Thermal photon and residual gas scattering of the electrons in the ILC RTML

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SLAC-TN-06-007
August 15, 2006

Prepared for the Department of Energy
under contract number DE-AC02-76SF0015

Abstract

The scattering of the primary beam electrons off of thermal photons and residual gas molecules in the projected International Linear Collider (ILC) is a potential source of beam haloes which must be collimated downstream of the linac.

In this report we give the analytic estimations of the individual input that each of the main scattering processes makes in the production of off-energy and large amplitude particles in the Damping Ring to Main Linac region (RTML).

1 Introduction

It has been shown [1], [2] that the performance of the single-pass electron accelerators that combine such critical features as high energy, high emittance and relatively low vacuum may be affected by the primary beam scattering. Thus, it is no wonder, that this issue concerns the operation of the ILC, which accelerates electrons up to 0.5 TeV, and that the scattering processes must be studied in details for each part of the ILC.

In this paper we limit our consideration by the RTML.

There are two possibilities to consider. The first possibility is the off-energy scatterings - the scattering of beam particles out of energy aperture (ϵ), which is about 5 GeV, the second possibility is the growth of the beam halo due to large angle scatterings. The large amplitude particles are scraped downstream of the linac and produce parasitic radiation. The tolerable radiation is produced by 10^{-5} fraction of the beam being scraped [3].

The four possible mechanisms of beam scattering [1] are: the scattering from blackbody thermal photons, inelastic scattering from residual gas molecules (beam-gas Bremsstrahlung), elastic beam-gas scatterings (relativistic Coulomb scattering) and the scattering off atomic electrons.

2 Simplified model of scattering in RTML

The RTML region starts downstream of the damping ring extraction system and continues until the upstream face of the first linac cryomodule. The RTML consists of the following parts, in order of longitudinal position:

- A set of 4 orthonormal skew quads which are used to eliminate coupling;
- A set of profile monitors which are used to tune the emittance and coupling of the beam;

- A betatron collimation system which eliminates transverse beam halo;
- A beam jitter measurement system for trajectory feedforward correction;
- A turnaround which delays the beam with respect to a line-of-sight cable to permit trajectory feedforward correction;
- A spin rotator which allows the polarization vector to be oriented to any direction desired by the experimenters;
- A 4-D multi-wire emittance measurement station which includes the steering dipoles for trajectory feedforward correction;
- A first stage bunch compressor which compresses the RMS bunch length to about 1 mm;
- A second stage bunch compressor which compresses the RMS bunch length to 150 to 300 um;
- A 2 x 2D emittance measurement station.

The length of the RTML is 1710 m, the beam energy varies from 5 to 15 GeV. Figure 1 shows the beam energy and beta functions versus the longitudinal position in the RTML.

For our estimations we will split the RTML in 5 regions: from 0 to 250 meters with average beta function $\beta_{x,y} = 23$ m and beam energy 5 GeV; from 250 to 530 m with average $\beta_{x,y} = 3.6$ m and beam energy 5 GeV; from 530 to 780 m with average $\beta_{x,y} = 22$ m and beam energy 5 GeV; from 780 to 1430 m with average $\beta_{x,y} = 57$ m and beam energy growing from 5 to 15 GeV with the energy gradient 0.154 MeV/m; and from 1430 to 1710 m with average $\beta_{x,y} = 22$ m and beam energy 15 GeV. We will also ignore for now that a particle undergoing some scattering process, which involves loss of energy in the regions with high dispersion, can have additional amplitude offset in the scraper.

Making further simplifications by assuming that the scattering cross-section does not vary with the betatron phase and that the scattering process has an isotropic distribution in $x'y'$ space one can derive the expression for the minimum scattering angle required to leave the betatron aperture of the collimators positioned downstream of the linac [1]:

$$\theta_{e_{\min}}(z) \geq 2\frac{a_{scr}}{\sqrt{\beta(z)\beta_{scr}}}\sqrt{\frac{E_{scr}}{E(z)}} \tag{1}$$

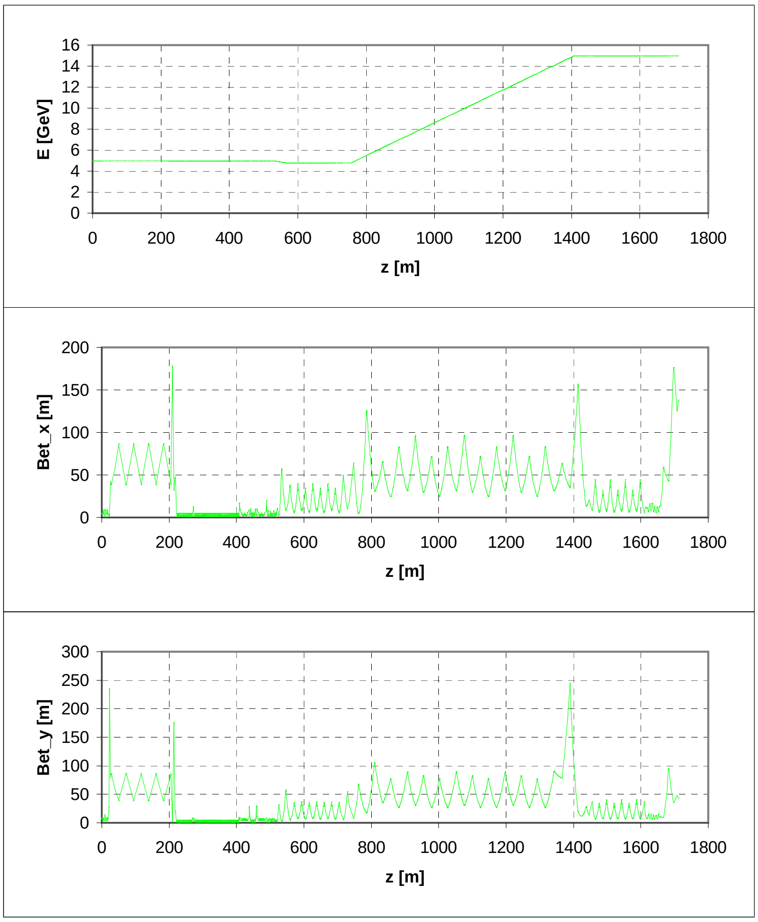

Figure 1: Beam energy and beta functions in the RTML.

where $a_{scr} = 200$ μm is the apperture of the scraper at the downstream end of the linac, $\beta_{scr} = 500$ m is the beta function in the scraper, $E_{scr} = 500$ GeV is the beam energy in the scraper, $\beta(z)$ and $E(z)$ are the beta function and beam energy in the RTML and z is the longitudinal coordinate of the point where scattering occurs. The plot of $\theta_{e_{\min}}$ in the RTML is given in figure (2).

In our estimations we also do not take into account the fact that the large amplitude particle can be intercepted by beamline aperture somewhere in the linac, instead we assume that all scattered electrons that acquired an angle large enough to hit the scraper at the end of the linac will reach it.

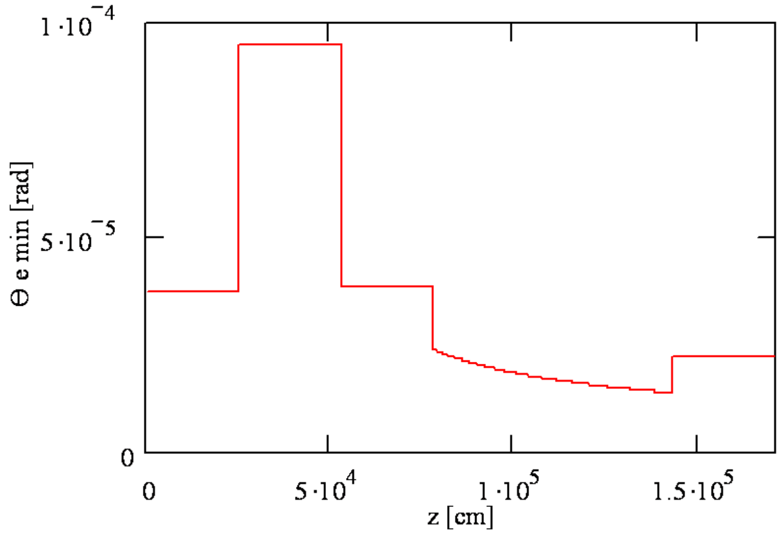

Figure 2: The minimum scattering angle ($\theta_{e_{\min}}$) required to the electron to be intercepted by the collimator downstream of the linac.

3 Calculus of scattering effects

3.1 Scattering from thermal photons

The differential cross-section for scattering of a beam electron and a photon is given by [2]:

$$\frac{d\sigma_{th}}{dy} = \frac{2\sigma_0}{x}\left(\frac{1}{1-y} + 1 - y - \frac{4y}{x(1-y)}\left(1 - \frac{y}{x(1-y)}\right)\right) \tag{2}$$

where $\sigma_0 = \dfrac{8\pi r_e^2}{3}$ is Thompson cross-section, r_e is classical electron radius, $x = \dfrac{4E\omega_0}{m_e^2c^4}\cos^2(\alpha/2)$, ω_0 is initial energy of the photon, α is the incident photon angle with respect to the beam in the laboratory frame and $\alpha = 0$ corresponds to the head-on collision , m_e is the mass of electron, c is the speed of light, $y = \omega_1/E$ is the relative energy of the scattered photon.

For the given α and ω_0 the relative energy loss is limited by the upper bound $y_{\max} = \dfrac{x}{1+x}$. We are interested in energy loss higher than energy aperture ϵ, therefore the lower limit for y is $y_{\min} = \epsilon/E$, and the scattering cross-section for the given α, ω_0 in the position z is:

$$\sigma_{th}(z,\alpha,\omega_0) = \int_{y_{\min}(E(z))}^{y_{\max}(x(E(z),\alpha,\omega_0))} \frac{d\sigma_{th}}{dy}(x(E(z),\alpha,\omega_0),y)dy \tag{3}$$

The relative number of electrons scattered out of energy aperture over the distance dl is given by ([4], Section 3.3.4):

$$\frac{1}{N}\frac{dN_E}{dl} = \int\int (1+\cos(\alpha)) \cdot \sigma_{th} \cdot dn_\gamma \frac{d\Omega}{4\pi} \tag{4}$$

where the Planck distribution for the spectral density of thermal photons is given by:

$$\frac{dn_\gamma}{d\omega_0} = \frac{\omega_0^2}{\pi^2(\hbar c)^3} \cdot \frac{1}{e^{\frac{\omega_0}{k_B T}} - 1} \tag{5}$$

here T is the temperature, $\hbar$ is the Planck constant and k_B is the Boltzmann constant.

As a matter of fact, the integration in formula 4 requires some additional precautions. The product $\omega_0 \cos^2(\alpha/2)$ must be less than $\frac{(m_e c^2)^2}{4E} \cdot \frac{y_{\min}}{1-y_{\min}}$ for the electron to be able to lose relative energy higher than $y_{\min}$. In other words, for the given ω_0 the physical values of α are limited by $\alpha_{\max} = 2\arccos\left(\sqrt{\frac{(m_e c^2)^2}{4E\omega_0} \cdot \frac{y_{\min}}{1-y_{\min}}}\right)$, and the lower limit in energy spectrum integral is $\omega_{\min} = \frac{(m_e c^2)^2}{4E} \cdot \frac{y_{\min}}{1-y_{\min}}$. Now we can rewrite 4 as:

$$\frac{\Delta N_E}{N} = \frac{1}{4\pi}\int_0^{L_{RTML}} \int_{\omega_{\min}(E(z)),y_{\min}(E(z))}^{\infty} \int_0^{\alpha_{\max}(E(z),\omega_0,y_{\min}(E(z)))}$$
$$(1+\cos(\alpha))\sin(\alpha) \cdot \sigma_{th}(z,\alpha,\omega_0) \cdot \frac{dn_\gamma}{d\omega_0}(\omega_0) d\alpha d\omega_0 dl \tag{6}$$

where L_{RTML} is the length of the Damping Ring to Main Linac region.

The numerical integration of (6) gives the negligibly small relative number of scatterings: 10^{-48}. This result is not very surprising, since for the low beam energies the cross-section of scatterings in which electron loses at least 5 GeV is nonzero only in case of $\omega_0 >> \bar{\omega}_0$ (where $\bar{\omega}_0 = 2.7 k_B T = 0.07$ eV for $T = 300$ K). On other hand, for $\omega_0 > \bar{\omega}_0$ $dn_\gamma/d\omega_0$ falls off exponentially and is effectively zero above $20\bar{\omega}_0$. The plot (3) shows the normalized cross-section of the scattering of an electron from thermal photons out of energy aperture for the beam energies up to 500 GeV. As one can see, the electron-photon scattering starts giving any noticeable result at the energies higher than 30 GeV.

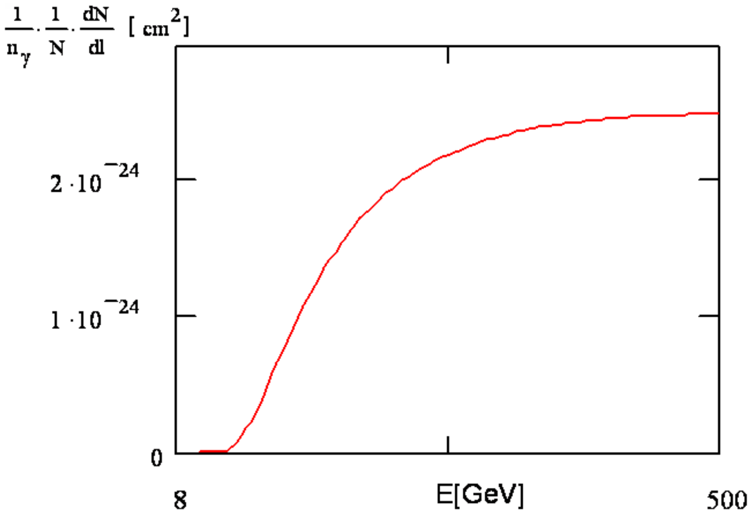

Figure 3: The "normalized cross-section" versus beam energy (here $n_\gamma = \int_0^\infty \frac{dn_\gamma}{d\omega_0} d\omega_0$).

The scattered electron will emerge with some recoil angle θ_e relative to its initial trajectory. The opening angle of the scattered photons in the laboratory frame is [1]:

$$\theta_\gamma = \frac{\sqrt{1 + \frac{\gamma\omega_0}{m_e c^2}}}{\gamma} \tag{7}$$

here γ is the relativistic γ-function.

The maximum possible value of energy lost by electron in the process of scattering is $\Delta E = E \cdot x/(1+x)$ and in approximation of small angles $\theta_e = \frac{\omega_0 + \Delta E}{E - \Delta E} \cdot \theta_\gamma$. For the parameters in the range of interest the maximum achievable recoil angle is:

$$\theta_e \approx \frac{4\omega_0\gamma}{E} \tag{8}$$

For $\bar{\omega}_0$ (8) gives an angle of about 550 nrad which is about two orders of magnitude less than the typical value of $\theta_{e_{\min}}$ in the RTML (see figure (2)).

Thus, the scattering of electrons from thermal photons gives negligible input in both large angle scatterings and off-energy scatterings.

3.2 Beam-Gas Bremsstrahlung

In beam-gas Bremsstrahlung an electron collides with an atom and emits a photon which carries away a fraction of electron's energy. The differential cross-section of this process is approximately given by [2]:

$$\frac{d\sigma_{Br}}{dy} = 4\alpha^* r_e^2 Z^2 \ln(183 \cdot Z^{-1/3}) \left(\frac{4}{3y} - \frac{4}{3} + y \right) \tag{9}$$

where α^* is a fine structure constant, $Z = 7$ is an atomic number for N_2, and y is the relative energy loss of an electron. The integration of (9) from $y_{\min}$ to 1 gives the total cross-section for electrons scattered out of energy aperture:

$$\sigma_{Br} = 4\alpha^* r_e^2 Z^2 \ln(183 \cdot Z^{-1/3}) \left(\frac{4}{3} y_{\min} - \frac{4}{3} \ln(y_{\min}) - \frac{y_{\min}^2}{2} - \frac{5}{6} \right) \tag{10}$$

Therefore, the relative number of off-energy scatterings through the RTML is simply given by:

$$\frac{\Delta N_E}{N} = n_{gas} \int_0^{L_{RTML}} \sigma_{Br}(y_{\min}(E(z)))dz = 2 \cdot 10^{-11} \tag{11}$$

where the density of the gas $n_{gas}[\mathrm{m}^{-3}] = 3.2 \cdot 10^{22} \cdot P[\mathrm{Torr}] \cdot N_{atom}$, number of atoms N_{atom} per molecule for N_2 is 2 and $P = 10$ nTorr.

The normalized angular distribution of the scattered photons is given by [1]:

$$f_\gamma = \frac{1}{0.391\gamma - 0.239} \cdot \frac{\theta_\gamma}{(\theta_\gamma^2 + \gamma^{-2})^2} \tag{12}$$

In the small angle limit the conservation of transverse momentum gives $y = \dfrac{\theta_e}{\theta_e + \theta_\gamma}$. Therefore, for the given relative energy loss there is the minimum angle of the scattered photon ($\theta_{\gamma_{\min}}$) that allows an electron to be scattered by an angle larger than $\theta_{e_{\min}}$: $\theta_{\gamma_{\min}} = \dfrac{1-y}{y} \cdot \theta_{e_{\min}}$. Thus, the total cross-section for large angle scatterings of electrons through Bremsstrahlung is given by:

$$\sigma_{Br,\theta}(z) = \int_0^1 \frac{d\sigma_{Br}}{dy}(y) \int_{\theta_{\gamma_{\min}}(y, \theta_{e_{\min}}(z))}^{\pi} f_\gamma(\theta_\gamma, \gamma(z)) \frac{\sin(\theta_\gamma)}{2} d\theta_\gamma dy \tag{13}$$

The plot of $\sigma_{Br,\theta}(z)$ in the RTML is given in figure (4).

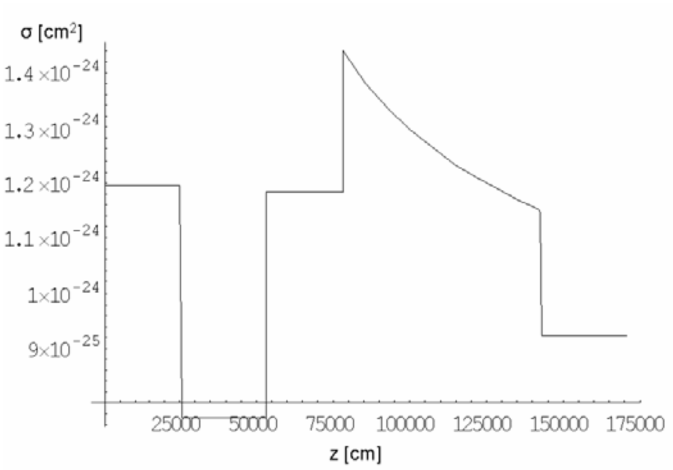

Figure 4: The cross-section of large angle scatterings through beam-gas Bremsstrahlung process in the RTML.

Integrating (13) numerically one obtains for the relative number of large angle scatterings:

$$\frac{\Delta N_\theta}{N} = n_{gas} \int_0^{L_{RTML}} \sigma_{Br,\theta}(z) dz = 1.2 \cdot 10^{-10} \tag{14}$$

3.3 Coulomb scattering

Since the Coulomb scattering is an elastic process, the scattered electrons are not a problem for the energy collimation.

As for the large angle scatterings, the cross-section for elastic scattering is given by ([4], Section 3.3.1):

$$\frac{d\sigma_C}{d\Omega} = \left(\frac{Z\alpha^*\hbar c}{2(E+m_e c^2)\beta^2}\right)^2 \cdot \frac{1-\beta^2\sin^2(\theta/2)}{\sin^4(\theta/2)} \tag{15}$$

where $\beta = \sqrt{1-\gamma^{-2}}$ is the rapidity.

Integrating (15) above the minimum angle $\theta_{e_{\min}}$ we get the Coulomb cross-section for the large angle scatterings:

$$\sigma_C = \pi \left(\frac{Z\alpha^*\hbar c}{(E+m_e c^2)\beta^2}\right)^2 \cdot \left(\frac{1}{\sin^2(\theta_{e_{\min}}/2)} + 2\beta^2 \ln(\sin(\theta_{e_{\min}}/2)) - 1\right) \tag{16}$$

Figure (5) shows the dependence of σ_C on the longitudinal coordinate in the Ring to Main Linac region.

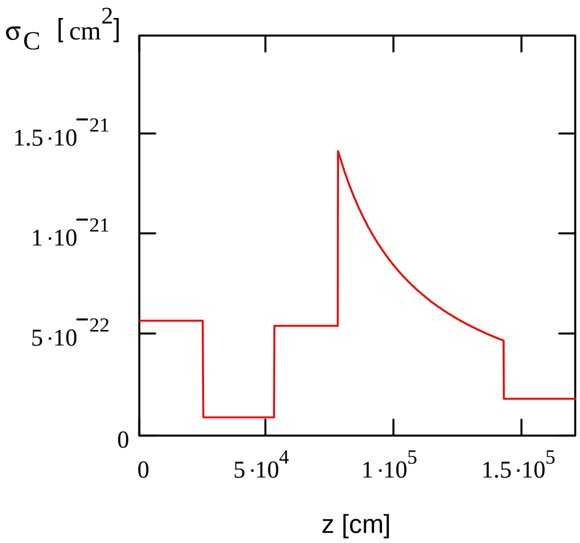

Figure 5: The Coulomb cross-section for large angle scatterings in theRTML.

The relative number of large angle scatterings is obtained by integration of (16) over the length of the RTML:

$$\frac{\Delta N_\theta}{N} = n_{gas} \int_0^{L_{RTML}} \sigma_C(\theta_{e_{\min}}(z), E(z), \beta(z))dz = 3.7 \cdot 10^{-8} \tag{17}$$

3.4 Scattering off atomic electrons

The differential cross-section for scattering on atomic electrons is ([4], Section 4.4):

$$\frac{d\sigma_{ee}}{dy} \approx \frac{2\pi Z r_e^2}{\gamma y^2} \tag{18}$$

therefore, the total cross-section for the scatterings with energy loss above $y_{\min}$ is:

$$\sigma_{ee} = \frac{2\pi Z r_e^2}{\gamma y_{\min}} \tag{19}$$

and the relative number of off-energy scatterings in the RTML is:

$$\frac{\Delta N_E}{N} = n_{gas} \int_0^{L_{RTML}} \sigma_{ee}(\gamma(z), y_{\min}(E(z)))dz = 4 \cdot 10^{-14} \tag{20}$$

That is, the effect of scattering from atomic electrons on the energy loss is negligible.

The scattering angle in the laboratory frame is given by [2]:

$$\tan(\theta) = \frac{1}{2\gamma} \cdot \frac{2\sqrt{y+y^2}}{1-2y} \tag{21}$$

Inverting (20) one can see that relative energy loss allowing an electron to be scattered into an angle θ is:

$$\begin{aligned} y &= \frac{4a^2 + 1 - \sqrt{12a^2+1}}{8a^2 - 2} \\ a &= \sqrt{\gamma/2} \cdot \tan(\theta) \end{aligned} \tag{22}$$

The relative number of large angle scatterings is given by integration of (18) with y given by (21) over the total length of the RTML:

$$\frac{\Delta N_\theta}{N} = n_{gas} \int_0^{L_{RTML}} \sigma_{ee}(\gamma(z), y(\gamma(z), \theta_{e_{\min}}(z)))dz = 5.2 \cdot 10^{-9} \tag{23}$$

The plot of large angle cross-section for the scattering off atomic electrons in the RTML is shown in figure (6)

3.5 Scattering effects for the beam accelerated to 250 GeV

Since the ILC is supposed to be functioning in different accelerating regimes, it is worth to repeat the above calculus for the beam accelerated up to 250 GeV. This gives E_{scr} of 0.25 TeV, therefore $\theta_{e_{\min}}$ must be scaled down by a factor of $\sqrt{2}$. Energy aperture ϵ, in its turn, is reduced from 5 GeV to 2.5 GeV. Taking into account these changes, we can apply the formulas derived in the previous sections.

The scattering off thermal photons in RTML still gives a negligible effect for 250 GeV final beam energy.

The relative number of beam-gas Bremsstrahlung off-energy scatterings in the RTML is now $6 \cdot 10^{-11}$, and the number of large angle scatterings is $1.4 \cdot 10^{-10}$.

For the Coulomb scattering the relative number of the electrons scattered by large angles is $7.4 \cdot 10^{-8}$.

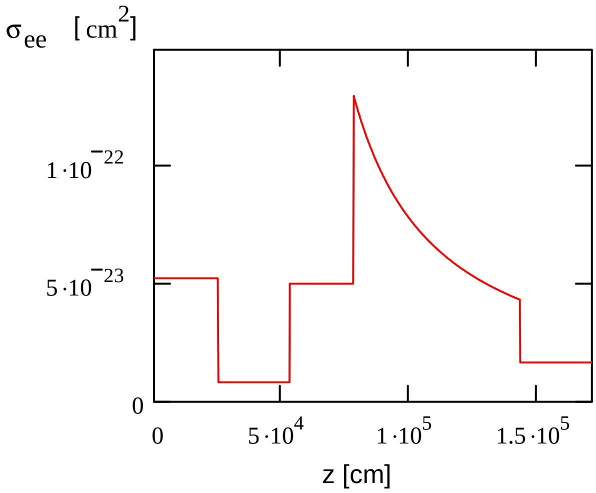

Figure 6: The cross-section for large angle scatterings off atomic electrons versus longitudinal coordinate.

Finally, for the scattering off atomic electrons the relative number of off-energy scatterings is $8 \cdot 10^{-14}$, and the relative number of large angle scatterings is $1.1 \cdot 10^{-8}$.

Thus, though the relative number of both large angle and off-energy scatterings has increased for 250 GeV final beam energy, the changes are quantitative not qualitative, since the total number of scatterings is much smaller than the tolerances.

3.6 Scattering in the regions with dispersion

In the previous sections we ignored the effect that dispersion (η) has on the off-energy scattering. An electron that loses the fraction of energy δ has both space and angular offset in the dispersive region:

$$x = \eta \cdot \delta, \quad \theta_x = \eta' \cdot \delta \tag{24}$$

Electron's transverse displacement at the location of the scraper due to its initial offset is given by:

$$x_{scr} = \sqrt{\frac{E}{E_{scr}}} \sqrt{\frac{\beta_{scr}}{\beta}} (\sin(\mu) + \alpha \cos(\mu)) x \tag{25}$$

here μ is the phase advance between the point were the scattering happens and the scraper, α is the Twiss correlation amplitude. The displacement due to initial angle is:

$$x_{scr} = \sqrt{\frac{E}{E_{scr}}}\sqrt{\beta_{scr}\beta}(\sin(\mu))\theta_x \tag{26}$$

Replacing μ with the RMS phase advance we find the root-mean-square of (30) and (26):

$$x_{scr} = \frac{1}{2}\sqrt{\frac{E}{E_{scr}}}\sqrt{\beta_{scr}\left(\eta^2\frac{(1+\alpha^2)}{\beta} + \eta'^2\beta\right)} \cdot \delta \tag{27}$$

Therefore, the minimum fractional energy loss in the dispersive region required to kick electron out of the scraping aperture is:

$$\delta_{\min} = 2a_{scr}\sqrt{\frac{E_{scr}\cdot\beta}{E\cdot\beta_{scr}}}\sqrt{\frac{1}{(1+\alpha^2)\eta^2 + \eta'^2\beta^2}} \tag{28}$$

The dispersion and α-function in the RTML are shown in plot (7).

Using (28) one can find $\delta_{\min}$ as a function of longitudinal coordinate in the RTML. The result for $\delta_{\min}$ is shown in figure (8).

For the purpose of estimation we will use the following average values of $\delta_{\min}$: $\delta_{\min} = 10^{-3}$ for z from 225 m to 405 m; $\delta_{\min} = 2 \cdot 10^{-3}$ from 440 m to 450 m; $\delta_{\min} = 10^{-3}$ from 570 m to 710 m; and $\delta_{\min} = 4 \cdot 10^{-3}$ from 1460 m to 1600 m.

We can find the relative number of the scatterings, which lead to the scraping electron out of the beam, by substituting the found values of $\delta_{\min}$ into equations (6), (11) and (20).

The scattering of electrons on thermal photons gives $3 \cdot 10^{-12}$ for the relative number of scatterings.

The relative number of beam-gas Bremsstrahlung scatterings is 10^{-10}.

For the scattering off atomic electrons the relative number of scatterings is $7 \cdot 10^{-12}$.

4 Conclusion

We considered the scattering of electrons in the Damping Ring to Main Linac region through four scattering processes: scattering off blackbody thermal photons, beam-gas Bremsstrahlung, Coulomb scattering and scattering off atomic electrons.

Combining the results of (6), (8), (11), (14), (17), (20), and(23) we see that the total relative number of the electrons scattered out of the energy aperture is:

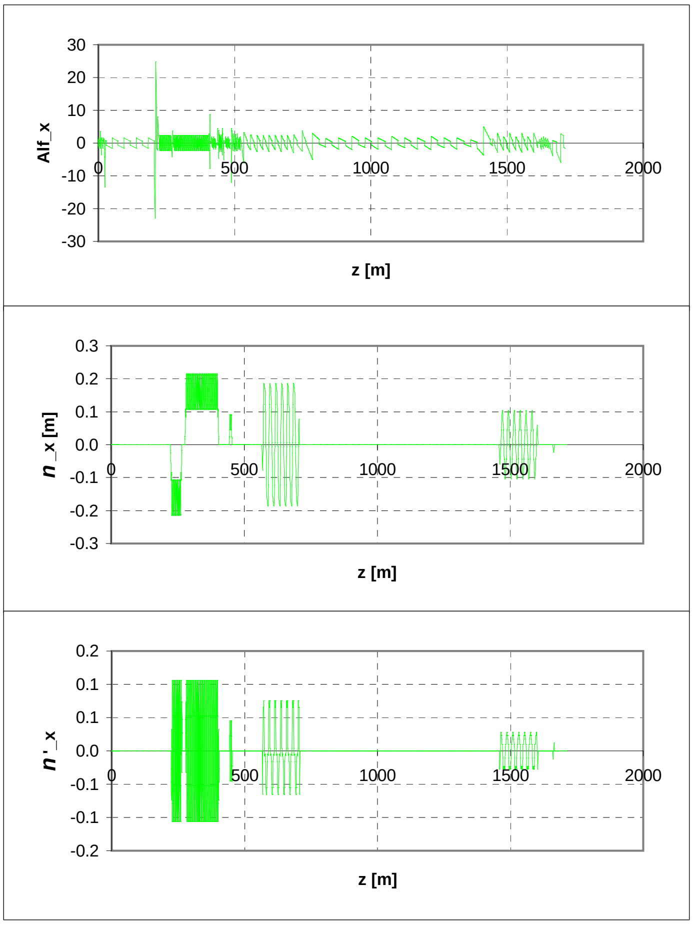

Figure 7: The dispersion and α-function in the RTML.

$$\frac{\Delta N_E}{N} \approx 2 \cdot 10^{-11} \tag{29}$$

for 500 GeV final beam energy and is $6 \cdot 10^{-11}$ for 250 GeV final beam energy.

The total relative number of the electrons that are scattered by large angle and will be intercepted by the scraper downstream of the linac is:

$$\frac{\Delta N_\theta}{N} \approx 4.2 \cdot 10^{-8} \tag{30}$$

for 500 GeV final beam energy and is $8.5 \cdot 10^{-8}$ for 250 GeV final beam energy.

Also, the off-energy scatterings in the dispersive regions produce an additional 10^{-10} relative number of scatterings.

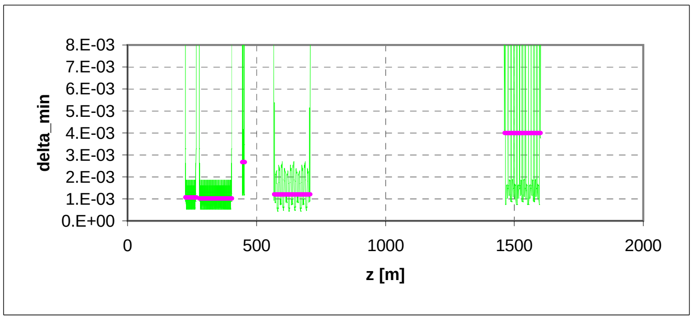

Figure 8: The minimum fractional energy loss in the dispersive region required to kick electron out of the scraping aperture. The green line is the result of numerical simulation for δ_{min}, the pink line is the average value of δ_{min} in different regions.

Therefore we are concluding that neither the scattering of electrons from the thermal photons nor the scattering from the residual gas in the RTML is a problem for operation of ILC.

5 Acknowledgements

The author thanks P. Tenenbaum for fruitful discussions and many helpful suggestions.

References

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