~~CONFIDENTIAL~~

MASTER

S. J. G-C
H. Smith
set up
Records

REON

ROCKET ENGINE OPERATIONS - NUCLEAR

REPORT NO. RN-S-0077

TO

AEC-NASA SPACE NUCLEAR PROPULSION OFFICE

DOSE PROBABILITY CONSIDERATIONS ASSOCIATED WITH
THE RE-ENTRY OF A FRAGMENTED
NUCLEAR ROCKET ENGINE (U)

NERVA PROGRAM

CONTRACT SNP-1

MAY 1964

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

AEROJET-GENERAL CORPORATION

AZUSA, CALIFORNIA • PHONES: AZUSA ED 4-6211, LOS ANGELES CU 3-6111

May 8, 1964

765:965

GDJ:rv

Mr. L. C. Corrington
Space Nuclear Propulsion Office
Cleveland Extension
National Aeronautics and Space Administration
21000 Brookpark Road
Cleveland 35, Ohio

Subject: AGC Report No. RN-S-0077, Dose Probability Considerations
Associated with the Re-Entry of a Fragmented Nuclear
Rocket Engine

Gentlemen:

The subject report has been prepared under Subtask 2.7, Contract SNP-1.

Distribution has been made as shown in Enclosure (2). Additional copies are available for distribution to Category "L" if SNPO-C desires.

Very truly yours,

AEROJET-GENERAL CORPORATION

H. E. Pharr
Manager of Contracts
Rocket Engine Operations - Nuclear

Enclosure:

- (1) Subject Report (8 copies)
- (2) Distribution List

If Enclosure (1) is withdrawn (or not attached), the classification of the correspondence will be downgraded to Unclassified.

AEROJET-GENERAL CORPORATION

AZUSA, CALIFORNIA • PHONES: AZUSA ED 4-6211, LOS ANGELES CU 3-6111

May 8, 1964

765:965

GDJ:rv

Mr. L. C. Corrington
Space Nuclear Propulsion Office
Cleveland Extension
National Aeronautics and Space Administration
21000 Brookpark Road
Cleveland 35, Ohio

Subject: AGC Report No. RN-S-0077, Dose Probability Considerations
Associated with the Re-Entry of a Fragmented Nuclear
Rocket Engine

Gentlemen:

The subject report has been prepared under Subtask 2.7, Contract SNP-1.

Distribution has been made as shown in Enclosure (2). Additional copies are available for distribution to Category "L" if SNPO-C desires.

Very truly yours,

AEROJET-GENERAL CORPORATION

H. E. Pharr
Manager of Contracts
Rocket Engine Operations - Nuclear

Enclosure:

- (1) Subject Report (8 copies)
- (2) Distribution List

If Enclosure (1) is withdrawn (or not attached), the classification of the correspondence will be downgraded to Unclassified.

DISTRIBUTION

P. Zigman
U. S. Naval Radiological Defense Laboratory
San Francisco 24, California

Earl W. Hesse
Lockheed Missile & Space Company
Nuclear Systems Program Division
Box 504
Sunnyvale, California

Major J. Rawers, MTO RS-3 - 3 copies
Chief, Technical Branch
Missile Operation Division
AFMTC - Patrick AFB
Florida

Dr. C. Rogers McCullough
Nuclear Utilities Services, Inc.
1730 M Street N. W.
Washington D. C. 20036

Morton I. Goldman
Nuclear Utilities Services, Inc.
1730 M. Street N. W.
Washington D. C. 20036

L. D. P. King - 3 copies
LASL
P. O. Box 1663
Los Alamos, New Mexico

Major R. C. Hock - 3 copies
National Aeronautics & Space Admin.
Launch Operations Center
Cocoa Beach, Florida

Harry A. Bechtol
Aberdeen Proving Ground
Aberdeen, Maryland

J. C. Whiton
National Aeronautics & Space Admin.
George C. Marshall Space Flight Center
Huntsville, Alabama

Lt. Col. R. S. Decker - 3 copies
Chief - Safety Branch
Space Nuclear Propulsion Office
U. S. Atomic Energy Commission
Washington 25, D. C.

H. Radd
Lockheed Missile & Space Company
Nuclear Systems Program Division
Box 504
Sunnyvale, California

L. C. Corrington - 8 Copies
Space Nuclear Propulsion Office
National Aeronautics and Space Admin.
21000 Brookpark Road
Cleveland 35, Ohio

C. T. Donnenworth - 3 copies
Lockheed Missile & Space Company
Sunnyvale, California

Col. W. S. Fellows - 2 copies
Chief, Nuclear Vehicle Projects Office
M-P&VA-N
National Aeronautics & Space
Administration
George C. Marshall Space Flight Center
Huntsville, Alabama

40

MASTER

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

REPORT NO. RN-S-0077

TO

AEC-NASA SPACE NUCLEAR PROPULSION OFFICE

DOSE PROBABILITY CONSIDERATIONS ASSOCIATED WITH
THE RE-ENTRY OF A FRAGMENTED
NUCLEAR ROCKET ENGINE (U)

ROCKET ENGINE OPERATIONS - NUCLEAR

NERVA PROGRAM

MAY 1964

CONTRACT SNP-1

SPECIAL REVIEW FINAL DETERMINATION	REVIEWERS	Class.	Date
	JRP	U	2/22/82
Class: <u>U</u>			

Classification cancelled (or changed to) _____
by authority of DOE
by H.F.C. TIC, date SEP 21

GROUP 4

Downgraded at 5
Declassified after 12 Years

"This document contains information affecting the national defense of the United States within the meaning of the espionage laws, title 18 U.S.C., Section 793 and 794. Its transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law."

AEROJET-GENERAL CORPORATION
A SUBSIDIARY OF THE GENERAL TIRE & RUBBER COMPANY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

CONFIDENTIAL

ABSTRACT

A dose-probability model is developed for the evaluation of external "whole-body" gamma exposures to the earth population resulting from fuel fragments produced by suborbital destruct of a nuclear rocket engine. The model is applied to a "worst-case" failure of a nuclear engine prior to achieving 100-nautical-mile orbit. Experimental and analytical data appearing in the literature are used where applicable.

J. B. Philipson, Manager
Safety Division - REON

Report No. RN-S-0077

FOREWORD

This report has been prepared in partial fulfillment of Subtask 2.7,
Contract SNP-1.

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION _____	1
II. DOSE-PROBABILITY MODEL _____	3
A. Particle Density Distribution _____	3
B. Model Derivation _____	6
C. Discussion _____	11
III. APPLICATION OF MODEL TO THE EVALUATION OF HAZARDS FROM RE-ENTRY DEBRIS _____	14
A. Problem Definition _____	14
B. Analysis _____	18
IV. SUMMARY AND CONCLUSIONS _____	23
List of References _____	25
APPENDIX A - EXACT PROBABILITY OF DOSE $\geq D$ RESULTING FROM A FIXED NUMBER OF PARTICLES	

LIST OF TABLES

<u>Table</u>		<u>Page</u>
1	Fuel Fragment Parameters _____	15
2	Population Data _____	17
3	Expected Number of Exposures _____	19

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Particle Dispersion Area, A_i (cm^2) _____	5
2	Model Geometry _____	6
3	Four-Day Dose from Core Fragments for 1-Meter Separation Distance _____	13
4	Particle-Size Distribution _____	16
5	Expected Number of People Receiving Dose $\geq D$ _____	21
6	Population Exposure Probabilities _____	22

LIST OF SYMBOLS

a	area of annulus as defined by Equation (4) and Figure 2, cm^2
A_i	dispersion area for class-i particles which contains 90% of the mass associated with class i, cm^2
c	expected number of particles contained in "a", particles
C	conversion factor from gamma-ray energy flux to dose rate, $\frac{\text{r}}{\text{sec}} \bigg/ \frac{\text{Mev}}{\text{cm}^2\text{-sec}}$
d	particle diameter, cm
$\bar{d}_i$	log mean diameter of particles in class i, cm
D	dose received during the time interval between t_1 and t_2 , roentgens
f_i	fraction of fuel volume which is fragmented into class i particles
i	particle class designation
j	subscript referring to those particle classes or portions thereof in which a single particle located at a distance of 1 meter from the dose point can produce a four-day dose $\geq D$
k	source constant defined by Equation (5), $\text{cm}^2\text{-r}$
$K(\tau, t)$	ratio of reactor decay gamma power to operating power, Mev/Mw-sec
K_n	modified Bessel function of the second kind and order n
m	number of radioactive particles of equivalent gamma-ray source strength, particles
M_i	total number of class-i particles produced by fragmentation of core, particles
n	number of people receiving a dose $\geq D$, people
N_i	total number of people in dispersion area A_i , people
N_e	expected number of people receiving a dose $\geq D$, people
p_n	probability that exactly n people receive a dose $\geq D$

LIST OF SYMBOLS (cont.)

P_m	probability that exactly m particles are contained in the area "a"
q	probability of receiving a dose $\geq D$ at the dose point
r_m	distance between m particles and the dose point, cm
t	time after reactor shut-down, sec
V	reactor core volume, cm^3
W	reactor operating power, Mw
α	circular area required to just reach the most distant of m particles, cm^2
β_i	average population density in the area A_i , people/ km^2
γ	Euler-Mascheroni constant equal to 0.577216
λ	mean particle density per unit area, particles/ cm^2
σ	standard deviation of a normal distribution
τ	reactor operating time, sec

I. INTRODUCTION

One of the most difficult safety problems associated with suborbital operation of a nuclear rocket engine, is the disposal of the highly radioactive core in a manner which does not expose the earth's population to undue hazards in the event of premature failure of the nuclear engine prior to orbital injection. An engine failure just prior to achieving 100-nautical-mile orbit could result in an excess of 10^8 curies being distributed over a portion of the earth's surface. Without dispersing this activity over a very large area or confining it to an acceptable deep-ocean disposal area, excessive population exposures could result.

A number of countermeasures have been investigated which would provide adequate safety without incurring excessive performance degradation in terms of weight penalties or reduction of mission reliability. The countermeasure concepts which have been studied include:

- (1) shaping of the ascent trajectory so that the nominal impact points on the earth's surface will never coincide with a major land mass,
- (2) auxiliary chemical propulsion to force an aborted nuclear stage to an acceptable deep-ocean disposal area or to boost the stage to a long-lifetime orbit,
- (3) a destruct system which will fragment and disperse the core over a large area of the earth's surface.

The weight penalties associated with trajectory shaping are relatively large compared to that required for a destruct system, while the limited reliability of an auxiliary chemical propulsion system would almost certainly require a backup, thus overriding the advantages to this concept. From the results of analyses performed to date, a destruct system appears to be the most desirable countermeasure against suborbital-start hazards.

In order to provide the information upon which to base the requirements for a destruct countermeasure system, the nature of the hazards associated with the re-entry of the fragmented fuel must be thoroughly understood. Furthermore, "acceptable" dose limits and exposure probabilities suitable for application to the problem must be established. The purpose of this study is to develop a dose-probability model which can be used to calculate external "whole-body" gamma exposures resulting from fuel fragments produced by sub-orbital destruct of a nuclear engine. No attempt is made to evaluate beta skin dose or internal exposures which may result from inhalation or ingestion of small radioactive particles. However, as previously demonstrated by Decker and Goldman (Ref. 1), these additional modes of exposure are significant and must be considered in an overall evaluation of hazards associated with the re-entry of fragmented fuel. The present study serves as an extension of the earlier investigation of the external "whole body" gamma dose problem described in Reference 1.

II. DOSE-PROBABILITY MODEL

A. PARTICLE DENSITY DISTRIBUTION

The development of a calculational model to predict the probability of radiation exposure to the earth population following the re-entry from space of radioactive fuel fragments, must be predicated upon a known or assumed particle density distribution on the earth's surface. In a recent study performed by Lockheed (Ref. 2), density distributions were calculated from various size particles re-entering from an altitude of 100 nautical miles. The results were obtained using the following assumptions:

- (1) Destruct of the engine occurs 58 sec prior to achieving 100-nautical-mile orbit.
- (2) Altitude at destruct is approximately 100 nautical miles and the corresponding velocity is approximately 137 meters per sec less than orbital velocity.
- (3) Within each class of particles considered, the destruct velocity is random in direction (isotropic) with a magnitude given by $v = 29.0/d$, where v is the velocity in meters per sec and d is the particle diameter in centimeters.
- (4) A drag coefficient of 2.0 was selected for all particles and $W/C_D A$ was held constant within each particle class.
- (5) Re-entry burnup and atmospheric transport effects were neglected.

The main conclusions which can be drawn from the results presented in Reference 2 are:

- (1) Approximately 90% of the total mass associated with each particle class is distributed within an area which is less than 45% of the total dispersion area for that class.
- (2) Within this reduced area, the variations in the relative surface mass densities are:

<u>Relative Surface Mass Density* Range</u>	<u>Fraction of Reduced Area in Which Relative Surface Mass Density Exists</u>
<0.5	0
0.5 - 0.75	0.23
0.75 - 1.25	0.62
1.25 - 1.75	0.15
>1.75	0

* Normalized to a mean surface mass density of 1.0 over the reduced area.

Report No. RN-S-0077

As can be seen, the distribution of the mass associated with a particular particle class is fairly uniform over the reduced area.

- (3) As illustrated by the graph in Figure 1, the dispersion area which contains 90% of the mass associated with a particular particle class can be approximated by the expression

$$A_i = 9.1 \times 10^{14} \bar{d}_i^{-2.07}, \quad (1)$$

where A_i = dispersion area for class-i particles which contains 90% of the mass associated with class i, cm^2 ,

$\bar{d}_i$ = log mean diameter of particles in class i, cm.

On the basis of the above results, it is concluded for the purposes of the present study that particles within a particular particle class can be considered to be randomly distributed over a large finite area on the earth's surface. Using this assumption in the section which follows, an expression is derived for the probability of exposing a population group to a whole-body dose in excess of a specified value.

Figure 1

Particle Dispersion Area, A_i (cm^2)

B. MODEL DERIVATION

Consider a circle of radius r_m with m radioactive particles of equivalent gamma-ray source strength located on its circumference. Then the dose D received by a dose point located at the center of the circle is given by

$$D = \frac{km}{\pi r_m^2}, \quad (2)$$

where k is a source constant. For a specified value of D ,

$$\pi r_m^2 = \frac{km}{D}, \quad (3)$$

and the area of any annulus a_m , as defined by Figure 2, is given by

$$\begin{aligned} a_m &= \pi r_m^2 - \pi r_{m-1}^2 \\ &= \frac{km}{D} - \frac{k(m-1)}{D} \\ &= k/D = a. \end{aligned} \quad (4)$$

Figure 2

Model Geometry

Therefore, the areas of these annuli are independent of m .

Neglecting air attenuation and buildup, the constant, k, is defined by

$$k = \frac{CfW}{4M} \int_{t_1}^{t_2} K(\tau, t) dt, \quad (5)$$

where C = conversion factor from gamma-ray energy flux to dose rate,

$$\frac{r}{\text{sec}} \bigg/ \frac{\text{Mev}}{\text{cm}^2\text{-sec}}$$

f = fraction of reactor core which is fragmented into M particles of equivalent gamma-ray source strength

W = reactor operating power, Mw

K = ratio of decay gamma power to operating power, Mev/Mw-sec

τ = reactor operating time, sec

t = time after shut-down, sec.

For operating times of the order of 10^3 seconds and decay times ranging from 10^3 to 10^7 seconds, the ratio of decay gamma power to operating power can be approximated by (Ref. 3)

$$K(\tau, t) \approx 4.45 \times 10^{16} \tau t^{-1.2}. \quad (6)$$

Assuming that the average energy of the photons emitted by each particle is 0.75 Mev, then $C = 5.6 \times 10^{-10} \text{ cm}^2\text{r/Mev}$, and by substitution of Equation (6) into Equation (5)

$$k = 6.24 \times 10^6 \frac{fW\tau}{M} \int_{t_1}^{t_2} t^{-1.2} dt \quad (7)$$

$$= 3.12 \times 10^7 \frac{fW\tau}{M} (t_1^{-0.2} - t_2^{-0.2}). \quad (8)$$

Next, consider the probability of m or more particles being located in an annulus a. If a total of M particles are randomly distributed over a large area A, then the expected or mean number of particles contained in "a" is $c = aM/A$, which upon substitution of Equations (4) and (8) becomes

$$c = 3.12 \times 10^7 \frac{fW\tau}{AD} (t_1^{-0.2} - t_2^{-0.2}). \quad (9)$$

The probability that m particles are contained in "a" can be described by a Poisson distribution with mean value c , i.e.

$$P_m = \frac{e^{-c} c^m}{m!} . \quad (10)$$

Thus, the probability of m or more particles in an annulus a is

$$P_{\geq m} = 1 - \sum_{m'=0}^{m-1} \frac{e^{-c} c^{m'}}{m'!} . \quad (11)$$

The probability of a person located at the dose point receiving a dose $\geq D$ is obtained by summing the probabilities associated with all particle distributions which individually produce a dose $\geq D$, i.e.,

$$\begin{aligned} P(\text{dose} \geq D) = & P(m \geq 1 \text{ in } a_1) + \\ & P(m = 0 \text{ in } a_1; \geq 2 \text{ in } a_2) + \\ & P(0 \text{ in } a_1; 1 \text{ in } a_2; \geq 2 \text{ in } a_3) + P(0 \text{ in } a_1; 0 \text{ in } a_2; \geq 3 \text{ in } a_3) + \\ & P(0 \text{ in } a_1; 0 \text{ in } a_2; 0 \text{ in } a_3; \geq 4 \text{ in } a_4) + \\ & P(0 \text{ in } a_1; 0 \text{ in } a_2; 1 \text{ in } a_3; \geq 3 \text{ in } a_4) + \\ & P(0 \text{ in } a_1; 1 \text{ in } a_2; 0 \text{ in } a_3; \geq 2 \text{ in } a_4) + \\ & P(0 \text{ in } a_1; 1 \text{ in } a_2; 1 \text{ in } a_3; \geq 1 \text{ in } a_4) + \\ & P(0 \text{ in } a_1; 0 \text{ in } a_2; 2 \text{ in } a_3; \geq 2 \text{ in } a_4) + \dots \end{aligned} \quad (12)$$

Since the areas are independent of each other, i.e., $a_1 = a_2 = a_m = k/D$, Equation (12) may be written as

$$\begin{aligned} P(\text{dose} \geq D) = & P(\geq 1 \text{ in } a) + P(0 \text{ in } a)P(\geq 2 \text{ in } a) + P(0 \text{ in } a)P(1 \text{ in } a)P(\geq 2 \text{ in } a) + \\ & P(0 \text{ in } a)P(0 \text{ in } a)P(\geq 3 \text{ in } a) + \\ & P(0 \text{ in } a)P(0 \text{ in } a)P(0 \text{ in } a)P(\geq 4 \text{ in } a) + \\ & P(0 \text{ in } a)P(0 \text{ in } a)P(1 \text{ in } a)P(\geq 3 \text{ in } a) + \\ & P(0 \text{ in } a)P(1 \text{ in } a)P(0 \text{ in } a)P(\geq 2 \text{ in } a) + \\ & P(0 \text{ in } a)P(1 \text{ in } a)P(1 \text{ in } a)P(\geq 1 \text{ in } a) + \\ & P(0 \text{ in } a)P(0 \text{ in } a)P(2 \text{ in } a)P(\geq 2 \text{ in } a) + \dots \end{aligned} \quad (13)$$

$$\begin{aligned}
&= P_{\geq 1}^{+} P_{\geq 2}^{+} P_{\geq 2}^{+} P_{\geq 2}^{+} P_{\geq 3}^{+} P_{\geq 4}^{+} P_{\geq 3}^{+} + \\
&\quad P_{\geq 2}^{+} P_{\geq 2}^{+} P_{\geq 1}^{+} P_{\geq 1}^{+} P_{\geq 2}^{+} + \dots
\end{aligned} \tag{14}$$

$$= q. \quad (15)$$

For values of $c \ll 1$, it will now be shown that q is approximately equal to c . From Equations (10) and (11),

$$\left. \begin{aligned} P_0 &= e^{-c} \\ P_1 &= ce^{-c} \\ P_2 &= \frac{c^2}{2} e^{-c} \\ P_3 &= \frac{c^3}{6} e^{-c} \end{aligned} \right\} \begin{aligned} P_{\geq 1} &= 1 - e^{-c} \\ P_{\geq 2} &= 1 - e^{-c}(1 + c) \\ P_{\geq 3} &= 1 - e^{-c} \left(1 + c + \frac{c^2}{2} \right) \\ P_{\geq 4} &= 1 - e^{-c} \left(1 + c + \frac{c^2}{2} + \frac{c^3}{6} \right) \end{aligned} \quad (16)$$

Substituting these values into Equation (14) and collecting terms gives

$$q = 1 - e^{-4c} \left(1 + 3c + 4c^2 + \frac{7c^3}{6} \right) + \dots \quad (17)$$

which, upon expansion of the exponential, reduces to

$$q = c + \frac{3c^3}{2} + \dots \text{higher order terms} \quad (18)$$

$$\approx c \text{ for } c \ll 1. \quad (19)$$

Having obtained the value of q , an expression for the probability of exposure to at least n people out of a total of N people in the large area A is readily determined. Since the probability of exposure to any one of the N people is independent of the exposures received by the others, then, the probability of exactly n people receiving a dose $>D$ follows the binomial distribution, i.e.,

$$p_n = \frac{N! q^n (1-q)^{N-n}}{n! (N-n)!}, \quad (20)$$

and the probability of at least n people receiving a dose $\geq D$ is

$$p_{\geq n} = 1 - \sum_{j=0}^{n-1} p_j \quad (21)$$

The expected number of people who will receive a dose $\geq D$ is given by

$$N_e = \sum_{n=0}^N n p_n \quad (22)$$

$$= \sum_{n=0}^N \frac{N! q^n (1-q)^{N-n}}{(n-1)! (N-n)!} \quad (23)$$

$$= Nq. \quad (24)$$

For those cases in which $n \ll N \gg 1$ and $q \ll 1$, Equation (20) can be closely approximated by a Poisson distribution, i.e.,

$$p_n = \frac{(Nq)^n e^{-Nq}}{n!} ; n \ll N \gg 1, q \ll 1. \quad (25)$$

When n , N , and $(N-n)$ are all larger than ~ 10 , a normal distribution provides a good approximation to Equation (20), i.e.,

$$p_n = \frac{1}{\sigma \sqrt{2\pi}} e^{-1/2 \left(\frac{n-Nq}{\sigma} \right)^2} ; n, N, (N-n) > 10, \quad (26)$$

where σ , the standard deviation, is defined by $\sqrt{Nq(1-q)}$. The probability of at least n people being exposed becomes

$$p_{\geq n} \approx \left. \begin{aligned} & \frac{1}{\sigma \sqrt{2\pi}} \int_n^{\infty} e^{-1/2 \left(\frac{n-Nq}{\sigma} \right)^2} dn \\ & \approx \frac{1}{\sqrt{2\pi}} \int_{\frac{n-Nq}{\sigma}}^{\infty} e^{-\frac{u^2}{2}} du, \end{aligned} \right\} n, N, (N-n) > 10 \quad (27)$$

$$(28)$$

which can be readily evaluated using standard probability tables (Ref. 4).

C. DISCUSSION

1. Accuracy of $P(\text{dose} \geq D)$

The expression derived in the previous section for the probability of a dose $\geq D$, Equation (14), underestimates the actual probability of this occurrence. This underestimate becomes apparent from an examination of the terms in Equation (12). For example, there exists a finite, though small, probability of one particle in area a_2 and one particle in a_3 producing a dose $\geq D$. However, the probability of this event is not accounted for by Equation (12), thus resulting in an underestimate of the magnitude of q .

The nature of the error in q can best be seen by consideration of the following hypothetical experiment. Set up a field of concentric circles and impose on the field a distribution of particles of the required density. Calculate the total dose at the center of the field by assuming that all of the particles within each annulus are on the outside circle defining the annulus. By repeating this experiment a sufficient number of times, an "observed" distribution of dose can be developed. Another distribution can be developed at the same time by measuring the distance from the center of the field to each particle and calculating the total dose using the measured separation distances. This distribution can be called the "actual" distribution of dose since its validity depends only upon the validity of the equation relating dose to radius. The distribution of dose determined by the first method will be biased such that the probability of a dose $\geq D$ obtained by integrating over the tail of the first distribution will always be less than the probability of a dose $\geq D$ obtained in the same manner from the actual dose distribution. Thus, the method of concentric circles described above, and used in the derivation of q , represents a lower bound to the exact probability function.

A more rigorous approach to the calculation of q is presented in Appendix A, where it is shown that for small values of c , the approach taken in Section II,B, provides a very close approximation to the exact probability of receiving a dose $\geq D$.

2. Upper Limit on Dose

In the development of the dose-probability equations presented in Section II,B, it was assumed that the total dose produced by a particle is

received at a point; thus, as the separation distance between the particle and the dose point approaches zero, the dose approaches infinity. In actual practice, the maximum dose a particle can produce is limited by the physical dimensions of the "whole body," i.e., there is some minimum effective separation distance which can be achieved between a particle and a whole body of finite size. For the purposes of this study it will be assumed that the maximum dose which a particle can produce is equal to the dose measured at a point 1 meter from the particle. Thus, all particles of a given class which are located within 1 meter of a person will produce the maximum dose corresponding to that particular particle class.

The expression for the maximum dose associated with a particle of diameter d is obtained by combining Equations (2) and (8), letting r equal 100 cm, and replacing f/M , the fraction of the core activity contained by a single particle, by $\pi d^3/6V$, which results in

$$D_{\max} = 520 \frac{W \pi d^3}{V} (t_1^{-0.2} - t_2^{-0.2}). \quad (29)$$

If the dose is defined as that received within four days following engine operation, and the total energy produced by the reactor during operation is 1 Mw-sec/cc of core volume, then the maximum value of the dose corresponding to the 1-meter separation distance is

$$D_{\max} = 520 d^3 (t_1^{-0.2} - 0.078). \quad (30)$$

This expression is plotted in Figure 3 for times t_1 ranging from 1 minute to 1 day after reactor shut-down: The curves shown will be used in Section III to determine the minimum diameter of particles singly capable of producing a whole body dose $\geq D$ for a given re-entry time t_1 .

Figure 3

Four-Day Dose from Core Fragments for 1-Meter Separation Distance

III. APPLICATION OF MODEL TO THE EVALUATION OF HAZARDS FROM RE-ENTRY DEBRIS

In this section, the dose-probability model developed in Section II,B, is used to estimate the magnitude of the external gamma-dose hazard to the earth population resulting from suborbital destruct of a nuclear rocket engine. The assumed size distribution of the fuel fragments following destruct is based upon APG-2 test data (Ref. 5) and upon an extrapolation of these data to smaller sizes made by Decker and Goldman (Ref. 1). The fragment dispersion data were taken from the Lockheed results (Ref. 2) which are described in Section II,A.

A. PROBLEM DEFINITION

A nuclear rocket engine is assumed to have operated suborbitally at a power level of 1000 Mw for a period of 500 seconds before a vehicle failure requires the engine-destruct system to be activated. Destruct occurs at an altitude of approximately 100 nautical miles. The reactor core is fragmented according to the particle-size distribution postulated by Ref. 1 and presented in Table 1 and Figure 4. The dispersion areas presented in Table 1 for each particle class were calculated from Equation (1). Since the amount of fuel-fragment burnup expected during re-entry is small (Refs. 6, 7), burnup effects were neglected.

The destruct event was assumed to occur at a location in space such that the centers of the dispersion areas for each particle class coincide with Johannesburg, the city with the largest population in South Africa. Though this assumption represents a worst case, the analysis performed by Lockheed (Ref. 2) indicates that this situation can occur for a launch azimuth from Cape Kennedy of approximately 104° . Using the population density data contained in Ref. 8 and the shapes of the dispersion areas reported in Ref. 2, the total number of people contained within the dispersion area for particle classes 1 through 6 was estimated. These results along with the average population densities for each area are presented in Table 2. For purposes of comparison, the average population density of South Africa is approximately 6.7 people/km^2 (17.4 people/mi^2).

TABLE 1
FUEL FRAGMENT PARAMETERS

Particle Class, i	Particle Diameter, d _i (cm)		Fraction of Fuel Volume, f _i	No. of Fuel Particles, M _i	Dispersion Area A _i x 10 ⁻¹⁰ (km ²)
	Range	Log Mean			
1	>2.54	3.59	0.0002	4.14	6.5 x 10 ³
2	1.27 - 2.54 (1")	1.80	0.0116	1.90 x 10 ³	2.7 x 10 ⁴
3	0.635 - 1.27 (1/2")	0.899	0.1030	1.36 x 10 ⁵	1.1 x 10 ⁵
4	0.317 - 0.635 (1/4")	0.449	0.2130	2.22 x 10 ⁶	4.7 x 10 ⁵
5	0.159 - 0.317 (1/8")	0.224	0.1280	1.11 x 10 ⁷	2.0 x 10 ⁶
6	0.10 - 0.159 (1/16")	0.127	0.0242	1.12 x 10 ⁷	6.5 x 10 ⁶
7	0.08 - 0.10	0.0895	0.0200	2.66 x 10 ⁷	1.3 x 10 ⁷
8	0.04 - 0.08	0.0566	0.1250	6.56 x 10 ⁸	3.5 x 10 ⁷
9	0.02 - 0.04	0.0283	0.1000	4.23 x 10 ⁹	1.5 x 10 ⁸
10	0.01 - 0.02	0.0141	0.0700	2.36 x 10 ¹⁰	**
11	0.005 - 0.01	0.00707	0.0550	1.49 x 10 ¹¹	-
12	0.001 - 0.005	0.00224	0.0750	5.97 x 10 ¹²	-
13	<0.001	0.000316	0.0750	2.25 x 10 ¹⁵	-

* Assumed fuel volume is 5 x 10⁵ cc.

** Area overlap occurs through encirclement of earth (Ref. 1).

~~CONFIDENTIAL~~

Report No. RN-S-0077

Figure 4

Particle-Size Distribution

TABLE 2

POPULATION DATA

Particle Class, i	Dispersion Area, $A_i \times 10^{-10} \text{ (km}^2\text{)}$	Fraction of A_i which is Inhabited	Total No. People in Dispersion Area, * N_i	Average Population Density, ** $\beta_i = N_i / (A_i \times 10^{-10})$
				(people/km ²)
1	6.5×10^3	1.0	1.77×10^6	272.
2	2.7×10^4	1.0	2.65×10^6	98.2
3	1.1×10^5	1.0	3.37×10^6	30.6
4	4.7×10^5	0.68	3.77×10^6	8.38
5	2.0×10^6	0.32	6.44×10^6	3.32
6	6.5×10^6	0.20	1.21×10^7	1.86

*Center of area is assumed to coincide with Johannesburg, South Africa.

**Average population density of South Africa is approximately 6.7 people/km².

B. ANALYSIS

The analysis will be limited to consideration of external "whole body" gamma exposures greater than or equal to 1 roentgen which occur within four days after reactor shut-down. From the problem description given above, the average energy release per unit volume of the core before reactor shut-down is $500 \text{ sec} \times 1000 \text{ Mw} / 5 \times 10^5 \text{ cc} = 1 \text{ Mw-sec/cc}$. Thus, from Figure 3, a single particle with a diameter less than approximately 1/16-inch cannot produce a four-day dose ≥ 1 roentgen. Since the values of c for each particle class are very much less than 1 for $D \geq 1 \text{ r}$ (as will be shown below), exposure probabilities resulting from multiple particle interactions can be neglected.* Consequently, particles with diameters less than 1/16-inch will have a negligible probability of producing a dose $\geq 1 \text{ r}$ and therefore will not be considered in the analysis.

The time required for re-entry of spherical particles larger than 1/16-inch diameter can vary between a few minutes and several hours depending upon initial altitude, velocity, and particle diameter (Ref. 9). However, as illustrated by Figure 3, the variation in the four-day dose for re-entry times ranging from 15 minutes to one hour is relatively small (about 50%). Therefore, for all particles with diameters larger than 1/16-inch, a single re-entry time of approximately 15 minutes can be used without significantly decreasing the accuracy of the results.

Upon substitution of $t_1 = 900 \text{ sec}$ (15 min), $t_2 = 345,000 \text{ sec}$ (4 days), and $W_T = 5 \times 10^5 \text{ Mw-sec}$ into Equation (9), the expression for c_i becomes

$$c_i = 3.12 \times 10^7 \times 5 \times 10^5 (0.257 - 0.078) \frac{f_i}{A_i D} \quad (31)$$

$$= 2.78 \times 10^{12} \frac{f_i}{A_i D} \quad (32)$$

Values of $c_i D$ for particle classes 1 through 6 are given in Table 3. Since for each particle class shown $c_i \ll 1$ for any $D \geq 1 \text{ r}$, then $q_i \approx c_i$ [see Equation (19)], and the expected number of people who will receive a dose $\geq D$ becomes

* See Appendix A.

TABLE 3

EXPECTED NUMBER OF EXPOSURES

Particle Class, i	$c_i D$	Expected Number of People Receiving Dose $\geq D$					
		$D = 1r$	$D = 10r$	$D = 25r$	$D = 100r$	$D = 400r$	$D = 1000r$
1	8.56×10^{-5}	15.2	1.5	0.6	0.15	0.038	0.015
2	1.20×10^{-3}	318	31.8	12.7	3.18	0.164	0.007
3	2.60×10^{-3}	876	87.6	33.4	0.95	-	-
4	1.26×10^{-3}	475	15.2	-	-	-	-
5	1.78×10^{-4}	54.5	-	-	-	-	-
6	1.03×10^{-5}	-	-	-	-	-	-
TOTALS		1739	136.1	45.7	4.28	0.202	0.022

$$N_e (\geq D) = \sum_j c_j N_j, \quad c_i \ll 1 \quad (33)$$

$$= \frac{278}{D} \sum_j f_j \beta_j, \quad c_i \ll 1 \quad (34)$$

where β_j is the average population density given in Table 2. The subscript j refers to those particle classes or portions thereof in which a single particle located at a distance of 1 meter from the dose point can produce a four-day dose $\geq D$. For example, from Figure 3, the minimum-size particle which can produce a four-day dose of 10r given that $t_1 = 15$ minutes, is a class-4 particle with a diameter of 0.48 cm. Figure 4 illustrates that the fraction of the core which is fragmented into particles with diameters ≥ 0.48 cm is 0.180. Thus, for this

Report No. RN-S-0077

particular case, $f_{j=4} = 0.180 - 0.1030 - 0.0116 - 0.0002 = 0.065$ and the expected number of people who will receive a dose $\geq 10r$ is

$$N_e (\geq 10) = \frac{278}{10} (0.065 \times 8.32 + 0.1030 \times 30.6 + 0.0116 \times 98.2 + 0.0002 \times 272)$$

$$= 15.2 + 87.6 + 31.8 + 1.5 = 136.1.$$

Values for $N_e (\geq D)$ for each particle class are given in Table 3. The function $N_e (\geq D)$ is plotted in Figure 5 for values of D between $1r$ and $1000r$. For purposes of comparison, $N_e (\geq D)$ was also evaluated assuming that (1) all core particles are large enough to produce a dose $\geq D$, and (2) all particles are uniformly dispersed in a single area which has a population density corresponding to the average value for South Africa; i.e., 6.7 people/km^2 . This result is shown by the dashed curve on Figure 5.

The probabilities of at least n people receiving a dose $\geq D$ were calculated from Equations (21), (25), and (28) using the expected number of exposures and the corresponding doses given in Table 3. Equation (28) was used for $D = 1, 10, \text{ and } 25r$, and Equations (21) and (25) were used for $D = 100, 400, \text{ and } 1000r$. The results of the calculation for doses of $1, 10, 25, \text{ and } 100r$ are presented in Figure 6. Exposure probabilities for doses of 400 and $1000r$ are given below.

<u>D(r)</u>	<u>P $\geq$ n</u>		
	<u>n = 1</u>	<u>n = 2</u>	<u>n = 3</u>
400	0.183	0.018	0.0013
1000	0.0217	0.00024	-

Assuming $400r$ to represent the mid-lethal dose, the probability of at least 1 death from radiation exposure would be approximately 0.1, and at least 2 deaths, 0.01. It should be recognized, however, that the probability of the postulated vehicle failure must be factored into these exposure-probability estimates in order to obtain the actual probability of exposure or hazard for the overall flight operation.

Figure 5

Expected Number of People Receiving Dose $\geq D$

Figure 6

Population Exposure Probabilities

IV. SUMMARY AND CONCLUSIONS

A dose-probability model has been developed for the evaluation of external "whole-body" gamma exposures to the earth population resulting from fuel fragments produced by suborbital destruct of a nuclear rocket engine. The model has been applied to the calculation of exposure probabilities for a "worst-case" failure of a nuclear engine prior to achieving 100-nautical-mile orbit. Applicable experimental and analytical data appearing in the literature were used wherever possible. The results of the analysis indicate that, out of the several million people located in the assumed contamination areas, only 46 people are expected to receive a dose larger than 25r and only 4 people are expected to receive a dose larger than 100r. The probability of at least 1 person receiving more than 400r in the event of the postulated accident is 0.18. The probability of at least 2 exposures greater than 400r drops to 0.018.

The exposure probabilities calculated in this study apply specifically to the accident condition postulated in Section III,A as opposed to an overall flight operation. Reference 2 estimates that, for a typical flight profile involving suborbital start of a nuclear engine, the total probability of all vehicle failures which could result in the contamination of South Africa, is less than 0.01. Thus, the probabilities of exposure resulting from the assumed flight operation are less than those calculated for the accident condition by at least two orders of magnitude. For example, the probability of at least one person receiving a dose $\geq 400r$ for the flight reduces to 0.0018. Furthermore, it should be recognized that by careful selection of the launch azimuth and the flight trajectory, the population densities contained within the particle dispersion areas can be substantially reduced over those assumed for this study. Certainly, large population centers such as Johannesburg can be avoided with a concomitant reduction in the expected number of exposures should suborbital destruct be required.

Another factor which will have a significant effect on the magnitude of the hazards predicted by the dose-probability model, is the normal movement of personnel within the contaminated area. The model developed assumes that people remain stationary during the four-day exposure period. Normal movement during this period will reduce the magnitude of the exposures to individuals but will increase proportionately the number of people exposed. The overall effect will be to

substantially reduce the predicted number of large exposures. Thus, it appears that additional effort directed at improving the model to account for this effect may be warranted.

Some of the significant conclusions which can be drawn from the results of this study and which are important to the development of countermeasure criteria for suborbital operation of a nuclear engine, are summarized below:

- (1) The expected number of people receiving a dose $\geq D$ is proportional to the fraction of the core which is fragmented into particles individually capable of producing a dose $\geq D$ [see Equation (34)].
- (2) The expected number of people receiving a dose $\geq D$ from a given size particle is directly proportional to the average population density in the dispersion area for that particle size [see Equation (34)].
- (3) For a particle size distribution similar to that assumed, the expected number of exposures to a dose $\geq D$ decreases more rapidly than $1/D$ (see Figure 5).
- (4) Doses in excess of $1r$ result primarily from single particle interactions with the population.
- (5) Exposure probabilities predicted using the model developed herein will be high because of neglecting the effects of population movement, fuel burnup, and fission-product diffusion.

LIST OF REFERENCES

1. R. W. Decker Jr., and M. I. Goldman, "Particle Re-Entry Interactions," NUS-167, 20 January 1964. (U)
2. "Nuclear Vehicle Flight Safety Study - Summary Report for Phases I and II," LMSC-BO61396, 31 July 1963. (C-RD)
3. M. R. Smith, "The Activity of the Fission Products of U^{235} (Program 408)," XDC 60-1-157, 10 December 1959. (U)
4. E. S. Pearson and H. O. Hartley, "Biometrika Tables for Statisticians, Volume I," University Press, Cambridge, 1958. (U)
5. "Feasibility Test of a Weapon-Launched Destruct System for a Nuclear Rocket Engine," APG Report No. DPS-1060, September 1963. (C-RD)
6. J. M. Bridges, H. G. Hargrove and D. Hoecker, "Summary Report of the Theoretical, Analytical, and Experimental Studies Performed in the Investigation of the Re-Entry Burnup of NERVA Fuel," WANL-TNR-135, September 1963. (U)
7. M. J. Engel, et al, "The Effects of Atmospheric Re-Entry on Graphite Particles," General Electric Company, January 1964. (C-RD)
8. "Hammond's Ambassador World Atlas," C. S. Hammond & Co., Maplewood, N. J., 1954. (U)
9. H. G. Hargrove and D. Hoecker, "Flight Mechanics of NERVA Re-Entry and Orbital Lifetime Studies," WANL-TME-635, December 1963. (U)
10. E. Parzen, "Modern Probability Theory and Its Application," John Wiley & Sons, Inc., New York, 1960. (U)
11. W. Groebner and N. Hofreiter, "Integraltafel, Zweiter Teil Bestimmte Integrale," Springer-Verlag, Vienna, 1950. (U)
12. "British Association Mathematical Tables, Vol. IV; Bessel Functions. Part I, Functions of Order Zero and Unity," University Press, Cambridge, 1950. (U)

APPENDIX AEXACT PROBABILITY OF DOSE $\geq D$ RESULTING FROM
A FIXED NUMBER OF PARTICLES1. Single Particle Probabilities

Consider a dose D resulting from m particles of equal gamma-ray source strength located at a distance r from the dose point. Then the dose can be defined by the expression

$$D = \frac{mk}{\alpha}, \quad \dots(A-1)$$

where k is a constant and α is the area of the circle defined by the radius r . A dose $\geq D$ will occur when the m particles are on the circumference or within the circular area α .

Assume that particles are distributed in an area α' in accordance with a Poisson distribution which has a mean value $\lambda\alpha'$, where λ is the particle density per unit area. Then the area α required to just reach the most distant of m particles obeys a gamma probability law with parameters m and λ and with a continuous probability density function defined by (Ref. 10).

$$f(\alpha | m, \lambda) = \frac{\lambda(\lambda\alpha)^{m-1}}{(m-1)!} e^{-\lambda\alpha}. \quad \dots(A-2)$$

Thus, the probability of $\alpha \leq \alpha'$ for a given m and λ is

$$P(\alpha \leq \alpha') = \int_0^{\alpha'} f(\alpha | m, \lambda) d\alpha. \quad \dots(A-3)$$

Defining D_m as the dose from only the most distant of m particles, and substituting $\alpha = k/D_m$ and $\alpha' = k/D$ into Equation (A-3), the probability of a dose $\geq D$ from the most distant of m particles is given by

$$P_m = \int_0^{k/D} \frac{\lambda}{(m-1)!} \left(\frac{\lambda k}{D_m} \right)^{m-1} \exp \left(- \frac{\lambda k}{D_m} \right) d \left(\frac{k}{D_m} \right). \quad \dots (A-4)$$

Letting $c_m = \lambda k/D_m$ and $c = \lambda k/D$, Equation (A-4) becomes

$$P_m = \int_0^c \frac{(c_m)^{m-1}}{(m-1)!} e^{-c_m} dc_m. \quad \dots (A-5)$$

This expression is readily integrable for specific values of m . For example, the probabilities associated with receiving a dose $\geq D$ from each of the three particles with the greatest probability of being near the dose point are:

$$\begin{aligned} P_1 &= 1 - e^{-c} \\ P_2 &= 1 - (c+1)e^{-c} \\ P_3 &= 1 - 1/2 (c^2 + 2c + 2)e^{-c} \end{aligned} \quad \dots (A-6)$$

which for small values of c become

$$\left. \begin{aligned} P_1 &= c \\ P_2 &= c^2/2 \\ P_3 &= c^3/6 \end{aligned} \right\} c \ll 1 \quad \dots (A-7)$$

Thus, it is apparent that for small values of c , the dose will result primarily from a single particle.

2. Multiple Particle Probabilities

To obtain the exact probability of a dose $\geq D$ resulting from m particles, it is necessary to sum the doses from the 1st, 2nd, ..., m th most distant particles. The cumulative distribution function for the total dose D

would thus be obtained from the convolution of the m different cumulative distribution functions for the variables $D_1, D_2, \dots, D_m$. This process becomes extremely difficult for a large number of particles; however, for two particles the method is relatively straightforward and provides some additional insight into the validity of the model developed in Section II of this report. The development of the two-particle probability is presented below.

The probability of a dose $\geq D$ resulting from the sum of the dose contributions from the two particles with the greatest probability of being near the dose point can be expressed as the probability of the first particle producing a dose $D_1 \geq D$ irrespective of the dose produced by the second particle, plus the probability of the first particle producing a dose D_1 between zero and D given that the second particle produces a dose $D_2 \geq D - D_1$. Thus, with the use of Equation (A-5), the probability of the two particles producing a dose $\geq D$ can be expressed as

$$P_{1,2} = \int_0^{\frac{\lambda k}{D}} e^{-c_1} + \int_{\frac{\lambda k}{D}}^{\infty} dc_1 \int_0^{\frac{\lambda k}{D_2}} e^{-c_1} \cdot c_2 e^{-c_2} dc_2, \dots (A-8)$$

where $D_1 + D_2 \geq D$, and the first term of Equation (A-8) is equal to P_1 [see Equation (A-5)]. Therefore,

$$P_{1,2} = P_1 + \int_0^{\infty} e^{-c_1} dc_1 \int_0^{\frac{c_1 c}{c_1 - c}} c_2 e^{-c_2} dc_2 \dots (A-9)$$

$$= P_1 + e^{-c} - \int_c^{\infty} \left(\frac{c_1 c}{c_1 - c} + 1 \right) e^{-\frac{c_1 c}{c_1 - c} - c_1} dc_1. \dots (A-10)$$

Substituting $k = c_1 - c$,

$$P_{1,2} = P_1 + e^{-2} - 2ce^{-2c} \left[\frac{c}{2} \int_0^{\infty} e^{-k} - \frac{c^2}{k} \frac{dk}{k} + \frac{c(c+1)}{2} \int_0^{\infty} e^{-k} - \frac{c^2}{k} \frac{dk}{k^2} \right], \dots (A-11)$$

Reference 11 gives for the first integral

$$\int_0^{\infty} e^{-k - \frac{c^2}{k}} \frac{dk}{k} = 2K_0(2c), \quad \dots(A-12)$$

where $K_0(2c)$ is a modified Bessel function of the second kind and zero order. A recursion relationship for higher order functions of this type, as given by Reference 12, is

$$xK_{n+1}(x) = nK_n(x) - xK_n'(x). \quad \dots(A-13)$$

Thus,

$$K_1(2c) = -K_0'(2c) = \frac{d}{d(2c)} \left[-1/2 \int_0^{\infty} e^{-k - \frac{c^2}{k}} \frac{dk}{k} \right] \quad \dots(A-14)$$

$$= \frac{c}{2} \int_0^{\infty} e^{-k - \frac{c^2}{k}} \frac{dk}{k^2}, \quad \dots(A-15)$$

so that Equation (A-11) reduces to

$$P_{1,2} = P_1 + e^{-c} - 2ce^{-2c} [c K_0(2c) + (c+1) K_1(2c)]. \quad \dots(A-16)$$

The functions $K_0(2c)$ and $K_1(2c)$ are equal to (Ref. 12)

$$K_0(2c) = -[\gamma + \ln(c)] \sum_{r=0}^{\infty} \frac{c^{2r}}{(r!)^2} + \sum_{r=1}^{\infty} \frac{c^{2r}}{(r!)^2} \left(1 + \frac{1}{2} + \dots + \frac{1}{r} \right) \quad \dots(A-17)$$

$$K_1(2c) = [\gamma + \ln(c)] \sum_{r=0}^{\infty} \frac{c^{2r+1}}{r! (r+1)!} + \frac{1}{2c} - \sum_{r=1}^{\infty} \frac{c^{2r-1}}{(r-1)! r!} \left(1 + \frac{1}{2} + \dots + \frac{1}{r} - \frac{1}{2r} \right), \quad \dots(A-18)$$

$$\gamma = 0.5772 \dots$$

which, for very small values of c , become

$$\left. \begin{aligned} K_0(2c) &\cong -\gamma - \ln(c) \\ K_1(2c) &\cong -\frac{1}{2c} + c \left[\ln(c) + \gamma - \frac{1}{2} \right] \end{aligned} \right\} c \ll 1 \quad \dots(A-19)$$

Thus, for $c \ll 1$

$$\left. \begin{aligned} P_{1,2} &\cong P_1 + e^{-c} -(c+1)e^{-2c} + c^2 e^{-2c} \left[1 - 2c \ln(c) \right] \\ &\cong P_1 + e^{-c} P_2 + c^2 e^{-2c} \left[1 - 2c \ln(c) \right] \\ &\cong P_1 + 3P_2 \cong P_1. \end{aligned} \right\} \quad \dots(A-20)$$

Therefore, it can be stated that, for values of $c \ll 1$, a dose $\geq D$ is attributable almost exclusively to the dose contribution from a single particle, and the probability of receiving this dose is

$$P(\text{dose} \geq D) \cong c, \text{ for } c \ll 1. \quad \dots(A-21)$$