentum $P_L > 0.6$ GeV, by which most of the entering tracks are eliminated.

A. Results on Cross Sections

1) For neutrino cross sections (in units of $10^{-38}$ cm$^2$/nucleon) we obtain:

<table>
<thead>
<tr>
<th>E(GeV)</th>
<th>Elastic</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1.5</td>
<td>0.39 ± 0.13</td>
<td>1.42 ± 0.42</td>
</tr>
<tr>
<td>1.5-2</td>
<td>0.41 ± 0.08</td>
<td>1.42 ± 0.32</td>
</tr>
<tr>
<td>2-3</td>
<td>0.35 ± 0.07</td>
<td>1.62 ± 0.26</td>
</tr>
<tr>
<td>3-4</td>
<td>0.47 ± 0.11</td>
<td>2.58 ± 0.42</td>
</tr>
<tr>
<td>4-5</td>
<td>0.30 ± 0.10</td>
<td>2.23 ± 0.43</td>
</tr>
<tr>
<td>5-6</td>
<td>4.60 ± 1.0</td>
<td></td>
</tr>
<tr>
<td>6-7</td>
<td>7.6 ± 1.9</td>
<td></td>
</tr>
<tr>
<td>7-8</td>
<td>7.0 ± 2.1</td>
<td></td>
</tr>
<tr>
<td>8-9</td>
<td>8.2 ± 2.5</td>
<td></td>
</tr>
<tr>
<td>9-10</td>
<td>7.4 ± 2.5</td>
<td></td>
</tr>
</tbody>
</table>

A linear fit between 1 to 10 GeV gives $\sigma = (0.70 ± 0.14) E_\nu \times 10^{-38}$ cm$^2$ per nucleon.

The errors given above take into account the statistical as well as the systematic error coming mainly from the uncertainty in the flux (about 15%).

2) For antineutrino cross section (in $10^{-38}$ cm$^2$/nucleon) we obtain:

<table>
<thead>
<tr>
<th>E(GeV)</th>
<th>Elastic</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1.5</td>
<td>0.153 ± 0.045</td>
<td>0.349 ± 0.1</td>
</tr>
<tr>
<td>1.5-2</td>
<td>0.20 ± 0.04</td>
<td>0.48 ± 0.12</td>
</tr>
<tr>
<td>2-3</td>
<td>0.200 ± 0.036</td>
<td>0.635 ± 0.1</td>
</tr>
<tr>
<td>3-4</td>
<td>0.27 ± 0.05</td>
<td>0.93 ± 0.16</td>
</tr>
<tr>
<td>4-5</td>
<td>1.12 ± 0.22</td>
<td></td>
</tr>
<tr>
<td>5-6</td>
<td>1.68 ± 0.38</td>
<td></td>
</tr>
<tr>
<td>6-9</td>
<td>2.75 ± 0.65</td>
<td></td>
</tr>
</tbody>
</table>
A linear fit is possible for $E > 1.\text{GeV}$

$$\sigma = (0.274 \pm 0.054) \times 10^{-38} \text{ cm}^2 \text{ per nucleon.}$$

E. Results on the Ratio of Antineutrino to Neutrino Total Cross Sections $R = \sigma(\bar{\nu})/\sigma(\nu)$

For the calculation of $R$, errors due to flux uncertainties partially cancel out and reduce to $\%5\%$. From the above data we obtain

<table>
<thead>
<tr>
<th>$E$ (GeV)</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1.5</td>
<td>0.243 ± 0.028</td>
</tr>
<tr>
<td>1.5-2</td>
<td>0.338 ± 0.034</td>
</tr>
<tr>
<td>2-3</td>
<td>0.39 ± 0.037</td>
</tr>
<tr>
<td>3-4</td>
<td>0.36 ± 0.04</td>
</tr>
<tr>
<td>4-5</td>
<td>0.50 ± 0.09</td>
</tr>
<tr>
<td>5-6</td>
<td>0.33 ± 0.08</td>
</tr>
<tr>
<td>6-9</td>
<td>0.34 ± 0.08</td>
</tr>
</tbody>
</table>

The average value of $R$ for $E > 2 \text{ GeV}$ is

$$R_{av} = 0.377 \pm 0.023.$$  

It is evident that these data exclude the value $R = 1$ predicted by some models (diffraction model or spin equal 0 constituents).

All results have to be taken as preliminary.

**Discussion**

K. Schultze (Aachen): Taking the measured values for the total cross sections as the asymptotic linear rise, our results support the usual quark parton model with fractional charges: From the quark-parton model one gets our upper limit for the sum of the $\nu$ and $\bar{\nu}$ cross sections \( (\sigma_{\nu} + \sigma_{\bar{\nu}})_{\text{quark parton}} = 1.2 E_{\nu} \times 10^{-38} \text{ cm}^2/\text{nucleon}, \)

Feynman, Balaton Conference, 1972 (same as $\#140$, this Conference)

The preliminary result quoted is 20% below this upper limit. The final values might be somewhat higher (if later corrected for energy loss by neutral secondaries) but certainly not as much as 20%. With present experimental uncertainties, the result supports the quark-parton model.

Alternatively, one can exclude integrally charged constituents with spin 1/2 and isospin < 1. Nachtmann calculated for these constituents a corresponding cross section which is smaller than the experimental result by a factor of roughly two [Phys. Rev. D5, 686 (1972)].

In terms of the triplet quark-parton model the ratio $\sigma_{\bar{\nu}}/\sigma_{\nu}$ measures the ratio of antiquarks and quarks in the nucleon. The measured value $\sigma_{\bar{\nu}}/\sigma_{\nu}$ is somewhat above 1/3. It indicates that the average number of antiquarks is small. A rough calculation shows $N_{\bar{q}}/N_q < 1/4$.

R. P. Feynman (Cal. Tech.): Yes, and we can add that not only does the quark model give an upper limit to the total of neutrino and antineutrino cross sections, but it suggests that the correct answer must be (probably within 10%) of this upper limit—so it is virtually a complete prediction. (One need only add a reasonable guess that the strange quark partons in the non-strange proton do not carry as much momentum as, say, the down quarks.) Thus, these results constitute, as Schultze emphasized, our first real test of the hypothesis that the quarks carry the expected fractional charges.
### WEAK INTERACTIONS III - MESON DECAY

<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Non-Leptonic Decays of $K$ Mesons ($K \to 3\pi$, $K \to 2\pi$, Lifetimes)</td>
<td>U. Nauenberg (Colorado)</td>
</tr>
<tr>
<td>2. $\Delta Q/\Delta S$ in $K_{S}^{0}$ Decays and Interference in $K_{L}^{0}$, $K_{S}^{0} \to 3\pi$</td>
<td>G. Manning (Rutherford)</td>
</tr>
<tr>
<td>3. $K_{S}^{0}$ Form Factors</td>
<td>S. Wojcicki (Stanford)</td>
</tr>
<tr>
<td>4. New Measurement of $</td>
<td>\eta_{+}</td>
</tr>
<tr>
<td>5. Interference Effects in $K^{0}$ and $\bar{K}^{0} \to 2\pi$</td>
<td>A. Wattenberg (Illinois)</td>
</tr>
<tr>
<td>6. Radiative Decay of $K$ Meson</td>
<td>T. Kycia (BNL)</td>
</tr>
<tr>
<td>7. $K f_{4}$ and Rare Decay Modes</td>
<td>A. Mann (Pennsylvania)</td>
</tr>
<tr>
<td>8. $K \to 2\pi$ Puzzle</td>
<td>D. Nygren (Columbia)</td>
</tr>
</tbody>
</table>
This summary covers the topics in the session which because of lack of time could not be included in Prof. Rubbia's rapporteur talk on weak interactions. The sections on $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ are based on the notes and talk of Prof. U. Nauenberg; the section on $\Delta Q/\Delta S$ and $\eta_{+0}$ on the notes and talk of Dr. G. Manning.
I. K\(_\nu\) Form Factors

Introduction

The K\(_\nu\) form factor situation as of 1971 has been summarized in a comprehensive article by Chouinet et al.\(^{(1)}\). Accordingly we shall discuss here only the most recent experiments with a special emphasis on the new results presented at this conference.

As is well known, in the V-A theory of weak interactions the matrix element for K\(_\nu\) decays can be written as

\[
M = \frac{\mathcal{G}}{\sqrt{2}} \sin \theta \langle \pi |V_\mu|K\rangle \bar{v}_\nu \gamma_\mu (1 + \gamma_5) u_\nu
\]

and the hadronic part can be expanded (assuming the validity of \(\Delta Q/\Delta S\) rule)

\[
\langle \pi |V_\mu|K\rangle \propto (k + q)_\mu f_+ (t) + (k - q)_\mu f_- (t).
\]

\(k\) and \(q\) are the four vectors of \(K\) and \(\pi\) respectively and \(t\) is the momentum transfer between the \(K\) and the \(\pi\) mesons given by

\[
t = (k-q)^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi
\]

The form factors - \(f_+\) and \(f_-\) - are the only undetermined parameters in the theory and on invariance principles they can only depend on \(t\). If \(\Delta I=1/2\) rule is valid in these decays, then the form factors must be the same for \(K^0\) and \(K^+\) decays. Finally, in all the physically measurable quantities \(f_- (t)\) always enters multiplied by the mass of the lepton squared; thus that form factor can be neglected for the \(K_{\nu e}\) decays.

Since the range of \(t\) covered in K\(_\nu\) decays is rather small, one can expect the behavior of form factors in the physical region to be relatively
smooth; accordingly one parametrizes $f_+(t)$ and $f_-(t)$ by linear dependence on $t$, i.e.

$$f_+(t) = f_+(0) [1 + \lambda_+ t]; \quad f_-(t) = f_-(0)[1 + \lambda_- t]$$

where $t$ is expressed in units of $\frac{\hbar}{m}$. 

Sometimes it is convenient to define the form factor ratio

$$\xi(t) = \frac{f_-(t)}{f_+(t)}$$

and express the functional dependence of $\xi$ as

$$\xi(t) = \xi(0) + \Lambda t$$

On theoretical grounds, a convenient linear combination is the divergence from factor, defined by

$$f_0(t) = f_+(t) + f_-(t) \frac{t}{m_K^2 - m_\pi^2}$$

which is customarily expanded as

$$f_0(t) = f(0) [1 + \lambda_0 t]$$

Keeping only first-order terms in $f_+(t)$ and $f_0(t)$ the parameters defined above are related as follows

$$\xi(0) = (m_K^2 - m_\pi^2)(\lambda_0 - \lambda_+)$$

$$\lambda_+ = 0$$

$$f_+(0) = f(0) \text{ if } \xi(0) \text{ is finite}$$

$$\Lambda = -\xi(0) \lambda_+$$
Experimental Situation

There are four kinds of measurements that one can make that have a bearing on $K^0_L$ form factors:

1) The measurement of Dalitz plot density for $K^0_L$ decays allows one to determine $f_+(t)$. More specifically one can check the hypothesis of linear dependence of $f_+(t)$ and measure $\lambda_+$. The recent measurements of $\lambda_+$ are summarized in Tables I and II.

In general, the overall agreement between various experiments has been rather poor, and thus it is not clear how meaningful is the world average. Furthermore, it is possible that a quadratic term in the expansion of $f_+(t)$ may be necessary, since inclusion of such a term reduces the discrepancy between various experiments.\(^{(1)}\)

2) The experimental situation on the branching ratio $R$, defined by

$$R = \frac{\Gamma(K_{\mu 3}^-)/\Gamma(K_{e 3}^-)}{\Gamma(K_{\mu 3}^+)/\Gamma(K_{e 3}^+)}$$

is summarized in Table III. Several comments are in order here. There are serious inconsistencies between various measurements of $\Gamma(K^+ \rightarrow \pi^0 \mu^+ \nu)$. For example, the results of X2 collaboration, $(2.80 \pm 0.11)$% is more than three standard deviations away from the average of other experiments. If that experiment were removed, the value of $R$ for $K^+$ would be greater than the value of $R$ for $K^0$. On the other hand, one can make a counter argument, namely that the X2 collaboration experiment is especially clean and that the value of $R$ for $K^0$ appears to be decreasing with time. One can probably summarize by saying that the experimental situation on branching ratios

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is still far from settled, and it is probably too early to conclude that a difference between $R$ for $K^0$ and $K^+$ has been definitely established.

The relationship between $R$ and $\lambda_0$, assuming linear parametrization of form factors is exhibited in Table IV. It is clear that the dependence on $\lambda_+$ is very weak.

3) To the extent that time reversal invariance is a good symmetry principle, the muon polarization in $K \to \pi \mu \nu$ decay must be in the $\mu \pi$ plane. Furthermore, for every point on the Dalitz plot, there exists a direction along which the muon is 100% polarized, and this direction is completely specified by the value of $\xi(t)$. Thus measurement of this direction corresponds to a measurement of $\xi$. The significant measurements of muon polarization for both $K^0$ and $K^+ \pi \mu \nu$ decays are summarized in Table V.

4) An analysis of the Dalitz plot in $K_{\mu 2}$ decay can yield information on both $f_+$ and $\xi$ (or $f_+$ and $f$). Unfortunately, these two parameters are strongly correlated, and the answers that one obtains depend strongly on the parametrization used.

The most recent $K^0_{\mu 2}$ experiments have had a sufficiently large number of events to do a parameter independent fit, i.e. analysis of the distribution in a small enough band of $T_K$ (i.e. $t$) so that the variation of the form factors in this band can be ignored. The fitted value of $f_+^2$ and $\xi$ (or $f$) in individual bands can then be used to extract the $t$ dependence of these form factors.
The recent $K^+$ and $K^0$ Dalitz plot analysis experiments are summarized in Table VI; the values of $\xi(t)$ extracted from the recent high statistics experiments on $K^0 \rightarrow \pi \mu \nu$ decay are exhibited in Fig. 1. Special attention needs to be paid to the assumption about form factor dependence that were used in obtaining the values quoted in Table VI because of previously mentioned correlations.

**Contributions to This Conference**

We would next like to summarize the three contributions to this conference that have a bearing on $K_{\ell 3}$ form factors:

1) The Manchester University group has studied $K^0 \rightarrow e \mu \nu$ decay using the same apparatus that has been used in the $K^0 \rightarrow \pi \mu \nu$ and $K^+ \rightarrow \mu \nu$ studies. The problem of quadratic ambiguity in solving for the $K$ momentum was resolved by always taking the lower of the two possible momenta. This resulted in the choice of the right solution about 80% of the time. As the apparatus had no means of identifying electrons, each event was plotted twice on the Dalitz plot, corresponding to two different possible mass assignments. A satisfactory $\chi^2(36.3$ for 33 degrees of freedom) to V-A theory is obtained, yielding a value of $\lambda_+$ of

$$\lambda_+ = 0.055 \pm 0.010$$

2) A SLAC group has obtained a new measurement of both the $K^0 \rightarrow \pi \mu \nu / K^0 \rightarrow \pi \epsilon \nu$ branching ratio and a value of $\lambda_+ (4)$. This hydrogen bubble chamber experiment has studied a final sample (after cuts) of 21,023 $K^0$ decays, obtained by exposing the SLAC 40" chamber to a beam of $K^0_L$'s. The
experiment employs no mass identification, but relies solely on kinematical quantities to obtain separation of various decay modes. More specifically, if $p_{1T}$ and $p_{2T}$ are the transverse momenta of the two charged tracks, with $p_{1T} > p_{2T}$, then a quantity $c$

$$c = m_K - \sqrt{p_{1T}^2 + m_\pi^2} - \sqrt{p_{2T}^2 + m_\mu^2}$$

is clearly an invariant, with the property that

$$c^2 - p_T^2 > 0$$

for all $K_\mu_3$, if experiment resolution can be neglected ($p_T$ is the transverse momentum of the missing neutral). On the other hand, because of low electron mass, this quantity can be negative for $K_{e_3}$ decays. Thus by means of cuts on $(p_{o'})^2$ and on $c^2 - p_T^2$, one can obtain separation between the three major decay modes of $K_2^0$. This separation is exhibited in Fig. 2, which shows a scatter plot of these two variables. A Monte Carlo calculation, which takes proper account of all measurement errors as well as cuts applied to the data, is then used to extract the relative amounts of $K_{\mu_3}^0$, $K_{e_3}^-$, and $K_{\mu_3}^-$ in the data. The comparison between the data and the Monte Carlo for $(p_{o'})^2$ distribution is illustrated in Fig. 3. The good agreement between the two verifies that the Monte Carlo adequately describes the data. The isolated $K_{e_3}$ events populate all regions of the Dalitz plot, and have the property that the higher laboratory momentum track is the pion 94% of the time. Thus the experiment is also able to obtain a value of $\lambda_+$. The results of the experiment are
\[ R = 0.686 \pm 0.046 \]
\[ \Gamma(K \rightarrow \pi^+ \pi^- \pi^0)/\Gamma(K \rightarrow \text{all charged}) = 0.150 \pm 0.004 \]
\[ \lambda_+ = 0.041 \pm 0.013 \]

3) The Stanford-Santa Cruz collaboration\(^{(5)}\) has presented preliminary results from a very high statistics study of \(K^0 \rightarrow \pi \mu \nu\) Dalitz plot using the SLAC \(K^0\) spectrometer facility\(^{(6)}\). Use of time of flight and knowledge of momentum spectrum allowed the choice of a correct solution about 80% of the time. Muons were identified by the requirement of penetration through 42" of Pb. The contamination due to \(\pi \rightarrow \mu \nu\) decays is subtracted by means of a Monte Carlo calculation and amounts to about 6% of the data. As required by such a large statistics experiment (about \(10^6\) events and \(1.2 \times 10^6\) Monte Carlo events) extensive checks have been made of the agreement between the Monte Carlo predictions and the real data. No significant discrepancies have been observed that would affect the results. Radiative corrections, as calculated by Ginsberg\(^{(7)}\) have been applied to the data. Unparametrized fit has been made to the data grouped in 5 MeV by 5 MeV bins to both \(f_+\) and \(\xi\) and \(f_+\) and \(f\). The results of these fits are exhibited in Fig. 4. The errors quoted are purely statistical. The authors conservatively estimate that systematic effects could at most double the statistical errors. The linear parametrization of the resulting \(f_+\) values yields

\[ \lambda_+ = 0.052 \pm 0.005 \]

where the quoted error is again purely statistical.
Summary

The experimental situation appears rather confused. On the one hand there are the polarization measurements yielding values of $\xi$ around -1.2; on the other, the new Stanford-Santa Cruz experiment yields values of $\xi$ around -0.1 for $t > 2.5$ with an anomalous rise as $t \to 0$. The $K^0$ branching ratio measurements tend to agree with the Stanford-Santa Cruz results. The $K^+$ branching ratio measurements and the analyses of $K^+$ Dalitz plots fall roughly between these two extreme situations. The other $K^0$ Dalitz plot experiments yield strongly negative $\xi$ values, but also rather high $\lambda_+$ values. Forcing the fits to give $\lambda_+$ in agreement with the world average, would certainly raise the values of $\xi$, but it is not clear exactly how much.

Of great theoretical importance is the test of Callan-Treiman relation which predicts $f(m_K^2) = f_\pi/f_K = 1.27 \pm 0.03$. Since the prediction refers to non-physical region, a great deal of literature has been recently devoted to predicting the behavior of the $f$ form factor in the physical region. The early experimental indications, based mainly on the strength of the polarization measurements, indicated that $f$ had a rather strong negative slope at $t=0$ in contradiction with most of the theoretical ideas. The Stanford-Santa Cruz experiment, however, indicates that the $f$ form factor converges toward the Callan-Treiman point, but appears to have difficulty with meeting the requirement that $f_+ (0) = f(0)$. The current high statistics $K^0$ polarization experiments, will hopefully help to clarify the present situation.
II. K→2π Decays

We would like to summarize the situation on K→2π decays with a view to extracting information on possible contributions of ΔI=5/2 amplitude. There have been two sets of contributions relevant to this problem. Firstly, new data on ππ phase shifts has been presented at this conference (9). Secondly, if the newly reported values of K_s lifetime (10) hold up, then some of the old measurements of K_s branching ratios would have to be reevaluated. As an example, the value quoted by Baltay et al. (11) would change from 2.22 to 2.16 if the lifetime were 0.899 x 10^{-10} sec. For the purpose of the discussion below, however, we shall not modify the present world average value of the branching ratio.

From the ππ scattering studies we obtain

\[ \theta_s^0 = (44 \pm 5)^0 \]
\[ \theta_s^2 = (-7.7 \pm 1.2)^0 \]
\[ \theta_s^2 - \theta_s^0 = (-51.7 \pm 5)^0 \]
\[ \cos(\theta_s^2 - \theta_s^0) = 0.619 \pm 0.052 \]

Using the accepted world average value we have

\[ \frac{\Gamma(K^+\rightarrow \pi^+\pi^0)}{\Gamma(K_S^0 \rightarrow 2\pi)} = \frac{|\sqrt{3/2} \ a_2 - \sqrt{1/3} \ a_3|^2}{|A_0|^2} = (14.6 \pm 0.2) \times 10^{-4} \]

where

- \[ A_0 = \Delta I=1/2 \] amplitude, taken to be real
- \[ a_2 = \Delta I=3/2 \] amplitude
- \[ a_3 = \Delta I=5/2 \] amplitude

and

\[ A_2 = a_2 + a_3 \]
Thus we have

$$\frac{|\sqrt{3/2} a_3 - \sqrt{1/3} a_2|}{A_0} = (3.83 \pm 0.03) \times 10^{-2}$$

From $K_S^0$ branching ratio value we obtain

$$\frac{\Gamma(K_S^0 \rightarrow \pi^+ \pi^-)}{\Gamma(K_S^0 \rightarrow \pi^0 \pi^0)} = 2.21 \pm 0.03 = 0.986 \left[2 + 6 \sqrt{2} \frac{R_e A_2}{A_0} \cos (\theta_5^2 - \theta_5^0)\right]$$

and from the smallness of CP violating amplitudes and near equality of $\eta_{00}$ and $\eta_{4+}$ we can conclude

$$\frac{I_m A_2}{A_0} \ll \frac{R_e A_2}{A_0} \approx \frac{|A_2|}{A_0}$$

With this approximation we obtain

$$\frac{|a_3 + a_2|}{A_0} = (4.57 \pm 0.44) \times 10^{-2}$$

which combined with the result on partial rates of $K^+$ and $K_S^0$ yields

$$\frac{a_3}{A_0} = (0.08 \pm 0.28) \times 10^{-2}$$

$$\frac{a_3}{A_0} = (4.49 \pm 0.55) \times 10^{-2}$$
There have been 3 contributions to this conference on this subject.

1) The Colorado-U.C. Santa Cruz-Stanford collaboration has presented \(^{(12)}\) preliminary results on the measurement of \(K_L \rightarrow \pi^+ \pi^- \pi^0\) matrix element. The observed quadratic term does not seem to be large enough in comparison with the linear terms, and thus it appears that the matrix element itself needs a negative quadratic term.

2) The Columbia group \(^{(13)}\) has studied the matrix element in the \(\eta \rightarrow \pi^+ \pi^- \pi^0\) decay. Their quadratic term in the fit to \(|M|^2\) is consistent with zero, implying that the matrix element requires a quadratic term.

3) The Princeton group has measured \(^{(14)}\) the \(\tau\) decay matrix element and although they see a quadratic term in \(|M|^2\), it is consistent with a linear expansion of the matrix element.

The data on all the 3\(\pi\) decay modes of the K mesons and the \(\eta\) is shown in Table VII. The following conclusions can be drawn from the measured values of the linear term:

1) The \(\tau\) and the \(\tau'\) linear terms in \((S_2 - S_0)\) expansion are not in the 1: -2 ratio as required if the three pions are in pure \(I=1\) state.

2) The \(\tau\) and \(K_L\) linear terms in \((S_3 - S_0)\) expansion are not in the 1: -2 ratio as required by the \(\Delta I = 1/2\) rule. One possible model to account for this would be the decay of \(K_L\) through a virtual \(\eta\) intermediate state. Then the violation would be electromagnetic in character.

3) The values of linear coefficient appear in reasonable agreement with the current algebra predictions \(^{(15)}\), considering the approximations used in deriving the theoretical result.
IV. Status of $\eta_{\pi^0}$, $\eta_{\rho\rho\rho}$, and $\Delta Q/\Delta S$ in $K_e^-$ Decays

The experimental situation on $\Delta Q/\Delta S$ is summarized in Table VIII for $K_{e3}^-$ decays and in Table IX for $K_{\mu3}^+$ decays.

The new world averages, including the contributions to this conference are:

\[
\begin{align*}
\text{Re } X &= 0.025 \pm 0.02 \\
\text{Im } X &= 0.008 \pm 0.016 \\
\text{Re } X &= 0.08 \pm 0.07 - 0.09 \\
\text{Im } X &= 0.01 \pm 0.10
\end{align*}
\]

for $K_{e3}^-$ decays

for $K_{\mu3}^+$ decays

The status of $\eta_{\pi^0}$ is summarized in Table X. No world average is given because of highly non-gaussian nature of the errors in individual experiments.

Finally there has been reported\(^{(38)}\) the first measurement of $\eta_{\rho\rho\rho}$.

The 90% confidence limit value is

\[
\frac{\Gamma(K_{e3}^- \rightarrow \pi^0 \pi^0 \pi^0)}{\Gamma(K_{\mu3}^+ \rightarrow \pi^0 \pi^0 \pi^0)} < 1.2
\]
References


6. R. Coombes et al., Nucl. Inst. and Meth. 93, 317 (1972).


16. M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, C. Filippi-Filosofo, H. Huzita, P. Mattioli and G. Miari, Nuovo Cimento 38, 684 (1965). The result quoted was re-evaluated using a more recent value of the \( K_L^0 - K_S^0 \) mass difference: M. Baldo-Ceolin, private communication.


Table I

Determinations of $\lambda_+^\pm$ in the Decay $K^+ \rightarrow \pi^0 e^+\nu$

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\lambda_+^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of older experiments</td>
<td>$0.030 \pm 0.007$</td>
</tr>
<tr>
<td>E. R. Botterill et al., PRL 31B, 325 (1970)</td>
<td>$0.045 \pm 0.015$</td>
</tr>
<tr>
<td>H. J. Steiner et al., Amsterdam Conference</td>
<td>$0.027 \pm 0.010$</td>
</tr>
<tr>
<td>I.-H. Chiang et al., PR DC, 1254 (1972)</td>
<td>$0.029 \pm 0.011$</td>
</tr>
<tr>
<td>World average</td>
<td>$0.031 \pm 0.004$</td>
</tr>
</tbody>
</table>

Table II

Determinations of $\lambda_+^\mp$ in the Decay $K^0 \rightarrow \pi^\pm e^-\nu$

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\lambda_+^\mp$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of older experiments</td>
<td>$0.017 \pm 0.007$</td>
</tr>
<tr>
<td>C.Y. Chien et al., PL 35B, 261 (1971)</td>
<td>$0.05 \pm 0.01$</td>
</tr>
<tr>
<td>V. Bisi et al., PL 36B, 533 (1971)</td>
<td>$0.023 \pm 0.005$</td>
</tr>
<tr>
<td>Neuhofer et al., Amsterdam Conference (preliminary)</td>
<td>$0.034 \pm 0.014$</td>
</tr>
<tr>
<td>C.Y. Chien et al., II, Brookhaven Conference 1972</td>
<td>$0.044 \pm 0.006$</td>
</tr>
<tr>
<td>M. G. Albrow et al., This Conference, paper #392</td>
<td>$0.055 \pm 0.010$</td>
</tr>
<tr>
<td>G. Brandenburg et al., This Conference, paper #819</td>
<td>$0.041 \pm 0.013$</td>
</tr>
<tr>
<td>World average</td>
<td>$0.033 \pm 0.003$</td>
</tr>
</tbody>
</table>
Table III
Summary of Branching Ratio Results

\( K^+ \) Experiments

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \Gamma(K^+ \rightarrow \pi^0 \mu^+ \nu)/\Gamma(\text{all}) )</th>
<th>( \Gamma(\nu^+ \rightarrow e^+ \nu)/\Gamma(\text{all}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compilation of Chounet et al.</td>
<td>((4.85 \pm 0.07))%</td>
<td>((3.04 \pm 0.08))%</td>
</tr>
<tr>
<td>I.-H. Chang et al., PR Do, 1254 (1972)</td>
<td>((4.36 \pm 0.10))%</td>
<td>((3.33 \pm 0.16))%</td>
</tr>
<tr>
<td>World average</td>
<td>((4.85 \pm 0.06))%</td>
<td>((3.08 \pm 0.07))%</td>
</tr>
</tbody>
</table>

\[ R = \frac{\Gamma(K^+ \rightarrow \pi^0 \mu^+ \nu)}{\Gamma(K^+ \rightarrow \pi^0 e^+ \nu)} = 0.635 \pm 0.016 \]

\( K^0 \) Experiments

<table>
<thead>
<tr>
<th>Reference</th>
<th>( R = \frac{\Gamma(K^0 \rightarrow \pi^0 \nu)/\Gamma(K^0 \rightarrow e \nu)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compilation of Chounet et al.</td>
<td>0.684 \pm 0.018</td>
</tr>
<tr>
<td>G. Brandenburg et al., This Conference, paper #819</td>
<td>0.686 \pm 0.046</td>
</tr>
<tr>
<td>World average</td>
<td>0.684 \pm 0.017</td>
</tr>
</tbody>
</table>

Table IV
Relationship of \( R \) to \( \lambda_0 \) as function of \( \lambda_+ \)

<table>
<thead>
<tr>
<th>( R )</th>
<th>( \lambda_+ )</th>
<th>( \lambda_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.635 \pm 0.016</td>
<td>0.025</td>
<td>-0.005 \pm 0.013</td>
</tr>
<tr>
<td></td>
<td>0.050</td>
<td>-0.009 \pm 0.014</td>
</tr>
<tr>
<td>0.684 \pm 0.017</td>
<td>0.025</td>
<td>0.026 \pm 0.011</td>
</tr>
<tr>
<td></td>
<td>0.050</td>
<td>0.030 \pm 0.012</td>
</tr>
</tbody>
</table>
### Table V

Summary of $K \rightarrow \pi \mu \nu$ Polarization Measurements

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Charge</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. C. Callahan et al., PR 150, 1153 (1966)</td>
<td>$K^+$</td>
<td>-1.4 ± 1.8</td>
</tr>
<tr>
<td>D. Cuts et al., PR 184, 1380 (1969)</td>
<td>$K^+$</td>
<td>-0.95 ± 0.3</td>
</tr>
<tr>
<td>D. Haidt et al., PR 30, 10 (1971)</td>
<td>$K^+$</td>
<td>-1.0 ± 0.3</td>
</tr>
<tr>
<td>L. B. Auerbach et al., PRL 17, 930 (1966)</td>
<td>$K^0$</td>
<td>-1.2 ± 0.5</td>
</tr>
<tr>
<td>R. J. Abrams et al., PR 176, 1603 (1968)</td>
<td>$K^0$</td>
<td>-1.6 ± 0.5</td>
</tr>
<tr>
<td>J. A. Helland et al., PRL 21, 257 (1968)</td>
<td>$K^0$</td>
<td>-1.75 ± 0.5</td>
</tr>
</tbody>
</table>

### Table VI

Summary of $K^+ \rightarrow \pi^o \mu^+ \nu$ and $K^0 \rightarrow \pi^o \mu^- \nu$ Dalitz Plot Analyses

<table>
<thead>
<tr>
<th>Charge</th>
<th>Experiment</th>
<th>$\lambda_+$</th>
<th>$\xi$</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+$</td>
<td>D. Haidt et al., PR D3, 10 (1971)</td>
<td>0.055±0.025</td>
<td>$\xi(0) = -0.5 \pm 1.5$</td>
<td>$\lambda_+ = 0.029$</td>
</tr>
<tr>
<td>$K^+$</td>
<td>D. Haidt et al., PR D3, 10 (1971)</td>
<td>$\xi(0) = 0.0 \pm 2.0$</td>
<td>$\xi(6.8) = 0.3 \pm 0.21$</td>
<td>$\lambda_+ = 0.0$</td>
</tr>
<tr>
<td>$K^+$</td>
<td>C. Ankenbrandt et al., PRL 28, 1472 (1972)</td>
<td>0.024±0.022</td>
<td>$\xi(0) = 0.62 \pm 0.28$</td>
<td>$\lambda_+ = 0.0$</td>
</tr>
<tr>
<td>$K^+$</td>
<td>T.-K. Chiang et al., PR D6, 1254 (1972)</td>
<td>$\xi = -0.09 \pm 0.28$</td>
<td>$\lambda_+ = \lambda_- = 0.03$</td>
<td></td>
</tr>
<tr>
<td>$K^0$</td>
<td>M.G. Albrow et al., 0.085±0.015</td>
<td>See Fig. 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^0$</td>
<td>E. Dally et al., submitted to Nucl. Ph.</td>
<td>0.11±0.05</td>
<td>See Fig. 1</td>
<td></td>
</tr>
<tr>
<td>$K^0$</td>
<td>G. Donaldson et al., This Conference paper #779.</td>
<td>0.052±0.010</td>
<td>See Fig. 4</td>
<td></td>
</tr>
</tbody>
</table>
Table VII

Comparison of $K_L$ and $\eta$ Decays

\[
|M|^2 = 1 + aY + b Y^2
\]

\begin{align*}
\begin{array}{lrrrr}
 & a & b & a^2/4 \\
\tau & 0.2752 \pm 0.0033 & 0.025 \pm 0.010 & 0.01893 \pm 0.000044 \\
K_L & -0.8737 \pm 0.0052 & 0.1311 \pm 0.0099 & 0.1908 \pm 0.0022 \\
\eta & -1.08 \pm 0.01 & 0.03 \pm 0.03 & 0.292 \pm 0.006 \\
\end{array}
\end{align*}

\[
|M|^2 = 1 + a \frac{Q}{M_K} Y + b \left( \frac{Q}{M_K} \right)^2 Y^2
\]

\begin{align*}
\begin{array}{lrrrr}
 & a & b & a^2/4 \\
\tau & 1.809 \pm 0.021 & 1.11 \pm 0.44 & 0.815 \pm 0.019 \\
K_L & -5.198 \pm 0.031 & 4.64 \pm 0.35 & 6.75 \pm 0.080 \\
\end{array}
\end{align*}

\[
|M|^2 = 1 + g \left( \frac{S_s - S_o}{m} \right)^2
\]

\begin{align*}
\begin{array}{lrrrr}
 & g \text{ (new data)} & g \text{ (Particle Tables)} \\
\tau & -0.2169 \pm 0.0026 & -0.2057 \pm 0.0069 \\
K_L & 0.6484 \pm 0.0038 & 0.603 \pm 0.03 \\
\tau' & & 0.527 \pm 0.017 \\
\end{array}
\end{align*}
Table VIII
Summary of previous experiments to test the $\Delta S = \Delta Q$ rule in $K_{e3}$ decay

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Group</th>
<th>Technique</th>
<th>$K^0$ production reaction</th>
<th>Events</th>
<th>Re $x$</th>
<th>Im $x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Padua</td>
<td>HLBC</td>
<td>$K^+n \rightarrow K^0p$</td>
<td>152</td>
<td>-0.01 ± 0.16</td>
<td>-0.30 ± 0.15</td>
</tr>
<tr>
<td>17</td>
<td>Paris</td>
<td>HLBC</td>
<td>$K^+n \rightarrow K^0p$</td>
<td>315</td>
<td>0.035 ± 0.11</td>
<td>0.21 ± 0.11</td>
</tr>
<tr>
<td>18</td>
<td>Columbia/Rutgers</td>
<td>HBC</td>
<td>$\bar{p}p$ annihilation</td>
<td>45</td>
<td>-0.08 ± 0.28</td>
<td>0.24 ± 0.40</td>
</tr>
<tr>
<td>19</td>
<td>Pennsylvania</td>
<td>Spk.ch.</td>
<td>$\pi^- p \rightarrow \Lambda K^0$</td>
<td>116</td>
<td>0.17 ± 0.16</td>
<td>0.0 ± 0.25</td>
</tr>
<tr>
<td>20</td>
<td>Carnegie/BNL</td>
<td>DBC</td>
<td>$K^+d \rightarrow K^0pp$</td>
<td>454</td>
<td>0.12 ± 0.09</td>
<td>-0.08 ± 0.07</td>
</tr>
<tr>
<td>21</td>
<td>LRL</td>
<td>HBC</td>
<td>$K^-p + K^0n$</td>
<td>242</td>
<td>0.22 ± 0.07</td>
<td>-0.08 ± 0.08</td>
</tr>
<tr>
<td>22</td>
<td>La Jolla</td>
<td>Spk.ch.</td>
<td>$K^+n \rightarrow K^0p$</td>
<td>686</td>
<td>0.09 ± 0.15</td>
<td>-0.11 ± 0.11</td>
</tr>
<tr>
<td>23</td>
<td>CERN</td>
<td>HBC</td>
<td>$\bar{p}p$ annihilation</td>
<td>121</td>
<td>0.09 ± 0.13</td>
<td>0.22 ± 0.37</td>
</tr>
<tr>
<td>24</td>
<td>Cal.Tech.</td>
<td>Spk.ch.</td>
<td>$\pi^-$ on brass target</td>
<td>1079</td>
<td>-0.07 ± 0.04</td>
<td>0.11 ± 0.09</td>
</tr>
<tr>
<td>25</td>
<td>Illinois/Northeastern</td>
<td>Spk.ch.</td>
<td>$\pi^- p + \Lambda K^0$</td>
<td>342 $K_{e3}$</td>
<td>-0.13 ± 0.11</td>
<td>-0.04 ± 0.16</td>
</tr>
<tr>
<td>26</td>
<td>CERN/Orsay/Vienna</td>
<td>Spk.ch.</td>
<td>$K^+p + K^0pn^+$</td>
<td>5800</td>
<td>0.05 ± 0.025</td>
<td>-0.01 ± 0.02</td>
</tr>
<tr>
<td>27</td>
<td>Padua/Wisconsin</td>
<td>HLBC</td>
<td>$K^+n \rightarrow K^0p$</td>
<td>312</td>
<td>0.11 ± 0.07</td>
<td>0.04 ± 0.09</td>
</tr>
<tr>
<td>Ref.</td>
<td>Group</td>
<td>Technique</td>
<td>$K^0$ production reaction</td>
<td>Events</td>
<td>Re $X$</td>
<td>Im $X$</td>
</tr>
<tr>
<td>------</td>
<td>------------------------</td>
<td>-----------</td>
<td>--------------------------</td>
<td>--------</td>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>28</td>
<td>Salcay/CERN/Oslo</td>
<td>HBC</td>
<td>$K^+ p \rightarrow \pi^+ pK^0$</td>
<td>306</td>
<td>$0.05 \pm 0.06$</td>
<td>$0.08 \pm 0.06$</td>
</tr>
<tr>
<td>29</td>
<td>RHEL/Cambridge</td>
<td>Sp.ch.</td>
<td>$\pi^- p \rightarrow \Lambda^0 K$</td>
<td>1367</td>
<td>$-0.03 \pm 0.07$</td>
<td>$0.09 \pm 0.08$</td>
</tr>
</tbody>
</table>

**World Average**

$\text{Re } X = 0.025 \pm 0.02$

$\text{Im } X = 0.008 \pm 0.016$
<table>
<thead>
<tr>
<th>Ref.</th>
<th>No. Events</th>
<th>Technique</th>
<th>Re $x_\mu$</th>
<th>Im $x_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>38</td>
<td>HBC</td>
<td>0.19 +.13</td>
<td>-.12 +.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.19 -.18</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>100</td>
<td>SC</td>
<td>.04 +.10</td>
<td>0.12 +.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-.04 -.13</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>74</td>
<td>HBC</td>
<td>.10 +.13</td>
<td>-.03 +.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-.10 -.16</td>
<td></td>
</tr>
</tbody>
</table>
Table X

\[ \eta_{+-0} \]

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Events</th>
<th>Technique</th>
<th>Re ( \eta_{+-0} )</th>
<th>Im ( \eta_{+-0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>50</td>
<td>HBC</td>
<td>0.05 ± .3</td>
<td>-0.15 ± .45</td>
</tr>
<tr>
<td>33</td>
<td>50</td>
<td>HBC</td>
<td>2.75 ± .65</td>
<td>0.5 ± .7</td>
</tr>
<tr>
<td>34</td>
<td>99</td>
<td>DBC</td>
<td>0.47 ± .3</td>
<td>-0.12 ± .44</td>
</tr>
<tr>
<td>35</td>
<td>99</td>
<td>S.C.</td>
<td>-0.09 ± .19</td>
<td>0.56 ± .43</td>
</tr>
<tr>
<td>36</td>
<td>384</td>
<td>S.C.</td>
<td>0.13 ± .17</td>
<td>0.17 ± .27</td>
</tr>
<tr>
<td>37</td>
<td>180</td>
<td>HBC</td>
<td>0.17 ± .17</td>
<td>0.01 ± .38</td>
</tr>
</tbody>
</table>

-233-
Fig 1. Values of $\hat{s}(t)$ vs $t$ from recent experiments in $K^0 \rightarrow \mu\nu$. 

-234-
Fig 2 Separation of the decay modes of $K_2^0$ in SLAC data
Fig 3 Comparison between experimental data and Monte-Carlo distribution (full line) in SLAC data.
Fig. 4. Parameters derived from the high-statistics data of the Stanford-Santa Cruz experiment (5).
Weak Interactions IV: Theory of Weak Interactions
(Phenomenology)

Organizer: M. K. Gaillard
Scientific Secretaries: S. H. Aronson
P. L. Pritchett
E. I. Rosenberg
<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Leptonic Interactions and W-Production</td>
<td>J. Smith (SUNY, Stony Brook)</td>
</tr>
<tr>
<td>2. The Neutrino Mass and Astrophysics</td>
<td>N. Cabibbo (Rome)</td>
</tr>
<tr>
<td>3. Radiative Corrections</td>
<td>A. Sirlin (NYU)</td>
</tr>
<tr>
<td>4. Parity Violation in Nuclear Physics</td>
<td>E. Fischbach (Purdue, Indianapolis)</td>
</tr>
<tr>
<td>5. CP Violation and $K_L \rightarrow \mu \mu$</td>
<td>L. Wolfenstein (Carnegie Mellon)</td>
</tr>
<tr>
<td>6. Current Algebra and Chiral Symmetry</td>
<td>V. I. Zakharov (ITEP, Moscow)</td>
</tr>
<tr>
<td>6.1 On the Violation of the $\Delta f = 1/2$ Rule in Nonleptonic K DECAYS</td>
<td>P. Auvil (Northwestern)</td>
</tr>
<tr>
<td>6.2 Status of Chiral Symmetry Breaking</td>
<td>M. K. Gaillard (Orsay/CERN)</td>
</tr>
<tr>
<td>6.3 Status of $K_{13}$ Form Factors</td>
<td></td>
</tr>
</tbody>
</table>
INTRODUCTION

The availability of high energy neutrino beams and the possibility of a renormalizable theory of weak interactions offer interesting prospects for the understanding of weak interactions. The session included discussions both of effects which can be studied in high energy leptonic interactions and of radiative and higher order weak corrections to low-energy processes. These discussions are reported below. Insofar as they touch on the new gauge theories of weak and electromagnetic interactions, they complement the work reported in the session on formal theory. The interesting possibility of limiting the neutrino mass from astrophysical considerations is also discussed.

The remainder of the discussion in this session consisted essentially of status reports on outstanding problems in weak interactions, parity violation in nuclear physics and various aspects of K decay. Unfortunately, the continued state of flux of experimental results does not allow many conclusions to be drawn from this conference.

The use of current algebra together with the detailed experimental studies of nonleptonic K decays now available can in principle provide some insight on the origin of isotopic spin selection rules, this analysis is discussed along with various difficulties of interpretation. The study of leptonic K decay in conjunction with current algebra bears closely on the problem of chiral symmetry breaking. We have therefore also included a brief summary of the present state of understanding of chiral symmetry breaking derived from hadron physics.

1 LEPTONIC INTERACTIONS AND W-BOSON PRODUCTION* (#'s 6 7 8)

(J Smith SUNY Stony Brook New York)

In the last two years there has been considerable interest in purely leptonic interactions and W-boson production. The present situation regarding $\bar{\nu}_e e^-$ scattering has already been reviewed by B W Lee so we concentrate on neutrino trident production and neutrino production of spin zero and spin one charged W bosons. We will also discuss production of neutral W bosons in $e^+ e^-$ collisions.

1.1 Trident Production

As you all know Gurr, Remes and Sobel 4 have now pushed the cross section for the scattering of electron antineutrinos on electrons down to almost the level predicted by the V-A theory of Feynman and Gell-Mann 2. Their experimental result is $\sigma_{\text{exp}}/\sigma_{\text{V-A}} < 1.9$ which puts an upper bound on the size of the neutral current allowed in this reaction. The local limit of Weinberg's Lagrangian 3 for $\bar{\nu}_e e^-$ scattering is

---

*Supported in part by National Science Foundation Grant No. GP-32998X
and the Chen Lee analysis of the data gives $\sin^2 \theta \leq 0.35$. Unfortunately the present experimental limit on $\nu_e^e$ scattering is not good enough to place further restriction on $\sin^2 \theta$. The limit found by Albright for this process is $\sigma_{\exp}^\mu (\nu_\mu + e^- \rightarrow \nu_\mu + e^-) \leq 0.4 \sigma_{\nu-A}^\mu (\nu_\mu + e^- \rightarrow \nu_\mu + e^-)$. Neutral current effects in reactions involving muons cannot be measured in scattering processes so we have to turn to neutrino trident production in a Coulomb field, i.e.,

$$\nu_\mu + Z \rightarrow \nu_\mu + \mu^+ + \mu^- + Z.$$  \hspace{1cm} (1.2)

The present experimental limit on the coupling constant for this reaction is only consistent with a $\nu_\mu$ coupling constant $G_{\mu} < 20 \text{ G}_{\text{F}}$. Figure 1.1 shows the relevant diagrams for the reaction in a

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{feynman_diagrams.png}
\caption{Feynman diagrams for $\nu_\mu + Z \rightarrow \nu_\mu + \mu^+ + \mu^- + Z$ in a theory with charged and neutral W bosons.}
\end{figure}
Weinberg type theory involving charged and neutral W bosons. If the boson masses are very large, then the weak interaction part collapses to an effective coupling like that given in Eq. (1) and the number of Feynman diagrams reduces from five to two. Hence we need to know the muon spectra for reaction (2) in a theory with an arbitrary admixture of vector and axial vector couplings. Unfortunately the recent calculations\(^7\) of lepton spectra for reaction (2) were limited to the case of a V-A interaction. The extension of these calculations to include the local limit of a Weinberg type theory has been made by Brown, Hobbs, Stanko, and Smith.\(^8\) The characteristic feature of the results can be understood very easily from Fig. 1.2 which shows the \(\mu^-\) energy spectrum in reaction (2) for arbitrary vector and axial vector couplings and 50 GeV neutrinos incident upon a proton target. Obviously the \(C_V C_A\) term is negative for low energies and positive for high energies. The corresponding curves for the \(\mu^+\) are identical except for a switch in the sign of the \(C_V C_A\) terms in the effective Lagrangian, the energy spectra for the muons either add constructively or destructively at low energies. In particular, for V-A theory with \(C_V = C_A = 1\), the average energy of the \(\mu^-\) is larger than that of the \(\mu^+\), and therefore the average angle of \(\mu^-\) is smaller than that of the \(\mu^+\).

Note also that the integrated total cross section receives very little contribution from the \(C_V C_A\) term because the energy spectrum changes sign in the center of the range of integration. Fig. 1.3

**Fig. 1.2** $\frac{d\sigma}{dE_{\mu^-}}$ in cm$^2$/GeV versus $E_{\mu^-}$ in GeV for production by 50-GeV neutrinos on iron with a coherent form factor.
Fig 1.3 Total cross sections (per nucleon) for $\nu_\mu + Z \rightarrow \nu_\mu + \mu^+ + \mu^- + Z$ in two theories with $C_V = C_A = 1$ (solid lines) and $C_V = 1.2$, $C_A = 0.5$ (dashed lines) and an iron target. The final total cross section is therefore

$$\sigma(Fe) = \sigma_{coh} + \sigma_{incoh}$$

shows the total coherent and incoherent cross sections (per nucleon) for reaction (2) calculated for an iron target. The solid line gives the V-A result and the dashed line the prediction of a model with $C_V = 1.2$ and $C_A = 0.5$. It is also interesting to look at the corresponding reaction with an electron pair, i.e.,

$$\nu_\mu + Z \rightarrow \nu_\mu + e^+ + e^- + Z.$$  \hspace{1cm} (4.3)

Figure 1.4 shows the coherent cross section (per proton) for this reaction in Weinberg's model. The target is an iron nucleus. For comparison the corresponding cross sections for $\nu_e + Z \rightarrow \nu_e + e^+ + e^- + Z$ are also given for (1) standard V-A theory, and (2) Weinberg's model. Obviously the cross section for the $\nu_\mu$ induced reaction is at least one order of magnitude smaller than the cross sections for the $\nu_e$ induced reaction.
Fig. 1.4. Total cross sections for $\nu + Z \rightarrow \nu + e^+ + e^- + Z$ with an iron nucleus. These cross sections are for the coherent scattering and have been multiplied by $Z^2$. (1) and (2) represent the reaction $\nu + Z \rightarrow \nu + e^+ + e^- + Z$ in V-A theory and in Weinberg's model. The neutral current prediction for $\nu + Z \rightarrow \nu + e'^+ + e'^- + Z$ is also shown.

1.2 Production of Charged W Bosons.

In the event that charged vector bosons do exist with masses below 15 GeV, then they will certainly be produced in neutrino experiments at NAL. If they decay leptonically then the cross section for reaction (2) will be much larger because the process is now mediated by an on-shell vector boson. The calculations of Brown, Hobbs, and Smith\textsuperscript{9} show that a charged vector boson produced in a Coulomb field via the reaction

\[
\nu_\mu + Z \rightarrow \mu^- + W^+ + Z
\]

is at least 90% left circularly polarized for the energy and mass region available at NAL, \textsuperscript{11} this "following the neutrino helicity sense" was first discussed by Bell and Veltman.\textsuperscript{10} The average
left-handed polarization $\bar{\rho}_{-+}$ is plotted in Fig. 1.5. Hence the decay $\mu^+$ angular distribution in the

![Graph](image)

Fig. 1.5. Plot of the average left-handed polarization $\bar{\rho}_{-+}$ as a function of the neutrino energy for a free proton and $\kappa = +1, 0, -1$.

$W$ boson's rest frame is $(1 - \cos \theta^*)^2$ where $\theta^*$ is the angle of the $\mu^+$ with respect to the original direction of the $W^+$ in the laboratory. The signal for production of real $W$ bosons is therefore the opposite of the signal for reaction (2) in V-A theory, i.e., the $\mu^+$ meson has a larger average energy than the $\mu^-$. Figure 1.6 gives a comparison of the two distributions. If the $W$ boson decays
Fig. 1.6. Distributions in the muon energies for $\mu^+\mu^-$ production off a proton with 15-GeV neutrinos. The solid lines are the results for the calculation which includes a W boson with a mass of 2 GeV and the dashed lines are results from the trident production in V-A theory.

hadronically then it may still be possible to detect its presence by a clustering of events in the $q^2$ versus $\nu$ plot for deep inelastic neutrino scattering.\textsuperscript{11}

In the model of T. D. Lee\textsuperscript{12} a negative-metric spin-zero charged W boson is introduced to make weak interaction theory renormalizable. The W propagator is therefore

$$D_{\mu\nu}(q^2) = \frac{\epsilon_{\mu\nu}}{q^2-M_1^2} + \frac{q_{\mu} q_{\nu}}{M_2^2} \left( \frac{1}{q^2-M_1^2} - \frac{1}{q^2-M_0^2} \right).$$

The decay amplitude for a $W_0$ particle into a lepton pair now vanishes for zero lepton mass. If this boson decays via hadronic channels, then, after being produced via the reaction

$$\nu_{\mu} + Z \rightarrow \mu^- + W_0^+ + Z,$$

its decay may give a clustering of events in a plot of $q^2$ versus $\nu$. Unfortunately both the signal and the total cross section are sensitive functions of the ratio between the boson masses. In general, as shown in Fig. 1.7 the total cross section for (5) is less than that for (4) by at least one order of magnitude.\textsuperscript{13} Also the signal in $q^2$-$\nu$ plot can only be seen if the mass of the spin one boson is much larger than that of the spin zero boson. Figure 1.8, taken from the work of Turner and Barish,\textsuperscript{13} shows the $q^2$-$\nu$ plot for $M_0 = 2$ GeV/c$^2$ and $M_1 = 15$ GeV/c$^2$. The neutrino energy is 300 GeV and the target an iron nucleus.
Fig 17 (a) Cross sections per proton for scattering off free protons (---), and coherent scattering off neon (-----), and lead (--), for $W_0^-$ production with $M_1 = 5 \text{ GeV}$ and $M_0 = 2.5 \text{ GeV}$, and $W_1^-$ production with $M_1 = 5 \text{ GeV}$ (b) Cross sections $\sigma_0 (---), \sigma_0^h (- - - -), \sigma_0^p (- -)$, for $W_0^\pm$ production with $M_1 = 10 \text{ GeV}$ and $M_0 = 5 \text{ GeV}$, and for $W_1^\pm$ production with $M_1 = 10 \text{ GeV}$ (c) Cross sections $\sigma_p (---)$ and $\sigma_{\text{inel}} (- -)$ for $W_1^\pm$ production with $M_1 = 5 \text{ GeV}$, and $W_0^\pm$ production with $M_1 = 5 \text{ GeV}$, $M_0 = 2.5 \text{ GeV}$ $[\sigma_p (- - -), \sigma_{\text{inel}} (- -)]$, and with $M_0 = M_1 = 5 \text{ GeV}$ $[\sigma_p (- - -), \sigma_{\text{inel}} (- - -)]$, $\kappa = 0$
Fig 18 $q^2$-$\nu$ distribution of 1000 neutrino inelastic events (a) and $W_0$ production events (b) normalized relative to the inelastic background (a) for $M_0 = 2$ GeV/c$^2$, $M_f = 15$ GeV/c$^2$, $K = 0$ GeV and $\nu_W = 0$. The spike signature is more pronounced for larger $M_0$ masses.

1.3 Production of Neutral $W$ Bosons.

All the reactions considered so far test weak interaction theory in a "low energy" domain. One way to obtain information on weak interactions at higher energy is to use electron positron colliding beams. However, the electromagnetic interactions are expected to dominate over the weak interaction. For example, the reaction $e^+e^- \rightarrow \mu^+\mu^-$ is already allowed electromagnetically in the one photon exchange approximation. Therefore, unless some special circumstances diminish the size of the one photon exchange diagram, it will be impossible to see any weak interaction effects.
It has been known for some time\textsuperscript{14} that circulating beams in an \(e^+e^-\) storage ring have very large transverse polarizations \(|P_+|, |P_-|\), along or opposite to the direction of the magnetic field due to synchrotron radiation. If we use a set of axes where \(z\) is the direction of the beam \(x\) the direction of the electron polarization, and the muon is emitted with a polar angle \(\theta\) and azimuthal angle \(\phi\) then the differential cross section for \(e^+e^- \rightarrow \mu^+\mu^-\) in the one photon exchange approximation is

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8\pi} \left[ 2 - \sin^2\theta (1 + |P_+| |P_-| \cos 2\phi) \right].
\] (1.6)

Now if \(|P_+| = |P_-| = 0.92\) as expected theoretically then for \(\theta = \frac{\pi}{2}\) and \(\phi = 0\) or \(\pi\), the one photon exchange diagram is severely reduced. Hence, for such special kinematical conditions, the interference term between the weak and electromagnetic interactions may be observable. This interference as calculated from the diagram in Fig. 1.9 adds an additional term to Eq. (6), namely

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \epsilon(s)}{8\pi} \left[ 2 - \sin^2\theta (1 + |P_+| |P_-| \cos 2\phi) + 2 \cos \theta \right],
\] (1.7)

where \(P = \pm 1\) is the muon helicity and

\[
\epsilon(s) = \sqrt{\frac{2G_0 M_W^2}{4\pi \alpha}} \frac{s}{(s-M_W^2)^2}.
\]

The longitudinal polarization of the outgoing muons \(P_L\) is dependent upon \(\epsilon(s)\) and is a maximum at \(\phi = 0\) or \(\pi\). Mann, Cline, and Reeder\textsuperscript{15} propose to look for such effects at SLAC. Two photon exchange radiative corrections also give rise to a longitudinal polarization of the outgoing muons, but they have already been calculated.\textsuperscript{16} A more convenient parameter to measure is probably the asymmetry

\[
A(\theta, \phi) = \frac{d\sigma(\theta, \phi) - d\sigma(\pi-\theta, \phi)}{d\sigma(\theta, \phi) + d\sigma(\pi-\theta, \phi)}
\]

for the spin averaged cross section. However, the contribution to the asymmetry from two photon exchange diagrams has not yet been calculated. If \(G_D = G_E\) and \(s = 64 \text{ GeV}^2\) then Cung, Mann, and Paschos\textsuperscript{17} predict that \(P_L (\phi = 0, \theta = 65^\circ) = 3.1\%\) and \(A(\phi = 0, \theta = 65^\circ) = -2\%\). Measuring such small effects may prove very difficult.
REFERENCES


2. NEUTRINO MASS AND ASTROPHYSICS (#295)

(N. S. Cabibbo, Enrico Fermi Institute, Roma, and G. Marx, Eötvös University, Budapest)

Measuring the weak interaction parameters in the laboratory is a hard undertaking just as a consequence of the small coupling constant. For example, our experimental information about the neutrino and neutretto rest masses is rather crude:

\[ m(\nu_e) \leq 60 \text{ eV} \quad m(\nu_\mu) \leq 1.2 \text{ MeV}. \]

It has been stressed that, similarly to gravity, weak phenomena might play a decisive role outside of the laboratory, on an astronomical scale.

The most disturbing problem of neutrino science is the solar neutrino puzzle. In the R. Davis experiment the capture rate of solar neutrinos on $^{37}$Cl is measured to be
Rate \( < 10^{-36} \, s^{-1} \)

in contrast with the value

\[
Rate_{(th)} = 9 \times 10^{-36} \, s^{-1}
\]

predicted by the astrophysical sun model and by the conventional theory of weak interactions. If weak interaction physics were responsible for this discrepancy, the most straightforward explanation would be that the neutrinos produced at the center of the sun do not reach the earth. Pontecorvo suggested that due to an \( L_e \) and \( L_{\mu} \) violating coupling a \( \nu_e \leftrightarrow \nu_{\mu} \) oscillation occurred. This predicts \( Rate = -1/2 \, Rate_{(th)} \) as the most probable value. In the optimistic case one can get \( Rate = 1/7 \, Rate_{(th)} \) In order to have \( Rate = 1/7 \, Rate_{(th)} \) one needs further neutrinos and an oscillation among them \( \nu_e \leftrightarrow \nu_{\mu} \leftrightarrow \nu_{\tau} \).

Another explanation of the solar neutrino puzzle would be that the solar neutrinos decay on their way to the earth. The decay products may be neutrinos lighter than the electron neutrino and massless bosons according to the scheme \( \nu_e \rightarrow \nu_l + \phi \).

Both explanations need a transmutation time \(< 7 \, \text{min} \) (the flight time of light from sun to earth). The transmutation time of neutrinos can be finite only if they are massive particles (\( m_{\nu} = 0 \) would mean \( \nu_c \) and an infinite time dilatation). This conclusion raises again the question: would it be possible to obtain limits on the neutrino and neutrino rest masses more accurate than those achieved in laboratory experiments?

Zeldovich first emphasized that empirical cosmology offers an approach to obtaining a value or at least an upper limit for the neutrino and neutrino rest masses. This idea was exploited by a more elaborate calculation of G. Marx and A. S. Szalay and quite recently in a note by R. Cowieson and J. McClelland. In the early state of the hot universe, above the temperature \( 10^{12} K \) all the leptons were in thermodynamical equilibrium with each other. Later the muons annihilated and the neutrinos were decoupled from the plasma. The main neutrette interactions are \( \nu_{\mu} + e^- \rightarrow e + e^- \), \( \nu_{\mu} + n \rightarrow p + \mu^- \), \( \nu_{\mu} + e^- \rightarrow \nu_{\mu} \) via virtual muon pair or via neutral vector boson. The most conservative decoupling temperature for the neutrino \((\nu_{\mu} + e^- \rightarrow e + \nu_{\mu})\) is \( T_d = 1.2 \times 10^{44} K \) that for the neutrino is \( T'_d = 0.18 \times 10^{44} K \)

The occupation number for fermions or bosons is given by

\[
n(E) = \left( \frac{\sqrt{m_j^2 + p^2}}{kT} \right)^{-1}
\]

where \( T \) is given as a function of the scale factor (or world radius) \( R \) by the adiabatic condition \( S \propto \frac{\rho}{R} \) at decoupling \( T_d \) and \( p = \lambda^{-1} - R^{-1} \) after decoupling. So one can calculate pressure and mass density in terms of the scale factor \( R \). This enables us to integrate the Einstein equation

\[
R^2 \frac{\dot{R}}{R} = \frac{8\pi G}{3} \rho(R) - \frac{\Sigma n(E)}{\int n(E)} <E_0>
\]

if the scale factor at decoupling \( R_d \) is given. The integration extends from \( T_d \) to the present value of the photon temperature \( T_d = 2.7 K \). The corresponding age of the universe \( t_0 \) is the Hubble
constant $H_0 = \frac{\ddot{R}(t_0)}{R(t_0)}$, the deceleration value $q_0 = \frac{-\ddot{R}(t_0)R(t_0)}{\dot{R}(t_0)^2}$ come out as a result of the computation. If the astronomers supply us with accurate information about the actual value of $t_0$, $H_0$, and $q_0$, the unknown masses $m(\nu_e)$, $m(\nu_\mu)$ and the unknown initial value $R_d$ can be calculated.

(If it is the most convenient procedure to start the calculation from the neutrino decoupling temperature $T_d$. The corresponding radius is $R_d$.)

Zel'dovich concluded from the age of the moon rocks (from $t_0 > 4.5 \times 10^9$ y) that $m(\nu_e) < 200$ eV. Marx and Szalay\textsuperscript{6} concluded from the recent values $\dot{H}_0^{-1} = (18.4 \pm 2) \times 10^9$ y and $q_0 < 2$ that $m(\nu_\mu) < 140$ eV. Cowsik and McClelland\textsuperscript{7} concluded from the value $q_0 = 0.94 \pm 0.4$ that $\Sigma m(\nu) < 66$ eV. These mass limits are by four orders of magnitude more accurate than those obtained in laboratory measurements.\textsuperscript{1}

![Fig. 2.1. Constraints on the neutrino mass from various astrophysical measurements; the allowed region is shaded.](image-url)
It is difficult to improve this method to give a much better accuracy. Pontecorvo suggested that the delay of the neutrino impulse after the optical flash of a supernova explosion may give the speed of the electron neutrinos, consequently also their rest masses. Such a neutrino watching of supernova explosion is, however, a music of the future. We must first catch sight of the sun with our neutrino eyes.

REFERENCES


3. RADIATIVE CORRECTIONS (#115)

(A. Sirlin, New York University)

The most interesting recent developments in the field of second-order corrections to weak interactions are the results obtained in the framework of Weinberg's unified model of weak and electromagnetic interactions. It is clear that similar studies can be extended in principle to other spontaneously broken gauge symmetries of the general class discussed by Salam, Weinberg, 't Hooft, Lee, Zinn-Justin, and others.

Muon Decay

It is convenient to first focus attention on muon decay, as this process has been accurately studied experimentally, and it is devoid of obvious strong interaction complications.

The first interesting result, obtained by Rajasekaran, 4 Lee, 5 and Appelquist, Primack, and Quinn 6 states that the second-order corrections to μ decay in the unitary gauge 7 of the Weinberg theory are finite after renormalization of the coupling constants. That is, a certain class of divergent contributions can be "absorbed" in the definitition of the renormalized coupling constants \( g_{\nu \gamma} \) and \( g_{\mu \nu \gamma} \). There are other divergent residual contributions but these cancel among themselves.

Rajasekaran and Lee study directly the divergent parts of the Feynman integrals, while Appelquist, Primack, and Quinn use an ingenious dispersive approach.

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To give an idea of the problem, consider the Feynman diagram of Fig. 3.1(a). To calculate this matrix element one studies the vertex function \( \bar{u} \lambda_\mu \Lambda_\lambda(q) u \) represented in Fig. 3.1(b). Setting \( q^2 = m_W^2 \), \( \epsilon^\lambda \epsilon_\lambda = 0 \) (\( \epsilon^\lambda \) is the polarization vector of a physical \( W \)) corresponds to the case in which the \( W \) meson is on its mass shell. Defining

\[
\bar{u} \lambda_\mu \Lambda_\lambda(m_{sh}) u \left( \frac{1}{Z_1} - 1 \right) \bar{v} \lambda_\mu \gamma_5 u,
\]

we can write

\[
\bar{u} \lambda_\mu \Lambda(q) u - \left( \frac{1}{Z_1} - 1 \right) \bar{u} \lambda_\mu \gamma_5 u + \bar{u} \lambda_\mu \left( \Lambda(q) - \Lambda(m_{sh}) \right) u,
\]

where \( m_{sh} \) is an abbreviation for "mass shell of \( W \)."

The first term on the right-hand member of Eq. (2) can be absorbed in the renormalized coupling constant \( g_{\mu \nu} \). The second term is still divergent. This feature is characteristic of the spontaneously broken gauge theories. We call such contributions "residual divergences". In a similar manner we can study the diagram of Fig. 3.1(c). Here two subtractions are made corresponding to mass and field renormalizations of the \( W \) meson. Again, a residual divergence is left over. Finally we can study skeleton diagrams such as that depicted in Fig. 3.1(d) which is not associated with any coupling constant or field renormalizations. The task is then to show that all the residual divergences and the skeleton divergences cancel exactly.

---

**Fig 3.1** Some diagrams contributing to the second-order corrections to \( \mu \) decay in Weinberg's model. \( W \) and \( Z \) are the charged and neutral massive vector bosons.
In the Weinberg model, there appear diagrams involving the W and Z mesons (these are the charged and neutral massive vector bosons) as in Fig. 3.4, diagrams involving W and γ and those involving W and the scalar meson φ. The second interesting result obtained by the above-mentioned authors is that after renormalization of the coupling constants $g_{\text{ev}_W}$ and $g_{\mu\nu\mu\nu}$, the contributions involving $WZ$, $W\gamma$, and $W\phi$ are separately convergent. That is, the cancellations mentioned above occur independently for the graphs involving $WZ$, $W\gamma$, and $W\phi$. Similar results have been obtained by Lee for $\nu$ lepton scattering.

Electron-Muon Universality and the Ratio $g_{\text{ev}_W}/g_{\mu\nu\mu\nu}$

There exists, however, a rather interesting point in which the cooperation between the electromagnetic and weak interactions becomes essential. This concerns the ratio $g_{\text{ev}_W}/g_{\mu\nu\mu\nu}$ of the two renormalized coupling constants. In terms of the bare coupling constant $g_0$, we have

$$g_{\text{ev}_W} = g_0 \sqrt{Z_{2e}} \sqrt{Z_{2\nu_e}} \sqrt{Z_{3W}/Z_{4\text{ev}_W}}$$

$$g_{\mu\nu\mu\nu} = g_0 \sqrt{Z_{2\mu}} \sqrt{Z_{2\nu\mu}} \sqrt{Z_{3W}/Z_{4\mu\nu\mu\nu}},$$

where the $Z_{2}$'s and $Z_{3}$ correspond to field renormalizations and the $Z_{4}$'s are the vertex renormalizations. Note that the bare coupling constant $g_0$ is the same in both equations as a consequence of the $e\mu$ universality implicit to lowest order in the Lagrangian density, and that this property is still valid in the presence of the spontaneous symmetry breaking. In computing the ratio $g_{\text{ev}_W}/g_{\mu\nu\mu\nu}$, $g_0$ drops out. This implies that this ratio can be calculated in this theory as it only involves the renormalization constants. Furthermore, because $g_{\text{ev}_W}$ and $g_{\mu\nu\mu\nu}$ are renormalized coupling constants, the answer must be finite. This has been verified to order $g^3$ in explicit calculations in the unitary gauge carried out independently by Appelquist, Primack, and Quimby and by Bollini, Giambiagi, and the present author. The calculations have been done using the n dimensional regularization method introduced independently by Veltman and 't Hooft and by Bollini and Giambiagi and require the consideration of the diagrams depicted in Fig. 3.2. The individual diagrams give rise to divergent terms proportional to $m_\ell^2/m_W^2$ and $m_\ell^2/m_Z^2$ ($m_\ell$ is the lepton mass), but these contributions cancel out when the various graphs are taken into account. This makes the calculation of $g_{\text{ev}_W}/g_{\mu\nu\mu\nu}$ convergent to order $g^3$. It is clear that some other ratios of renormalized coupling constants can be computed as well. As an example, the finiteness of $g_{\text{ev}_W}/g_{\mu\nu\mu\nu}$ has been verified to order $g^3$. Note that the fact that the theory gives a finite answer for these ratios of renormalized coupling constants is a consequence of the delicate interplay of weak and electromagnetic interactions! The fact that they can be calculated to higher orders is due to the $e\mu$ universality implicit in the primary interaction.

Practical Considerations.

If we now focus our attention on the more practical side, a relevant question is whether the contributions involving the W, Z, and φ mesons may give significant finite contributions. Because $m_W^2$, $m_Z^2$, $m_\phi^2 >> m_\mu^2$, the effect of the new particles on the shape of the electron spectrum and the energy dependence of the angular distribution is negligible. To be sure, there are large
corrections to these observables arising from the photon contributions but these would be essentially the same as the old radiative corrections calculated in the local theory. There may be, however, new contributions to the overall rate and the total asymmetry. One way to express the effect on the rate is the following: if we define $G_{\mu \text{exp}}$ by means of the relation

$$\frac{1}{\tau} \left( \frac{G_{\mu \text{exp}}}{m_{\mu}} \right)^2 \frac{m_{\mu}}{192 \pi^3} = \frac{1}{3},$$

(3.5)

where $\tau$ is the experimental muon lifetime, then up to order $g^4$ one can write

$$\frac{G_{\mu \text{exp}}}{\sqrt{2}} \frac{2}{8 m_w^2} \left[ 1 + \frac{1}{4} g^2 \lambda^+ \right],$$

(3.6)

where $g$ is a suitably defined renormalized $e\nu w$ coupling constant.

Appelquist, Primack, and Quinn have studied the finite contributions to $\lambda$ arising from the $WZ$ and lepton cuts in their dispersive calculation. The answer depends on the ratio $R = m_w^2/m_z^2$. They find that the contribution of these cuts to $\lambda$ is a fraction of a percent for $0.65 < R < 0.9$. Values of $R \leq 0.65$ are excluded by the analysis of $\nu e$ scattering by Chen and Lee. Note, however, that as $R \to 1$ the perturbation expansion breaks down because to lowest order $g^2 \alpha^2/(1 - R)$, so that the terms of order $g^4$ can become appreciably larger as $R$ approaches 1! Appelquist, Primack, and
Quinn have also discussed the contributions of the W cuts and are presently studying the effects of the Wγ cuts. The latter are more difficult to evaluate because of infrared divergences and soft-photon contributions. Although a formula such as Eq. (6) expresses the second-order corrections to the lowest-order relation $g / 8m = \mu^2$, it is clearly not very interesting by itself. To get something more one must at least compute the rate for a second process and check some principle of universality such as the $\mu$ and $\beta$ decay universality.

Some preliminary work on second-order corrections to hadronic $\beta$ decay has been carried out by Lee on the basis of the assumption that the $Z$ meson couples to the Weinberg current

$$J^Z - \mu^3 - A^3 - \frac{g^2}{g^2 + g^2} \beta^2 \mu,$$

where $g'$ is the second constant characteristic of the SU(2)$_L \times$ U(1) gauge theory.

The interesting thing is to try to compute the corrections to the ratio of the rates of $\beta$ and $\mu$ decay, something that has not been done so far. Although the general arguments of renormalizability indicate that the result should be finite in the broken gauge theories, the evaluation of the finite parts, if at all possible, may depend on the way in which the hadrons and the Cabibbo angle are introduced into the theory. If the old calculations of radiative corrections in the W theory can serve as a guide, it is very likely that large corrections of order $\alpha S\sin^2 \theta$ and $\alpha S\sin^2 \theta$ may arise in these corrections. The verification of this statement must await, however, a rather complete calculation. It is interesting to note that relatively large corrections of the order of a few percent are indeed necessary at present to satisfy the comparison of Cabibbo's universality with the experimental data.

**Conventional Theory: Radiative Corrections to the Goldberger-Treiman Relation**

In the framework of conventional theory some results have been obtained recently regarding the second-order electromagnetic corrections to the Goldberger-Treiman relation. This problem is mainly of conceptual interest, as it is rather natural to attribute the departure of the Goldberger-Treiman relation from exact validity to the breaking of SU(2) × SU(2) in the strong interactions, rather than to electromagnetic effects.

Under restrictive assumptions it is possible to show that in the presence of electromagnetism one can write the Goldberger-Treiman relation as

$$g_A'(m + m_\pi) = \sqrt{g_A} g_{\pi^0} + \delta^h + \alpha C' + O(\alpha \epsilon),$$

where $g_A$, $f_A$, $m_\pi$, $m_\rho$, $g_{\pi^0}$ are observable quantities renormalized by electromagnetism, $\delta^h$ represents the purely hadronic corrections to zeroth order in $\alpha$, $C'$ is a finite quantity and $\epsilon$ is the parameter that characterizes SU(2) × SU(2) breaking. This expression means that all the divergent terms of order $\alpha$ have been absorbed in the definition of the physical quantities. Nothing can be said in this approach about the terms of order $O(\alpha \epsilon)$. An alternative way to express this limitation is to state that the argument makes literal use of the PCAC approximation, namely, hadronic matrix elements involving $A^{\mu\pi^+}(0)$ at $q^2 = 0$ ($A^{\mu\pi^+}$ is the axial vector current) are approximated by the contribution of the pion pole, neglecting the error in such a procedure.

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The derivation assumes the validity of the divergence equation
\[
\partial_\mu A_\mu^\pi (x) = -\sqrt{2} \varepsilon^{\tau \pi} (x) + ieA_\mu^\pi (x) a^\pi (x), \tag{3.9}
\]
where \( a^\pi (x) \) and \( \varepsilon^{\tau \pi} (x) \) are the electromagnetic field and the pseudoscalar density responsible for the breaking of \( SU(2) \times SU(2) \) in the strong interactions. More importantly, the argument assumes the existence of operator product expansions at short distances and Wilson's enumeration of the fields of dimensionality \( d \leq 4 \).

REFERENCES AND FOOTNOTES

2. A. Salam, Proceedings of the Eighth Nobel Symposium (Almqvist and Wiksell, Stockholm (1968)).
5. For an extensive discussion of spontaneously broken gauge theories, see B. W. Lee's talk at the Plenary Sessions of this Conference as well as the reports by many authors at the Parallel Sessions. In particular, the report by J. R. Primack discusses several aspects of higher-order effects, with a somewhat different emphasis.
9. We recall that the unitary gauge is that gauge in which the would-be Goldstone bosons of the theory have been transformed away, so that only the fields corresponding to the physical particles appear explicitly in the Lagrangian density.
17. See, for example, A. Sirlin in Proceedings of the International Conference on Weak Interactions at CERN (1969), and references cited therein.
The principal motivation for studying parity violation in nuclear physics is as a probe of models of the weak Hamiltonian $H_w$. It can be shown \(^4\) that even models of $H_w$ which are explicitly constructed to agree in their predictions for low energy leptonic and semileptonic processes, such as $\mu \rightarrow e\nu\bar{\nu}$ or $n \rightarrow p\nu\bar{\nu}$, will not in general agree in their predictions for nonleptonic processes such as $\Lambda \rightarrow p\pi$. From a theoretical point of view, the most sensitive means of discriminating among models of $H_w$ is by measuring the ratio

$$R = \frac{a(\Lambda_0)}{a(n^-)}$$

where $a(\Lambda_0)$ is the S-wave (parity-violating) amplitude for $\Lambda \rightarrow p\pi$, and $a(n^-)$ is the S-wave amplitude for the (virtual) strangeness-conserving weak decay $n \rightarrow p\pi^-$. Direct calculations indicate that $R = R(H_w)$ can vary by more than an order of magnitude from one model to another. \(^2\) $a(n^-)$ can be studied experimentally through the parity-violating (p-v) effects it induces in the nucleon-nucleon interaction, as shown in Fig. 4.1. In order to extract $a(n^-)$ from such experiments, one must be able to calculate the p-v contributions from other mesons. It can be shown, however, that from the low-lying $J^P = 0^-$ and $1^-$ nonets only $\pi^\pm$, $\rho^\pm$, $\rho^0$, $\omega$, and $\phi$ contribute to $V_{12}$. The momenta of the various particles are given in parentheses.

![Diagram](image)

**Fig 4.1** One-meson-exchange contributions to the parity-violating internucleon potential $V_{12}$. The shaded circles indicate a weak parity-violating interaction while the shaded square denotes a purely strong vertex. $M$ represents a scalar (S), pseudoscalar (P), or vector (V) meson. From the low-lying $J^P = 0^-$ and $1^-$ nonets only $\pi^\pm$, $\rho^\pm$, $\rho^0$, $\omega$, and $\phi$ contribute to $V_{12}$. The effect of $V_{12}$ in nuclei is to admix "wrong-parity" states into nuclear levels which would otherwise be eigenfunctions of the angular momentum $J$ and parity $P$. Hence in the presence of $V_{12}$, nuclear levels have the structure

$$\psi = \psi(J^P) + \mathcal{F}\psi(-J^P),$$

where $\mathcal{F} \sim V_{12}$ is a small coefficient whose exact magnitude depends both on models of $H_w$ (through $V_{12}$) and on details of nuclear structure. The presence of contributions proportional to $\mathcal{F}$ give rise to apparent violations of parity selection rules in nuclear transitions, such as the observation of circular polarization of emitted photons. Thus working backwards from experiment, we can extract information about $H_w$ from p-v nuclear transitions.

A comparison of present day theory and experiment with reference to two particular p-v transitions is given in Table 4.1. It is seen that there is a systematic tendency of the theoretical values to be substantially lower than the experimental values, when the conventional (Cabibbo) model of $H_w$ is used. However, one cannot conclude from this that the "true" model of $H_w$ is one other than the Cabibbo model, since on symmetry grounds it can be shown that only the "$\rho$-exchange"
Table 4.1. Comparison of Theory and Experiment (Cabibbo Model of $H_w$).

<table>
<thead>
<tr>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n + p \rightarrow d + \gamma$</td>
<td>$P = (6.1 - 9.0) \times 10^{-6}$</td>
</tr>
<tr>
<td>Hadjimichael/Fischbach</td>
<td>$P = (1.30 \pm 0.45) \times 10^{-6}$ Lobashov 1972</td>
</tr>
<tr>
<td>$10^{-9} \leq</td>
<td>P</td>
</tr>
<tr>
<td>Danilov</td>
<td>$P = (9.6 - 12.8) \times 10^{-6}$ Vinh Mau/Desplanques</td>
</tr>
<tr>
<td>Tadić</td>
<td>$P \geq (5 \times 10^{-5})$ Missimer</td>
</tr>
<tr>
<td>Partovi</td>
<td></td>
</tr>
<tr>
<td>$181\text{Ta}(482\text{keV})$</td>
<td>$= (17.1 - 19.5) \times 10^{-6}$</td>
</tr>
<tr>
<td>$P = (6.1 - 9.0) \times 10^{-6}$ Tadić/Eman</td>
<td></td>
</tr>
<tr>
<td>$= (9.6 - 12.8) \times 10^{-6}$ Vinh Mau/Desplanques</td>
<td></td>
</tr>
<tr>
<td>$= (17.1 - 19.5) \times 10^{-6}$ Missimer</td>
<td></td>
</tr>
<tr>
<td>$= (0.035 - 0.059) \times 10^{-6}$ Gari et al.</td>
<td></td>
</tr>
</tbody>
</table>

References for Table 4.1.

Contribution to $V_{12}$ can be responsible for the nonzero value of $P_\gamma$ in $n + p \rightarrow d + \gamma$, and this contribution (in contradistinction to that from pion exchange) varies little from one model to another. The resolution of this particular discrepancy is not clear at the moment, although some work in this direction by Danilov was reported to the conference by Professor Lobashov.

We note in passing that in another contribution to this conference, J. L. Alberi, R. Wilson, and I. Schröder have reported a value $P_\gamma = (6.0 \pm 1.5) \times 10^{-4}$ for radiation following the combined 8.51- and 9.04-MeV transitions in $^{114}$Cd following thermal neutron capture.

The general relationship among the various theoretical concepts of weak interactions and nuclear physics and the experiments on parity violation in nuclei is indicated schematically in Fig. 4.2. Table 4.2 summarizes the isospin character of the parity-violating potential $V_{12}$ tested by different nuclear transitions.

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Fig. 4.2. Outline of the present scheme for extracting information about $H_w$ from parity violating nuclear interactions.
Table 42. Isospin Analysis of Experiments in Parity-Violation. Cross (X) indicates the isospin term(s) in $V_{12}$ to which the experiment is sensitive.

| Experiment | $|\Delta T| = 0$ | $|\Delta T| = 1$ | $|\Delta T| = 2$ | Performed |
|------------|----------------|----------------|----------------|------------|
| $\alpha$ decay | X | | | Yes |
| $^{16}_8(8.88 \text{ MeV}) \rightarrow ^{12}_4 \text{C} + \alpha$ | X | | | Yes |
| $n + p \rightarrow d + \gamma$ | X | | | Yes |
| circular polarization | | | | |
| $n + p \rightarrow d + \gamma$ | X | | | No |
| angular asymmetry | | | | In progress |
| $n + d \rightarrow t + \gamma$ | X | X | | |
| circular polarization | | | | |
| Complex nuclei ($^{184}_{71}\text{Ta}$, etc.) | X | X | X | Yes |
| $\alpha + d \rightarrow ^6\text{Li} + \gamma$ | X | | | Yes |
| Self conjugate nuclei ($^{18}_{8}\text{F}$, $^{10}_{5}\text{B}$, etc.) | X | | | No |
| $e + d \rightarrow e + d$ | X | | | No |

5. CP VIOLATION AND $K_L \rightarrow \mu^+\mu^-$ PUZZLE (#’s 85, 184, 256, 373)

(L. Wolfenstein, Carnegie-Mellon University, Pittsburgh)

Experimental results on CP violation in $K^0$ decay have continued to converge on the superweak values. Two measurements give $|\eta_{00}/\eta_{-+}|$ equal to unity with an accuracy of 5 to 10%. The phase $\phi_{-+}$ equals the superweak value of 43° within 5 to 10% and the charge asymmetry in $K_{f3}$ decays is in agreement with the prediction $2|\eta_{-+}| \cos 43^\circ$. No CP or T violation has been discovered in other processes. It remains important to emphasize that there exist many milliweak theories that make the same predictions as the superweak for $K^0$ decays but predict CP or T violation in some other process at the level of parts per thousand. However, present tests of CP or T violation outside of $K^0$ decays do not go down to this level. It is also noted that tests of CPT invariance are in many cases no better.

Many theoretical papers have been published to try to explain the Berkeley result which gives an upper limit of $4.8 \times 10^{-9}$ on the rate for $K_L \rightarrow \mu^+\mu^-$. The most attractive possibility was the suggestion of Christ and Lee that CP violation was involved. The idea was that if $K_1$ and $K_2$ were the CP eigenstates so that

$$K_L = K_2 + \epsilon K_1$$

$$K_S = K_4 + \epsilon K_2$$

then the expected imaginary amplitude $K_2 \rightarrow \mu^+\mu^-$ (through the $2\gamma$ state) might be cancelled out by the $K_4 \rightarrow \mu^+\mu^-$ amplitude (large enough to be significant even when multiplied by $\epsilon$). This requires
two new physical effects--a large direct coupling of $K_\lambda$ to $\mu^+\mu^-$ and a CP violating decay amplitude (in contradiction with the superweak theory) for either $K_\lambda$ or $K_\mu$ to $\mu^+\mu^-$. The idea seems reasonable if these two new physical effects are the result of a single new effective interaction which allows the CP violating decay $K_\lambda \to \mu^+\mu^-$ in the $S_0$ state. Such an effective interaction is

$$H' = G' \sin \theta \frac{A_\lambda}{c} \left[ \bar{u} Y_\lambda (4 + \gamma_5) u \right]$$

+ possible e term + possible hadron term

The required value of $G'$ is $10^{-2}G$ and thus $H'$ fits the requirements of a milliweak theory of CP violation. The most distinctive feature of this model is the large branching ratio (greater than $10 \times 10^{-7}$) predicted for $K_\lambda \to \mu^+\mu^-$. (This prediction follows from a unitarity branching ratio of $6 \times 10^{-9}$ for $K_L \to \mu\mu$ and an experimental upper limit of $1.8 \times 10^{-9}$. If these numbers were $5.4 \times 10^{-9}$ and $2 \times 10^{-9}$, respectively, the value $10 \times 10^{-7}$ would be reduced to $7 \times 10^{-7}$.) An upper limit for this branching ratio reported at this conference is $4 \times 10^{-7}$.

The effective form given by Eq. (5 1) may occur naturally in some existing milliweak theories of CP violation. For example, just such an interaction occurs in the Okubo theory and indeed the order of magnitude could have been predicted ahead of time by this theory. If the branching ratio for $K_\lambda \to \mu^+\mu^-$ should turn out to be sufficiently low, on the other hand, it would indicate the failure of the Okubo model independently of the $K_\lambda \to \mu\mu$ puzzle.

If the Christ-Lee explanation is ruled out then there are no attractive resolutions of the $K_L \to \mu^+\mu^-$ puzzle. The most promising solutions would be those involving some kind of anomalous muon interaction, the search for such anomalies remains important in any case. The most extreme suggestions are those involving CPT violation, violation of the superposition principle, or the existence of extra $K^0$ mesons.

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4. S. Barshay, contribution #373.
5. S. Nakamura, contribution #256.

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6. CURRENT ALGEBRA AND CHIRAL SYMMETRY

6.1 On the Violation of the $\Delta I = 1/2$ Rule in Nonleptonic $K$ Decays (#'s 156, 360, 822, 876)

(V. I. Zakharov, Institute for Theoretical and Experimental Physics, Moscow)

Two experimental papers presented to this Conference report measurements of the matrix elements for $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ and $K^0_L \rightarrow \pi^+ \pi^- \pi^0$ decays, respectively. These works are based on very large statistics and a careful study of possible systematic errors. According to the results obtained the $\Delta I = 1/2$ rule is badly violated for these decays.

If one defines the slopes of pion spectra $b$ as coefficients in the expansion of the matrix element

$$M = a \left[ 1 + b \frac{P_1 \cdot (2P_2 - P_3)}{m_K^2} \right].$$

$p_K, p_1, p_2, p_3$ being the four-momenta of the kaon, even and odd pions respectively, the ratio of the slopes in the above decays is predicted by the $\Delta I = 1/2$ rule

$$\frac{b(0^{-+})}{b(-++)} = -2.$$  \hspace{1cm} (6.1.2)

Experimentally this ratio is equal to

$$\frac{b(0^{-+})}{b(-++)} = -2.89 \pm 0.05,$$  \hspace{1cm} (6.1.3)

where the error corresponds to the uncertainties of the values of the slopes obtained in Refs. 1-2 and is not averaged over other data.

A large violation is in fact expected on the basis of current algebra calculations. According to Ref. 3

$$\frac{b(0^0)}{2b(+--)} + 1 - \frac{b(0^-)}{b(--)} + 1 = \frac{3cA^{3/2}}{a(+--)b(+--)} \approx -0.37,$$  \hspace{1cm} (6.1.4)

where $c = 1.7 \, m_{\pi}^{-1}$ and $A^{3/2}$ is the amplitude for $K^+ \rightarrow \pi^+ \pi^0$ decay. If the $\Delta I = 1/2$ rule is valid the left-hand side of Eq. (6.1.4) vanishes. If, however, experimental numbers are used a large violation is predicted.

The theoretical value of the ratio $b(+0)/b(+--)$ is close to the current experimental result (3). The predicted equality of $b(+0)$ and $b(+00)$ now seems to be violated.

$$\frac{b(0^0)}{b(+0)} \bigg|_{\text{exp}} \approx 1.15.$$  \hspace{1cm} (6.1.5)

In this connection new experiments on measuring $b(+00)$ with accuracy comparable to that achieved in Refs. 1-2 are desirable.

Although experimental data tend to agree with Eq. (6.1.4) I will spend some time discussing the assumptions involved as they are far from fully justified at present.

---

**The experimental numbers used in Eq. (6.1.4) are as follows:** $b(+--) = 0.9, A^{3/2} = 1.77 \cdot 10^{-5}$ MeV, $a(+--) = 1.92 \cdot 10^{-5}$ The value of $b(+--)$ is slightly different from that used in Ref. 3.
Assumptions.

Relations (6.1.4) are obtained by applying the standard soft pion technique to \( K \rightarrow 3\pi \) decays.\(^5\) The derivation involves two assumptions which I would like to call desirable ones in the sense that their check would give important information on the structure of the weak Hamiltonian and one further assumption which has no fundamental importance and is rather suspect, but still necessary.

a) The equal time commutator of the weak Hamiltonian with the axial "charge" is assumed to be equal to

\[
[H_w, A] = [H_w, V],
\]

(6.1.6)

where \( V \) is the vector charge, or the generator of the group of isotopic rotations. In particular, this assumption is satisfied if the Hamiltonian is constructed from the \( V-A \) currents.

Using Eq. (6.1.6) and the PCAC hypothesis it is possible to relate the amplitudes with \( \Delta I = \frac{3}{2} \) in \( K \rightarrow 3\pi \) and \( K \rightarrow 2\pi \) decays. Furthermore,

b) it is assumed that decay \( K^+ \rightarrow \pi^+ \pi^0 \) is caused by the \( \Delta I = \frac{3}{2} \) part of the weak Hamiltonian rather than by the radiative corrections. This is true if the observed decays are a manifestation of the weak interaction which is a product of charged currents.

Using assumptions a) and b) it is possible to calculate the amplitude at the unphysical point where the pion four-momentum is zero. To relate this prediction to measurable quantities one relies on the assumption

c) that the transition amplitude with \( \Delta I = \frac{3}{2} \) is a linear function of the total pion energy. This assumption may be wrong if quadratic terms are large.

Extrapolation of the Amplitude and Interactions in the Final State.

Linear extrapolation with real coefficients may also be wrong if the interaction in the final state is strong. Indeed, the amplitude for \( K \rightarrow 2\pi \) decay contains phase shifts at the total two-pion energy \( m_K \):

\[
\begin{align*}
A(K^+_s \rightarrow \pi^+ \pi^-) &= A_{\frac{1}{2}} e^{i\delta_0} e^{\frac{2}{3} A_{\frac{3}{2}}} e^{i\delta_2} \\
A(K^+ \rightarrow \pi^+ \pi^0) &= A_{\frac{3}{2}} e^{i\delta_2}.
\end{align*}
\]

(6.1.7)

According to current estimate the phase shift \( \delta_0 \) in the state \( I = 0 \) is large. However, when relating the \( \Delta I = \frac{3}{2} \) amplitude in \( K_s^+ \rightarrow 3\pi \) decay to that in \( K^+ \rightarrow \pi^+ \pi^0 \) one assumes only that \( \delta_2 = 0 \) which is consistent with experiment. From this point of view the extrapolation of the \( \Delta I = \frac{3}{2} \) part is safer than the extrapolation of the \( \Delta I = \frac{1}{2} \) part. However, corrections to the \( \Delta I = \frac{3}{2} \) amplitude due to rescattering could also be large since only in the limit \( p_\pi = 0 \) are the other two pions in a pure \( I = 2 \) state. Thus, the neglect of the final state interaction is not obviously inconsistent as is the case for \( \Delta I = \frac{1}{2} \) amplitude; it might be inconsistent, however. Let me note that corrections to current algebra predictions were studied by Neveu and Scherk.\(^8\)

It is worth emphasizing also that experimental rather than theoretical numbers for \( a(\pm -) \) and \( b(+- -) \) are used in the right-hand side of Eq. (6.1.4). The use of theoretical values (see e.g., 4, 7) makes a 50% difference in the predicted contribution of \( \Delta I = \frac{3}{2} \) amplitude.
Some Ambiguities in Testing the $\Delta I = 1/2$ Rule in $K \to 3\pi$ Decays

It was realized some time ago that it is far from simple to check the $\Delta I = 1/2$ rule in $K \to 3\pi$ decays with an accuracy better than say $10\%$ \cite{9,10}. The hope is that the predicted violation of the $\Delta I = 1/2$ rule is too large to be obscured by these uncertainties. But still it is important to discuss them furthermore, relations which are very briefly mentioned below may become most interesting in the coming years. Thus, it seems appropriate to list the following uncertainties.

a) Electromagnetic mass splitting and large values of slopes make it impossible to formulate predictions of the $\Delta I = 1/2$ rule with a good accuracy in the case of rates \cite{9,10}. Starting from completely equivalent matrix elements in the limit of isotopic symmetry one obtains different predictions when physical masses are used. In this connection it is worth mentioning that the corresponding uncertainty in the values of the product $a_b$, which represents the coefficient in the invariant expansion of the matrix element, arises only from quadratic terms. Since not so much is known about these terms one may hope that they are small. In this case it is better to compare with theory not the ratio of coefficients $b$, as is being done now, but the ratio of the products $a_b$.

b) Final state interactions make it impossible to expand the matrix element in powers of energy. The nonanalytic terms can be expressed in terms of $\pi \pi$ scattering lengths \cite{11} (for details see a review article \cite{12} and papers \cite{10,13} for application of the general theory to $K \to 3\pi$ decays).

c) The Coulomb correction changes the value of the decay slope in $\tau$ decay by $-10\%$. One feels uneasy about this since calculation of the simplest Feynman graphs does not indicate that the Coulomb correction really dominates the electromagnetic contribution to the $\Delta I = 3/2$ amplitude \cite{14}.

$K \to 3\pi$ Decays and $\pi \pi$ Scattering Lengths

In Ref. 10 a relation was found which survives uncertainties a) and b) mentioned in the preceding section

$$\frac{\gamma(\pi^+ \pi^-)}{4\gamma(000)} - \frac{3\gamma(\pi^0 \pi^0)}{2\gamma(000)} = -0.186(a_2 - a_0)^2 Q,$$  \hspace{1cm} (6.18)$$

where $\gamma$ stands for the rates divided by the phase volume factors, $a_0$ and $a_2$ are the $\pi \pi$ scattering lengths for 1 0 and 2, and $Q$ is the energy release.

This relation was used with a fit to the observed data, including nonanalytic terms in the matrix element, in order to extract $\pi \pi$ scattering lengths. Such a determination of scattering lengths from $\tau$ decay has been presented to the present conference \cite{15}.

From comparison of various analyses it seems, however, that the rescattering effects can be distinguished from the contribution of terms of the next order in energy only by the shape of the spectrum at small relative two-pion energy (see Ref. 13). There is some indication of structure according to the paper of Ref. 2. I was strongly advised, however, by Nauenberg (who presented the paper to the Conference) not to use data for small relative energy because of large experimental uncertainties. It would be very important to clarify the experimental situation.

One may suppress the effect of $\pi \pi$ scattering by considering the part of the Dalitz plot corresponding to large relative energies \cite{16,17}. 

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Current Algebra and Radiative Corrections to the $\Delta I = \frac{1}{2}$ Rule.

In a paper presented to this Conference Sarker\(^\text{18}\) has calculated the contribution of radiative corrections to the $\Delta I = \frac{1}{2}$ part of the Hamiltonian which can induce $K^+ \rightarrow \pi^+ \pi^0$ decay. In a simple model he was able to show that these corrections account for the observed $K^+ \rightarrow \pi^+ \pi^0$ decay rate. Some assumptions are made, however, which are difficult to justify. Moreover, a similar but much more involved study of the same problem by Dolgov\(^\text{19}\) has not shown dominance of electromagnetic corrections over the contribution of the $\Delta I = \frac{3}{2}$ part of the Hamiltonian. According to the calculation of Preparata\(^\text{20}\) the decay is certainly due to the $\Delta I - \frac{3}{2}$ piece of the current $\times$ current Hamiltonian.

It is my feeling that current algebra calculations are not powerful enough to clarify the origin of $K^+ \rightarrow \pi^+ \pi^0$ decay.

Back to Relation (6.1.4).

After this brief discussion of possible uncertainties I would like to return to relation (6.1.4) and say that none of these uncertainties are sure to invalidate its derivation. Things may happen to be simple and we may see a large violation of the $\Delta I - \frac{1}{2}$ rule more or less in accordance with this relation as experimental numbers now indicate.

In any case, a test relation (6.1.4) gives a very rare opportunity to get some insight into the structure of the weak Hamiltonian. Confirmation of this prediction would imply a lot. Unfortunately, the failure of this relation to confront the experimental data could be interpreted in different ways.

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Some Aspects of Chiral Symmetry Breaking for the Strong Interactions (#'s 113, 131, 665, 707)

(Paul R. Auvil, Northwestern University, Evanston, Illinois)

We take the simplest possible point of view, neglecting scale-invariance breaking and possible nonanalytic terms in perturbation expansions, and assuming an SU(3) symmetric vacuum, SU(3) classification of states to lowest order, equality of the pion, kaon, and eta PCAC constants, and octet dominance for the symmetry breaking part of H. From the Hamiltonian, \( H' \), where \( H' \) is the SU(3) \( \times \) SU(3) breaking piece, we consider the prediction of four basic quantities:

- **Meson Masses**
  \[
  \mu_\sigma^2 = -\frac{1}{F^2} <0 \left| \left[ F_5^\sigma, \left[ F_5^\sigma, H' \right] \right] \right| 0 >
  \]

- **Pion-Pion Scattering Lengths**
  \[
  a_0^{(0)} = \frac{1}{9\pi m_\pi} \left( 5A + 16 \right) \frac{m_\pi}{F^2}
  \]
  where \( A = \frac{4}{m_\pi F^2} <0 \left| \left[ F_5^3, \left[ F_5^3, \left[ F_5^3, H' \right] \right] \right] \right| 0 >
  \]

- **Baryon Masses**
  \[
  M_\alpha M_\alpha + B_\alpha \left| H' \right| B_\alpha
  \]

- **Nucleon Sigma Term**
  \[
  \sigma_N = \left< \left| F_5^3, \left[ F_5^3, H' \right] \right| \right>_N
  \]

The former two are in the zero four-momentum limit and involve vacuum reduced matrix elements, whereas the latter depend on Baryon reduced matrix elements.

In discussing models of \( H' \), it is useful to decompose it under the SU(2) \( \times \) SU(2) subgroup because if \( H' \) transforms as \((0, 0) + \) a pure \((1, 1)\) representation, then \( A = \frac{4}{5}\left[ 3(1+1) - 1 \right] \). In the GOR model, \( H' \sim (0, 0) + (4/2, 1/2) \) so that \( A = 4 \left( A - 1 \right) \) is a measure of the \( T = 2 \) component of the sigma commutator \( \left[ F_5^3, H' \right] \) which leads to \( a_0^{(0)} = 0.16/m_\pi, F = 94 \text{ MeV} \). (The Goldberger-Treiman...
The value of $F = 85$ MeV yields $a_0^{(0)} = 0.19/m_\pi$. The next simplest possibility, namely, $H'\sim (0, 0) + (1, 1)$, has $\Lambda = 4$ or $a_0^{(0)} = 0.26/m_\pi$. The experimental evidence from analysis of $K^0_L$ decays finds $a_0^{(0)} = 0.6 \pm 0.25$ and $a_0^{(0)} = 0.4 \pm 0.1$. These large values of $a_0^{(0)}$ may indicate that the $(3, 3^*) + (3^*, 3)$ breaking scheme of GOR needs modification. However, two things should be noted. The next two simplest possibilities are either $(8, 8)$ or $(6, 6^*) + (6^*, 6)$, but each of these used alone leads to $\Lambda < 0$, which is unsatisfactory. Second, the on-shell and unitary corrections to the chiral estimate of $a_0^{(0)}$ may be large. A recent calculation suggests a 30% enhancement.

It may well be that this is sufficient to bring the $\Lambda = 4 \{(3, 3^*) + (3^*, 3)\}$ model into agreement with experiment, and it is clear that this is true for an $\Lambda = 4$ theory. Such a Hamiltonian can be constructed satisfactorily from $(3, 3^*) + (3^*, 3)$ plus $(8, 8)$ or $(6, 6^*) + (6^*, 6)$ or both.

The theoretical understanding of low-energy theorems for pion-nucleon scattering is now quite good, and the question of any possible ambiguities from $t$-channel unitarity have been resolved. However, the actual evaluation of these sum rules, which lead to a value of the nucleon sigma term, has yielded a wide range of values (20-440 MeV). The $(3, 3^*) + (3^*, 3)$ scheme prefers a small sigma term (~20 MeV), but a more general scheme as mentioned above can accommodate any value. It is clear that until more accurate evaluations of the nucleon sigma term and its SU(3) analogues are available, it is impossible to reach any definitive conclusions about the nature of $H'$. Additional information can also be obtained from pion production reactions but more assumptions are involved.

Finally, we should remark that both scale invariance with the possible existence of a scalar dilation and the probable nonanalyticity of perturbations about the chiral limit are important questions to be kept in mind. Each of these can potentially provide significant corrections to the analysis of scattering lengths and the sigma term. It is also possible that the Wemberg-Salam models of the weak and electromagnetic interactions may provide a natural method of introducing chiral symmetry breaking.

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20 Contributed papers Nos. 11, 588, and 744 also discuss chiral symmetry breaking but time did not permit their inclusion in this brief review.

6.3 $K_{e3}$ Form Factors (#’s 745, 779)

(M. K. Gaillard, Orsay/CERN)

Although there has always been a certain amount of disparity among data on $K_{e3}$ decay, recent analyses\(^1\) indicated an apparent trend toward consistency of results. The new data presented here show a reversal of that trend and point in a direction which is closer to theoretical expectations based on chiral symmetry. So before drawing final conclusions we must wait until a number of mutually consistent experiments tell us what the form factors really are.

The form factors have been defined in the experimental report on $K_{e3}$ decay. Here, we simply emphasize that for purposes of theoretical interpretation the relevant form factors are $f_+(t)$ and $f(t)$, which are the amplitudes for spin one and spin zero exchange to the leptons respectively.

As is well known, current algebra together with PCAC implies a relation for form factors at an unphysical value of the momentum transfer:

\[ f(M^2) = \frac{f_0}{f_\pi} + O(\mu^2/M^2), \]  

(6.3.1)
where \( \mu \) and \( M \) are the pion and kaon masses, respectively. Since we know from comparison of various decay rates (assuming the usual Cabibbo theory) that

\[
f(0) \equiv f_+^2(0) = (f_+^2(f_+^1/1.27 \pm 0.03)^{-1}
\]

the soft pion theorem (6.3.1) tells us that the function \( f(t) \) must increase by about 30% between \( t = 0 \) and the soft pion point \( t = M^2 \). A linear extrapolation gives the prediction

\[
\lambda = 0.0 \mu^{-2}
\]

whereas a linear fit to data available prior to this conference favored a negative slope.

However, recent experiments have had sufficient statistics to extract the function \( f(t) \) in an unparametrized form; Fig. 6.3.4 shows the results of three Dalitz plot studies. The older results

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**Fig 6.3.1** Recent determinations of the divergence form factor. full circles - Stanford-Santa Cruz, full squares - JHU-UCLA-SLAC, open squares - Manchester-Daresbury, shaded area - theoretical bound.
clearly favor a form factor which decreases with $t$ in the physical region, agreement with the soft pion prediction

$$f(M^2)/f_p(0) \approx 1.27$$

would then require a dip. However, the new, more precise, results presented at this Conference appear to extrapolate smoothly to approximate agreement with the soft pion prediction, although for low $t$ values the extrapolation to $f(0)/f_p(0)$ is (as required by the analytic properties of $f_p$) does not appear to be smooth

Theoretical procedures for extrapolation to the soft pion point have been reviewed extensively elsewhere \textsuperscript{1} They include the use of Ward identities and dispersion relations, various smoothness assumptions, and assumptions about the nature of chiral symmetry breaking. In most of these treatments $f(t)$ is found to be a smoothly rising function, particularly under the assumption of $(3, ar{3})+(3, 3)$ breaking with the symmetry breaking parameter $c$ near its chiral $SU_2$ value $c = \frac{1}{\sqrt{2}}$. In a recent paper by Ecker, smoothness assumptions were replaced by the assumptions of unsubtracted propagators for currents and their divergences and finite (c number) Schwinger terms. Using analyticity and unitarity constraints he confirms this general prediction, as do Ali et al. \textsuperscript{3} in a closely related work presented to this conference.

Another recent development \textsuperscript{4} is in the derivation of rigorous bounds on the divergence form factor under a few well-defined assumptions. Li and Pagels first pointed out that the positive definite property of the absorptive part $\rho(t)$ of the scalar spectral function

$$\Delta(t) \frac{1}{2} \int d^4x \ e^{ipx} <\Sigma(x, \bar{\Sigma}(0)> \quad (6.3.3)$$

allows one to bound $\rho(t)$ in terms of its lowest-lying contribution which is the $K\pi$ state

$$<\bar{\Sigma}_\mu \Sigma_\mu |K\pi> (M^2 - \mu^2) f(t)$$

$$t \geq (M_\tau \mu)^2 \quad (6.3.4)$$

$$\rho(t) \geq K(t) |f(t)|^2$$

where $K(t)$ is a kinematic factor. Then if $\Delta(t)$ is unsubtracted and if $\Delta(0)$ is known or bounded from above, Eq. (6.3.5) can be used to bound $f(t)$ in the decay region in terms of

$$f(0) \int \rho(t) \ dt/t$$

In the $(3, \bar{3})+(3, 3)$ symmetry breaking model $\Delta(0)$ can be determined as a function of the symmetry breaking parameter $c$, or bounded independently of $c$, if pole dominance is assumed for the pion and kaon propagators. Using techniques developed by Okubo and Shih, Bourrely derived the bounds shown in Fig. 6.3.1, under the following assumptions

a) $\Delta(t)$ is unsubtracted,

b) $\Delta(0)$ is determined as above,

c) $f(t)$ satisfies the soft pion theorem.
If the third assumption is dropped the bounds are of course loosened and \( f(t) \) can have a slightly negative slope. However, if information on \( K\pi \) phase shifts is used, the slope is again constrained to be positive under assumptions a) and b).

If \( \Delta(t) \) requires subtractions bounds may still be obtained but they are generally weaker and \( \Delta(t) \) must be known at more than one point. For example, Shih and Okubo have dropped assumptions a) and b) and assumed only that \( \Delta(t) \) has a Breit-Wigner form in the region of the \( K \) mass. They again find that \( f(t) \) must have a positive slope unless assumption c) is also dropped.

Hopefully, the form of \( f(t) \) will be reasonably well established empirically in the not too distant future. Then this information can be used to bound \( \Delta(t) \), i.e., the bounds may be inverted. This additional information can be used with Ward identities which also relate \( \Delta(t) \) to \( f(t) \) in studying chiral symmetry breaking. That is, in determining the compatibility of a set of assumptions with experiment, not only the empirical form of \( f(t) \) must be reproduced, but \( \Delta(t) \) must satisfy the "empirical bounds".

REFERENCES

5. C. Bourrely, private communication.
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The session was organized into four parts. The first part dealt with the dispersion relation approach to high-energy weak interactions. Valentin Ivanovich Zakharov gave a comprehensive survey of this field, which was pioneered by Pomeranchuk and has been actively pursued by the Moscow school of Okun and collaborators and by Appelquist and Bjorken, among others.

The remaining parts all dealt with one or another aspects of unified gauge theory of weak and electromagnetic interactions. The second part was devoted to the exposition and discussion of the renormalizability, regularization, and anomalies. Bruno Zumino gave the main lecture for this part, followed by a more specialized discussion of William A. Bardeen. Bardeen gave a hitherto unpublished result of his on the dimensional regularization and the Ward-Takahashi identity anomalies. Bardeen also gave a compact algorithm for dimensional regularization of Feynman integrals with any number of loops which is apparently due to Lautrup. Zumino's lecture was extremely comprehensive and up-to-date on this rather complicated subject.

The third part actually consisted of two subjects both of which were presented by J. D. Bjorken. The first subject was the phenomenology of heavy leptons required by some models of gauge theory of weak interactions. He has ably summarized the results of C. H. Llewellyn-Smith and himself. I did not ask him for their manuscript since it will soon be published. This was followed by his detailed analysis of model building along the strategy of spontaneously broken-gauge symmetry and his discussion of very new results of Pati and Salam, Bars et al., and Coleman et al.

Last, Joel R. Primack gave a definitive survey of all existing higher-order calculations in any of these models. He emphasized various physical constraints imposed on model building by consideration of higher-order effects such as the anomalous magnetic moment of the muon, $K^0-L^0$ mass difference and induced strangeness changing neutral current effects.

Each part was followed by comments and questions from the floor. No record was kept of these discussions, but one discussion following Zumino's presentation, made a deep impression on me. (The following is my understanding of what was said.) The discussion had to do with the Adler-Bell-Jackiw anomaly and the view that while it will destroy renormalizability, its physical effects will occur, for instance, in order $e^6$, and models with anomalies should not be rejected just for this account. André Martin registered exception to this view, emphasizing that one need introduce an infinite number of counterterms beyond this order to specify a complete theory. Bjorken countered this argument saying (according to my interpretation) that the occurrence of anomalies may have to do with the breakdown of the asymptotic expansion in coupling constant, so that (1) the first few finite terms may still be an excellent approximation to the real answer and...
(2) it would be unjustified to reject a perfectly sensible theory on the ground that the power series expansion in coupling constant breaks down at some high order. Nothing was resolved by this exchange, but we had an explication of two important opposing views.

Each speaker in this session graciously agreed to provide a written text of his talk, and all have fulfilled their pledges. I am grateful to them for their exemplary acts and I am happy to have their texts reproduced in this tome.
I. Introduction

This talk is devoted to a dispersion approach to weak interactions at high energy. There are two kinds of problems which will be touched upon:

1. Asymptotic bounds on the total cross section.
2. Higher-order weak interactions at finite energy.

The first problem arises from the fact that for weak interactions at high energy massless-particle (neutrino) exchange may be essential while most of the results of the theory of dispersion relations are valid only in the case of finite masses. In particular, the derivation of the Froissart bound depends on this assumption. The proper generalization of these results still represents a challenge to the theorists. Some asymptotic bounds obtained so far will be discussed at the end of the talk.

As for the application of dispersion relations to higher-order weak interactions, the hopes are high now that they will never be needed for this purpose. Indeed, if perturbative-type models of weak interactions are valid, dispersion relations for weak interactions will play a subordinate, if any, role.

Experimentally, however, there is no single piece of information which compels us to accept these models. So far weak processes are well described by the lowest-order interaction. For conventional theories it is difficult to incorporate this simple picture both at low and high energies; if one tries perturbative calculations with cut-off or introduces new particles higher-order corrections are large unless the cut-off or masses of new particles are small.

If, however, the cross section of weak interaction continues to grow with energy and no new particles are produced, we will presumably be forced to say that there is some mystery (symmetry) about coupling constants which makes them small and universal. The unitarity condition and dispersion relations will then suggest themselves as a model-independent approach to weak interactions.

In the applications discussed below dispersion relations with two subtractions are mostly used. The four-fermion coupling constants are considered here to be subtraction terms, any hope of calculating them being abandoned. The emphasis is made instead on terms of next order in energy which are assumed to be dispersive.

II. Pomeranchuk's Relation

Dispersion relations were first applied to weak interactions at high energy by Pomeranchuk. He realized that the dispersion approach provides us with a model-independent connection between the low- and high-energy behavior of the amplitudes of weak interactions.

Indeed, let us imagine that starting from some energy $s_0$ the total cross section of weak interactions becomes large. For simplicity we assume also that it is equal to a constant $\sigma_0$ at higher energies (see Fig. 1). In what way would it affect the amplitude at low energy? To answer this question let us calculate the dispersion contribution $A^{(2)}$ coming from $s' > s_0$. It is easy to find that for $t = 0$

$$\frac{A^{(2)}}{A^{(1)}} \sim \frac{s_0}{s_0(4\sqrt{2G})},$$  

where $A^{(1)}$ stands for amplitude of pointlike four-fermion interaction and is equal to $4\sqrt{2G}$.

Through some positivity condition it can be shown that this correction to the lowest-order amplitude due to the contribution of $s' > s_0$ cannot be cancelled out by other pieces of the dispersion integral so that Eq. (1) gives a lower bound on the correction. Pomeranchuk also found the correction to $dA^{(1)}/dt$ and showed it is more sensitive to the value of $\sigma_0$.

III. At What Energy Can Weak Interactions Become Strong?

Equation (1) was used by Pomeranchuk to answer just this question. Up to now no sign of the presence of the $A^{(2)}$ term has been found. If we turn to consideration of $\nu \nu \rightarrow \nu \nu$ scattering, it is safe to say that for $s \gtrsim 10 \text{ GeV}^2$ the ratio of the amplitudes $A(\nu \nu \rightarrow \nu \nu)/A(\nu n \rightarrow \nu p)$ is less than or equal to unity. From Eq. (1) we learn then that the total cross section of weak interaction can be comparable to the total cross section of strong interaction $\left(\sigma_0 \sim m_N^{-2}\right)$ only at an energy squared

$$s_0 \gtrsim 10^5 \text{ GeV}^2.$$  

In paper it was assumed that $A^{(2)}/A^{(1)} \lesssim 1$ up to $s \sim G^{-4}$ and was concluded that

$$s_0 \gtrsim \left(Gm_N^{-2}\right)^2 \text{ GeV}^2 = 10^{10} \text{ GeV}^2.$$  

Up to now we used dispersion relations for forward scattering to estimate $A^{(2)}$. Let us turn now to the discussion of the $t$-dependence of the amplitude and for simplicity let us consider first the case of lepton-lepton scattering. Then, in the lowest-order weak scattering proceeds via a single partial wave and the $t$-dependence of the amplitude is very smooth. This is not true for the dispersive correction $A^{(2)}$. Indeed, this correction arises from the dispersion contribution of high energies and reflects the structure of the amplitude at these energies. We assumed that the cross section at high energy is large, this implies a sharp $t$-dependence of the amplitude. As far as we assume the cross section to be a constant at $s' > s_0$, it is natural to expect that the $t$-dependence factorizes out and for some region of $t$ can be approximated by an exponential function. In this way we come to the conclusion that

$$A^{(2)}(t) \sim \exp\left(-\frac{-t}{t_{\text{eff}}^\text{eff}}\right), \quad t_{\text{eff}}^\text{eff} \sim \sigma_0^{-1}$$  

Since existing experimental data on lepton-lepton scattering are very poor, Eq. (4) does not help directly to improve the bound (2) on $s_0$ obtained above. However, Eq. (4) shows that at small energies there should exist some sort of halo with radius of order $\sqrt[3]{\sigma_0^{-1}}$. If one considers it to be
unacceptable from a theoretical point of view, then $s_0$ can be pushed to infinity to cancel the contribution of high energies by a factor $s/s_0$ [see Eq. (1)] .

A somewhat more conservative point of view is to allow the long-range forces introduced by $A^{(2)}$ to be comparable in strength with long-range forces arising from hadron exchange in higher orders in weak interactions. It still restricts possible value of $s_0$ severely. We would like, however, not to use additional theoretical assumptions and stick to bound (2).

For lepton-hadron scattering the situation is even more complicated since it is not clear how one can distinguish between the damping factor (4) and the usual form factor.

IV. Model of Strongly Interacting W-Bosons

Appelquist and Goldman have observed that the amplitude of elastic scattering will become rather large at NAL energies if W-bosons have strong pairwise interaction with hadrons (see also Bjorken's Lecture in Erevan).

Indeed, in this case the cross section increases promptly once production of real W-bosons is possible (see Fig. 2). Roughly speaking, we have

$$s_0 \sim \frac{m_{W}^2}{\sigma} g^2 \sigma_{\text{strong}} \equiv G m_W^2 \sigma_{\text{strong}},$$

where $g$ is the semiweak coupling constant. Then Eq. (1) gives

$$\frac{A^{(2)}}{A^{(1)}} \sim \frac{g \sigma_{\text{strong}}}{4\sqrt{2}}$$

which is independent of the mass of W-boson (as far as $G m_W^2 \leq 1$) and is rather large. A more accurate calculation of the graph (Fig. 2) reduces the estimate (6) somewhat but still for $E_{\nu} \approx 50$ GeV $\sigma(vp \rightarrow vp)$ and $\sigma(vn \rightarrow np)$ are comparable if $\sigma_{\text{strong}} \sim 1$ mb.

Dispersion relations were also used to pose the problem of damping higher-order effects in this model (see Ref. 3).

V. General Form of Amplitude of Weak Interactions at Low Energy

For a more detailed discussion we need now a better understanding of the structure of $A^{(2)}$.

The problem is to describe the corrections in a model-independent way without referring to any dynamical calculations. An example of such a description is provided by an amplitude of non-relativistic scattering at low energy

$$A \sim \frac{1}{\frac{1}{a} + i\sqrt{E}},$$

where $E$ is the energy and $a$ is the scattering length.

Somewhat similar formulas can be obtained for weak interactions at low energy. The difference is that several GeV (perhaps even 100 GeV) is still a "low" energy for weak interactions.

It is reasonable therefore to consider the interaction of massless particles to simplify the formulae. In the case of elastic $e^- e^+$ scattering we have in the second order $^4, ^5$
\[ A(ee \rightarrow ee) = \frac{2G^2}{3\pi^2} \left( t \ln \frac{-t}{\Lambda} + u \ln \frac{-u}{\Lambda} \right) + \text{possible contact term}, \quad (8) \]

where \( \ln \Lambda \) is some parameter, \( t \) and \( u \) are the energy squared in annihilation channels.

Equation (8) is a general one in the same sense as Eq. (7), it keeps all the terms of the second order in \( s, t \) and satisfies the unitarity condition in this approximation. For sufficiently low \( s \) it is surely valid provided that there are no neutral weak currents which give a contribution to the unitarity sum comparable with that of the \( \nu \bar{\nu} \) intermediate state (see Fig. 3).

Terms of the third order in \( s, t \) can be described in the same way \(^4,5\) with more parameters entering the game. For the terms of higher order the calculation has not been tried yet.

**VI. Dispersion Sum Rule for \( \ln \Lambda \)**

Expanding a dispersion relation with two subtractions in powers of \( s \) and comparing the result with Eq. (8) one readily obtains

\[ \ln \frac{\Lambda}{\tilde{s}} = 6\pi \int_{\tilde{s}}^{\infty} \frac{\sigma^+(s)}{(Gs)^2} \frac{ds}{s} + 6\pi \int_{0}^{\tilde{s}} \frac{ds}{(Gs)^2} \left[ \sigma^+(s) - \sigma^+_\text{theor}(s) \right], \quad (9) \]

where

\[ \sigma^+(s) = \left[ \sigma^\text{weak}_{\text{tot}}(e^+e^-) + \sigma^\text{weak}_{\text{tot}}(e^-e^+) \right], \quad \sigma^+_\text{theor} = \frac{G^2}{6\pi} \quad \tilde{s} \text{ is arbitrary} \]

and the dependence of \( \ln \Lambda \) on \( \tilde{s} \) is just superficial.

This sum rule gives the parameter \( \ln \Lambda \) which represents the cut-off in perturbative calculations in terms of integral over the total cross section. It is a generalization of Eq (4) which keeps the contribution of intermediate energies.

Any violation of Eq. (9) would imply the violation of dispersion relations with two subtractions. Because the integrand of the first term in rhs of Eq (9)—the only one where integration extends to infinity—is positive such a violation, if it exists, has a chance to be established at finite energies.

Terms of higher order in the expansion of the amplitude at low energies can also be expressed in terms of some dispersive integrals. In particular, the coefficients of an expansion in powers of \( t \) are given by

\[ \int \frac{d^n A(s,t)}{dt^n} \bigg|_{t=0} \frac{ds}{s^2} \quad (10) \]

which by virtue of the inequality \(^6\)

\[ \frac{d^n A(s,t)}{dt^n} \geq \text{Const} \quad \sigma^n_{\text{tot}} \quad (11) \]

depends most crucially on the total cross section at high energy. In an explicit form such representations were obtained for terms of the third order. In higher orders the problem is to isolate the singularity of the amplitude at \( t = 0 \) connected with massless particle exchange so as to make the derivative in \( t \) meaningful.
VII. Dispersion Sum Rule for Fermi Constant $G$

Up to now the constant $G$ was treated as a subtraction constant. If there exist dispersion relations with one subtraction then the constant $G$ can be represented as an integral of the difference of total cross sections of particle and antiparticle interactions 

$$G = \frac{1}{\pi \sqrt{s}} \int_0^\infty \frac{ds}{s} \left[ \sigma_{\text{tot}}^{\text{ve}}(s) - \sigma_{\text{tot}}^{\text{ve}}(s) \right]$$  \hspace{1cm} (12)

It is worth noting that even if this dispersion integral is convergent it is not excluded that some constant should be added to the rhs of Eq. (12). The absence of this constant is an additional assumption needed to derive representation (12).

According to paper, a dispersion sum rule for the constant $G$ can be obtained even in the case of two subtractions. To this end one should consider the dispersion relation for the modulus of amplitude and its phase. According to paper there are no zeros in the upper half plane if dispersion relations with two subtractions are valid and masses of particles are kept zero. Then there are no arbitrary constants in the dispersion representations for modulus and phase. As a result the following sum rule arises

$$\int_0^\infty \frac{ds}{s} \ln \left| \frac{A_{\text{ve}}}{A_{\text{ve}}} \right| = -\frac{\sqrt{2}}{12} G$$  \hspace{1cm} (13)

which becomes an inequality if the number of subtractions is larger than two.

VIII. Weak Interaction of Colliding Beams with Energy $10^{-2}-10^{-3}$ GeV

If the Fermi coupling constant $G$ provides the only energy scale inherent to weak interactions, experimental investigation of weak processes at energies $s \approx G^{-1}$ will become imperative. By virtue of the unitarity condition these are the energies where higher-order corrections should be noticeable.

There exist some plans for constructing colliding beams with energy $\sim 100$ GeV to probe weak interactions. These are for lepton-lepton beams in Erevan and Novosibirsk and for proton-proton beams at Brookhaven.

The formulae obtained above may be useful to expose general features of such experiments and, later, to provide a framework for analyzing the results.

Let us consider the simplest case of $e^+ e^-$ elastic scattering. It is easy to guess that the second-order weak and electromagnetic amplitudes become comparable to each other at large momentum transfer if

$$f_4^2(s) \approx \alpha \text{ where } f_4(s) = \frac{G s}{6\sqrt{2}},$$  \hspace{1cm} (14)

$f_4$ being the partial-wave amplitude of $e^+ e^- \rightarrow \nu \bar{\nu}$ annihilation (in the lowest order only the partial wave with $j = 1$ is different from zero).

This guess can be checked by calculating the imaginary part (see Fig. 3) which is uniquely determined in terms of constant $G$. By retaining the imaginary part only one obtains a lower bound on the weak cross section. It turns out that this lower bound equals the electromagnetic
differential cross section for θ = 90° at an energy of 225 GeV, the corresponding cross section being \(10^{-37}\) cm\(^2\)/sr.

The measurement of the real part which is not predicted would provide us with the knowledge of some integral of the total cross section [see Eq. (9)]. Higher-order corrections are presumably smaller by the factor \(f_1 \sim \sqrt{\sigma}\). Thus, for such energies some kind of perturbation treatment could be applicable and for the whole range of energies and angles the scattering amplitude is expected to be described by a single parameter \(ln \Lambda\).

A more detailed presentation of the same problems can be found in Ref. 4. In particular, the last paper listed in this reference deals with electromagnetic corrections of higher order.

**IX. Long-Range Forces and Weak Interactions at High Energies**

Up to now dispersion relations with two subtractions at \(t \leq 0\) were used. We are going to discuss now the validity of this assumption. The problem, as was already mentioned in the introduction, is that long-range forces arising from massless-particle exchange may result in rapidly growing total cross sections and invalidate dispersion relations.

It is quite clear that in general when massless-particle exchange is taken into account it is impossible to obtain any bound on the cross section. It is sufficient to say that photon exchange results in an infinite cross section. However, in the case of weak interactions arguments can be presented in favor of dispersion relations with two subtractions. The idea is that in the case of weak interactions the long-range forces are not so important because they arise from exchange of two spin 1/2 particles (neutrinos).

Just to show how this idea can work let us start with a very crude consideration. The amplitude corresponding to the simplest graph with exchange of massless particles (see Fig. 4) is proportional to

\[ A \sim G^2 \ln t \quad (15) \]

The partial-wave amplitudes corresponding to this expression for large \(j\) are given by

\[ f_j \sim G^2 j^{-4} \quad (16) \]

For large enough \(j\) one could hope that this calculation is sensible. For smaller \(j\), \(f_j\) is larger than unity according to Eq. (16) and the calculation is senseless. For such \(j\) we use only the unitarity condition \(f_j \leq 1\). As a result the partial-wave amplitudes are given by the curve on Fig. 5. It is clear that the cross section is of the order

\[ \sigma \sim R^2 \sim j_0^2 / s, \quad (17) \]

where \(j_0\) stands for such \(j\) that \(f_j \sim 1\) according to Eq. (16). Finally, we obtain

\[ \sigma \sim G, \quad (18) \]

and, thus, the cross section is rather small, despite the long-range effects.
X. Asymptotic Bound on the Total Cross Section $\sigma(s) < s^{1/3}$

By formalizing the consideration of the preceding section it is possible to obtain the bound quoted in the title of the section. To give an idea of the derivation let us briefly outline one of the proofs of the Froissart bound in the case of strong interactions.

We assume that there exist dispersion relations with a finite number of subtractions for $t \leq 0$ (in the case of massless-particle exchange this assumption is still awaiting for approval or disapproval from axiomatic field theory).

For the sake of definiteness we start from dispersion relations with two subtractions, which, rather symbolically, can be written as

$$\text{Re}A(s,t) - \text{Re}A(0,t) + s \text{Re}A'(0,t) + \frac{2}{\pi} \int \frac{ds' \text{Im}A(s',t)}{(s' - s) s'^2} + \text{left-cut term} \quad (19)$$

Let us now differentiate this relation with respect to $t$ at $t = 0$. It can be shown that the order of integrating over $s'$ and differentiating in $t$ may be interchanged and we come to an integral of $d^nA(s,t)/dt^n |_{t=0}$. By virtue of relation (11) this derivative is bounded from below by $s_0^{n+1}(s)$.

Thus, one comes to the conclusion that the integral of any power of the total cross section is convergent. This rules out a cross section growing as any power of $s$. To establish the $\ln s$ factor in the Froissart bound a more refined consideration is required, but hereafter we omit the $\ln s$ factors.

So far strong interactions were considered. Where does this proof fail in the case of weak interactions? The answer to this question is that for weak interactions the amplitude is nonanalytic at $t \to 0$ because of massless-particle exchange and some derivatives just do not exist.

The singularity at $t = 0$ is rather mild, however. The simplest graph discussed in the preceding section contains $t$ Int but it depends on $s$ linearly and is absorbed into the subtraction term in the dispersion relations in $s$. For the graphs depending non trivially on $s$ it can be shown that the singular part of the amplitude is proportional to $t^2$, so that the second derivative exists (in neglect of Int terms). Assuming that the same is true for the total amplitude we come to the asymptotic bound $\sigma(s) < s^{1/3}$.

To summarize, the bound

$$\sigma(s) < s^{1/3} \quad s \to \infty$$

follows from two assumptions.

a) there exist dispersion relations with finite number of subtractions for $t \leq 0$.

b) the singularity of the total amplitude at $t = 0$ is given by singularities of separate Feynman graphs.

XI. Two-Particle Exchange

If one believes that long-range forces arising from two massless-particle exchange in $t$-channel are most important some further progress can be reached. The point is that in this case the singularity of the amplitude can be studied in more detail by means of the Mandelstam representation for the double spectral function.
\[
\rho(s, t) = \int \frac{dz_1 dz_2}{\sqrt{z_1^2 + z_2^2 + z_1^2 z_2^2 - 2z_1 z_2 z - 1}} \left[ A_s A_u \left( \frac{A_s A_u}{s_1^2} + \frac{A_s}{s_2} u_1 u_2 \right) \right] \quad (20)
\]

where \(A_s\) and \(A_u\) are imaginary parts of the amplitude in the \(s-\) and \(u-\) channel, \(s_1\) and \(s_2\) are the energies of the upper and lower blocks of the graph of Fig. 6.

Possible values of \(s_1, s_2\) are constrained by the condition \(s_1, s_2 \leq s, 4s_1 s_2 \leq s t\). If \(s t\) is very large then \(s_1\) and \(s_2\) are large. If \(s t\) is small then at least \(s_1\) or \(s_2\) is also small. In the former case \(A_s\) can be replaced by its asymptotic value, while in the latter case at least \(A_s\) or \(A_u\) is described by low-energy representation (8).

As the imaginary part is proportional to \(s^2\) at small energies \(\rho(s, t)\) is proportional to \(t^2\) for small \(s, t\), in agreement with general remarks made in the preceding section about the character of the singularity at small \(t\).

For large values of \(s, t\) the answer may be represented in the form

\[
\rho(s, t) \sim (st)^{\alpha+1} \int_0^{\sqrt{s}/t} \int_0^{4/y} \frac{dx dy}{\sqrt{1 - 4xy}}, \quad (21)
\]

where it was assumed that asymptotically \(A_s \sim s^\alpha\). Equation (21) was first obtained by Rajaraman (let us notice, however, that the upper limits of integration over \(x, y\) in Ref. 9 were erroneously put to be equal to infinity). The integral over \(x, y\) in Eq. (21) contains a logarithmic factor but it is not essential for future analysis. What is essential is that the ratio of \(\rho(s, t)\) and \(A_s\) contains a factor \(t^\alpha\). If \(A_s\) is asymptotically proportional to \(s^\alpha\),

XII. Asymptotic Bounds on the Total Cross Section \(s^{-1} \leq \sigma(s) \leq s^0\)

If one assumes that the two-particle intermediate state dominates the unitarity condition at small \(t\) and that there exist dispersion relations in \(s\) for positive \(t\), some arguments in favor of the bounds quoted above can be given. The lower bound, as noticed by Anselm and Gribov, is virtually contained in a paper of Gribov and Pomeranchuk (1962). The upper bound was obtained first by Rajaraman and discussed in Ref. 12.

In both cases Eq. (21) is used and at small \(t\) the imaginary part \(A_s(t, s)\) is replaced by its optical value.

Then, if the cross section is falling faster than \(s^{-1}\), the imaginary part in \(t\) of \(A_s\) contains according to (21) factor \(t^{-\epsilon}\) \((\epsilon > 0)\) as compared with \(A_s\) itself and this is inconsistent for \(t \to 0\). In this way the lower bound arises.

To present the argument against growing a cross section we should notice first that if the cross section is growing as some power of \(s\), then the effective value of \(t\) should fall at least as the same power of \(s\). Otherwise, the elastic cross section is larger than the total cross section. Indeed,
\[ a_{ee} \sim \frac{\text{Im} A(s, t-0)}{s^2} t_{\text{eff}}, \quad a_{ee} \ll a_{\text{tot}} \sim s^\alpha \]  

(22)

and

\[ t_{\text{eff}} \leq s^{-\alpha} \]

With this information in hand we see that \( \rho(s, t) \) is small as compared with \( A_s \) for \( t - t_{\text{eff}} \) and it is plausible that it cannot feed the growing cross section.

To realize this idea Rajaraman calculated first the potential as a function of \( A_s \) and then, in the eikonal representation, \( A_s \) as a function of the potential. The result of the calculation is a selfconsistency condition. This condition cannot be satisfied unless \( \sigma(s) \) is not growing asymptotically. The weak point of this derivation is that the potential is determined from \( \rho(s, t) \) through a dispersion integral in \( s \) at fixed \( t \) However, for \( s \) tending to infinity any finite \( t \) becomes much larger than \( t_{\text{eff}} \) which falls as some power of \( s \). For such \( s \) replacement of \( A_s \) by its optical value to calculate \( \rho(s, t) \) is not justified and, strictly speaking, there is no selfconsistency condition.

This problem was studied in detail in paper and it was found that this difficulty can be overcome and shown that up to possible logarithmic factors the cross section is bounded by a constant. The most essential assumptions are the use of dispersion relations in \( s \) for positive \( t \) and dominance of two-particle intermediate states in the \( t \)-channel unitarity sum up to \( t(s) - s^\epsilon [1/\sigma(s)] \), \( \sigma(s) \sim s^\alpha \), \( \epsilon \) and \( \alpha \) being positive numbers. It is worth emphasizing once more that to obtain this bound much stronger assumptions are needed than those used to derive the bound \( \sigma(s) < s^{1/3} \)

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V. N. Gribov, A. D. Dolgov, L. B. Okun, and V. I. Zakharov, Nucl. Phys., to be published.
I. Introduction

Quantum electrodynamics is accurate to a few parts in a million. Its remarkable success is due to the smallness of the coupling constant and to the possibility of renormalizing the perturbation-theory divergences. The usual theory of weak interactions, on the other hand, is not renormalizable. Therefore, in spite of the smallness of the coupling constant \((Gm^2 = 10^{-5})\) higher-order calculations are meaningless. The quadratic divergences present a problem not only because the theory is not finite, but because it is not clear how the higher-order corrections can be smaller than the lowest order. For instance, if a process goes like \(G\) in lowest order, the next order will go like \(G^2\Lambda^2\), where \(\Lambda\) is a cutoff, and it will not be smaller unless \(G\Lambda^2\) is appreciably smaller than unity. For some processes, this kind of interpretation requires rather small values of \(\Lambda\), an admission that the theory fails at uncomfortably small energies.

One may hope that a unified theory of weak and electromagnetic interactions based on a gauge group will be renormalizable because of the cancellations due to the Yang-Mills relations among the couplings. Furthermore, because of the properties of the nonabelian gauge group, it could explain the universality of both interactions. Since the intermediate boson must be taken to be massive and the theory of Yang-Mills fields with mass is not renormalizable, a special procedure had to be found—the Higgs mechanism; as described below, a renormalizable theory seems possible. Universality, on the other hand, has been only partially achieved. In the presently known models either the electromagnetic or the weak universality has to be put in by hand.  

II. Spontaneously Broken Gauge Groups

The Higgs Lagrangian

\[ L = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} (\partial_{\mu} A_{\nu} - i g A_{\nu})^2 - \mu^2 |\phi|^2 - h |\phi|^4 \]

is invariant under the gauge transformation

\[ A_{\mu} \to A_{\mu} + \partial_{\mu} A, \quad \phi \to e^{igA} \phi \]

Treating it at first classically (tree diagrams) we distinguish the two situations \(\mu^2 > 0\) and \(\mu^2 < 0\) (in both cases \(h > 0\)). In the second case the potential

\[ V(\phi) = \mu^2 |\phi|^2 + h |\phi|^4 \]

has a minimum for \(|\phi| = \lambda = (-\mu^2/h)^{1/2}\) and the complex field \(\phi\) has a nonvanishing vacuum expectation value. Choosing it to be real, \(\langle \phi \rangle = \lambda\), we see that the solution no longer has the symmetry
of the equations. Rather, the group transforms the particular solution chosen into an infinity of
equivalent solutions, none of which exhibits the gauge symmetry. The symmetry is spontaneously
broken. In the absence of gauge fields the spontaneous breaking of a symmetry implies the exist-
ence of massless bosons (Goldstone bosons), as many as the number of generators of the original
group which are no longer conserved. In the presence of gauge fields some or all of these mass-
less bosons disappear from the physical spectrum and provide instead the additional degrees of
freedom necessary to give a mass to a corresponding number of gauge fields. In the Higgs model
this can be seen by introducing the fields $\chi$ and $\Theta$ through

$$\phi = \frac{1}{\sqrt{2}} (\lambda + \chi) e^{i\Theta/\lambda}.$$ 

Under the gauge transformation, the field $\Theta$ transforms as

$$\Theta \rightarrow \Theta + g\alpha \Lambda,$$

while the field $\chi$ and the vector field

$$B_\mu = \Lambda_\mu - \frac{1}{g\alpha} \partial_\mu \Theta$$

are invariant. The Lagrangian can be expressed completely in terms of these gauge-invariant
fields

$$L = -\frac{1}{4} B_\mu B^\mu - \frac{1}{2} (g\alpha)^2 B_\mu B^\mu - \frac{1}{2} (\Theta \chi)^2 - \frac{1}{4} (3h\lambda^2 + \mu^2) \chi^2$$

$$- \frac{1}{4} \lambda^4 - \frac{2}{g^2} B_\mu B^\mu (2\lambda \chi + \chi^2).$$

We see that the phase $\Theta$ has disappeared and at the same time the vector field has acquired a mass
$m = g\lambda$. The gauge group is no longer visible, although the particular relations among the coupling
constants are a reminder of the original gauge invariance.

The pervading idea of the work described here is that if a theory is renormalizable in the
symmetric case ($\mu^2 < 0$), it will also be renormalizable in the case of spontaneous symmetry
breaking. Since the original Higgs Lagrangian is renormalizable in the symmetric case, we
expect the new form to correspond to a renormalizable theory. In the new form the vector meson
propagator

$$\mathcal{B}_\mu \mathcal{B}_\nu \sim \left( \delta_{\mu\nu} + \frac{\mu \cdot \nu}{m^2} \right) \frac{1}{k^2 + m^2}$$

will give rise to highly divergent Feynman diagrams. Cancellations will hopefully take place which
will render the theory renormalizable. This kind of quantization which uses the conventional
vector meson propagator and no unphysical fields gives rise to a manifestly unitary $S$-matrix and
is therefore called the U-formalism. In this formulation renormalizability is the hard thing to
show.

An alternative quantization scheme leaves the unphysical degrees of freedom and makes
explicit use of the gauge group. Here renormalizability is obvious by simple power counting and
therefore one speaks about the R-formalism. Unitarity, however has to be proved. This can be
done either by showing that the unphysical singularities cancel as a consequence of the Ward identities of the group, or by showing that the R-formalism is equivalent to the unitary U-formalism. Let us write

$$\phi = \frac{1}{\sqrt{2}} (\lambda + \phi_1^* + i\phi_2^*)$$

The real fields $\phi_1$ and $\phi_2$ have vanishing vacuum expectation value and transform into each other under the gauge group. One can fix the gauge by taking as Lagrangian

$$L = \frac{1}{2} \xi (\partial_\mu \phi_1 + \frac{m}{\xi} \phi_2)^2,$$

where $L$ is the Higgs Lagrangian expressed in terms of $\phi_1$ and $\phi_2$. Other gauge conditions are possible but the above is especially convenient because it cancels the bilinear $A_\mu \partial^\mu \phi$ term in $L$ generated by the shift in the scalar field. Now the vector meson has a propagator with good high-momentum behavior

$$A^\mu A_\mu \sim \left[ \delta_{\mu \nu} + \frac{k_\mu k_\nu (1 - \xi)}{\xi k^2 + m^2} \right] \frac{1}{k^2 + m^2},$$

while the would-be Goldstone boson $\phi_2$ has the propagator

$$\phi_2 \phi_2 \sim \frac{1}{k^2 + m^2 / \xi}.$$

Observe that the vector meson propagator differs from its unitary counterpart given earlier by

$$\frac{k_\mu k_\nu}{m^2 (k^2 + m^2 / \xi)}.$$

This scalar ghost must cancel in all amplitudes against the Goldstone ghost to give a unitary $S$-matrix. Notice that for $\xi \to 0$, the vector propagator tends to the unitary form while the Goldstone ghost acquires infinite mass and drops out of the theory. Therefore, there exists a one-parameter set of renormalizable gauges, starting from $\xi = \infty$ (Landau gauge), which tends in the limit of $\xi \to 0$ to the unitary gauge. The on-mass-shell $S$-matrix elements must be independent of the particular gauge chosen. These considerations provide the basis for a proof of equivalence between the R- and the U-formalism.

Let us already mention here that in the nonabelian case the proof is complicated by the occurrence, in the correct Feynman rules of the Feynman-Faddeyev-Popov ghosts. It turns out that the interaction between these ghosts and the residual physical scalar (the analogue of $\phi_1$ above) is proportional to $1/\xi$. Therefore, the ghost loops with external physical scalar lines give a contribution which diverges in the limit as $\xi \to 0$. In this case one must combine the ghost loops with all other diagrams contributing to the same amplitude. The $S$-matrix element itself is well behaved as $\xi \to 0$. Finally, let us remark that for a satisfactory proof one must first regularize and subtract the divergences. These points will be discussed later.

The Higgs model has been shown by B. W. Lee to be renormalizable to all orders. The generalization of the Higgs mechanism to the nonabelian case is due to Kibble.
III. The Weinberg-Salam Model

If one tries to construct a minimal gauge theory containing the known currents and only the presently known leptons, one is led almost unavoidably to the group $SU(2) \times U(1)$. Consider, for instance, the electromagnetic current $\overline{e} \gamma_\mu e$ of the electron and the charged weak current $\overline{e} \gamma_\mu (1+\gamma_3)/2 \nu e$ with its hermitian conjugate. In terms of the left-handed doublet

$$L = \left( \begin{array}{c} v_e \\ e \end{array} \right)$$

the charged current is

$$\overline{e} \gamma_\mu \frac{1+\gamma_5}{2} \nu e = \overline{L} \gamma_\mu \tau^L e.$$ 

Together with its hermitian conjugate, it closes to an $SU(3)$ structure with

$$\overline{L} \gamma_\mu \tau^L e = \overline{e} \gamma_\mu \frac{1+\gamma_5}{2} \nu e - \overline{e} \gamma_\mu \frac{1+\gamma_5}{2} e.$$

Since this neutral current differs from the electromagnetic current, one must introduce at least one more neutral current. Using the right-handed singlet

$$R = \frac{1-\gamma_5}{2} e,$$

one notices that

$$-\overline{e} \gamma_\mu e = \frac{1}{2} \overline{L} \gamma_\mu \tau^L L - \frac{1}{2} \overline{L} \gamma_\mu L - \overline{R} \gamma_\mu R$$

and that the current $\frac{1}{2} \overline{L} \gamma_\mu L + \overline{R} \gamma_\mu R$ commutes with the above $SU(2)$ group. The Yang-Mills interaction is written in the form

$$\frac{g_5}{2} \overline{L} \gamma_\mu \tau^L L - g'(\frac{1}{2} \overline{L} \gamma_\mu L + \overline{R} \gamma_\mu R) B_\mu ,$$

where $B_\mu$ is the vector field of the $U(1)$ group. The charged intermediate boson field is identified as

$$W_\mu = \frac{4}{\sqrt{2}} (A_{\mu 1} + i A_{\mu 2}),$$

while the photon field is fixed by the fact that it couples to the electromagnetic current. It is given by

$$A_\mu = (g'A_{\mu 3} + gB_\mu)/(g^2 + g'^2)^{\frac{1}{2}},$$

while the orthogonal combination

$$Z_\mu = (g'A_{\mu 3} - g'B_\mu)/(g^2 + g'^2)^{\frac{1}{2}}.$$
describes a neutral intermediate boson. One finds easily that the electron charge satisfies

\[ \frac{1}{\alpha^2} + \frac{1}{\alpha'}^2 + \frac{1}{g^2} \]

from which it follows that \( g > e > g' > e \). The muon is treated in a perfectly analogous way.

In this model neutral currents are very important. In particular, \( v_e \) scattering and \( v_\mu \) scattering are consequences of the theory. To make the model realistic one must find a way to generalize it to hadrons so as to explain why \( \Delta S \neq 2 \) neutral currents are suppressed experimentally by a factor of \( 10^{-4} \sim 10^{-5} \) in the amplitude with respect to charged-current processes. The same applies to \( \Delta S = 2 \) transitions. This can be done by using an idea due to Glashow, Iliopoulos, and Maiani, who make use of cancellations between intermediate states with ordinary hadrons and states with new charmed hadrons (or quarks).

The vector mesons \( W^\pm \) and \( Z^0 \) must be given masses by means of the Higgs mechanism. One takes a scalar doublet

\[ \begin{pmatrix} \phi^+ \\ \phi \\ 0 \end{pmatrix} \]

and writes its Lagrangian

\[ -\left( \partial^\mu \phi^+ \right) \left( \frac{g}{2} \gamma^\mu \vec{A} \right) \frac{\lambda}{\sqrt{2}} \left( \frac{1}{\lambda} \phi \right) \]

When \( \phi \) due to its self-interaction acquires a vacuum expectation value

\[ \langle \phi \rangle = \left( \begin{pmatrix} 0 \\ \lambda/\sqrt{2} \end{pmatrix} \right) \]

the vectors acquire masses \( m_W = \frac{1}{2} \lambda g \), \( m_Z = \frac{1}{2} \lambda (g^2 + g'^2) \). Identifying the Fermi constant from

\[ \frac{G}{\sqrt{2}} = \frac{\lambda}{8 \sqrt{2} \lambda} = \frac{g^2}{4 \lambda} \]

one finds \( m_Z^2 > m_W > 372 \) GeV. The photon is of course massless. Observe that

\[ \frac{m_W^2}{m_Z^2} = \frac{g^2}{g'^2} \]

This is an example of a zeroth order relation among observable parameters which may be corrected but should remain finite when loops are included (see later).

IV Anomalies

We have mentioned that the proof of unitarity in the R formalism makes use of the Ward identities to prove that the unphysical ghost singularities do not appear in the S-matrix. In theories like the Weinberg-Salam model in which the gauge group involves chiral transformations, the Ward identities may develop anomalies and it is easy to see that the presence of anomalies...
spoils the proof of unitarity Fortunately it is possible to arrange things so that the anomalies due
to the various fundamental spinor fields cancel. This has been shown for one-loop anomalies by
Bouchiat, Floiopoulos and Meyer, by Gross and Jackiw, and by Wess and the author The
more difficult question of cancellation in higher orders has been considered by Bardeen, who
reached the conclusion that the anomalies will not cause difficulties in higher order if they cancel
in the one-loop case

The one-loop anomaly is relatively easy to treat. Its general form in the nonabelian case
has been given by Bardeen The relevant property here is that it is proportional to the symbol
\[ d_{abc} = \text{Tr} \left( \lambda^a \lambda^b \lambda^c \right) \]
of the gauge group. In the case of the Weinberg-Salam model the interaction can be written in
general (for leptons as for quarks) in the form
\[ \mathcal{L} = \frac{e}{2} C^a \cdot \bar{\chi} \chi + \frac{e^f}{2} \left( C_0 B \right)_\mu \chi_L - \bar{R} \gamma_\mu g^f QB \chi_R, \]
where \( Q \) is the (diagonal) charge matrix \( C_0 = C_3 - 2Q \) and \( L \) and \( R \) are the left-handed and right-handed parts of the spinor fields which we imagine arranged in a single column \( \psi \) containing all
leptons and quarks. The matrices \( C_4, C_2, \) and \( C_3 \) of the SU(2) algebra will therefore have, in
general, a highly reduced form. Separating the vector and the axial vector part in the above interac­
tion, one can write it as
\[ \mathcal{L} = \gamma_\mu V_{\mu} \psi + \bar{\psi} \gamma_5 A_{\mu} \psi, \]
where
\[ V_{\mu} = \frac{e}{2} C^a \cdot \bar{\chi} \chi + \frac{e^f}{2} C_3 B^\mu + g^f \left( -Q + i C^f B^\mu \right) \]
\[ A_{\mu} = \frac{e}{4} C^a \cdot \bar{\chi} \chi + \frac{e^f}{4} C_2 B^\mu \]
Now we use the fact that the anomaly can always be eliminated from the vector Ward identities by
a suitable definition of the one-loop amplitudes (which always allow redefinition by contact terms)
The axial vector anomaly will cancel if the corresponding \( d_{abc} \) symbol vanishes. This gives rise here to the condition \( \text{Tr} C_3^2 Q = 0 \). In all models based on SU(2) \times U(1) which have been proposed, the eigenvalues of the matrix \( C_3 \) are such that this equation simplifies to \( \text{Tr} Q = 0 \). So, if the sum
of the charges of all elementary spinor fields vanishes, the one-loop anomaly cancels. This con­
dition can be satisfied by arranging the quark charges in a suitable way. However it is not
possible, e.g., to cancel the electron against the muon since they have the same charge.

A further restriction to be considered in model building is that the purely hadronic part of
the anomaly should give the right \( \pi^0 \rightarrow 2\gamma \) decay amplitude in both magnitude and sign. This re­
quires that \( 2 \text{Tr} T_3 Q^2 = 1 \), where \( T_3 \) is the hadronic isospin matrix and only the hadronic part of
the charge matrix is taken in the trace.

Georgi and Glashow have considered the general question of anomaly-free gauge group.
The anomaly of a gauge group is proportional to the \( d_{abc} \) symbol of the representation to which
one assigns the fundamental spinor fields. Now, pseudoreal representations (equivalent to their
conjugates) have vanishing $d_{abc}$. Groups which have only pseudoreal representations cannot have anomalies. This leads to a classification of gauge groups which are safe for model building. On the other hand, "vector-like" models in which the interaction can be transformed into a purely vector interaction by redefining the spinor fields have no anomaly since the one-loop amplitudes can always be defined so as to satisfy anomaly-free vector Ward identities.

Absence or cancellation of the anomalies is necessary to obtain a finite unitary $S$-matrix. On the other hand, the anomalies would begin to cause problems only in a relatively high order (sixth order), and one may ask oneself if the constraint on model building is not being taken too seriously since the essential physical requirement is perhaps not renormalizability but rather the smallness of the second order with respect to the first. The constraint given by the $\pi^0 \rightarrow 2\gamma$ amplitude may also possibly be relaxed if different explanations of the process turn out to be correct.

V. Regularization

An important step in the proof of finiteness and unitarity is that of regularizing the theory in a gauge-invariant way. Most known regularization methods violate the Yang-Mills gauge invariance. Two regularization methods which preserve gauge invariance will be described here.

The first is the use of higher-order covariant derivatives due to Slavnov and Lee and Zinn-Justin. As an example, in the Yang-Mills theory one can take the Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 - \frac{\alpha}{4\Lambda^2} \left( D_\sigma F_{\mu\nu}^{\sigma} \cdot D_\rho F_{\mu\nu}^{\rho} - \frac{\beta}{4\Lambda^4} \left( D_\sigma^2 F_{\mu\nu}^{\sigma} \cdot D_{\rho\lambda} F_{\mu\nu}^{\rho\lambda} \right) \right)$$

The first term is the usual Yang-Mills Lagrangian. The second and third term contain derivatives up to sixth order and give rise to a regularized propagator

$$A_{\mu\nu} \sim \left( \frac{\Lambda}{k^2} \right)^2 \frac{1}{k^2} \left[ 1 + \frac{k^2}{\Lambda^2} + \beta \left( \frac{k^2}{\Lambda^2} \right)^2 \right] + \text{gauge-dependent terms}$$

At the same time, the need to use covariant derivatives in the second and third term of the Lagrangian introduces additional interactions of maximum dimension eight. The occurrence of additional interactions limits the order of derivatives since interactions of too high dimension offset the advantage due to the higher derivatives. With the above choice one finds that only one-loop diagrams with two, three, and four external lines are primitively divergent (like $\Lambda^2$, $\Lambda$, and $\log \Lambda$ respectively). Although not all divergences are regularized, the divergence of the theory is sufficiently reduced and the divergences identified and isolated so as to become amenable to treatment. Lee and Zinn-Justin use this regularization in combination with the use of regulator fields for scalars and spinors.

The second regularization method is the $n$-dimensional regularization of 't Hooft and Veltman, which had been used earlier in a different context by Bollini and Giambiagi and others. A Feynman integral can be transformed by the standard parameter method. For a one-loop diagram, for instance, one obtains an integral like
where \( n \) is the dimension of space-time. The divergence of such an integral for \( n \neq 4 \) appears in the right-hand side as a pole in the gamma function at \( n = 4 \). For instance, if the original integral was logarithmically divergent, one would find a pole at \( n = 4 \); if it was quadratically divergent, one would find poles at \( n = 2 \) and \( n = 4 \). With \( n \) away from the poles, the integral is regularized and one can evaluate the Feynman graph by using formulas such as \( g^{\mu \nu} \frac{1}{n} k^\mu k^\nu - \frac{1}{n} k^2 g^{\mu \nu} \) etc. The important point is that 't Hooft and Veltman were able to prove that this regularization method respects the Ward identities coming from gauge invariance. It also provides a way of defining a subtraction procedure by subtracting the poles at \( n = 4 \) with appropriate residues so that the amplitude becomes finite for \( n = 4 \).

The \( n \)-dimensional regularization method can be used in connection with either the U- or the R-formalism. It seems to be the best available at this time. In the case of theories with spinors and chiral gauge groups, it cannot be applied to the spinor loops because of the special properties of the matrix \( \gamma_5 \) in four dimensions. It must then be supplemented by another regularization method such as the Pauli Villars regularization for the spinor loops.

**VI. Renormalization**

Different aspects of the renormalization program for a theory with a spontaneously broken gauge group have been studied among others by 't Hooft, 't Hooft and Veltman, B. W. Lee, and Zum-Justin, Ross and J. C. Taylor. From the work of these authors emerges the possibility of a proof of renormalizability and unitarity to all orders in the R-formalism. No attempt has been made to give a proof in the U-formalism although a number of concrete calculations have been performed in the U-formalism in the one-loop approximation with finite results after renormalization.

The proof in the R-formalism is too involved to be reported here. The main idea which is simple is to make use of an invariant regularization procedure, subtract the infinities, and then prove the unitarity of the renormalized on-mass-shell amplitudes either by using the Ward identities in the particular renormalizable gauge being used or by proving gauge independence and the equivalence to the U-formalism. Because of the length of its expression, we renounce writing here a Lagrangian with all necessary counterterms. Let us only observe that, contrary to the familiar situation in electrodynamics, the simplest choice for the finite parameters entering in the Lagrangian (coupling constants) does not correspond to a simple and direct physical meaning in terms of observable processes. Furthermore, the simplest choice of renormalized fields does not correspond to normalized fields so that additional multiplicative renormalizations will be needed in the S-matrix elements. Finally, let us observe that the field and coupling constant renormalization are not inversely proportional.
and the additional renormalization constant $Z'$ is infinite in general. This means that the renormalized field transforms under the gauge group by a transformation with an infinite coefficient and the Ward identities for the renormalized quantities are correspondingly complicated.

**VII. Mass Relations**

In a renormalizable model such as the Weinberg-Salam model, all renormalized masses are finite since there must exist a sufficient number of counterterms to obtain this result. However, the masses and mass differences are arbitrary. Are there situations in which finite calculable mass relations arise? Weinberg\textsuperscript{25} and Georgi and Glashow\textsuperscript{26} have discussed this possibility. Suppose that in a particular model a relation among masses is satisfied to zeroth order (tree graphs and tadpoles). Suppose further that the mass relation is valid to zeroth-order for all renormalizable Lagrangians which are invariant under the given gauge group and which are constructed with a given set of fields. In general, the original gauge group will be spontaneously broken down to a subgroup, but let us consider the case in which the mass relation is not a consequence of the invariance under the subgroup. Then the inclusion of loops will be expected to modify the mass relation. Nevertheless, if the theory is renormalizable, the corrections must be automatically finite since one has already taken into account all counterterms allowed by the original gauge group. Zeroth-order mass relations of the kind described here can arise because of the special representation content of the scalar multiplet giving rise to the Higgs phenomenon (which may imply for instance that certain spinor fields do not couple to a scalar which has a non-vanishing vacuum expectation value and therefore have no zeroth-order mass) or because of the particular dynamics of the scalar multiplet (which may imply that a particular scalar does not acquire a nonvanishing vacuum expectation value although this is not a consequence of group theoretic arguments).

The ideas outlined here open the possibility of understanding, within the framework of spontaneously broken gauge theories, mass relations such as the Gell-Mann-Okubo relation. They generate the hope that one might find models in which, e.g., the neutron-proton mass difference or the electron-mass are calculable. For instance, if in a model the electron-mass vanishes to zeroth-order it could come out proportional to $e^2 m_u$ in the one-loop approximation. Unfortunately so far these are only possibilities in principle since no realistic models have been found.

Finally, let us point out that the above considerations apply equally well to relations involving not only masses but also other observable parameters such as coupling constants.

**VIII. Conclusion**

The renormalization program appears to work. However, it seems fair to say that none of the models suggested for a unified theory of weak and electromagnetic interactions is physically satisfactory. The experience gathered poses very strong constraints on model building. If a model exists which satisfies them all, it has a good chance of being the right theory. We must continue to look for it.
The author is very grateful to T W. Applequist, W A Bardeen, J D Bjorken, B W Lee, G 't Hooft, J. R. Primack, M. Veltman, and J Wess for discussing with him one or the other of the subjects treated in this manuscript.

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REGULARIZATION OF GAUGE FIELD THEORIES*

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Yang-Mills gauge field theories form the basis of the new class of renormalizable theories
of weak and electromagnetic interactions. The renormalizability of these theories* depends
in crucially upon the high degree of symmetry implied by the exact local gauge invariance of the
initial lagrangian. Unfortunately, the divergences encountered in the Feynman amplitudes of per­
turbation theory may prevent the implementation of these symmetries in higher order. Indeed,
the well-known Adler, Bell, and Jackiw Ward-identity anomalies of the free spinor loop provide
an example where the full gauge symmetry of the initial lagrangian cannot be maintained.

The known spinor-loop anomalies already place dynamical restrictions on "realistic" models
of weak interactions. We must now ask whether it is possible to avoid new anomalies either in
higher order or in the Yang-Mills structure which would destroy the renormalizability of these
theories. Using the work of the past year, we will show that this question can be given a positive
answer. We conclude that if the theory is chosen as to avoid the anomalies of the free spinor loop
then no further anomalies are present in the theory.

Our procedure will be to devise a regularization scheme for the Feynman amplitudes which
preserves the local gauge symmetry at every stage. Ignoring for a moment the problem of
including spinor particles, we examine two methods proposed for regularizing meson amplitudes.

An approach discussed by B. W. Lee and J. Zinn-Justin4 involves the addition of higher
dimension, but gauge-invariant, terms in the lagrangian to both the kinetic energy and interaction
terms. An analysis of the Feynman amplitudes shows that all of the more complicated amplitudes
are regularized by this mechanism and the gauge invariance of the unregularized amplitudes can
be directly established.

A simpler, more intuitive approach has recently been proposed by 't Hooft and Veltman5
and others.6 They show that Feynman amplitudes can be defined in noninteger or even complex
dimensions of space-time. To any finite order in perturbation theory, the amplitudes are mero­
morphic functions of the dimension variable with a simple pole structure on the positive real axis.
For scalar particles, this regularization corresponds to a modification of the phase-space integrals.
For the gauge theories of mesons considered here, the gauge symmetry implies algebraic rela­
tions between different amplitudes. As these algebraic manipulations are not sensitive to the
dimension of space-time, the gauge relations are easily established for amplitudes regularized by
this method. A simple algorithm for constructing amplitudes in n dimensions will be discussed
at the end of this talk.

*A preliminary version of this talk was presented at the Colloquium on Yang-Mills Theories held
at C.N.R.S., Marseilles, June 1972.
†Alfred P. Sloan Foundation Fellow

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We now wish to discuss the problem of regularizing theories involving spinor particles and particularly the anomaly problem. We remark that the n-dimensional technique can be simply extended to theories such as electrodynamics as discussed by 't Hooft and Veltman. However, weak-interaction theories usually involve the axial vector current, and no successful extension of $\gamma_5$, or more precisely the pseudotensor, to complex dimensions is possible in general.

We may avoid this problem by observing that the regularization of only the meson part of any graph is almost sufficient to regularize the whole graph. Consider the graph in Fig. 1 where an arbitrary meson blob $M$ connects the spinor line $AB$ with spinor loops $C$ and $D$. If we have sufficiently regularized the meson integrations, we observe that there are no further divergences associated with the spinor line. Also since the spinor loops are connected by meson lines, there are no overlapping divergences. Hence, we may study independently the divergences of each spinor-loop subgraph. There is only a polynomial ambiguity in each spinor-loop subgraph, and it is simple in principle to give a prescription for renormalizing these subgraphs. There are no further divergences.

We turn to the gauge relations between graphs such as in Fig. 1. As described previously there are at least two methods of giving a gauge-invariant regularization of the meson part of the graph. As we have not modified the structure of the spinor line, gauge relations involving manipulations along the spinor line are preserved as the regularized meson integrations are not divergent. Gauge relations also involve manipulation of the spinor loops. If the spinor loops can be given a gauge-invariant definition, then the whole graph can be given a definition consistent with the gauge relations. Hence only the lowest-order spinor-loop anomalies need be cancelled to obtain a gauge-invariant regularization procedure for the whole graph.

Fig. 1. A general Feynman graph with spinor line $AB$, spinor loops $C$ and $D$, and a meson blob $M$. 

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We can give a simple rule for regularization within the n-dimensional procedure. The calculation is to be done in n-dimensions but the $\gamma$-matrices must be kept four dimensional (they have only the first four components of an n-vector). Hence even if a spinor propagator carries an n-dimensional momentum it only depends upon the first four components of that n-vector. The spinor-loop integration is therefore reduced to a four-dimensional integration. The small spinor loops must be computed consistent with all relevant gauge relations (the cancellation of the free spinor-loop anomalies is the only dynamical constraint on the theory). The gauge relations never involve momenta coming out of spinor line or loop and hence the fact that $\gamma$-matrices are four dimensional does not destroy any of the gauge relations.

We now briefly remark upon the actual construction of n-dimensional amplitudes. It is obvious that the operation $\int d^n P$ has no immediate meaning if $n = 100 + 37i$. 't Hooft and Veltman give a definition by integrating explicitly the first k integer components of momentum and doing the remaining $n-k$ components via a volume integral which is defined through integrations by parts. An equivalent method has been suggested by Lautrup, which exploits the use of differential regularizer acting on the integrand. Hence the n-dimensional integral may be defined in terms of a normal four-dimensional integral. The n-dimensional amplitude for a single-loop graph is defined by

$$A(n, P) \int d^n t \quad \Gamma(t^2, f, P)$$

$$= \int d^4 t \quad R_n \Gamma(t^2, f, P)$$

where $R_n = (\frac{1}{n \cdot t^2})^{2 - \frac{n}{2}}$.

The integrand $\Gamma$ is to be computed as if you were in n dimensions. In practice it means only $g^{\mu\nu} g_{\mu\nu}$ n instead of 4 in doing the algebra.

This procedure can be generalized to the many-loop case. We have

$$A(n, P) \int d^n t \quad - \int d^n t \quad \Gamma(t^2, f, f, P)$$

$$\int d^4 t \quad - \int d^4 t \quad R_n \Gamma(t^2, f, f, P)$$

where $R_n = (\det A)^{2 - \frac{n}{2}}$ and $A = a_{ij}$, $a_{ij} = \frac{1}{2^i} \pi \delta_{f_i}^2 f_j$ where $i \neq j$

$$\frac{1}{2^i} \pi \delta_{f_i}^2$$

where $i = j$.

Again the integrand is computed with $g^{\mu\nu} g_{\mu\nu}$ n. It should be noted that the differential operator acts only on the meson part of the graph. With these definitions it is easy to show that all n-dimensional rules for shifting momenta, symmetric integration and relabelling loop momenta are allowed.

For simple graphs this procedure is similar to that employed by Speer. The important difference, however, for Yang-Mills theories is that the n-dimensional technique regularizes the whole meson blob at once.
We have established that a gauge-invariant regularization of Yang-Mills theories can be made. The only dynamical constraint on the theory comes from cancellation of the anomalies of the free spinor loops. The n-dimensional regularization scheme is intuitively pretty and simple to use in practical calculations.

Acknowledgment

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THEORIES OF WEAK AND ELECTROMAGNETIC INTERACTIONS
EMPLOYING THE HIGGS PHENOMENON*

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I. Introduction

The topic of this report is the status of efforts to make a realistic renormalizable theory of weak and electromagnetic interactions embodying the Higgs spontaneous-breakdown mechanism. I have found two possible ways of evaluating them. The first criterion is whether the theories are "believable," and the second is whether the theories are "not in contradiction with experiment." They are, to be sure, quite a few of the latter. They are useful as encouragement for experimentalists to go out and look for neutral currents and heavy leptons and also as a vehicle by which we theorists may gain experience on where some of the pivotal computational and conceptual issues lie.

I don't know any "believable" theories. This is of course a subjective statement. Different people will define believability in different ways. For me, a believable theory of this class will necessarily lead to at least some gain in understanding of Great Questions, such as origin of lepton mass (and in particular the smallness of $m_e/m_\mu$, $m_e/m_\tau$, and $m_e$), the nature of chiral and SU(3) symmetry breaking, and the origin of the Cabibbo angle. All these issues are fully entangled in the fabric of any model. No model makes progress in understanding them.

In the following we assume as known the basic ideas of the Higgs mechanism and the structure of the model of Weinberg. Generalizations either use the same gauge group SU(2) x U(1), a smaller one [SU(2)], or a bigger one. We consider these in turn, and then consider a few general ideas which may eventually aid in understanding the existence of a hierarchy of masses.

II. Theories Using SU(2) x U(1)

Many straightforward variants of the Weinberg model can be built by assigning the known leptons $\nu_e$, $e^+_L$, $e^-_L$, and corresponding muons and $\nu_{\mu}$ to various SU(2) "weak isospin" representations. Only the original version [doublet $(\nu, e^-)_L$ plus singlet $e^+_R$] avoids introduction of new heavy leptons. The models can be classified by $I_L$ and $I_R$, the isospins of the multiplets containing $(\nu, e^-)_L$ and $e^-_R$ respectively. Models with all combinations of $I_L \leq 1$ and $I_R \leq 1$ have by now been constructed, along with their generalizations to hadrons. For example, B Lee and Prentki and Zumino consider putting $e^+_e$, $\nu_e$ into a triplet

\[
\begin{pmatrix}
E^+ \\
\nu_e \\
e^-
\end{pmatrix}
\]

(with $e^+_R$, $e^-_R$ singlets) at the cost of introducing an extra heavy lepton. Prentki and Zumino also consider a two-doublet scheme.

*This work was done under auspices of the United States Atomic Energy Commission.
(with $E_R^+, E_R^0, e_R^-$ singlets)

Three other such models, namely (2-2, 3-2, and 2-3), along with the original 2-1 Weinberg model, all use the same Higgs doublet and the same universal couplings of $(e_L^-, \nu_e)$ to $W^\pm$. Thus any of the four may be used for the electron system, any for the muon system, and any for the generalization to hadrons, making instantly $4^3 = 64$ possible theories. Theories with $L_L = 1$ have, by the way, a special virtue: the $\nu_e$ is central member of the multiplet and therefore couples neither to $Z$ nor to $A$. This helps to avoid embarrassing conflicts with experiment.

The typical heavy leptons in these models are $E^+$ and/or $E^0$, which possess the same lepton number as $e^-$. Their phenomenology is discussed by Gerstein and Folomeskin and in Ref. 5. The typical Higgs scalar meson in these theories has properties similar to the one in Weinberg's model. One exception occurs in the 3-1 model, where a doubly charged $\phi^{++}$ exists. If heavy enough, it decays into $E^+e^+$ rapidly; if it is light it decays in second order to $e^+e^+\nu\bar{\nu}$, $e^+\mu^+\nu\bar{\nu}$, $e^+\tau^+\nu\bar{\nu}$, $\pi^+\pi^+$, etc. The vector masses $m_W, m_Z$ have masses in the range $(m_W > 37 \text{ GeV})$ given in the original model. In all these theories, the pattern of the lepton masses is introduced by brute force, and this already gives these theories a low believability.

**III Generalization to Hadrons**

The generalizations of these models to hadrons are constructed in terms of local constituent fields, treated as free fields. Were any of the models good enough, one could probably deal with them at the more abstract level of current algebra, operator product structures, etc. But we are still some way from that level of sophistication.

The Glashow-Iliopoulos-Maiani SU(4) idea, used by Weinberg for the original model, can be employed to generalize any of these lepton models to hadrons without getting into instant trouble with $\Delta S = 1$ neutral currents. The idea is to construct the scheme for the $(p, n^1)$ doublet ($n^1 = n \cos \theta_C + \lambda \sin \theta_C$) in parallel with the way used for the $(\nu_e, e)$ doublet. Then introduce a new doublet $(q, \lambda^1)$ with $q$ a new "heavy" constituent (and $\lambda^1 = \lambda \cos \theta_C - n \sin \theta_C$), such that there is permutation symmetry under $(p, n^1) \leftrightarrow (q, \lambda^1)$. Thus lowest-order neutral currents come only in the $\Delta S = 0$ combination

$$n^1 \Gamma p + \lambda^{1 \dagger} \Gamma \lambda = n^{\dagger} \Gamma n + \lambda^{\dagger} \Gamma \lambda$$

Induced $\Delta S = 1, 2$ neutral current effects in higher order (Fig 4) still make trouble. and limit from above the bare mass of the extra $q$ most likely to a few GeV.
Thus, in parallel with the new heavy leptons, one might have a "new spectroscopy" of heavy hadrons with a threshold mass of a few GeV.

However, these models look even more contrived than those for leptons, and just tend to make their believability rating even lower.

**IV Models Based on SU(2)**

Georgi and Glashow\(^7\) succeeded in totally eliminating the Z from the original scheme by choosing \(L = I_R - 1\) for the fermions. One must also mix in another singlet \(I = 0\) left-handed field. The triplets are

\[
\begin{pmatrix}
E^+ \\
\nu_e \sin \alpha + E^0 \cos \alpha \\
e^-
\end{pmatrix}
\begin{pmatrix}
E^+ \\
E^0 \\
e^-
\end{pmatrix}_L
\]

and the singlet is obvious. The Higgs fields are a Hermitian triplet, only the neutral member remains a physical particle after spontaneous breakdown. A boon for some future generation of experimentalists (if the theory lasts that long) is the prediction that \(m_W < 53\) GeV. The believability rating of the model plummets toward zero as one notices that \(m_\nu\) is given as the difference of two terms, one a bare mass term \(= m_{\nu / 2}\), the other emerging from the spontaneous breakdown \(\psi^\dagger \psi \sim m_{\psi / 2}\). No rationale for the miraculous cancellation is given.

The direct generalization to hadrons is awkward. Two triplets

\[
\begin{pmatrix}
p \\
\sin \alpha + Q \cos \alpha \\
P
\end{pmatrix}_L
\begin{pmatrix}
p \\
Q \\
P
\end{pmatrix}_R
\]

along with three singlets do the job. This makes five basic hadron constituents \(p^+, n^0, \lambda^0, Q^+, P^\mp\). However, the induced effects shown in Fig 1 ruin that scheme.\(^10\) To repair the damage one can again go back to the Glashow-Hiopoulos-Maiani trick. This works at the price of three more constituents. At this point, one might as well try nine, in particular the Han-Nambu model, which might actually have some relevance to hadron spectroscopy. Lapin\(^11\) has built such a scheme in which the observable [SU(3) singlet] weak current is of Cabibbo form and the induced effects of Fig 1 vanish. The fermion weak isospin triplets are
Universality with leptons is attained by choosing a "natural" mixing angle for $\nu_e$ and $E_0$ of 45° (in that case $m_W = 37$ GeV). Although one must cope with the perplexing nature of the SU(3)$^f$ in this model, this model is probably the best SU(2) model existing. However, it still seems quite artificial and arbitrary in its construction and to my mind does not raise the believability of the SU(2) models very much.

**V Bigger Groups**

With at least three (four-component) leptonic degrees of freedom, it is very tempting to try bigger gauge groups in which both Weinberg's $Y$ and $T$ are embedded. However, bigger groups of gauge fields will have more neutral members, potentially making the neutral-current problem even worse. Weinberg suggested that a way of living with this might be to make most of the gauge particles superheavy ($m \geq 10^3 - 10^4$ GeV$^2$), leaving only $W^\pm$, $Z$ and maybe a few others relatively light ($m \leq 100$ GeV). This is not quite as disgusting as it first sounds. After all, lepton mass in these models typically arises from an input Yukawa coupling $g \bar{\psi} \psi \phi$ turning into a mass term $g \bar{\psi} \psi \phi$ upon spontaneous breakdown. If the various couplings $g$ are roughly universal in strength, then we must accept a hierarchy of Higgs expectation values:

$$<\phi_e> < <\phi_\mu> < <\phi_{heavlep}>$$

corresponding to the observed hierarchy of lepton masses. Since masses of gauge particles occur in the same way,
this may give a corresponding hierarchy of masses for the W's as well

But the "light" W's generate a subgroup of the full-gauge group. This follows from general considerations, as \( \langle \phi \rangle \rightarrow 0 \), the masses of the light W's go to zero. They become the gauge bosons for the residual unbroken symmetry in the theory. Why, then, bother with the big group? Because the big group may be simple and the subgroup not simple, there will consequently be relations between the commuting subgroups which one would not have seen otherwise. Weinberg, for example, argues that the mixing angle \( \theta \) in his model should be 30° if his SU(2) \( \times U(1) \) is a descendant of SU(3) \( \times SU(3) \). Such schemes and many others, e.g., [SU(4)] have been invented, but space mercifully forbids their description here.

**VI Can the Strong Interactions be Unified Too?**

Bars, Halpern, and Yoshimura, in a very ingenious and interesting paper, have tried to incorporate the strong interactions into this kind of theory. For leptons they take the original theory of Weinberg [although there is nothing to stop one from using many of the SU(2) \( \times U(1) \) variants]. For the strong interactions they introduce Yang-Mills nonets of massless vector and axial mesons (\( \rho, A_1 \), etc.) coupled to a triplet of quarks and to a (3, \( \overline{3} \)) multiplet of scalar and pseudoscalar mesons (\( \sigma, \pi \), etc.) which undergo spontaneous breakdown. The hadrons do not directly interact with W, Z, or A. Hadronic, weak and electromagnetic interactions are induced by introducing two additional 3 \( \times \) 4 complex matrices of \( J = 0 \) Higgs fields \( M_L, M_R \) which interact invariantly with \( \rho, A_1 \), etc. as well as with W, A, Z. These also undergo spontaneous breakdown, which provides the necessary link of W and A to hadrons (see Fig 3).

Fig 3 Mixing of W with vector hadron states

Thus weak and electromagnetic interactions of hadrons proceed primarily through a vector-dominance mechanism, although there will also be contributions from all the M-meson currents. The usual (3, \( \overline{3} \)) + (3, \( \overline{3} \)) SU(3) breaking is nicely incorporated, and there is no problem in lowest order with \( \Delta S = 1 \) neutral currents.

The overall idea is very attractive, but there are many questions to be clarified. Among them are the following:

1. All the M-mesons (I count 27 degrees of freedom) are hadrons, because they couple strongly to \( \rho, A_1 \), etc. Will strong radiative corrections involving M's destroy the universality of the hadronic weak and electromagnetic couplings?

2. Do the \( K_L - K_S \) mass difference and \( \Gamma(K_L \rightarrow \mu^+ \mu^-) \) computed, say, in one-loop approximation, converge only because of the presence of the M-multiplet? If so, the mass of the M's may well be bounded above by a few GeV. If this is the case, one must consider their impact upon hadron spectroscopy. The M's become just as much the constituents of hadrons as the heavy \( J = 1/2 \) constituents of previous models.
The authors point out that the hadronic current algebra looks more like field algebra and may come into conflict with deep-inelastic electroproduction phenomenology. I personally feel this is too high-order a speculation to worry about yet. But it is important to know what the current algebra is for the scheme and whether it implies lepton-hadron universality of weak interactions.

VII Another Unification Scheme

Pati and Salam are considering another scheme to unify strong interactions which starts with the Han-Nambu group $\text{SU}(3)_L \times \text{SU}(3)_R \times \text{SU}(3)_t$, where $\text{SU}(3)_t$ is the symmetry group ("color") which mixes the three triplets, and $\text{SU}(3)_{L,R}$ is the ordinary hadron symmetry. They propose to do the following:

1. Extend the $\text{SU}(3)_L \times \text{SU}(3)_R \times \text{SU}(3)_t$ to $\text{SU}(4)_L \times \text{SU}(4)_R \times \text{SU}(4)_t$
2. Use the $\text{SU}(3)_t$ subgroup of gauge fields for generating the strong force.
3. Unify leptons and hadron spinor degrees of freedom by putting them into a $(1, 4, \bar{4}) + (4, 1, \bar{4})$ representation shown schematically below.

\[
\begin{pmatrix}
\text{Han Nambu} \\
\text{net} \\
\text{New hadron} \\
\text{triplet}
\end{pmatrix}
\begin{pmatrix}
v_e \\
e \\
\mu \\
v_\mu
\end{pmatrix}
\]

4. Use an $\text{SU}(2)_L$ subgroup of the ordinary $\text{SU}(4)_L$ for the weak-interaction gauge particles. The photon is a mixture of the gauge fields of the $\text{SU}(4)_{L,R}$ and $\text{SU}(4)_t$. The remaining 33 gauge degrees of freedom are not used.

VIII The Hierarchy of Lepton Masses

We have already touched upon the question of the hierarchy of lepton masses and the possibility that it may be connected to a hierarchy of vacuum expectation values of the Higgs fields. However, typically

\[m > g < \phi >,\]

and $g$ is uncomputable in higher orders and can be only prescribed by a renormalization procedure. In other words, renormalizability never guarantees the calculability of masses or even of mass differences. Weinberg, however, points out an important criterion for calculability of mass. A mass contribution generated by a spontaneous breakdown mechanism can only be divergent if its group structure is consistent with some Yukawa $\bar{\psi} \psi < \phi >$ or self-mass counterterm $\bar{\psi} \psi$ possessing the full Yang-Mills symmetry of the unbroken theory. Otherwise the theory couldn't be renormalizable. Weinberg furthermore exhibits, in unitary gauge, how the mechanism works. Georgi and Glashow have developed this further and have given examples of calculable mass shifts as well as a simple mnemonic for this calculability criterion. The fermion self-energy $\Sigma$ is a function of the Higgs expectation value <\phi>. To determine calculability, formally expand $\Sigma$ diagrammatically as a power series in <\phi>, treating <\phi> as an external J = 0 virtual particle of zero total four-momentum. Then the mass is calculable if all diagrams are superficially convergent, i.e., the first two terms in the expansion are not present. By judicious choice of the
representation content of the scalar and spinor multiplets, one may hope that the $\mu$ and $e$ masses emerge as higher-order radiative corrections and are finite. But thus far the best that has been accomplished are some clever but highly unrealistic examples invented by Georgi and Glashow.

IX Spontaneous Breakdown as a Consequence of Radiative Corrections

If one hopes to make a hierarchy of Higgs expectation values, say with $<\phi>_{\text{small}} \ll <\phi>_{\text{large}}$, it is necessary to know how to study the spontaneous breakdown in higher orders. This was done by Jona-Lasinio long ago. Instead of minimization of the potential $V(\phi) = \mu^2 \phi^2 / 2 + (\lambda \phi^4 / 4!)$ which occurs in the Lagrangian density, one should minimize the sum of all connected one-particle irreducible vacuum diagrams with only $\phi$ lines of zero total momentum as external lines.

Coleman and E. Wigner study the original Higgs model with the single-loop diagrams included. For very small $\mu^2$ and $\lambda$, the vector-boson loop in Fig. 4 is the most important term, and after mass renormalization and transformation

$$V(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{1}{24} \lambda \phi^4$$

Fig 4. Effective potential for spontaneous breakdown condition to unitary gauge, they find

$$V(\phi) = \frac{\lambda_0 \phi^4}{4!} + \frac{1}{2} \mu_0 \phi^2 - \frac{3}{8} \phi^4 \log \frac{M^2}{\phi^2}$$

For small $\phi$ the induced potential is attractive, and the minimum at $\phi = 0$ exists, producing the Higgs phenomenon. The parameters of the theory $\mu_v$ (vector mass), $\mu_s$ (scalar mass), and renormalized $\lambda$ are determined by

$$\mu_v^2 = \frac{\lambda_0}{2} \phi^2 \quad \frac{\partial V}{\partial \phi} = 0$$

$$\mu_s^2 = \frac{\lambda_0}{8} \phi^2 \quad \frac{\partial^4 V}{\partial \phi^4} = \lambda$$

all evaluated at $\phi = <\phi>$. One finds

$$\lambda_0 = \lambda - 18 \alpha^2 \log \frac{M^2}{<\phi>^2} - 75 \alpha^2$$

$$\left( \frac{\mu_s^2}{\mu_v^2} \right) = \frac{1}{4 \pi \alpha} \left( \frac{\lambda}{3} - 16 \alpha^2 \right) \quad \left( \frac{\mu_0}{\mu_v} \right)^2 = \frac{1}{4 \pi \alpha} \left( 11 \alpha^2 - \frac{\lambda}{6} \right)$$

Coleman and Wigner prefer to study the special case $\mu_0 = 0$. That leads to

$$\lambda = 66 \alpha^2 \quad \left( \frac{\mu_s^2}{\mu_v^2} \right) = \frac{3 \alpha}{2 \pi}$$

and a rudimentary hierarchy of masses emerges.

Perhaps in complicated nonabelian theories a hierarchy of scalar expectation values can emerge by a similar mechanism. However, no such example has yet been found.

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X. Conclusions

The last three ideas discussed, namely inclusion of strong interactions into the scheme, of the calculability of fermion masses generated in higher orders of $\alpha$, and of spontaneous breakdown itself considered in higher orders, are all quite new and can perhaps get us out of the apparent dead end into which the more pedestrian generalizations have tended to lead us. In particular, little effort has yet been made in trying to synthesize some of these ideas perhaps one should also throw CP violation into the soup. Thus, while the history of model-building efforts is a little discouraging, there is still cause for encouragement with regard for the future.

But one must also keep in mind that, despite the calculability, the intrinsic beauty, and the potential for economy of description present in this class of theories, it is quite possible that nature may simply choose something else.

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I Introduction

The standard phenomenology of weak interactions arises, in the models we will consider here, from the interaction of charged V-A leptonic and hadronic currents via charged massive vector bosons $W^\pm$. If the masses of the vector bosons and the nonconservation of the currents are induced by spontaneous breakdown of the gauge symmetry, the models remain renormalizable. However, additional particles and interactions not contemplated in the usual phenomenology are inevitably introduced in the various models: Higgs scalars, massive neutral vector bosons, massive leptons and "charmed" hadrons. Moreover, because higher-order corrections are finite in these models and should consequently be taken seriously, it is essential to check whether these corrections—as well as the effects of the new particles and interactions—are indeed small enough so that no conflict with experiment arises. Of course, it is first of all necessary to verify that higher-order corrections are indeed finite and well-behaved.

R and U Formalisms

As was initially emphasized by 't Hooft, gauge freedom allows one to formulate spontaneously broken gauge theories so that the vector propagator is well-behaved ($\propto k^{-2}$) at high energy, at the price of having to carry along unphysical fields whose contributions eventually cancel out of physical S-matrix elements ("R formalism"). Alternatively, one can eliminate the unphysical degrees of freedom, the vector propagator then has the canonical form with bad high-energy behavior ($\propto k^0$) ("U formalism"). (See Table I.) Since the U and R formulations are formally equivalent, the theory possesses renormalizability (manifest in the R formalism) together with unitarity (manifest in the U formalism).

Although the investigation of spontaneously broken gauge-field theories is still young, more than a dozen papers on higher-order corrections have so far appeared (mostly within the past few months). Many of these investigations have been motivated by the desire to verify by explicit calculation the good properties indicated for these theories by formal arguments.

The first papers I shall discuss undertake to demonstrate explicitly the cancellation of divergences necessary for finiteness of physical amplitudes in the U formalism. Weinberg pointed out that in processes such as $\nu + \bar{\nu} \rightarrow W^+ + W^-$, the behavior of the cross section is $-1/\sqrt{s}$ for $s \gg M_W^2$, in contrast to the $\sqrt{s}$ growth at high energy in conventional intermediate vector boson models. This good high-energy behavior is a result of the gauge symmetry of Weinberg's model. The Yang-Mills structure causes the cancellation only of the leading high-energy behavior, however, which is necessary but not sufficient for renormalizability. As Appelquist and Quinn showed in the context of a simplified abelian gauge model (one neutral massive vector
meson), the inclusion of effects due to the Higgs scalar meson is in general necessary for the cancellation of nonleading divergences.

Consider, for example, the contribution of the $2B$ ($B =$ neutral vector boson) intermediate state to lepton-antilepton scattering in the abelian model (see Fig. 1). The absorptive parts from the box and crossed-box graphs each grow as $t$ but have opposite sign. As a result of this cancellation, the sum of the contributions from these graphs is $-m_l^2$ as $t \to \infty$, where $m_l$ is the mass of the lepton $l$, so that a dispersion relation in $t$ would diverge logarithmically if these were the only contributions. It is only after the inclusion of the graphs containing the scalar Higgs meson that the dispersion relation is convergent without subtraction. The same cancellation of divergences is observed if one calculates the Feynman graphs directly, without dispersing. This cancellation is, of course, a consequence of the spontaneous symmetry breaking which is incorporated in this model. Current conservation is violated only by mass terms generated by the Higgs scalar boson.

Several authors have more recently discussed the cancellation of divergences in the $U$ formalism in more realistic nonabelian gauge models. These models, which contain a multiplet of vector mesons ($W^\pm$, the photon, and possibly other massive vector mesons), are even more divergent than the abelian model because of the vector meson self-interaction. For example, individual one-loop graphs (Fig. 2) contributing to the vector propagator are superficially as much as sextically ($A^5$) divergent and actually quadratically ($A^4$) divergent. Although Green's functions are more convergent than individual graphs which contribute to them, they nevertheless remain divergent in the $U$ formalism. For example, the vector propagator remains logarithmically divergent (except at the renormalization point) in order $g^2$, even after mass and wave-function renormalizations have been performed. Additional cancellations occur in calculating physical amplitudes, as in Fig. 1, and render them convergent.

In order to go on and actually calculate finite expressions in the $U$ formalism, it is necessary to use a powerful regulation method—such as 't Hooft and Veltman have developed—in order to define the highly divergent individual Feynman graphs. Alternatively, one can sidestep this problem by calculating dispersively. Absorptive parts from one-loop graphs are always well-defined, if these are added so as to cancel the worst high-energy behavior, the resulting dispersion integral is then convergent after the usual renormalization subtractions. Yet another alternative which has been suggested is to go into position space and shift the derivatives in the numerator of the vector propagator onto the currents by partial integration, giving mass terms. In practice, however, it appears to be easiest to calculate higher-order corrections in the $R$ formalism, particularly in the 't Hooft gauge.

II. Calculations: Static Quantities

I shall first discuss explicit calculations of static quantities in the various weak-interaction models and then discuss higher-order contributions to certain weak scattering and decay phenomena.

Models

The models in which explicit calculations have been performed are 1) the Weinberg-Salam $SU(2) \times U(1)$ model, 2) a modified version of this model due to Lee and Prentki and Zumino.
In which neutral neutrino processes like $\nu + p \rightarrow \nu + X$ are eliminated in lowest order, and
3) the O(3) model due to Georgi and Glashow. The Weinberg and LPZ models possess a neutral
massive vector boson $Z$ in addition to $W^\pm$ and the photon, while in the Georgi-Glashow model the
only neutral vector boson is the photon. Higher-order effects can be considerably larger in the
Georgi-Glashow O(3) model than in the SU(2) x U(1) models, both because the $W^\pm$ mass can be
small in the O(3) model, while it must be large in the SU(2) x U(1) models, and also because the
couplings of the leptons to the neutral Higgs scalar are potentially much larger in the O(3) model.

A number of authors have calculated the weak contributions to the anomalous magnetic
moment of the muon in one or more of these models, their results are summarized in Table II.
Bardeen, Gastmans, and Lainrup also calculated the static properties of the $W^\pm$, and they and
other authors have emphasized the care which must be taken to avoid a spurious neutrino charge
and to eliminate other calculational ambiguities.

Fujikawa, Lee, and Sanda have given a particularly complete discussion, their $R$ gauge
generalizes the $\epsilon$ -limiting regulation of Lee and Yang, but with the added virtue that both by
formal argument and by explicit calculation physical results are independent of their gauge param­
der $\epsilon$. The agreement of the various calculational techniques employed by different authors tes­
tifies to the soundness of theoretical understanding of how to deal with spontaneously broken gauge
field theories. We should mention, however, that Fukuda and Sasaki obtain the result 7/3 for
the $W$ graph in the Weinberg model by means of a sideways dispersion relation, if this result is
correct, the dispersion relation evidently requires a subtraction despite the fact that it converges
Incidentally, this graph was calculated long ago in intermediate vector boson theory, and it was
realized then that the graph gives a convergent answer only if the charged vector boson $W^\pm$ has a
gyromagnetic ratio $g_W = 2$ in lowest order. It was also known that the higher-order corrections
to the quadrupole moment of the $W^\pm$ are finite only for $g_W = 2$. In gauge field theories of weak
and electromagnetic interactions, the $W$-photon coupling is fixed by the Yang-Mills interaction so
that $g_W = 2$ automatically.

The close agreement between the measured values of $g_\mu$ and $g_\mu$ and the predictions of quan­
tum electrodynamics implies that any weak contribution to the lepton magnetic moments must be
very small. The experimental values are $[a = (1/2)(g - 2)]^{26}$

$$\begin{align*}
(a_\mu)_{\text{expt}} &= (11661.6 \pm 3.1) \times 10^{-7} \\
(a_e)_{\text{expt}} &= (1159657.8 \pm 3.5) \times 10^{-7}
\end{align*}$$

The contributions of QED are $^{26}$

$$\begin{align*}
(a_\mu)_\text{QED} &= (11658.1 \pm 0.2) \times 10^{-7} \\
(a_e)_\text{QED} &= (1159655.0 \pm 3.1) \times 10^{-7}
\end{align*}$$

The strong corrections to the QED calculations are estimated to be $(0.65 \pm 0.1) \times 10^{-7}$ for $a_\mu$ and
negligibly small for $a_e$; thus,

$$\begin{align*}
(a_\mu)_{\text{expt}} - (a_\mu)_\text{QED} = (a_\mu)_{\text{strong corrections}} &= (2.8 \pm 3.1) \times 10^{-7}
\end{align*}$$
It thus seems reasonable to conclude that

\[ -3 \times 10^{-7} \leq (a_\mu)_{\text{weak}} \leq 9 \times 10^{-7} \]

\[ -5 \times 10^{-9} \leq (a_e)_{\text{weak}} \leq 11 \times 10^{-9}, \]

allowing for a discrepancy of two standard deviations in each case.

It is evident from the results displayed in Table II that in both the Weinberg and LPZ models, the weak contributions to \( a_\mu \) are smaller than \( 10^{-8} \). This is much smaller than the current experimental errors and is comparable to the uncertainty in the hadronic contribution to \( a_\mu \). Such small effects would be very difficult to observe. The contribution to \( a_e \) in these models is smaller by \( (m_e/m_\mu)^2 \) and is thus entirely negligible.

By contrast, in the Georgi-Glashow model the weak contributions to both \( a_\mu \) and \( a_e \) can be comparable to the experimental limits for sufficiently small values of \( M_W \) and \( m_\phi \) and sufficiently large values of \( m_{Y^+}, m_{X^+} \). Thus, allowed values of the masses in this model are constrained by experiment. If we assume that \( m_{Y^+} \) and \( m_{Y^0} = (m_{Y^+} - m_\mu)/2 \cos \beta \) are both larger than 0.5 GeV, consistent with the nonobservation of these heavy leptons in kaon decay or at accelerators, then from the lower limit on \( (a_e)_{\text{weak}} \) we conclude that

\[ M_W > 10 \text{ GeV} \]

We could get a stronger lower bound on \( M_W \) from \( a_\mu \) if it were not for the fact that the \( \phi \) graph contribution is comparable to that of the \( Y^0 \) graph and opposite in sign. If we can from some other considerations deduce that \( m_{Y^+}m_\mu/m_\phi^2 < 1 \), so that the \( \phi \) graph contribution is negligible, then we can conclude that

\[ M_W > 20 \text{ GeV} \]

It also follows in this case that the mass of the charged heavy lepton \( Y^+ \) is effectively bounded above by

\[ m_{Y^+} < 7 \text{ GeV} \]

for \( M_W < 50 \text{ GeV} \). (Although the actual upper limit on possible values of \( M_W \) in this model is 53 GeV, this is a singular limit of the model.)

Clearly, although the experimental data are already astonishingly accurate, any improvement in the determination of \( g_\mu \) or \( g_e \) could be very helpful in constraining theoretical models of weak interactions.

### III  Muon Decay

Appelquist, Primack, and Quinn\(^1\) have calculated the first-order weak corrections to the muon decay amplitude in the Weinberg model. These corrections are found to be of order \( G_F \), consequently, the muon decay rate is changed by about 0.5% for values of the parameter

\[ R = M_W^2/M_Z^2 - \cos^2 \theta_W \text{ not too close to unity} \]

(Experimental limits on \( \sigma(\nu_\mu + e \rightarrow \nu_\mu + e) \) imply \( R > 0.65 \). Since the model is so constructed that

\[ g^2 - e^2/(1 - R), \]

\[ (a_e)_{\text{expt}} - (a_e)_{\text{qed}} = (2.8 \pm 0.6) \times 10^{-9} \]
where $g$ is the lowest-order ($e W$) coupling constant, it follows that for $R \to 1$ the perturbation expansion in $g$ breaks down."

Because of the large masses of the intermediate vector bosons, the weak corrections to muon decay have a negligible effect on the shape of the final electron spectrum. This is to be contrasted with the effect of soft photons, which changes the spectrum significantly (the effective shape parameter $\rho$ is changed from $3/4$ to $0.69$). There is no comparable soft photon enhancement of the electromagnetic corrections ($W$-photon intermediate states) to the muon decay rate, however, these corrections are therefore again of order $\alpha$.

The higher-order weak and electromagnetic corrections to $\beta$-decay are similar to those for muon decay, but difficult to calculate precisely because of the presence of strong interactions. In comparing muon- and $\beta$-decay, there is the additional uncertainty introduced by the Cabibbo angle. It thus unfortunately appears to be impossible to detect the small weak corrections to the muon decay rate until other weak processes can be calculated and measured with precision comparable to muon decay.

Certain features of the muon decay calculation may be of wider interest. The calculation was performed dispersively in the $U$ formalism, and such complications as the instability of the $W^+$ against decay into $\pi\nu$ were treated in some detail. The calculation also shows explicitly that neither the limit $M \to m$ nor the limit $m_\phi \to \infty$ may be taken without destroying the renormalizability of the Weinberg model, both limits leading to logarithmic divergences in muon decay.

IV Higher-Order Corrections to Neutral Current Processes

The observed weak-interaction processes are represented in the conventional phenomenology as effectively arising from the interaction of charged $V-A$ currents. Processes which would correspond to neutral current interactions appear experimentally to be suppressed. In the Weinberg $SU(2) \times U(1)$ model, however, such processes can occur in lowest order, mediated by the massive neutral vector boson $Z$. Moreover, in any model such processes can occur through the neutral Higgs meson or in higher order through $W^+W^-, \bar{\nu} \nu$ etc, intermediate states.

The upper bounds presently available on purely leptonic neutral current processes -- $e \nu_\mu \bar{\nu}_\mu + e \nu_\mu \bar{\nu}_\mu + e \nu_\mu + e \nu_\mu$ -- are only moderately restrictive. The experimental bounds are somewhat more restrictive for $\Delta S = 0, \Delta Q = 0$ semileptonic processes. In particular, the upper bound for $\nu_\mu + p \to \mu + X$ has now diminished sufficiently to make serious trouble for the Weinberg model. Most decisive is the situation for neutral-current effects in strangeness-changing semileptonic processes, such as $K_L \to \mu\nu_\mu$ or $K^+ \to \pi^+ \nu_\mu$ and in the $K^+ - K_0$ mass difference. These data are so restrictive that one takes it as a principle of model building to banish strangeness-changing neutral vector boson couplings altogether. Even then it is still necessary to examine Higgs scalar exchange and higher-order effects to see whether the models can be made consistent with the experimental constraints.

Neutral Scattering Processes

Neutral current processes involving neutrinos are eliminated in lowest order in the LPZ model, and the weak neutral current is entirely absent in the Georgi-Glashow model. Furthermore, the zero (or very small) neutrino mass dictates that the Higgs scalar coupling to the neutrinos must vanish (or nearly vanish) in lowest order. The first nonvanishing contribution
to neutrino scattering in these models consequently comes from one-loop graphs. These have been calculated by Fujikawa, Lee, Sanda, and Treiman and by Bouchiat, Iliopoulos, and Meyer (See Table III). On the basis of dimensional considerations, such higher-order contributions are expected to be of order $G(G \lambda^2)$. In the spontaneously broken gauge models of weak interactions, the vector boson mass acts as an effective cutoff since the underlying gauge symmetry is effectively recovered for energies larger than this mass. Since $e^2/M_W^2 \sim G$ in these gauge models of weak and electromagnetic interactions, it follows that higher-order contributions will ordinarily be of order $G \alpha$. This was found to be the case in the muon decay calculation, and it is also true for neutrino scattering processes; these processes are thus neither enhanced nor suppressed. Consequently, in models where neutrino elastic scattering is eliminated in lowest order, the cross section will be greatly suppressed (by $\sigma^2$, apart from logarithms). Experimental observation of these processes would thus be of crucial importance in pointing us toward the correct theory of weak interactions.

Strangeness-Changing Processes

The $\Delta S \neq 0$, $\Delta Q = 0$ processes impose the strongest restrictions on models of weak interactions. Lee, Primack, and Treiman have studied the constraints imposed by the processes $K^+ \to \pi^+ \bar{e}e$, $K_L^+ \to \bar{\mu}\mu$, and the $K^+ - K_L^+$ mass difference.

There is no theoretical principle in the broken gauge symmetry scheme that relates the masses of the Higgs particles to the masses of other particles in the models. Thus, it can always be arranged that observable effects from Higgs particle exchange are as small as one wishes. If there are nonvanishing lowest-order couplings of the neutral Higgs scalar to neutral strangeness-changing combinations, then strong lower limits on the Higgs particle mass are implied by the strong experimental upper limits on the decays $K_L^+ \to \bar{\mu}\mu$ (constrains the pseudoscalar coupling $\phi_{\pi\pi}^n$) and $K^+ - \pi^+ \bar{H}$ (constrains the scalar coupling $\phi_{\pi n}$). In the "5-quark" $O(3)$ model of Georgi and Glashow, for example, the $\phi_{\pi n}$ couplings are given in terms of the $\lambda, n$ quark masses. Consideration of $K^+ \to \pi^+ \bar{e}e$ then gives the constraint

$$m_\phi (\text{GeV}) > 10 \left( \frac{m_\lambda + m_n}{m_\lambda - m_n} \right)^{1/2}$$

Higher-order effects, arising for example from $W^+W^-$ exchange, are less easily suppressed by adjustment of free parameters. As was the case for the processes already discussed, the natural size for such higher-order amplitudes is $-i G \alpha$. Explicit calculation in the Georgi-Glashow "5-quark" model therefore gives

$$R = \frac{\Gamma(K_L^+ \to \bar{\mu}\mu)}{\Gamma(K_L^+ \to \text{all})} = 3 \times 10^{-4},$$

independent of $M_W$ or other adjustable parameters. This result is clearly inconsistent with experiment and rules out the "5-quark" model.
The only way to avoid this catastrophe appears to be through the introduction into the model of a new symmetry which forbids the occurrence of \( n + \bar{n} \rightarrow W^+ + W^- \), then neutral strangeness-changing amplitudes will be proportional to \( G(\Delta m^2/M_W^2) \), where \( \Delta m^2 \) is a parameter which measures the breaking of this new symmetry. The prototype of this idea is a model due to Glashow, Iliopoulos, and Maiani, in which an extra "charmed" quark is added to the usual (integrand-charged) \( p, n, \lambda \), and the current is

\[
J^{(+)\mu} - \bar{p} \gamma_\mu (1 - \gamma_5) n_c + \bar{p} \gamma_\mu (1 - \gamma_5) \lambda_c,
\]

where \( n_c \) and \( \lambda_c \) are the Cabibbo-rotated combinations

\[
n_c = n \cos \theta_c + \lambda \sin \theta_c, \\
\lambda_c = n \sin \theta_c + \lambda \cos \theta_c.
\]

Then, insofar as the masses of \( p \) and \( p' \) are equal, the contributions from the two graphs (Fig 4) for \( n + \bar{n} \rightarrow W^+ + W^- \) exactly cancel.

Let us imagine for the sake of estimation that the process \( K_L \rightarrow \mu \nu \) takes place through the effective quark-quark interaction arising from \( W^+ W^- \) exchange, as pictured in Fig 5. Evaluating the \( K_L \) matrix element by PCAC, the branching ratio of this decay is found to be

\[
R = \frac{\Gamma(K_L \rightarrow \mu \nu)}{\Gamma(K_L \rightarrow \text{all})} \approx 10^{-3} \left( \frac{m \Delta m}{M_W^2} \right)^2 \left( \frac{M}{M_W} \right)^2
\]

in an O(3) "8-quark" model, which incorporates the Glashow, Iliopoulos, Maiani device, here \( m \) is a typical quark mass and \( \Delta m \) is the typical mass difference between "charmed" and "normal" quarks. The experimental value of this branching ratio is presently obtained by the Berkeley group, obtained an upper limit of \( R < 1.8 \times 10^{-9} \), but the BNL-Columbia-NYU group have reported to this conference a much larger result \( R = 10 \times 10^{-9} \). The \( 2\gamma \) intermediate state is expected, on the basis of the measured rate of \( K_L \rightarrow 2\gamma \), to give a branching ratio of at least \( (6 \pm 1) \times 10^{-9} \). To be definite, I shall assume that the branching ratio equals this "naive unitarity bound." It then follows that

\[
\frac{2m \Delta m}{M_W^2} < 5 \times 10^{-3} \quad (\star)
\]

If \( \Delta m \approx 1 \text{ GeV} \) (since charmed hadrons have not been seen), this implies

\[
m (\text{GeV}) < 7 \sin^2 \beta,
\]

where \( M_W (\text{GeV}) = 53 \sin \beta \) in the O(3) model. A result similar to Eq (\star) is obtained in the Weinberg or LPZ models.

An analogous calculation of \( K_1 \rightarrow K_2 \) gives the following result for the \( K_1 - K_2 \) mass difference \( \Delta m_K \)

\[
\frac{\Delta m_K}{m_K} = (7.14 \pm 0.05) \times 10^{-15} = 3 \times 10^{-14} \left( \frac{\Delta m}{1 \text{ GeV}} \right)^2
\]

in the 8-quark or LPZ model. This implies that the difference between "charmed" and un-charmed quark masses is \( \Delta m = 0.5 \text{ GeV} \). The approximations inherent in this calculation--or,
more generally, the theoretical uncertainties introduced by the presence of strong interactions--
prevent the drawing of sharp quantitative conclusions here. It is clear, however, that the con-
straint following from these $\Delta S \neq 0$, $\Delta Q = 0$ processes are indeed very strong. Models which do
not incorporate a suppression mechanism such as that of Glashow, Iliopoulos, and Maiani, give
results which are unacceptably large.

V Conclusions

I can summarize briefly the conclusions that follow from the higher-order calculations which
have been done thus far. First, the results are finite and well-defined: the spontaneous-symmetry
breaking scheme is verified to be as good as it is advertised to be. And second, higher-order weak
and electromagnetic corrections are of the same order of magnitude in both cases the second-order amplitude is typically $\sim G_\alpha$.

Unfortunately, none of the models which yet exists is sufficiently attractive to be taken
seriously, particularly in the hadronic sector. The calculations I have discussed here will never
theless hopefully be useful in guiding the construction of better models and in elucidating the physi-
cal consequences which follow from them.

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Symposium (Almqvist and Wiksell, Stockholm, 1968)
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The value \( g = 2 \) is the "natural" value for a particle of any spin; in two respects:
1) for \( g = 2 \), the spin-flip Compton amplitude vanishes as \( \omega \to 0 \) faster than \( \omega \), consequently, the Drell-Hearn integral \( \text{[Phys Rev Letters 16, 908 (1966)]} \) vanishes.
2) for \( g = 2 \), the magnetic precession and the Thomas precession of a spinning particle moving in an electromagnetic field exactly cancel, indeed, it is this fact which is used in high-precision determinations of \( g - 2 \) [For details and references, see, e.g., S J Brodsky and J R Primack, Ann Phys (N Y ) 52, 315 (1969), especially pp 360-362]


28. See the review by M Perl, SLAC preprint, July 1972, prepared for the Seminar on the \( \mu - e \) Problem, Moscow

29. For example, in the Georgi-Glashow "5-quark" model, the existence of a substantial \( \phi \bar{N} \) coupling and the absence of \( K^+ \to \pi^+ \eta \) imply just such a constraint (see Ref 18)


31. Earlier data are reviewed by H H Chen and B W Lee, Phys Rev D5, 1874 (1972)

32. The present situation is reviewed in the rapporteur's report of D H Perkins, these proceedings.


34. Note that \( M_Z \to \infty \) for fixed \( M_W \), corresponds to \( R = M_W^2 / M_Z^2 = g^2 / (g^2 + g'^2) \to 0 \), or \( g' \to \infty \)


36. C Bouchiat, J Ilipoulos, and P Meyer, Orsay preprint, August 1972. The authors of Ref 16 concur with the results of this reference (erratum to Ref 16, September 1972)

37. However, J L Gervais and A Neveu, Orsay preprint, 1972, have shown that an abelian broken-gauge-symmetry theory with the Higgs mass equal to the vector mass is effectively obtained as the zero-slope limit of a dual theory


40. This data is reviewed in the rapporteur's talk of C Rubbia. If the new value of \( | \eta_{\pi^0} | \) is used, both the Berkeley and the BNL values are increased about 40%, as noted by Rubbia

41. L M Sehgal, Phys Rev 183, 1514 (1969), C Quigg and J D Jackson, Lawrence Berkeley Laboratory UCRL-18487. See also the recent reviews by H Stern and M K Gaillard, Ann of Phys (N Y ), to be published; A D Dolgov, L B Okun, and V I Zakharov, Usp Fiz Nauk, to be published.
Note Added in Proof: Renormalization of the Weinberg model in the U formalism has also been discussed by A. Sirlin (these Proceedings), by C. G. Bolliini, J. J. Giambiagi, and A. Sirlin (NYU Preprint, November, 1972), and by T. W. Appelquist, J. R. Primack, and H. R. Quinn (Harvard Preprint, November, 1972). The latter paper also completes the calculation of μ decay begun in Ref. 11. D. A. Ross and J. C. Taylor (Oxford Preprints 45/72, 46/72) discuss renormalization and calculate second-order corrections to μ decay in the 't Hooft gauge.

### Table I: Comparison of R and U Formalisms

<table>
<thead>
<tr>
<th></th>
<th>R Formalism</th>
<th>U Formalism</th>
</tr>
</thead>
</table>
| **propagator of massive vector boson** | \[ \frac{g_{\mu
u}}{k^2 - M_W^2} + \text{gauge terms} - \frac{k \cdot k}{k^4} \] | \[ \frac{k \cdot k}{k^2 - M_W^2} - \frac{M_W^2}{k^2} \] |
|                      | good high-k behavior, ghost                       | bad high-k behavior, no ghost                     |
| **scalar fields**    | unphysical scalar fields present                 | only physical components remain, others removed by gauge transformation |
| **Green's functions after renormalization** | non-gauge invariant and "non-unitary" away from mass shell, but finite | divergent away from mass shell, but "unitary" |
| **S-matrix**         | finite gauge-invariant and unitary after cancellation of unphysical contributions | unitary, finite after cancellation of divergences |
| **calculational difficulties** | presence of many Feynman graphs involving unphysical scalars (but these are often negligible since scalar-fermion couplings are \(-\epsilon m/M \) where \( m = \text{fermion mass} \) and \( M = \text{heavy vector mass} \)) | ultraviolet divergences necessitate use of powerful regulation scheme (e.g., 't Hooft and Veltman's) or dispersive calculation |

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Table II. Weak Contributions to Muon Magnetic Moment.

<table>
<thead>
<tr>
<th>Contributions</th>
<th>Result$^a$ for $a = \frac{1}{2}(g_\mu - 2)$</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weinberg SU(2) $\times$ U(1)</td>
<td>$\left( R \equiv \frac{g^2}{g^2 + g'^2} = \cos^2 \theta_W = \frac{M_W^2}{M_Z^2} \right)$</td>
<td>$\frac{10}{3}$</td>
</tr>
<tr>
<td>Lee, Prentki, and Zumino</td>
<td>$\left( R \equiv \frac{g^2}{g^2 + g'^2} = \cos^2 \theta_{LPZ} = \frac{1}{2} \frac{W^2}{M_Z^2} \right)$</td>
<td>$\frac{4}{3}(4R^2 - 6R + 1)$</td>
</tr>
<tr>
<td>Georgi and Glashow O(3)</td>
<td>$\left( M_W = 53 \text{ GeV} \sin \beta; \quad \gamma^+\gamma^0 \quad \text{are heavy muons} \right)$</td>
<td>$\frac{10}{3}$</td>
</tr>
</tbody>
</table>

$^a$The factor $G = m^2_{\mu}/8\sqrt{2}\pi = 1.16 \times 10^{-9}$ multiplies each entry. $F$ and $G$ are functions of order 1 for reasonable values of their arguments:

$$F(x) = 1 + \frac{3}{(1 - x)^2} \left( 1 - 3x - \frac{2x^2}{1 - x} \ln x \right), \quad G(y) = \int_0^1 d\lambda \frac{2\lambda^2 - \lambda^3}{\lambda^2 y - \lambda + 1}.$$
### Neutrino Scattering Processes

<table>
<thead>
<tr>
<th>Model</th>
<th>Effective Interaction$^a$ for $\nu_\mu + e \rightarrow \nu_\mu + e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weinberg SU(2) × U(1)</td>
<td>( \frac{G}{\sqrt{2}} \left( \bar{\nu}<em>\alpha \gamma^\nu (1 - \gamma^5) \nu \right) \left( \bar{e}</em>\alpha \gamma^\nu (1 - 4 \sin^2 \theta_W \gamma^5) e \right) )</td>
</tr>
<tr>
<td>Lee, Prentki, and Zumino SU(2) × U(1)</td>
<td>( \frac{G}{\sqrt{2}} \frac{3\alpha}{4\pi} \left( \frac{M_W}{53 \text{ GeV}} \right)^2 \left[ \bar{\nu}<em>\alpha \gamma^\nu (1 - \gamma^5) \nu \right] \left( \bar{e}</em>\alpha \gamma^\nu (1 - \gamma^5) e \right) + T )</td>
</tr>
<tr>
<td>Georgi and Glashow O(3)</td>
<td>( \frac{G}{\sqrt{2}} \frac{3\alpha}{2\pi} \left[ \bar{\nu}<em>\alpha \gamma^\nu (1 - \gamma^5) \nu \right] \left( \bar{e}</em>\alpha \gamma^\nu e \right) + T )</td>
</tr>
</tbody>
</table>

$^a$ Here \( T = (G/F^2)(\alpha/\pi)\left[ \bar{\nu}_\alpha \gamma^\nu (1 - \gamma^5) \nu \right] \left( \bar{e}_\alpha \gamma^\nu e \right) \left( \frac{2}{3} \ln \left( m_\nu / m_e \right) + F(q^2) \right) \), where \( F(0) = 0 \) \( \left[ F(q^2) \right] \) is given in Ref. 33. The results for the LPZ and Georgi-Glashow models are obtained under the approximation that external momenta and lepton masses are negligible compared to \( M_W \). The effective interaction for $\nu_\mu + p \rightarrow \nu_\mu + p$ is given by the replacement $e \rightarrow p \cos \theta_c$, neglecting hadron structure.

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**Key:**
- lepton \( \ell \)
- vector B
- Higgs scalar \( \phi \)

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**Fig. 1** $W^+W^-$ intermediate-state contribution to fermion-fermion scattering in abelian model, indicating cancellation of divergence in calculating the effective contact term $(\bar{U}_4U_2)(\bar{U}_3U_1)$

**Fig. 2** Some one-loop contributions to the W propagator in the nonabelian model, with the U-formalism degree of divergence indicated.

**Fig. 3** Higgs scalar (\( \phi \)) exchange contributions to $K^0 \rightarrow \ell\bar{\ell}$, $K^+ \rightarrow \pi^+\ell\bar{\ell}$
Fig. 4. The Glashow, Iliopoulos, Maiani mechanism causes these graphs to cancel, except for a contribution proportional to \((m_\nu - m_\nu')_p\).

Fig. 5. Graphs contributing to the effective \((\bar{\mu}\nu)(\bar{\nu}\mu)\) interaction used in computing \(K_L \rightarrow \mu\nu\) in the \(O(3)\) "8-quark" model.
### PARTICLE SEARCHES

<table>
<thead>
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<th>Speaker</th>
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<td>1. Search for Quarks at the ISR (No report provided)</td>
<td>B. Hyams (CERN)</td>
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*This contribution, based on work done jointly with Blankenbecler and Gunion, appears at the end of Strong Interaction Dynamics II, Vol. I.*
The problem of the existence of heavy leptons has been considered in a number of papers.\textsuperscript{1-10} If heavy leptons have masses $M > 1$ GeV and only interact electromagnetically and weakly, then they should be comparatively short-lived ($T \sim 10^{-11}$ sec).\textsuperscript{5,7} However, stable or long-lived heavy leptons may exist in nature. Such a possibility may require the introduction of new quantum numbers other than for $\mu$ and $e$. If the "subleptonic" number is conserved, then the lighter of two particles $\lambda^+$, $\lambda^0$ will be stable. It is quite possible that charged $\lambda$ leptons exist without their own neutrinos. Then they also will be stable. Finally, there may exist stable leptons with an unusual combination of quantum numbers (e.g., lepton charge and an integral spin).\textsuperscript{2}

If subleptonic number is not strictly conserved, then these new particles might be long-lived rather than stable, e.g., if nonconservation of leptonic number takes place because of the same interaction that violates CP invariance, then for a milliweak violation, the lifetime of a heavy lepton with $M$ between 1 and 4 GeV will be $10^{-5} - 10^{-9}$ sec, and for a superweak violation it will be $10^{-5} - 10^{-4}$ sec.\textsuperscript{8}

Heavy leptons may be produced in pairs in electromagnetic interactions in photon beams or in virtual electromagnetic processes which take place in hadron collisions. Using the data obtained by the Lederman group on production of muonic pairs with large invariant masses,\textsuperscript{11} one can estimate the total production cross section for lepton pairs with mass $M_{\lambda}$ in proton-nucleon interactions:

$$\sigma_{\lambda}(M_{\lambda}) = \int \frac{dQ}{dQ} \left[ \frac{1}{2M_{\lambda}} \left( 1 - \frac{4M_{\lambda}^2}{Q^2} \right) \left( 1 + \frac{2M_{\lambda}^2}{Q^2} \right) \right] dQ.$$  \hspace{1cm} (1)

Here $(d\sigma/dQ)_{\pm \pm}$ is the mass spectrum of muonic pairs produced in p-n collisions. The spectrum for proton energy $E_p = 70$ GeV is determined by extrapolation using scale invariance from the experimental data obtained at $E_p = 28.5$ GeV.\textsuperscript{11} $Q$ is the effective mass of the lepton pair and $Q_{\text{max}} = \sqrt{s} - 2M_p$. The calculated results $\sigma_{\lambda}(M_{\lambda})_{\text{theor}}$ for $E_p = 70$ GeV are presented in Fig. 1 and Table I.

Searches for heavy particles with lifetimes between $10^{-7}$ and $10^{-8}$ sec have been made in many experiments at proton accelerators.\textsuperscript{13-21} However, these experiments were not sufficiently sensitive to conclude that heavy, quasi-stable leptons do not exist in some particular mass range. More definite results have been obtained in the experiments at electron accelerators.\textsuperscript{22-25} They conclude that stable and quasi-stable ($T \geq 10^{-8}$ sec) heavy leptons with the masses $M\lambda \leq 4$ GeV do not exist.
In the present experiment, performed at the IHEP 70-GeV accelerator, a search for heavy, charged, quasi-stable leptons was made in the mass range from 1 GeV up to 4-5 GeV. These leptons could be produced in virtual electromagnetic processes when 70-GeV primary protons strike the internal Al target of the accelerator.

A schematic diagram of the experiment is presented in Fig. 2. The experiment used the following basic units: a) a removable filter, located close to the accelerator target. Its thickness (410 g/cm$^2$ of plastic) was chosen as a compromise between the maximum absorption of hadrons in the beam and the minimum loss in the acceptance of the setup caused by multiple scattering in the filter. b) Iron multilayer shielding at the end of the beam with total thickness 6430 g/cm$^2$. c) Scintillation counters $S_1 - S_7$, $M_1 - M_6$ along the beam and guard counters with holes $A_1 - A_3$. d) Gas threshold Cerenkov counters $C_1 - C_4$. e) A hadron detector HD, i.e., a sandwich of scintillation counters alternating with iron plates. Pulse-height analysis of the signals from the counters allowed the separation of leptons from hadronic events with nuclear showers (the rejection factor was $K \geq 15$).

The particles which passed through all counters could be either muons with the energy $> 12$ GeV, or heavy leptons. The gas threshold Cerenkov counters $C_1 - C_4$ were used to suppress the background from muons.

In our experiment the background was caused mainly by accidental coincidences between a slow particle in the beam which stopped in the iron shielding and was not detected by the counters $C_1 - C_3$ and a particle in the muon halo which had counted in counters $M_1 - M_6$. To suppress this accidental background there were many counters, operating in coincidence, much shielding, the guard counters, the hadron detector and the Cerenkov counter $C_4$, which was placed in the middle of the iron filter to protect the tail of the setup from muonic halo. The good time resolution of the multichannel system (±1 nsec), necessary for suppression of the background was obtained by using a multitrace fast scope.

The beam channel consisted of a quadrupole doublet and a deflecting magnet which compensated for the changing accelerator field during each pulse. The acceptance of the beam channel was $\Delta \theta \Delta s \approx 250 \mu \text{sr-GeV/c}$. During the experiment the beam was tuned for 25 GeV/c to optimize the conditions for the heavy-lepton search.

If the first filter was out of the beam, the flux was $1.0 \times 10^8$ particles for $10^{12}$ protons on the target. About 95% of these particles were $\pi$ mesons with a momentum spread of $\Delta p \approx 3$ GeV/c. A plastic filter was used at the beginning of the beam channel during the heavy-lepton search, reducing the rates in the detector by almost two orders of magnitude.

The gas pressure in the Cerenkov counters $C_4 - C_4$ was chosen such that the apparatus was sensitive for all heavy leptons with mass $M_\lambda \approx 1.0$ GeV.

In about 50 hours of accelerator time about $1-16 \cdot 10^9$ particles went through the detector. This corresponds to an effective flux of $n_\pi = 0.89 \cdot 10^{12}$ pions in the solid angle of the detector. In this time, 650 triggers of a preliminary selection system were recorded.

The final selection of the events was performed by analyzing oscillograms and digitally-recorded data. From this analysis it was clear that each detected event did not satisfy at least several criteria for heavy lepton identification. Only two events did not have signals from Cerenkov. 

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counters C on the oscillogram (in fact, in these two cases the signals were very weak), and there was no trigger without at least one of the guard counters A - A on. A very important selection criterion is that signals from the counters as recorded by the oscilloscope be in time with each other as almost all the triggers were due to accidental coincidences. In fact, the limiting factor in the determination of the total cross sections was the accelerator time and not background.

At the final stage of analysis the number of selection criteria was reduced the information from the counters S, the guard counters A - A, and the hadron detector was not used. By doing this, the detection efficiency of the apparatus improved (from $\epsilon = 0.33$ up to $\epsilon = 0.40$), although all the detected events were excluded by the remaining selection criteria.

Thus, no heavy lepton with mass $M_\lambda \geq 1$ GeV passing through the setup was observed. The resulting value for the upper bound of the differential cross section for heavy lepton production in proton-nucleon interactions at 70 GeV is (with 90% confidence level)

$$\frac{d^2\sigma}{d\Omega dp} (p = 25 \text{ GeV/c}, \theta = 0, M_\lambda > 4 \text{ GeV}) |_{90\%} \approx \frac{2.3}{N_\pi} \frac{d^2\sigma}{d\Omega dp} (p = 25 \text{ GeV/c}, \theta = 0),$$

$$1.6 \times 10^{-37} \text{ cm}^2/\text{sr GeV/c}$$

Here the multiplier 2.3 corresponds to a 90% confidence level of the upper-bound estimation and $\epsilon = 0.40$ is the total detection efficiency. The quantity $\epsilon = \epsilon_1 \epsilon_2$, where $\epsilon_1 = 0.47$ represents losses of leptons due to multiple scattering in the first filter, and $\epsilon_2 = 0.85$ is the basic efficiency of the apparatus.

The upper bounds of the total cross sections for heavy-lepton production were obtained using angular and momentum distributions of particles calculated with the proton model of leptonic pair production in nucleon-nucleon collisions (see Fig. 3). The values of the upper bound for the total cross sections are presented in Table I and Fig. 1 (curve 3). A comparison of these upper bounds with the expected values of the total cross sections for heavy-lepton production (curve 1 in Fig. 1) allows one to establish the upper bounds for the heavy-lepton lifetime. The corresponding data are also presented in Table I.

The results of the present experiment show that in nature there are no quasi-stable, heavy-charged leptons with masses from $M_\lambda = 1.0$ GeV (with $T \geq 2 \times 10^{-9}$ sec) up to $M_\lambda = 4.25$ GeV (with $T \geq 4 \times 10^{-8}$ sec) and up to $M_\lambda = 4.5$ GeV (with $T \geq 4 \times 10^{-7}$ sec).

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Table I. Theoretical Estimations of the Cross Sections for Heavy Charged Lepton Production in Nucleon-Nucleon Collisions at $E = 70$ GeV

$\sigma(\mathcal{M}_\lambda)$, the Upper Bounds on the Total Cross Sections for Heavy Lepton Production (at 90% Confidence Level) $\sigma(\mathcal{M}_\lambda)|_{90\%}$ and Upper Bounds of the Lifetime of Heavy Leptons.

<table>
<thead>
<tr>
<th>$M_\lambda$ (GeV)</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.25</th>
<th>4.5</th>
</tr>
</thead>
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<tr>
<td>$\sigma(\mathcal{M}<em>\lambda)</em>{th}$ ($cm^2$)</td>
<td>$3.5 \times 10^{-34}$</td>
<td>$5.7 \times 10^{-35}$</td>
<td>$2.1 \times 10^{-35}$</td>
<td>$9.9 \times 10^{-36}$</td>
<td>$4.4 \times 10^{-36}$</td>
<td>$1.4 \times 10^{-36}$</td>
<td>$2.4 \times 10^{-37}$</td>
<td>$6.8 \times 10^{-38}$</td>
<td>$6.3 \times 10^{-39}$</td>
</tr>
<tr>
<td>$\sigma(\mathcal{M}_\lambda)</td>
<td>_{90%}$ ($cm^2$)</td>
<td>$7.5 \times 10^{-38}$</td>
<td>$7.2 \times 10^{-38}$</td>
<td>$6.9 \times 10^{-38}$</td>
<td>$5.5 \times 10^{-38}$</td>
<td>$4.0 \times 10^{-38}$</td>
<td>$2.7 \times 10^{-38}$</td>
<td>$4.4 \times 10^{-38}$</td>
<td>$9.6 \times 10^{-39}$</td>
</tr>
<tr>
<td>$R = \sigma(\mathcal{M}_\lambda)</td>
<td>_{90%}$</td>
<td>4700</td>
<td>790</td>
<td>300</td>
<td>180</td>
<td>110</td>
<td>52</td>
<td>17</td>
<td>6.0</td>
</tr>
<tr>
<td>$T_\lambda$ (sec)</td>
<td>$1.8 \times 10^{-9}$</td>
<td>$3.4 \times 10^{-9}$</td>
<td>$5.3 \times 10^{-9}$</td>
<td>$7.3 \times 10^{-9}$</td>
<td>$9.8 \times 10^{-9}$</td>
<td>$1.3 \times 10^{-8}$</td>
<td>$2.1 \times 10^{-8}$</td>
<td>$3.6 \times 10^{-8}$</td>
<td>$3.6 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Note: 1. $\sigma(\mathcal{M}_\lambda)_{th}$ calculated with Eq. (1).
2. A full path-length from the target to the end of the setup is $L = 110$ m; the upper bound for lifetime of heavy lepton is determined from the ratio

$$T_\lambda = \frac{L}{\gamma_\lambda \sqrt{2m \rho}} = \frac{367.10^{-9}}{\gamma_\lambda \sqrt{2m \rho}} \text{ sec.} \quad \gamma_\lambda \text{ is Lorentz factor of heavy lepton.}$$
Fig. 1. Total cross sections for heavy-lepton production in pN collisions at the energy $E_p = 70$ GeV:

a) expected cross sections $\sigma_{\lambda}(M_{\lambda})_{\text{theor.}}$ calculated with formula (1) - curve 1;

b) upper bound for total cross sections (90% confidence level $\sigma_{\lambda}(M_{\lambda})_{90\%}$, determined with a parton model - curve 3; for comparison there is given an upper bound for total cross sections $\sigma_{\lambda}(M_{\lambda})_{90\%}$, determined with the phase-space model - curve 2.
Fig. 2. The experiment layout. $D_1$ is a magnet; $K_1 - K_4$ are collimators; $L_1, L_2$ are quadrupole lenses; $S_1 - S_7, M_4 - M_6, A_1 - A_3$ are scintillation counters; $F$ is a monitor; $S_M$ is an additional counter; $C_1 - C_4$ are gas threshold Čerenkov counters; $HD$ is a hadron detector; $\phi$ is a removable filter made of plastic.

Fig. 3. Heavy lepton differential spectra

$$\frac{d^2 \sigma}{dp \Omega}/\sigma_{\text{tot}} \left( \frac{1}{\text{GeV/c} \cdot \text{sr}} \right)$$

$M_\lambda = 1 \, \text{GeV}$

$M_\lambda = 3 \, \text{GeV}$

$M_\lambda = 4 \, \text{GeV}$

$M_\lambda = 4.5 \, \text{GeV}$

Fig. 3. Heavy lepton differential spectra

$$\left[ \frac{d^2 \sigma_\lambda}{dp \Omega}, \sigma_\lambda \right]_{\text{theor}}.$$
The aim of this investigation is a search for muons arising in the decay of hypothetic intermediate $W$ bosons produced in nucleon-nucleon collisions. The first stage was devoted to the study of the energy spectrum of muons produced within the target and not related to pion and kaon decays. Muons with large transverse momenta between 1.7 and 3.3 GeV/c produced in bombarding the internal copper target of the IHEP accelerator by 70-GeV protons were detected.

The layout of the experiment is presented in Fig. 1. At a $9^\circ$ angle to the beam there was a special muon guide made up of units of magnetized iron, where muons were focused and decelerated. The muon guide consisted of focusing lenses (iron cylinders of 20-cm diameter and 75-cm length in which current created axially symmetric magnetic induction) and magnets with closed cores which deflected the beam by $4.5^\circ$. An effective transport of positive or negative muons with the energy from 12 to 25 GeV captured in the space angle $3 \cdot 10^{-3}$ sr took place until a muon stopped. As shown schematically in Fig. 2, between the target and the first focus of the muon guide there was a moveable nuclear absorber (copper blocks, 20 or 80 cm thick) with which one could change the decay path $Z$ of the pions and kaons (distance between target and absorber) from 25 to 85 cm. Muons in the muon guide were selected by the coincidence of two gas Cerenkov counters (4.5 m and 1.3-m long) and a system of scintillation counters between elements of the beam guide. The muon energy $E_\mu$ was calculated by their path in the iron of magnets and lenses.

A special system of monitors and an auxiliary copper target located in the accelerator chamber at a distance approximately 20 m after the main target downstream were used to determine muon contamination not arising in proton interaction with the target. Guiding of the proton beam onto the auxiliary target allowed one to define various background components which are present in measurements with the main target; they also allowed optimizing accelerator operation for minimum background.

Experimentally, the muon fluxes through different sections of the muon guide were measured simultaneously for different decay paths $Z$ at both signs of the magnetic field of the muon guide $N_\mu^+(E_\mu, Z)$. If pion and kaon decays are the only source of muons, the dependence of the quantity $N_\mu^+(E_\mu, Z)$ on $Z$ should be proportional to $(Z + \lambda)$ where $\lambda$ is an effective nuclear absorption length of pions and kaons in copper. Presence of muonic components not related to pion and kaon decays $X_\mu^\pm(E)$ is defined by a nonzero ordinate at $Z = -\lambda$.

The typical results of measurements for decay curves and energy spectra for muons are presented in Fig. 2 and 3. The thickness of the nuclear absorber was chosen to be 80 cm in our measurements. A correction for the background which at $Z = 85$ cm is less than 1% for $\mu^+$ mesons and about 5% for $\mu^-$ mesons was made on the experimental values of $N_\mu^+(E_\mu, Z)$.

In our experiment the value for the effective nuclear absorption length $\lambda(E_\mu)$ for pions and kaons in copper was directly determined by comparing the decay curves measured at two values
of nuclear absorber thickness, 80 cm and 20 cm $\lambda^\mu(E)$ as thus determined are given in Fig 4. For comparison there are also given the nuclear lengths for pion and kaon absorptions obtained from the known experimental values of the cross sections for pion and kaon absorption in copper $^{2,3}$

For further analysis we used experimental values of $\lambda^\mu(E)$ obtained in our experiment.

From Fig 2 it is obvious that in the whole energy range under investigation there is observed a muon $X^\mu(E)$ component, not related to pion and kaon decay, and its intensity corresponds to a differential cross section for production on a nucleon of the order of magnitude $10^{-32}$ to $10^{-36}$ cm$^2$/sr at muon energies from 12 to 24 GeV.

The series of control experiments and calculated estimations to clarify possible obvious causes for a spurious $X^\mu$ component, such as muon multiple scattering in the nuclear absorber, hadron secondary interaction in the walls of the accelerator chamber and in the nuclear absorber, etc. had shown that the studied phenomena could not result in a considerable contribution to the quantity $X^\mu$. Thus, with some uncertainty corresponding to the great experimental difficulties of direct measurement of such background effects, it would be possible to consider the component $X^\mu$ to be generally formed within the target. The dependence of $X^\mu$ on energy is presented in Fig 5.

A possible source of the observable component may be the process of $\mu^+\mu^-$ pair of electromagnetic production or $W$-boson production with further decay in $W \rightarrow \mu\nu$. In the first case the components $X^+ = X^-$ should be equal in the whole energy interval studied, and longitudinal muon polarization should be absent. In the case of $W$-boson production, one should expect an excess of $\mu^+$ mesons, i.e., $X^+ > X^-$, the presence of some bumps in the energy spectrum, and longitudinal positive polarization of $\mu^+$ mesons $^4$.

The calculations in the framework of the phase space model show that in the geometry of our experiment, the energy spectrum of muons in $W \rightarrow \mu\nu$ decay should have a width of several GeV and the maximum of the distribution should be given by $P\sin\theta = M_w/2$, where $P$ is muon momentum, $\theta$ is the muon emission angle in the lab system with respect to initial proton momentum, and $M_w$ is the $W$-boson mass. Theoretical estimates of the differential cross section for muon production in $W$-boson decay, made on the basis of the parton model and a phase-space model with normalization for experimental data on $\mu^+\mu^-$ pair production, $^5$ provide the value $\frac{d\sigma}{dE} \approx 10^{-35} - 10^{-36}$ cm$^2$ GeV$^{-1}$ sr$^{-1}$ in the order of magnitude for the studied range of mass $M_w$.

From the quoted experimental data, one can conclude that the $X^\mu(E)$ component seems to be connected mainly with electromagnetic production of muonic pairs. There were no obvious indications to the existence of $W$ boson. Proceeding from the above results, upper limits on the cross section for $W$ production were obtained for the mass interval from 3 to 10 GeV/c and are presented in Fig 6.

A final conclusion of the nature of the observed component $X^\mu(E)$ can be made on the basis of measurements of the sign and degree of muon polarization. We hope to measure polarization by studying the asymmetry of $(\mu e)$ decays with stopping muons in the 100x120x800-cm streamer chamber containing 60 polyethylene plates with total thickness 400 g/cm$^2$.
Fig. 1. The experimental layout: L-lenses; T1-T2-target; M1-M4-monitors; BM-beam bending magnets; C1-C2-Cerenkov counters; S1-S12-scintillation counters; MA-accelerator magnets; P-proton beam. In the inset, the arrow indicates the direction of electric current; letters \( B_L(R) \) and \( R \) denote tangential magnetic induction in the lens body and its radius respectively. The graph shows the linear radial dependence of \( B_L(R) \).
Fig. 2. Experimental dependence of the muon flux on the decay length for three different muon energies. The straight lines are drawn by the least-square method. The measurement errors at $E_{\mu} > 12.3$ GeV and $E_{\mu} > 17.25$ GeV do not exceed the size of the experimental points. The straight lines, corresponding to different muon energies, are displaced in the ordinate axis.
Fig. 3. The energy (range) spectra of muons detected in the muon guide when magnetic induction in the beam-bending magnets is $B_{BM} = 14$ kG and the magnetic induction of the lens periphery is $B_L(R_0) = 14$ kG.
Fig. 4. The measured effective length of pion and kaon nuclear absorption in copper. Solid lines illustrate dependence of effective length or nuclear absorption on energy from the results of Refs. 2 and 3.
Fig. 5. The energy spectrum $X^\pm(E)$ normalized to positive muon flux at $Z = 85$ cm.
Fig. 6. Experimental limits of the differential cross section for W-boson production depending on its mass. Dashed area corresponds to uncertainties of estimations.
SEARCH FOR MAGNETIC MONOPOLES AT THE 70-GeV IPHE PROTON SYNCHROTRON (#903)

Presented by V. Lebedev
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Moscow, USSR

The result of a search for magnetic-charged particles at the 70-GeV IPHE proton accelerator is presented in this report. Using the "ferromagnetic trap" method, the upper limit of the magnetic monopole production cross section in proton-nucleon collisions \( \sigma(95\%) \leq 1.4 \times 10^{-43} \text{ cm}^2 \) was found.

With the 70-GeV IPHE proton accelerator, monopoles with masses up to five proton masses can be produced on a hydrogen target. Figure 1 presents the layout of the present experiment based on the monopole ability to be accumulated in ferromagnetic samples (4) which were put into the gap between the vacuum chamber (2) and the magnet pole (1). The total length of the samples was about 480 mm. Since the target was situated inside the accelerator magnet \( (H = 12000 \text{ G}) \) only monopoles arrived at the samples.

The calculations\(^2,3\) show that in the present experiment monopoles with the charge of \( g \geq 51.4e \) should be slowed down in the ferromagnetic foils to thermal velocity. It is necessary to apply a magnetic field substantially exceeding saturation induction in order to remove the monopole from the ferromagnetic samples. For example, for Permendur \( H \approx 44 \text{ kG} \) when \( g = 68.5e \).

The experiment was performed in two stages. At the first stage the collection of the monopoles was carried out in ferromagnetic foils. At the second stage the extraction of the monopoles from ferromagnetic traps was done with the help of a high pulsed magnetic field. The nuclear emulsion \( Br_2 \) of 400\(\mu\) thickness was used as a detector. These operations were separated in space and time. Thus it was possible to perform a long-term exposure of samples in the accelerator without high background arising in the photoemulsion.

About 35% of exposed ferromagnetic foils underwent the action of a 300-kG magnetic field in the impulse magnet.\(^4\) In Fig. 2 a schematic drawing of the detection system is presented. The ferromagnetic foils were placed at the edge of the magnet at a distance of 13 cm from the center. At the opposite edge of the magnet two transverse layers of emulsion 80 mm in diameter were installed. Just behind it the emulsion-chamber of dimensions 55x45x30 mm was placed. The monopoles would lose the energy obtained from acceleration by the 26-cm path in the magnet between 1 to 3 cm of nuclear emulsion.

In addition, to check the possibility of an anomalous interaction of the monopoles with material, a part of the exposed foils were placed in \( \approx 800 \text{ kG} \) pulsed magnetic field. During the scanning of emulsion layers no tracks were found which crossed both layers with ionization noticeably greater than that of a relativistic proton. The result of our experiment is presented in Fig. 3 by curve E.
The "ferromagnetic trap" method permits a search for magnetic monopoles at the Batavia accelerator with mass up to 10 proton masses for $g \geq 68.5e$ and with mass up to 13 proton masses for $g \geq 137e$.

The authors would like to thank Professor V M Galitsky for participation in the analysis of the monopole interaction with media, Professor A A. Logunov, A A. Naumov, and Yu. D Prokoshkin for help in organization of the experiment, A. A Zhuravlev, A P Kurov, and the staff of IPHE and also V. I Nikitkin, A. V Telnov, A M Guschin, V D. Riabov, E. S Bar'mova, E O Shliapnikova, I. S Pisanko, V P. Smakh'on, and V. S. Hlestov for help in this experiment.

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Fig. 1. Layout of apparatus: 1-accelerator magnet pole; 2-accelerator vacuum chamber; 3-target; 4-detector samples; 5-tungsten plate; 6-copper foil; 7-permalloy 50 H foil; 8-permalloy 79 HM foil; 9-Permendur foil.
Fig. 2. The detection system.

Fig. 3. Results of monopole searches: A - search for the monopole in the earth atmosphere; B - search for the monopole in the magnetic ocean minerals; C - search for the monopole in the lunar matter; D - experiments on the 30-GeV accelerator; E - results of the present work.
PRELIMINARY EXPERIMENTAL RESULTS ON A SEARCH FOR THE DIRAC MONOPOLE AT THE 70 GeV IHEP SYNCHROTRON USING VAVILOV-CERENKOV RADIATION (#877)

Presented by Yu. A. Shcherbakov
Joint Institute for Nuclear Research
Dubna, USSR

The hypothesis on the possible existence of monopoles with the magnetic charge $g = 68.5 \, \text{e}$ advanced by Dirac has been subjected to practically continuous experimentation. So far, these experiments have yielded negative results. We consider the pair production of monopole-antimonopole $(g, \overline{g})$ with masses up to $5.5 \, \text{MeV}$ and velocities above the emission threshold of Vavilov-Cerenkov radiation in a transparent medium with $n = 1.5$, which could occur at the 70-GeV Serpukhov proton synchrotron. The use of Vavilov-Cerenkov radiation facilitates the search for magnetic charges among the large number of electrically charged particles because of the following properties:

1. The ratio of the Vavilov-Cerenkov radiation intensities (in a medium with refractive index $n = \sqrt{\varepsilon}$) from a fast monopole with magnetic charge $g$ and a particle with electric charge, at the same particle velocity, is given by $W_g/W_e = g^2 \varepsilon/e^2$. At $g = 68.5 \, \text{e}$ ($m = 1$), $\sqrt{\varepsilon} = 1.5$, $W_g = 10^4 W_e$.

2. As Frank first noted, the polarization of Vavilov-Cerenkov radiation from the $g$ and $e$ charges differs in rotation of the electric vector field by $90^\circ$.

The distinctive feature of our experiment resides in the fact that the detection of monopoles is done with the help of Vavilov-Cerenkov radiation immediately after their appearance in the combined target radiator. This allows one to search for even unstable magnetic charges with lifetime $T_g > 10^{-10}$ sec. Based on the above-mentioned criteria, the experimental device for a Dirac monopole search has been designed and placed in the internal beam of the Serpukhov PS. The schematic drawing of this device is shown in Fig. 1.

The 70-GeV proton beam is slowly directed to the conical target radiator (see Fig. 2) fabricated from radiation resistant quartz ("Herasil I"). The Vavilov-Cerenkov radiation generated in the target is ejected from the vacuum chamber of our experimental apparatus through eight windows. Each window is connected to the so-called "optical sleeve" which consists of the lens, polaroid sheets, and a system of plane and conical mirrors. Each optical sleeve extracts a part of the radiation cone (within $\pm 4^\circ$ in azimuth) which is focused and collected on the photocathode of 58AVP photomultiplier tubes located at an angle of $100^\circ$ to the direction of proton beam. The polaroids are oriented so that in six "sleeves" the radiation has to be emitted from the magnetic charges, and in two others from the charged particles. To separate the Vavilov-Cerenkov radiation from the primary proton beam, a screen of 6.5-cm width is installed in the focal plane of each sleeve. The monitoring of the incident proton-beam intensity on the target was performed by two monitors (of three scintillation counters each, with $7 \times 7 \times 30\,\text{mm}$ crystals) which record the particle scattering at $90^\circ$ to the 70-GeV proton beam.
The electronics block diagram is shown in Fig. 3. The amplitude analysis obtained from the latter dynodes of all eight photomultipliers has been performed with the help of two five-beam oscillographs which were triggered after a six-fold coincidence of the pulses with amplitude above 250 photoelectrons from the 58AVP photocathodes. The monopole production events might be followed by the simultaneous appearance of large pulses in six sleeves (with I-shaped polaroids) and the absence of those in two other sleeves (with "II"-shaped polaroids).

The apparatus efficiency for fast monopole detection (\( g \) and \( \overline{g} \)) was calculated by the Monte-Carlo method taking account of the kinematics for \( P + N \rightarrow P' + N' + g + \overline{g} \), the internuclear motion inside the \( \text{SiO}_2 \) target, as well as the apparatus design features and the shape of the 70-GeV proton beam at the target. For a magnetic charge with \( g = 68.5e \) and mass from \( 3 \text{ M}_p \) to \( 5 \text{ M}_p \), the detection efficiency ranged from \( \eta = 0.1 \) to 0.2. From the multiplicity \( k \) produced by 70-GeV protons incident on the \( 9 \text{ g/cm}^2 \) \( \text{SiO}_2 \) target-radiator, \( \langle k \rangle = 8.3 \) calculated and obtained from the experiment, the integrated proton flux was calculated to be \( \sim 3.5 \times 10^{15} \).

In processing the data from the five-beam oscillographs it has been found that not a single event satisfying the above-mentioned criteria of \( g \) detection was observed. This gives an upper limit of the monopole production cross section, by 70-GeV protons on \( \text{SiO}_2 \) target nuclei, of \( \sigma(g, \overline{g}) < 8 \times 10^{-40} \text{ cm}^2 \) (95%) for \( m \) between 4 and 5 \( \text{ M}_p \), and \( \sigma(g, \overline{g}) < 1.6 \times 10^{-39} \text{ cm}^2 \) (95%) for \( m = 3 \text{ M}_p \) respectively. Our experiment is still in progress.

References

4. M. Frank, In Memory of S. I. Vavilov (Moscow, 1952), p 193
Fig. 1. Schematic view of the experimental device: 1 - photomultiplier 58AVP (the diameter of the photocathode is 110 mm); 2 - conical mirrors; 3 - the screen; 4 - plane mirrors; 5 - film polaroids; 6 - the collecting lens with \( f = 150 \) cm; 7 - transparent glass; 8 and 13 - the connecting tubes; 9 - the light-proof housing of the "optical sleeve"; 10 - photomultiplier magnetic shielding; 11 - vacuum chamber of the experimental device; 12 - the fused silicon target-radiator; 14 - titanium pumps; 15 - pumping system; 16 - lead shielding; 17 - lock for target-radiator lead-in.

Fig. 2. The conical fused silicon target-radiator: 1 - 70-GeV proton beam; 2 - plane blacked face; 3 - Vavilov-Cerenkov radiation.
Fig. 3. Electronics block diagram: 1 - $A_4$, $D_4 + A_8$, $D_8$ anodes, last dynodes of the photomultipliers; 2 - $d_1 + d_8$ discriminators; 3 - $DL_1$, $DL_2$ delay lines; 4 - $CC_1 + CC_2$ coincidence circuits (with $T = 5 \times 10^{-9}$ sec and $T = 10^{-8}$ sec); 5 - 5RO (1 and 2) five-beam oscillographs; 6 - 5Scalers; 7 - DP digital printer; 8 - $C_{123}$, $C_{678}$, $C_{123678}$ trigger pulses from coincidence circuits.
We will describe results on the distribution of large transverse momenta of particles produced at 90° in pp collisions at the CERN intersecting storage ring, for center-of-mass energies between 23.2 and 52.4 GeV.

The transverse momentum distributions will be given for:
1. all particles up to 3 GeV/c
2. γ rays between 0.6 and 2 GeV/c
3. pions above 3 GeV/c transverse momentum

I. Experimental Arrangement

The experimental setup is shown in Fig. 1. It is conceived for the detection of charged particles at 90° of the intersecting beams. A set of magnetostrictive wire spark chambers in front and in the back of a magnet determines the deflection of charged particles in the magnetic field. The magnet aperture is 150 × 40 cm². The field integral is 300 kG·cm. The trigger is defined by a coincidence between the three counter hodoscopes H₁, H₂, and H₃ at 20 cm, 115 cm, and 328 cm from the intersecting beams, respectively. These hodoscopes consist of 6, 10, and 18 scintillation counters whose size are 10 × 10 cm², 10 × 20 cm², and 10 × 65 cm², respectively. The particles are identified by their momentum and time of flight. The time of flight is taken between H₁ and H₂. A second time of flight between H₂ and H₃ is used to check that the velocity measurement is not taken between uncorrelated pulses. The mean solid angle of the apparatus is 85 milli-steradians.

The selection of high-speed particles is obtained by a Čerenkov counter located inside the magnet. This consists of 8 elliptical mirrors. The Čerenkov design is shown in Fig. 2. The image of the intersection region of the two beams is formed on eight phototubes corresponding to the eight mirrors. Each mirror sees the intersecting region in a certain angular range and therefore provides a correspondence between the track direction and the mirrors. At each trigger, the location of the phototubes that triggered and the wire-chamber information are written on tape and provide a very effective tool to reject wrong triggers in the analysis.

Determination of the Electron and High Transverse-Momentum Pion Spectra

In this case, the Čerenkov signal is required in the trigger. The Čerenkov was usually filled with isobutane and the threshold for pion detection was 2.7 GeV/c. The major source of electrons is the converted gamma rays in the vacuum pipe. The pair electrons are separated by the magnetic field. If the two electrons have enough energy so that they are not completely swept by the magnetic field but emerge through the magnet, the gamma ray is easily identified and their momenta are determined. For low-momentum gamma rays, only one of the two electron pair emerges after the magnet. In this case, pulse-height analysis in the hodoscope counters H₁ and
$H_2$ permits the discrimination between one and two particles in the same scintillation counter. Therefore it provides a separation between converted gamma rays and single-charged particles. Figure 3 shows the pulse-height spectrum obtained. Sometimes the Cerenkov counter is triggered by a gamma ray converted inside whereas the particle that traversed the apparatus was a pion.

The first rejection of this type of event is to check if the right Cerenkov mirror has been hit. There remains an ambiguity if the pion is in the vicinity of the converted gamma ray. In this case the electrons are recognized by pulse height in a shower counter. This shower counter is formed by a sandwich of scintillators and lead. There is a total of 2.5 radiation lengths. In front of this counter there are 3 radiation lengths of shower chambers. The mean pulse height of a 1 GeV/c electron is five times bigger than a minimum ionizing particle. The number of expected electrons being small, optical shower chambers are triggered if the pulse height in the sandwich counters goes beyond 5 times minimum ionization and will allow a visualization of the shower development. The converted gamma rays have been used to calibrate the shower-counter response.

Figure 4 shows the particle momentum spectra in such type of runs. The fast falloff below 1 GeV/c corresponds to converted gamma-ray electrons. At 3 GeV/c the rise in particle yield is due to pions above the Cerenkov threshold. As explained previously, a certain number of triggers come from gamma rays converted inside the Cerenkov whereas the particle that traversed the apparatus was a pion below threshold. These events are recognized by the fact that the wrong mirror has been hit. The dashed curve represents the particle spectrum when the wrong mirror has been hit. We have verified that this particle presents a minimum ionization peak in the shower counter, and the amplitude in $H_1$ and $H_2$ corresponds to a single particle. Using this spectrum, it is possible to interpolate how many wrong triggers are under the right mirror hit. The result is curve 3 in Fig. 4. It is to be emphasized that above 3 GeV/c no observed event had a wrong mirror hit and therefore all particles above 3 GeV/c are pions that triggered the Cerenkov because at this momentum we are above the Cerenkov threshold.

Data Reduction

The horizontal projection of a particle track is obtained with four chambers in front of the magnet and four chambers behind. A reconstructed track must have at least three aligned sparks on each side of the magnet. Furthermore, the trajectory continuity of the particles requires the two track segments to cross in the vicinity of the magnet center. Four chambers in front of the magnet and two behind determine the vertical projection. In this projection at least four aligned sparks are required with a minimum of one spark in the rear. Four chambers with an angle of 20° with respect to the horizontal plane allow the horizontal and vertical association in case of multitrack events. The fraction of multitrack events in this experiment is 1.5%. Advantage is taken of the spark-chamber redundancy to measure their efficiency by using the events themselves to determine the track reconstruction efficiency. In order to correct a possible bias in the momentum distribution, this efficiency is measured as a function of momentum. Above 200 MeV/c the track reconstruction efficiency was found to be better than 97%. In the majority of the runs, it was better than 99%. In the momentum range of this experiment, the resolution $\Delta p/p$ is 5% and the uncertainty is essentially due to multiple scattering in the different parts of the apparatus.

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To see if events thus identified originate from true beam-beam interactions, the reconstructed track is projected back to the beam-intersecting region. In this way, horizontal and vertical profiles are obtained (Fig 5), which permit a separation between beam-beam and beam-gas or beam-wall interactions. The particle momentum distribution in the vicinity of the beam-beam interaction region allows background correction.

II Results

Transverse Momentum Spectra Above 1.2 GeV/c

The first approach is to determine the transverse momentum spectra of all particles up to 3 GeV/c. Up to 1.2 GeV/c, the major part of particle productions is due to pion production. Above 1.2 GeV/c, the invariant cross section \(\frac{\left(E/p^2\right)d^2\sigma}{dp d\Omega}\), can be approximated by \(\frac{1}{p}d\sigma/dp d\Omega\). The result is shown in Fig 6 for positively charged particles, and in Fig 7 for negatives. The remarkable feature is a considerable slowdown of the exponential decrease. This was already apparent just for pion production below 1.2 GeV/c because an exponential of the form \(A e^{-Bp_T + C p_T^2}\) gave a better fit to the data with \(C\) positive. It could still be objected that this slowdown is due to protons and antiprotons alone, since the slope below 1.2 GeV/c for protons and antiprotons is smaller than the slope of 6 (GeV/c)^{-1} for pions. But the data taken with the Cerenkov in the trigger permit to measure the cross section above 3 GeV/c transverse momentum for pions alone. These data are shown in Fig 8. The value of the invariant cross section at 3.5 GeV/c is a factor 200 bigger than the one obtained by the extrapolation of the pion data in the form \(A e^{-Bp_T}\) between 0.2 and 0.8 GeV/c. We observed 73 events in \(\pi^+\) and 68 events in \(\pi^-\) above 3 GeV/c. To determine the slope, these events have been added. Figure 8 shows the result. Because there is a threshold effect, the data have been corrected by the Cerenkov efficiency at threshold. To avoid uncertainty in the slope determination, only events above 3.5 GeV/c transverse momentum have been retained. The slope between 3.5 and 5 GeV/c transverse momentum is 2.5 ± 0.4 (GeV/c)^{-1} in sharp contrast to 6 (GeV/c)^{-1}.

The gamma rays converted in the vacuum pipe have been also detected and gave us the opportunity to measure their spectrum. The gamma-ray spectrum is obtained up to 2 GeV/c. Assuming that the majority of gamma rays are originated from \(\pi^0\), we can calculate the \(\pi^0\) transverse momentum distribution using the formula

\[
\frac{dN_{\pi^0}}{dp_T^2} = \frac{1}{4} \frac{d}{dp_T} \left( \frac{dN_Y}{dp_T} \right)
\]

Figure 9 shows the gamma rays and the calculated \(\pi^0\) spectrum. The \(\pi^0\) spectrum is shown also in Fig. 10 for comparison with charged pion spectrum. This \(\pi^0\) spectrum confirms the slowdown in the exponential decrease in the pion transverse momentum distribution.
Fig. 1. Experimental layout.
Fig. 2. Design of Cerenkov Counter.
Fig. 3. Pulse-height spectrum in hodoscope counters.
Fig. 4. Momentum spectra of detected particles.
Fig. 5(a). Horizontal beam profile.
Fig. 5(b). Vertical beam profile.
Fig. 6. Invariant cross section for positive particles at several cm energies.
Fig. 7. Invariant cross section for negative particles at several cm energies.
Fig. 8. Decreased slope for high transverse momentum pions.

\[ \pi^+ + \pi^- \]

\[ \sqrt{s} = 44.4 \text{ GeV} \]

\[ x = 0 \]
Fig. 9. Observed gamma-ray spectrum and deduced neutral pion spectrum.
Fig. 10. Neutral and charged pions spectra compared.
OBSERVATION OF MUONS AT LARGE ANGLES FROM CERN ISR

Presented by G. MANNING


*** University of Bristol
** University College London
* Rutherford High Energy Laboratory
† University of Hawaii
†† University of California

1) Introduction

A search is being carried out for muons with large transverse momenta produced in colliding beam interactions at the CEFN ISR. Such observations could provide evidence for heavy virtual photon production with subsequent muon pair production, or possibly could indicate the existence of the Intermediate Vector Boson (W) or heavy leptons decaying into a muonic mode.

Other sources of high momentum muons are expected as backgrounds. First, cosmic rays will pass through the detector during the experiment and some fraction of these may trigger the equipment. Secondly, pions and kaons will be produced in the region of the intersection and a certain fraction of these will decay giving rise to muons which then enter the detector.
2) **The Detector**

The detector consists of a large array of magnetized steel plates which are interleaved with optical spark chambers and scintillation counters (see Figs. 1 and 2). Four counter planes (S, B, C and D) are in coincidence to define a highly penetrating particle with a transverse momentum of at least 1.6 GeV/c. A muon must have at least 2.1 GeV/c to traverse the complete detection system.

The magnetic field in the steel plates has a mean value of about 15kG and has the effect of focussing positive particles towards the centre of the equipment. Monte Carlo studies have been carried out to determine the acceptance of the equipment for positive and negative muons (see Fig. 3). The same studies suggest that a resolution of ±15% is obtainable.

In order to reduce the cosmic ray contribution to triggers, a ring of directional perspex Cerenkov counters has been set up on the opposite side of the intersection region (see Fig. 2). These counters will veto forward downward-going cosmic rays whilst passing any particles moving out from the intersection region. Backward-going cosmic rays are rejected by time-of-flight measurements in the scintillation counter banks. In addition, sets of magnetostrictive wire spark chambers have been placed on either side of the intersection region so that any remaining cosmic ray tracks can be removed by rejecting
events in which a co-linear track can be formed from the two sets of wire chambers.

A lead absorber (see Figs. 1 and 2) has been inserted in front of the muon detector to reduce strongly interacting particles. This absorber may be moved towards and away from the intersection so that the available decay path for muons can be varied.

3. Data Taking

The provisional results presented in this report are based on a sample of the data in which triggers were only allowed from the top half of the equipment. This permitted rejection of cosmic ray events in the trigger using time of flight alone - the Cherenkov counters are required to reject cosmic ray triggers from the lower half of the equipment. Data from the optical spark chambers was recorded on 70 mm film and has been measured on an HPD. Data from the magnetostrictive chambers was recorded on magnetic tape via a PDP8 computer, and the two records were later combined.

The analysis discussed below is based on the following types of runs:

i) beam x beam. Absorber Near (43 cm from intersection point)

ii) beam x beam. Absorber Back (65 cm from intersection point)

iii) one ISR beam only.

iv) cosmic rays (with ISR off).

v) cosmic rays with S counters fed by a pulse generator.
(This last run was included as it was realised that it is possible to have triggers originating from a random (but probably machine induced) count on the S counters (see Fig. 1) together with a genuine cosmic ray anywhere through the counter system BCD).

Results are reported from data taken at 26 x 26 GeV.

4) Results and Analysis

Data has been analysed from runs for a total live time of \(\sim 11\) hours at ISR currents of about 5 amp on 6 amps. The film has been visually scanned for events with tracks. The HPD digitisings have been automatically track followed and the momentum determined by fitting to the observed track.*

The tracks are extrapolated back to a vertical plane through the intersection region and events have been selected which come from the region of the intersection-liberal cuts of \(\pm 40\) cm horizontally and \(\pm 30\) cm vertically have been made. Events are also required to have a root mean square error \(< 1.0\) cm in the momentum fitting procedure.

Identical selection criteria are used for the cosmic ray data, random S data and single beam data.

Table 1 shows the data for absorber near and absorber back together with the corrections applied for background. A
differential cross section averaged over the detector is given but it should be noted that the distribution is not isotropic within the detector.

It will be noted in Table I that the number of cosmic ray selected events (of both types) to be expected during the 26 GeV data taking was $3.9 \pm 1$. In fact, during the run, with the aid of the magnetostrictive chambers 4 events were classified as of cosmic origin.

It can also be seen that the ratio of cross sections with the absorber near and back is $2.1 \pm 0.4$ and this is consistent with a simple estimate of the ratio expected from decay muons from $\pi$'s and $K$'s ($2.0 \pm 0.3$).

Figure 4 shows the momentum spectrum of accepted events from 2 to 6 GeV/c. Events with momenta greater than 6 GeV/c have been grouped together.

\[
\int_{6}^{\infty} \frac{d^2\sigma}{d\Omega dp} dp = (9.0 \pm 3.0) \times 10^{-33} \text{ cm}^2/\text{sr. for absorber back}
\]

\[
= (6.6 \pm 1.4) \times 10^{-33} \text{ cm}^2/\text{sr. for absorber near}
\]

These events, 6.7% of the total tracks, have been rescanned by physicists. Half of them behave as one would expect for high energy muons. The rest seem to be straight tracks, compatible with high momentum, but they stop within the detector. For muons this requires a momentum less than 2 GeV/c.
Until the origin of the "stopping" particles is understood, it is not possible to identify any events as muons. A few particles penetrate to the back of the detector traversing six 10 cm thick iron plates and twelve spark chambers after the last trigger counter without visibly interacting. An upper limit can be quoted for the production of muons of momenta $>6$ GeV/c

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{muons with } p > 6 \text{ GeV/c}} \leq 4 \times 10^{-33} \text{ cm}^2/\text{sr.}$$

It is not possible to account for this upper limit in terms of $\pi$ decay; but it is possible that there are enough kaons to do so - the kaon yield has not been measured at large momentum.

Possible explanations of the high momentum "stopping" tracks are:

a) The rear spark chambers are inefficient. We do not believe this as the measured efficiency is greater than 98% and all chambers are driven in parallel.

b) The particle is a muon of low momentum ($< 2$ GeV/c) but multiple scattering makes it appear straight. This cannot be completely ruled out until further checks have been made but it is considered very unlikely.

c) The particles are a residue of strongly interacting particles that have penetrated the 50 cm of lead, 70 cm of iron and 8 cm of aluminum (for normal incidence) as charged particles to satisfy the trigger requirement. They then interact with their normal interaction probability. An approximate calculation of the penetration, allowing for
the typical angle of the particle to the plates, gives an attenuation of \( \approx 3300 \). The observed rate then requires 
\[
\frac{d\sigma}{d\Omega} \sim 10^{-29} \text{ cm}^2
\]
for all strongly interacting particles with \( p > 6 \text{ GeV/c} \) averaged over the acceptance of our detector.

At 90° for all charged particles \( \frac{d^2\sigma}{d\Omega dp} \sim 6 \times 10^{-31} \text{ cm}^2/\text{sr} \)
GeV/c for \( p_T = 4 \text{ GeV/c} \) and it hardly falls by \( p_T = 5 \text{ GeV/c} \)
and hence integration could give a sufficient cross section.

It is surprising that energetic hadrons do not give a visible star on interacting but the event is only observed at intervals corresponding to 10 cm thick iron plates. Two events have been seen with a large angle kink in the track in the last spark chamber.

d) The particles are weakly interacting and the charged decay products have a short range while most of the energy is carried off by a neutral particle. A lifetime in the laboratory of \( \sim 10^{-9} \text{ sec.} \) is required and only \( \sim 1/3000 \) will survive to counter D and satisfy the trigger requirement.

The production cross section required would therefore be \( \sim 10^{-29} \text{ cm}^2/\text{sr} \) which makes this explanation seem unlikely.

e) The particles are very massive and hence have a short range but high momentum - masses of several GeV are required. If the stopped particle had a lifetime greater than the sensitive time of the spark chambers its decay would not be observed.

The evidence is at the moment preliminary, the cross sections are uncertain, the results are interesting and are at the
moment no more than encouraging.

FOOTNOTE AND REFERENCES

*Only 40% of the tracks are found and fitted by the automatic system. Those events found and lost are compared using information taken at the scanning stage (approximate direction and stopping point) and this indicates that there is no strong bias in the selection and hence the cross sections quoted are corrected for this loss. For rare events this could mean an overestimate of the cross section by a factor of 2.5 if all rare events are found or an underestimate if rare events are preferentially lost.


**TABLE 1 Results at 26 GeV/c**

<table>
<thead>
<tr>
<th></th>
<th>Absorber near (43 cm from intersection point)</th>
<th>Absorber back (65 cm from intersection point)</th>
<th>Luminosity x time</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>visual tracks</td>
<td>fitted events</td>
<td>visual tracks</td>
</tr>
<tr>
<td><strong>Signal + background</strong></td>
<td>1063 ± 33</td>
<td>623 ± 41</td>
<td>439 ± 21</td>
</tr>
<tr>
<td><strong>Cosmic rays</strong></td>
<td>10.4 ± 1.3</td>
<td>3.0 ± 0.9</td>
<td>3.2 ± 0.4</td>
</tr>
<tr>
<td><strong>Random S</strong></td>
<td>85.5 ± 4.4</td>
<td>8.2 ± 1.6</td>
<td>27 ± 1.4</td>
</tr>
<tr>
<td><strong>Single beam</strong></td>
<td>309 ± 63</td>
<td>0 ± 78</td>
<td>122 ± 0.4</td>
</tr>
<tr>
<td><strong>Signal</strong></td>
<td>658 ± 89</td>
<td>612 ± 88</td>
<td>287 ± 39</td>
</tr>
<tr>
<td>$\frac{d^2}{d\Omega}$</td>
<td>$(7.4 \pm 1) \times 10^{-32}$</td>
<td>$(6.8 \pm 1) \times 10^{-32}$</td>
<td>$(15 \pm 2) \times 10^{-32}$</td>
</tr>
</tbody>
</table>

**Note**: No momentum selection has been made on the fitted events.
SCINTILLATION COUNTERS

CERENKOV COUNTERS
MAGNETOSTRICTIVE SPARK CHAMBERS
OPTICAL SPARK CHAMBERS.

Fig. 1. Muon detector, plan view.
Fig. 2. Muon detector, side el on.
Fig. 3. Acceptance as a function of momentum.
Fig. 4. Momentum spectrum of accepted events.
Searching for tachyon monopoles is like betting on the daily double: there is only a small chance for either horse to win. Yet the very fact that neither faster-than-light particles nor magnetic monopoles has been seen suggests that we may not be looking for them in the right way. We start by observing that the particles which travel slower than light are all electrically charged or neutral. Those which travel at the velocity of light are neutral. We speculate that particles traveling faster than light will be neutral or magnetically charged. We assume that this charge will be near that of the Dirac monopole. We do not know of any theoretical problems which this model explains. However, some years ago Leonard Parker proposed that electrons and protons moving with infinite velocity would appear magnetically charged to a stationary observer.

If you wish to catch a strange beast, you have to make some assumptions about its lair and habits. Unfortunately, we have had to make a number of assumptions. These are documented in a forthcoming paper. Let us concentrate on a single important assumption.

Since the tachyon monopole (TM) moves faster than light, it should emit Cerenkov radiation. We detect the TM by this radiation. You recall that Cerenkov radiation is predominantly blue. This is a consequence of the fact that the energy radiated per unit frequency varies like $\omega^2$. The total energy radiated is given by

$$\frac{dE}{ds} = Z^2 \int_0^\infty \omega d\omega,$$

where $Z$ is the charge of the TM in units of the Dirac monopole.

How can an ultraviolet catastrophe be avoided? Two cutoffs have been proposed:

1. The TM has a diameter comparable to that of a proton and consequently cannot emit light of wavelength shorter than a Fermi or

2. The TM cannot emit a photon having energy greater than it itself possesses.

In the former case the energy of the TM becomes negative in a microscopic distance from the source. In the latter case, the energy approaches zero asymptotically.

* Work performed under auspices of the United States Atomic Energy Commission
Ultimately nature will tell us which, if either, of these cutoffs is correct. For the moment we assume that the energy of the TM approaches but does not go below zero. Even then, the tachyon’s energy is reduced to an energy of an electron volt in a distance of only $10^{-6}$ cm from its point of production.

To raise the tachyon’s energy to a level where it can emit visible photons, we place a longitudinal magnetic field in the path of the tachyon. The tachyon takes magnetostatic energy from this field and promptly emits it as light. The energy of the tachyon rises to a stationary value $E_0$ at which the rate of gaining energy from the magnetic field is exactly equal to the rate of energy lost to Cerenkov radiation. This process is illustrated by the following equations:

$$ E_0 = \frac{h}{c} + ZgH $$

$$. E_0 = \frac{hc}{Zg} = 3eV, if Z = 1, g (emu) = 69e (esu) and H = 400 Oe. $$

The universal curve below summarizes the spectrum for Cerenkov radiation. Since in any experiment the magnetic field is fixed it is convenient to view the charge of the TM as a free parameter. The minimum detectable charge $Z_{\text{min}}$ is determined by the condition that the TM emit enough photons in the visible part of the spectrum to be detectable. The maximum charge $Z_{\text{max}}$ is determined by the condition that the energy of the TM, $(E_0)$, must be greater than the energy of a visible photon.

We know of no theory which predicts a likely source for tachyons. We looked for TM’s produced in the lead shield of a 20,000 Ci $^{60}$Co source. The geometry of the experiment is shown in Fig. 1.
We did not see any TM's. Some previous monopole searches can be viewed as searches for tachyon monopoles. Particularly relevant are those searches where the monopole was collected in flight by a solenoid and accelerated into a detector. If the "fringing" magnetic field at the detector were high enough, a TM would emit visible Cerenkov radiation.

We have analyzed experiments of this type and have found that indeed two such experiments would have detected a singly-charged tachyon monopole. Two others would have detected a TM having a charge appreciably different from the Dirac value (see Table I). We find that the cross section for photoproduction of tachyon monopoles in lead is

\[ \sigma(\gamma + \text{Pb} \to \text{TM}) < 0.6 \times 10^{-36} \text{cm}^2. \]

Table I. Monopole Searches Interpreted as Searches for TM.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Source</th>
<th>( Z_{\text{min}} )</th>
<th>( Z_{\text{max}} )</th>
</tr>
</thead>
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<td>Carithers et al.</td>
<td>Cosmic Ray</td>
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<td>20</td>
</tr>
<tr>
<td>Fidecaro et al.</td>
<td>Accelerator</td>
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<td>Purcell et al.</td>
<td>Accelerator</td>
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<td>0.4</td>
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<td>Bradner et al.</td>
<td>Accelerator</td>
<td>6</td>
<td>160</td>
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<td>This Experiment</td>
<td>(^{56}\text{Co} \gamma \text{ Rays} )</td>
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<td>4</td>
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References

3. This technique is strictly analogous to that used by David, Kreisler, and Alvager to detect electrically-charged tachyons, Phys. Rev. 183, 1132 (1969).
Fig. 1. Experimental arrangement
<table>
<thead>
<tr>
<th>Topic</th>
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<tbody>
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<td>A. I. Amatun (Erevan)</td>
</tr>
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<td>3. 12-Ft. Bubble Chamber at ANL</td>
<td>M. Derrick (ANL)</td>
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<td>S. Derenzo (LBL)</td>
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<td>8. Special Spectrometer Magnets at the ISR</td>
<td>J. Sens (CERN/FOM)</td>
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<td>9. Ω-Magnet Project, CERN</td>
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<td>10. The SPEAR Magnetic Detector</td>
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<td>M. Schwartz (Stanford/SLAC)</td>
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<tr>
<td>12. Pattern Recognition Processor and Its Application to Straight Line Reconstruction of Spark Chamber Data</td>
<td>J. Solomon (U. Ill., Chicago Circle)</td>
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<td>13. Streamer Chambers</td>
<td>R. Mozley (SLAC)</td>
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<td>14. A Streamer Chamber Filled with $^3$He</td>
<td>Yu. Shcherbakov (Dubna)</td>
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<td>15. An Effective Mass Spectrometer</td>
<td>R. Diebold (ANL)</td>
</tr>
<tr>
<td>16. Track-Sensitive Hydrogen Target in a Neon-Hydrogen Bubble Chamber (No summary available)</td>
<td>D. J. Miller (University College, London)</td>
</tr>
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Transition radiation is produced when a charged particle passes an interface between two media of different dielectric constants. Consider, for example, a charged particle approaching a homogeneous dielectric foil:

![Diagram showing transition radiation](image)

The processes shown above reverse as the particle leaves the foil. It is clear that charges on the surface are being accelerated and will radiate. As the incident particle becomes relativistic and its electric field becomes compressed in the transverse plane, the motion and resultant radiation extend to ever higher frequencies, and the total energy radiated increases nearly in proportion to \( \gamma \). Since the radiation leaving the foil travels with velocity \( c \) and the particle with velocity \( \beta c = c \), the angle of maximum intensity is nearly straight forward:

\[
\theta_{\text{peak}} \approx \sin \theta_{\text{peak}} \approx \frac{\sqrt{c^2 - \beta^2 c^2}}{c} = \frac{1}{\gamma}
\]

![Diagram showing angle of maximum intensity](image)
Unlike Cerenkov radiation, transition radiation is generated whenever $\epsilon \neq 1$, including the important x-ray region where $\epsilon < 1$

Transition radiation was first predicted by Ginzburg and Frank in 1946 and found by Goldsmith and Jelley in 1959. Lilienfeld also observed it in 1919, but its nature was not recognized at that time. The recent interest in it is largely due to the possibility of using it to measure $\gamma$. Then with a momentum or energy measurement (ym), highly relativistic particles can be identified through their rest mass. Pioneering work has been done by Russian theorists and experimental groups and by the Brookhaven group in the US. We report here the identification of individual particles by the detection of x-ray transition radiation. First, however, we list some useful formulas.

### Some Useful Formulas

1. **Radiation from a perpendicular traversal of a single interface** ($\epsilon_1 \to \epsilon_2$):

$$\frac{d^2W}{d\theta dE} = \frac{\alpha^2}{2\pi} \frac{\sin^2 \theta \cos^2 \theta}{\epsilon_1 \epsilon_2 (1 - \beta^2 \epsilon_1 \epsilon_2)^2}$$

Here $\alpha = 1/137$, $\beta = v/c$, $\epsilon_i$ = dielectric constant of medium $i$, $E = h\omega / (4\pi N_{e}^{0} / m)$ = 29/$(\rho Z/A)^{1/2}$ electron volts. For $\beta \to 1$ this simplifies in the x-ray region to

$$\frac{d^2W}{d\theta dE} = \frac{\alpha^2}{2\pi} \frac{1}{\omega_1^2 + \theta^2 + \omega_2^2 / \omega_2^2}$$

where $\omega_1$ and $\omega_2$ are the specific plasma frequencies. We now specialize to gas-solid interfaces and set $\omega_1 = 0$, $\omega_2 = \omega_p$.

2. **Radiation from a stack of plates** Formulas are given in Ref 4, but modifications must be made to allow for absorption of the soft x rays. A computer program is usually needed for useful results.

3. **Integrated radiation from a single surface-vacuum interface**

$$W = \frac{1}{3} \alpha \gamma h \omega_p$$

4. **Limiting frequency** $dW/dE$ has dropped to half its low frequency value at

$$\omega_{\text{lim}} = \frac{1}{2} \gamma \omega_p$$

typically in the 10 keV region for NAL energies. Since typical transition radiation quanta have

$$E = \frac{1}{2} h \omega_{\text{lim}} = \frac{1}{4} \gamma h \omega_p$$

the total energy divided by the typical energy is

$$\frac{W}{E} = \frac{1}{3} \alpha \gamma h \omega_p$$

$\alpha$ photons are emitted per interface.
Formation zone two interfaces must be separated by more than about
\[ Z_{\text{formation}} = \frac{2c}{\omega \left( \frac{1}{2} + \theta^2 + \frac{\omega_p}{\omega} \right)} \]  

(8)
to prevent cancellation of the radiation from the two surfaces. Referring to the first figure, the cancellation can be seen to come from the identical motion of the + and - charges on the two surfaces as \( a \to 0 \). Cancellation is prevented by differing radiation and particle paths (\( \theta^2 \) term), and by the departure from \( c \) of both the particle velocity \( (1 - \beta \approx \frac{1}{\gamma^2}) \) term) and the radiation-phase velocity \( \left( \frac{\omega_p^2}{\omega} \right) \) term. For example, in the solid [see (6)],

\[ \left( \frac{\omega_p}{\omega} \right)^2 = \left( \frac{\omega_p}{\frac{1}{4} \gamma \omega_p} \right)^2 \frac{16}{\gamma^2}, \]

and the plasma term dominates the \( \theta^2 \) and \( \frac{1}{\gamma^2} \) terms, giving for 400-GeV pions,

\[ Z_{\text{formation}} = \frac{2c}{\omega \left( \frac{1}{2} + \theta^2 \right)} = \frac{2c}{\frac{1}{4} \gamma \omega_p} = \frac{c}{2} \gamma \approx 10^{-3} \text{ cm} \]

If the foils are separated by gas or vacuum, \( \omega_p = 0 \) there and the necessary separation distance is about an order of magnitude larger.

**Radiators**

The major fact of life is given by (7)--\( \gamma \) photons are generated per interface. Thus typical radiator stacks must have \( \geq 10^2 \) foils. The full \( \gamma \) dependence for detected energy will not be realized unless 1) the detector is sensitive to energies up to \( \gamma \omega_p \) and 2) the foil thickness and spacing exceed the corresponding \( Z_{\text{formation}} \). An upper limit to the number and thickness of foils is set, however, by the self-absorption of radiation, primarily through the photoelectric effect. Thus, to achieve adequate statistics, it is often necessary to have repeated radiator-detector sets. Because of its narrow angular distribution, the transition radiation must be detected in the presence of ionization from the particle track unless a long drift distance or magnetic separation is used. Both of these are practical for particle identification in beam lines only if repeated radiators and detectors are used. Some recent measurements have been taken with crystalline radiators, but most commonly, low-Z foils (to reduce photoelectric absorption) of as high a density as possible (to increase \( e^+ \) are used. Beryllium would be ideal, but the usual accommodation with reality normally results in the use of separated Mylar, polyethylene, or aluminum foils. A more compact (2 mm rather than 200 mm) though less efficient radiator could be constructed of alternate high and low density foils since \( Z_{\text{formation}} \) is so much less for solids.

Constructive interference effects between separate foils are possible without excessive mechanical tolerances since the near equality of the particle and wave velocities increases the tolerances from \( \Delta Z < < \frac{1}{\gamma \omega} \) to \( \Delta Z < < Z_{\text{formation}} \). No significant interference effects would be expected with electrons because multiple Coulomb scattering changes their direction by \( \theta_{\text{peak}} = 1/\gamma \) every few foils. Multiple Coulomb scattering also plays a role in slowing the \( \gamma \) dependence above.
\[ \gamma = \frac{16 L_{\text{radiation}}}{c} \left( \frac{M_{\text{incident}} c^2}{21 \text{ MeV}} \right)^{2/3} \]  

(9)

to \( \gamma^{2/3} \), since the formation zone increases with \( \gamma \) while the mean distance for the particle to scatter through an angle \( \theta_{\text{peak}} = 1/\gamma \) is independent of \( \gamma \)

### Detectors

Most any detector that works in the energy region of 1-100 keV can be used when the transition radiation is separated from the particle track, including streamer chambers, sodium iodide scintillators, solid-state detectors, and proportional chambers. With no separation, the detector must be thin, since the absolute magnitude of the Landau fluctuations increases with thickness. Thus far, gas proportional chambers have been used exclusively. Krypton and xenon (with 5-10% CH<sub>4</sub>) are the best gases because of their large photoelectric cross sections. In many cases, the lower Z of krypton is compensated by its lower dE/dx and strategically located K edge (14 keV). Its slight radioactivity (~500 Hz/liter) normally causes no problems. Multiwire liquid xenon detectors now being developed might have one additional useful property—sufficient resolution to separate the transition radiation from the particle track within a reasonable drift distance.

### Some Experimental Results

Figure 3 shows results from a run in a 3 GeV/c beam at the Bevatron. Sum counts from 11 multiwire proportional chambers are shown for pions and electrons without radiators and electrons with a radiator set [100-1/2 mil (0.0005 in) Mylar foils spaced 30 mils apart] placed in front of each chamber. The shift from pions to electrons without radiators is due to the relativistic rise in ionization energy loss. Electrons with and without an equivalent absorber (a single piece of 50-mil Mylar) give the same distribution, indicating, as expected, that bremsstrahlung is not important. A further rise, due to transition radiation, is seen when radiator assemblies are placed in front of each chamber. 85% of the equivalent absorber events are below a discrimination threshold set near the cross-over point. 85% of the radiator events are above the same threshold.

### Uses

From work done so far, it is clear that a modest extension of present techniques (more chambers, slightly better radiators, more sophisticated treatment of the data) will permit the identification of particles with \( \gamma \approx 2000-3000 \) at the several per cent level, for example, 400 GeV/c pions in the presence of 400 GeV/c K and p. The main advantages of transition radiation detectors besides their usefulness at very high values of \( \gamma \) lie in their wide-angular acceptance (Cerenkov counters at NAL energies typically limit at angles of \( 10^{-3} \) to \( 10^{-5} \) radians) and their potential ability to handle several simultaneous particles—both important in studying reaction products. These advantages are shared by particle identifiers using the relativistic rise in dE/dx which should work up to \( \gamma \)'s of ~200. There still is a gap between 200 and 2000. Closing it with an effective, wide-angle detector remains a challenge for the future.

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References

1. V. L. Ginzburg and I. M. Frank, JETP 16, 15 (1946)
2. P. Goldsmith and J. V. Jelley, Phil. Mag. 4, 836 (1959)
3. J. E. Lilienfeld, Physik Zeit. 20, 280 (1919)
5. A. I. Alikhanyan, Adventures in Experimental Physics, 1, 407 (1972) and earlier papers cited in Ref. 4.
7. F. Harris et al., The Experimental Identification of Individual Particles by the Observation of Transition Radiation in the X-Ray Region, submitted to Nucl. Instr. and Methods.
Fig. 3. Equal area spectra of energy loss in 11 chambers (krypton + methane) without saturation correction. Saturation in some of the readout channels causes a systematic underestimate of the number of large values.
High-energy particle detectors on the basis of x-ray transition radiation (XTR) as explored till now possess low efficiency of particle registration and small relative aperture. The particle detector described below takes advantage of the detection of energy released in the bulk of a gas by the absorption of XTR-quanta, as well as the ionization of the gas by the high-energy primary particle. High registration efficiency of ultra high-energy particles could be achieved using XTR techniques, the relative aperture being rather high.

This method, first proposed in 1961, was experimentally investigated only recently. The philosophy of the method consists in registration by a single detector of x-ray quanta both the charged particle and the transition radiation generated by the particle in a laminar medium. The yield of the detector will be proportional to the total energy released due to the absorption of XTR and the ionization losses of a particle in the same detector. While the intensity of XTR depends strongly on \( \gamma - E/mc^2 \), the ionization losses of ultrarelativistic particles are practically independent of \( \gamma \). The yield of detector for small values of \( \gamma \) is defined, consequently, by ionization losses and for large \( \gamma \) basically by the absorption of XTR.

The detection of radiation and particles in high-energy particle detectors under examination was by means of gaseous xenon scintillator enclosed in an aluminum container with round 100-m thick mylar windows of 6 cm in diameter through which the radiation and registered particles passed. The photomultiplier, with its photocathode right in the gaseous medium, was inserted from the side of the container, the thickness of the scintillator being 4 cm of xenon at the pressure of 1.6 atm.

As the wavelengths of the light emitted in a gaseous scintillator are in the far ultraviolet, the coating of the inner side of the container and the photocathode by a spectrum-shifter was provided to match the radiation spectrum and the spectral characteristic of the photomultiplier. Special attention was given to gas cleaning, as the impurities cause the sharp reduction of scintillation intensity. With that end in view, the scintillation gas has been continuously purged in a 600° C hot calcium chip by means of natural circulation.

The energy resolution of the xenon gaseous scintillator, at xenon pressure of 1.6 atm and the energy of quanta ~24 keV was 70 per cent. One, the light yield of such a scintillator equals that of NaI(Tl) crystal.

The high-energy particle detector assembly, consisting of a laminar medium followed by the xenon gaseous scintillator, was exposed to 31-GeV electrons of the Serpukhov proton accelerator. The electron beam was separated by an array of scintillation counters. The laminar medium was composed of 1000, 10-μ-thick mylar films 0.7-mm distant from each other. The measurements were carried out with and without the laminar medium to check the contribution of background events. The xenon scintillator output signals were transmitted through a pulse stretcher and a
scintillation array controlled linear gate to a 128-channel pulse-height analyzer

In Fig 2 the energy distribution of events in xenon measured with the laminar medium (crosses) and without it (points) is given. In the first case, the maximum number of events, i.e., the most probable value of energy released in xenon due to the absorption of XTR-quanta and the ionization losses corresponds to 125 keV. The calculations of the probable value of ionization losses in xenon give the figure of 48 keV. Hence, 77 keV can be attributed to the absorption of XTR.

The ratio of the total number of events in an interval of energy release of 75-200 keV to the number of electrons traversing the laminar medium, i.e., the efficiency of electron registration by transition radiation, was 0.865±0.095. The particle registration probability as measured in the absence of the laminar medium was 0.110±0.013. A part of these events is due to the tail of the Landau distribution, another part is due to electron bremsstrahlung in a 12.5 g/cm² liquid hydrogen target used in another experiment and installed in front of the laminar medium.

The energy distributions in xenon due to XTR and the ionization losses of 31-GeV electrons as well as that due to the ionization losses only were calculated for our detection instrument by the Monte-Carlo technique. The Monte-Carlo calculations are in good agreement with the experimental results.

These data indicate that XTR detection by a xenon scintillator allows identification of ultrahigh-energy particles in the region of \( \gamma \geq 10^3-10^4 \).

In conclusion, the authors wish to express their gratitude to G. Ts. Avakian and M. S. Kocharyan for the help in detector assembly development, to A. S. Belowsov and N. Boodanov of Lebedev Physical Institute for their assistance during the run, and also to the staff of the IHEP accelerator.

References

8. A. I. Alikhanian et al., IHEP preprint OP-STF 70-105
Fig. 1  Schematic of gaseous xenon scintillator 1-photomultiplier, 2-magnetic shield, 3-teflon 0-ring, 4-container.

Fig. 2. The distribution of the energy release with the lammar medium (crosses) and without it (points).
The 12-ft bubble chamber shown in Fig. 1 has been in operation at the Argonne ZGS since 1970. The chamber itself is 3.76 m in diameter (visible) and 1.8 m in height. Four cameras are used which employ lenses with 140° field of view. Besides being the largest working bubble chamber in the world, it also uses the largest superconducting magnet. The field of 18 kG has been mapped and fitted to a polynomial. The results for three of the eleven horizontal planes are shown in Fig. 2(a) which shows the field uniformity is good to a few per cent. Figure 2(b) shows the deviation of the fitted values of field from the measured values.

The momentum accuracy $\Delta p/p$ on charged tracks is found to be $\sim 0.5$ to 1%, which is the same as for the 2-m chambers. This results from a setting error in space ($\epsilon$) of $\sim 300 \mu$. One particularly useful feature of the large size of the chamber is the $\gamma$-ray conversion of $\sim 10\%$. This has been used in two of the experiments performed so far.

By the ZGS shutdown in the spring of 1972, over 1 million pictures had been taken. The experiments were studies of $\nu$ interactions in $H_2$ and $D_2$, pp interactions at 8 and 12 GeV/c and $K_L^0$ decays. At this time the expansion system had reached 1.5 million pulses without any major failures, which is a measure of the picture-taking efficiency. The omega bellows that provides the flexible member for the expansion has performed extremely well. It was inspected in June of 1972 and found to be in excellent condition and was left in place for future operation.

The entire chamber was expanded for 30,000 triple and double-pulse modes in March 1972. Track sensitivity was reached even for pulses as close at 150 msec apart. No major difficulties were encountered in any system. In the future, we hope to operate in the double or triple pulse mode whenever the beam conditions allow.

Figure 3 shows the beam arrangement that presently exists at the chamber. At the top is the rf-separated K beam, in the center the $\nu$ beam and below entering through the side beam window of the chamber the $K_L^0$ beam. The latter will be rebuilt as a low-energy K and $\pi$ beam next spring.

Further details of the precision and other properties of the chamber may be found in K. Jaeger, "Performance and Operating Characteristics of the 12-Foot Bubble Chamber," ANL/HEP 7210.

Work performed under auspices of the United States Atomic Energy Commission.
Fig 1 Argonne 12-ft bubble chamber
Fig. 2. Magnetic-field homogeneity of the 12-ft chamber.
Fig. 3. Present bubble-chamber beam
LIQUID XENON-FILLED WIRE CHAMBERS
Presented by S. E. Derenzo
Lawrence Berkeley Laboratory
University of California
Berkeley, California

Abstract

We describe several types of small liquid xenon-filled chambers, each optimized for a particular property such as a real-time spatial resolution of ±15 μ, a time resolution of ±10⁻⁷ sec, or a pulse height of 10⁻¹² coulomb. Larger chambers combining all these properties will be of great value at NAL energies, and we describe some of the techniques necessary for their construction.

Introduction

In 1968, Luis Alvarez suggested that a filmless particle detector with +10 μ space resolution could be developed by using a liquified noble gas as the detection medium. He reasoned that a thousand-fold increase in density (when compared with gas-filled wire chambers) would simultaneously permit a decrease in detector thickness and an increase in ion statistics. Other resolution-limiting factors such as electron diffusion and 5-rays would also be significantly reduced. Operation at one atmosphere pressure would allow the technique to be used over large areas and readout would be simplified if the initial ionization could be amplified in the liquid.

Single Wire Chambers

We have been operating liquid xenon-filled single-wire proportional chambers for two years. Figure 1 shows our single-wire chamber design, and Fig. 2 shows the pulse height of the 279-keV photopeak as a function of operating voltage for 2.9-, 3.5-, and 5.0-μ anode wires. The chamber counts well and the pulse-height curves are reproducible to ±10% every time the chamber is filled.

As mentioned in Ref. 2, the single-wire chamber produces two classes of avalanche pulses. The first type are proportional in size to the initial ionization and rise in ~2 × 10⁻⁷ sec [see Fig. 3(a)]. The second "Geiger" type are larger (~2 × 10⁻¹² C), uniform in size, and rise more rapidly [see Fig. 3(b)]. The rise time shown in Fig. 3(b) is limited by the rise time of our charge amplifier. When the "Geiger" pulses are observed directly on an oscilloscope, the true rise time of ~10⁻⁸ sec may be seen (see Fig. 4).

Using the two collinear gamma rays from a ²²Na source, we measured the time resolution of a single-wire liquid xenon chamber to be ±10⁻⁷ sec. During this test electronegative impurities restricted the effective diameter of the chamber to approximately 1 mm. For details see Ref. 5.

For decades the gas filling in wire chambers has included quenching agents to suppress sparking and to increase the size of the pulses. We have found that 2000 ppm ethylene (C₂H₄) in liquid xenon also permits higher operating voltage and larger pulses. This comparison is shown in Fig. 5.

Work supported by the United States Atomic Energy Commission.
Multi-Wire Proportional Chambers

We have designed and built a liquid xenon multi-wire proportional chamber specifically for detecting gamma rays in nuclear medicine. (For gamma rays in the 0.15 to 10-MeV range this approach provides an unprecedented combination of detection efficiency, spatial resolution, and potential for up-scaling.) This chamber is shown in Fig. 6 and consists of 24 3.5-μ wires spot-welded to Kovar pins sealed with glass to holes in the ceramic base. The wires are spaced 2.8 mm apart, and the wire plane is centered between two cathode planes spaced 15 mm apart. The gross detection efficiency for 280-keV gamma rays is 65%. At an operating potential of -4500 V the liquid gain is 10, and under these conditions the spatial resolution was measured to be 3 mm FWHM, demonstrating that the wires amplify independently without the addition of a quenching agent.

For the development of a chamber possessing high spatial resolution for charged particles, however, it is very important to demonstrate amplification by arrays of much more closely spaced wires. To this end we ran three arrays of five wires 3.5 μ in diameter with spacings of 150, 300, and 1000 μ between opposing cathodes spaced 5 mm apart in a small test chamber. With a 1000-μ spacing, gain sets in at 3800 V, reaching 10 at 5000 V. A further increase in voltage at the 1000-μ spacing and observation of gain at the 150- and 300-μ spacings was prevented by the occurrence of sparks between the flat cathode and the wires of the anode. Very recently we have learned that these sparks are initiated by field emission from the cathode. We will continue studies with closely spaced anodes (50 to 300 μ apart) using specially polished cathodes and the addition of ethylene to suppress sparking and provide larger gain.

In previous papers we have reported on schemes for producing closely spaced narrow conducting strips on an insulating substrate. Heidenham Corporation can produce sub-micron chrome lines on glass or Mylar. In addition, we have made Noryl pressings with sharp ridges 50 μ apart and then vacuum deposited metal at a grazing angle to produce conductive strips several μ wide at the top of the ridges (see Fig. 7).

Recently we have devised a method of attaching fine wire to a substrate in such a way that 98% of the wire does not touch the substrate. A Noryl pressing is made, producing sharp ridges 100 μ wide, 100 μ high, and 1 mm apart. Then 5-μ wire is wound around the ridges at right angles. The assembly is placed in a magnetic field while a current is passed through the wire. This serves to heat the wire and press it into the edge of the ridge. The result is shown in Fig. 8.

Multi-Wire Ionization Chambers

Electron avalanche in the liquid is essential in reducing the complexity and cost of the readout. It is quite possible, however, using present technology, to build a liquid xenon multi-wire ionization chamber having no liquid gain with a space resolution of ±10 μ and a time resolution of ±20 nsec. Unfortunately the readout requires that a low-noise charge-sensitive amplifier be attached to each wire, increasing the cost by approximately $20 per wire, and severely limiting its applications in physics experiments. Figure 9 shows the excellent spatial resolution that we have measured for the ionization mode.
Gamma-Ray Detection

The density and atomic number of liquid xenon make it attractive for the absorption and detection of gamma rays. In order to study the energy resolution obtainable, we built an ionization chamber with a Frisch grid. The chamber and experimental setup are shown in Fig. 10. The best resolution for 279-keV gamma rays obtained thus far is 10.5% FWHM (shown in Fig. 11), comparable to NaI(Tl).

The amount of liquid xenon required for the absorption of a multi-GeV electromagnetic shower is quite expensive ($3/cc) but there is hope for the future. For every megawatt-year generated by nuclear power reactors, 57 grams of Xe are produced. The only important contamination is 10.76-yr $^{85}$Kr, which can be reduced to levels of 10 pCi per STP liter Xe by distillation followed by a series of dilutions (with atmospheric Kr) and redistillations. C. A. Rohrmann estimates that by 1980 the annual recovery of xenon will exceed 10 tons per year at a cost of 25¢ per liquid cc.

Summary

We have developed several versions of liquid xenon-filled chambers, each optimized for a particular property such as unexcelled real time spatial resolution ($\pm 15 \mu$), good energy resolution ($\pm 5\%$ for 279-keV gamma rays), good pulse height (10$^{-12}$ coulomb for minimum ionizing particles passing through 0.7 mm liquid Xe), or good time resolution ($\pm 10^{-7}$ sec for a 1-mm-thick chamber). We are working on the technology to allow us to combine several such properties in chambers covering useful areas.

Acknowledgments

We are indebted to Joe Savignano, Tony Vuletich, and Buck Buckingham for essential contributions to all phases of this work.

This work was supported in part by the U.S. Atomic Energy Commission and in part by NASA.

Appendix - Selected Properties of Liquid Xenon

<table>
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<th>Value</th>
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<tr>
<td>Boiling point</td>
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</tr>
<tr>
<td>Freezing point</td>
<td>-111.8°C (1 atm)</td>
</tr>
<tr>
<td>Density</td>
<td>3.057 g/cc</td>
</tr>
<tr>
<td>STP gas/boiling-liquid volume ratio</td>
<td>518.9 (1 atm)</td>
</tr>
<tr>
<td>$\text{e}^-$ drift velocity$^{10}$</td>
<td>$\begin{cases} 1 \times 10^5 \text{ cm/sec at 60 V/cm} \ 2 \times 10^5 \text{ cm/sec at 500 V/cm} \ 3 \times 10^5 \text{ cm/sec from 3 to 60 kV/cm} \end{cases}$</td>
</tr>
<tr>
<td>Ion pairs per 100 $\mu$</td>
<td>2000 (minimum ionization)</td>
</tr>
<tr>
<td>Energy loss per 100 $\mu$</td>
<td>43 keV (minimum ionization)</td>
</tr>
<tr>
<td>Radiation length</td>
<td>25 mm</td>
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<td>Collision length</td>
<td>450 mm</td>
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<tr>
<td>Present cost</td>
<td>$3/cc</td>
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</table>

References

Recently workers at Dubna have reported on some excellent new work using a single-wire proportional chamber filled with high-pressure gases and solidified noble gases: A. F. Pisarev, V. G. Pisarev, and G. S. Revenko, Dubna Reports JINR - P13 - 6450 and JINR - P13 - 6449 (1972).


Heidenhain Corporation, 8225 Traunreut, West Germany.

Noryl is a class of thermoplastic resins manufactured by General Electric, Noryl Avenue, Selkirk, New York 12158.

The collimated alpha source used in Ref. 2 is described in Lawrence Radiation Laboratory Report UCRL-20857 (1971).


To vacuum and gas supply

BNC (signal out)

Guard ring

Glass insulation

Cathode region

High voltage feed-through

Fine wire anode

Guard ring

Glass feed-through

Fig. 1. Liquid xenon single-wire proportional chamber.
Fig. 2. Pulse height vs voltage in liquid xenon single-wire proportional chamber with 8-mm-diam cathode and 2.9-, 3.5-, and 5.0-μ-diam anodes. Signal is 279-keV photopeak from 203 Hg source.
Fig. 3. Pulses from chamber of Fig. 1 seen on a charge amplifier with a gain of 0.4 V/pC. A: proportional pulse. B: "Geiger" pulse.
Fig. 4. "Geiger"-type pulses from chamber of Fig. 1 seen directly on an oscilloscope. A: 1-megohm load. B: 125-ohm load.
Fig. 5. Pulse height vs voltage in single-wire chamber for 5-μ anode in pure liquid xenon and in liquid xenon containing ~2000 ppm C₂H₄.
Fig 6  Gamma-ray chamber 15 mm thick, containing 24 3 5-μ-diam wires and 24 cathode strips. A shows ceramic chamber wall and support for anode wires and cathode strips.
Fig. 6. Gamma-ray chamber 15 mm thick, containing 24 3.5-μ-diam wires and 24 cathode strips. B* shows assembled chamber.
Fig. 7. Array of conducting strips produced by vacuum depositing metal at a grazing angle onto a series of Noryl ridges 40 μ wide and 100 μ apart.

Fig. 8. Array of 5-μ wires bonded to Noryl ridges 1 mm apart. Current is passed through the wire in a magnetic field to heat the wire and press it into the ridges.
Fig. 9. Image of a 38-μ wide collimated alpha source detected by a 0.7-mm-thick liquid xenon ionization chamber. (See Refs. 2 and 6 for details.)
Fig. 10. Liquid xenon ionization chamber with Frisch grid and collimated gamma source.
Fig. 11. Pulse-height spectrum for 279-keV gamma rays seen in setup of Fig. 10. FWHM is 10.5%, comparable to NaI(Tl).
In this paper we report the results of designing and investigation of a new particle detector—the crystal wire counter. A number of physicists have been trying to design counters with liquid media. However, it has been found that the performance of these counters was not reliable.

In this connection, one of the authors of this paper has proposed a new design based on the substitution for the entire liquid medium of the solid frozen crystal substance. This kind of counter has been the subject of the present research. The counter had a cylindrical brass cathode, 6-mm in diameter and a gold-plated tungsten wire anode 10 μm in diameter. The working space of the counter has been observed through the end windows. In our experiments we have used solid argon and xenon. They were produced in the following steps: the gas was first condensed in the counter above the triple point, then the liquid was slowly cooled to the crystallization point and gradually frozen.

In these experiments the amplitude and counting properties of the device were determined with the help of the gamma quanta from Co$^{60}$. The main results of these investigations with transparent crystals are shown in Figs. 1 and 2. It is seen that the argon counter has the single amplitude and counting properties while the characteristics of the xenon counter are within a certain region of values.

In each experiment with xenon we obtained the well-determined characteristics, yet varying in part from one freezing to another. The set of these characteristics obtained in a number of measurements lie within the shaded regions. It is clear from these amplitude properties that the counter has three distinct regions: the ionization region up to 2 kV, the proportional region up to 5.2 kV, and the saturation region above 5.2 kV. The coefficient of amplification in the proportional region was ~150. The counting curve had a 3 kV wide plateau with a low slope. For imperfect crystals (not highly transparent) electron multiplication has also been observed. However, the crystals were rapidly charged by the positive space charge, and the pulse amplitudes were noticeably decreased. To restore the sensitivity of the counter after recording a large number of pulses, it was necessary to apply, for a short time, a small field of opposite polarity. For the perfect crystals (highly transparent) the mobility of positive ions was found to be $10^{-1} \text{cm}^2 \text{v}^{-1} \text{sec}^{-1}$ and $10^{-1} - 10^{-2} \text{cm}^2 \text{v}^{-1} \text{sec}^{-1}$ for argon and xenon respectively.

The results obtained from these experiments can serve as the basis for design of proportional wire chambers of large dimension where the wires will be rigidly fixed in space due to their freezing in the crystal.

The specific particle ionization in the crystals allows a small space between the wires in the chamber that will provide the high time and spatial resolution superior to those of the gas-filled wire chambers.
References


Fig. 1. Amplitude properties: 1) argon counter; 2) region of the xenon counter values.

Fig. 2. Counting properties: 1) argon counter; 2) region of the xenon counter values.
TOTAL ABSORPTION NaI DETECTORS

Presented by E. B. Hughes
High Energy Physics Laboratory
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Introduction

This discussion concerns only homogeneous NaI(Tl) total absorption detectors. The aim is
to demonstrate that such detectors can be used in high-energy physics experiments and that appli­
cations which utilize their unique properties can be found. There are two types of NaI(Tl) total
absorption detectors: TASC detectors for the detection of electrons and γ rays and TANC detectors
for the detection of hadrons. These will be discussed in turn.

TASC Detectors

A considerable familiarity with the properties of TASC detectors has been acquired during
the last two years[1, 2] and this has led to the physics applications which are presently underway.
Figure 1 summarizes the properties of a simple TASC detector.

One application of the TASC detector in a high-energy physics experiment[3] is to the meas­
urement of the inclusive cross section for π0's in the reactions π± p → π± X. This experiment has
recently been completed at SLAC at an incident pion energy of 14 GeV. A schematic diagram of
the experimental apparatus is shown in Fig. 2. The requirements of this experiment necessitated
modifications to the simple TASC design shown in Fig. 4 of Ref. 2. In particular, the experiment
required two distinct crystal segments, a much larger aperture and the inclusion of a position
sensing device. The NaI(Tl) crystal assemblies, initially available to this experiment, are of
two types: either 10 inches or 3 5 inches in thickness. Both types are 30 inches in diameter and
equipped with radial viewing ports for phototubes. While radial ports are needed in some appli­
cations of these crystals, they do not provide the resolution and uniformity of response desired
for the TASC detector. These qualities, particularly the latter, are much improved if rear
viewing ports are provided. The modified assemblies as used in the experiment are shown in
Fig. 3. Each assembly, one 20 inches in thickness and the other 3 1/2 inches, is provided with a
glass end plate through which the crystal can be viewed, either with phototubes coupled directly to
the glass or through a diffusion box. The 20-inch crystal consists of two 10-inch crystals in
optical contact. An overall view of the apparatus is shown in Fig. 4.

The properties of the TASC detectors as used in the experiment are illustrated in Fig. 5.
Due to the modified TASC design, the energy resolution has been degraded relative to that provided
by the simple detector shown in Fig. 1. This degradation is principally connected with the use of
two crystal segments, one of which is especially thin relative to its diameter. With more study it
may be possible to remove some of this degradation. Figures 6 and 7 show typical results obtained
with the τ0 spectrometer.

An upcoming application[4] of the TASC detector is to physics experiments at SPEAR (the
Stanford Positron-Electron Asymmetric Storage Ring). The apparatus is described in Ref. 2.
The two TASC detectors employed permit a clean identification of certain of the final states to be made \((e^+ e^- \rightarrow e^+ e^-\) and \(e^+ e^- \rightarrow \gamma \gamma\)) through a precise measurement of the detected energy, which for these states is expected to equal that in the initial state. This experiment imposes a new measurement requirement on the TASC detector, namely to determine within each detector the direction of an electron or \(\gamma\) ray at energies of about 2 GeV. Presently provision is made for a drift space within each detector for this purpose. The final design remains to be determined. It represents, however, another departure from the simple TASC detector, which must be done in a way such that the fundamental properties of the detector are retained.

**TANC Detectors**

During the last two years a volume of NaI(Tl) sufficiently large to make a significant test of the TANC detector idea has been accumulated at Stanford. There presently exists a detector 70-1/2 inches long and 30 inches in diameter (4.8×2.0 nuclear absorption units). A picture of this detector is shown in Fig. 8. It will be tested shortly at SLAC energies and later at NAL. From previous studies a mean energy containment of about 80% at 8 GeV is expected, limited principally by radial leakage. There is no reliable estimate of the energy resolution. However, it should be distinctly better than anything observed in the past.  

**Acknowledgment**

The author acknowledges the contributions to this work of his colleagues J. F. Crawford, R. L. Ford, R. Hofstadter, L. H. O'Neill, R. F. Schilling, and R. Wedemeyer.

**References**

Fig. 1. The energy resolution and uniformity of a simple TASC detector. The calculated resolution due to leakage fluctuations as a function of electron energy is also shown.
Fig. 2. A schematic diagram of the experimental apparatus used in the measurement of $\pi^+ p \rightarrow \pi^0 X$.

Fig. 3. Photograph of 30-in. diameter NaI(Tl) assemblies 10 in. and 3-1/2 in. in thickness equipped with glass windows on one end face.
Fig 4 An overall view of the $\pi^0$ spectrometer used in the measurement of $\pi^\pm p \rightarrow \pi^0 X$.

![Electron Resolution vs Energy](image)

**Electron Resolution vs Energy**

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$E^{-1/4}$

![Resolution vs Displacement at 2 GeV](image)

**Resolution vs Displacement at 2 GeV**

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Fig 5 The energy resolution and uniformity of the modified TASC detector used in the $\pi^0$ spectrometer. The dashed line in the upper plot reproduces the performance of the simple detector shown in Fig 1. The bump in the uniformity plot at a radial displacement of 6 in is correlated with a ring of phototubes located at a radius of 6 in.
Fig. 6. A typical $\gamma\gamma$ mass spectrum recorded by the $\pi^0$ spectrometer. This is an on-line result and not fully corrected.
Fig. 7. A typical $\pi^0$ energy spectrum recorded by the $\pi^0$ spectrometer, uncorrected by detection efficiency.
Fig 8 Photograph of a NaI(Tl) TANC detector 70 1/2 in in length and 30 in in diameter located in a secondary beam line at SLAC
A HIGH ENERGY NEUTRON DETECTOR USING PROPORTIONAL WIRE CHAMBERS

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An experiment to study the properties of negative hyperons produced by the Brookhaven National Laboratory Alternating Gradient Synchrotron required a neutron detector to identify the fast (15-20 GeV/c) neutrons resulting from hyperon decays of interest, e.g., $\Sigma^- \rightarrow n\bar{\nu}_\tau$. Figure 1 shows a schematic of the apparatus. The extracted proton beam (about $5 \times 10^9$ protons per pulse at 28 GeV/c) is allowed to strike a small target at the entrance of the magnetic channel. Negative hyperons, $\Sigma^-$, $\Xi^-$, $\Omega^-$, produced in the forward direction traverse the channel and have their position recorded by a special high-resolution spark chamber and the channel exit. Most of the hyperons that decay downstream of this chamber have their charged decay particles momentum-analyzed by the two spectrometer magnets and four clusters of magnetostrictive spark chambers. The fast forward neutrons produced in some decays are then detected by the device described here. A measurement of the neutron position and hence angle, along with a rough measurement of its energy, is desired.

As shown in Fig. 2 the detector consists of two parts, the neutron position detector and the neutron calorimeter. The first section contains alternating layers of 3.2-cm thick steel plate, 0.6-cm thick plastic scintillator, and a pair of orthogonal proportional wire chambers. There are five such layers each having a cross section of about 60 cm x 60 cm. The detector consists of 10 proportional wire chambers, 5 steel plates, and 6 scintillation counters. The active area of the proportional chambers is 48 cm x 48 cm. Each chamber (see Fig. 3) contains 48 signal wires (25 micron diameter gold-plated tungsten) which are 1-cm apart. The spacing between the wire plane and the outer electrodes is 8 mm. The outer electrodes are made of 50 micron copper foils on 1 5-mm thick epoxy glass sheets. The same material is also used around the chamber as an electromagnetic shield. The calorimeter was built by the University of Michigan group and consists of alternating layers of steel plates (3.8-cm thick) and plastic scintillators (0.6-cm thick). It contained over four interaction lengths of material.

Neutrons of momentum 13 to 21 GeV/c interact in the steel plates to give charged hadronic and electromagnetic cascades. The position of these cascades is given by the proportional wire chambers. The scintillators in both the position detector and the calorimeter are summed to give pulse-height information, hence a measurement of the energy deposited. This pulse height is used in the trigger and also recorded by the data-collection system.

A similar type of detector with a single converter plate was reported. G. Couignet et al. obtained ±2.2 mm accuracy in determining the interaction vertex of neutrons in a carbon converter with an efficiency of 6.5%.

* Operated by Universities Research Association, Inc. for the United States Atomic Energy Commission.
A typical picture of a hadronic and electromagnetic cascade resulting in the detector from the interactions of a 15 GeV/c pion and neutron respectively is shown in Figs 4(a) and (b). In this picture each row of a vertical projection or a horizontal projection corresponds to a proportional chamber plane and each (1) represents a wire pulse. These data along with all other relevant data concerning the event were recorded on magnetic tape by our DEC PDP 15 computer. Even a cursory perusal of these projection plots indicated that showers were indeed being detected, and a measurement of the neutron interaction was possible. A spatial resolution of \( \sigma = 7 \) mm with a 61% total detection efficiency in determining the neutron positions was achieved. See Fig. 5. The exact determination of this position is a pattern-recognition problem which is discussed in a separate paper. Better spatial resolutions can be obtained with proportional chambers having smaller wire spacing. The energy resolution of the detector is determined to be about 20% for hadrons of 15 to 20 GeV/c. The resolution of the proportional chambers was 35 nsec at HWHM.

Figure 4(c) shows a picture of a \( \mu \) track obtained from the detector. Muons are used for determining the relative alignment of the proportional chambers. An obvious identification of the muons is that they leave straight tracks or single tracks with a 5-ray branch formation in the detector.

This idea may be applied to detection of all neutral particles and gammas. High repetition-rate capability, high multi-track efficiency, and good time resolution are some of the advantages of this neutron detector over detectors using optical spark chambers. The spatial and energy resolutions of this type of detector are expected to be improved at higher energies since the forward cone angle of the hadronic and electromagnetic cascades will be narrower.

The authors would like to express their appreciation to J. Lach, J. Marx, A. Roberts, J. Sandweiss, and W. J. Willis for very useful discussions and to J. Bloomquist, B. Lombardi, E. Steigmeyer, and I. J. Winters for their help in constructing the detector and the readout system.

References

1. AGS Proposal #430, Study of Production and Decay of High Energy \( \Xi^- \) and \( \Omega^- \)
2. W. J. Willis et al., Nucl. Instr. and Methods 91, 33 (1971)
6. W. Tanenbaum, to be published in Nucl. Instr. and Methods
Fig. 1. Schematic view of the experimental layout.
Fig 2  Top view of the neutron detector
Fig. 3. A simplified cross-sectional view of the proportional chamber.
Fig. 4. A typical picture of a hadronic and electromagnetic cascade resulting in the detector from the interaction of a) a 15 GeV/c π; b) a neutron; c) picture of μ track.
Fig. 5. A probability distribution of the reconstruction of a neutron-interaction point.
Two remarks may be made concerning the CERN-ISR and instrumentation in general. First, in parallel with the construction of the storage rings proper, a multitude of instruments, mostly related to vacuum and beam diagnostics, has been developed; apart from being a pair of storage rings, the ISR is in fact a laboratory for instrumentation in its own right. Second, at the ISR, centre of mass energies are reached which are equivalent to up to 1500 GeV laboratory energies on a stationary target, while nevertheless no secondary particle can have a momentum larger than ~ 30 GeV/c. Hence "standard" techniques for particle identification and momentum measurements are in principle adequate for studying the collision products. Nevertheless, a number of "unconventional" devices have in fact been installed at the various crossing points, in order to cope with problems arising from the proximity of the primary beams, the need to detect secondaries produced at very small angles and the desire to have large acceptance in order to detect as many particles as possible produced in a given collision, or to have sensitivity for very rare processes. In the following I will describe, briefly and without detail, the central vacuum chamber and four magnets, forming part of physics experiments presently under way or in preparation.

A. VACUUM CHAMBER AT THE CROSSING POINT.

A device of key importance to all experiments is the vacuum chamber at the crossing point. Fig. 1 shows the solution adopted for most intersections. The figure shows a double-conically shaped vessel with corrugated walls of 0.2 mm thickness. Its overall size is the smallest compatible with complete symmetry around an axis, formed by
the bisector of the two crossing vacuum pipes. As a result of this
symmetry four simple supporting rods (two are visible in fig. 1) are
sufficient to prevent collapse under evacuation. The chamber is
1280 mm long, 200 mm in diameter at the centre, 325 mm at the ends.
Its size and wall thickness have been chosen so as to satisfy the
requirements of minimum dead space between crossing point and detectors
and of minimum material at any angle in the path of the secondaries.
This chamber has been installed in all but one intersection, where
physics experiments are being done.

B. SMALL APERTURE, MOBILE, CURRENT-SEPTUM MAGNETS.

Fig. 2 shows two septum magnets, marked MNP34 and MNP35, installed
near crossing point No.2. The purpose of these magnets is to intercept
secondaries produced at small angles in the vertical plane and bend
them upwards towards a set of three conventional magnets (not shown
in fig. 1), placed on top of the main ring magnets (p. 163 in fig. 2
and further to the left). The apertures of all five magnets of the
spectrometer are roughly matched. The septum magnets are mobile, the
other magnets are in fixed positions. By (computer-controlled) rotation
and lift of the septum magnets particles produced at angles between
25 and 180 mrad can be intercepted. At each position the system accepts
± 8 mrad. The accepted interval in momentum is roughly ± 15%.

In MNP34, the septum consists of four current carrying conductors
mounted side by side along the bottom of the magnet. The conductors
are 14.5 x 14.5 mm each, with a 8 mm Ø bore for cooling, and are
electrically in series. The magnet is fed with 20 KA at 30 V the max.
magnetic field is 1.7T. At maximum current the current density is
~ 100A/mm²; high speed water cooling (~ 15 m/sec) is applied in order
to keep the temperature rise below 40°C. The magnet is 1.7 m long, the
overall cross-section is 34 x 38 cm, the aperture is 6 cm wide x 10
cm high. The magnet is mounted on a horizontal shaft (behind the
magnet, not visible in fig. 2), whose bearing block is placed on a
motor driven platform with a total travel of 60 cm. A second motor
allows for rotation up to 170 mrad around the shaft via a push-pull
mechanism.
MNP 35 is similar in concept, but roughly twice in size. The septum consists of ten conductors, 24 x 24 mm each, mounted in two layers of five. The power is ~ 20 KA x 30 V, max. field is 1.8 T. The length is 3 m, overall cross-section 108 x 78 cm, aperture 12 cm wide x 22 cm high. The magnet is mounted on a shaft which is held from a platform above the yoke, see fig. 2. The total vertical travel is 75 cm, the maximum angle 170 mrad.

Both magnets have been in operation for approximately 8 months. The apertures are large enough to obtain an event rate of several per second at ~ 50 mrad and ~ 5A beam current per ring, on the other hand small enough to intercept at most one secondary per collision in more than 99% of the cases, in spite of the high multiplicities encountered at ISR energies. One useful consequence of the high length/cross-section ratio is that the $|Bd|_1$ varies negligibly across the gap, hence the entire magnet is represented by just one number in the off-line event analysis programs.

C. LARGE APERTURE, IRON-SEPTUM MAGNETS.

Fig. 3 shows a cross-sectional view of a large aperture magnet, with a septum plate made of iron. Two such magnets are mounted in an experiment, one above the primary beam in one downstream section of a crossing region, one below the primary beam, in the other downstream section.

Each magnet has a magnetic length of ~ 2.6 m and an overall cross-section of 2.3 x 1.6 m. The useful aperture is 80 x 62 cm, the weight is 50 tons, the power is 1KA x 600 V, The coil consists of 14.5 x 14.5 mm conductors, arranged in 38 pancakes, 532 turns in total. The magnet can be run in two ways: with an iron septum and a field limited to 0.5 T and with an iron + cobalt septum and a field up to 1.1 T.

The two magnets of this type have been operational for ~ 8 months, in an experiment designed to measure elastic scattering out to large (~ 100 mrad) angles.
D. THE SPLIT-FIELD MAGNET.

One of the crossing points is presently being equipped with a large, general purpose magnet system, in order to be able to measure momenta and angles of a number of secondaries produced in a given collision. The system consists of five magnets and a number of passive magnetic channels, see fig. 4.

The central magnet consists of one set of polepieces extending over ~ 5 m above and below one downstream arm, and another set on the other downstream arm. The fields are in opposite directions, the return yoke is common. Since the primary beams pass through the field region, separate magnets are required to restore their orbits. The beam therefore passes through an upstream compensator magnet, the two sections of the main magnet, where the fields are opposite and a downstream compensator magnet. In order to protect the beams against magnetic edge effects, the main magnet entry and exit sections of the vacuum pipes are surrounded by magnetic channels.

The central magnet extends over ~ 5 m at either side of the crossing point. The gap height is 1.1 m, the width 3.5 m at the centre, 2 m at the ends. The useful magnetic volume is 28 m$^3$. The field is 1.1 T for a power of 6.25 KA × 650 V. The weight is ~ 900 ton. Pillars carry the top part of the magnet and withstand the magnetic forces (in total 1600 ton).

The upstream compensator magnets are relatively small, with an overall width of ~ 1 m, length 45 cm. The useful aperture is 7.5 cm high by 36 cm wide. The field is 1.8 T, the power 1 KA × 44 V.

The downstream compensator magnets are larger than the upstream ones, since they serve the double purpose of compensating the primary beams and momentum analyzing secondaries produced at small angles. They are 1.5 m long, 3 m wide and have a useful aperture of 40 cm height and ~ 90 cm width. The field is 1.5 T, the power 2 KA × 150 V.

The beam channels have the form of long rectangular tubes of variable section surrounding the vacuum pipes, protruding into the central magnet from the upstream sides. They are ~ 3.5 m long and
split into zones of different thickness. Over ~ 50 cm the two horizontal sides of the tube can be (remotely) inclined, so as to compensate gradients at different field levels.

The magnet is presently being assembled in the vicinity of crossing point No. 4. The central magnet as well as the downstream compensators will be equipped with a number of multiwire proportional chambers for the simultaneous detection of several collision products. More than ten experiments using the split field magnet facility are presently being prepared.

E. LARGE SIZE IRON/SPARK CHAMBER SANDWICH.

One of the detectors designed to search for intermediate bosons consists of a set of magnetized iron plates, interleaved with optical spark chambers. Fig. 5 shows a side view of the plates during installation. The ISR beampipe and one ring magnet are visible on the right.

The set up is symmetric with respect to a vertical plane through the crossing point perpendicular to the bisector of the rings. 13 plates are mounted at each side of this plane. Each plate is 10 cm thick, 2 m wide, 4 m high. A bundle of 64 (24 x 24 mm) current conductors are located such as to describe a closed loop in the plane of symmetry: the conductors pass over the top, come down to half height at the crossing point, return to the back of the plates through the space between the two sets and go up from mid height to top at the back of the last plate, thus closing the loop. The field in the top and bottom half of the plates thus has opposite sign. The power is 500 A at ~ 50 V. The field is highly non uniform, it drops from 17 K gauss at the centre to 11 K gauss at the edges. The weight of the entire assembly is ~ 300 ton.

This assembly has been operational for approximately one year. It is used to measure the momenta of high energy muons, which penetrate several or all of the plates. Strongly interacting particles are rejected by absorbing them in a thick lead absorber placed between the first plate and the crossing point.
Fig 1 A bi-cone the thin-walled intersection region vacuum chamber used for physics experiments.

Fig 2 The small-aperture, mobile septum-magnet spectrometer. The septum magnets MNP-34 and MNP-35 bend secondary particles in the vertical plane.
Fig. 3. Large-aperture, iron-septum magnet spectrometer
Fig. 4. General-purpose large split-field magnet system.
Fig 5  Very large magnetized iron spark chamber system
This short report summarizes the present status of the Omega spectrometer which first took beam in June, 1972. The Omega magnet has a useful field volume of $3 \times 1.5 \times 1.5$ m and is located in the West Hall at CERN. It has superconducting coils which were tested at the design current of 4800 amps before the beam was first transported to the hall. The special feature of the coils is that liquid helium is flowed through the hollow windings, and this has proved to be very successful. Currently only one coil is mounted giving a field of 1.1 T instead of the 1.8 T full field.

Up to the time of the Conference there have been five three-day test runs with beam during which the plumbicon television camera readout system for the 100 optical spark-chamber gaps has been tested as well as the several triggering devices around the magnet. The plumbicon system was developed at the Rutherford Laboratory by a Birmingham, RHEL, Westfield collaboration with the help of the Rutherford Laboratory electronics and apparatus groups. The cameras have achieved the expected spatial accuracy of 0.5 mm and are adequately efficient for two- and four-prong events though some improvement is desirable for higher multiplicities. This may come from further work on the spark chambers.

The three pairs of cameras as well as the beam-proportional chambers, hodoscopes, and Cerenkov counters are read into an EMR 6130 on-line computer while each of the triggering systems uses a PDP-11 for checking purposes. When a particular user is triggering the system, trigger data from his PDP-11 are transferred to the EMR and written on to magnetic tape along with the spark chamber and beam information for each event. Also available either on-line or off-line is a CII 10070 computer where most of the program development on the track-finding and geometrical reconstruction programs has been carried out and where users can perform more sophisticated checks on their data using a complete chain of analysis programs. The bulk of the analysis is however expected to take place outside CERN at the user's home laboratories.

Figure 1 shows a schematic plan view of the apparatus. Two experiments, one by the Birmingham, RHEL, Westfield group and the other by a Bari, Bonn, CERN, Daresbury, Liverpool, Milan collaboration are studying meson resonances with slow neutron and slow proton-recoil triggers respectively. In the latter case the protons pass through the specially designed thin walls at one side of the spark-chamber frames. Two other groups are making use of the atmospheric pressure threshold Cerenkov counter built at Saclay and its associated hodoscopes to identify fast protons or antiprotons coming from the decay of forward lambda or antilambda particles. The lambda trigger of the CERN, ETH, Freiburg, Karlsruhe group is designed to study baryon exchange and uses a small veto counter immediately after the 30-cm hydrogen target. The Glasgow-Saclay collaboration's anti-lambda trigger is to study mesons decaying into baryon-antibaryon pairs. Initial tests have shown the Cerenkov counter to have an efficiency for pions above 99% and
good uniformity. Also shown in Fig. 1 is a test $K^+$ detector from Imperial College, London, intended for a $Z^*$ experiment not yet approved, and several proportional chambers which are not yet installed.
Figure 2 is a general view of the apparatus with the spark chambers withdrawn and Fig 3 shows them in place inside the magnet. Two examples of events from 8 GeV/c $\pi^-$ mesons seen on the EMR 6130 on line display and taken with the neutron and fast lambda triggers are shown in Fig 4. In one of these the fiducial marks on either side of each 10 gap module are seen. These are used to correct the nonlinearities of the plumbicon scan. In the other photograph the path of the proton from the lambda decay is shown between the two hodoscopes. The lambda mass has been reconstructed on-line.

Time is available in October for preliminary data taking when several hundred thousand triggers should be recorded for test purposes. Systematic evaluation of the device will then be possible. The main data taking will start in February 1973 by which time the second coil and other missing items will be in place.

Fig 2 View of Omega magnet with spark chambers withdrawn from interior

Fig 3 Spark chambers installed in magnet gap
Fig. 4(a) Omega on-line display of antilambda event.

Fig. 4(b) Associated production event with kaon and lambda decays.
THE SPEAR MAGNETIC DETECTOR*

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A large magnetic detector is being built for the SPEAR electron-positron storage ring facility by a group of physicists from the Stanford Linear Accelerator Center and the Lawrence Berkeley Laboratory. The detector, described in this report, is presently under construction and is scheduled for completion late in 1972 with the experimental physics program commencing early in 1973.

The major design criteria evolved from the desire to emphasize the study of hadronic final states produced by single-photon annihilation of $e^+e^-$ pairs and are summarized in the following requirements: (1) Large solid-angle acceptance in order to minimize systematic errors in estimating total cross sections and to insure maximum trigger efficiency; (2) A magnetic field to identify specific two-body final states, to put additional constraints on multiparticle final states, and to allow measurement of single-particle inclusive spectra; (3) Provision for particle identification so that electromagnetic final states can be separated from hadronic ones and for the separation of specific hadronic final states; (4) A selective trigger system capable of rejecting beam-gas backgrounds, cosmic rays, and elastic $e^+e^-$ events so that the overall data rate is kept to an acceptably low level; (5) Flexibility in the apparatus to be able to exploit any important new processes encountered at the higher energies of SPEAR.

*Work supported by the U. S. Atomic Energy Commission.
The final design for the detector involves a large solenoidal magnet (the average magnetic field is 4 kG) containing a system of wire spark chambers and scintillation counters. The detector will be interfaced to an XDS Sigma-5 computer which will log event data on magnetic tape and will provide on-line data analysis. A schematic drawing of the detector is given in the attached figure; important design parameters are summarized in the table.

A particle emanating from the interaction region within the polar angle interval $45^\circ \rightarrow 135^\circ$, traverses in sequence: (1) The beam vacuum chamber; (2) A cylindrical multiwire proportional chamber which is used in the trigger; (3) Eight gaps of small-angle stereo, cylindrical wire spark chambers furnishing the momentum measurements; (4) A cylindrical array of scintillation counters providing the basic trigger and time-of-flight information; (5) The aluminum coil of the magnet; (6) A cylindrical array of Pb-scintillator shower counters; (7) The 20-cm-thick flux return iron; (8) Two gaps of planar wire spark chambers normally used for muon identification. Overlapping spark chambers, constructed using printed-circuit techniques, are mounted near the ends of the cylindrical spark chambers to detect particles with polar angles as small as $15^\circ$.

Unique design features of the detector include the large octagonal iron box that serves as magnetic flux return, hadron filter, and support structure and the cylindrical wire spark chambers which have a common external support structure, eliminating massive support structures within the region of momentum measurement, thus giving complete azimuthal coverage.
TABLE

DESIGN PARAMETERS FOR THE SPEAR MAGNETIC DETECTOR

<table>
<thead>
<tr>
<th>Magnet</th>
<th>$B = 4 \text{ kG} \pm 1%$ over the region $\begin{cases} 45^\circ \leq \theta \leq 135^\circ \ 0^\circ \leq \phi \leq 360^\circ \end{cases}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil Diameter</td>
<td>3.2 m</td>
</tr>
<tr>
<td>Coil Length</td>
<td>3.56 m</td>
</tr>
</tbody>
</table>

Acceptance Trigger and Particle Identification System:

- $4^\circ \leq \phi \leq 360^\circ$
- $45^\circ \leq \phi \leq 135^\circ$
- $15^\circ \leq \theta \leq 165^\circ$

Momentum Resolution ($45^\circ - 135^\circ$):

- Single Tracks $\frac{\Delta p}{p} \approx 2\% - 6\%$ for $p \sim 1 \text{ GeV/c} \rightarrow 3 \text{ GeV/c}$
- Global Fit to Two-Body Final State $\frac{\Delta p}{p} \approx 2\%$

Particle Identification ($45^\circ - 135^\circ$) (3 Standard Deviation Separation):

- Single Tracks (Time of Flight)
  - $\pi$-$K$: $p \lesssim 0.7 \text{ GeV/c}$
  - $K$-$p$: $p \lesssim 1.2 \text{ GeV/c}$
- Two-Body Final State (Momentum)
  - $\pi$-$K$: $p \lesssim 1.5 \text{ GeV/c}$
  - $K$-$p$: All momenta encountered at SPEAR
Fig 1. Schematic drawing of the SPEAR detector
We have developed a modular bus-oriented computing system with the aim of carrying out rapid on-line analysis of wire spark chamber data. This system makes use of a set of bins each of which is equipped with a data transfer bus very similar in architecture to that of the DEC PDP-11. The PDP-11 serves to send out routing and operating instructions to the various computing modules in the system. Because of the possibility of building great speed and flexibility into these modules and because of the inherent parallelism of the system we have been able to achieve a speed of analysis which is substantially greater than that of the IBM 360/91. Typically we expect to analyze a set of 20 spark chambers, each with up to 16 tracks, in under 5 milliseconds. In this analysis we will determine all direction cosines and momenta and transfer the results to a PDP-9 for logging on magnetic tape.

One of the important features of the system is the ability to break the bus under program control so as to allow portions of the system to operate independently of one another and in parallel. In this way we can process the front and rear x and y spark chambers independently. The modules we have developed are flexible in having their instructions microprogrammable for specific application.

*Work supported by the United States Atomic Energy Commission.*
A general purpose hardware processor is described for use in conjunction with a digital computer to speed up reconstruction of straight line tracks from data generated in wire spark chamber experiments. The processor receives both data and operating instructions from the computer. With a 50 nsec cycle time it executes a search algorithm which simultaneously tests many possible combinations of data. It quickly sorts out the points that are likely candidates for straight lines; the computer makes the final decision. 100 spark chamber events taken at the Argonne National Laboratory were analyzed using a pure software approach and compared with an analysis using responses from the Pattern Recognition Processor as simulated by a separate program. For equivalent operations the latter performed approximately four times as fast. The expectation that the Pattern Recognition Processor will do even better as the complexity of the events increases.
The major difference between the Stanford approach and this one is that theirs is a purely digital approach using a very fast version of the computer off to the side. Ours, however, is based more on a pictorial approach using the computer and the processor together and each part doing what it can do best and relying on the other to simplify its task. A major attribute of the system is its built in flexibility, utilizing hardware whose tasks may be altered remotely by the computer. The first attempt was in straight line analysis, but searching for curves may be implemented with the same hardware requiring only a change in software. The system further has the attributes of speed and comparative low cost. We also envision that this approach can be successfully applied to a larger category of pattern recognition problems.
It is not my purpose to describe any of the new technology relating to streamer chambers but instead I shall try to convince you that the streamer chamber should no longer be considered an exotic and possibly unreliable tool.

A year ago chambers had been used in Russia, at DESY, at PPA, and SLAC. Only DESY had successfully used a liquid-hydrogen target in their chamber. Now the LBL, ANL, and SLAC chambers have completed successful data-taking runs with liquid-hydrogen targets. Several chambers are now running routinely and most are envisioned as developing into facilities operated in a manner similar to bubble chambers.

I am unfamiliar with the Russian work except to note that a 5-meter chamber is being constructed for use at Serpukhov; at CERN a chamber is being used for studies with a hyperon beam, and I list below the completed data-taking runs of some other chambers (Table I).

As far as chamber performance is concerned I can speak only of the SLAC experience. We have obtained a reconstruction accuracy of about 0.5 mm in real space. This figure includes the entire photographic process, optical corrections, magnetic field measurement and fitting, and the measurement process. From our analysis it is clear that this value is dominated by systematic errors and that improvement can be made to reduce this to about 0.3 mm.

In the photoproduction experiment on which these values are based, the available 0.5-mm accuracy resulted in a photon missing mass resolution of \( M_y^2 = 0.06 \text{ GeV}^2 \) full width at half maximum for the bremsstrahlung region between 5.5 and 7 GeV. A Gaussian fit to these data yielded a mean mass for the \( K^0 \) of 498.7 ± 0.4 MeV with a standard deviation of 5.7 ± 0.5 MeV.

A competitor of the streamer chamber is the rapid-cycling bubble chamber, for which we can anticipate a successful development; hence a comparison is useful. A major difference in performance is of course the lack of a visible interaction vertex in the streamer chamber. Gaseous target diameters of ~1 cm are practical and allow the observation of protons with momenta greater than 70 MeV/c, very similar to that obtainable using a bubble chamber. A liquid-hydrogen target of 1.3-cm diameter has been used at SLAC. Here the minimum observable momentum of a proton is ~200 MeV/c.

For the observation of ionization density a bubble chamber will prove superior to a streamer chamber. Moreover, in the streamer-chamber experiment quoted above, it was found that for tracks having laboratory momenta less than 1 GeV/c, the proton could be distinguished from a pion 86% of the time.

The streamer chamber is usually large enough to allow the measurement of events without the use of auxiliary equipment, except for tracks of very high momentum. In contrast, the small diameter of a rapid-cycling bubble chamber makes such equipment essential even for low momentum tracks.

* Work supported by the United States Atomic Energy Commission.
A major advantage of the streamer chamber is its data rate. For experiments where backgrounds determine the allowable rates one can compare the relative number of sensitive time intervals for the two devices.

At accelerators with a poor duty cycle, such as SLAC, the time intervals may be defined by the accelerator. At SLAC there are 360 pulses a second to which a streamer chamber can potentially be sensitive although 180 pulses is a more reasonable available number. Here the rapid cycling chamber will have a relatively good sensitivity depending on its cycle rate, possibly 60 to 100 pps. In such circumstances the streamer chamber must compete by its larger target length and lesser need for external measuring devices.

At high duty cycle accelerators one can operate a streamer chamber with its memory time reduced to ~2 μsec. In a one second spill it can be sensitive to 500,000 time intervals. The rapid-cycling bubble chamber on the other hand will still be confined to its pulse rate of 60-100 pps and hence very much at a disadvantage.

It is for this reason, in addition to the lack of multiple scattering and interaction problems, that we advocate the use of streamer chambers at high energy accelerators such as NAL.

### TABLE I. Streamer-Chamber Exposures Completed to Date

<table>
<thead>
<tr>
<th>Laboratory</th>
<th>Chamber Length</th>
<th>Target</th>
<th>Experiment</th>
<th>Number of Pictures</th>
<th>Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPA</td>
<td>76 cm</td>
<td>-</td>
<td>( K_L^0 \rightarrow \eta + \pi^+ + \pi^- )</td>
<td>60,000</td>
<td>3,000</td>
</tr>
<tr>
<td>DESY</td>
<td>80 cm</td>
<td>Liquid Hydrogen 3 cm</td>
<td>Photoproduction with Tagged Photons 1.6-6.3 GeV</td>
<td>150,000</td>
<td>40,000</td>
</tr>
<tr>
<td>DESY</td>
<td>1.5 meters</td>
<td>Liquid Hydrogen 9 cm</td>
<td>Inelastic Electron Scattering</td>
<td>200,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Berkeley</td>
<td>1.2 meters</td>
<td>Liquid Hydrogen 30 cm</td>
<td>( \pi^- p \rightarrow nX^0 ) Forward</td>
<td>360,000</td>
<td>60,000</td>
</tr>
<tr>
<td>Argonne</td>
<td>1.5 meters</td>
<td>Liquid Hydrogen 30 cm</td>
<td>( \pi^- p \rightarrow p + ? ) Forward</td>
<td>150,000</td>
<td>70,000</td>
</tr>
<tr>
<td>SLAC</td>
<td>2 meters</td>
<td>Gas Hydrogen 6 atm</td>
<td>Photoproduction with 16 GeV Bremsstrahlung</td>
<td>60,000</td>
<td>12,000</td>
</tr>
<tr>
<td>SLAC</td>
<td>2 meters</td>
<td>Gas Hydrogen 8 atm</td>
<td>Photoproduction with 18 GeV Bremsstrahlung</td>
<td>600,000</td>
<td>100,000</td>
</tr>
<tr>
<td>SLAC</td>
<td>2 meters</td>
<td>-</td>
<td>( K_L^0 \rightarrow \pi^+ \nu \nu ) ( E_{\nu} &gt; 10 )</td>
<td>( 2.5 \times 10^6 )</td>
<td>( 5 \times 10^5 )</td>
</tr>
<tr>
<td>SLAC</td>
<td>2 meters</td>
<td>Liquid Hydrogen 40 cm</td>
<td>( K^- p \rightarrow Hyperon )</td>
<td>( 2.3 \times 10^6 )</td>
<td>( \sim 6 \times 10^5 )</td>
</tr>
</tbody>
</table>
It has previously been shown\textsuperscript{1} that a necessary degree of localization and luminosity of tracks in a streamer chamber filled with helium may be obtained by using small admixtures of hydrocarbons in the gas and using high voltage pulses with a pulse width some hundreds of nanoseconds long.

For a number of experiments on pion scattering it is essential to have a streamer chamber filled with $^3\text{He}$ wherein short-range particles are detectable. The use of such a gas filling does not alter the principles of the discharge mechanism; there arises only the necessity of preventing losses of the isotope and providing the possibility of its purification for refilling the chamber. As in the case of $^4\text{He}$ a gas density as high as possible is desirable, therefore the chamber must be operated at elevated pressures.

The construction of the chamber was briefly described in paper\textsuperscript{2} and has been slightly changed. The streamer chamber is 50 cm in diameter and 12-cm high. The cylinder used for the chamber is made of 1-mm thick plexiglas and is closed by an optical glass plate. A vacuum rubber seal is used. Mylar windows are used at the entrance ($\phi$ 80-mm) and exit ($80 \times 200$ mm\textsuperscript{2}) of the beam. The windows are 0.1-mm thick.

A hodoscope of scintillation counters covering 0.12 of a 4\textpi solid angle surrounds the streamer chamber. The trigger for the chamber is given by a pulse from the hodoscope when a scattered (elastically or inelastically) pion passes through one of the counters.

The streamer chamber was placed together with the hodoscope in a stainless steel vessel designed to hold a pressure of 8 bars. The general view of the experimental setup is shown in Fig. 1.

The system used for filling the chamber except for some changes was the same as previously described in paper.\textsuperscript{3} The filling and purification system made it possible to add necessary amounts of $\alpha$-pinene and SF\textsubscript{6} which is used for regulating the memory time of the chamber. The working pressure of the chamber was 4 bars. The concentration of tritium after purification of the filling gas was $10^{-15}$ and therefore there was practically no background from tritium decays. The chamber was kept being evacuated to a pressure of 0.1 mbar during several days before the filling. The admixture of $^4\text{He}$ was not greater than 0.02%. There was some amount of carbon and hydrogen nuclei ($\leq 0.5\%$) in the gas due to the $\alpha$-pinene (up to 0.6 cm\textsuperscript{3} of the liquid) used for the regulation of the discharge structure. During the regeneration of the gas the admixtures were frozen down in an absorbent carbon trap at the temperature of liquid nitrogen. The stainless steel vessel was filled with nitrogen, and the pressure in the outer volume of the chamber was kept equal to 10-20 mbars less than the pressure in the streamer chamber itself. In order to avoid possible losses of the $^3\text{He}$ an emergency system is installed allowing fast evacuation of the gas from both the inner and the outer volumes of the experimental device in case of leakages.
A twenty-stage pulse generator of the Arkadiev-Marx type was used as a source of high voltage supply to the chamber. The capacitor sections were charged to a voltage of 40 kV stabilized within 0.2%. The high voltage pulse amplitude was 500 kV, the rise-time was 25 nsec, and the pulse width was 0.5 μsec. The output capacity of the pulse generator was 320 pF. The brightness of the tracks in the chamber obtained with such characteristics of the pulse generator permitted direct photography on a film with a sensitivity $S_{0.85} = 800$ units COST USSR (corresponding to 1000 ASA) through an f/2.8 lens aperture. The total time delay of the high voltage pulse relative to the passing of the particles through the chamber was 1.2 μsec.

The experimental setup was used for investigation of elastic and inelastic scattering of pions on $^3$He at an energy of 100-200 MeV. Figure 2 shows a typical event of an inelastic interaction of a pion with a nucleon of the $^3$He nucleus. Usually the chamber was exposed without refilling for 48 hours. During such runs the brightness of the tracks and the memory time remained practically unchanged after about two hundred thousand discharges. It was noticed that due to the impulse discharge the gas temperature within the inner volume of the chamber increased. For a repetition cycle equal to 0.3 sec, the corresponding increase of pressure was 10 mbar during the time required for the exposure of one film (~15 min). Usually a beam of pions 4 cm in diameter with an intensity of $10^4$ sec$^{-1}$ was passed through the chamber. One good event of elastic or inelastic scattering was registered per 10-15 triggers of the chamber. The repetition rate of the apparatus was limited by the photocameras and was practically 0.2 sec.

More than 200 thousand pictures were obtained during the exposure of the chamber filled with $^3$He and about 1 million pictures were taken with $^4$He.

The previously observed dependence of the track structure upon the ionization density remains the same for many prong events, and this is very essential for the identification of the events.

Figure 3 shows a typical five prong event of an interaction of a pion with the $^4$He nucleus. This star was taken from a group of events being analyzed in the search for double charge exchange processes of the pions.

In particle beams with fluxes of $10^4$ sec$^{-1}$ or more the statistics of events can be obtained with a greater rate with a helium streamer chamber than with a liquid helium bubble chamber of the same dimensions. The possibility of selecting certain processes and observing tracks of short range recoil nuclei is also an important advantage of the streamer chamber. This possibility of observing tracks of recoil nuclei is useful for the selection of elastic scattering events, and of coherent production processes, and widens the range of applications of such chambers in experiments at both intermediate and high energies.

References

Fig. 1. General view of experimental setup.
Fig 2  Picture of an inelastic $\pi^-^3\text{He}$ scattering event at 100 MeV, $\pi^- + ^3\text{He} \rightarrow \pi^- + p + d$
Fig. 3. Five prong event of $\pi^- + ^4$He interaction, $\pi^+ + ^4$He $\rightarrow \pi^- + 4p$. 
Argonne Effective Mass Spectrometer Facility

R. Diebold

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Argonne, Illinois 60439

The facility consists of a large aperture magnet surrounded by magnetostrictive wire spark chambers and associated equipment. It was designed and built as a collaborative effort of two Argonne groups and is shown in Fig. 1.

The beam supplied to the facility is unseparated with maximum momentum 6 GeV/c. A six-counter hodoscope at the first focus tagged the momentum to ± 0.2% and four threshold Čerenkov counters identified the beam particles as fast (\(\pi, \mu, e\)), K or proton. A maximum of 200,000 beam particles per 0.7-sec spill passed through the apparatus, with special gating to avoid extra beam tracks.

Two liquid hydrogen targets (8' and 20" long, 2" diameter) were mounted on moveable carts along with associated counters and/or spark chambers. This allowed a rapid changeover of experiments.

The magnet is one of three SCM-105 magnets at the ZGS. The gap was opened to 26 inches, giving a central field of 11.4 kG for 1 MW of power and a 40-inch effective length. A 40-counter hodoscope at the magnet exit was used in the trigger to count the number of charged particles traversing the magnet.

The spark chambers were constructed by epoxying flattened 4 mil aluminum wire onto 1 mil Mylar having 1 mil aluminum foil backing for even distribution of the high voltage pulse. The wire spacing ranged from 40/inch (K1,2,3) to 20/inch (K0,4). Each spark chamber had two Mylar sandwiches glued onto a G-10 fiberglass-epoxy frame, and a total thickness of about 0.0018 radiation lengths (including the neon gas). Inside the chambers inside the magnet, the Mylar sheets and wires were continued on out the rear of the magnet to a low field region for readout. The magnetostrictive readout lines of all chambers

Fig. 1 Plan view of the Effective Mass Spectrometer as used with a neutral trigger to study \(\pi^- p \rightarrow K^0 \Lambda\).
Fig. 2 Missing mass distribution showing $\Lambda - \Sigma^0$ separation.

Operating parameters for good multiple-track efficiency were optimized in the laboratory using a $^{106}\text{Ru}$ beta source together with permanent magnets to select and guide the 2-MeV electrons. The spark chamber gas was recirculated through a purifier and bubbled through ethanol at -5°C and 2 psig. High voltage pulses of 7 kV and 60 nsec FWHM were followed by a pulsed clearing field of 200 V for 6 msec on top of an 80 V dc clearing field. Up to 50 triggers per 0.7-sec spill were recorded with a minimum dead time of 10 msec. The chamber sensitive time was about 1 µsec, and the high voltage pulse was applied 400 nsec after each event.

The K3 chambers had only one readout per gap (giving horizontal information), these chambers made an important contribution to the good momentum resolution of the system. All other chambers had two readouts giving horizontal and vertical information, the vertical information from the k1 and k2 chambers came from wires oriented ± 30° from vertical (also used to resolve ambiguities). Chamber resolution was about ± 0.5 mm.

The counter and spark chamber information was read into an EMR-6050 computer via SAC electronics, up to four sparks per readout were digitized with 10 mil least count. The computer had 32 K of 24-bit word memory, 1.9 µsec cycle time, floating point hardware, priority interrupts, a CRT display, two fast tape units, and other peripherals. It not only logged the raw data onto tape, but also analyzed a sample of the events (typically 10 events in the 3 seconds available between beam spills) and displayed the results on the CRT.

An essential feature of the spectrometer is its good resolution. Fig. 2 shows the $\Lambda, \Sigma^0$ missing mass separation for the reactions $\pi^- p \rightarrow K^0(\Lambda, \Sigma^0)$ with $K^0 \rightarrow \pi^+ \pi^-$ being detected by the spectrometer. The separation is very clean at 3 GeV/c (rms width $\pm 13$ MeV), at 6 GeV/c the peaks have merged ($\pm 34$ MeV), but the cross sections can still be unambiguously determined with a computer fit. The good effective mass resolution (typically ± 3 or 4 MeV with little dependence on energy) for two-particle systems coming through the spectrometer, combined with the capability for high statistics, make the spectrometer ideal for studying line structure in $\pi^- p, K^0 K^+, n^- K^+$, and pp mass spectra. A large Cerenkov counter (not shown in Fig. 1) was recently placed behind the K5 chambers to aid in $K^0 K^+$ and pp studies.

The spectrometer first received beam in...
May 1971, and data were collected for five experiments during the period August 1971 to April 1972. Several papers reporting results (mainly preliminary) from these experiments were presented to this conference.

1,2) Wicklund et al (No. 355 and 734), "Comparison of Particle-Antiparticle Elastic Scattering from 3 to 6 GeV/c." About 300,000 good events were obtained, mainly in the cross-over region, with some data out to -t 1.5 GeV^2. Final results at 7 GeV/c are shown in Fig. 3 and have been submitted to Phys. Rev. Letters.

3) Ayres et al (No. 736), "Structure in the π^0 π^0 Mass Spectrum." Eventually more than 500,000 events are expected, including results from both π^+ and π^- incident on deuterium, allowing a detailed study of ρω interference.

4) Ayres et al (No. 736), "Meson Resonance Studies with K^+K^- and pp Spectra." The data will give information on π^+p - φπ, f ↔ A_2 interference, etc., with 24,000 events at 6 GeV/c, as well as data at 4 and 5 GeV/c. About 8,000 pp events were obtained at 6 GeV/c.

5) Knasel et al (No. 459), "Λ and Nucleon Resonance Production in pp Collisions at 6 GeV/c." About 1,000 Λ events were found in the data analyzed so far (~15% of total) as well as a large number of Δ events.

6) Yovanovitch et al (No. 298), "Λ Polarization in Semi-Inclusive π^+p Reactions from 3 to 6 GeV/c." Results are based on about 33,000 events.

7) Yovanovitch et al (No. 299), "The Reactions π^+p - K^0Λ^0 and π^-p - K^-Λ^0 at 3, 4, 5 and 6 GeV/c." About 46,000 events were found in the region -t < 0.7 GeV^2, data on Λ production as well will be analyzed.

8) Yovanovitch et al (No. 300), "The Baryon Exchange Process π^-p - ΛπK^0 at 3, 4, 5 and 6 GeV/c." About 4,000 ΣK events were used to measure both dσ/dt and the Λ polarization.

The facility has a busy future ahead with seven proposed experiments waiting to be run. Fortunately, experiments can be run in a fast, efficient manner since the system is well understood and debugged, and these experiments represent less than a year of running.

References


Organizer: J. P. Blewett

Accelerators and Storage Rings
# FUTURE ACCELERATORS AND STORAGE RINGS

*(Panel Discussion)*

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<th>Speaker</th>
</tr>
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<td>1. The Future of Weak and Electromagnetic Interactions</td>
<td>S. Berman (SLAC)</td>
</tr>
<tr>
<td>2. The Future of Strong Interactions</td>
<td>S. J. Lindenbaum (BNL)</td>
</tr>
<tr>
<td>3. New European Ideas</td>
<td>B. Gregory (Ecole Polytechnique)</td>
</tr>
<tr>
<td>4. Electron-Proton Interactions at DESY</td>
<td>W. Paul (DESY)</td>
</tr>
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<td>5. Recycling SLAC, SPEAR, and PEP</td>
<td>B. Richter (SLAC)</td>
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<td>6. ISA</td>
<td>F. Mills (BNL)</td>
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<td>7. Future Developments at NAL</td>
<td>R. R. Wilson (NAL)</td>
</tr>
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</table>
The panel discussion on Accelerators and Storage Rings ranged from brief presentations of the present status of major accelerators to reasonably accurate forecasts of the near future and more tentative speculation about the more distant future. The speakers included B. Gregory, former Director General of CERN, now at the Ecole Polytechnique, W. Paul, Director of DESY; S. Berman and B. Richter of SLAC, R. R. Wilson, Director of NAL, and S. J. Lindenbaum and F. E. Mills of Brookhaven. The Session Organizer was J. P. Blewett of Brookhaven.

The stage was set by S. Berman and S. J. Lindenbaum who outlined the areas in high-energy physics in which new goals will be set for accelerator and storage-ring designers. Both placed major emphasis on the importance of colliding beams. S. Berman began by quoting Feynman's acceptance speech for the Richtmyer award last February in which he said, "The experiments on deep-inelastic scattering are most physically explained by supposing that the proton is composed of a minute substructure of elementary point particles (partons)". To probe into this structure we need a delicate precise tool of higher energy than now is available. The ideal approach seems to be to study collisions between high-energy electrons and high-energy protons.

With a positron-electron-proton colliding-beam system as proposed by SLAC and LBL under the name of PEP four areas would become accessible:

1. Deep-inelastic scattering of electrons on protons The question as to whether scaling is confirmed or violated at higher energies can be answered with the most profound consequences whatever is the answer. For example, a violation of scaling would mean a new energy scale for hadronic phenomena such as the threshold for parton production.

2. Weak interactions Extrapolation of coupling constants from the energy range of beta-decay to the CERN and Brookhaven neutrino experiments yields consistent results. If this concept can be extrapolated to the 50-100 GeV in the center-of-mass proposed for the new colliding-beam machines, the weak interactions could become stronger than the electromagnetic forces. Reactions of interest include production of neutrinos and hadrons in electron-protons collisions (the neutrino's appearance would follow from the hadron momentum imbalance) and production of W particles in electron-proton collisions. W-particle production would probably be more copious in electron-positron collisions but must be identified in the presence of a large strong-interaction background.

3. Photoproduction An electron scattered from a proton will produce a spectrum of real photons which will make possible a study of photon-proton reactions.

4. Electron-positron scattering and colliding-beam experiments. All production processes involving hadrons, leptons, and photons can be studied.

S. Berman made the significant observation that at the 1968 Conference about 15 per cent of the papers related to weak and electromagnetic interactions. In 1970 that fraction had increased to 25 per cent and at the present Conference it has reached 50 per cent.
S. J. Lindenbaum concentrated on strong interactions and supported the conclusion of the previous speaker that a major step upward in center-of-mass energy is highly desirable. Most present models--Regge, scaling, limiting fragmentation, and others--predict asymptotic behavior. A review of present experimental data does not yet show a violation of asymptotic behavior and S. J. Lindenbaum does not believe that an accelerator of an energy as high as 2000 GeV will necessarily indicate a breakdown of such behavior. He does feel, however, that a proton-proton colliding-beam system equivalent to a $10^5$ - $10^6$-GeV accelerator has an excellent chance either of establishing asymptotic behavior or of showing a new energy domain. Many interesting searches would become possible with the 200-GeV proton-proton intersecting storage accelerators (ISABELLE) proposed at Brookhaven. The existence of quarks and of partons or other particles of masses between 10 and 100 GeV could be demonstrated.

CERN's status and plans were presented by B. Gregory who began by mentioning CERN's smallest and largest rings. The PS booster has accelerated protons to 800 MeV and has injected a beam into the PS, and the SPS (super-proton synchrotron) group has placed orders for enough magnets to bring the machine to an energy of 200 GeV. The original 1976 completion date still holds for the first phase of the machine construction. The choice of magnets for the second phase is still in doubt. If the gaps in the ring are filled with room-temperature magnets, the peak energy can be doubled. If, on the other hand, a conversion is made to superconducting magnets, it is conceivable that the machine might reach an energy of 1000 GeV. Superconducting magnets are under study by the Rutherford Laboratory, by Saclay, and by the Nuclear Research Center at Karlsruhe. All three are working in concert with European industry. A decision on whether or not to accept superconducting magnets will be made during 1973.

The ISR has been run to a maximum current of 12.5 A; normal operation is at 7 or 8 A. Maximum luminosity is $1.2 \times 10^{30}$ cm$^{-2}$ sec$^{-1}$ which, the speaker emphasized, is four orders of magnitude lower than peak luminosities quoted in recent speculations about new machines. Two recent ISR runs are of interest. At the end of June the two ISR beam's were accelerated to 31 GeV, yielding a center-of-mass energy equal to that of a 2000-GeV accelerator bombarding a stationary target. In a second run, the two beams were run at different energies for the convenience of the users.

After establishing the fact that in presenting more advanced speculations, he does not necessarily represent CERN, the CERN Council, or a consensus of European opinion, B. Gregory turned to the distant future. Nothing new will be started until the SPS is in operation, but this will be in a few years and more advanced thinking can well begin now. Two ISR studies were quoted—one on p-p rings and one on e-p colliding-beam systems. For the p-p system, the assumptions were circulating proton currents of 15 A, an allowable tune shift due to beam-beam interactions of 0.01 units, a normalized emittance of $30 \mu$ rad-m, a beta-value at the beam crossing of 3 m and a length of beam crossing of 0.67 m. With these choices the luminosity of the system is $10^{33} E/100$ cm$^{-2}$ sec$^{-1}$ where $E$ is energy in GeV, this figure is consistent with figures derived in Brookhaven's ISABELLE study. Could this system be mounted in the ISR tunnel? The conclusion was that with superconducting magnets yielding fields of 5 - 6 T, the maximum energy would be 120 GeV.
The electron-proton study fixed the proton energy at 400 GeV and the electron energy at 135 GeV to give a center-of-mass energy of 90 GeV. Other parameters chosen were $10^{13}$ particles in each ring, radii of 1100 m for the proton ring and 55 m for the electron ring, beta values of 1 m for the proton ring and 0.1 m for the electron ring, 3 MW of rf power in the electron ring and bunches of both protons and electrons 20-cm long. The luminosity in the system would be $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$.

The details of these advanced studies can be found in CERN reports ISR Th 72/33 and ISR Th 72/16.

It seems to be felt at CERN that the very different parameters that optimize the p-p and e-p systems indicate that it will be very difficult to build a combined system in which both types of collisions can take place.

In conclusion B. Gregory pointed out that before a new project can be started there are scientific, political, technical, and economic problems to be solved.

A more modest, but less distant project for studying e-p collisions was described by W. Paul. At DESY, work will begin next year on injection of protons into one of DESY’s electron-positron storage rings. A 1-GeV proton synchrotron having a Van de Graaff injector would serve as the source of protons. The first e-p collisions could be observed in 1976 or so.

Stanford plans for recirculating the beam from the 20-GeV linear accelerator were summarized by B. Richter who reported also on the status of the 3-GeV electron-positron ring SPEAR, and PEP, the positron-electron-proton rings under study by SLAC and the Lawrence Berkeley Laboratory.

The Recirculating Linear Accelerator or RLA will have three possible operating modes. In the high-energy mode the accelerated beam from the linac will be bent through 180° in such a fashion that it can be returned down the linac tunnel to be reinjected into the linac and accelerated again. Maximum energy will be more than 43 GeV, current will be $7 \times 10^{13}$ electrons per second, the repetition rate will be 360 pulses per second as it is now and the duty cycle will also be unchanged at $5 \times 10^{-4}$. In a "high duty cycle" mode, the recirculating system will be used as a storage ring with a small amount of beam peeled out on each recirculation. Maximum energy in this mode will be about 20 GeV. The current as before will be $7 \times 10^{13}$ electrons per second. The repetition rate would be 43 kHz and the duty cycle would be 0.07. The third operating mode would be the present linac mode with a maximum energy of about 25 GeV and a current $2 \times 10^{14}$ electrons per second. Completion of the RLA could be within 2 1/2 years from authorization.

At the time of the Conference the SPEAR ring had been in operation for 11 weeks. Operation has been at 1.5 GeV where currents of 50 mA have been reached (limited by coherent instabilities) and, for currents of 20 mA in each beam the luminosity attained is $2 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$. Steady progress is expected toward the design figures of 2.8 GeV, 250 mA of circulating current and a luminosity of $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$. Single beam instabilities observed are always coherent and therefore controllable. Beam-beam instabilities seem to permit tune shifts up to 0.03, in good agreement with the allowable figures observed elsewhere of about 0.025. An experimental program using SPEAR will begin at the end of 1972.
PEP, the positron-electron-proton system, has been studied to the extent that it seems certain that there are no real problems and the system can indeed be built. Energies contemplated are 15 GeV for the electrons and positrons and 72 GeV for the protons. If high-field superconducting magnets were used, the proton energy could be increased to about 150 GeV. Both rings would be housed in the same tunnel having a radius of about 200 m. Luminosities for both electron-positron and for electron-proton collisions would be about $10^{32}$ cm$^{-2}$ sec$^{-1}$. A proposal for PEP will be submitted in time that construction could be approved for FY 1976.

Brookhaven's 200-GeV Intersecting Storage Accelerators (ISABELLE) were the subject of a talk by F. E. Mills. Here the initial objective is production of two 200-GeV proton beams by acceleration of 30-GeV beams injected from the AGS. High luminosity is to be achieved by the automatic adiabatic damping that occurs during acceleration, small crossing angles, reduction in beta values at the intersection points, and bunching of the proton beams. Reasonable values of machine parameters lead to luminosities of the order of $10^{33}$ cm$^{-2}$ sec$^{-1}$. Brookhaven is confident that the ring magnet systems can be superconducting, operated initially at 40 kG. Six straight sections will be included, two will be 400-m long and the other four will have lengths of 220 m. Options to be exploited later will include addition of an electron ring to make possible e-p collisions, acceleration of deuterons on one or both rings, and possibly acceleration of antiprotons.

The final speaker was R. R. Wilson, who presented a scheme for converting the NAL accelerator from the present 200-300 GeV in the Laboratory to a colliding-beam system having 200 GeV in the center-of-mass. First, the present machine would be converted into a 1000-GeV accelerator by addition in the present ring of a set of superconducting magnets capable of operation at 45 kG. The beam at maximum energy in the present ring would be transferred to the superconducting ring and accelerated further to 1000 GeV. The 1000-GeV ring would be transferred to a bypass, where it would undergo collisions with a 10-GeV beam accelerated in the present booster and stored in a small storage ring. The 10 vs 1000-GeV collisions would make available 200 GeV in the center-of-mass. The luminosity was estimated at $10^{30}$ cm$^{-2}$ sec$^{-1}$. A factor of 100 in counting rate would however be gained by the asymmetric nature of the collision which would collimate collision products in the direction of the 1000-GeV beam.
Contributed Papers
AXIOMATIC FIELD THEORY

(papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

17 A Proof of the Pomeranchuk Theorem and Related Theorems Deduced From Unitarity
T.N. Truong and W.S. Lam

26 Generalizations of the Goldstone and the Coleman Theorems
Y. Dothan and E. Gal-Ezer

* 82 Generalized Free Fields and the Representations of Weyl Group
J. Lukierski; K. Sienkiewicz

** 83 Lee Model with V-Particle having Continuous Spectrum of Asymptotic Masses
J. Lukierski and M. Oziewicz (Acta Physica Polonica B3,231(1972))

** 84 On the Definition of Interacting and Asymptotic Currents in the Quantum Field Theory of Currents

273 Properties of the ( :φ^4:)_{1+1} Interaction Hamiltonian
B. Gidas

274 Unitary Renormalization in Model Field Theories
B. Gidas

312 Asymptotic Equality of Cross-Sections for Line-Reversed Reactions
H. Cornille; A. Martin

313 A "Pomeranchuk" Theorem for Elastic Diffraction Peaks
H. Cornille; A. Martin

314 Constraints on the Phase of Scattering Amplitudes due to Positivity
H. Cornille; A. Martin

376 Analytic Continuation in the Dimension of Space-Time
G.M. Cicuta

497 Minimally Non-Local Electromagnetic Interactions
M. Petrag

* 607 The Relativistic Theory of the Charged Oscillator
V.I. Belinicher

* 608 Relativistic Wave Equations for Particles of Arbitrary Spin Interacting with an Electromagnetic Field.
V.I. Belinicher

637 Remarks on Conformal Invariance
J.A. Swieca, A.H. Völkel

787 A Rigorous Parametric Dispersion Representation with Three-Channel Symmetry
G. Auberson, N.N. Khuri

788 Recent Progress on Analyticity of n Point Amplitudes
A. Martin

855 The Generalized Goldstone Theorem
A.N. Vassilev

913 Energy-Momentum of a Gravitational Field
V.N. Polomeshkin

On Automodel Asymptotic in Quantum Field Theory II
N.N. Bogolubov, A.N. Tavkhelidze, V.S. Vladimirov
CURRENTS I

ELECTRON-POSITRON INTERACTIONS

(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

87 Continuation from Deep Electroproduction to Deep Electron-Positron Annihilation
R. Gatto; P. Menotti; I. Vendramin

89 Deep Electroproduction and Deep Electron-Positron Annihilation
R. Gatto; P. Menotti; I. Vendramin

107 The Process $e^+e^- \rightarrow e^+e^- + \text{Hadrons} \quad \text{(General Formalism and } \pi \pi \text{ Production)}$
C.J. Brown, D.H. Lyth

108 Unitarity and the Reaction $\gamma\gamma \rightarrow \mu^+\mu^-$
D.H. Lyth

119 Electron-Positron Annihilations into Hadrons at High Energies
H. Cheng, T.T. Wu

178 Initial Results on $e^+e^- \rightarrow \mu^+\mu^-$

194 The Problem of Continuation from Deep Inelastic Electron Scattering to Deep Electron-Positron Annihilation
R. Gatto, G. Preparata

296 Possible Method of Searching for a Weak Neutral Current in $e^+e^- \rightarrow \mu^+\mu^-$
V.K. Cung; A.K. Mann; E.A. Paschos

374 Quasi Two Body $e^+e^- \rightarrow \text{Annihilation}$
G. Kramer, T.F. Walsh

456 Multihadronic Cross Sections from $e^+e^- \rightarrow \text{Annihilation}$ at Adone in the Energy Range 1.3 - 3.0 GeV

560 Multi Hadron Production by Electron Positron Colliding Beams and the Probable Existence of the $\rho'(1600)$

561 Preliminary Evidence for the Decay Scheme $\rho'(1600) \rightarrow \rho^0(765) + \pi^0(\pi n800)$

562 Multi-Hadron Production by Electron-Positron Beams Colliding with a Total Energy of 3 GeV
F. Ceradini, M. Conversi, S. D'Angelo, L. Paoluzi, R. Santonico; G. Barbarino, R. Visentin

724 Decay Correlations of Heavy Leptons in $e^+e^- \rightarrow \mu^+\mu^-$
Y-S. Tsai (Phys. Rev. D4, 2821(1971))

**840 Charge Asymmetry of the Muon Angular Distribution in the Process $e^+e^- \rightarrow e^+\mu^-$
I.B. Khriplovich
Observation of Multihadronic Events in $e^+e^-$ Collisions at the Energy of 1.18 - 1.34 GeV
L.M. Kurdadze, A.P. Onuchin, S.I. Serednyakov, V.A. Sidorov, S.I. Eidelman

The Measurements of the Pion and Nucleon Form-Factors in a Timelike Region of 1.72 $f^2$, 2.23 $f^2$, and 2.88 $f^2$ Momentum Transfers

Polarization Effects in Weak Interactions in Colliding $e^+e^-$ Beams at High Energies
G.V. Grigoryan, V.A. Khoze

Measurement of the Electron-Positron Annihilation Cross-Section into $\pi^+\pi^-$ and $K^+K^-$ Pairs at the Total Energy 1.18-1.34 GeV

Observation of Multihadronic Events in $e^+e^-$ Collisions at the Energy of 1.18 - 1.34 GeV
L.M. Kurdadze, A.P. Onuchin, S.I. Serednyakov, V.A. Sidorov, S.I. Eidelman
CURRENTS II

DEEP INELASTIC SCATTERING (ELECTRONS, MUONS, NEUTRINOS)

(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

310 Study of π⁻ Inclusive Production in γd Interactions with a 7.5 GeV Linearly Polarized Photon Beam

317 The Döppler Effect in the Extraction of Total Neutron Cross Sections from Deuterium Data with Particular Emphasis on Asymptotic Hadron and Deep Inelastic Electron Scattering
G.E. West

323 A High-Quality Muon Beam at SLAC for High-Energy Lepton-Hadron Scattering Work
S.M. Flatte, C.A. Heusoch, A. Seiden

401 Deep Inelastic Scattering of Polarized Electrons by Polarized Protons

** 412 A study of the Inclusive Reaction yp → π⁺ (Anything) with Polarized Photons at 2.8, 4.7, and 9.3 GeV

420 A Measurement of the Pion Form Factor at k² = -2.0 GeV²

440 Preliminary Results on the Inclusive Electroproduction of Hadrons

548 Inelastic Electron-Proton and Electron-Neutron Scattering in the Resonance Region
M. Röberling, J. Moritz, K.H. Schmidt, D. Wegener, D. Zeller; J. Bleckwenn

632 Test of Scaling in Muon-Nucleon Inelastic Interactions at Small q²
P.L. Jain, R.D. Malucci, M.J. Potoczak

633 Inclusive Reaction μ + Nucleon → π + Anything at 10.1 and 15.8 GeV/c
P.L. Jain, Z. Ahmad, R.D. Malucci, M.J. Potoczak, B. Girard

639 Electroproduction in a Streamer Chamber at 7.2 GeV and 0.3 < Q² < 1.5 GeV² Part I: Experimental Setup, Multiplicities, and Inclusive π⁻ Spectra

640 Electroproduction in a Streamer Chamber at 7.2 GeV and 0.3 < Q² < 1.5 GeV² Part II: Study of Δ⁺ and η Production in the Reaction ep → epπ⁺π⁻
Electroproduction of $\pi^-\Delta^{++}(1236)$ and of $\pi^+\Delta^0(1236)$ on Hydrogen
I. Dammann, C. Driver, K. Heinloth, G. Hofmann, P. Janata, P. Karow,
D. Lüke, D. Schmidt, G. Specht

Inclusive Photoproduction of Pions, Kaons, and Protons at 6 GeV
H. Burfeindt, G. Buschhorn, H. Genzel, P. Heide, U. Kötz, K.-H. Mess,
P. Schmäser, B. Sonne, G. Vogel, B.H. Wiik

Measurement of Inclusive Photoproduction at 3.2 GeV and Comparison
with Electroproduction
H. Burfeindt, G. Buschhorn, H. Genzel, P. Heide, U. Kötz, K.-H. Mess,
P. Schmäser, B. Sonne, G. Vogel, B.H. Wiik

Inclusive Photoproduction of $\pi^+$ and $p$ at Energies Up to 6.3 GeV,
and Comparison to Electroproduction
W. Struczinski; P. Dittmann, V. Eckardt, P. Joos, A. Ladage, H. Meyer,
B. Naroska, D. Notz, S. Yellin; G. Hentschel, J. Knobloch, E. Rabe;
S. Brandt, M. Grimm, D. Pollmann; I. Derado, R. Meinke, F. Schacht,
H. Strobl

A Note on Muon Energy Loss by Photonuclear Interaction
M. Strovink

Inclusive $\pi^+$ and $\pi^-$ Distributions in Electroproduction on Protons
I. Dammann, C. Driver, K. Heinloth, G. Hofmann, P. Janata, P. Karow,
D. Lüke, D. Schmidt, G. Specht

A Measurement of Muon-Pair Photoproduction in the Deep Inelastic Region
J.P. Davis, S. Hayes, R. Imlay, P.C. Stein, P.J. Wanderer

$\pi^0$ Electroproduction at the First Resonance at Momentum Transfers
$q^2 = 0.6$, 1. and 1.56 GeV$^2$
J.C. Alder, F.W. Brasse, E. Chazelas, W. Fehrenbach, W. Flauger,
K.H. Frank, E. Ganssauge, J. Gayler, V. Korbel, J. May, M. Merkwitz;
A. Courau, G. Tristram, J. Valentín

Measurements of Inclusive Hadron Momentum Distributions in Deep Inelastic Electroproduction
J.C. Alder, F.W. Brasse, E. Chazelas, W. Fehrenbach, W. Flauger,
K.H. Frank, E. Ganssauge J. Gayler, V. Korbel, W. Krechlok, J. May,
M. Merkwitz, P.D. Zimmerman

Deep Inelastic Electron Scattering from Hydrogen and Deuterium
A. Bodek, M. Breidenbach, W. Ditzler, J. Elias, J. Friedman,
H. Kendall, J. Poucher, E. Riordan, M. Sogard, D. Coward, E. Bloom,

Inelastic Proton Distributions in Coincidence Electroproduction
L. Ahrens, K. Berkelman, G. Brown, D.G. Cassel, W.R. Francis,
P. Garbincius, D. Harding, D.L. Hartill, J.L. Hartmann, R.L. Loveless,
R.C. Rohlfs, D.H. White; A.J. Sadoff

Preliminary Report on a Study of Scaling in the Inclusive
Electroproduction Reactions $e^- + p \rightarrow e^- + \pi^- + X$
C.J. Bebek, C.N. Brown, C.A. Lichtenstein, M. Herzlinger,
F.M. Pipkin, K. Sisterson; D.L. Hartill; N. Hicks

Analysis of Photo and Electroproduction Data Against $\omega_W$
F.W. Brasse, E. Chazelas, W. Fehrenbach, K.H. Frank, E. Ganssauge,
J. Gayler, V. Korbel, J. May, M. Merkwitz; V. Rittenberg,
H.R. Rubinstein

Separation of $\sigma_\pi$ and $\sigma_\omega$ at $q^2 \sim 1$ (GeV/c)$^2$ in the Resonance Region
J.C. Alder, F.W. Brasse, E. Chazelas, W. Fehrenbach, W. Flauger,
K.H. Frank, E. Ganssauge, J. Gayler, W. Krechlok, V. Korbel, J. May,
M. Merkwitz, P.D. Zimmerman
Production of $K^0_L$ Mesons and Neutrons from Electrons on Beryllium Above 10 GeV

A Study of Hadronic Final States from Inelastic Muon Scattering in a Hybrid Bubble Chamber Experiment
CURRENTS III

SCALING PHENOMENA (AND OTHER THEORETICAL MATTERS)

(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

11 Spontaneous Breakings of Chiral Symmetries II. Mass Relations and Particle Mixings
G. Cicogna, F. Strocchi, R. Vergara Caffarelli

19 Surface Dominance in Deep Inelastic Electron Scattering
H. Goldberg

22 Anomalies of Bilocal Operators
H.J. Schnitzer

** 29 Influence of Higher-Order Cut Contributions and of the Asymptotic Behaviour of the \( \pi \pi \) Scattering Phase Shift on the Decreasing Properties of the Isovector Nucleon Form Factors
U. Brail, R. Rodenberg (Nuovo Cimento 8A, 381 (1972))

** 30 Threshold Relations for Inelastic Neutrino-Nucleon Interactions
J. Cleymans, R. Rodenberg (Phys. Rev. D5, 1205 (1972))

31 Scaling Behaviour in a Quark-Parton Model with Second Class Currents
B-R. Kim, R. Rodenberg

32 Deep Inelastic Lepton-Nucleon Scattering and the Core Structure of Nucleons in the Framework of the Quark-Parton Model
B-R. Kim, R. Rodenberg

* 37 The Effect of Virtual Photon Mass on Transverse Momentum Distributions in Inclusive Processes
H.D. Ibarbanel, J.B. Kogut

39 Polarization Effects in Neutrino and AntiNeutrino Scattering. I - General Results
M. Gourdin

40 Inelastic Lepton (Anti) Lepton Scattering and Two Photon Exchange Approximation
G. Bonneau, M. Gourdin, F. Martin

41 Semi-Inclusive Reactions Induced by Leptons
M. Gourdin

58 Electromagnetic Form Factors and Vector Meson Coupling Constants
N. Zovko

* 70 Composite Theory of Large Angle Scattering and New Tests of Parton Concepts
J.F. Gunion, S.J. Brodsky, R. Blankenbecler

* 71 Composite Theory of Inclusive Scattering at Large Transverse Momenta
J.F. Gunion, S.J. Brodsky, R. Blankenbecler

* 72 Phenomenology of Photon Processes, Vector Dominance and Crucial Tests for Parton Models
S.J. Brodsky, F.E. Close, J.F. Gunion

* 73 A Test for Fractionally Charged Partons from Deep Inelastic Bremsstrahlung in the Scaling Region
S.J. Brodsky, J.F. Gunion, R.L. Jaffe

* 79 Is the Adler Sum Rule for Inelastic Lepton-Hadron Processes Correct?
J.D. Bjorken, S.F. Tuan
Continuation from Deep Electroproduction to Deep Electron-Positron Annihilation
R. Gatto; P. Menotti; I. Vendramin

Analytic Continuation of Scaling Functions
P. Menotti

Deep Electroproduction and Deep-Electron-Positron Annihilation
R. Gatto; P. Menotti; I. Vendramin

A Comparison Between the Bilocal Algebra on the Light Cone and the Parton Model
M. Chaichian (Nucl. Phys. B42, 333(1972))

Various Aspects of Divergent Electromagnetic Self-Masses
K. Morita

The Process $ee \to ee + \text{Hadrons}$ (General Formalism and $\pi\pi$ Production)
C.J. Brown, D.H. Lyth

Mass Divergences and Callan-Symanzik Equations in Quantum Electrodynamics
A. Sirlin

Light Cone Constraints and Two Asymptotic Domains in Lepton-Nucleon and Meson-Nucleon Scattering
T. Das, L.K. Pandit, P. Roy

Scaling and the Asymptotic Behaviour of Form Factors
P.P. Divakaran, G. Rajasekaran

What Neutrinos Can Tell Us About Partons
R.P. Feynman

Scaling Behavior in a Four Dimensional Solvable Field Theory
W.S. Hellman, J. Oliver

Symmetry Relations for Deep Inelastic Processes
H.J. Lipkin; E.A. Paschos

The Nature of Hadronic Constituents and the Asymptotic Behavior of Off-Shell Form Factors
J. Sucher; C.H. Woo

A Family of Sum Rules for Weak Electromagnetic and Strong Processes
L. Žukaszuk

New Constraints on the Pion Electromagnetic Form Factor and the Chou-Yang Model
Z. Aiduk, L. Žukaszuk

On the Neutrino-Electron Cross Sections at High Energies
L. Žukaszuk

Direct Derivation of the Soft Pion Relation Between the $\gamma + 3\pi$ and $\pi^0 + \gamma\gamma$ Amplitudes
C. Ryan

The Problem of Continuation from Deep Inelastic Electron Scattering to Deep Electron-Positron Annihilation
R. Gatto, G. Preparata

Electromagnetic Mass Differences
D.P. Majumdar, J. Stern, Y. Tomozawa

Light Cone Current Algebra and Regge Couplings
M. Testa
The Algebra of Vertex Strengths and the Light Cone
N. Cabibbo; M. Testa

Physical Unitarization of Indefinite Metric Theories by Shadow State
Summation, Bjorken Scaling, and Light Quarks
C.A. Nelson

Low Energy Limit in Hard Pion Amplitudes and Magnetic Moment of
Charged p-Mesons
R. Shtokhamer, P. Singer

Generalized Vector Dominance and Inelastic Electron-Nucleon
Scattering II: The Neutron-to-Proton Ratio
J.J. Sakurai, D. Schildknecht

Generalized Vector Dominance and Inelastic Electron-Proton Scattering
J.J. Sakurai, D. Schildknecht (Phys. Lett. 40B, 121 (1972))

Relation Between the Axial Vector and Magnetic Moment Form Factors
of Nucleon
Riazuddin; Fayyazuddin

Light Cone Approach to Structure Functions in a Theory of Weak and
Electromagnetic Interactions
Riazuddin; Fayyazuddin

Inclusive Spectra in Deeply Inelastic ep Collisions
C. Quigg, J-M. Wang

Optimal Lower Bounds on the Hadronic Contribution to the Muon
Anomalous Magnetic Moment
G. Nenciu, I. Raszillier

Quasi Two Body e+e− Annihilation
G. Kramer, T.F. Walsh

Generalized Vector Meson Dominance of Electromagnetic Interactions
in a Relativistic Quark Model
M. Böhm; H. Joos, M. Krammer

Reciprocal Bootstrap on the Light Cone
A. Zee

Spectral Function Sum Rules and ω-φ Mixing
B.G. Kenny

Quantum Electrodynamics and Renormalization Theory in the Infinite
Momentum Frame
S.J. Brodsky, R. Roskies

Unified Light Cone Treatment of Scaling and a Positivity Constraint
on Short Distance Behaviour
K.M. Bitar

Light Cone Behavior of Perturbation Theory
N. Christ; B. Hasslacher; A. Mueller

Some Speculations on High Energy Quantum Electrodynamics
K. Johnson; M. Baker

Broken Scale Invariance, Current Algebra, and Massive "Gravitation"
I. General Formulation
P. Nath, R. Arnowitt, M.H. Friedman

Scaling Conditions and Applications
M.H. Friedman, P. Nath, R. Arnowitt
A Theory of Currents
R. Perrin

Status of the Cabibbo-Radicati Sum Rule and a New Test of Vector Meson Dominance
G.J. Gounaris

Testing Triplet Models
H. Suura; T.F. Walsh; B-L. Young

Light Cone Algebra, Field-Current Identity and Deep Inelastic Scattering
R. Arnowitt, M.H. Friedman, P. Nath

The Scaling Limit of Longitudinal Virtual Compton Cross Sections
J.E. Mandula

Sum Rules for Real Parts of Current-Particle Scattering Amplitudes
R.L. Heimann, A.J.G. Hey, J.E. Mandula

Tests for Neutral Currents in Neutrino Reactions
E.A. Paschos, L. Wolfenstein

Applications of the Quark Parton Model in One Particle Inclusive Leptonic Induced Reactions
M. Gronau, F. Ravndal, Y. Zarmi

Bounds on Deep Inelastic Structure Functions
S. Pallua, B. Renner

Can the Adler Inelastic Lepton-Hadron Sum Rule Ever Be Satisfied?
J.J. Sakurai, H.B. Thacker; S.F. Tuan

Canonical Scaling and Conformal Invariance

Manifestly Conformal Covariant Operator-Product Expansion
S. Ferrara, A.F. Grillo; R. Gatto (Nuovo Cimento 2, 1363(1971))

Conformal Algebra in Two Space-Time Dimensions and the Thirring Model
S. Ferrara, A.F. Grillo; R. Gatto

Symmetry Principles and Constraints on Deep Inelastic Structure Functions
M. Chaichian, S. Pallua

Iso-Scalar Part of Nucleon Electromagnetic Form Factor
S. Furuichi

On a Fixed Pole in Virtual Compton Scattering Amplitude
Yu.A. Rakov, V.A. Tsarev ( Yadern. Fiz. 15, 1032(1972))

Recent Theoretical Work- e^+e^- Annihilation and Continuation from Inelastic Electron Scattering
S.D. Drell

Generalized Vector Dominance and Inelastic Electron Nucleon Scattering. III: The Small \omega' Region
J.J. Sakurai; D. Schildknecht

Theoretical Problems in Deep Inelastic Scattering
E. Leutwyler, P. Ottersen

Light Cone Structure of Infinite Component Fields
H. Bebič, V. Gorgé, H. Leutwyler
637 Remarks on Conformal Invariance
J.A. Swieca, A.H. Völkel

** 657 Deep Inelastic Lepton-Nucleon Interactions
C.P. Wang; A.L.L. Lin (Nuovo Cimento 4,93(1972)

685 Neutron Polarizability and the n-e Scattering Length
J. Bernabeu, T.E.O. Ericson

* 690 Threshold Relations for Inelastic Scattering of Polarized Leptons from Polarized Nucleons
J. Cleymans

* 701 Axial Vector Anomalies and the Scaling Property of Field Theory
A. Zee

* 706 Coincidence Electroproduction Reactions and Positivity Restrictions
A. De Rújula; M.G. Doncel; E. de Rafael

* 722 Improved Weizsacker-Williams Method and Its Application to Lepton and W-Boson Pair Production
K.J. Kim, Y.-S. Tsai

729 Scaling, Light-Cone Singularities and Asymptotic Behaviour of the Jost-Lehmann Spectral Function
P. Stichel

* 732 Finiteness of Electromagnetic Mass Shift in Light-Cone Algebra
Fayyazuddin; Riazuddin

866 On W_1 and W_2 Functions in Deep Inelastic e-p Scattering
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* 867 On A Scaling Law for the Proton Form Factors
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918 Parton-Antiparton Contributions to Photon-Photon Scattering
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942 Inclusive Proton Distributions in Electroproduction

967 Existence of A Short-Range Electromagnetic Interaction at High Energy
D.O. Caldwell, V.B. Elings, W.P. Hesse, R.J. Morrison, F.V. Murphy

972 Finite Energy Sum Rules and Light Cone Commutators
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* 982 Hole Fragmentation in Deep-Inelastic Processes
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(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

80 Calculation of the Induced Pseudotensor Term in A Beta Decay
P.L. Pritchett, N.G. Deshpande

280 A Measurement of Radiative A Decays

431 Weak Mesonic Currents of Second Class
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457 Measurement of the Up-Down Asymmetries in the Beta Decay of Polarized Lambda Hyperons

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239 Experimental Limit on the Neutral Current in the Semileptonic Processes
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333 Neutrinless Double Beta Decay of $^{76}$ Ge-
E. Fiorini, A. Pullia; G. Bertolini, F. Cappellani, G. Restelli

473 Search for Neutral Currents in $\nu_p$ Interactions
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523 Program of Neutrino Experiments at LAMPF
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540 Tests for Neutral Currents in Neutrino Reactions
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781 Bubble Chamber Study of the Elastic Neutrino Reaction $\nu_e + d \rightarrow p + p + \bar{p}$
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* 782 Observation of "Elastic" Hyperon Production by Antineutrinos
T. Eichten, H. Faisser, S. Kabe, W. Krenz, J. von Krogh, J. Morfin,
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D.C. Cundy, D. Haidt, P. Musset, U. Nguyen-Khac, S. Natali,
V. Brisson, B. Degrange, M. Hagenauer, F. Jacquet, L. Kluberg,
P. Petiau; E. Bellotti, S. Bonetti, M. Brini-Penzo, C. Conta,
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J. P. Vialle; F. Bullock, E.H.S. Burhop, M.J. Esten, T.W. Jones,

786 Limit on Strangeness Conserving Neutral Current
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805 $\nu_p \rightarrow \Delta^++$: Comparison with Theory
P.A. Schreiner, F. Von Hippel

821 Study of the Reaction $\nu_p \rightarrow \Delta^++$
J. Campbell, G. Charlton, Y. Cho, M. Derrick, R. Engelmann,
J. Fehkovich, L. Hyman, K. Jaeger, D. Jankowski, A. Mann, U. Mehtani,
B. Musgrave, P. Schreiner, T. Wangler, J. Whitmore, H. Yuta

943 Preliminary Result on the Ratio of Antineutrino to Neutrino Total Cross Sections
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(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

132 Observed Difference in the Ranges of Positive and Negative Muons

156 The \( \Delta I = 3/2, 5/2 \) Contributions in \( K \to 2\pi \) Decays and \( (\delta_0 - \delta_2) \) S-Wave \( \pi-\pi \) Phase Shifts
A.Q. Sarker

213 A Search for the Process \( K^+ + \mu^+ + \nu + \bar{\nu} + \bar{\nu} \)
G.D. Cable, R.H. Hildebrand, C.Y. Pang; R. Stiening

214 A Renewed Search for the Process \( K^+ + \pi^+ + \nu + \bar{\nu} \)
G.D. Cable, R.H. Hildebrand, C.Y. Pang; R. Stiening

** 215 Limits on the \( K^+ + \pi^+ + \nu + \bar{\nu} \) and \( K^+ + \pi^+ + \eta \) Decay Rates
J.H. Klems, R.H. Hildebrand; R. Stiening (Phys. Rev. D1, 66(1971))

266 Experimental Study of \( K^o + \pi^+ \pi^- \pi^0 \) Decays and Search for CP Violation
F. James, L. Montanet, E. Paul, P. Saetre, D.M. Sendall; P. Bertranet, G. Burgun, E. Lesquoy, A. Muller, E. Pauli, S. Zylberajch; O. Skjeggestad

267 Measurement of the \( K^o \) Mean Life
O. Skjeggestad; P. James, L. Montanet, E. Paul, P. Saetre, D.M. Sendall; G. Burgun, E. Lesquoy, A. Muller, E. Pauli, S. Zylberajch

268 Experimental Study of the Selection Rule \( \Delta S = \Delta Q \) in \( K^o_{G3} \) and \( K^o_{U3} \) Decays
G. Burgun, E. Lesquoy, A. Muller, E. Pauli, S. Zylberajch; F. James, L. Montanet, E. Paul, P. Saetre, D.M. Sendall; O. Skjeggestad

** 360 Experimental Study of the \( \tau^\pm \) Decay Matrix Element

392 A Study of the Decay \( K^{o} + \pi^0 \)

480 Experimental Comparison of CP Violation in \( K^{+} + \pi^- \) and \( K^{0} + \pi^- \) Decays
D. Banner, J. Frank, M. Gormley, L.J. Koester, S. Raither, J.H. Smith, A. Wattenberg

641 The Importance of a Measurement of the Decay \( K^{o} + \mu^+ + \mu^- + \gamma \)
M.J. Longo

687 Search for the Decay \( K_0^+ \to 2\mu \)

688 A Measurement of the \( Ke2/Ku2 \) Branching Ratio

779 A Measurement of the Form Factors in \( K^o \) Decay
Measurement of the Charge Asymmetry in the Decay $K^0 \rightarrow \pi^+ \nu \bar{\nu}$

A Measurement of the $K^0_s/K^0$ Branching Ratio and the $K^0_s$ Form Factors

Experimental Determination of the $K_\pi^+ K^- \pi^0$ Decay Matrix Element
R. Messner, A. Franklin, R. Morse, U. Nauenberg; D. Dorfan, D. Hitlin, J. Liu, R. Piccioni

Search for a Rare $K_S$ Decays $K_S \rightarrow 2\pi$, $K_S \rightarrow 3\pi^0$

The Search for the $\mu^+ + e^- + e^- + e^- + e^-$ Decay
S.M. Korenchenko, B.F. Kostin, G.V. Mielmacher, K.G. Nekrasov, V.S. Smirnov

A Search for the Decays of Short-Lived Kaons into Two Muons

A Search for the $K^0 \rightarrow \pi^+ e^- \nu \bar{\nu}$ Decay by Means of the Arrangement SKM-100 with a 1m Streamer Chamber
WEAK INTERACTIONS IV:
THEORY OF WEAK INTERACTIONS (PHENOMENOLOGY)

(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

** 6
Intermediate Boson. II. Theoretical Muon Spectra in High-Energy Neutrino Experiments

** 7
Intermediate Boson. I. Theoretical Production Cross Sections in High-Energy Neutrino and Muon Experiments

8
Intermediate Boson. III. Virtual Boson Effects in Neutrino Trident Production
R.W. Brown; R.H. Hobbs; J. Smith, N. Stanko

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Spontaneous Breakings of Chiral Symmetries II. Mass Relations and Particle Mixings
G. Cicogna; P. Strocchi; R. Vergara Caffarelli

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Broken Algebra of Currents and the Cabibbo Angle
R. Oehme

23
Renormalization of the SU3 X SU3 Sigma Model
L.-H. Chan, R.W. Haymaker

24
Closed Loop Corrections to the SU3 X SU3 Sigma Model - One and Two Point Functions
L.-H. Chan, R.W. Haymaker

25
Meson Dynamics in the Renormalized SU3 X SU3 Sigma Model
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** 28
New Results on p + p -> d + γ, and Time-Reversal Invariance
D.F. Bartlett, C.E. Friedberg, F.E. Goldhagen, K. Goulianos
(Phys. Rev. Lett. 27, 881 (1971))

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Virtual Neutrino Effects
P. Budini

74
Theory of Leptons
K. Tennakone, S. Pakvasa

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On Neutrinos of Non-Zero Rest Mass
S. Pakvasa, K. Tennakone

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A Unified Explanation of the K0 -> 2µ Puzzle, CP Non-Conservation, and the Excess Muon Anomaly of Utah?
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Neutrino Spectrum and the Origin of the Cabibbo Angle
K. Tennakone, S. Pakvasa

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Improved Test of CPT-Invariance in Neutral Kaon Decays
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πρ Scattering and the πN Sigma Term
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Ur-Citon; a Unified Scheme of Hadrons, and Its Application to Weak Interactions and Electromagnetic Form Factors
S. Ishida, K. Konno, K. Nakamura, M. Oda, Y. Yamazaki

Non-Leptonic Hyperon Decays and Ur-citon Scheme
S. Ishida, K. Nakamura, M. Oda (Prog. of Theor. Phys. 47, 304 (1972))

New Universality of Weak Interactions and Ur-citon Scheme
S. Ishida, M. Oda

The ΔI = 3/2, 5/2 Contributions in K → 2π Decays and (δ_0 - δ_2)
S-Wave π-π Phase Shifts
A.Q. Sarker

Report on W Boson Model of Weak Interactions with Maximal CP Violation
R.E. Marshak

K^+ → μ^+μ^- Puzzle and the Two K-Meson Theory
N. Nakazawa, S. Nakamura, S. Sato

Parity Violation in Neutron Capture Gamma Rays
J.L. Alberi, R. Wilson; I.G. Schroder

Cosmological Limit on Neutretto Mass
G. Marx, A.S. Szalay

Possible Method of Searching for a WeakNeutral Current in e^+ + e^- → μ^+ + μ^-
V.K. Cung; A.K. Mann; E.A. Paschos

Suggested Appearances of the Heavy Leptons
S. Mikamo, H. Yoshih

The K^+ - K^0 Mass Difference in a Chiral Approach
Riazuddin; Fayyazuddin

A Possible Difference Between the Muon- and Electron-Nuclear Interactions
S. Barshay

Octet Dominance in Non-Leptonic Decays
Riazuddin

On the Diagonal Part (ūν_μ)(ν_μū) in the Theory of Leptonic Weak Interactions
K. Koike, M. Konuma, K. Kurata, K. Sugano

Heavy Lepton Production in Inclusive Neutrino Reactions
C.H. Albright

K^0 Decays, Unitarity and TCP
B. Stech

Optimal Extrapolation of Pion-Nucleon Scattering: The σ Term Re-Examined
C.C. Shih; H.K. Shepard

W-Parity, Octet Dominance and the Breaking of Chiral Symmetry
A. McDonald, S.P. Rosen, T.K. Kuo

Improved Weizsacker-Williams Method and Its Application to Lepton and W-Boson Pair Production
K.J. Kim, Y-S. Tsai

Photoproduction of Electrons, Muons and Heavy Leptons
K.J. Kim, Y-S. Tsai
Deformed Current Algebra and Computation of the Cabibbo Angle
R. Oehme

(8,8) Symmetry Breaking and the Decay Rate η' → ηππ
A. Ali

Unitarized Hard-Meson Calculation of K_3; Form Factors
A. Ali; F. Hussain; M.S.K. Razmi

Nonleptonic Hyperon Decays, Current Algebra, and Decuplet Dominance
M.D. Scadron

Role of Weak Interaction Higher Order Corrections in Decays
K^0 → π^0γ, K → π^0e^+e^-
E.P. Shabalin

Charge Asymmetry of the Muon Angular Distribution in the Process
e^+e^- → μ^+μ^-
I.B. Khriplovich

Double Differential Cross-Sections for Secondary Leptons in W-Boson
Production in Neutrino Beam with Energy E_ν < 50 GeV
V.V. Makeev, P.L. Nevskiy, Yu.P. Nikitin, G.V. Rozhov, A.A. Sokolov

One More Possible Explanation of the Inconsistency of the Measured
Decay Rates K_L^0 → 2γ and K_S^0 → 2μ
A.L. Lyubimov

On Multifermion Interaction
V.N. Polomeshkin, S.S. Gerstein

On the Question of Heavy Leptons
S.S. Gerstein, L.S. Landsberg, V.N. Polomeshkin

On Possible Check of Scheme with Multiplicative Leptonic Quantum
Numbers in High Energy Neutrino Experiments
S.S. Gershtein, V.N. Polomeshkin

Atmospheric Neutrino Induced Muon Flux and the Neutrino Nucleon
Interactions at High Energies
Z. Kunszt

Nonet Dominance and Isospin Constraints on μ-Pair and W-Boson
Production
Z. Kunszt, R.M. Muradyan

Mixing, K_L → 2γ and the K_L - K_S Mass Difference
M.D. Scadron
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FORMAL WEAK INTERACTION THEORY

(papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference

12 A New Approach to Quantum Numbers in Elementary Particle Physics
   A. Ramakrishnan

35 A Renormalizable Model of Weak and Electromagnetic Interactions
   with CP-Violation
   R.N. Mohapatra

86 Higher Order Corrections to Weak Processes
   H. Horváth and G. Pócsik

135 Divergences of the Higher Order Corrections to $\mu$ Decay in the
   Gauge Theory
   G. Rajasekaran

159 Lepton-Hadron Symmetry Breaking and the Cabibbo Rotation
   W.F. Palmer

162 A Family of Sum Rules for Weak Electromagnetic and Strong Processes
   L. Sukaszuk

165 On the Neutrino-Electron Cross Sections at High Energies
   L. Sukaszuk

** 182 The Case for a Quartet Model of Hadrons
   C.E. Carlson; P.G.O. Freund (Phys. Lett. 39B, 349(1972))

183 Lepton Number Conserving and Non-Conserving Weak Interactions
   P.G.O. Freund

285 Gauge Theory of Strong, Weak, and Electromagnetic Interactions
   I. Bars, M.B. Halpern, and M. Yoshimura

319 Light Cone Approach to Structure Functions in a Theory of Weak and
   Electromagnetic Interactions
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363 On Quantization and Gauge Freedom in a Theory with Spontaneously
   Broken Symmetry
   Y.-P. Yao

* 368 Optimal Lower Bounds on the Hadronic Contribution to the Muon
   Anomalous Magnetic Moment. G. Nenciu and I. Raszillier

376 Analytic Continuation in the Dimension of Space-Time
   G.M. Cicuta

* 396 Proton-Neutron Mass Difference in a Unified Theory of Weak and
   Electromagnetic Interactions
   Fayyazuddin; Riazuddin

441 A Model of Weak and Electromagnetic Interactions with No New Quarks
   Y. Achiman

** 526 Analytic Renormalization Via Continuous Space Dimension
   G.M. Cicuta and E. Montaldi (Nuovo Cimento 4, 329(1972))

* 569 A Theory of Universal Weak Interaction of Leptons
   A.T. Filippov

* 732 Finiteness of Electromagnetic Mass Shift in Light-Cone Algebra
   Fayyazuddin; Riazuddin

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Limit on Strangeness Conserving Neutral Current
E.A. Paschos and B.W. Lee

The Weak Interaction Hamiltonian in L-Matrix Theory
A. Ramakrishnan (Jour. of Math. Analy. and Applic. 37,432(1972))

Symmetry and Renormalizability of the Theories of Massive Vector Fields
A.I. Vainshtein

Neutrino Electromagnetic Formfactor in the K-Matrix Approach
D.V. Galtsov, N.S. Nikitina, A.N. Safronov

Clashing Symmetries in Unified Descriptions of Electromagnetic and Weak Interactions, and the Case for the Han-Nambu Model
H.J. Lipkin
PARTICLE SEARCHES

(Papers designated by a single asterisk were submitted elsewhere for publication; papers designated with a double asterisk had been published prior to the conference)

311 Suggested Appearances of the Heavy Leptons
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* 362 Search for Tachyon Monopoles
D.F. Bartlett, M.D. Lahana

584 Heavy Lepton Production in Inclusive Neutrino Reactions
C.H. Albright

631 Search for Muonium-Antimuonium Transition in Free Space

683 Search for the Utah Effect with the UCSD Cosmic Ray Spectrometer
T. Burnett, L. LaMay, G.E. Masek, T. Maung, E. Miller, H. Ruderman, W. Vernon

* 686 A Search for Quarks at the CERN Intersecting Storage Rings

848 Search for Heavy Quasistable Leptons

877 The Preliminary Experimental Results on a Search for the Dirac Monopole at the 70 GeV IHEP Synchrotron Using Vavilov-Cerenkov Radiation
V.P. Zrelov, L. Kollarova, D. Kollar, V.P. Lupil'tsev, P. Pavlovič, J. Ružička, V.I. Sidorova, R.F. Shabashov, P. Šulek, R. Janík

898 On Electromagnetic Mechanism of W-Meson Pairs Production by High Energy Particles
V.A. Khoze, N.L. Ter-Isaakian

903 Search for Magnetic Monopole at 70 GeV IPHE Proton Synchrotron

915 On the Question of Heavy Leptons
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137 Telecommunications with Muon Beams
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323 A High-Quality Muon Beam at SLAC for High-Energy Lepton-Hadron Scattering Work
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421 Deuteron Stripping at 6 BeV/c and Production of a Tagged Neutron Beam

460 Tests of Two Inch Diameter Photomultiplier Tubes with Extended Ultraviolet Response
A. Gingle, T.M. Knasel, E. Swallow and R. Winston; R. Sumner

461 Design of a High Rejection Self-Collimating Cerenkov Counter for the NAL Hyperon Beam
R.L. Sumner, T.M. Knasel, E.C. Swallow and R. Winston

702 A System of Large Driftchambers
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703 Two Dimensional Read-Out of Driftchambers
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727 A High Energy Neutron Detector Using Proportional Wire Chambers
M. Atac; R. Majka and S. Dhawan

838 Particle Shower Detector + The Multiple Streamer Chamber

859 Quantum Transitions of Relativistic Electrons in a Superstrong Magnetic Field
A.A. Sokolov, I.M. Ternov, V.Ch. Zhukovskii

860 On a Possibility of Producing of the Collimated \( \gamma \) Quanta Beam in the Homogeneous Magnetic Field
A.A. Sokolov, I.M. Ternov, D.V. Galtsov, V.Ch. Zhukovskii

870 The New Particle Detector-Crystal Wire Counter
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904 300 Kilogauss Impulse Magnet
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934 The Detection of X-Ray Transition Radiation of 31 GeV Electrons

935 Superhigh Energy Particles Separation by Method of Transition Radiation
A.I. Alikhanian, E.S. Beliakov, G.M. Garibian, M.P. Lorikian, K.Zh. Markarian, K.K. Shikhliarov