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Inferred Limits on Lepton Flavor Violating Decays of the K_S

L. Bellantoni
Fermi National Accelerator Laboratory, Batavia IL 60510, U.S.A.

Strong model independent upper bounds on $Br\left(K_S \to \pi^0 e\mu\right)$ may be inferred from recent experimental limits on $Br\left(K_L \to \pi^0 e\mu\right)$ and $Br\left(K^+ \to \pi^+ e^+ \mu^-\right)$. From this result, upper bounds for $Br\left(K_S \to e\mu\right)$ may be obtained for a broad class of models. Models outside of this class seem unlikely.

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I. Introduction

A lengthy program of experimental searches for lepton flavor violation (LFV) decays in kaons has recently lead to strong limits on the non-existence of these decays[1234]:

$$\begin{aligned} Br\left(K^+ \to \pi^+ \mu^+ e^-\right) &< 1.2 \times 10^{-11} \\ Br\left(K^+ \to \pi^+ \mu^- e^+\right) &< 5.2 \times 10^{-10} \\ Br\left(K_L \to \mu e\right) &< 4.7 \times 10^{-12} \\ Br\left(K_L \to \pi^0 \mu e\right) &< 3.3 \times 10^{-10}. \end{aligned} \tag{1}$$

LFV decays in K^- and K_S have not been studied. In K^- decays, there is no referent phase caused by *e.g.* strong interactions and the interference that leads to *CP* violation cannot occur. Accordingly, even if *C* or *P* are violated individually, after integration over phase space $\Gamma\left(K^+ \to \pi^+ \mu^+ e^-\right) = \Gamma\left(K^- \to \pi^- \mu^- e^+\right)$ and

$\Gamma\left(K^+ \to \pi^+ \mu^- e^+\right) = \Gamma\left(K^- \to \pi^- \mu^+ e^-\right)$. For the K_S, where decays proceed from an admixture of $S=+1$ and $S=-1$, bounds may be found from limits on decays from the (different) admixture found in K_L and from the pure $S=-1$ amplitudes found in K^+ decays.

II. The decay $K_S \to \pi^0 \mu e$

With the convention $CP\left|K^0\right\rangle = -\left|\overline{K^0}\right\rangle$,

$$\Gamma\left(K_L \to \pi^0 \mu e\right) \propto \left| \frac{(1+\varepsilon)\left\langle \pi^0 \mu e \middle| K^0 \right\rangle + (1-\varepsilon)\left\langle \pi^0 \mu e \middle| \overline{K^0} \right\rangle}{\sqrt{2}\sqrt{1+|\varepsilon|^2}} \right|^2 \tag{2}$$

where μe refers to either $\mu^+ e^-$ or $\mu^- e^+$ and ε is the familiar indirect *CP* violation parameter. In the known semileptonic K_L decays, the interference between the K^0 and $\overline{K^0}$ amplitudes shown in Eqn. (2) does not occur because the charge of the muon or electron distinguishes $S=+1$ and $S=-1$ amplitudes. If, *e.g.*, generation number as conventionally defined is a conserved quantity then interference will also not occur. In that case, $\Gamma\left(K_S \to \pi^0 \mu e\right) = \Gamma\left(K_L \to \pi^0 \mu e\right)$ and $Br\left(K_S \to \pi^0 \mu e\right) < 5.7 \times 10^{-13}$ at the 90% C.L.

Letting $\Psi_3 \equiv \left\langle \pi^0 \mu e \middle| \overline{K^0} \right\rangle \Big/ \left\langle \pi^0 \mu e \middle| K^0 \right\rangle$ parameterize the inequality of decays from the $S=+1$ and the $S=-1$ initial state components,

$$\frac{\Gamma\left(K_L \to \pi^0 \mu e\right)}{\Gamma\left(K^+ \to \pi^+ \mu e\right)} = r \frac{|1+\varepsilon|^2 + 2\Re\left[(1+\varepsilon)^*(1-\varepsilon)\Psi_3\right] + \left|\Psi_3(1-\varepsilon)\right|^2}{4\left(1+|\varepsilon|^2\right)}, \tag{3}$$

where r differs from unity due to isospin violation. Final state $\pi^0 - \eta$ mixing contributes[5] 2.2% to r, and phase space effects (for unit matrix element) are 3.1%, giving $r \cong 1.054$. A similar equation to Eqn. (3) holds for $\Gamma\left(K_S \to \pi^0 \mu e\right)$, but the interference term enters with the opposite sign.

Eqn. (3) constrains Ψ_3 to a circle on the Argand plane. A conservative upper bound on $Br\left(K_S \to \pi^0 \mu e\right)$ may be obtained by scanning the experimentally permitted ranges for $Br\left(K_L \to \pi^0 \mu e\right)$ and $Br\left(K^+ \to \pi^+ \mu e\right)$ and selecting the value of Ψ_3 that maximizes $Br\left(K_S \to \pi^0 \mu e\right)$. That maximum is

$$Br\left(K_S \to \pi^0 e\mu\right) = \tau_S \left(\sqrt{\Gamma\left(K_L \to \pi^0 e\mu\right)} + \sqrt{\Gamma\left(K^+ \to \pi^+ e\mu\right)} \sqrt{\frac{r}{\left(1+|\varepsilon|^2\right)\left|1-\varepsilon\right|^2} \left\{ \left(1-|\varepsilon|^2\right)^2 + \left(2\Im(\varepsilon)\right)^2 \right\}} \right)^2 \tag{6}$$

leading to the bounds $Br\left(K_S \to \pi^0 \mu^+ e^-\right) < 1.1 \times 10^{-12}$ and $Br\left(K_S \to \pi^0 \mu^- e^+\right) < 7.6 \times 10^{-12}$.

It is interesting to note that Ψ_3, which parameterizes the difference between the decays of the CP conjugate states K^0 and $\overline{K^0}$ into the same final state can be constrained with (at least conceptually) simple branching ratio measurements.

III. The Decay $K_S \to \mu e$

The tree level matrix element for $K_S \to \mu e$, assuming new LFV physics at a large mass scale, is

$$
\begin{aligned}
M_2 = &\langle 0|H_{WK}(P)|K\rangle \quad \left[\bar{u}(p_\mu)(a_S + a_P\gamma^5)v(p_e)\right] \\
&+\langle 0|H_{WK}(A^\alpha)|K\rangle \quad \left[\bar{u}(p_\mu)(a_V + a_A\gamma^5)\gamma_\alpha v(p_e)\right]
\end{aligned}
\tag{7}
$$

where the a_i are the coupling constants of the new physics and $\langle 0|H_{WK}(S,P,V^\alpha,A^\alpha,T^{\alpha\beta})|K\rangle$ are (Scalar, Pseudoscalar, *etc.*) hadronic currents. The coupling constants a_i can be different for $S=+1$ and $S=-1$ initial state decays; for neutral kaons some admixture of these couplings is appropriate. That admixture depends on the presence or absence of interference effects, but in either case, similar effects also appear in the leptonic current squared for the $\pi^0\mu e$ final state. I redefine the coupling constants to be the appropriate admixture for a K_S decay and then constrain them using the bound found above for $K_S \to \pi^0\mu e$.

The tree level matrix element for $K_S \to \pi^0\mu e$ is

$$
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M_3 = &\langle 0|H_{WK}(S)|K\rangle \quad \left[\bar{u}(p_\mu)(a_S + a_P\gamma^5)v(p_e)\right] \\
&+\langle 0|H_{WK}(V^\alpha)|K\rangle \quad \left[\bar{u}(p_\mu)(a_V + a_A\gamma^5)\gamma_\alpha v(p_e)\right]. \\
&+(\mathit{tensor\ terms})
\end{aligned}
\tag{8}
$$

Tensor terms could conceivably enter with canceling phase in M_3, suppressing $K_S \to \pi^0\mu e$ but not $K_S \to \mu e$. Consider first the class of models for LFV in which Eqn. (8), without the tensor terms, provides a good description.

Measured standard model decays provide two of the hadronic currents in Eqns. (7,8). The value $f_K = 159.9\,MeV$ is taken for the kaon decay constant[6]. The value $f_+(0) = 0.967$, based on an average of four analyses[5789], and form factors based on the

$e-\mu$ averaged quadratic fit of KTeV[10] are used. From the two measured hadronic currents, covariant derivatives provide the other two:

$$\langle 0|\bar{s}\gamma^5 d|K^0\rangle = {[p_K]_\alpha \langle 0|\bar{s}\gamma^5\gamma^\alpha d|K^0\rangle} \Big/ {(m_s+m_d)}, \tag{9}$$

$$\langle \pi|\bar{s}d|K^0\rangle = {[p_\pi - p_K]_\alpha \langle \pi|\bar{s}\gamma^\alpha d|K^0\rangle} \Big/ {(m_s-m_d)}, \tag{10}$$

and similarly for the currents of the $\overline{K^0}$. Only the comparatively well-known ratio of current masses $(m_s-m_d)/(m_s+m_d)$ is needed in what follows; the value 0.90 ± 0.01, corresponding to m_s/m_d ratios between 17 and 22 is taken[6].

The evaluation of partial widths is straightforward. For $K_S \to \pi^0\mu e$,

$$\Gamma_3^0 = \frac{1}{2m_k}\int d\Phi\,\overline{\left|M_3^0\right|^2} = \left(9.06\times10^{13}\ MeV^7\right)\frac{|a_S|^2+|a_P|^2}{(m_s-m_d)^2} \tag{11}$$

$$\Gamma_3^1 = \frac{1}{2m_k}\int d\Phi\,\overline{\left|M_3^1\right|^2} = \left(5.48\times10^{8}\ MeV^5\right)\left(|a_V|^2+|a_A|^2\right)$$

$$\Gamma_3^i = \frac{1}{2m_k}\int d\Phi\,\overline{\left|M_3^i\right|^2} = \left(2.59\times10^{11}\ MeV^6\right)\frac{\Re\left(a_A a_P^* - a_V a_S^*\right)}{(m_s-m_d)}.$$

The corresponding expressions for $K_S \to \mu e$ are

$$\Gamma_2^0 = \left(2.83\times10^{17}\ MeV^7\right)\frac{|a_S|^2+|a_P|^2}{(m_s+m_d)^2} \qquad \Gamma_2^1 = \left(5.15\times10^{9}\ MeV^5\right)\left(|a_V|^2+|a_A|^2\right) \tag{12}$$

$$\Gamma_3^i = \left(-2.42\times10^{13}\ MeV^6\right)\frac{\Re\left(a_A a_P^* - a_V a_S^*\right)}{(m_s-m_d)}.$$

The sum $\Gamma_3^0 + \Gamma_3^1 + \Gamma_3^i$ is constrained by $\Gamma\left(K_S \to \pi^0 \mu e\right)$; within that constraint scanning in the variables $\alpha_0 = |a_S|^2 + |a_P|^2$, $\alpha_1 = |a_V|^2 + |a_A|^2$ and $\alpha_i = \Re\left(a_A a_P^* - a_V a_S^*\right)$ determines the upper bound for $Br\left(K_S \to \mu e\right)$. The results are in Table I for the three constraints given in section II. Also listed are results in the two special cases where (a) only scalar and pseudoscalar couplings are permitted and (b) only vector and axial vector couplings are permitted.

Constraint	$Br\left(K_S \to \mu e\right)$ (Full Form)	$Br\left(K_S \to \mu e\right)$ (S, P couplings)	$Br\left(K_S \to \mu e\right)$ (V, A couplings)
No interference	4.0×10^{-10}	2.9×10^{-10}	1.1×10^{-11}
$Br\left(K_S \to \pi^0 \mu^+ e^-\right)$	7.6×10^{-10}	5.6×10^{-10}	2.1×10^{-11}
$Br\left(K_S \to \pi^0 \mu^- e^+\right)$	5.3×10^{-9}	3.8×10^{-9}	1.4×10^{-10}

Upper bounds on $K_S \to \mu e$ for the class of models described in the text.

If the tensor terms of Eqn. (8) are large, or if the new physics of LFV is not at a high mass scale, the proceeding argument is invalid. However, the experimental limit $\Gamma\left(K_L \to \mu e\right) < 6.0 \times 10^{-26}\, MeV$ is a tight constraint. In order for the ratio

$$\frac{\Gamma\left(K_S \to \mu e\right)}{\Gamma\left(K_L \to \mu e\right)} = \frac{\left|(1+\varepsilon)\left\langle \mu e \middle| K^0 \right\rangle - (1-\varepsilon)\left\langle \mu e \middle| \overline{K^0} \right\rangle\right|^2}{\left|(1+\varepsilon)\left\langle \mu e \middle| K^0 \right\rangle + (1-\varepsilon)\left\langle \mu e \middle| \overline{K^0} \right\rangle\right|^2} \qquad (13)$$

to be large, the amplitudes $\left\langle \mu e \middle| K^0 \right\rangle$ and $\left\langle \mu e \middle| \overline{K^0} \right\rangle$ must satisfy, to high precision, the constraint

$$\Psi_2 = \left\langle \mu e \middle| \overline{K^0} \right\rangle \Big/ \left\langle \mu e \middle| K^0 \right\rangle = (\varepsilon + 1) \big/ (\varepsilon - 1) . \qquad (14)$$

There is little *a priori* reason to imagine that LFV in the kaon sector would satisfy such a simple relationship with the unrelated phenomena of indirect *CP* violation.

IV. Conclusions

The upper bounds for $Br(K_S \to \pi^0 \mu e)$ are a few 10^{-12}, and an order of magnitude lower if generation number is conserved. In a broad class of models, the upper bounds on $Br(K_S \to \mu e)$ are a few 10^{-9}, and again an order of magnitude lower if generation number is conserved. Evasion of this latter limit is conceivable but unlikely. K_S branching ratios as low as a few 10^{-9} have been successfully measured, and KLOE may ultimately obtain[11] a sample approaching 10^9 reconstructed $K_S \to \pi^+\pi^-$ decays. To bring all of the limits described here within experimental reach however, a difficult experimental program would clearly be needed.

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