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D. A. White, B. J. Fasenfest

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Performance of Low-Rank QR Approximation of the Finite Element Biot-Savart Law

Daniel A. White and Benjamin J. Fasenfest
Lawrence Livermore National Laboratory
L-154, 5000 East Ave, Livermore CA USA
dwhite@llnl.gov

Abstract— We are concerned with the computation of magnetic fields from known electric currents in the finite element setting. In finite element eddy current simulations it is necessary to prescribe the magnetic field (or potential, depending upon the formulation) on the conductor boundary. In situations where the magnetic field is due to a distributed current density, the Biot-Savart law can be used, eliminating the need to mesh the non-conducting regions. Computation of the Biot-Savart law can be significantly accelerated using a low-rank QR approximation. We review the low-rank QR method and report performance on selected problems.

I. Introduction

There are many different finite element formulations for transient eddy current simulation. In this paper we employ a magnetic vector potential formulation as described in [1]. In order to completely specify the problem it is necessary to apply boundary conditions, and for this particular formulation the boundary conditions are either the inhomogeneous Dirichlet condition $n \times A = A_{bc}$ or inhomogeneous Neumann condition $n \times B = B_{bc}$. These boundary conditions on the conductor of interest can be computed using the Biot-Savart law,

$$\frac{\mu}{4\pi}\int_{\Omega}\frac{J}{R}dV = A_{bc}$$

or

$$\frac{\mu}{4\pi}\int_{\Omega}\frac{J\times R}{R^3}dV = B_{bc}\,.$$

The QR procedure is independent of the kernel and can be applied to either law, for brevity we will focus on the former equation that gives A_{bc}. Applying the Galerkin procedure and employing the basis function expansions $A_{bc}(r) = \sum \alpha_i W_i(r)$ and $J(r) = \sum c_i N_i(r)$ yields the discrete Biot-Savart law $\mathbf{M\alpha} = \mathbf{Kc}$, where the bulk of the computational effort is in computing and applying the dense matrix **K**, where **K** is given by

$$K_{kj} = \frac{\mu}{4\pi}\int_{\Gamma}\int_{\Omega}\frac{N_k(r')\bullet W_j(r)}{|r-r'|}dr'dr$$

II. Low-Rank QR Approximation

The matrix **K** is compressed using a low-rank QR decomposition. Consider the geometry shown in Figure 1, where the mesh has been decomposed into 24 partitions using a graph-based algorithm [2]. The **K** matrix is then a block 24 × 24 matrix. Most blocks represent distant interactions and hence have a low-rank QR decomposition. The low-rank QR decomposition is computed by first selecting ***r*** rows and columns of the block, where ***r*** is the rank, and applying the modified Gram-Schmidt procedure [3] to compute Q and R. The rank ***r*** is adaptive, based on an error tolerance. The row and column selection procedure is somewhat ad-hoc and is a modification of that used in [4] for electrostatic applications. Blocks of **K** that represent near interactions are not low-rank, however these blocks can be sub-partitioned, and the low-rank QR decomposition can be applied to the low-rank sub-blocks. This process is applied recursively until the size of the partitions decreases below an a-priori threshold, e.g. 50 elements.

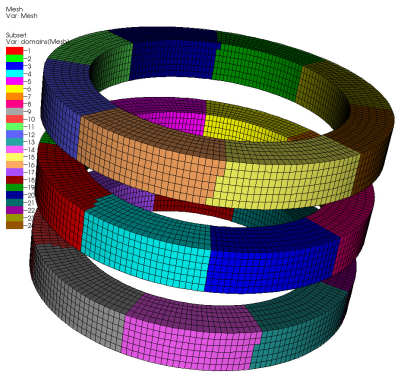

Figure 1. A finite element mesh of 3 coils, partitioned into 24 partitions for QR compression.

The mesh in Figure 1 has 20736 volume elements. Using the recursive low-rank QR compression, with only 2 levels of recursion, resulted in a compression of 109× for this specific problem. The user-specified error tolerance for this specific example was 10^{-3} and the rank of the sub-blocks varied from 4 < ***r*** < 25. It is important to note that as the computational mesh is refined, that rank ***r*** does not increase, hence the method asymptotically approaches *O(n)log(n)*.

III. References

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