FERMILAB-PUB-05-433-AD

Synchrotron Frequency Spread Independence of Bunched-Beam Stochastic Cooling at the Fermilab Recycler

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Abstract

It is generally accepted that longitudinal stochastic cooling of bunched beams is not possible without a synchrotron frequency spread. Experiments in the Recycler storage ring (Fermilab) demonstrate the opposite: with an antiproton bunch in a parabolic potential well (no synchrotron frequency spread), the cooling was almost as efficient as in a trapezoidal potential well (with a relative synchrotron frequency spread of $\sim 100\%$). A possible explanation is that, at Recycler parameters, diffusion processes are sufficient to provide particle mixing.

PACS numbers: 29.20.-c, 29.27.-a

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I. INTRODUCTION

Bunched beam stochastic cooling has been experimentally demonstrated in ICE (CERN) and in the Antiproton Accumulator (Fermilab) [1, 2]. However, this method was not used in operations. In ICE, only 15000 antiprotons were successfully cooled in the bunched mode. In the Antiproton Accumulator, the beam was only partially bunched and a substantial amount of beam was outside of the RF bucket both before and after cooling. Bunched beam cooling was unsuccessfully attempted at both the CERN Sp$\overline{\text{p}}$S and the Fermilab Tevatron. A bunched beam stochastic cooling system is currently being developed for RHIC [3].

The synchrotron frequency spread is considered to be an essential parameter for the bunched beam stochastic cooling [4]. If this spread is too small, the mutual shielding of particles in a sample would make the individual particles nearly invisible to the stochastic cooling system, thus suppressing the cooling process as well as the thermal- and Schottky-noise diffusion. Accordingly, a maximal achievable cooling rate should be proportional to the synchrotron frequency spread. For an ideal parabolic potential well, there is no synchrotron frequency spread and cooling should not be possible. This conclusion sharply disagrees with recent observations at the FNAL Recycler antiproton storage ring.

II. OBSERVATIONS

The 3.3 km, 8.9 GeV/c Recycler ring is equipped with a broad-band RF system [5], capable of generating arbitrary RF waveforms up to several MHz with maximum amplitude of 4 kV peak-to-peak. Figure 1 and Figure 2 show the measured RF waveforms and bunch structures typically used in the Recycler ring during the longitudinal cooling process. Table 1 summarizes the main parameters of the Recycler storage ring and the specific parameters of the antiproton beam for these studies.

An experimental study was performed for comparison of cooling in a barrier and linear RF shapes, with the antiproton intensity of 30×10^{10}. The beam was first cooled in a barrier bucket that was 4.8 μs long and the RF voltage amplitude was 4 kV peak-to-peak (Figure 2). Beam momentum spread evolution during cooling is shown in Figure 3. The cooling process can be described as a time evolution of the energy spread,

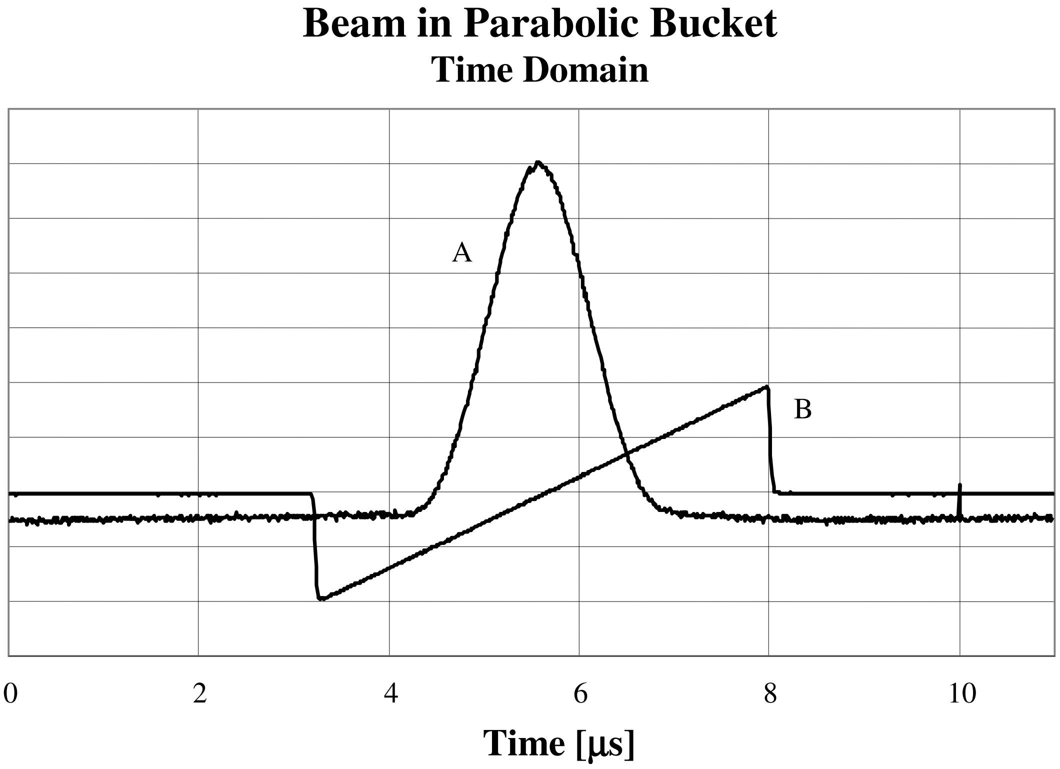

Figure 1: The measured longitudinal bunch distribution (A, arbitrary units) and the linear-ramp RF waveform (B, 2kV peak-to-peak) for antiprotons in the Recycler for one revolution period.

$$\sigma_E^2(t) = \left(\sigma_0^2 - \frac{D_{SC}}{2\lambda}\right) e^{(-2\lambda t)} + \frac{D_{SC}}{2\lambda}, \tag{1}$$

where σ_0 is the initial momentum spread, D_{SC} is the diffusion with stochastic cooling and λ is the cooling rate. For the barrier bucket beam cooling data of Figure 3, the fitted values are: $\lambda = 1.10\,\text{hour}^{-1}$, $D_{SC} = 17.83\,\text{MeV}^2/\text{hour}$. The final (equilibrium) energy spread is $(D_{SC}/2\lambda)^{1/2} = 2.85\,\text{MeV}$.

After the beam had reached an equilibrium, it was adiabatically placed within a linear RF waveform (parabolic potential well). The RF gradient was $0.417\,\text{kV}/\mu\text{s}$. At these parameters, the longitudinal beta-function $\beta_{\tau E}$ is $0.16\,\mu\text{s}/\text{MeV}$ ($\sigma_\tau = \beta_{\tau E}\sigma_E$), and the synchrotron period is 1.05s. After the adiabatic bunching, the bunch energy spread increased to 4.6 MeV. The beam continued to be cooled until a new equilibrium energy spread was obtained. Settings for the stochastic cooling were not changed during these operations. As it is seen in Figure 3, the final energy spread obtained for the parabolic bucket is 2.80 MeV, the same as with the barrier bucket RF. The parabolic-bucket cooling process can be parameterized similar to

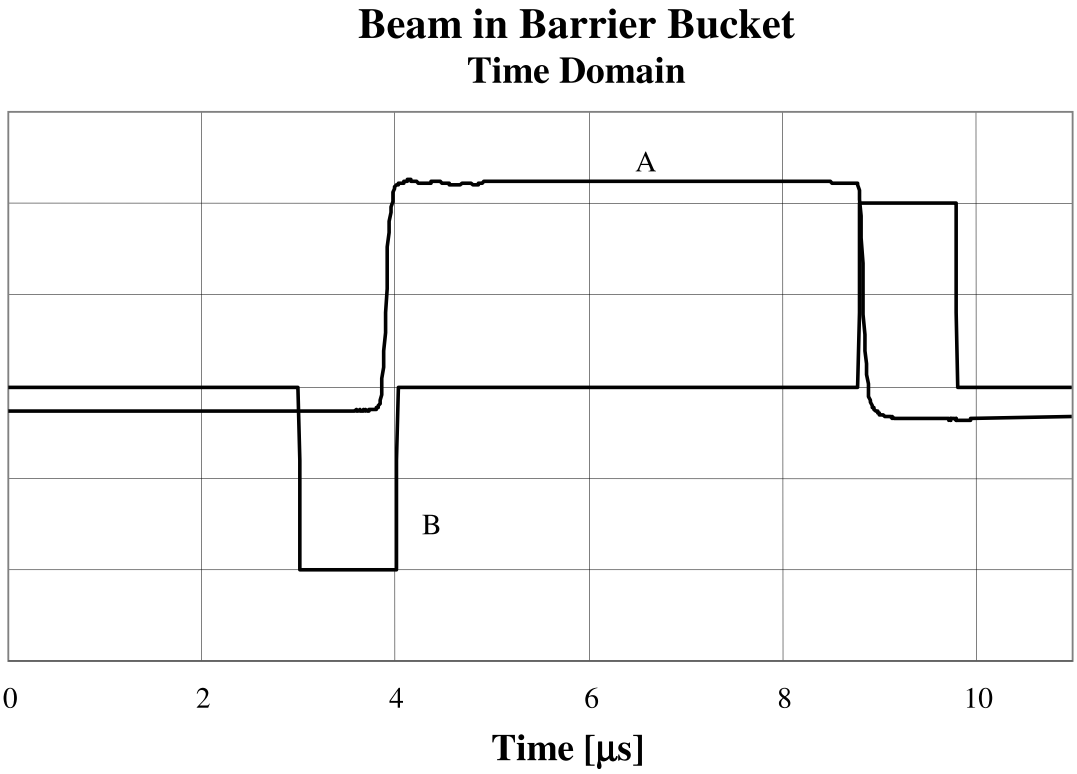

Figure 2: The measured longitudinal bunch distribution (A, arbitrary units) and the barrier-pulse RF waveform (B, 2 kV peak-to-peak) for antiprotons in the Recycler for one revolution period.

the barrier-bucket case:

$$\sigma_E^2(t) = \left(\sigma_0^2 - \frac{D_{SC}}{2\lambda}\right) e^{(-\lambda t)} + \frac{D_{SC}}{2\lambda}. \tag{2}$$

The difference between the barrier bucket Eq. (1) and linear bucket Eq. (2) parameterizations reflects a kinematic aspect: both cooling and diffusion rates of the harmonic oscillator are twice lower than the same sources give for the uniform motion. For the linear-bucket data of Figure 3, the fitted values are: $\lambda = 0.86\,\mathrm{hour}^{-1}$, $D_{SC} = 13.39\,\mathrm{MeV}^2/\mathrm{hour}$. The final (equilibrium) energy spread is $(D_{SC}/2\lambda)^{1/2} = 2.79\,\mathrm{MeV}$. The numerical fit errors do not exceed 1%.

The difference between the barrier and parabolic-bucket cooling rates is about 20%. Consideration of this difference would involve a detailed analysis of systematic experimental differences which is beyond the scope of this paper. Neglecting this difference, it can be stated that the cooling rate seen for the linear RF waveform (no synchrotron frequency spread) is approximately the same as for the barrier bucket, where the relative synchrotron frequency spread is $\sim 100\%$.

Table I: Main parameters

Parameter	**Symbol**	**Value**	**Units**
Recycler Ring			
Circumference	C	3320	m
Momentum	p	8.9	GeV/c
Slippage factor	η	-0.0086	
Emittance(n, 95%)	$\epsilon_{\rm n}$	$\sim 3-5$	μm
Average β-function	$\beta_{\rm ave}$	30	m
Momentum Cooling System			
Number of antiprotons	N	30	10^{10}
Bandwidth	W	1.5	GHz
Cooling time	τ	$\sim 1-2$	hour
Momentum spread, rms	σ_p/p	$3 \cdot 10^{-4}$	

In this paper we suggest that there is a mechanism, particularly effective for the Recycler ring, that makes stochastic cooling insensitive to the synchrotron frequency spread.

III. DIFFUSIVE RANDOMIZATION

A stochastic cooling system detects individual particles through macroscopic beam samples. If the sample randomizes too slowly, the particles have enough time to shield each other, becoming invisible to the stochastic cooling system, thus suppressing the cooling process. This is why efficient cooling requires that the sample randomization does not take longer than the time required for the cooling system to establish shielding. In an unbunched beam, this randomization naturally takes place as particles move with respect to each other due to the beam momentum spread. In a parabolic potential well, particles oscillate with the same synchrotron frequency so the related randomization process is infinitely long.

If one assumes that the synchrotron frequency spread is the only reason for the sample to be randomized, it would follow that this spread limits the maximal cooling rates. However, there are other sources for the sample randomization - microscopic diffusion processes.

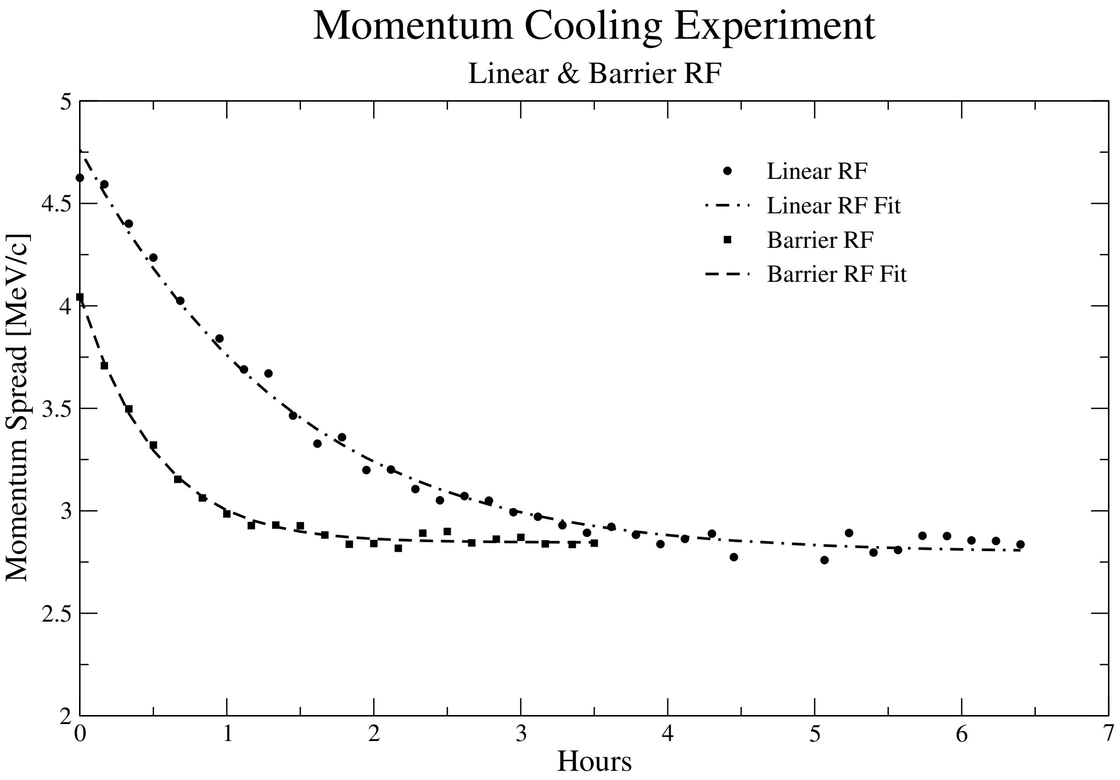

Figure 3: Evolution of the beam r.m.s. momentum spread for the linear and barrier RF buckets. Points are the measured data, the lines are the fits.

Because of the diffusion, particles randomly travel along the bunch, even if the potential well is perfectly parabolic. The cooling bandwidth is normally in a GHz range; thus, only a few centimeters of random walk moves a particle out of its initial sample. A dominant source of the longitudinal diffusion in intense beams is intrabeam scattering. In principle, RF noise and beam-gas scattering may contribute as well. It should be pointed out that the diffusion related to the stochastic cooling process itself (the pick-up thermal noise and the Schottky noise) is also suppressed by beam shielding since it acts on the beam through macroscopic beam samples. Thus, only the microscopic, single particle diffusion effects can lead to sample randomization.

A particle subject to diffusion, D, randomly walks from its unperturbed position, $\langle \delta s^2 \rangle = Dt$. This leads to a bunch length growth rate of $\tau_b^{-1} = \frac{1}{\sigma^2}\frac{d\sigma^2}{dt} = \frac{D}{\sigma^2}$. For sample lengths much less than the bunch length, $l \cong \frac{c}{2\pi W} \ll \sigma$, this diffusion rate causes a sample randomization rate of

$$\tau_{\text{sample}}^{-1} \cong \frac{D}{l^2} \cong \tau_b^{-1}\left(\frac{\sigma}{l}\right)^2 \gg \tau_b^{-1}. \tag{3}$$

If this diffusion-related randomization is faster than the synchrotron period, T, the particles of the initial sample will never assemble together regardless of the synchrotron frequency spread. Thus, the synchrotron frequency spread is insignificant for particle mixing if

$$\frac{T}{\tau_b}\left(\frac{\sigma}{l}\right)^2 \geq 1. \tag{4}$$

For the Recycler storage ring, this condition is satisfied with a substantial margin. Taking $T \cong 0.7\,\text{s}$, $\sigma \cong 100\,\text{m}$, $l \cong 5\,\text{cm}$ for Recycler beam in a parabolic bucket, longitudinal diffusion processes are not particularly constrained, $\tau_b \leq 777\,\text{hrs}$, by Eq. (4), which can also be rewritten as

$$\frac{\sigma}{l} \geq \left(\frac{c\eta(\sigma_p/p)\tau_b}{2\pi l}\right)^{1/3}. \tag{5}$$

In this form, it is clear that the significance of the diffusive randomization is mainly determined by the number of samples per bunch, $\frac{\sigma}{l}$. The right hand side of Eq. (5) contains parameters with a power of 1/3 and does not vary much from one machine to another. For the Recycler parameters and beam conditions in Figure 3, the bunch length growth rate from IBS is $\tau_b \simeq 3\text{hrs}$ (see, e.g., Refs. [6, 7]). The right hand side of Eq. (5) is then ~ 600, while the left hand side is an order of magnitude greater. In comparison, at nominal RHIC parameters [8, 9], the right hand side of Eq. (5) is then ~ 880, while the left hand side is an order of magnitude less.

When Eq. (4) is satisfied, the sample mixing occurs in less than a synchrotron period and stochastic cooling is essentially identical to the coasting beam cooling with the same phase space density. If Eq. (4) is not satisfied, the diffusive rate Eq. (3) is slower than the synchrotron period, $T = 1/f_s = 2\pi/\omega_s$, and has to be compared with the randomization rate due to the linear and non-linear synchrotron frequency spread, $(\sigma/l)\delta\omega_s$. Sample mixing would then be determined by the faster process of these two.

IV. CONCLUSIONS

For certain conditions, the synchrotron frequency spread can be insignificant for bunched beam longitudinal stochastic cooling. Namely, the synchrotron frequency spread plays no role when the sample randomization is determined by the diffusive motion of the cooled particles rather than the synchrotron frequency spread. This condition can be satisfied if the bunch length is much longer than the sample length.

The Recycler, with a bunch length of $\sim 100\,\mathrm{m}$ and a stochastic cooling sample length of$\sim$ 5cm meets this condition. Conversely, Eq. (4) is not satisfied in RHIC where the bunch length and stochastic cooling sample length are [8, 9], $\sim 30\,\mathrm{cm}$ and $\sim 6\,\mathrm{mm}$ respectively. For the RHIC stochastic cooling project, diffusive randomization is insignificant and the only source for mixing is the synchrotron frequency spread as implied in [9].

Acknowledgments

The authors are thankful to B. Chase and P. Joireman for providing the data shown in Figure 1 and Figure 2. The authors acknowledge useful discussions with V. Lebedev.

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