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This paper has been prepared for the American Nuclear Society National Topical Meeting on Nuclear Engineering Education, the proceedings of which will appear in a special issue of Nuclear Technology.

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ABSTRACT

The role of Nuclear Engineering Education in the application of computers to controlled fusion research can be a very important one. In the near future the use of computers in the numerical modelling of fusion systems should increase substantially. A recent study group has identified five categories of computational models to study the physics of magnetically confined plasmas. A comparable number of types of models for engineering studies are called for. The development and application of computer codes to implement these models is a vital step in reaching the goal of fusion power. In order to meet the needs of the fusion program the National CTR Computer Center has been established at the Lawrence Livermore Laboratory. A large central computing facility is linked to smaller computing centers at each of the major CTR laboratories by a communications network. The crucial element that is needed for success is trained personnel. The number of people with knowledge of plasma science and engineering that are trained in numerical methods and computer science is quite small, and must be increased substantially in the next few years. Nuclear Engineering departments should encourage students to enter this field and provide the necessary courses and research programs in fusion computing.

INTRODUCTION

Large-scale digital computers will play an increasingly important role in future controlled fusion research and in the development of practical fusion reactors. The plasma state exhibits such a diverse variety of physical phenomena that only through extensive use of high-speed computers can the interplay of all factors that affect the performance of a fusion system be modelled.

The behavior of a plasma confined by a magnetic field is simulated by a variety of numerical models. Some models used on a short time scale give detailed knowledge of the plasma on a microscopic scale, while other models used on much longer time scales compute macroscopic properties of the plasma dynamics. All of these models are under continual development, but in the next few years there should be a substantial increase in the development and use of numerical models in order to meet the needs of the fusion power program.

A 1974 study group on the "Applications of Computers to CTR" sponsored by the AEC/DCTR identified five categories of computer codes used to model the physics of fusion devices.

1. Time-dependent macroscopic (fluid) codes.
2. Time-independent macroscopic codes.
3. Vlasov and particle codes.
4. Fokker-Planck codes.
5. Hybrid codes.

In later sections of this report we shall examine these problem areas in more detail and consider their importance to the CTR program and discuss future goals in each area.

There are also a wide variety of engineering applications which require extensive computing. These applications will become more and and more important as we move into the reactor development phase. We shall also discuss some of these problems in more detail later in the report.

Computers are expected to play an important role in the acquisition and analysis of experimental data as well as the control of experiments.

In order to meet the needs of the controlled fusion program the National CTR Computer Center has been established at the Lawrence Livermore Laboratory. This Center will be connected to all major CTR laboratories, as well as selected universities, by means of a wide-band communications network. The increased computing power made available by the Center will accelerate the development of the theoretical models and associated computer codes needed to predict the behavior of plasma confinement systems and the operating characteristics of fusion power reactors. A more detailed description of the Center will be given later in this report.

COMPUTER MODELS OF A MAGNETICALLY CONFINED PLASMA

1. Time-dependent macroscopic codes

The complex nature of the MHD fluid equations of motion is such that our understanding of the macroscopic behavior of realistic toroidal plasma devices has been most efficiently advanced by numerical studies of simplified fluid models. When one considers the additional complicating features involved in realistic boundary conditions, the presence of divertors, and extensions to non-axisymmetric systems, it becomes clear that many new physical phenomena await investigation.

The range of time and space scales of the various physical phenomena leads to a fairly natural division from the physics point of view--(a) fast time scale; and (b) diffusion time scale. Such a division is also natural from the numerical point of view as the techniques involved in the solution for each category are different. Not only do the classification of the model equations change (i.e., from hyperbolic to parabolic), but it is clear that even with Class IV and V computer hardware, one would not, for example, attempt to reach confinement time scales in a Tokamak simulation by time-stepping a code designed to follow fast compressional Alfvén waves.

a. Simulation on the Fast Time Scale

Detailed comparison of experimental data from Scyllacs and pinches with theory, taking due account of experimental complications (plasma heating, compression, progression of equilibria, stability, atomic processes, etc.), will depend on the development of 2-D and 3-D (two and three dimensional) versions of codes analogous to the 1-D Hain-Roberts code.[1] Most of the physical phenomena important here lie in the fast MHD timescale (nanoseconds to microseconds).

For Tokamak configurations, the corresponding effects occur primarily on rather longer time scales, milliseconds, and are discussed in subsection (b). However, the questions of stability of Tokamak discharges toward MHD modes are very important for achieving efficiency in a reactor system and for minimizing the cost of feasibility devices. One example of great interest is the area of the early stages of a Tokamak discharge and the formation and destruction of magnetic surfaces.

Here also the relevant times are on the fast MHD time scale.

The potentially high impact of simulation techniques at the compressional Alfven time scale lies in the possibility of determining the ranges of plasma parameters in which MHD modes (e.g., local, ballooning, kink, tearing modes) are stable, or sufficiently so to be tolerable. Factors which affect the existence and severity of such modes are discharge shape, distribution of plasma pressure, current and magnetic field. location of conducting walls, dissipative effects in the plasma and walls, self limitation of modes from nonlinear effects, possibilities for stabilization due to feedback, finite particle orbit size effects, etc. In addition to studies of specific effects, composite models will be needed in which all the effects are treated simultaneously and accurately to yield results with all the relevant physics taken into account.

Some descriptions of codes in this category are given in Refs. 2-16.

b. *Simulation on the Diffusion Time Scale*

In order to simulate the time evolution of a plasma in a magnetic confinement device over most of its lifetime--from tens to hundreds of milliseconds--a set of partial differential equations of the diffusion type must be solved. Typical dependent variables are the number densities and temperatures of each particle species, current densities, and magnetic fields. The transport coefficients such as thermal conductivity, electrical resistivity, and diffusion coefficients are obtained from the best available theories, e.g., neo-classical, but the codes should also have the capability of easily changing the form of the coefficients in order to develop phenomenological models or take into account results of particle codes.

In the past few years, a considerable effort has been devoted, at several laboratories, to the numerical solution of one-dimensional (radial) transport equations for toroidal plasmas. This effort provides an excellent means of comparing theory with experiment. Recent developments of these codes have concentrated on the inclusion of neutrals and impurities in the models, and the use of empirical transport coefficients.

In the next five years there will be a great increase in the use of these codes and considerable refinement of the models will be required. Other physical effects will include an increase in the number of ion species--e.g., deuterons, tritons, and α-particles--in order to simulate DT burning experiments. The effects of radiation and atomic and molecular processes will be included.

The biggest change, however, and the greatest demand on the talents of the computational physicist and programmer will be the increase in dimensions required in these models. In order to realistically simulate actual devices with non-circular cross sections, neutral beam lines, and divertors, two- and three-dimensional codes will be required.

Some descriptions of codes in this category are given in Refs. 17-23.

2. Time-Independent Macroscopic Codes

It is normally necessary to develop time-independent codes to support the design and operation of each major CTR experiment. These include codes used to compute and study prospective equilibrium plasma configurations and determine their stability from δW-type calculations (especially important in high beta applications).

Experimental devices incorporating the idea of axial symmetry in a torus appear to be capable of plasma confinement for times which are of great interest. One reason for this result is the assurance of equilibria in such devices as predicted by MHD and guiding-center theories. Two-dimensional models for plasma equilibria have been used to compute the equilibrium fields for various containment schemes including Astron,[24] Levitron,[25] Tokamak,[26] and the stuffed cusp.[27]

Several containment schemes, including the stabilized mirror devices, do not possess an ignorable coordinate, allowing a reduction of the equilibrium equations to two dimensions. Codes have been developed to handle the general case for open containment--a three-dimensional code that solves for plasma equilibria in open-field geometries, which allows analysis of most minimum-B mirror systems.[28,29]

The critical problems in this area are the following:

(a) Develop two and three dimensional plasma equilibrium codes.

(b) Develop δW codes to determine the stability of various plasma configurations.

(c) Apply these codes to determine what plasma equilibria exist which look promising from their stability properties and from the plasma that they can confine.

(d) Investigate the effects of divertors and other boundaries.

Some descriptions of equilibria calculations are given in Refs. 24-31 and time-independent stability codes are described in Refs. 32-34.

[illegible]. Vlasov and Particle Codes

Particle codes are the most fundamental in that they emulate nature, following in detail the motion of particles under the influence of their self-consistent electric and magnetic fields, as well as any externally imposed fields. These codes give the most detailed results; they give phase-space distribution functions, fluctuation and wave spectra, and even the orbits of individual particles. They are ideal for providing detailed information on the growth and saturation of strong instabilities and the effects of turbulence. Such turbulence can give rise to many important macroscopic effects--for example, the anomalous absorption of waves and the transport of plasma, energy, and momentum. Information on absorption properties helps determine the relative merits of different types of turbulent heating, e.g., what type of distribution functions are produced, what type of turbulent spectra are produced, and what turbulent transport is caused by various wave heating methods. Information on transport produced by turbulence is essential for fluid and hybrid codes because these codes require a knowledge of transport coefficients which result from microscopic processes. Particle codes are also valuable in checking theories and the approximations that go into them. They provide clues to the important nonlinear mechanisms which saturate instabilities and help establish an intuitive understanding of plasma behavior. Because these codes must follow the behavior of plasma on the plasma-frequency time scale and on the Debye-length space scale they are not suitable for modeling large systems for long time periods.

Despite the fact that some applications of particle codes to realistic problems will require considerable development over a long period of time, many urgent problems can be carried out with reasonably straightforward extensions of existing techniques.

The following problems which are critical to the progress of the CTR program can be usefully attacked with particle codes:

(a) Determining the stability of plasmas heated by neutral injection or containing energetic reaction products.

(b) Determining how the loss rate from mirrors is affected by the loss-cone and high-frequency instabilities.

(c) Understanding the operation of divertors and determining the effects of boundaries on plasma behavior.

(d) Determining the effectiveness of heating by means of parametric instabilities using various types of waves (lower hybrid, ion cyclotron, etc.).

(e) Determining the effects of trapped particle instabilities in Tokamaks and other closed geometries.

(f) Simulation of current penetration in fusion discharges.

(g) Determining the nature and magnitude of particle diffusion and other transport processes due to nonlinear collective effects.

Three-dimensional codes and calculations will be limited by computing time and the size of systems which can be handled. It is difficult to predict the exact extent of progress in an effort which is in its infancy and just beginning to make contact with laboratory experiments. The size of the effort required to make rapid progress on all of the many critical problems is very large. Availability of people trained in both plasma physics and computing techniques will be one of the most important limitation.

Some description of particle codes and results are given in Refs. [illegible].

[illegible]. *Fokker-Planck Codes*

In the simulation of magnetically confined plasmas where the ions are not Maxwellian and where a knowledge of the distribution functions is important, kinetic equations must be solved. The proposition that a stable mirror plasma will yield net thermonuclear power depends on the rate at which particles are lost out the ends of the device. At the densities and energies typical of mirror machines, the end losses are due primarily to the scattering of charged particles into the loss cone in velocity space by classical Coulomb collisions. The kinetic equation describing this process is the Boltzmann equation with Fokker-Planck collision terms.

The use of this equation is not restricted to mirror systems. The heating of plasmas by energetic neutral beams, the thermalization of α-particles in DT plasmas, the study of runaway electrons in Tokamaks, and the performance of two-energy component fusion reactors are other examples where the solution of the Fokker-Planck equation is required.

The problem is to solve a nonlinear partial differential equation for the distribution function of each charged species in the plasma, and there are seven independent variables (three spatial coordinates, three velocity coordinates, and time). Such an equation, even for a single species, exceeds the capability of any present computer so several simplifying assumptions are therefore required to treat the problem. Typical approximations that are made in present day codes are to neglect spatial dependence and to assume that the distribution functions are azimuthally invariant in velocity space (about the direction of the magnetic field) and can be represented by the lowest angular eigenfunction.

The goal in this area is to develop multi-species codes which are fully two-dimensional in velocity space and include the spatial dependence of the magnetic field. The development of two-dimensional (v, μ) multi-species codes is already completed. The next step is to introduce one space dimension (r) to such codes. Such codes should be in production use in one year. The goal for the five-year period (1975-1979) should be to include realistic spatial dependence in Fokker-Planck codes. This will certainly require Class V and VI computers.

Descriptions of Fokker-Planck codes and results are given in Refs. 40-44.

5. Hybrid Codes

There will be a critical need for codes which can best be described as Hybrid Codes; these are codes which combine the good features of fluid codes with the good features of particle codes, but which can avoid some of the weaknesses of each. The advantage of a particle code is that it contains the most complete treatment of the physics. Its disadvantage also stems from this feature because it is forced to follow the development of the plasma on the fastest time scale and shortest space scale at which significant plasma phenomena occur. These scales are typically much shorter than the time and size scale in thermonuclear plasma devices. It is clear that even the fastest computer would not be able to follow the plasma development in a microscopic way over the length of time or over the spatial volume of practical interest. The feature of fluid codes which is attractive is that they treat the plasma on a coarser scale and hence need many fewer time steps and spatial points. On the other hand, the usual conditions required for a fluid treatment (short mean free paths and frequent collisions) are not realized for a thermonuclear plasma. The motions of certain classes of particles are often crucial. One example is the motion of trapped particles, which cause the trapped particle instability. Further, the generation and damping of waves and the effectiveness of waves in causing diffusion and energy transport will depend critically on resonance phenomena between particles and waves. It is clear that proper treatment of such phenomena will require some reasonably accurate description of the important class of particles. On the other hand, it should be possible to treat the rest of the plasma by means of fluid equations. Thus some mixed description should be effective and economical.

The relevance of hybrid codes to thermonuclear plasma devices is really the same as the relevance of fluid codes and particle codes. It appears that the impact of developments in this area will be large and, perhaps, even crucial. Codes of this type will probably give the most accurate overall description of the plasma which is practical and hence may ultimately be called upon to predict the performance of various plasma devices and reactors. It is difficult to assess just what the developments will be or how soon they will impact the program because this is an area which is still very much in its infancy; not many things have been tried, and it is not yet known how successful this approach will be.

Another class of hybrid code which promises to be very useful is the coupling of a Fokker-Planck code to a plasma transport (diffusion) code. In neutral-beam-heated Tokamaks and the two-energy-component toroidal fusion test reactor there is a warm Maxwellian background plasma which can be described by macroscopic transport equations and an energetic species which should be described by the Fokker-Planck equation. The coupling of these systems is by means of sources of particles and energy in the multi-species transport code and a time-dependent Maxwellian target plasma in the multi-species Fokker-Planck code. Codes of this type are solved on the same time scale (milliseconds) separately so there does not appear to be any difficulty in this regard.

The first step is to add r dependence to a (v, μ) multi-species Fokker-Planck code and then couple it with an already existing 1-D transport code of the type described in 1b.

The long-range goal would be the coupling of a 2-D transport code to a Fokker-Planck code which included two space dimensions.

Some existing hybrid codes are described in Refs. 44-46.

ENGINEERING AND TECHNOLOGY STUDIES

Engineering and technology studies will become more and more important as we move towards a DT burner and into the reactor development phase. Among the critical engineering problems are the following:

(a) Development of 1- and 2-D transport codes, including burning reaction products and radiation.

(b) Neutron reaction and transport studies in the blanket and reactor structure using 2- and 3-D Monte Carlo calculations.

(c) Studies of wall and divertor effects, sputtering, plasma and heat loss to the divertor, effectiveness of divertors in preventing the penetration of impurities, etc.

(d) Specific machine design calculations, especially parametric studies with optimization in view.

(e) Investigations of control systems.

(f) Structural studies, stress, deflections, heat loads, induced activity, etc.

(g) Safety studies, possible faults and failures and their effects.

(h) Analysis of the operation of direct converters.

The near-term goals are to provide the required calculations for the design, construction, and operation of upcoming and existing experimental devices. The long-term goals will be similar but, in addition, we must be sure that the required codes are available for the very large experiments which will be built and that the required manpower and computing capability exist.

With proper support there appears to be no reason why the critical engineering calculations cannot be carried out. Class IV computers are capable of performing nearly all of these studies. Class V computers will probably be required for the parameter studies and reactor system studies needed. Large engineering calculations requiring at least Class IV and possibly Class V computers will continue as long as machines are designed and built.

NATIONAL CTR COMPUTER CENTER

The initial configuration of the CTR Computer Center, which will be operational in late 1975, consists of a Control Data Corporation 7600 computer located at Livermore. This type of computer is referred to as a Class IV computer and will have 500K words of large core memory and 64K words of small core memory as well as disc storage. This machine will be fully dedicated to fusion computing and will be used by all of the CTR installations.

The CTR projects at Los Alamos, Oak Ridge, Princeton, General Atomic Corp., and Livermore will each have a User Service Center (USC) which consists of a Digital Equipment Co. PDP10KI computer, disc packs, printers, and other peripheral equipment. The PDP10, which is referred to as a Class II computer will have 64K words of fast memory.

Each of the UCS's will be connected to the central computer at LLL by means of leased 50 kilobit/sec lines. The USC's will serve as local computers for small jobs and experimental data processing and as remote job entry terminals to the 7600 for large numerical calculations of the type described in previous sections of this report.

Future expansion of the CTR computer network will include smaller User Service Centers at universities engaged in CTR research. These mini-USC's will be linked to the network at the nearest USC by means of communication lines of somewhat smaller capacity. In addition the computing capability of the Center at LLL will be increased by memory expansion of the CDC 7600 and later by another Class IV computer or possibly a Class VI computer if such a machine is available.

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