**DIFFUSION COUPLED (ELECTROCHEMICAL)
SELF-POTENTIAL EFFECTS IN GEOTHERMAL AREAS**

by

W. R. Sill

Work performed under Contract No.

DE-AC07-80ID12079

Department of Geology and Geophysics

**University of Utah
Salt Lake City, Utah (USA)**

AUGUST 1982

Prepared for
DEPARTMENT OF ENERGY
Division of Geothermal Energy

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

**Diffusion Coupled (Electrochemical) Self-Potential
Effects in Geothermal Areas**

by

**W. R. Sill
University of Utah
Department of Geology and Geophysics
Salt Lake City, Utah 84112**

**Prepared for the Department of Energy, Division of
Geothermal Energy under contract DE-AC07-80ID12079**

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.....	1
INTRODUCTION.....	2
COUPLED DIFFUSION AND ELECTRIC TRANSPORT.....	4
DIFFUSION CROSSCOUPLING PARAMETERS.....	9
GEOHERMAL MODEL.....	12
NUMERICAL MODEL RESULTS.....	16
SUMMARY.....	19
REFERENCES.....	20
LIST OF FIGURES.....	22
LIST OF TABLES.....	24

ABSTRACT

Electrochemical effects caused by the diffusion of geothermal brines are investigated as a possible source mechanism for the generation of self-potential. One of the difficulties associated with the modeling of the coupled salt flow and electric current problem is that the coupled equations are nonlinear because the electrical conductivity depends on the ambient salt concentration. The linearized solutions correspond to the case where the concentration variations are small compared to the background concentration. The linear case is not of much interest in geothermal areas where the brines may be much more concentrated than the normal groundwater.

The investigation of the coupled transport problem in this paper shows that the solution of the salt flow problem gives a concentration distribution that is very close to that derived from the solution of the ambipolar diffusion problem. In pure ambipolar diffusion, the local electric field is such that the cations and anions drift with the same velocity and the total electric current is zero. When ambipolar diffusion is an adequate description of the transport of solute ions, the salt flow and electric current problems can be solved in a manner analogous to the coupled fluid flow and heat flow problems as described in Sill (1981, 1982a). The results of model calculations indicate that this mechanism can generate only small self-potentials (~ 10 mv).

INTRODUCTION

Electrochemical effects due to concentration gradients of minor groundwater constituents (Fe^{2+} , Fe^{3+}) are generally accepted to be the mechanism for the generation of self-potential anomalies associated with conducting ore bodies (Sato and Mooney, 1960). On the other hand, electrochemical self-potential caused by the diffusion of geothermal brines has not received much attention as a possible source mechanism in geothermal systems. Corwin and Hoover (1979), in their discussion of the use of self-potential in geothermal exploration, consider electrochemical effects mostly in terms of a noise source.

Nourbehecht (1963) modeled the diffusion potential generated by vertical concentration gradients. For a dike model, he estimated the typical anomaly magnitude of about 20 mv. In this modeling, he did not explicitly consider the effects of the variable concentration on the electrical conductivity, which is appropriate if the concentration of the diffusing ions is small compared to the background concentration. The solution technique used by Nourbehecht (1963) requires an a priori definition of the concentration distribution and the electrical conductivity model.

In a geothermal system, the brines can be much more saline than the groundwater and thus the electrical conductivity structure depends strongly on the concentration distribution. This dependence introduces nonlinearity into the coupled salt flow and electric current equations.

This paper presents a study of the nonlinear, coupled transport problem that is applicable to geothermal systems with high salinities. The results of

the first part of the investigation indicate that the salt flow problem can usually be adequately described by assuming ambipolar diffusion. The results of numerical calculations based on a simple geothermal model indicate that electrochemical effects due to the diffusion of the brine are capable of generating only small self-potential anomalies.

COUPLED DIFFUSION AND ELECTRIC TRANSPORT

The steady state fluxes of cations (Γ_+) and anions (Γ_-) which are driven by concentration gradients (∇C_+ , ∇C_-) and electric potential gradients (∇V) are given by

$$\Gamma_+ = \frac{-kT}{e} \frac{U_+}{Z_+} \nabla C_+ - U_+ C_+ \nabla V \quad (1)$$

$$\Gamma_- = \frac{-kT}{e} \frac{U_-}{Z_-} \nabla C_- - U_- C_- \nabla V, \quad (2)$$

where k = Boltzmann constant,
 T = absolute temperature,
 e = electronic charge,
 $U_{+,-}$ = effective mobility,
and $Z_{+,-}$ = ion valence.

In rocks, the effective ion mobility depends on the porosity and pore structure (geometrical effects) and on the distribution of ions in the double layer at the pore water/silicate interface (electrochemical effects). Geometrical effects reduce the effective mobilities of both cations and anions relative to their values in ordinary electrolytic solutions. These effects are described by an electrical formation factor which can be related to the porosity through Archie's Law. Electrochemical effects often result in excess cations in the double layer (Ishido and Mizutani, 1981) and this produces an increase in the effective cation mobility. Using the condition of charge neutrality

$$Z_+ C_+ + Z_- C_- = 0, \quad (3)$$

and
$$Z_+ = -Z_- = 1.$$

The concentrations can be expressed in the following form

$$C_+ = C_- = C_0 + C, \quad (4)$$

where C_0 is a constant term and C is a function of position. The flux equations (1) and (2) can then be combined into a salt flow ($\Gamma_s = \Gamma_+ + \Gamma_-$) which describes the flux of the solute ions and a total electric current ($J_T = e\Gamma_+ - e\Gamma_-$) which describes the flux of charge. These fluxes are given by

$$e\Gamma_s = -\sigma_+ C_1 - \sigma_-(1 + C/C_0) V, \quad (5)$$

$$J_T = -\sigma_- C_1 - \sigma_+(1 + C/C_0) V, \quad (6)$$

where

$$\sigma_+ = eU_+ C_0 (1 + \alpha), \quad (7)$$

$$\sigma_- = eU_- C_0 (1 - \alpha), \quad (8)$$

$$\alpha = \left| \frac{U_-}{U_+} \right|, \quad (9)$$

and

$$C_1 = \frac{kT}{e} \frac{C}{C_0}. \quad (10)$$

Multiplication of the salt flux by the electronic charge converts it to electrical units (amp/m²) and so the concentration C_1 is also expressed in electrical units (volts). Equations (5) and (6) are the coupled equations that describe the flow system. The equations are coupled by the σ_- conductivity, and the presence of the C/C_0 terms makes them nonlinear. The equations decouple when σ_- is zero or when the ion mobilities are equal. In ordinary electrolytic solutions the mobilities of cations and anions are very nearly the same, and if they are identical, then the salt flow is described by diffusion under the influence of a concentration gradient and the electric

current and potential are zero. In rocks, the presence of excess cations in the diffuse layer at the interface between the silicates (especially clays) and the pore water can give rise to an effective cation mobility which is larger than the anion mobility. Therefore the flows in geological materials can be more strongly coupled than in pure electrolytic solutions. The crosscoupling conductivity σ_- will be discussed in more detail in a subsequent section.

The equations also decouple when the total current is zero and this is the case of pure ambipolar diffusion (Krall and Trivelpiece, 1973). For pure ambipolar diffusion, equations (5) and (6) can be rewritten as

$$e\Gamma_s = -\sigma_+ \left(1 - \frac{\sigma_-}{\sigma_+}\right) \frac{C_1}{2} ; J_T = 0 , \quad (11)$$

$$V = -\frac{\sigma_-}{\sigma_+} \frac{C_1}{(1 + C/C_0)} ; J_T = 0 . \quad (12)$$

In this case, the potential gradient (equation (12)) is in opposition to the concentration gradient and its strength is such that the cations and anions drift with the same velocity. Pure ambipolar diffusion is a special case which requires an appropriate geometry such that the total current can be zero everywhere. It is also an ideal limit that any coupled flow system would tend toward, since it is a state of minimum energy dissipation.

Pure ambipolar diffusion is possible in many geometries where the salt flow and the current flow are either parallel or perpendicular to all the boundaries where there are changes in the conductivities. A simple example is one-dimensional flow with boundaries perpendicular to the flow. In the general case, with an arbitrary geometry, the state of pure ambipolar diffusion with the total current zero everywhere may not be permissible.

For self-potential problems, we are interested in the electrical effects generated by the diffusion of ions and the divergence conditions (from equations (5) and (6)) are

$$e \nabla \cdot \Gamma_s = eS = -\nabla \cdot [\sigma_+ \nabla C_1] - \nabla \cdot [\sigma_- (1 + C/C_0) \nabla V], \quad (13)$$

$$\text{and } \nabla \cdot J_T = 0 = -\nabla \cdot [\sigma_- \nabla C_1] - \nabla \cdot [\sigma_+ (1 + C/C_0) \nabla V]. \quad (14)$$

The external sources for the salt flow are given by S and for the self-potential problem there are no external sources for the total current. One way of viewing equation (13) is to consider the second term on the right as an additional source term for the concentration, and in equation (14) the first term can be considered as the source for the potential. This approach suggests an iterative method of solution:

$$\nabla \cdot [\sigma_+ \nabla C_1(n)] = -eS - \nabla \cdot [\sigma_- (1 + C(n-1)/C_0) \nabla V(n-1)], \quad (15)$$

$$\nabla \cdot [\sigma_+ (1 + C(n)/C_0) \nabla V(n)] = -\nabla \cdot [\sigma_- (1 + C(n)/C_0) \nabla C_1(n)], \quad (16)$$

where $C_1(n)$ and $V(n)$ are the solutions at the n^{th} iteration, and the first iteration could start with $V(0) = 0$. The solution of the divergence equations (15) and (16) for the transmission surface algorithm is outlined in Sill (1981 and 1982a). This iterative procedure was first implemented for the linear case ($C/C_0 \ll 1$) and was found to converge quite rapidly for σ_-/σ_+ not too close to 1. Not surprisingly, considering the previous discussion, it was observed that the solutions gave concentrations essentially the same as that for pure ambipolar diffusion.

In geothermal problems the linear case is of lesser importance since geothermal waters are often of a much greater concentration than the

surrounding groundwater. If in these non-linear problems we anticipate a final state close to ambipolar diffusion, then equations (11) and (12) will provide a much better starting point for the iterative solution. In terms of the ambipolar flow, equation (5) can be rewritten as:

$$e\Gamma_s = -\sigma_+ \left(1 - \frac{\sigma_-}{\sigma_+} \frac{\nabla^2 C_1}{2}\right) + \frac{\sigma_-}{\sigma_+} J_T \quad (17)$$

The second term on the right gives an indication of how far the true salt flux deviates from the pure ambipolar one. Since the flow system tends toward ambipolar diffusion, where J_T is zero, the difference term is usually small. Therefore the starting point for the non-linear case will be taken from the solution of the divergence of the ambipolar equations (11 and 12):

$$\nabla \cdot \left[\sigma_+ \left(1 - \frac{\sigma_-}{\sigma_+} \frac{\nabla^2 C_1}{2}\right) \right] = -eS \quad (18)$$

$$\nabla \cdot [\sigma_+ (1 + C(1)/C_0) \nabla C_1] = -\nabla \cdot [\sigma_- C_1] \quad (19)$$

Equations (15) and (16) are then used in the subsequent iterations.

As a test of this iterative solution the model of Figure 1 was run. The concentration and potential on the surface at $x = 0$ are shown in the bottom of the figure as a function of iteration number. The changes in both the concentration and potential are around a percent. The difference term, $J_T \sigma_- / \sigma_+$ (equation (17)), which indicates how far the salt flow deviates from the ambipolar case is around ten percent. Considering the accuracy of a few percent for the numerical technique, the results from initial guess (equations (18) and (19)) are probably sufficiently accurate for most cases of interest.

DIFFUSION CROSSCOUPLING PARAMETERS

Experimental measurements of the diffusion crosscoupling are usually made by observing the voltage developed across a rock sample when it is subject to a known concentration gradient with $J_T = 0$. Under these conditions, equation (6) can be rearranged to give the diffusion potential coefficient, defined by

$$-(\frac{\nabla V}{\nabla} \log C) = 2.3 \frac{kT}{e} \frac{\sigma_-}{\sigma_+} ; J_T = 0. \quad (20)$$

Measurements of the diffusion potential for 20 rock samples (Nourbehecht, 1963) showed a range from -5 to 45 mv/decade. Most of the samples, which included sedimentary, igneous and metamorphic rocks, fell in the range from 5 to 10 mv/decade. A larger suite of measurements (75 samples) is reported in Nourbehecht and Madden (1965) with most of the samples falling in the range from 8 to 14 mv/decade. A coefficient of 10 mv/decade corresponds to $\sigma_-/\sigma_+ = .17$ ($U_-/U_+ = .7$) and 40 mv/decade corresponds to $\sigma_-/\sigma_+ = .67$ ($U_-/U_+ = .2$). Thus the crosscoupling effects in most of these rocks are modest, however the larger values for the diffusion potentials correspond to an extreme reduction in the effective anion mobility when compared to the cation mobility.

As noted previously, the contribution by the excess cations in the double layer to the transport can be viewed as an increase in the effective cation mobility. The effects of these double-layer ions can be treated in an approximate way by considering that the average cation concentration in the rock pores is increased so that the cation and anion concentrations are given by

$$C_+ = C + C^* , \quad (21)$$

$$C_- = C \quad , \quad (22)$$

Where C^* is the double-layer cation excess when it is averaged over the pore volume. This concentration can also be expressed as

$$C^* = Qv/e \quad , \quad (23)$$

where Qv is the cation exchange capacity per unit pore volume as in the Waxman and Smits (1968) model. The electric field transport terms (Γ_{+e} , Γ_{-e}) in equations (1) and (2) can then be expressed as

$$\Gamma_{+e} = - Fe^{-1} (U'_+ C + U^* C^*) \nabla V \quad , \quad (24)$$

and
$$\Gamma_{-e} = - Fe^{-1} U'_- C \nabla V \quad , \quad (25)$$

where Fe = electrical formation factor,

$U'_{+,-}$ = ion mobility in the bulk electrolyte,

U^* = ion mobility in the double layer.

In equations (24) and (25) the geometrical effects in the effective mobility have been combined into the electrical formation factor which can be expressed in terms of the porosity (ϕ) and the porosity exponent (m) by Archie's Law:

$$Fe = \phi^{-m} \quad . \quad (26)$$

Comparison of equations (24) and (25) with (1) and (2) shows that the effective cation mobility is increased by the contribution from double-layer cations while the anion mobility is unchanged. When the bulk electrolyte mobilities have equal magnitudes ($U'_+ = -U'_- = U$), the sum of equations (24) and (25) give their contribution to the salt flux as

$$e\Gamma_s = - Fe^{-1} eU^* C^* \nabla V \quad , \quad (27)$$

and the difference gives their contribution to the current as

$$J = - F_e^{-1} (2eUC + eU^*C^*)\nabla V . \quad (28)$$

The conductivity in equation (28) is in the form of a Waxman-Smits model (Waxman and Smits, 1968):

$$\sigma = Fe^{-1} (\sigma_w + \sigma_s) , \quad (29)$$

where $\sigma_w = 2eUC$ is the bulk pore water conductivity,

and $\sigma_s = eU^*C^*$ is the surface conductivity.

Comparison of equations (27) and (28) with equations (5) and (6) indicates that σ_+ depends on the contributions from both the bulk and surface conductivities, and the crosscoupling conductivity σ_- depends on the surface conduction. A more detailed derivation based on the transport in a capillary, as was done in Sill (1982b) for other crosscoupling parameters, leads to the same conclusions even though the final results are considerably more complicated than equations (24) and (25) or equations (27) and (28).

GEOHERMAL MODEL

In the simplest of geothermal models we must consider the model parameters for the reservoir and the host rocks. Almost by definition, the reservoir has a relatively high porosity and permeability and the host rocks that confine the system are, at least in part, of low porosity and permeability. In Sill (1982b), a model for estimating primary and crosscoupling parameters is developed. This model is based on transport in capillaries and the representation of the pore structure in a rock by a network of capillaries with a lognormal distribution of pore sizes. According to this model, the surface conductivity (σ_s) which governs the diffusion crosscoupling and the permeability (κ) are given by

$$\sigma_s = 2\sigma^* / \exp(m_x),$$

$$\kappa = Fe^{-1} \exp(2m_x)/8,$$

where σ^* is the surface conductance and m_x is the mean of the lognormal distribution of pore sizes. These equations show that an increase in permeability due to an increase in the mean pore size will result in a reduction of the surface conductivity. Since the surface conductivity determines the magnitude of the diffusion crosscoupling conductivity (σ_-), we should expect the crosscoupling to be larger in the relatively impermeable host compared to the more permeable reservoir, if the surface conductance (σ^*) is similar in the two regions. The surface conductance (σ^*) depends on the potential at the pore water-silicate interface and it is typically larger for clays compared to other silicates. Thus the distribution of clays or the amount of clay alteration in the vicinity of the geothermal system can also

have a significant effect.

As a specific example we can consider the geothermal system at Roosevelt Hot Springs, Utah (RHS). At RHS the host rocks consist of granites and gneisses and the reservoir at depth consists of the same rocks in a more fractured and porous condition (Ward et al., 1978; Nielson et al., 1978). Hydrothermal alteration is associated with the fractures and the clay alteration decreases with depth. The maximum temperature in the reservoir is close to 250°C and the salinity of the fluid is around 0.1 molar compared with a typical groundwater salinity of around 0.01 molar.

Electrical measurements at RHS show that away from the geothermal system the host rocks have a conductivity approaching 0.002 s/m (Ward and Sill, 1976). The surface conductivity can be estimated using equations (29) and (26). If we assume a porosity of 5 percent and $\sigma_w = 0.1$ s/m (0.01 m), then $\sigma_s = 0.7$ s/m. In the host rocks the surface conductivity is then much larger than the bulk pore water conductivity. An estimate from equations (27), (28), and (24) of the diffusion coupling as measured by the ratio σ_-/σ_+ is

$$\frac{\sigma_-}{\sigma_+} = \frac{\sigma_s}{\sigma_w + \sigma_s} \quad (32)$$

For the host rocks this ratio is close to 0.9, which corresponds to a diffusion potential (equation (20)) of 50 mv/decade, however the use of equations (29) and (32) probably results in an overestimate.

In the reservoir, below the strong clay alteration zone, the conductivity is near 0.05 s/m. The major factors contributing to this increase in conductivity are the increase in porosity due to fracturing, the increase in conductivity of the reservoir fluid ($\sigma_w = 1.0$ s/m) and the increase in

temperature. The maximum temperature corresponds to an increase in conductivity by about a factor of 5 so that a porosity of about 10 percent would account for the increase in conductivity without requiring any contribution from the surface conductivity. This decrease in surface conductivity is also substantiated by a consideration of the permeability required to permit convection in the system. Convection in a porous medium will take place when the Rayleigh number (Ra) satisfies the condition (Combarrous and Bories, 1975)

$$Ra = \alpha \beta g \Delta T \Delta Z \kappa / \eta > 4\pi^2, \quad (33)$$

where α = rate of change of fluid density with temperature,

β = thermal diffusivity,

g = acceleration of gravity,

ΔT = temperature change over the vertical distance ΔZ ,

κ = permeability,

η = fluid viscosity.

From equation (33), convection will take place in a system with a temperature change of 250°C over a depth of 1 km when the permeability is greater than 10^{-14} m^2 (10 millidarcy). In the host rocks, with a lognormal mean such that $\exp(m_x) = 10^{-6} \text{ m}$ (Brace, 1977), the permeability at 5 percent porosity is (equation (31)) about $3 \times 10^{-16} \text{ m}^2$ (.3 millidarcy) which is too small for convection to take place. In the reservoir, the increase in porosity will result in some increase in permeability, but an increase in mean size will also be required if convection is to take place. If we consider an increase in pore size to 10^{-5} m , the reservoir permeability at 10 percent porosity is 10^{-13} m^2 (100 millidarcy) which is large enough to satisfy the convection

criterion. Equation (30) shows that the required increase in mean pore size will reduce the contribution from the surface conductivity. Under the above conditions, and assuming that (σ^*) in the reservoir and the host have similar values at the same temperature, then the ratio of σ_s/σ_w in the reservoir is around 0.1. This gives a diffusion potential of 5 mv/decade which is much less than that in the host.

Near the surface, in the clay alteration zone, the surface conduction may be greatly enhanced so that the diffusion coupling as measured by σ_-/σ_+ may be close to 1. The picture that emerges is then one of a conductive reservoir with a small diffusion crosscoupling parameter, surrounded by a more resistive host with a larger crosscoupling. Near the surface, a conductive clay alteration zone with a large crosscoupling parameter can also be present.

NUMERICAL MODEL RESULTS

In order to determine the magnitude of diffusion coupled SP effects in a geothermal system the two-dimensional model of Figure 2 was used, with the model parameters given in Table 1. The point source (3-dimensional) locations for the salt flow are shown by the crosses in Figure 2. Medium 1 represents the reservoir and medium 2 is a transition region from the reservoir and medium 2 is a transition region from the reservoir to the host (medium 3). Medium 5 represents an overburden and medium 4 is included so that a clay alteration zone can be represented. The values for the parameters (σ_+ and σ_-/σ_+) and the physical size are in accord with the discussion in the previous section for the system at Roosevelt Hot Springs. The σ_+ parameter is the total electrical conductivity at the reference concentration which is taken as $C_0 = .01$ molar. The effects of concentration changes, due to salt flow sources in the reservoir, are included in the algorithm for the solution of the electric potential problem. The effects of temperature on the electrical properties are included in a gross fashion by assuming that the average temperatures in the reservoir (1), transition region (2), and host (3) are around 250°C, 100°C, and 20°C respectively. The σ_+ parameters, as given in Table 1, include these temperature effects.

The results for model 1 are shown contoured in the vertical plane through the sources in Figures 3 and 4. In Figure 3, the results are given in terms of the concentration ratio C/C_0 . For the reference concentration of $C_0 = 0.1$ molar, the 10 contour gives a concentration of $C = 1.0$ molar. This would then correspond to the brine-saturated part of the reservoir at Roosevelt Hot Springs, and in the model it extends from a depth of 250 m to 1250 m. The SP,

shown in Figure 4, reaches a minimum of -10. mv at a depth of 750 m at the boundary of the reservoir where the crosscoupling (σ_-/σ_+) goes from 0.0 to 0.8. The strongest induced electrical sources are also at this point. The SP anomaly is quite small, due to the high ambient conductivity in the region of the reservoir which results from the increased concentrations. In the vicinity of the reservoir, including the overburden, the ratio C/C_0 is near 10. This increases the ambient conductivity to around 0.05 s/m compared to its reference value of about 0.005 s/m. The surface SP profiles for the models of Table 1 are shown in Figure 5. The SP for model 1 reaches its minimum value (-5.8 mv) over the center of the reservoir. In model 1a, the effects of a conductive overburden are illustrated. In this model, the concentrations near the reservoir are reduced by about 20 to 40 percent compared to those of model 1, Figure 3. The ambient conductivity in the reservoir is about the same as in model 1 (0.05 s/m) but now the overburden is around 0.2 s/m. The increase ($\sim 4x$) in the overburden conductivity over the reservoir reduces the anomaly to -2.5 mv (Figure 5). Model 2 represents the case where there is a conductive region in the overburden due to clay alteration ($\sigma_-/\sigma_+ = 0.8$). In this model the concentrations are almost the same as model 1. Figure 5, model 2, shows that in this case the increase in the crosscoupling near the surface compensates for the increase in conductivity and the anomaly over the center of the reservoir increases to -7.6 mv.

In the previous models, the crosscoupling ratio σ_-/σ_+ in the reservoir has been set to zero. This result is in general agreement with the discussion in the previous section where it was shown that the increase in the permeability should lead to a drastic decrease in the surface conduction.

However, it was noted that the presence of some clay alteration in the reservoir would tend to increase the surface conduction and the ratio σ_-/σ_+ . Model 3 is an attempt to include such effects by increasing the crosscoupling ratio σ_-/σ_+ in the reservoir from 0.0 to 0.5. Figure 5, model 3, shows that the maximum anomaly is increased to -8.8 mv. Model 3a illustrates the nonlinear response of the voltage to the source strengths or concentrations. The only difference between models 3 and 3a is a doubling of the source strengths in the latter. The concentrations in model 3a increase by a factor of two but the increase in potential is less than 10 percent.

SUMMARY

Tests of an iterative solution to the nonlinear, coupled diffusion and electric transport problem indicate that in many cases a very good approximation to the concentration distribution is given by the solution to the pure ambipolar diffusion problem. When the ambipolar approximation is adequate the self-potential can be calculated on the first iteration, saving considerable computational effort.

In the geothermal models tested, the SP anomalies were small, with none exceeding 10 mv in magnitude and all were negative. At Roosevelt Hot Springs, which the models were intended to emulate, the negative SP anomalies often exceed -100 mv over large areas in the vicinity of the reservoir (Sill and Johng, 1979). Increasing the source strength to the point that the model concentration exceeds that observed for the reservoir does not help much because of the nonlinear response of the SP. This leveling off of the response is due to the increase in conductivity as the concentration increases.

The model results indicate that diffusion coupled SP effects are not likely to be an important source mechanism for geothermal areas like Roosevelt Hot Springs.

REFERENCES

- Brace, W. F., 1977, Permeability from resistivity and pore shape: *J. Geophys. Res.*, v. 82, p. 3343-3349.
- Combarnous, M. A., and Bories, S. A., 1975, Hydrothermal convection in saturated porous media, in *Advances in Hydrosience*, v. 10, ed. V. T. Chow, Academic Press, New York.
- Corwin, R. F., and Hoover, D. B., 1979, The self-potential method in geothermal exploration: *Geophysics*, v. 44, p. 226-245.
- Ishido, T., and Mizutani, H., 1981, Experimental and theoretical basis of electrokinetic phenomena in rock-water systems and its application to geophysics: *J. Geophys. Res.*, v. 86, p. 1763.
- Krall, N. A., and Trivelpiece, A. W., 1973, *Principles of plasma physics*: McGraw-Hill, New York.
- Nielson, D. L., Sibbett, B. S., McKinney, D. B., Hulen, J. B., Moore, J. N., and Samberg, S. M., 1978, *Geology of Roosevelt Hot Springs KGRA, Beaver County, Utah*: Earth Science Laboratory, University of Utah Report #ID0/78-1701.b.1.1.3 (ESL-12).
- Nourbehecht B., 1963, Irreversible thermodynamic effects in inhomogeneous media and their application in certain geoelectric problems: Ph.D. thesis, MIT, Cambridge.
- Nourbehecht, B., and Madden, T. R., 1965, Irreversible thermodynamics in inhomogeneous media and geoelectric applications, unpublished manuscript.
- Sato, M., and Mooney, H. M., 1960, The electrochemical mechanism of sulfide self-potentials: *Geophysics*, v. 25, p. 226-249.
- Sill, W. R., 1981, Self-potential modeling from primary flows: Dept. Geology and Geophysics, Univ. of Utah, DOE report no. DOE/ID/12079-42.
- Sill, W. R., 1982a, Self-potential modeling: *Geophysics*, v. 47, to be published.
- Sill, W. R., 1982b, A model for the crosscoupling parameters of rocks: Univ. of Utah, DOE report # DOE/ID/12079-69.
- Sill, W. R., and Johng, D. S., 1979, Self-potential survey, Roosevelt Hot Springs, Utah: DOE/DGE topical report, contract DE-AC07-78ET28392, University of Utah.
- Ward, S. H., and Sill, W. R., 1976, Dipole-dipole resistivity delineation of the near-surface zone at the Roosevelt Hot Springs area: Univ. of Utah ERDA report # 76-1 contract EY-76-S-07-1601.

Ward, S. H., Parry, W. T., Nash, W. P., Sill, W. R., Cook, K. L., Smith, R. B., Chapman, D. S., Brown, F. H., Whelan, J. A., and Bowman, J. R., 1978, A summary of the geology, geochemistry and geophysics of the Roosevelt Hot Springs thermal area, Utah: *Geophysics*, v. 47, p. 1515.

Waxman, Motlo, and Smits, L. J. M., 1968, Electrical conductivities in oil-bearing shaly sands: *Soc. Pet. Eng. J.*, v. 8, p. 107-122.

List of Tables

Table 1. Model parameters for geometric model in Figure 2.

Table 1. Model parameters for models 1, 2, and 3.

σ_+ is the total electrical conductivity at the reference concentration of $C_0 = 0.01$ molar. σ_-/σ_+ is the cross-coupling ratio.

Model 1		
Medium	σ_+ (s/m)	σ_-/σ_+
1	5×10^{-3}	0.0
2	4×10^{-3}	0.8
3	2×10^{-3}	0.8
4	5×10^{-3}	0.0
5	5×10^{-3}	0.0

Model 1a		
Medium	σ_+ (s/m)	σ_-/σ_+
1	5×10^{-3}	0.0
2	4×10^{-3}	0.8
3	2×10^{-3}	0.8
4	2×10^{-2}	0.0
5	2×10^{-2}	0.0

Model 2		
Medium	σ_+ (s/m)	σ_-/σ_+
1	5×10^{-3}	0.0
2	4×10^{-3}	0.8
3	2×10^{-3}	0.8
4	2×10^{-2}	0.8
5	5×10^{-3}	0.0

Model 3		
Medium	σ_+ (s/m)	σ_-/σ_+
1	5×10^{-3}	0.5
2	4×10^{-3}	0.8
3	2×10^{-3}	0.8
4	2×10^{-2}	0.8
5	5×10^{-3}	0.0

List of Figures

Figure 1. Top - model used for the convergence test.

Bottom - convergence of the concentration and potential for a point on the surface over the center of the body.

Figure 2. Geothermal model used in numerical calculations. The media parameters for the various models are given in Table 1. The crosses show source point locations.

Figure 3. Contours of concentration in the vertical plane for model 1.

Figure 4. Contours of self-potential in the vertical plane for model 1.

Figure 5. Surface profiles of the self-potential for the various geothermal models.

Figure 1

GEO THERMAL MODEL

Figure 2

MODEL I
CONTOURS OF C/C_0
--- RESERVOIR

Figure 3

MODEL I
CONTOURS OF SP(mV)
--- RESERVOIR

Figure 4

SELF - POTENTIAL PROFILES

Figure 5