A Lower Bound on Neutrino Mass and Its Implication on The Z-burst Scenario

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We show that the cascade limit on ultra high energy cosmic neutrino (UHECν) flux imposes a lower bound on the neutrino mass provided that super-GZK events of ultra high energy cosmic rays (UHECRs) are produced from Z-bursts. Based on the data from HiRes and AGASA, the obtained neutrino mass lower bound violates its existing cosmological upper bound. We conclude that the Z-burst cannot be the dominant source for the observed super-GZK UHECR events. This is consistent with the recent ANITA-lite data.

PACS numbers:

I. INTRODUCTION

Big bang cosmology predicts the existence of both cosmic microwave background (CMB) and cosmic neutrino background (CνB). Ultra high energy cosmic protons are expected to interact effectively with the CMB photons, predominantly through the photopion production at ∆-hyperon resonance, and would lose their energies rapidly with the attenuation length around 50Mpc. As such the ultra high energy cosmic ray (UHECR) spectrum is predicted to exhibit a cutoff — the so called GZK cutoff [1, 2] — around the threshold energy \( \sim 4 \times 10^{20} \text{eV} \). While observations from the HiRes experiment is consistent with the notion of GZK cutoff [3], the AGASA data appears to suggest the opposite [4, 5]. This leads to many speculations as to whether the GZK cutoff really exists, and if not what is the nature of these super-GZK events.

Existing models for super-GZK UHECRs are usually categorized into top-down and bottom-up scenarios. The top-down scenario assumes the existence of super-massive exotic elementary particles based on theories beyond the standard model. The major challenge of this scenario lies in the demand for a fine-tuned decay and/or annihilation rate and the lack of physical evidence for their existence. On the other hand, the bottom-up scenario, which assumes ordinary particles as the UHECRs, faces the challenge of providing an effective mechanism to accelerate particles to ultra high energies. Even if an effective “cosmic accelerator” can be identified, the issue of cosmic transport dictated by the GZK mechanism remains, as there does not seem to exist identifiable sources within our local super cluster (\( \sim 50\text{Mpc} \)) for the detected events.

To circumvent this difficulty, it was suggested that the Z-burst, the resonant annihilation of the ultra high energy cosmic neutrino (UHECν) with the CνB into a Z boson and its subsequent decay into ultra high energy protons [6, 7, 8], that occurs within our local super cluster can account for UHECRs beyond the GZK-cutoff [9, 10]. With its mean-free-path comparable to the present Hubble radius, the UHECν serves as a cosmic messenger that can avoid the GZK proton attenuation problem without invoking particle theory beyond the standard model. For the Z-burst to happen, the UHECν must be at a resonant energy,

\[
E_{\text{res}} = \frac{M_Z^2}{2m_\nu} \approx 4 \times 10^{21} \left(\frac{1\text{eV}}{m_\nu}\right) \text{eV},
\]

which depends on the neutrino rest mass, \( m_\nu \). Here \( M_Z \) denotes the mass of the Z boson. If the Z-burst mechanism is indeed responsible for the observed UHECR super-GZK spectrum, then there must exist a constraint on the neutrino mass via the above relation.

Based on the Z-burst scenario, two groups have derived bounds on neutrino masses using AGASA data with different strategies. Fodor, Katz and Ringwald [11] deduce the Z-burst spectrum from the AGASA data by parameterizing the transition from the non-burst to the burst component near and above the “ankle” of the UHECR spectrum. Gelmini, Varieschi and Weiler [12] derive their bound by requiring the Z-burst not to overproduce non-observation events beyond the AGASA end-point energy. Our strategy, instead, is to invoke an upper limit on the UHECν flux so as to obtain an upper bound on the required resonant energy \( E_{\text{res}} \), which can in turn be translated into a lower bound on the neutrino mass.

In this paper we derive a lower bound on the neutrino mass based on the assumption that Z-burst mechanism saturates the observed UHECR super-GZK spec-
trum. Our deduced neutrino mass lower bound, however, turns out to be higher than the existing upper bound deduced from cosmological considerations. We thus conclude that the Z-burst mechanism cannot be responsible for the super-GZK UHECR spectrum. Our conclusion agrees with that from the recent ANITA-lite experiment [13].

II. UHECR FLUX AND Z-BURST

Assume that all observed super-GZK UHECR proton events are induced from Z-bursts. The observed super-GZK proton flux must be smaller than the total Z-burst proton yield in the universe since there must be events occurred outside our local GZK-sphere (∼ 50 Mpc) which could not reach the Earth. Furthermore, in order for Z-burst events to saturate the observed super-GZK spectrum, it is inevitable that they are oversupplied since there must be some Z-burst protons that are generated at energies below the GZK-cutoff. Therefore,

\[ I_{p,>GZK}^{\text{obs}} \leq I_{\nu}^{Z}, \] (2)

where \( I_{p,>GZK}^{\text{obs}} \) is the total observed super-GZK proton flux with energy exceeding the GZK-cutoff and \( I_{\nu}^{Z} \) the total proton flux from Z-bursts, both in units of cm\(^{-2}\)s\(^{-1}\)sr\(^{-1}\).

Though observations [14] cannot completely rule out the possible contribution to the super-GZK UHECR spectrum by UHE photons, experimental data [14] suggests that protons saturate the super-GZK flux, \( I_{p,>GZK}^{\text{obs}} = I_{\nu}^{GZK} \), at 2\(\sigma\) confidence level. Then in terms of the total observed UHECR flux, Eq. (2) can be written as

\[ I_{p,>GZK}^{\text{obs}} \leq I_{\nu}^{Z}. \] (3)

The AGASA experiment has accumulated 57 events above \( 4 \times 10^{19} \text{eV} \) with a total exposure of \( \sim 4 \times 10^{20} \text{cm}^{2}\text{sr} \) [15]. This translates into an observational super-GZK flux,

\[ I_{GZK}^{\text{obs}} \simeq 1.43 \times 10^{-19} \text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}. \] (4)

It can be shown that a fitting spectrum with a power-law index \(-2.78\) [14] reproduces the above flux.

III. Z-BURST YIELD

Now we deduce the total Z-burst proton yield within a relevant cosmic volume. Solar and atmospheric data on neutrino oscillations indicate that the oscillation lengths are much shorter than the solar distance. So for cosmic neutrinos their population among the 3 flavors should be equalized. The total UHEC\(\nu\) flux is thus simply 3 times that for a single neutrino flavor. We further assume that UHEC\(\nu\) fluxes are the same for neutrinos and antineutrinos. By definition,

\[ I_{p,>GZK}^{Z} = 3\xi_{p+\nu}\int_{0}^{R_{\max}} dr \int_{0}^{\infty} dE F(E, r) \times \sigma_{\nu\bar{\nu}}(E = \frac{s}{2m_{\nu}}) \text{Br}(Z \rightarrow \text{hadrons}) n_{\nu}(r). \] (5)

Here \( F(E, r) \) is the UHEC\(\nu\) flux at energy \( E \) and distance \( r \) from the Earth, \( n_{\nu}(r) \) is the number density of the C\(\nu\)B, \( \sigma_{\nu\bar{\nu}}(s) \) the neutrino-antineutrino cross section at \( s = 2m_{\nu}E \), Br\((Z \rightarrow \text{hadrons}) \) the branching ratio, and \( \xi_{p+\nu} \) the multiplicity of nucleons per Z-burst.

For completeness, our integration should include all neutrinos and Z-burst events in the universe. Such a treatment tends to be over-conservative as the protons deduced from Z-bursts outside of our local GZK-sphere may hardly survive. The complete but ultra-conservative treatment is discussed in the appendix. A physically reasonable yet much simplified calculation can be carried out by neglecting the contributions outside of our local GZK-sphere. This amounts to replacing the maximum distance \( R_{\max} \) in our integration by the radius of our local GZK-sphere \((R_{GZK} \sim 50\text{Mpc}) \). As the distance under consideration is much more local, all the \( r \)-dependence can be ignored:

\[ I_{p,>GZK}^{Z} = 3\xi_{p+\nu} R_{GZK} n_{\nu} \text{Br}(Z \rightarrow \text{hadrons}) \times \int_{0}^{\infty} dE F(E) \sigma_{\nu\bar{\nu}}(E), \] (6)

where \( n_{\nu}(r) = n_{\nu} = 112\text{cm}^{-3} \) is the neutrino-antineutrino number density per flavor at present.

The UHEC\(\nu\) flux is commonly assumed to follow a power-law energy spectrum

\[ F(E) = F_{0}E^{-\alpha}, \] (7)

with \( F_{0} \) being the normalization factor.

Using \( E = sE_{\text{res}}/M_{Z}^{2} \), we can write the energy integration as

\[ \int_{0}^{\infty} dE F(E) \sigma_{\nu\bar{\nu}}(s = 2mE) = E_{\text{res}} \int_{0}^{\infty} \frac{ds}{M_{Z}^{2}} F(sE_{\text{res}}/M_{Z}^{2}) \sigma_{\nu\bar{\nu}}(s). \] (8)

As the neutrino-antineutrino annihilation cross section is sharply peaked at the Z-resonance, it acts essentially like a \( \delta \)-function in the integration over the energy of the UHEC\(\nu\). We therefore introduce the energy-averaged cross section [3] [14]

\[ \langle \sigma_{\nu\bar{\nu}} \rangle \equiv \int \frac{ds}{M_{Z}^{2}} \sigma_{\nu\bar{\nu}}(s) = 2\pi\sqrt{2} G_{F} = 40.4\text{nb}, \] (9)
which is the effective cross section for all neutrinos within the resonance range \((E_{\text{res}}(1-\Gamma_Z/M_Z), E_{\text{res}}(1+\Gamma_Z/M_Z))\) and simplify the integration of Eq. (8) as

\[
E_{\text{res}} \int_0^\infty \frac{ds}{M_Z^2} F(s E_{\text{res}}/M_Z^2) \sigma_{\nu\nu}(s) \\
\simeq F_0 E_{\text{res}}^{-\alpha} \int_0^\infty \frac{ds}{M_Z^2} \sigma_{\nu\nu}(s) \\
= F_0 E_{\text{res}}^{-\alpha} \langle \sigma_{\nu\nu} \rangle. \tag{10}
\]

Putting everything together we find

\[
I_{p\nu}^Z = R_{\text{GZK}} n_0 \xi_{\nu p+n} F_0 \left(\frac{M_Z^2}{2m_\nu}\right)^{1-\alpha} \\
\times \langle \sigma_{\nu\nu} \rangle \text{Br}(Z \to \text{hadrons}). \tag{11}
\]

The experimental data \(^{17}\) gives the branching ratio \(\text{Br}(Z \to \text{hadrons}) = (69.89 \pm 0.07)\%\). \(^{12}\)

The final proton multiplicity per Z-burst was calculated by Fodor, Katz and Ringwald(see \(^{18}\) and references therein) as a function of the proton momentum distribution and by Gelmini, Varieschi and Weiler using the event generator PYTHIA \(^{10}\). They obtain \(\xi_{\nu p+n} \approx 2.04\) and 1.6, respectively. We take the former value in this paper.

### IV. CASCADE LIMIT AND NEUTRINO MASS BOUND

We now invoke the cascade limit to constrain the UHEC\(\nu\) flux \(^{20\, 21}\). This is permissible due to the fact that neutrino productions must always be accompanied by photons and electrons. The cascades are induced while these photons or electrons interact with low energy background radiations such as the CMB in extra galactic space and the infrared radiation inside the galaxy. The photons so induced would further cascade and eventually pile up in the energy range of 10MeV-100GeV with a spectrum \(\propto E^{-2}\), which is consistent with the EGRET observation \(^{22}\). The estimated average energy density in this range is \(\omega_{\text{EGRET}} \approx 2 \times 10^{-6}\text{eV/cm}^3\). This provides an upper bound on the UHE neutrino flux,

\[
E^2 F(E) < \frac{c}{4\pi} \omega_{\text{EGRET}}. \tag{13}
\]

To be prudent, we do not assume the exact value of \(\alpha = 2\) for the power-law index, but instead leave \(\alpha\) as a free parameter, knowing that its value should be close to 2. Thus the parameter \(F_0\) can be substituted with an upper bound as follows:

\[
F_0 < \frac{c}{4\pi} \omega_{\text{EGRET}}/E_{\text{min}}^{2-\alpha}, \quad \alpha \geq 2, \\
F_0 < \frac{c}{4\pi} \omega_{\text{EGRET}}/E_{\text{max}}^{2-\alpha}, \quad \alpha < 2, \tag{14}
\]

where \(E_{\text{max}}\) and \(E_{\text{min}}\) are the maximum and minimum energies of the UHE\(\nu\) spectrum.

Implementing the cascade limit condition, and inserting all the relevant physical quantities discussed in the previous section, Eq. (4) becomes

\[
m_\nu > 28.7 \frac{E_{\text{res}}}{E_{\text{min}}} \alpha^{-2}\text{eV}, \quad \alpha \geq 2, \tag{15}
\]

\[
m_\nu > 28.7 \frac{E_{\text{max}}}{E_{\text{res}}} \alpha^{-2}\text{eV}, \quad \alpha < 2. \tag{15}
\]

Note that in this expression not all the \(m_\nu\) dependence were grouped to the LHS, as \(E_{\text{res}}\) clearly depends on \(m_\nu\). Nevertheless this expression has an advantage in that \(E_{\text{res}}/E_{\text{min}} \geq 1\) and \(E_{\text{max}}/E_{\text{res}} \geq 1\) by definition.

An explicit \(m_\nu\) lower bound can be obtained by moving all the \(m_\nu\) dependence to the LHS. We then find

\[
m_\nu > \frac{1}{2} (A^{1-\alpha} E_{\text{min}}^{2-\alpha} M_Z^2), \quad \alpha \geq 2, \tag{16}
\]

\[
m_\nu > \frac{1}{2} (A^{1-\alpha} E_{\text{max}}^{2-\alpha} M_Z^2), \quad \alpha < 2, \tag{17}
\]

where

\[
A = \frac{I_{p\nu}^{\text{obs}}}{\xi_{p+n}^{\nu} m_\nu R_{\text{GZK}}} \tag{18}
\]

The mass bound is dependent on the power-law index \(\alpha\) and the values of \(E_{\text{max}}\) or \(E_{\text{min}}\). Our limited knowledge on the UHE\(\nu\) renders large uncertainty in the determination of \(E_{\text{max}}\) and \(E_{\text{min}}\). One thing which is certain, however, is that the resonant energy must lie in between \(E_{\text{max}}\) and \(E_{\text{min}}\) for the Z-burst to happen. Eq. (15) indicates that the minimum value of our bound corresponds to the situation where either \(E_{\text{max}}\) or \(E_{\text{min}}\) equals \(E_{\text{res}}\), or \(\alpha = 2\). Since we should look for the lowest possible lower bound, we put \(E_{\text{res}} = E_{\text{max}} = E_{\text{min}}\) in our estimate and arrive at our neutrino mass lower bound

\[
m_\nu > 28.7^{+11.8}_{-10.6}\text{eV} \quad (R_{\text{max}} = R_{\text{GZK}} \approx 50\text{Mpc}), \tag{19}
\]

where the error comes from fitting the AGASA data \(^{16}\).

Recent WMAP \(^{23}\) measurement of the CMB fluctuations has deduced a strong upper limit on neutrino masses, \(\Sigma_i m_{\nu_i} < 0.69\text{eV}\). Since any single neutrino mass \(\sim 0.04\text{eV}\) implies a near mass-degeneracy for all three active neutrinos, one concludes \(m_{\nu} < 0.23\text{eV}\). Two analyses \(^{24\, 25}\) which include data from WMAP, 2dF, SDSS, and galaxy cluster surveys have arrived at a bound of \(\Sigma_i m_{\nu_i} \lesssim 0.7\text{eV}\). Another analysis \(^{26}\) using CMB and LSS data gives \(\Sigma_i m_{\nu_i} \lesssim 1\text{eV}\), but finds a stronger bound \(\Sigma_i m_{\nu_i} \lesssim 0.6\text{eV}\) when priors from supernova data and Hubble Key Project are included. These newer results are close to the original WMAP bound. All these analyses converge to a cosmological upper bound of \(m_{\nu} \lesssim 0.23\text{eV}\), which is 2 order of magnitudes smaller than the lowest possible lower bound we have derived.
V. IMPLICATION

Our derivation is based on two assumptions: the saturation of the observed super-GZK UHECR flux by the Z-burst mechanism and the cascade upper limit on the maximum UHE\nu flux.

Since the cascade limit is deduced from the cascades of photons accompanying the neutrino production, it is valid for all sources. Not only astrophysical accelerators (e.g. GRBs, AGNs, SNe, etc.) but also top-down sources, such as topological defects, superheavy X particles, dark matter, etc., are all contributing to this limit as long as the photons are co-produced alongside with neutrinos. It is generally believed\cite{27, 28} that the limit as long as the photons are co-produced alongside particles, dark matter, etc., are all contributing to this sources, such as topological defects, superheavy X

According to the big bang cosmology, the number density of CνB is \( n_\nu(z) = n_\nu(1+z)^3 \text{cm}^{-3} \), where \( n_\nu = 112 \text{cm}^{-3} \) is the neutrino-antineutrino number density at present. The UHE\nu flux is now assumed to follow a power-law energy spectrum with cosmological evolution of the source included \cite{29, 30} and can be parameterized as
\[
F(E, r) = F(E) f(z) = F_0 E^{-\alpha} f_0(1+z)^\beta, \tag{A2}
\]
where \( f_0 \) is the normalization factor for the source evolution determined by the condition
\[
\int_0^{z_{\text{max}}} f(z) dz = 1, \tag{A3}
\]
with \( z_{\text{max}} \) being the redshift of the most distant source.

A more sophisticated distribution function has been introduced, based on the star formation rate and the GRB site distribution \cite{31}
\[
f(z) = f_0 \frac{1 + a_1}{(1+z)^{-a_2} + a_1(1+z)^{a_3}}, \tag{A4}
\]
where \( a_1, a_2 \) and \( a_3 \) are fitting parameters. It can be shown that our neutrino mass bound is insensitive between these two choices of distribution functions and for simplicity we will invoke Eq.\,(8) in our subsequent discussion. The separation of variables allows us to carry out the UHE\nu energy and distance integrations independently.

The integration over propagation distance involves several sources of cosmic evolution,
\[
\int_0^{z_{\text{max}}} cH_0^{-1} \frac{n_\nu(1+z)^3 f(z) dz}{(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \equiv cH_0^{-1} n_\nu f(z_{\text{max}}, \beta), \tag{A5}
\]
where simple analytic result is not readily attainable. Table\,[3] displays values of \( f(z_{\text{max}}, \beta) \) for selected choices of \( z_{\text{max}} \) and \( \beta \). It is clear that \( f(z_{\text{max}}, \beta) \) is reasonably insensitive to \( z_{\text{max}} \) and \( \beta \).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\( \beta \) & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\hline
\( z_{\text{max}} \) & 1.147 & 1.54 & 1.62 & 1.69 & 1.76 & 1.82 & 1.88 \\
\hline
1 & 1.65 & 1.81 & 2.00 & 2.18 & 2.35 & 2.49 & 2.59 \\
\hline
2 & 3.13 & 1.98 & 2.27 & 2.56 & 2.81 & 2.99 & 3.12 \\
\hline
\end{tabular}
\caption{Values of \( f(z_{\text{max}}, \beta) \) for selected \( z_{\text{max}} \) and \( \beta \).}
\end{table}

APPENDIX A: A COMPLETE BUT ULTRA-CONSERVATIVE VERSION

For completeness, the integration over distance must be carried out to include the cosmological evolution of the CνB number density and the UHE\nu flux. Since the dependence on the propagation distance \( r \) of the UHE\nu fluxes and the CνB number density can be expressed more straightforwardly in terms of the redshift parameter \( z \), we make the following change of variables
\[
dr = -\frac{cH_0^{-1}}{(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} dz, \tag{A1}
\]
where \( \Omega_m \approx 0.3 \) and \( \Omega_\Lambda \approx 0.7 \) are the present matter and dark energy densities in units of the critical density, respectively.
With the knowledge of \( f(z_{\max}, \beta) \), the Z-burst yield, Eq. 7, becomes

\[
I_{\nu}^Z = cH_0^{-1}n_\nu(f(z_{\max}, \beta)\xi_{p+n}\nu F_0 \frac{M^2_\nu}{2m_\nu})^{1-\alpha} \times \langle \sigma_{\nu\nu} \rangle Br(Z \rightarrow \text{hadrons}). \tag{A6}
\]

Correspondingly, the lower bound on neutrino mass is changed to

\[
m_\nu > 1.25f(z_{\max}, \beta)^{-1} \frac{E_{\text{res}}}{E_{\text{min}}} \alpha^{-2} \text{eV}, \quad \alpha \geq 2,
\]

\[
m_\nu > 1.25f(z_{\max}, \beta)^{-1} \frac{E_{\max}}{E_{\text{res}}} \alpha^{-2} \text{eV}, \quad \alpha < 2. \tag{A7}
\]

while Eqs. 10 and 17 remain the same form with

\[
A = I_{>\text{GZK}}^{\text{obs}} \left[ \frac{c}{4\pi} \omega_{\text{EGRET}} cH_0^{-1} n_\nu \xi_{p+n} \langle \sigma_{\nu\nu} \rangle Br(Z \rightarrow \text{hadrons}) \right]^{-1}. \tag{A8}
\]

The mass bound is now dependent on the numerical values of the evolution factor \( f(z_{\max}, \beta) \) as well (see Table II). Again we look for the lowest possible lower bound by putting \( E_{\text{res}} = E_{\max} = E_{\text{min}} \) in our estimate. As we have shown, \( f(z_{\max}, \beta) \) is of the order 1 and is insensitive to \( z_{\max} \) and \( \beta \). With the choice of \( f(z_{\max}, \beta) = f(3,0) = 2.56 \), we arrive at our neutrino mass lower bound

\[
m_\nu > 0.49^{+0.20+0.53}_{-0.18-0.14} \text{eV} \quad (R_{\max} \sim cH_0^{-1}). \tag{A9}
\]

The former error comes again from the AGASA data and the latter from the uncertainty of the evolution factor \( f(z_{\max}, \beta) \).

### Table II: Values of mass lower bound for selected \( z_{\max} \) and \( \beta \) with \( \alpha = 2 \) energy spectrum.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{\max} )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( f(z_{\max}, \beta) )</td>
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<td>1.65</td>
<td>1.73</td>
<td>1.69</td>
<td>2.18</td>
<td>2.56</td>
</tr>
<tr>
<td>mass bound</td>
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<td>0.76</td>
<td>0.73</td>
<td>0.74</td>
<td>0.58</td>
<td>0.49</td>
</tr>
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