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THE APPLICATION OF A SUM RULE
TO A REGGE MODEL FOR THE
HIGH ENERGY πN SYSTEM

by

James Vincent Beaupre
and
Charles L. Hammer

August 1967

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THE APPLICATION OF A SUM RULE TO A REGGE MODEL FOR THE HIGH ENERGY πN SYSTEM*

James Vincent Beaupre and Charles L. Hammer

ABSTRACT

Theoretical $\pi^\pm \rho$ total and elastic, and charge exchange differential cross sections, and polarizations, and were fitted to 259 high energy data points, under the assumptions of either a four Regge pole model or a three pole and a cut model. The generalized Restignoli - Sertorio - Toller sum rule, generalized to include the cut, is used to constrain the charge exchange forward amplitude under the assumptions that this amplitude is dominated by either the ρ and ρ' poles or by the ρ pole and a $\rho - P$ cut. The resulting fits, having χ^2 's of 438 for the $P + P' + \rho + \rho'$ model and 426 for the $P + P' + \rho + \text{cut}$ model, generally agreed with the data. However the use of the sum rule strongly depressed the calculated charge exchange polarization for both models.

* This report is based on a Ph. D. thesis submitted by James Vincent Beaupre, August 1967, to Iowa State University, Ames, Iowa.

I. INTRODUCTION

Even today, many years after the discovery of the pion and with the proliferation of elementary particles, pion-nucleon interactions are studied with high interest because the $0^{-}1/2^{+}$ spin system is the simplest example of the strong interactions. As a consequence of this continuing interest, πN elastic and charge exchange processes are theoretically and experimentally the best known strong interactions. The wealth of experimental data allows an almost immediate confirmation or rejection of model predictions, and, because the data themselves exhibit the same gross features as more complicated reactions*, πN scattering is the touchstone of the high energy theoretical physicist. If a model works for the πN system, it is almost immediately applied to quasi-elastic processes. It is, then, of the utmost importance to really understand πN scattering in all its details.

This thesis will be concerned with the explanation of the experimental observations at high energies of the following three reactions:

$$\pi^{+}P \rightarrow \pi^{+}P$$

$$\pi^{-}P \rightarrow \pi^{-}P$$

$$\pi^{-}P \rightarrow \pi^{0}n$$

The aim of this work is to examine the total cross-sections, differential cross-sections and polarizations near the forward direction in these reactions in the multi-Bev energy range under the assumptions of the Regge

* For example, the πP high energy elastic scattering data is similar to the KP elastic data in many respects (1), and the πP charge exchange data is similar to the $\pi P \rightarrow \pi \Delta$ data (2).

model for the scattering amplitudes.

Although the Regge theory is perhaps unsatisfying from theoretical and aesthetic points of view, it, nevertheless, gives the best agreement to experiment in the 5 - 20 Bev energy range. There are, however, discrepancies between some Regge πN models and the experimental data.

In the energy range 1 - 2 Bev, as shown in Figure 1, resonances either dominate or are the significant features of the πN total and differential cross-sections (3). From 2 - 5 Bev, there are still resonances present but the smooth background for the resonances provides most of the cross-section. From 5 - 20 Bev, it appears that the resonances have disappeared and there is little or no structure in the observed total and differential cross-sections. It is to be expected that this smoothness is a result of the asymptotic nature of the energies, and that in this region the Regge hypothesis would have great success. The total cross-sections are described within the experimental errors by the simple, but accurate empirical expression (4):

$$\sigma_T = a_0 + \sum_{n=1}^2 a_n (E/E_0)^{\alpha_n - 1} \quad (1.1)$$

where E is the incident pion laboratory energy and the a 's and the α 's are constants. These data are shown in Figure 2. A similar relation appears to be true for the KN system (1).

Near the forward direction almost all elastic processes can be crudely described by a differential cross-section of form (5):

$$\frac{d\sigma}{dt} \cong f(E)e^{At} \quad (1.2)$$

where t is the negative square of the momentum transfer. For πN scattering, $f(E)$ is essentially the square of the largest term in the total cross-section formula, roughly constant and of the order of 40 mb/Bev^2 as can be seen from Figures 3 and 4.

The pion nucleon charge exchange process (6) and some quasi-two body reactions (2) have a similar crude t dependence

$$\frac{d\sigma}{dt} \approx f(E)(1 + Bt)e^{At} \quad (1.3)$$

where for $\pi^+P \rightarrow \pi^0n$, $f(E)$ is of the order of $.4 \text{ mb/Bev}^2$ and strongly decreasing with the energy as shown in Figure 5. For small $-t$, there is a general vanishing of the spin-slip amplitudes which are proportional to $\sqrt{-t}$. From this observation and the optical theorem one may conclude that σ_T and $\frac{d\sigma}{dt}$ (elastic) exhibit essentially the properties of the imaginary part of the non-spin-flip amplitude. The extra t dependence in the charge exchange differential cross-section, which is observable at small $-t$, implies the possibility of a large charge exchange spin-flipping force.

A more direct measurement of the spin-flipping forces comes from polarization measurements. Both π^+P and π^-P elastic polarizations are large and change signs a few times in a small t region as shown in Figures 6 and 7. The π^+P and π^-P polarizations are almost the negatives of each other, and the absolute value of the maximum polarization is about 15% (7). The difference in sign suggests that the isopin, $I = 1$, forces are measured primarily in elastic polarization experiments. The charge exchange reaction $\pi P \rightarrow \pi^0n$, shown in Figure 8, has a positive relatively flat polarization of about 15% for small $-t$ (8). Because of the largeness of the polarizations, even though most of the elastic cross-sections come from

the non-spin-flip amplitude, spin-flip forces must be accounted for in the 5 - 20 Bev region.

Other smaller effects in the high energy πN system are the "cross-over" effect and various "dip and bump" structures at $-t \sim 1$ Bev. The "cross-over" effect is the changing of sign of the difference of the π^+P and π^-P elastic differential cross-sections at $t \sim -.05 \text{ Bev}^2$. For $|t| < 0.05 \text{ Bev}^2$ the π^-P cross-section is slightly larger than that of π^+P , but, for $|t| > 0.05 \text{ Bev}^2$, the π^+P cross-section is slightly greater. The most pronounced "dip and bump" structure is in the charge exchange reaction at $-t \sim 0.6 \text{ Bev}^2$ (see Figure 5).

These are the general features to be explained. A three pole model proposed by Chiu, Rarita and Phillips (9) explains all the data quite well, except that it predicts a zero charge exchange polarization. The present data are clearly not consistent with zero polarization and so the amplitudes must be modified.

To bring the theory into closer agreement with experiment a new term may be added to the three pole Regge model (9). There are two simple ways to do this, either with another Regge pole (10) or with a Regge cut (11). The parameters of the new pole or cut are not free, and are, in fact, constrained to obey the forward dispersion relations, which relate the low energy structure in the πN total cross sections to the high energy Regge parameters. This is a very important constraint since it has been shown (12) that both the extra pole (10) and the cut (11) violate this constraint, and because, as will be described in the next chapter, these dispersion relations are a special case of the theoretical basis for the

relativistic Regge theory. This thesis will study the πN system with the addition of either a new pole or a cut to the three pole model subject to the forward dispersion relation constraint, in hopes of determining whether or not the new contribution can be made consistent with both the data and the dispersion relation constraint.

II. REGGE THEORY

In this chapter a description of the Regge theory is given and its relevance to high energy two body reactions is indicated. Due to the vast amount of research effort that has been expended on Regge poles, amplitudes, and philosophy, it is impossible to give even a detailed summary of the Regge theory in a few pages. This chapter is intended to summarize the general Regge result with a minimum of detail and to outline how the basic Regge results can be applied to the πN system.

Regge and others (13, 14) considered the following problem: If the non-relativistic potential scattering amplitude is described by the usual partial wave expansion,

$$f(E, \theta) = \sum_{\ell=0}^{\infty} (2\ell+1) f_{\ell}(E) P_{\ell}(\cos\theta) \quad (2.1)$$

in a $\cos\theta$ domain (generally an ellipse) which contains the line $-1 \leq \cos\theta \leq 1$, what is the analytic continuation of the series, in $\cos\theta$, and, in particular, how does $f(E, \theta)$ behave for large $|\cos\theta|$? Regge performed a Sommerfeld-Watson transformation (15, p. 279) of the amplitude and arrived at the Regge representation for $f(E, \theta)$ (14)

$$\begin{aligned} f(E, \theta) = & \frac{1}{2} \int_{-1/2 - i\infty}^{-1/2 + i\infty} d\ell \frac{(2\ell+1) f(\ell, E) P_{\ell}(-\cos\theta)}{\sin \pi \ell} \\ & + \sum_{n=1}^N \beta_n(E) P_{\alpha_n}(-\cos\theta) \\ & + \left(\begin{array}{l} \text{contributions from other singularities} \\ \text{to the right of } \text{Re } \ell = -1/2 \text{ in the complex } \ell\text{-plane} \end{array} \right) \end{aligned} \quad (2.2)$$

where the α_n are the poles of $f(\ell, E)$ such that

$$\operatorname{Re} \alpha_n > -1/2, \quad (2.3)$$

and if ℓ is an integer one has

$$f(\ell, E) = f_\ell(E) \quad (2.4)$$

Regge also shows that if there are only pole singularities to the right of $\operatorname{Re} \ell = -1/2$ in the ℓ plane then the large $\cos\theta$ behavior of $f(E, \theta)$ is given by

$$f(E, \theta) \sim \sum_n \beta_n' (\cos\theta)^{\alpha_n} + O(\cos\theta)^{-1/2} \quad (2.5)$$

The relationship of the asymptotic form of Equation 2.5 to the high energy scattering amplitude comes from the use of crossing symmetry, which states that the invariant scalar amplitude for the crossed (t-channel) process (16, p. 9, 17, 18)

$$A + \bar{C} \rightarrow \bar{B} + D, \quad (2.6)$$

where the anti-particles of B and C are denoted by $\bar{B}$ and $\bar{C}$, is also the invariant amplitude for the direct (s channel) process,

$$A + B \rightarrow C + D \quad (2.7)$$

The invariant scalar amplitude, F , is considered to be a function of the two invariant scalars s and t which are given by

$$\begin{aligned} s &= (p_A + p_D)^2 \\ t &= (p_A + p_C)^2 \end{aligned} \quad (2.8)$$

where the momenta, p_N , are defined by Figure 9. For the direct reaction (s channel), s and t have the following interpretations:

$$\begin{aligned} s &= (\text{center of mass energy})^2 \\ t &= -(\text{momentum transfer})^2 \end{aligned} \quad (2.9)$$

The large s, small negative t limit therefore corresponds to high energy, near forward scattering in the s channel. In the t channel the interpretations are interchanged:

$$\begin{aligned} s &= -(\text{momentum transfer})^2 \\ t &= (\text{center of mass energy})^2 \end{aligned} \quad (2.10)$$

Hence given the invariant amplitude $F(s, t)$ in the t channel, one can find $F(s, t)$ for the s channel by performing an analytic continuation in s and t from the kinematically allowed region of the t channel to the kinematically allowed region of the s channel.

In the t channel for the pion-nucleon system ($\pi\pi \rightarrow \bar{N}N$), the cosine of the scattering angle is given by

$$\cos\theta_t = \frac{s + p^2 + q^2}{2pq} \quad (2.11)$$

where $p(q)$ is the magnitude of the nucleon (pion) 3-momentum in the center of mass system, and t is given by

$$t = 4(p^2 + M^2) = 4(q^2 + \mu^2) \quad (2.12)$$

where $M(\mu)$ is the nucleon (pion) mass. When one evaluates $\cos\theta_t$ for $t = 0$ and large s, one finds

$$-\cos\theta_t = \frac{E}{\mu} \gg 1 \quad (2.13)$$

where E is the s channel incident pion laboratory energy. This limit and crossing symmetry relates the large $|\cos\theta_t|$ behavior of the t channel scattering amplitude to the s channel high energy forward scattering amplitude. For scattering functions that obey the Regge representation (Equation 2.2) with only pole singularities, and for

$$\cos\theta = \cos\theta_t \quad (2.14)$$

one finds the s channel high energy near forward scattering amplitude to be given by

$$F(s,t) \sim \sum_{n=1}^N \beta'_n (E/\mu) \alpha_n(t) \quad (2.15)$$

where the $\alpha_n(t)$ are the poles of the generalized partial wave coefficients for the t channel. Regge and de Alfaro (13, p. 67) related the $\alpha_n(t)$ to the resonances and bound states present in the t channel, so that each pole is associated with a particle with definite quantum numbers.

It is to be recalled that Regge's work applies to spinless particles interacting through a potential. In realistic high energy problems, there are no potentials and the particles have spin. Chew and Frautschi (19, 20) gave a set of conditions which provide a reasonable basis for applying the Regge results to realistic high energy problems. These conditions are the following:

1. The invariant scalar amplitudes obey the Mandelstam hypothesis (18), which states that the invariant amplitude, $F(s,t)$, is related to another scattering function $\bar{A}(s,t,u)$ by

$$F(s,t) = \bar{A}(s,t, \sum_{i=1}^4 m_i^2 - s - t) \quad (2.16)$$

where m_i is the mass of the i^{th} particle in the reaction. The new scattering function, $\bar{A}(s,t,u)$, has pole singularities in the s , t , and u planes corresponding to bound states in the s , t , and u^* channels and cuts in each plane from the (center of mass threshold energy)² of each channel to $+\infty$ along the real axis.

2. All Regge poles, $\alpha_n(t)$, in the ℓ -plane are functions of t only.

Condition 1 allows a representation of $\bar{A}(s,t,u)$ to be written as a dispersion relation, where the discontinuities across the cuts and the residues of the poles determine $\bar{A}(s,t,u)$. At $t = 0$, one obtains the forward dispersion relations which will be used later on as a constraint on the amplitudes. Condition 2 removes arbitrary functions which may occur if the dispersion relations are written in subtracted form.

Gell-Mann et al. (21) using the above conditions provided a general way to write the Regge amplitudes in the helicity formalism for reactions involving spin. However for clarity, the Reggization method developed by Singh (22) for the πN system will be used in what follows. Both methods are equivalent (22), although the final Gell-Mann formulae are more explicit.

Singh takes the Frazer and Fulco (23) partial wave expansions of the t channel ($\pi\pi \rightarrow \bar{N}N$) helicity amplitudes of Jacob and Wick (24) and Reggizes these amplitudes. The t channel helicity amplitudes are related to the s channel helicity amplitudes by a crossing matrix (25) which allows one

*The u channel is another channel related to the s and t channels by crossing symmetry.

to discover, via Wang's hypothesis (26), certain facts about the analytic structure of the helicity amplitudes. Essentially, Singh's amplitudes and the Wang hypothesis form the basis for the amplitudes used in this thesis.

Frazer and Fulco (23) start by considering the S matrix formula for the t channel, $\pi\pi \rightarrow \bar{N}N$,

$$S_{fi} = -i(2\pi)^4 \delta^4(p_1 + p_2 - q_1 - q_2) \frac{M}{(4E_1 E_2 \omega_1 \omega_2)^{1/2}} \bar{u}(p_2)^T v(p_1) \quad (2.17)$$

where the T matrix is given by

$$T = -A(s, t) + \frac{i}{2} (q_1 - q_2)_\nu \gamma^\nu B(s, t) \quad (2.18)$$

where $A(s, t)$ and $B(s, t)$ are the usual invariant scattering functions and the p's, q's, E's and ω 's are defined in Figure 10, the schematic diagram for the process. The final state spinor is $u(p_2)$ and the anti-nucleon spinor is $v(p_1)$.

In the t channel center of mass system, where

$$|\vec{p}_1| = |\vec{p}_2| = p \quad (2.19)$$

$$|\vec{q}_1| = |\vec{q}_2| = q \quad (2.20)$$

$$E_1 = E_2 = E \quad (2.21)$$

$$\omega_1 = \omega_2 = \omega \quad (2.22)$$

and θ_t is the angle between $\vec{p}_1$ and $\vec{q}_1$, Frazer and Fulco concluded that the partial wave expansion of the non-spin-flip, and spin-flip, amplitudes are given by

$$g_{++} = \frac{2}{p\sqrt{t}} \sum_{J=0}^{\infty} (J+1/2)(pq)^J f_+^J(t) P_J(\cos\theta_t) \quad (2.23)$$

$$g_{+-} = -\frac{1}{p} \sum_{J=0}^{\infty} (pq)^J f_-^J(t) \frac{d}{d\theta_t} P_J(\cos\theta_t) \quad (2.24)$$

In terms of the invariant scalars A and B, these amplitudes are given by

$$g_{++} = \frac{-pA(s,t) + MqB(s,t) \cos\theta_t}{4\pi\sqrt{t}} \quad (2.25)$$

$$g_{+-} = \frac{qB(s,t) \sin\theta_t}{8\pi} \quad (2.26)$$

Using the Mandelstam representation (18) Singh found that the $f_{\pm}^{J\pm}(t)$, which are $f_{\pm}^J(t)$ for J ($\begin{smallmatrix} \text{even} \\ \text{odd} \end{smallmatrix}$), obeyed the following expression:

$$f_+^{J\pm} = -\frac{p^2}{4\pi^2} \frac{1}{(pq)^{J+1}} \int ds' Q_J\left(\frac{s'+p^2+q^2}{2pq}\right) \left[A_s^{\pm}(s',t) - \frac{s'+p^2+q^2}{2p^2} B_s^{\pm}(s',t)\right] \quad (2.27)$$

$$f_-^{J\pm} = \frac{1}{8\pi^2} \frac{1}{(pq)^J} \int ds' B_s^{\pm}(s',t) \left[Q_{J-1}\left(\frac{s'+p^2+q^2}{2pq}\right) - Q_{J+1}\left(\frac{s'+p^2+q^2}{2pq}\right)\right] \quad (2.28)$$

where $J\pm$ is an ($\begin{smallmatrix} \text{even} \\ \text{odd} \end{smallmatrix}$) integer, and $A_s^{\pm}(s,t)$ and $B_s^{\pm}(s,t)$ are related to the poles and discontinuities across the s and u cuts of $A(s,t)$ and $B(s,t)$, and Q_J is a Legendre function of the second kind.

The ($\begin{smallmatrix} \text{even} \\ \text{odd} \end{smallmatrix}$) J partial wave coefficients $f_+^{J\pm}(t)$ and $f_-^{J\pm}(t)$ are found to have separate analytic continuations in J . Bose statistics, when applied to the incoming pion, states that the even J partial waves come from the isospin $I = 1$ interaction and that the odd J partial waves come from the $I = 0$ interaction. Singh then forms the amplitudes,

$$A'^{\pm} = -\frac{4\pi}{p^2} \sum_{J=0}^{\infty} (pq)^J (J+1/2) f_+^{J\pm}(t) [P_J(\cos\theta_t) \pm P_J(-\cos\theta_t)] \quad (2.29)$$

$$B^{\pm} = 4\pi \sum_{J=0}^{\infty} (pq)^{J-1} f_-^{J\pm} [P_J'(\cos\theta_t) \pm P_J'(-\cos\theta_t)] \quad (2.30)$$

and performs the Sommerfeld-Watson transformation (15) to obtain the Regge representations.

Singh then concludes that the small angle high energy s channel behavior is given by

$$A'^{\pm} \sim \sum_{\alpha_{\pm}} C_{\alpha_{\pm}}(t) \left(\frac{s}{2M\mu}\right)^{\alpha_{\pm}} \left(\frac{1 \pm e^{-i\pi\alpha_{\pm}}}{\sin \pi\alpha_{\pm}}\right) \quad (2.31)$$

$$B^{\pm} \sim \sum_{\alpha_{\pm}} \alpha_{\pm} D_{\alpha_{\pm}}(t) \left(\frac{s}{2M\mu}\right)^{\alpha_{\pm}} \left(\frac{1 \pm e^{-i\pi\alpha_{\pm}}}{\sin \pi\alpha_{\pm}}\right) \quad (2.32)$$

where $\alpha_{\pm}$ are the leading poles ($\text{Re}\alpha_{\pm} > -1/2$) in the J plane in the analytic continuation in J of $f_+^{J\pm}$ and $f_-^{J\pm}$. Both $f_+^{J\pm}$ and $f_-^{J\pm}$ have the same poles, $\alpha_{\pm}$, and the $\alpha_{\pm}$ are functions of t . In terms of these amplitudes the helicity amplitudes are

$$g_{++} = \frac{p}{4\pi\sqrt{t}} (A'^+ + A'^-) = \frac{p}{4\pi\sqrt{t}} A' \quad (2.33)$$

$$g_{+-} = \frac{q \sin\theta_t}{8\pi} (B^+ + B^-) = \frac{q \sin\theta_t}{8\pi} B \quad (2.34)$$

Trueman and Wick (25) relate g_{++} and g_{+-} to the s channel helicity amplitudes, h_{++} and h_{+-} by a crossing matrix $K(s,t)$ so that one has

$$\sqrt{s} \begin{pmatrix} h_{++} \\ h_{+-} \end{pmatrix} = \sqrt{t} K(s,t) \begin{pmatrix} g_{++} \\ g_{+-} \end{pmatrix} \quad (2.35)$$

Following Wang (26), one can analyze the crossing relation (Equation 2.35) under the assumption that the reduced helicity amplitudes of Gell-Mann (21) are free of kinematic singularities, and can conclude that the functions $C_{\alpha_{\pm}}(t)$ and $D_{\alpha_{\pm}}(t)/p^2$ appearing in Equations 2.31 and 2.32 are analytic functions of t .

One can obtain the differential cross section and polarization formulas for the s channel in terms of the t channel amplitudes. These formulas are

$$\frac{d\sigma}{dt} = \frac{1}{16\pi p_L^2} \left[(1 - t/4M^2) |A'|^2 - \frac{t}{4M^2} \left(\frac{4M^2 p_L^2 + st}{4M^2 - t} \right) |B|^2 \right] \quad (2.36)$$

$$P = \frac{-\sin\theta_s}{16\pi\sqrt{s}} \frac{\text{Im } A'B^*}{(d\sigma/dt)} \quad (2.37)$$

where p_L is the s channel incident pion laboratory momentum. These formulas and the theory described in this section provide the basic approach to the πN phenomenology used in this thesis.

III. REGGE POLE PHENOMENOLOGY

Since the Regge idea gives quite unspecific information about the t behavior of the high energy s channel amplitudes, it is at first glance quite unattractive. Physical intuition does not yield very much quantitative insight into the mysterious J -plane singularities, which in the t channel give rise to resonances. The only thing that one knows is that

$$\alpha(m_R^2) \sim J \quad (3.1)$$

where J is the angular momentum of a resonance of mass m_R in J^{th} partial wave. The residues $C_{\alpha_{\pm}}(t)$ and $D_{\alpha_{\pm}}(t)$ are completely undetermined by the theory. Actually these are consequences of not putting enough physics into the calculation. At the present time no one can do this, and so the Regge theory is almost devoid of physics, and is, to the physicist, an almost useless result. Unfortunately, the same criticisms apply to almost every other facet of high energy strong interaction physics.

As a result of this unfortunate circumstance, one becomes involved in self-consistent calculations, for which some things are measured and used to predict other phenomena (16, p. 9): A famous calculation of this nature in low energy physics is the Chew-Low calculation (27) of $\gamma N \rightarrow \pi N$ given the $\pi p \rightarrow \pi p$ amplitudes. Another success of this sort is the famous $SU(3)$ mass formula. By using most of the data known about the particle masses, the mass of the Ω^- was predicted (28). Then the Ω^- was found (29), and its mass was within a few per cent of the calculated value. Still another application of this philosophy was the Rarita and Phillips work (1), where, by an extremely precise fitting of Regge amplitudes to the $\pi^{\pm} p$ total and differential cross sections, they were able to predict the $\pi^- p$ elastic

polarizations to within a few per cent.

This paper of Rarita and Phillips is perhaps the cornerstone of Regge phenomenology. Although other people have applied the Regge idea to certain individual reactions (6, 30, 31), Rarita and Phillips performed a 27 parameter fit to the Kp and πp , elastic and charge exchange, differential and total cross sections. This large number of fixed parameters is not a severe criticism of Rarita and Phillip's work. Although charge and $SU(3)$ symmetries exist, there are 12 amplitudes or 2^4 real functions of energy and momentum transfer which must describe the cross section data over a 5 to 20 Bev range. Five Regge poles were introduced, and ten parameters were used to specify them according to

$$\alpha(t) \sim \alpha(0) + \alpha'(0)t \quad (3.2)$$

Table 1 shows the poles used by Rarita and Phillips for the πN interactions, and the quantum numbers and resonances associated with these poles.

Table 1. Dominant Regge poles in the πN system

Pole	Symbol	Resonance	Resonance Quantum Numbers: I^P, J	Trajectories
Pomeranchuk	P	f(1250)	$0, ^+2$	$1+.34t^*$
Pomeranchino	P'	f(1500)	$0, ^+2$	$.65+.34t^*$
Rho	ρ	$\rho(760)$	$1, ^-1$	$.57+1.01t^*$

* These values come from the work of Chiu, Phillips and Rarita (9), a later version of Rarita and Phillips (1).

The resonances and particles do not lie precisely on the linear trajectories, however one finds that

$$J(\text{resonance}) \sim \alpha(m_R^2) \quad (3.3)$$

It is to be noted that $\alpha(t)$ is determined by the cross section data in the region $t < 0$, and that the resonances begin to occur in the neighborhood of $t \sim -0.5 \text{ BeV}^2$. The P pole is used to describe the asymptotic value of the total cross section and the P' pole is needed for the approach to the asymptotic total cross section. The ρ pole explains the difference between π^+p and π^-p scattering. Using isospin conservation, the amplitudes for $\pi^\pm p$ elastic ($\pm$) and $\pi^-p \rightarrow \pi^0 n$ (CEX) reactions are given symbolically for the three pole model by

$$(A', B)_\pm = P + P' \mp \rho \quad (3.4)$$

$$(A', B)_{\text{CEX}} = -\sqrt{2}\rho \quad (3.5)$$

where P, P' and ρ denote the relevant pole contributions to the amplitudes. The work of Rarita and Phillips (1) and Chiu, Phillips and Rarita (9) shows that the elastic interactions are dominated by P and P' ($P, P' \gg \rho$) and that the spin flip amplitudes, B, for P and P' are small compared to the non-flip amplitudes, A. Also the spin-flip ρ amplitude is large compared to the non-flip ρ amplitude. These amplitudes are roughly consistent with the experimental data, except that, contrary to experimental fact (8), a zero charge exchange polarization is predicted.

This prediction occurs because the polarization depends on the phase difference of the spin-flip and non-spin-flip amplitudes according to

$$P \propto \sin(\varphi_{A'} - \varphi_B) \quad (3.6)$$

Since single Regge pole amplitudes have the same phase (modulo: π), the charge exchange polarization, depending only on the ρ pole, is zero. The charge exchange polarization can be made nonzero if a correction is made to the ρ pole according to

$$\rho_{\text{old}} \rightarrow (\rho_{\text{new}} + Z), \quad (3.7)$$

where

$$|Z/\rho| \ll 1 \quad (3.8)$$

In such a case, one expects very small changes in the elastic total and differential cross sections because of the dominance of P and P' . Nevertheless, there are measurements which are more sensitive to Z , such as the charge exchange differential cross section and the elastic polarization. The most sensitive measurement of Z is the charge exchange polarization.

There are essentially two ways to introduce a Z which may remedy the calculated charge exchange polarization without drastically altering the agreement of the other quantities with experiment. The first method takes advantage of the high energies involved in the 5 to 20 Bev region. The energy dependence of Z is chosen so that

$$\lim_{E \rightarrow \infty} \frac{Z}{\rho} \xrightarrow{\text{rapidly}} 0. \quad (3.9)$$

This corresponds to the addition of the tails of resonances which occur in the s channel to the various amplitudes as was done by Logan and Sertorio (32) and Desai et al. (33) among others. Another way to have this same sort of energy limit is to introduce another Regge pole with the ρ quantum numbers such that its trajectory, $\alpha_{\rho'}(t)$, is smaller than the ρ trajectory, $\alpha_{\rho}(t)$, so that one has

$$(E/E_0)^{\alpha_{\rho'}(t)} \ll (E/E_0)^{\alpha_{\rho}(t)} \quad (3.10)$$

Logan, Beaupre and Sertorio (10) did this using

$$\alpha_{\rho}(t) = .58 + .90t \quad (3.11)$$

and

$$\alpha_{\rho'}(t) = .17 + .90t \quad (3.12)$$

The second mechanism is to introduce a Z with essentially the same energy dependence as the ρ , so that one has

$$\frac{Z}{\rho} = \left(\begin{smallmatrix} \text{small, slowly} \\ \text{varying function} \end{smallmatrix} \right) e^{i\varphi(E,t)} \quad (3.13)$$

Two separate models have been introduced which give this sort of behavior for Z. Both involve Regge cut-like behaviors as given by

$$Z = \frac{(E/E_0)^{\alpha_c(t)} e^{i\varphi_Z}}{a + \ln(E/E_0)} \quad (3.14)$$

where for small t one has

$$\alpha_c(t) \sim \alpha_{\rho}(t) \quad (3.15)$$

The $\ln(E/E_0)$ term in the energy range 5 - 20 Bev is slowly varying compared to $(E/E_0)^{\alpha_c(t)}$. Cohen-Tannoudji (34) and Chiu and Finkelstein (11) both propose this sort of Z, but the theoretical bases for the two models are quite different. Cohen-Tannoudji proposed that their Z comes from a unitarity correction to the amplitude, while the Chiu and Finkelstein Z is caused by a Regge cut.

Using the charge exchange polarization and differential cross section data as the model criterion, the two best models of each class are:

1. The cut model of Chiu and Finkelstein (11). The "cut" of Cohen-Tannoudji gives a zero in the polarization at $t \sim .2 \text{ Bev}^2$, which is not consistent with the most recent data (8).

2. The extra pole (ρ') model of Logan, Beaupre and Sertorio (10). The models containing resonances (32, 33, 35) are quite difficult to justify. Some models (33, 35) involve infinite sequences of baryon resonances and others (32, 35) use a Breit-Wigner resonance form many resonance widths away from the resonance. It has been suggested that the Breit-Wigner tail does not represent the resonance in this region* and that the pure mixing of s and t channel resonances may not be satisfactory†. Since the resonance question is quite complex and the validity of the procedures used is questionable, the models discussed in this thesis will be limited to the more or less clean addition of another pole (10) or cut (11).

Since the ρ' model (10) has not been used to fit the elastic data, it may not be consistent with them; the cut (11) was, however, fitted with the elastic data. Since both the ρ' and the cut are Regge models one expects these models to obey the Mandelstam representation, which at the present time, is so basic in the derivation of the relativistic Regge amplitudes. There is, at present, no way to test the adherence of the phenomenological Regge amplitudes to the full Mandelstam representation, however,

* Durand, Loyal, III, Physics Department, University of Wisconsin, Madison, Wisconsin. Comments on Reggology. Private communication. April, 1967.

† Goldberger, M. L. Institute for advanced Studies, Princeton, N. J. Private discussion about resonances as reported by L. Sertorio. Private communication. January, 1967.

sum rules (12, 36, 37) relating the low energy cross sections to the high energy Regge parameters, have been derived from a limiting case of the Mandelstam representation. Both the ρ' and Chiu and Finkelstein cut ignored these sum rules and violated them (12). In this thesis the sum rule of Restignoli, Sertorio and Toller (37), which relates the difference of the π^+p and π^-p total cross sections to the πN coupling constant, will be applied to the $I = 1$ amplitudes as a constraint on the $\rho + \rho'$ and $\rho + \text{cut}$ parameters. In the next chapter these models and this sum rule will be discussed in greater detail. Then a fit will be made to the charge exchange and elastic data which incorporates the sum rule.

IV. THE ρ' , CUT AND SUM RULE

Besides offering a clear explanation of the πp charge exchange polarization, the ρ' seems to be an almost universal panacea. The following quotation from Logan, Beaupre, and Sertorio (10) offers a good summary of other reasons for introducing an additional Regge pole.

... we shall review the independent support (i.e., support from sources other than the πp CEX polarization) for the introduction of the ρ' . This support comes from a variety of sources. The first of these is a recent analysis of Høgaasen and Fisher¹¹ of high-energy, nucleon-nucleon charge-exchange scattering where they show the introduction of a ρ' is necessary to obtain a consistent Regge-pole description of the forward differential cross-section and the total cross-section data. Evidence for the ρ' arises from two recent unitary symmetry analyses. Nelson¹² has constructed a mass operator for the $SU(3)$ harmonics which gives the correct masses of the well-known 1^- nonet of mesons. This operator also predicts a 1^- meson with the quantum numbers of the ρ at 984 MeV. A recent $SU(3) \otimes SU(3)$ calculation of Bose¹³ also predicts a ρ' of mass 1000 MeV. Further evidence for the ρ' comes from the failure to explain the behavior of the nucleon isovector electromagnetic form with just a single ρ . The introduction of a ρ' with a mass of 890 MeV in one analysis¹⁴ and 1100 MeV in another¹⁵ eliminates this discrepancy.

¹¹H. Høgaasen and W. Fisher, Phys. Letters 22, 516 (1966).

¹²T. Nelson, private communication.

¹³S. K. Bose, Nuovo Cimento 46A, 419 (1966).

¹⁴L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. 141, 1298 (1966).

¹⁵E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. 139, 3458 (1965). Also see an earlier work where ρ' was first introduced: A. P. Balachandran, P. G. O. Freund, and C. R. Schumacher, Phys. Rev. Letters 12, 209 (1964).

This missing-mass spectrometer experiment of Kienzle et al.¹⁶ indicates a narrow state at 963 ± 5 with $I \geq 1$ which they call the δ . Allen et al.¹⁷ have tentative evidence for an $I \geq 1$ dipion state of unknown spin and parity at ≈ 965 MeV. Both the SU(3) and form-factor arguments for the ρ' predict a mass in this energy region. We shall assume, therefore, for the purposes of our analysis that the state (or states) observed by Kienzle et al.¹⁶ and Allen et al.¹⁷ is the ρ' . If we further assume that the ρ' trajectory has a linear t dependence with the same slope as the ρ , we must choose $\alpha_{\rho'}(0) = 0.17$ in order that $\alpha(t = 963^2) = 1.0$. Extrapolating this trajectory to $\alpha_{\rho'} = 3$, we expect the first recurrence of the ρ' to occur at a mass of 1770 MeV, which is very close to the third peak of the R which Focacci et al.¹⁸ observe at 1748 MeV.

¹⁶W. Kienzle, B. C. Maglic, B. Leurat, F. Lefebures, D. Freitag, and H. R. Blieden, Phys. Letters 19, 438 (1965).

¹⁷D. D. Allen, G. P. Fisher, G. Godden, L. Marshall, and R. Sears, Phys. Letters 22, 543 (1966).

¹⁸M. N. Focacci et al., in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (unpublished).

The fit of Logan, Beaupre, and Sertorio (10) involving the ρ' was restricted to $-t < 0.4 \text{ BeV}^2$ and so did not involve a complicated parameterization of the amplitudes.

The amplitudes are given by

$$f = \sum_{j=\rho, \rho'} \frac{-M_{\mu} b_1^j}{4\pi\sqrt{s}} \left(\frac{E}{\mu}\right)^{\alpha_j} (i + \tan \pi\alpha_j/2) \quad (4.1)$$

$$\tilde{f} = \frac{\mu}{16\pi} \sum_{j=\rho, \rho'} (b_1^j - \alpha_j b_2^j) \left(\frac{E}{\mu}\right)^{\alpha_j} (i + \tan \pi\alpha_j/2) \quad (4.2)$$

and the formulas for the experimental quantities are

$$\frac{d\sigma}{dt} = \frac{\pi}{q} [|f|^2 + \sin^2\theta |\tilde{f}|^2] \quad (4.3)$$

$$P = \frac{-2 \sin\theta \operatorname{Im} f \tilde{f}^*}{|f|^2 + \sin^2\theta |\tilde{f}|^2} \quad (4.4)$$

The four b_i^j parameters were fitted to the small t data and both $d\sigma/dt$ and P were in reasonable accord with experiment.

The cut investigated by Chiu and Finkelstein (11) does not occur because of a resonance in the t channel, but is due to the more complicated diagram (38, 39, 40) shown in Figure 11. The more simple pole diagram is also shown.

For small t and large s the cut contribution to the scattering amplitudes is of form (11),

$$A'_C = \frac{R(t)(E/E_0)^{\alpha_c(t)} [i + \tan(\pi\alpha_c(t)/2)]}{\ln(E/E_0) - i\pi/2} \quad (4.5)$$

$$B_C = 0 \quad (4.6)$$

where

$$\alpha_c(t) \sim \alpha_\rho(0) + \frac{\alpha'_\rho(0) \alpha'_P(0) t}{\alpha'_\rho(0) + \alpha'_P(0)} \quad (4.7)$$

and

$$R(t) = [\alpha_c(t) + 1] a_0^C e^{a_1 t} \quad (4.8)$$

Because of the largeness of the ρ spin-flip amplitude, Chiu and Finkelstein choose the cut to have only a non-spin-flip amplitude. Unless the cut spin-flip amplitude is abnormally large, the polarization is essentially determined by the product,

$$A'_C B_\rho^* \quad (4.9)$$

The Chiu Finkelstein cut is also in adequate agreement with the charge exchange polarization data.

Neither the ρ' nor the cut satisfy the sum rules (12, 36, 37). Sertorio and Toller (36) derive the sum rule for Regge poles by considering the $\pi^\pm_{p,I=1}$ forward amplitudes, $F_\pm$. If these amplitudes obey the forward dispersion relations (41), which are the $t = 0$ Mandelstam relations, one has

$$\frac{RF_\pm(E)}{E} = \frac{RF_\pm(\mu)}{\mu} \pm \frac{2f^2}{\mu^2} \frac{k^2}{E^2 - (\mu^2/2M)^2} + \frac{k^2}{\pi} \int_0^\infty \frac{\text{Im } F_\pm(E') dk'^2}{E' k'^2 (k'^2 - k^2)} \quad (4.10)$$

where k is the incident pion laboratory momentum and

$$E^2 = k^2 + \mu^2. \quad (4.11)$$

If the integral on the right hand side of Equation 4.10 is broken up into two parts, the first, an integral from 0 to K^2 , and the second from K^2 to ∞ , and the K^2 to ∞ part is evaluated by using the Regge form,

$$F_\pm(E) = \pm \frac{E}{k} \sum_{i=\rho,\rho'} a_i (k/k_0)^{\alpha_i} [i + \tan \pi \alpha_i / 2], \quad (4.12)$$

$$k_0 = 1 \text{ BeV}/c \quad (4.13)$$

one obtains, for $k > K$, the relation,

$$RF_\pm(E) = \frac{E}{k} \sum_{i=\rho,\rho'} a_i \tan(\pi \alpha_i / 2) (k/k_0)^{\alpha_i} + Y_\pm. \quad (4.14)$$

If one sets $Y_\pm$ equal to zero for the first few orders of k^{-n} one obtains the sum rule

$$\sum_{i=\rho, \rho'} \frac{4\pi a_i}{(\alpha_i+1)k_0} \left(\frac{K}{k_0}\right)^{\alpha_i+1} = \frac{4\pi}{k_0} V(K) \quad (4.15)$$

where

$$V(K) = \frac{1}{2} \int_{\mu}^{E(K)} \text{Im}[F_+(E) - F_-(E)] dE/k_0 + \pi k_0 f^2 (1 - \mu^2/4M^2). \quad (4.16)$$

The term $V(K)$ is calculated numerically in Reference 36 and involves, via the optical theorem, an integral over the differences of the total $\pi^\pm p$ cross sections and a term depending on the experimental coupling constant, f^2 . The whole procedure is, of course, dependent on how close the Regge amplitudes are to the real amplitudes at momenta larger than K . At $K \sim 5$ Bev/c, the real amplitudes seem to assume the Regge asymptotic form, and Restignoli, Sertorio and Toller (37) evaluate $\frac{4\pi V(K)}{k_0}$ for

$$K = 5 \text{ Bev/c} \quad (4.17)$$

and find it to be equal to $-18.6 \pm .2\text{mb}$.

The amplitude for the cut contribution (11), to $F_+(E)$ is given by

$$F_c \sim \frac{-i(\alpha_c+1)b_c}{\cos \pi\alpha_c/2} \frac{E}{k} \int_{-\infty}^{\alpha_c} (E/E_0)^J e^{-i\pi J/2} dJ \quad (4.18)$$

The addition of the cut to the ρ amplitude results in the following sum rule,

$$\frac{4\pi}{k_0} \frac{a_\rho}{\alpha_\rho+1} \left(\frac{K}{k_0}\right)^{\alpha_\rho+1} - \frac{4\pi}{k_0} b_c \left(\frac{K}{k_0}\right)^{\alpha_c+1} B(K/k_0, \alpha_c) = \frac{4\pi}{k_0} V(K) \quad (4.19)$$

where $B(K/k_0, \alpha_c)$ is derived in the appendix and is given by

$$B(X, \alpha) = \text{Im} \left[\frac{i + \tan \pi\alpha/2}{\ln X - i\pi/2} z e^z E_1(z) \right] \quad (4.20)$$

where

$$z = -(\alpha + 1)(\ln X - i\pi/2) \quad (4.21)$$

and $E_1(z)$ is the exponential integral function (42).

In terms of the sum rules for poles (Equation 4.15) and for a pole and a cut (Equation 4.19), b_c and a_ρ may be solved for in terms of $V(K)$, the ρ parameters, and α_c and α_ρ' , respectively. This fact will be used in the fitting process to be described in the next chapter.

V. THE AMPLITUDES AND CALCULATIONS

In this chapter the choice of amplitudes is discussed and the least squares fitting procedure is described. The form of the amplitudes relies heavily on the forms used in Reference 9, but minor changes are introduced. The relevant formulas for the differential cross-section and polarization are given by:

$$\frac{d\sigma}{dt} = \frac{(1 - t/4M^2)}{16\pi p_L^2} [|\mathcal{A}|^2 + \left(\frac{p_L q \sin\theta_s}{4M^2}\right) |\mathcal{B}|^2] \quad (5.1)$$

and

$$P = \frac{(1 - t/4M^2) \sin\theta_s \operatorname{Im}\mathcal{A}\mathcal{B}^*}{32\pi M/s (d\sigma/dt)} \quad (5.2)$$

where t and s are the usual invariant variables used in Chapter 3, and p_L and q are the pion s channel momenta in the laboratory and center of mass systems respectively. The t channel non-spin-flip and spin-flip amplitudes are proportional to $\mathcal{A}$ and $\mathcal{B}$, respectively, which, of course, depend on the model and process. If $\mathcal{A}$ (and hence $\mathcal{B}$) have units of (millibarns)^{1/2} the total cross-section in millibarns is given by

$$\sigma_T = \frac{\operatorname{Im}\mathcal{A}(t=0)}{1.608 p_L} \sqrt{\text{mb}} \text{ Bev/c} \quad (5.3)$$

Recalling Equations 3.4 and 3.5, the pole contributions to $\mathcal{A}$ and $\mathcal{B}$ for each process are given by

$$\mathcal{A}(\pi^+P \rightarrow \pi^+P) = A_0(P,P') - A_1(\rho,Z) \quad (5.4)$$

$$\mathcal{B}(\pi^+P \rightarrow \pi^+P) = B_0(P,P') - B_1(\rho,Z) \quad (5.5)$$

$$\mathcal{A}(\pi^-P \rightarrow \pi^-P) = A_0(P,P') + A_1(\rho,Z) \quad (5.6)$$

$$\mathcal{B}(\pi^-P \rightarrow \pi^-P) = B_0(P,P') + B_1(\rho,Z) \quad (5.7)$$

$$\mathcal{A}(\pi^-P \rightarrow \pi^0n) = -\sqrt{2} A_1(\rho,Z) \quad (5.8)$$

$$\mathcal{B}(\pi^-P \rightarrow \pi^0n) = -\sqrt{2} B_1(\rho,Z) \quad (5.9)$$

The parametrization of $A_0(P,P')$, $B_0(P,P')$, $A_1(\rho,Z)$ and $B_1(\rho,Z)$ is given by the following equations:

$$\begin{aligned} A_0(P,P') = & C_1 e^{C_{01}t} \alpha_1(\alpha_1+1) E^{\alpha_1} \zeta_+(\alpha_1) \\ & + C_2 e^{C_{02}t} \alpha_2(\alpha_2+1) E^{\alpha_2} \zeta_+(\alpha_2) \end{aligned} \quad (5.10)$$

$$\begin{aligned} B_0(P,P') = & D_1 e^{D_{01}t} \alpha_1^2(\alpha_1+1) E^{\alpha_1-1} \zeta_+(\alpha_1) \\ & + D_2 e^{D_{02}t} \alpha_2^2(\alpha_2+1) E^{\alpha_2-1} \zeta_+(\alpha_2) \end{aligned} \quad (5.11)$$

$$A_1 = C_3(\alpha_3+1)\zeta_-(\alpha_3) E^{\alpha_3} [(1+H)e^{C_{13}t} - H] + A_Z \quad (5.12)$$

$$B_1 = D_3\alpha_3(\alpha_3+1)\zeta_-(\alpha_3) E^{\alpha_3-1} e^{D_{13}t} + B_Z \quad (5.13)$$

where the additional subscripts 1, 2, 3 stand for the P , P' and ρ quantities respectively and

$$\zeta_{\pm}(\alpha) = - \frac{\Gamma(\alpha+3/2)}{\Gamma(\alpha+1)} \frac{e^{-i\pi\alpha_{\pm 1}}}{\sin\pi\alpha}, \quad (5.14)$$

$$\alpha_j = \alpha_j(0) + \alpha'_j(0)t \quad (5.15)$$

and where E is measured in Bev.

The difference between the $I = 0$, and $I = 1$, α dependent factors appearing in each term can be attributed to the sort of coupling each pole exhibits in the amplitude. The P and P' are chosen to couple to the $\pi\pi$ and $\bar{N}N$ states in a "sensible" way. These terms follow from a discussion of the angular momentum selection rules in the crossed channel. In the t channel $\pi\pi \rightarrow \bar{N}N$, $\bar{N}N$ in a triplet state, the pion angular momentum L is $J \pm 1$ where J is the angular momentum between the $\bar{N}$ and N . A nonsense transition would be the nonvanishing of the L partial wave in the $L = J - 1$ case as $J \rightarrow 0$. A sense transition would be the vanishing of this partial wave amplitude (43, 21). Since α is the analytic continuation of the angular momentum for the P and P' "resonances", the "resonances" presumably occur below the t channel threshold (11) one can choose P and P' to couple according to sense or nonsense. See References 43, 21, 44 and 45 for details and further references. The extra α in the B amplitudes comes from the asymptotic form of the Legendre functions, $P'_\alpha(Z)$ and $P_\alpha(Z)$, as does the ratio of the f functions in $\zeta_\pm(\alpha)$.

The factors e^{Ct} and e^{Dt} are needed to obtain good agreement with the data and are proportional to the analytic continuation of the t channel coupling constants. The equivalent ρ factor, $(1-H)e^{C_{13}^t t} - H$, explains the cross-over effect.*

The trajectories, $\alpha_1(t)$ and $\alpha_3(t)$, are the same as those given in Reference 9 for the P and ρ poles respectively. Scanio (46) using another sum rule found that the P' intercept, $\alpha_2(0)$ should be 0.69. The ρ'

* See Chapter 1 and Reference 1.

intercept, $\alpha_4(0)$, is a parameter to be determined by the fitting process. The slopes of the P' and ρ' trajectories, $\alpha_2'(0)$ and $\alpha_4'(0)$, are chosen to be equal to the P and ρ slopes respectively. The cut trajectory is the same as that given in Reference 11. The trajectory parameters are the following:

$$\alpha_1(t) = 1.00 + .34t \quad (5.16)$$

$$\alpha_2(t) = 0.69 + .34t \quad (5.17)$$

$$\alpha_3(t) = 0.57 + 1.01t \quad (5.18)$$

$$\alpha_4(t) = \alpha_4(0) + 1.01t \quad (5.19)$$

$$\alpha_c(t) = 0.57 + .26t \quad (5.20)$$

where t is measured in units of $(\text{Bev})^2$.

The A_Z and B_Z are the contributions to A_1 and B_1 which come from the second ρ or the cut. They are parametrized in the following way:

$$A_{\rho'} = (\alpha_4 + 1) C_4 e^{C_{14} t} E^{\alpha_4} \zeta_-(\alpha_4) \quad (5.21)$$

$$B_{\rho'} = \alpha_4(\alpha_4 + 1) D_4 e^{D_{14} t} E^{\alpha_4 - 1} \zeta_-(\alpha_4) \quad (5.22)$$

$$A_c = (\alpha_c + 1) C_5 e^{C_{15} t} E^{\alpha_c} (i + \tan \pi \alpha_c / 2) / (\ln E - i\pi/2) \quad (5.23)$$

$$B_c = 0 \quad (5.24)$$

If one applies the $V(K/k_0)$ sum rule to the $\rho + z$ amplitudes, one obtains the following sum rules,

$\rho + \rho'$:

$$C_4(K/k_0)^{\alpha_4(0)+1} \text{Im}\zeta_-(\alpha_4(0)) + C_3(K/k_0)^{\alpha_3(0)+1} \text{Im}\zeta_-(\alpha_3(0)) \\ = -4\pi V(K/k_0) \quad (5.25)$$

and

$\rho + \text{cut}$:

$$C_5(K/k_0)^{\alpha_c(0)+1} B(K/k_0, \alpha_c(0)) + C_3(K/k_0)^{\alpha_3(0)+1} \text{Im}\zeta_-(\alpha_3(0)) \\ = -4\pi V(K/k_0) \quad (5.26)$$

Using the $\alpha_j(0)$ values given by Equation 5.18 to 5.20, and Reference 37's evaluation of $V(K/k_0)$, for $K/k_0 = 5$, C_4 is determined via the sum rule by the values of the free parameters C_3 and $\alpha_4(0)$, and C_5 is determined by the free parameter C_3 .

The problem is to find the best values of the free parameters. Using the I.B.M. 360 computer and the least squares fitting program MINFUN (47) the free parameters of the charge exchange amplitude for each model were fitted first. The free parameters determined for the $\rho + \rho'$ charge exchange model were the ρ parameters: C_3 , H , D_3 , C_{13} and D_{13} , and the ρ' parameters: $\alpha_4(0)$, D_4 , C_{14} and D_{14} . For the cut model there is only one free cut parameter C_{15} . The ρ parameters are the same as those given above.

All of the above parameters are determined by the charge exchange data (8, 48). This in turn determines the A_1 and B_1 amplitudes for each model. Since for either a ρ' or a cut model A_1 and B_1 are determined by the charge exchange data, the fit to the elastic amplitudes involves only the determination of the P and P' parameters which occur in A_0 and B_0 .

The P and P' parameters determined by the elastic data (7, 49, 50) are for the P pole: C_1 , C_{12} , D_1 and D_{12} , and for the P' pole: C_2 , C_{22} , D_2 and D_{22} .

Since the data are not fitted all at once, a bias is introduced into the calculation. This is not a serious criticism of the calculation because the main purpose is to see if the charge exchange polarization data can be fitted using the sum rules and if also a reasonable fit to the elastic data could be obtained. Due to the abundance of the elastic data (190 points used in the calculation) and the scarcity of the charge exchange polarization data (13 points existent), another bias, based purely on the number of points, would be introduced into the calculation if it were a simultaneous fitting of all the parameters. Through sheer weight of numbers it would no longer make any difference whether or not the charge exchange polarization points were included or not in an all over calculation. A second reason for splitting up the problem into the charge exchange and elastic parts is that the amount of computer time needed to obtain any useful information from a run would increase by a factor between two and four. The present programs need from six to ten minutes to sufficiently vary the fitting parameters over a given region in parameter space, to find new minima or interesting smaller regions of parameter space.

In the charge exchange problem χ^2 , the function to be minimized is defined by

$$\chi^2 = \sum_{\text{Points}} \left| \frac{(\frac{d\sigma}{dt})_{\text{EXP}} - (\frac{d\sigma}{dt})_{\text{CALC}}}{\text{ERROR}} \right|^2 + \sum_{\text{Points}} \left| \frac{P_{\text{CALC}} - P_{\text{EXP}}}{\text{ERROR}} \right|^2 \quad (5.27)$$

For the elastic problem, χ^2 was defined by

$$\begin{aligned} \chi^2 = & \sum_{\pi^+, \pi^-} \sum_{\text{Points}} \left| \frac{(\frac{d\sigma^\pm}{dt})_{\text{EXP}} - (\frac{d\sigma^\pm}{dt})_{\text{CALC}}}{\text{ERROR}} \right|^2 \\ & + \sum_{\text{Points}} \left| \frac{P_{\text{EXP}} - P_{\text{CALC}}}{\text{ERROR}} \right|^2 \\ & + \sum_{\text{Points}} \left| \frac{(\sigma_T^\pm)_{\text{EXP}} - (\sigma_T^\pm)_{\text{CALC}}}{\text{ERROR}} \right|^2 \end{aligned} \quad (5.28)$$

The data used in the calculation are as follows:

13 P(CEX) points (8),

56 $\frac{d\sigma}{dt}$ (CEX) points (48),

93 $\frac{d\sigma}{dt}(\pm)$ points (50),

83 P($\pm$) points (6),

and 16 $\sigma_T(\pm)$ points (49).

The energy range of the data points varied between 5.9 Bev and 20 Bev, and $-t$ varied between 0 and .85 (Bev)².

VI. RESULTS AND CONCLUSIONS

Following the procedure outlined in the last chapter, the least squares fitting of the free parameters resulted in the charge exchange parameters for the $\rho + \rho'$ and $\rho + \text{cut}$ models which are listed in Table 2. The solutions are plotted with the charge exchange data in Figures 5 and 8.

Table 2. The $\rho + \rho'$ and $\rho + \text{cut}$ charge exchange parameters

<u>$\rho + \rho'$</u>		<u>$\rho + \text{cut}$</u>	
$\alpha_4(0)$	= .379		
C_3	= 2.328 $\sqrt{\text{mb}}$	C_3	= 2.284 $\sqrt{\text{mb}}$
C_4	= -.38* $\sqrt{\text{mb}}$	C_5	= -.321 [†] $\sqrt{\text{mb}}$
H	= 2.34	H	= 2.86
D_3	= 74.6 $\sqrt{\text{mb}}$	D_3	= 75.8 $\sqrt{\text{mb}}$
D_4	= 16.7 $\sqrt{\text{mb}}$		
C_{13}	= 1.56 (Bev) ²	C_{13}	= 1.15 (Bev) ²
C_{14}	= .059(Bev) ²	C_{14}	= .79 (Bev) ²
D_{13}	= -.0975(Bev) ²	D_{13}	= -.185(Bev) ²
D_{14}	= .896(Bev) ²		
$\chi^2(\frac{d\sigma}{dt})$	= 91.8	$\chi^2(\frac{d\sigma}{dt})$	= 94.4
$\chi^2(\text{total})$	= 116.0	$\chi^2(\text{total})$	= 116.7
69 points		69 points	

* In contrast, C_4 , calculated from model III of Reference 10, is 3.2 $\sqrt{\text{mb}}$ for $\alpha_4(0) = .17$.

[†] The number corresponding to C_5 in Reference 11 is -1.01 $\sqrt{\text{mb}}$.

The most striking result of the ρ' and cut fits to the charge exchange data is that both models give almost identical fits to the cross section and polarization data, and further that the calculated polarization is too low. (See Figure 8). In spite of the huge errors in the polarization, if one accepts the average polarization to be $10-15\% \pm 5\%$ from 6 to 12 Bev, two poles, and a pole and a cut models, constrained by a sum rule simply do not provide enough polarization. The polarization for this case is dominated by the $A_Z B_\rho^*$ term (60% in the ρ' case and 100% for the cut). One therefore has

$$P \sim \text{Im}(A_Z B_\rho^*) \propto C_Z D_3 \quad (6.1)$$

The sum rule constraint severely prohibits C_Z from becoming large enough to fit experimental polarization. In the cut case a reasonable fit to the polarization data could be obtained, if, holding B_ρ fixed, one increases C_5 by a factor of about three. The sum rule, which almost insures a good fit to the difference of the π^+p total cross section, doesn't allow a large enough charge exchange polarization.

The fits to the charge exchange differential cross section were almost identical for each model and in quite reasonable agreement with the data. The B_ρ term in the differential cross section supplied most of the cross section for $|t| < .4 \text{ Bev}^2$.

The elastic solutions are given in Table 3 and depicted in Figures 2, 3, 4, 6 and 7. The two solutions result from the two different best fits ($\rho - \rho'$ and ρ cut) to the charge exchange data. There is essentially no difference in the A_0 and B_0 parameters between the two models. Both sets of C_1 and C_2 parameters satisfy Scanio's sum rule to within 1%.

Table 3. The elastic parameters determined by the elastic data and the $\rho + \rho'$ or $\rho + \text{cut}$ charge exchange parameters of Table 2

	$\rho + \rho'$	$\rho + \text{cut}$
$C_1(\sqrt{\text{mb}})$	9.67	9.63
$C_2(\sqrt{\text{mb}})$	22.75	22.80
$D_1(\sqrt{\text{mb}})$	-7.97	-7.97
$D_2(\sqrt{\text{mb}})$	-22.99	-23.00
$D_{11}(\text{Bev}^2)$	1.685	1.688
$C_{12}(\text{Bev}^2)$	5.334	5.338
$D_{11}(\text{Bev}^2)$	4.993	4.993
$D_{12}(\text{Bev}^2)$	7.945	7.942
$\chi^2(\frac{d\sigma^\pm}{dt})$	125.2	122.2
$\chi^2(P^\pm)$	187.9	178.6
$\chi^2(\sigma_T^\pm)$	8.8	8.7
no. of points	190	190
$\chi^2(\text{total})$	322	310

The elastic best fit parameters, shown above, produce the interesting relation that the calculated elastic polarization is for most values of t , approximately given by

$$P^\pm \propto \pm \text{Im}(A_0 B_1^*) \quad (6.2)$$

In the fits to the elastic polarization data one sees (Figures 5 and 7) that, as a general rule one has

$$|P_{\text{CALC}}^\pm| \leq |P_{\text{EXP}}^\pm| \quad (6.3)$$

From Equation 6.1, if one could increase $|B_1|$, which is solely determined by the charge exchange data, the elastic x^2 coming from the polarization could be decreased. The result of increasing $|B_1|$, which lowers the elastic x^2 , would, however, probably have a more drastic effect on the charge exchange x^2 . This is true because most of the charge exchange cross section is proportional to $|B_1|^2$. Hence the all over x^2 , the sum of x^2 (elastic) and x^2 (charge exchange), would remain about the same or increase a little because of the increase of $|B_1|$.

The elastic experimental polarization data seem to obey

$$P^+ + P^- \sim 0 \quad (6.4)$$

except for the data at 6 Bev. Here the experimental π^+p polarization is significantly larger than P^- . Also, the agreement of the 6 Bev π^+p polarization with the calculated polarization is poor. One can imagine that either the data is incorrect (which is quite unlikely) or that an energy of 6 Bev is too low for the asymptotic-in-energy Regge theory to explain the finer features of πp elastic scattering. At 5 Bev, total and differential cross sections are well explained by Regge theories, but 10 Bev appears to be the threshold of agreement for present calculated quantities with experiment. It would then appear that because the experimental errors seem to increase with energy and momentum transfer that perhaps the asymptotic-in-energy Regge theory is only testable, with present machines, in the quite finite and limited 10 to 15 Bev range.

An alternative to the above pessimistic arguments, also within the context of a Regge theory, would be the addition of the s channel resonances to the Regge amplitudes for the P , P' , ρ poles and the ρ' pole or

cut. Conceivably if one knew the behavior of the resonance tails, or if one knew there were an infinite number of resonances, and if one could take this knowledge into account in the sum rule and amplitudes, one may be able to resolve the charge exchange polarization paradox. Finally one could also make an attempt to study the other non-asymptotic terms which are usually thrown away in phenomenological studies. The background integral* is bounded in energy by $(E/E_0)^{-1/2}$. A ρ pole gives a contribution of about $(E/E_0)^{1/2}$. At $E = 20$ Bev, $E_0 = 1$ Bev, the ratio of the possible background energy dependence to the energy dependence is about 5%, which, other things being equal, is a non-negligible contribution to the amplitude. Furthermore a background term would have no restrictions on its phase (as does a pole or cut) and so could have a small imaginary part (to be consistent with the sum rule) and a large real part, so that the difference in the phase between the background and the pole could interfere constructively to increase the polarization over that coming from a pole or cut. If the background contributes to both the spin-flip and non-flip amplitudes, the phases of the two background terms would not have to be the same, which may also cause a constructive phase interference.

Another source of criticism of the Regge theory was pointed out in Chapter III. Aside from the α factors and the energy dependence of a Regge pole or cut which are fairly well understood, the factors in the residues, which are in some sense the continuation of the resonance coupling constant, like e^{Ct} are not well understood. For the ρ pole, the natural energy scale factor occurs in the amplitude (6), and this amplitude is proportional to

*The first term in Equation 2.2.

$$\left(\frac{E}{\mu}\right)^{\alpha+\alpha't} = (\text{constant}) \left(\frac{E}{E_0}\right)^{\alpha+\alpha't} e^{Ct} \quad (6.5)$$

where μ is the pion mass. Therefore one has

$$C_3 \sim \alpha' \ln(\mu/E_0) = 1.77^* \quad (6.6)$$

However, this is true only for the ρ pole, and only for small t .

In spite of the criticisms, that the energies aren't high enough for a Regge theory, that important parts of the amplitude are missing which influence the more sensitive calculated quantities, and that the residues are mysterious, the Regge model is still the best one available. It does very well, in its clumsy multi-parametered way, to explain and correlate $\pi^\pm p$ total and elastic cross sections and high energy elastic polarizations, and charge exchange differential cross sections. It does a poor job on the lower energy polarizations, and the charge exchange polarization, if the data are correct, is a puzzle.

* This number is evaluated using the parameters of Reference 11.

VII. LITERATURE CITED

1. Phillips, R. J. N. and Rarita, W. Regge pole models for high energy πN , $K N$ and $\bar{K} N$ scattering. *Phys. Rev.* 139:B1336. 1965.
2. Krammer, M. and Maor, U. Regge pole parameters for the reaction $\pi^+ + p \rightarrow \pi^0 + \Delta^{++}(1238)$. [To be published in *Nuovo Cimento* ca. 1967].
3. Diddens, A. N., Jenkins, E. W., Kycia, T. F., and Riley, K. F. Structure in the pion-proton total cross section between 2 and 3 Bev. *Phys. Rev. Letters* 10:262. 1963.
4. Höhler, G., Ebel, G., and Giesecke, J. The pion-nucleon forward scattering amplitude. *Z. Physik* 180:430. 1964.
5. Van Hove, L. Theory of strong interactions of elementary particles in the GEV region. Unpublished mimeographed paper presented at the International Symposium on Electron and Photon Interactions at High Energies, Hamburg, June, 1965. Geneva, Switzerland, CERN [European Council for Nuclear Research]. 1965.
6. Logan, R. K. Single Regge pole analysis of $\pi^- p$ charge exchange scattering. *Phys. Rev. Letters* 14:414. 1965.
7. Borghini, M., Coignet, G., Dick, L., Kuroda, K., di Lella, L., Michalowicz, A., Macq, P. C., and Olivier, J. C. Polarization parameter in $\pi^\pm p$ and pp elastic scattering from 6 to 12 Bev/c. *Phys. Letters* 24B:77. 1967.
8. Bonamy, P., Borgeaud, P., Bruneton, C., Falk-Vairant, P., Guisan, O., Sonderegger, P., Caverzasio, C., Guillaud, J. P., Schneider, J., Yvert, M., Mannelli, I., Sergiampietri, F., and Vincelli, L. $\pi^- p \rightarrow \pi^0 n$ polarization at 5.9 and 11.2 Bev/c. *Phys. Letters* 23:501. 1966.
9. Chiu, C. B., Phillips, R. J. N. and Rarita, W. πN polarization and Regge poles. *Phys. Rev.* 153:1485. 1967.
10. Logan, R. K., Beaupre, J., and Sertorio, L. On the charge exchange polarization and the possibility of a second ρ meson. *Phys. Rev. Letters* 18:259. 1967.
11. Chiu, C. B. and Finkelstein, J. Suggestive features in πN charge exchange polarization associated with Regge cuts. *Nuovo Cimento* 48:820. 1967.
12. Igi, Keiji and Matsuda, Satoshi. New sum rules and singularities in the complex J plane. *Phys. Rev. Letters* 18:625. 1967.

13. De Alfaro, V. and Regge, T. Potential scattering. New York, N. Y., John Wiley and Sons, Inc. 1965.
14. Regge, T. Introduction to complex orbital momenta. Nuovo Cimento 14:951. 1959.
15. Sommerfeld, A. Partial differential equations in physics. New York, New York, Academic Press, Inc. 1966.
16. Chew, G. F. S-matrix theory of strong interactions. New York, N. Y., W. A. Benjamin, Inc. 1962.
17. Chew, G. F. Double dispersion relations and unitarity as the basis for a dynamical theory of strong interactions. In Screation, G. R., ed. Dispersion relations. pp. 167-226. New York, N. Y., Interscience Publishers, Inc. 1961.
18. Mandelstam, S. Determination of the pion-nucleon scattering amplitude from dispersion relations and unitarity general theory. Phys. Rev. 112:1344. 1958.
19. Chew, G. F. and Frautschi, S. C. Regge trajectories and principle of maximum strength for strong interactions. Phys. Rev. Letters 8:41. 1962.
20. Chew, G. F. and Frautschi, S. C. Principle of equivalence for all strongly interacting particles within the s-matrix framework. Phys. Rev. Letters 7:394. 1961.
21. Gell-Mann, M., Goldberger, M. L., Low, F. E., Singh, V., and Zachariasen, F. Elementary particles of conventional field theory as Regge poles. IV. Phys. Rev. 133:B145. 1964.
22. Singh, V. Regge poles in πN scattering and in $\pi + \pi \rightarrow N + \bar{N}$. Phys. Rev. 129:1889. 1963.
23. Frazer, W. R. and Fulco, J. R. Partial wave dispersion relations. Phys. Rev. 117:1603. 1960.
24. Jacob, M. and Wick, G. C. On the general theory of collisions for particles with spin. Ann. Phys. 7:404. 1959.
25. Trueman, T. L. and Wick, G. C. Crossing relations for helicity amplitudes. Ann. Phys. 26:322. 1964.
26. Wang, Ling Lie Chau. General method of constructing helicity amplitudes free from kinematic singularities and zeroes. Phys. Rev. 142:1187. 1966.

27. Chew, G. F. and Low, F. E. Theory of photomeson production at low energies. *Phys. Rev.* 101:1579. 1956.
28. Gell-Mann, M. [Discussion at the 1962 International Conference on High Energy Physics at CERN]. *International Conference on High Energy Physics, Genève, 1962*:805. c1962.
29. Barnes, V. E. et al. Observation of a hyperon with strangeness minus three. *Phys. Rev. Letters* 12:204. 1964.
30. Höhler, G., Baacke, J., Schlaile, H., and Sonderegger, P. Analysis of πN charge exchange scattering from 4 to 18 BeV/c. *Phys. Letters* 20:79. 1966.
31. Arbab, F. and Chiu, C. B. Association between the dip in the $\pi p \rightarrow \pi^0 n$ high-energy angular distribution and the zero of the p trajectory. *Phys. Rev.* 147:1045. 1966.
32. Logan, R. K. and Sertorio, L. Regge pole analysis of πp charge exchange polarization. *Phys. Rev. Letters* 17:834. 1966.
33. Desai, R. Bipin, Gregorich, David T., Ramachandron, R. πN charge exchange polarization and the baryon trajectories in the direct channel. Mimeographed. Riverside, California, Physics Department, University of California. 1967.
34. Cohen-Tannoudji, G., Morel, A., and Navelet, H. A phenomenological analysis of high energy data in πN and $K N$ systems: shadow scattering and Regge singularities. Mimeographed. Saclay, France, Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay. 1966.
35. Altarelli, G., Borgese, A., Buccella, F., and Colocci, M. Regge recurrences and πp charge exchange polarization at high energies. Mimeographed. Firenze, Italy, Istituto di Fisica dell' Università di Firenze. 1967.
36. Sertorio, L. and Toller, M. On the evaluation of the real part of the forward pion-proton scattering amplitude in a Regge poles approximation. *Phys. Letters* 18:191. 1965.
37. Restignoli, M., Sertorio, L. and Toller, M. Regge-pole phenomenology and forward dispersion relations. *Phys. Rev.* 150:1389. 1966.
38. Mandelstam, S. Cuts in the angular momentum plane. I. *Nuovo Cimento* 30:1127. 1963.
39. Mandelstam, S. Cuts in the angular momentum plane. II. *Nuovo Cimento* 30:1148. 1963.

40. Rothe, Heinz J. Study of branch points in the angular momentum plane. Ph.D. thesis. Mimeographed. U.S. Atomic Energy Commission Report UCRL-17068 [Berkeley. University of California. Lawrence Radiation Laboratory]. 1966.
41. Goldberger, M. L., Miyazawa, H., and Oehme, R. Application of dispersion relations to pion-nucleon scattering. Phys. Rev. 99:986. 1955.
42. Gautschi, W. and Cahill, W. F. Exponential integral and related functions. In Abramowitz, M. and Stegun, I. A., eds. Handbook of mathematical functions. pp. 228-294. Washington, D.C., U.S. Government Printing Office. 1965.
43. Gell-Mann, M. Applications of Regge poles. International Conference on High Energy Physics, Geneva, Switzerland, 1962:533-542. c1962.
44. Arbab, F., Bali, N. F., and Dash, J. Ambiguities in the phenomenological determination of Regge pole parameters. Mimeographed. U.S. Atomic Energy Commission Report UCRL-17325 [Berkeley. University of California. Lawrence Radiation Laboratory]. 1967.
45. Wang, Ling Lie. Regge-pole formulas for differential cross sections of quasi-two-body πN and NN interactions. Phys. Rev. 153:1664. 1967.
46. Scanio, Joseph J. G. New determination of the P' Regge-trajectory intercept. Phys. Rev. 152:1337. 1966.
47. Pankhurst, R. J. MINFUN-program to minimize a general function of n variables. Mimeographed. Geneva, Switzerland, CERN Program Library, CERN [European Council for Nuclear Research], 1964.
48. Sonderegger, P., Kirz, J., Guisan, O., Falk-Vairant, P., Bruneton, C., Borgeaud, P., Stirling, A. V., Caverzasio, C., Guillaud, J. P., Yvert, M., and Amblard, B. A secondary peak at $t = -1(\text{Bev}/c)^2$ in the high energy π^-p charge exchange scattering. Phys. Letters 20:75. 1966.
49. Galbraith, W., Jenkins, E. W., Kycia, T. F., Leontic, B. A., Phillips, R. H., Read, A. L., and Rubinstein, R. Total cross sections of protons, antiprotons and π and K mesons on hydrogen and deuterium in the momentum range 6-22 BeV/c. Phys. Rev. 138:B913. 1965.

50. Harting, D., Blackall, P., Elsner, B., Helmholtz, A. C., Middelkoap, W. C., Powell, B., Zacharov, B., Zanella, P., Dalpiaz, P., Focacci, N. M., Focardi, S., Giacomelli, G., Monari, L., Beaney, J. A., Donald, R. A., Mason, P., Jones, L. W., Caldwell, D. O. π^+P and P-P elastic scattering at 8.5, 12.4, and 18.4 Bev/c. Nuovo Cimento 38:60. 1965.

IX. APPENDIX

Sertorio and Toller (36) derived the sum rules for Regge poles only. In this appendix, their derivation is generalized so that the cut contribution is easily calculable, and can be included in the sum rule.

Let $a(k/k_0)$ be the actual π^+p , $I = 1$, $t = 0$ scattering amplitude, such that if the total cross section for π^+p scattering is written as

$$\sigma_T = \sigma(I = 0) + \sigma(I = 1) \quad (A.1)$$

then one has

$$\sigma(I = 1) = 4\pi a(k/k_0)/k \quad (A.2)$$

where k is the incident pion laboratory momentum, and k_0 is chosen to be 1 BeV/c. Let $r(k/k_0)$ be the corresponding Regge amplitude such that

$$k_0 | r(k/k_0) - a(k/k_0) | < \eta/y \ll 1 \quad (A.3)$$

for

$$k/k_0 \geq x \quad (A.4)$$

One parametrizes $r(k/k_0)$ for a pole and a cut by

$$r(k/k_0) = EA(k/k_0)/k \quad (A.5)$$

which for $k \geq 5$ BeV/c is equal to $A(k/k_0)$ to an accuracy of order 0.01%.

For the pole and cut used in this thesis, $A(k/k_0)$ is given by

$$A(y) = b(i + \tan\pi\alpha/2)y^\alpha + \frac{c(i + \tan\pi\beta/2)y^\beta}{\ln y - i\pi/2} \quad (A.6)$$

where

$$y = k/k_0. \quad (A.7)$$

By considering Cauchy's theorem one sees that

$$\frac{2\pi i r(\sqrt{y^2 + i\gamma})}{\epsilon(\sqrt{y^2 + i\gamma})(y^2 + i\gamma)} = \int_C \frac{r(\sqrt{x})dx}{\epsilon(\sqrt{x})x(x - y^2 - i\gamma)} \quad (\text{A.8})$$

where C is the contour shown in Figure 12, and

$$x = (k/k_0)^2 \quad (\text{A.9})$$

and

$$\epsilon(y) = (y^2 + \mu^2/k_0^2)^{1/2} \quad (\text{A.10})$$

Choosing x^2 to be the radius of the circular contour, C_1 , shown in Figure 12, one can derive a dispersion relation for $r(y)$ by having

$$\gamma \rightarrow 0^+ \quad (\text{A.11})$$

and

$$y > x. \quad (\text{A.12})$$

This dispersion relation is given by

$$\frac{\pi R r(y)}{\epsilon(y)y^2} = \frac{1}{2} \text{Im} \int_{C_1} \frac{A(\sqrt{x})dx}{x^{3/2}(x - y^2)} + P \int_{x^2}^{\infty} \frac{\text{Im} r(\sqrt{x + i0})dx}{\epsilon(\sqrt{x})x(x - y^2)}, \quad (\text{A.13})$$

where one makes use of the fact that along the $x = 0$ to $+\infty$ cut that

$$r(\sqrt{x + i0}) + r(\sqrt{x - i0}) = 2i \text{Im} r(\sqrt{x + i0}) \quad (\text{A.14})$$

It is known from Reference 41 that the actual amplitude $a(k/k_0)$ obeys

$$\frac{\pi R a(y)}{\epsilon(y)y^2} = \frac{\pi k_0}{y^2} Y(y) + P \int_0^{\infty} \frac{\text{Im} a(\sqrt{x + i0})dx}{\epsilon(\sqrt{x})x(x - y^2)} \quad (\text{A.15})$$

where

$$k_o Y(y) = \frac{Ra(\mu/k_o)}{\mu/k_o} + \frac{2f^2}{\mu^2/k_o} \frac{y^2}{\epsilon^2(y) - (\mu^2/2Mk_o)^2} \quad (A.16)$$

Using the basic assumption that Equation A.3 is valid for large y and by subtracting Equation A.13 from Equation A.15 one obtains

$$\begin{aligned} \frac{1}{4} \text{Im} \int_{C_1} \frac{A(\sqrt{x})dx}{x^{3/2}(1 - x/y^2)} = & -\frac{\pi k_o}{2} Y(y) + \frac{1}{2} \int_0^{x^2} \frac{\text{Ima}(\sqrt{x} + i0)dx}{\epsilon(x)x(1 - x/y^2)} \\ & + O\left(\frac{\eta}{k_o y^2}\right) \end{aligned} \quad (A.17)$$

Since we have that y is larger than x and consequently $\sqrt{x}$, one may expand Equation A.17 in powers of $1/y^2$. If the Regge amplitude really approaches the real amplitude the η term may be neglected. The coefficients of y^0 and y^{-2} are then given by

$$\frac{1}{4} \text{Im} \int_{C_1} \frac{A(\sqrt{x})dx}{x^{3/2}} = -\frac{\pi}{2} \left(\frac{Ra(\mu/k_o)}{\mu/k_o} + \frac{2f^2 k_o}{\mu^2} \right) + \frac{1}{2} \int_0^{x^2} \frac{\text{Ima}(\sqrt{x} + i0)dx}{\epsilon(x)x} \quad (A.18)$$

$$\frac{1}{4} \text{Im} \int_{C_1} \frac{A(\sqrt{x})dx}{\sqrt{x}} = \frac{\pi}{2k_o} f^2 (1 - \mu^2/rM^2) + \frac{1}{2} \int_0^{x^2} \frac{\text{Ima}(\sqrt{x} + i0)dx}{\epsilon(x)} \quad (A.19)$$

The quantities on the right hand side of Equations A.18 and A.19 are, respectively $W(x)$ and $V(x)$ of Reference 37. Numerically it is difficult to evaluate $W(x)$ accurately, so Equation A.18 is not used. The evaluation of the pole term in the left hand side of Equation A.19 is trivial. If one makes the substitution

$$x = x^2 e^{i\varphi} \quad (A.20)$$

and uses

$$\frac{(i + \tan \beta\pi/2)y^\beta}{\ln y - i\pi/2} = \frac{2ie^{-i\pi\beta/2}}{\cos\pi\beta/2} \frac{y^\beta}{\ln y^2 - i\pi} \quad (\text{A.21})$$

one find that the cut term becomes proportional to

$$x^{\beta+1} \text{Im} \int_{-2\pi}^0 d\varphi \frac{e^{i(\beta+1)(\varphi-\pi)/2}}{\ln x^2 + i(\varphi-\pi)} \quad (\text{A.22})$$

which, using Reference 42, is easily evaluated. The final result for the pole and cut is

$$\frac{bx^{\alpha+1}}{\alpha+1} + \frac{cx^{\beta+1}}{\beta+1} B(x, \beta) = V(x) \quad (\text{A.23})$$

where

$$B(x, \beta) = \text{Im} \left[\frac{i + \tan\pi\beta/2}{\ln x - i\pi/2} ze^z E_1(z) \right] \quad (\text{A.24})$$

where

$$z = -(\beta + 1)(\ln x - i\pi/2) \quad (\text{A.25})$$

and $E_1(z)$ is the exponential integral function.

X. FIGURES

Figure 1. Low energy πp total cross-sections. The $\pi^{\pm} p$ low energy total cross sections vs. energy (3).

Figure 2. High energy πp total cross sections. The data (49) and the ρ' best fit solution of this thesis of the $\pi^\pm p$ total cross sections are plotted versus energy. The cut solution was almost the same as the ρ' solution. The ρ' and the cut are first discussed in Chapter III.

Figure 3. High energy π^+p elastic cross sections. The data (50) and the best ρ' solution of this thesis for the π^+p differential cross sections are plotted versus momentum transfer for various energies. The cut solution was almost identical to the ρ' solution. The ρ' and the cut are first discussed in Chapter III.

Figure 4. High energy π^-p elastic cross sections. The data (50) and the best fit ρ' solution of this thesis for the π^-p elastic differential cross sections are plotted versus momentum transfer for various energies. The cut solution was almost identical with the ρ' solution. The ρ' and the cut are first discussed in Chapter III.

Figure 5. High energy charge exchange cross sections. The data (48) and the best fit ρ' solution of this thesis for the $\pi^-p \rightarrow \pi^0n$ differential cross section are shown as functions of energy. The cut solution was almost identical with the ρ' solution. The ρ' and the cut are first discussed in Chapter III.

Figure 6. High energy $\pi^+ p$ elastic polarizations. The data (7) and the best fit ρ' solution of this thesis are shown as functions of momentum transfer. There is essentially no difference between the ρ' and cut solutions. The ρ' and the cut are first discussed in Chapter III.

Figure 7. High energy $\pi^- p$ elastic polarizations. The data (7) and the best fit ρ' solution of this thesis are shown as functions of momentum transfer. There is essentially no difference between the ρ' and the cut solution. The ρ' and the cut are first discussed in Chapter III.

Figure 8. High energy charge exchange polarization. The data (8) and the best fit ρ' and cut solutions are shown as functions of momentum transfer. The ρ' and the cut are first discussed in Chapter III.

Figure 9. Schematic diagram for defining s and t . In this diagram all the particles momenta point towards the interaction. Thus in the physical reaction $A+B \rightarrow C+D$, p_C and p_D are the negatives of the physical momenta of C and D .

Figure 10. The t channel for the πN system. The physical momenta of the incoming and outgoing particles are defined.

Figure 11. The diagrams yielding the ρ pole and ρ - P cut. In the t channel the ρ pole is just another way of describing the resonance, as shown in the upper diagram. In the lower diagram, the kinematics and kernels of the X-like exchange at each end of the Regge pole "resonances" force the amplitude coming from this diagram to be essentially given by a convolution type of integral between the two Regge poles.

Figure 12. The contour used to derive the sum rules.