HEAT TRANSFER AND PRESSURE DROP CALCULATIONS:

MINIMUM ACTIVATION ALUMINUM BLANKET

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June 1975

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INTRODUCTION

The purpose of the present report is to consider such analysis aspects of the design of the fusion reactor cooling system for the blanket as are concerned with the transmission of heat in solid components of the blanket and the transfer of heat from those components to a coolant across the solid-fluid interface. In addition pressure drop effects are also taken into account. As a result it will be seen that it is possible to determine the temperature distribution in various parts of the blanket as well as the coolant streams. Such a knowledge is of importance, for example, since a particular temperature may set a limit on the thermal efficiency or the use of a particular structural material. The work is a documentation of assumptions made and methods used in the "reference" design calculations as well as proposed methods to be used in the "option" designs.

The report consists of sections on the water and the He coolant systems - pressure drop and heat transfer effects. Appendix A contains a discussion of proposed analysis methods for several option designs. Appendix B contains the detailed reference design calculations [1].

I. Water Coolant Systems

A. Pressure Drop Effects.

For the water coolant system as depicted in Figs. I-1 and I-2, it is assumed that there are an equal number of inlet as well as outlet ducts, all in parallel. The pressure drop, ΔP , in the region depicted B, for laminar or turbulent flow is assumed to be given by

$$\Delta P = f \rho \frac{v^2 L}{2gD} \quad (I-1)$$

where f is the friction factor and v , the mean velocity. Equation I-1 assumes a fully-developed flow situation.

The mean velocity, v , is determined as follows: for each module knowing the amount of heat deposited in the aluminum due to neutron and gamma heating as well as the bremsstrahlung radiation and heat deposited in the water, we calculate the mass flow rate, m

$$m = \frac{\Delta Q_1}{\Delta h} \quad (I-2)$$

where Δh is the change of enthalpy, $h_{ex} - h_{in}$, and ΔQ is the total power (BTU/hr) deposited in the aluminum. Here h_{ex} is the exit and h_{in} the inlet enthalpies. These values are fixed by fixing the exit and inlet temperatures. Actually ΔQ should be modified, in this case increased, so as to account for the heat from the insulator and aluminum distributor plate. In other words as heat flows from the insulator and aluminum distributor plate to the helium stream, this serves to increase the temperature of the aluminum shell in contact with the helium. If sufficient water cooling of the aluminum is available the helium may be kept

at virtually the He inlet temperature on the downward pass of the helium in the annular region. As we note in a later section we make an approximation in calculating the heat leak.

Knowing m , from the conservation of mass we can determine the velocity v

$$v = \frac{m}{A\rho} \quad (I-3)$$

where $A = \pi \frac{D^2}{4}$ and ρ is the density. The density ρ is evaluated at the arithmetical mean of inlet and outlet temperatures. We are now in a position to calculate the Reynolds number, Re ,

$$Re = \frac{vD\rho}{\mu}$$

to establish whether the flow is laminar or turbulent.

The piping across the elliptical dished head is complicated in that the ducts must cross over. Consequently to calculate the pressure drop across the dished head we employ an equivalent system as depicted in Fig. I-3. The head loss, increased in fully-developed pipe flow through bends as a result of secondary flows, is expressed most conveniently by an equivalent length of straight pipe. The equivalent length depends on the relative radius of curvature of the bend. The pressure drop/bend may be expressed as

$$\Delta p = f \frac{l_{eq}}{D} \rho \frac{v^2}{2g} \quad (I-4)$$

for a given $\frac{r}{D}$, i.e., $\frac{l_{eq}}{D}$ is a function of $\frac{r}{D}$.

For entrance and exit pressure loss effects

$$\Delta p = K_{en} \rho \frac{v^2}{2g} \quad (I-5)$$

and

$$\Delta p = K_{ex} \rho \frac{v^2}{2g} \quad (I-6)$$

respectively.

Fig. I-3 also depicts the equivalent water coolant circuit used for calculational purposes for estimating pressure drop effects for water fed from the headers to the module and then exiting. The total pressure drop is given by

$$\begin{aligned} \frac{\Delta p}{\rho} = f \left(\frac{L_1 + L_2 + L_3 + L_4 + L_5}{D} \right) \frac{v^2}{2g} + 6f \frac{l_{eq}}{D} \frac{v^2}{2g} + \\ + K_{en} \frac{v^2}{2g} + K_{ex} \frac{v^2}{2g} = \left[f \left(\frac{L_1 + L_2 + L_3 + L_4 + L_5}{D} \right) + 6f \frac{l_{eq}}{D} + \right. \\ \left. + K_{en} + K_{ex} \right] \frac{v^2}{2g} \quad (I-7) \end{aligned}$$

B. Heat Transfer Effects

Of prime importance in the design of the module is to know if the aluminum shell can be sufficiently cooled as a consequence of internal heat generation.

a. Mixed-Mean Fluid Temperature

If the quantity $\frac{A}{m}$ is independent of x , the longitudinal distribution

of the coolant fluid mixed-mean temperature, T_f , may be determined from

$$h_f - h_o = \frac{\Delta Q_2}{m} \quad (I-8)$$

where ΔQ_2 is the net heat, actually heat rate, convected to and deposited in the water (heat from the aluminum as well as from neutron and gamma heating deposited in the water) over a given length x and h_f is the coolant fluid mixed-mean enthalpy. The fluid entrance enthalpy, h_o , at $x = 0$, is chosen as the datum. Equation I-8 gives the increase in the coolant enthalpy, or equivalently, the coolant temperature due to the heat added as the water flows along the channel. The heat deposition rates are given in Table 1.

b. Estimate of Pipe Wall Temperature

The local (solid) surface or wall temperature, T_s may be related to the mixed-mean fluid temperature, T_f , by

$$\frac{dq}{dA_h} = h (T_s - T_f) \quad (I-9)$$

where h is the heat transfer coefficient for the given conditions, dA_h is a small element of heat-transfer area, and dq , the heat flux at the wall. It will be assumed that h is independent of x , which is the case for ducts having a large ratio of length to diameter. If p_r is the passage perimeter, then $dA_h = p_r dx$, and Equation I-9 may be written as

$$dq = hp_r (T_s - T_f) dx. \quad (I-10)$$

T_s and T_f vary with x . Over a given length L we define an average value of T_s as well as T_f

$$\bar{T}_s \equiv \frac{1}{L} \int_L T_s dx, \quad \bar{T}_f \equiv \frac{1}{L} \int_L T_f dx \quad (I-11)$$

so that

$$\bar{T}_s = \bar{T}_f + \frac{1}{hpL} \int_L dq \quad (I-12)$$

To determine h along smooth ducts we employ the following Nusselt number relationship:

$$Nu = 0.023 Re^{0.8} Pr^{0.333} \quad (I-13)$$

where Pr , the Prandtl, $= \frac{\rho c}{k}$. Here c is the specific heat and k , the thermal conductivity. From the definition of Nu

$$Nu = \frac{hD}{k} \quad (I-14)$$

we find

$$h = 0.023 \frac{k}{D} Re^{0.8} Pr^{0.333} \quad (I-15)$$

C. Estimate of First Wall Aluminum Temperature

To obtain some estimate of the temperature of the aluminum which faces the plasma, we consider the following one-dimensional conduction model

which is consistent with the one-dimensional neutronics calculations. For reference see Figure I-4.

Assuming the one-dimensional heat conduction model with heat generation taken as an average over the length of interest, we have

$$\frac{d^2 T}{dx^2} = -\frac{Q}{k} \quad (I-16)$$

subject to boundary conditions

$$x = 0: -k \frac{dT}{dx} = P_o \quad (I-17a)$$

$$x = L: T = \bar{T}_s \quad (I-17b)$$

The solution to Equation I-16 is

$$T - \bar{T}_s = -\frac{Q}{k} \frac{(L-x)^2}{2} + \left(\frac{QL}{k} + \frac{P_o}{k} \right) (L-x) \quad (I-18)$$

The maximum value of T is obviously at $x = 0$:

$$T)_m - \bar{T}_s = \frac{L}{k} \left(\frac{LQ}{2} + P_o \right), \quad (I-19)$$

i.e., the value of the temperature of the aluminum which faces the plasma.

II. Helium Coolant System

A. Pressure Drop Effects

For the helium coolant system as depicted in Figure I-5 the helium flows downwards (or upwards depending on the location of the module) through an annulus, is turned as a consequence of the slope of the module and flows upwards (downwards) through the distributor plate, nonfluidized packed bed, and then pass the graphite rods.

The pressure drop in Region B for turbulent flow is assumed to be given by

$$\Delta p = f \frac{v^2 L}{2gD_{eq}} \quad (II-1)$$

where f is the friction factor and D_{eq} , the equivalent or hydraulic diameter.

In this case

$$D_{eq} = 4 \frac{\text{flow cross-section area}}{\text{wetted perimeter}}$$

so that for an annulus formed between two concentric ducts we have

$$D_{eq} = 4 \frac{(\pi/4)(D_2^2 - D_1^2)}{\pi(D_1 + D_2)} = D_2 - D_1 \quad (II-2)$$

The helium velocity is determined once the mass flow rate is known. The mass flow rate is first determined by considering the heat deposited in the module less that deposited in the aluminum and water. Assuming fixed inlet and outlet temperatures, T_{in} and T_o , respectively the mass flow rate is

$$m = \frac{\Delta Q}{c_p (T_o - T_{in})} \quad (II-3)$$

where ΔQ is the heat deposition rate and c_p the specific heat at constant pressure.

Knowing m the velocity is

$$v = \frac{m}{A\rho} \quad (\text{II-4})$$

where the area $A = \frac{\pi}{4} (D_2^2 - D_1^2)$ and ρ , the density evaluated at the inlet temperature. The Reynolds number, necessary to establish whether the flow is laminar or turbulent, is given by

$$Re = \frac{v D_{eq} \rho}{\mu} .$$

For the case of flow through the nonfluidized packed bed region, we assume for the pressure drop calculation the empirical relation

$$\frac{\Delta P}{L} \frac{d_p}{\rho V_o^2} \frac{\epsilon^3}{1-\epsilon} = 150 \frac{1-\epsilon}{Re_p} + 1.75 \quad (\text{II-5})$$

where L is the bed height, d_p the diameter of assumed spherical particles, ϵ is the porosity, ρ the density and V_o the velocity which the fluid would have if no particles were present (the approach velocity). The Reynolds number is based on this approach velocity V_o and the particle diameter

$$Re_p = \frac{V_o d_p \rho}{\mu}$$

Equation II-5 holds only as long as the diameter of the whole bed is larger than at least 10 times the particle diameter. This condition is met in our design. Since a significant amount of heat is deposited in the bed region thereby causing a large change in helium temperature across the bed, average values of properties based on the arithmetical mean of inlet and outlet temperature of the bed are used in the above calculation for pressure drop.

The approach velocity is determined as follows: assuming that the distributor plate through which the He passes is made up of a large number of passages, the He velocity is calculated as if it passes through a duct of diameter, d . See Figure I-6. From the conservation of mass, assuming that the density effect is small compared with the area change, the He velocity through the distributor plate is given by

$$v_{in} A_{in} = v_{dist} A_{dist} \quad (II-6)$$

so that

$$v_{dist} = \frac{v_{in} A_{in}}{A_{dist}} \quad (II-7)$$

where v_{in} and A_{in} are the inlet velocity and inlet area (beginning of the annular region), respectively and v_{dist} and A_{dist} are the He velocity through the distributor plate and area of the distributor plate, respectively. Actually v_{dist} should be slightly greater than the velocity as calculated above as a consequence of finite pores or ducts through the plate. The velocity as given by Equation II-7 is also the approach velocity for the bed pressure drop calculations.

The pressure drop calculation through the graphite rod region is again determined on the basis of fully developed flow, either laminar or turbulent depending on the Reynolds number. An equivalent diameter is used in the calculations which is a function of the flow passage shape. Actually the flow is probably not developed as it comes out of the bed region,. Nevertheless, the fully developed flow assumption should give a good estimate or rather the correct order of magnitude for the pressure drop. The pressure drop expression is given by Equation II-1 while the passage shape and equivalent diameter is discussed in Appendix B.

Knowing the voidage fraction in the graphite region, the inlet velocity to this region can be determined. From conservation of mass

$$v_{\text{dist}} A_{\text{dist}} = v_{\text{rod}} A_{\text{rod}} = v_{\text{rod}} j A_{\text{dist}} \quad (\text{II-8})$$

so that

$$v_{\text{rod}} = \frac{v_{\text{dist}}}{j} \quad (\text{II-9})$$

where v_{dist} and A_{dist} are the He velocity and area of the distributor plate ($\pi \frac{d^2}{4}$), respectively and v_{rod} is the inlet velocity to the graphite region while j is the voidage fraction.

B. Heat Transfer Effects

The He coolant is introduced so as to cool the hot neutron multiplying zone as well as the stacked graphite rod region. In addition the bred tritium diffuses into the He coolant stream.

a) Mixed-Mean Fluid Temperature

If the quantity $\frac{A}{m}$ is independent of x , the longitudinal distribution

of the helium coolant mixed-mean temperature, T_f , may be determined from

$$T_f - T_o = \frac{\Delta Q}{c_p m} \quad (\text{II-10})$$

where ΔQ is the net heat, actually heat rate, convected to the He over a given length x . We assume that heat deposited in the helium is negligible. T_f is the coolant fluid mixed-mean temperature. The He entrance temperature, T_o , at $x = 0$, is chosen as the datum. Equation II-10 gives the increase in coolant temperature due to the heat added as the He flows along the channel.

An estimate of the increase in He temperature as a consequence of heat convected to the stream from the insulator and Al distributor plate can now be made. In turn as the He temperature increases and becomes greater than the Al shell temperature heat is transferred to the Al shell. Equation II-10 may be used to estimate the He temperature increase across the Al distributor plate at the bottom of the module, through the bed region, and graphite rod region.

b) Heat Transfer Coefficient Estimate

To determine the heat transfer coefficient, h , along smooth ducts we employ the following Nusselt number relationship (assuming we have turbulent flow):

$$\text{Nu} = 0.021 \text{Re}^{0.8} \text{Pr}^{0.4} \quad (\text{II-11})$$

where Re is the Reynolds number, Pr the Prandtl number $= \frac{\rho c_p}{k}$. Here c_p is the specific heat at constant pressure and k , the thermal conductivity. From the definition of Nu

$$\text{Nu} = \frac{h d_{eq}}{k} \quad (\text{II-12})$$

we find

$$h = 0.021 \frac{k}{d_{eq}} Re^{0.8} Pr^{0.4} \quad (II-13)$$

Across the bed region it is assumed that

$$Nu = 0.8 Re_p^{0.7} Pr^{0.333} \quad (II-14)$$

so that

$$h = 0.8 \frac{k}{d_p} Re_p^{0.7} Pr^{0.333} \quad (II-15)$$

c) Estimate of Temperature in Graphite Rods

To obtain an estimate of the maximum temperature in the graphite rods, we assume a cylindrical geometry of radius r_0 with an average source $\bar{Q}$ surrounded by a fluid at coolant temperature T_f . See Figure I-7. For heat conduction in the radial direction, the general solution is

$$T = \frac{\bar{Q}r^2}{4k} + C_1 \ln r + C_2 \quad 0 \leq r \leq r_0 \quad (II-16)$$

The boundary conditions we impose are

$$r = 0: \quad \frac{dT}{dr} = 0 \quad (II-17a)$$

and

$$r = r_0: \quad T = T_1 \quad (II-17b)$$

so that the appropriate solution is

$$T = T_1 + \frac{\bar{Q}}{4k} (r_0^2 - r^2) \quad (\text{II-18})$$

where T is the temperature at the radial distance r . If T_0 is the temperature along the central axis, where $r = 0$, then

$$T_0 - T_1 = \frac{\bar{Q} r_0^2}{4k} \quad (\text{II-19})$$

for the temperature drop across the rod itself. Note that T_0 is also the maximum temperature.

T_1 is related to T_f by

$$T_1 = T_f + \frac{\Delta Q}{hA_h} \quad (\text{II-20})$$

where T_1 , T_f , and ΔQ are taken to be averages over a given length x . A_h is the heat transfer area and h the heat transfer coefficient.

d) Heat Leak Estimate

To obtain an estimate of the heat which flows as a consequence of temperature difference from the hot insulator and Al distributor plate to the "colder" Al shell we consider the radial heat flow through concentric cylinders of different thermal conductivity without internal heat generation. See Figure I-8. In the steady state, the rate of heat flow through each section is the same. The resulting expression for the rate of heat flow through two concentric cylinders then becomes

$$q = \frac{T_i - T_o}{\frac{1}{2\pi R_i \ell h_i} + \frac{1}{2\pi k_{12} \ell} \ln \frac{R}{R_i} + \frac{1}{2\pi k_{23} \ell} \ln \frac{R_o}{R} + \frac{1}{2\pi R_o \ell h_o}} \quad (\text{II-21})$$

A typical temperature profile across the solid structure on which Equation II-21 is based might look like that of Figure I-9. The important point is that the direction of heat flow is from the high temperature zone to the lower temperature zone.

Actually the heat flow situation including internal heating is more complicated. Depending on the magnitude of the internal heat generation, heat may be transferred through both faces at different rates. In addition a fraction of the total generated heat is added to or subtracted from the nongeneration heat flow depending on the surface in question. These features will be made clearer in the next section.

e) Analysis of Insulator Region

As was noted in the previous section the analysis to be presented is an attempt to refine the heat leak calculation as well as assess the approximations made. Certain simplifications are still made, though, in the analysis.

Radial heat flow through concentric cylinders of different thermal conductivity as well as internal heat generation is assumed. Referring to Figure I-8, the one-dimensional radial heat conduction equations with internal heat generation are

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{u_1 r}{k_{12}} = 0 \quad R_i \leq r \leq R \quad (\text{II-22a})$$

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{u_2 r}{k_{23}} = 0 \quad R \leq r \leq R_0 \quad (\text{II-22b})$$

subject to boundary conditions

$$r = R_i : T = T_i \quad (\text{II-23a})$$

$$r = R : T = T_R \quad (\text{II-23b})$$

$$r = R : k_{12} \left. \frac{dT}{dr} \right|_{R-} = k_{23} \left. \frac{dT}{dr} \right|_{R+} \quad (\text{II-23c})$$

$$r = R_0 : T = T_0 \quad (\text{II-23d})$$

Here u_1 and u_2 refer to an average internal heat generation rate per unit volume over a given length ℓ of insulator and Al distributor plate. T_i and T_0 are taken to be the mixed mean fluid temperature - the same as the inner and outer surface temperatures. We neglect the convective resistance terms.

Solving for the temperature, T , in the two regions of interest, we find

$$T - T_i = \frac{u_1}{4k_{12}} (R_i^2 - r^2) + \frac{1}{k_{12}} \ln \frac{r}{R_i} \left\{ \frac{1}{\frac{1}{k_{12}} \ln \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R}} \times \right. \\ \left. \left[\frac{u_1}{4k_{12}} \Delta R_{12} (R_i + R) + \frac{u_2}{4k_{23}} \Delta R_{23} (R + R_0) + \frac{R^2}{2} (u_1 - u_2) \frac{1}{k_{23}} \ln \frac{R_0}{R} \right. \right. \\ \left. \left. - (T_i - T_0) \right] \right\} \quad R_i \leq r \leq R \quad (\text{II-24a})$$

and

$$T - T_0 = \frac{u_2}{4k_{23}} (R_0^2 - r^2) + \frac{1}{k_{23}} \ln \frac{r}{R_0} \left\{ \frac{R^2}{2} (u_2 - u_1) + \frac{1}{\frac{1}{k_{12}} \ln \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R}} \times \right. \\ \left. \left[\frac{u_1}{4k_{12}} \Delta R_{12} (R_i + R) + \frac{u_2}{4k_{23}} \Delta R_{23} (R + R_0) + \frac{R^2}{2} (u_1 - u_2) \frac{1}{k_{23}} \ln \frac{R_0}{R} - (T_i - T_0) \right] \right\} \quad R \leq r \leq R_0 \quad (\text{II-24b})$$

The heat flux, per unit length, may be found at the two surfaces ($r = R_i$ and $r = R_0$) by using

$$\frac{q}{L} = -k \, 2\pi r \frac{dt}{dr} \quad (\text{II-25})$$

The results are found to be

$$\left(\frac{q}{L} \right)_{R_i} = 2\pi \left\{ \frac{u_1 R_i^2}{2} + \frac{1}{\frac{1}{k_{12}} \ln \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R}} \left[T_i - T_0 + \frac{R}{2} (u_2 - u_1) \times \right. \right. \\ \left. \left. \frac{1}{k_{23}} \ln \frac{R_0}{R} - \frac{u_1}{4k_{12}} \Delta R_{12} (R_i + R) - \frac{u_2}{4k_{23}} \Delta R_{23} (R + R_0) \right] \right\} \quad , \quad (\text{II-26a})$$

$$\left(\frac{q}{L}\right)_{R_0} = 2\pi \left\{ \frac{u_2}{2} (R_0^2 - R^2) + \frac{u_1 R^2}{2} + \frac{1}{\frac{1}{k_{12}} + \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R}} \left[T_i - T_0 + \right. \right. \\ \left. \left. + \frac{R^2}{2} (u_2 - u_1) \frac{1}{k_{23}} \ln \frac{R_0}{R} - \frac{u_1}{4k_{12}} \Delta R_{12} (R_i + R) - \frac{u_2}{4k_{23}} \Delta R_{23} (R + R_0) \right] \right\} \quad (\text{II-26b})$$

Defining $U \equiv \frac{T_i - T_0}{\frac{1}{k_{12}} \ln \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R}}$

and

$$q^* = \frac{1}{\frac{1}{k_{12}} \ln \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R}} \left[\frac{R^2}{2} (u_2 - u_1) \frac{1}{k_{23}} \ln \frac{R_0}{R} - \frac{u_1}{4k_{12}} \Delta R_{12} (R_i + R) - \frac{u_2}{4k_{23}} \Delta R_{23} (R + R_0) \right]$$

The heat flux terms become

$$\left(\frac{q}{L}\right)_{R_i} = 2\pi \left(\frac{u_1 R_i^2}{2} + U + q^* \right) \quad (\text{II-27a})$$

and

$$\left(\frac{q}{L}\right)_{R_0} = 2\pi \left[\frac{u_2}{2} (R_0^2 - R^2) + u_1 \frac{R^2}{2} + U + q^* \right] \quad (\text{II-27b})$$

Equation II-27b may be rewritten as

$$\left. \frac{q}{L} \right)_{R_0} = 2\pi \left[\frac{u_2}{2} (R_0^2 - R^2) + \frac{u_1}{2} (R^2 - R_i^2) + U + q^* + \frac{u_1}{2} R_i^2 \right] \quad (\text{II-28})$$

Defining the total average heat deposition rate in each region as

$$\bar{k}_2 = \pi (R_0^2 - R^2) L u_2 \text{ and } \bar{u}_1 = \pi (R^2 - R_i^2) L u_1 \quad (\text{II-29})$$

Equation II-28 becomes

$$q)_{R_0} = \bar{u}_2 + \bar{u}_1 + \frac{1}{\frac{1}{2\pi L}} U + 2\pi L (q^* + \frac{u_1}{2} R_i^2) \quad (\text{II-30})$$

This expression makes clear the nature of the approximation being made in the first estimate of the heat given off to the He stream, i.e.,

$$q)_{R_0} = \bar{u}_2 + \bar{u}_1 \quad (\text{II-31})$$

so that the error is in the neglect of the terms in

$$\frac{1}{\frac{1}{2\pi L}} U + 2\pi L (q^* + \frac{u_1 R_i^2}{2}) \quad .$$

The heat leak calculation is the term, neglecting convection terms,

$$\frac{U}{2\pi L} = \frac{T_i - T_0}{\frac{1}{2\pi L k_{12}} \ln \frac{R}{R_i} + \frac{1}{2\pi L k_{23}} \ln \frac{R_0}{R}} \quad (\text{II-32})$$

(compare with Equation II-21).

APPENDIX A

In this section we consider several modifications to the reference design and subsequent analysis techniques. One such design change is to cool the insulator by passing He through it. Another is to remove the water coolant ducts and cool the solid Al shell by passing He through the annular channel.

For the case of bleeding off the He through the insulator we estimate the change in He velocity through the annular region as follows: from Figure I-10 the dotted line represents the boundary of the control surface. We assume that a percentage, α , of the inlet mass flows through the insulator so that an overall mass balance of the He existing at the base of the annular channel, i.e., through the distributor plate, is simply

$$\rho_{in} A_{in} V_{in} = \sum_i \rho_i A_i V_i + \rho_D A_D V_D \quad (A-1)$$

$$\text{but } \sum_i \rho_i V_i A_i \equiv \alpha \rho_{in} A_{in} V_{in} \text{ and } \rho_B \approx \rho_{in}$$

therefore

$$V_D = (1-\alpha) \frac{A_{in}}{A_D} V_{in} \quad (A-2)$$

The velocity at the bottom of the annular channel is determined from

$$\rho_{in} A_{in} V_{in} = \sum_i \rho_i A_i V_i + \rho_B A_B V_B \quad (A-3)$$

$$\text{but } \sum_i \rho_i A_i v_i = \alpha \rho_{in} A_{in} v_{in}, \rho_{in} \tilde{v} \rho_B \text{ and } A_{in} = A_B$$

therefore

$$v_B = (1-\alpha) v_{in} \quad (A-4)$$

For the second design modification referred to above, the solid Al shell cooled by He with bleed-off through the insulator, the mixed-mean He temperature must be determined anew. Referring to Figure I-9 we make the following assumptions: assume that the heat deposited in the Al distributor plate and insulator is given off to the He being bleed off through the insulator so that the distributor plate appears as an adiabatic surface to the main He stream. This, of course, is an extreme since some heat should still leak to the main He stream. In addition to the calculation assuming an adiabatic surface we will make a calculation assuming a 5% heat leak to the He stream. The mixed-mean temperature is still determined by Equation II-10 with a different ΔQ though.

APPENDIX B

In this section we include the pressure drop and heat transfer calculations for the reference design.

I. Pressure Drop-Water Coolant

a) Mass Flow Rate Per Module

$$m = \frac{\Delta Q}{\Delta h}$$

ΔQ = heat deposited in the Al + heat deposited in the water = 6.061×10^5 BTU/hr

$$\Delta h = h_{ex} - h_{in}$$

enthalpy: $h_{ex} = 374.97$ BTU/lb at 400°F

$h_{in} = 341.29$ BTU/lb at 370°F

$$m = \frac{6.061 \times 10^5}{374.97 - 341.20} = 1.795 \times 10^4 \text{ lb/hr}$$

$$m/\text{coolant duct} = \frac{1.795 \times 10^4}{84} = 214 \text{ lb/hr}$$

b) Conservation of Mass

The mean velocity, v , is determined from $m = \rho av$.

where $v = \frac{m}{\rho A}$

$$\rho = 55 \frac{\text{lb}}{\text{ft}^3} \text{ evaluated at } \bar{T}_m = \frac{400 + 370}{2} = 385^\circ\text{F},$$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4} \left(\frac{3}{4} \right)^2 = 0.472 \text{ cm}^2 = 4.76 \times 10^{-4} \text{ ft}^2$$

so that

$$v = \frac{224}{55 (4.76 \times 10^{-4}) 3.6 \times 10^3} = 2.27 \frac{\text{ft}}{\text{s}}$$

c) Total Pressure Drop:

$$\begin{aligned} \Delta p = & \left[\frac{f (L_1 + L_2 + L_3 + L_4 + L_5)}{D} + 6f \left(\frac{L_{eq}}{D} \right) + \right. \\ & \left. + K_{en} + K_{ex} \right] \rho \frac{v^2}{2g} = \left[0.023 \left(\frac{26 + 70 + 60 + 70 + 26}{\frac{3}{4}} \right) + \right. \\ & \left. + 6 (0.023) \times (38) + \frac{1}{2} + 1 \right] \frac{(55)}{2g \times 144} (2.3)^2 = 0.434 \text{ psi} \end{aligned}$$

Pressure drop in region A:

$$\begin{aligned} \Delta P = & \left[f \left(\frac{L_1}{D} \right) + 2f \left(\frac{L_{eq}}{D} \right) + K_{en} \right] \rho \frac{v^2}{2g} = \left[0.023 \left(\frac{26}{\frac{3}{4}} \right) + \right. \\ & \left. 2 (0.023) (38) + 0.5 \right] \frac{55 (2.3)^2}{2g (144)} = 0.0914 \text{ psi} \end{aligned}$$

Pressure drop in region B:

$$\Delta P = f \left(\frac{L_2}{D} \right) \frac{\rho v^2}{2g} = 0.023 \left(\frac{70}{\frac{3}{4}} \right) \frac{54 (2.27)^2}{2g (144)} = 0.0644 \text{ psi}$$

Pressure drop in region C:

$$\Delta P = \left[f \left(\frac{L_3}{D} \right) + 2f \left(\frac{L_{eq}}{D} \right) \right] \rho \frac{v^2}{2g} = \left[0.023 \left(\frac{60}{\frac{3}{4}} \right) + 2 (0.023) 38 \right] \times \frac{54 (2.27)^2}{2g(144)} = 0.108 \text{ psi}$$

Pressure drop in region D:

$$\Delta P = f \left(\frac{L_4}{D} \right) \rho \frac{v^2}{2g} = 0.023 \left(\frac{70}{\frac{3}{4}} \right) \frac{54 (2.27)^2}{2g(144)} = 0.0644 \text{ psi}$$

Pressure drop in region E:

$$\Delta P = \left[f \left(\frac{L_5}{D} \right) + 2f \left(\frac{L_{eq}}{D} \right) + K_{ex} \right] \rho \frac{v^2}{2g} = \left[0.023 \left(\frac{26}{\frac{3}{4}} \right) + 2 (0.023) (38) + 1.0 \right] \frac{54 (2.27)^2}{2g(144)} = 0.106 \text{ psi}$$

II. Heat Transfer - Water Coolant

a) The mixed-mean temperature in the regions of interest are determined by Equation I-8. The heat deposition rates are given in Table 1.

Region B:

$$h_{fB} - h_0 = \frac{\Delta Q_B}{M}$$

assume that h_0 is evaluated at 370°F:

$$h_0 = 342.79 \text{ BTU/lb} \quad , \quad \Delta Q_B = 5.1778 \times 10^4 \text{ BTU/hr}$$

$$m = 1.795 \times 10^4 \text{ lb/hr}$$

$$h_{fB} = 342.79 + \frac{5.1778 \times 10^4}{1.795 \times 10^4} = 342.79 + 2.88 = 345.67 \text{ BTU/lb}$$

so that $T_{fB} \sim 374^\circ\text{F}$

ΔQ_B is one-half of the total heat deposited in the Al shell plus water in the inner and outer graphite regions. In other words it represents the heat deposition on the inlet side. While we could divide ΔQ_B by 84, the number of inlet water coolant ducts, to get an average heat deposition for each coolant duct and then divide by $m/\text{coolant duct}$ the result is the same as given by B-

Region C:

$$h_{fC} - h_{fB} = \frac{\Delta Q_C}{m}$$

$$\Delta Q_C = 5.02485 \times 10^5 \text{ BTU/hr} \sim \text{total heat deposited in first wall}$$

$$m = 1.795 \times 10^4 \text{ lb/hr}$$

$$h_{fC} - h_{fB} = 28 \text{ BTU/lb}$$

$$T_{fC} = 396^\circ\text{F}$$

b) The heat transfer coefficient, h , is determined by Equation I-15.

Knowing h permits us to estimate the Al shell wall temperature.

$$h = 0.023 \frac{k_f}{D} \text{Re}^{0.8} \text{Pr}^{0.333}$$

For $T_m = 385^\circ\text{F}$, $P_r = 1$ so that

$$h = 0.023 \frac{k_f}{D} \text{Re}^{0.8}$$

$$k_f = 0.385 \text{ BTU/hr } f^\circ\text{F}, \quad D = 0.0247 \text{ ft}$$

$$\text{Re} = 3.103 \times 10^4$$

so that

$$h = \frac{0.023 (0.385) (3.103 \times 10^4)^{0.8}}{0.0246} = 1411 \text{ BTU/hr } f^2\text{F}$$

c) The Al shell wall temperature is determined by Equation I-12. Actually what we obtain is an average over a given length. The heat rate, dq , in Equation I-12 is that conducted to or from the fluid and does not include the heat deposited in the water.

For region B:

$$\bar{T}_f = \frac{370 + 374}{2} = 372^\circ\text{F}$$

$$h = 1411 \text{ BTU/hr } f^2\text{F}, \quad L = 2.3 \text{ ft}, \quad P = \pi D = \pi \times \frac{3}{4} \times \frac{1}{30.48} = 0.07730 \text{ ft}$$

$$\int_L dq = 3.7657 \times 10^4 \text{ BTU/hr}$$

$$\bar{T}_s = 372 + \frac{4.37762 \times 10^5}{1411 (2.3) (.07730) (84)} = 385 + 24 = 409^\circ\text{F}$$

For Region D:

$$\bar{T}_s = 398 + 2 = 400^\circ\text{F}$$

d) An estimate of the temperature of the frontal area of the Al which faces the plasma is given by Equation I-19.

$$T)_m = \bar{T}_s + \frac{L}{k} \left(\frac{LQ}{2} + P_0 \right)$$

where $L = 0.855 \text{ cm}$, $k = 100 \text{ BTU/hr } ^\circ\text{F}$.

$$Q = \frac{0.68841}{AL} \times 10^5 \text{ BTU/hr cm}^3, \quad P_0 = \frac{3.00077 \times 10^5}{A} \text{ BTU/hr cm}^2$$

$A = \text{frontal area of the module} = 3547.1 \text{ cm}^2$

$$T)_m = 409 + \frac{h}{Ak} \frac{0.68841}{2} + 3.00077 \times 10^5 = 409 + 24 = 433^\circ\text{F}$$

Q has been determined by taking half of the heat deposited in the first wall (Al) since L is measured to the equivalent water coolant duct.

III. Pressure Drop ~ Helium Coolant

a) Mass Flow Rate Per Module

$$m = \frac{\Delta Q}{C_p \Delta T}$$

ΔQ = heat deposited in the module less that deposited in the Al shell and water = 1.63569×10^5 BTU/hr

$$C_p = 1.24 \text{ BTU/B}^\circ\text{F} \quad , \quad \Delta T = T_{\text{ex}} - T_{\text{in}}$$

$$T_{\text{ex}} = 1400^\circ\text{F}, \quad T_{\text{in}} = 350^\circ\text{F}$$

$$m = \frac{1.63569 \times 10^5}{1.24 (1400 - 350)} = 1.256 \times 10^3 \text{ lb/hr}$$

b) Conservation of Mass

The mean velocity, v , is determined from $m = \rho A v$.

$$v = \frac{m}{\rho A}$$

where $\rho = 0.140 \text{ lb/f}^3$ evaluated at $T = 350^\circ\text{F}$,

$$A = \frac{\pi}{4} (D_2^2 - D_1^2) = \frac{\pi}{4} (60^2 - 59^2) = 94.2 \text{ cm}^2 = 0.101 \text{ f}^2$$

so that

$$v = \frac{1.256 \times 10^3}{0.140 (0.101) (3.6 \times 10^3)} = 24.7 \text{ f/s}$$

The inlet Reynolds number, Re , based on the equivalent diameter, D_{eq} , necessary for pressure drop calculations is determined by

$$Re = \frac{v D_{eq}}{\nu}$$

where $v = 24.7 \text{ f/s}$, $D_{eq} = D_2 - D_1 = 1.0 \text{ cm} = 0.0328 \text{ ft}$,

$$\nu = 1.35 \times 10^{-4} \frac{\text{ft}^2}{\text{s}}$$

so that

$$Re = \frac{24.7 (0.0328)}{1.35 \times 10^{-4}} = 6001$$

c) Pressure Drop

The pressure drop through the annular duct is determined by Equation II-1.

$$\Delta p = f \rho \frac{v^2 L}{2g D_{eq}}$$

where $f = 0.079 Re^{-0.25}$, based on the Blasius equation, $= 0.079 (6001)^{-0.25}$

$= 0.0352$, $\rho = 0.140 \text{ lb/ft}^3$, $v = 24.7 \text{ f/s}$, $L = 79 \text{ cm}$, $D_{eq} = 0.0328 \text{ ft}$

so that

$$\Delta p = \frac{(0.0352)(0.140)(24.7)^2(79)}{2(32.2)(3.28 \times 10^{-2})(30.48)(144)} = 0.0256 \text{ psi}$$

Through the packed bed the pressure drop is determined by Equation II-5.

$$\frac{\Delta p}{L} \frac{d_p^3}{\rho v_0^2} \frac{\epsilon^3}{1-\epsilon} = 150 \frac{1-\epsilon}{Re_p} + 1.75$$

where $L = 8 \text{ cm}$, $d_p = 1 \text{ cm}$, $\rho = 0.096 \text{ lb/f}^3$, $v_0 = 3.313 \text{ f/s}$, $\epsilon = 0.35$, $\mu = 0.08 \text{ lb/ft-hr}$; μ and ρ are evaluated at $T_{avg} = 682^\circ\text{F}$. Actually the v_0 used here was the average through the bed.

$$Re_p = \frac{d_p v_0}{\mu} \rho = \frac{1}{30.48} \frac{(3.313)(0.096)(3600)}{0.08} = 469,$$

$$150 \frac{(1-\epsilon)}{Re_p} + 1.75 = \frac{150(0.65)}{469} + 1.75 = 0.208 + 1.75 = 1.96$$

$$\Delta p = \frac{1.96(8)(0.096)(3.313)^2(0.65)}{32.2(0.35)^3(144)} = 0.0539 \text{ psi}$$

Based on the inlet velocity, v_0 , where $v_0 = 1.035 \text{ f/s}$

$$\Delta p = 0.00654 \text{ psi}$$

roughly a factor of 10 difference.

The inlet velocity, V_0 , is determined by Equation II-7:

$$V_0 = V_{in} \frac{A_{in}}{A_{dist}} = \frac{24.7 (94.2)}{\pi (26.75)^2} = 1.035 \text{ f/s}$$

The average velocity is determined as follows: the velocity of the He exiting from the bed is given by Equation II-9,

$$V_{rod} = \frac{V_0}{J} = \frac{1.035}{0.185} = 5.59 \text{ f/s}$$

Consequently the average is simply

$$V_{avg} = \frac{V_0 + V_{rod}}{2} = 3.315 \text{ f/s}$$

Through the graphite rod region the pressure drop is determined by Equation II-1.

$$\Delta P = f \frac{\rho V^2 L}{2gD_{eq}}$$

$$Re = \frac{VD_{eq}}{\nu}$$

The flow passage is assumed to be in the shape of a cruciform. For example

Figure B-1

where the equivalent diameter is found to be

$$D_{eq} = 0.410 \text{ cm}$$

D_{eq} is determined from:

$$D_{eq} = \frac{4A}{P}$$

$$A = 4as + s^2, \quad P = 2a + s$$

$$a = 1 \text{ cm}, \quad s = 0.2154 \text{ cm}$$

$$\nu = 4.1 \times 10^{-4} \text{ ft}^2/\text{s} \text{ evaluated at } T_{avg} = 1165^\circ\text{F}$$

$$\nu = 5.59 \text{ f/s}$$

$$Re = \frac{5.59 (0.01345)}{4.1 \times 10^{-4}} = 183. \text{ This value of the Reynolds number}$$

implies that the flow is laminar:

$$f = \frac{64}{Re} = 0.349$$

$$\Delta P = \frac{0.349 (0.0671) (5.59)^2}{2g(144)} \frac{70}{0.410} = 0.0135 \text{ psi}$$

Decrease D_{eq} by 10%:

$$D_{eq} = 0.369 \text{ cm}$$

$$R_e = 165 \quad f = 0.388$$

$$\Delta P = 0.0166 \text{ psi}$$

Increase D_{eq} by 10%:

$$D_{eq} = 0.451 \text{ cm}$$

$$R_e = 202 \quad f = 0.317$$

$$\Delta p = 0.0111 \text{ psi}$$

V. Heat Transfer - Helium Coolant

a) The increase in the temperature in the annular channel as a consequence of heat given off by the distributor plate and insulator (due, simply, to internal heating) is given by Equation II-10.

$$\Delta T = T_f - T_0 = \frac{\Delta Q}{C_p m} = \frac{1.2827 \times 10^5}{1.24 (1.256 \times 10^3)} = 82^\circ\text{F}$$

Accounting for the additional heat transfer due to the temperature difference across the insulator and distributor plate increases the above value by 10°F . This calculation will be given in a later section.

Across the distributor plate at the base the temperature increase is

$$\Delta T = \frac{1.7652 \times 10^4}{1.24 (1.256 \times 10^3)} = 11^\circ\text{F}$$

This increase was not accounted for in the reference design case.

Across the bed region:

$$\Delta T = \frac{7.75498 \times 10^5}{1.25 (1.256 \times 10^3)} = 498^\circ\text{F}$$

b) The heat transfer coefficient, h , in the annular passage is determined by Equation II-13.

$$h = 0.021 \frac{k}{D_{eq}} Re^{0.8} Pr^{0.4}$$

$$k_f = 0.115 \text{ BTU/hrf}^\circ\text{F evaluated at } 400^\circ\text{F};$$

$$D_{eq} = 1.0 \text{ cm} = 3.28 \times 10^{-2} \text{ ft}$$

$$Pr = 0.70; \quad Re = 6001$$

$$h = 67 \text{ BTU/hr f}^2 \text{ }^\circ\text{F}$$

Since the flow through the rod region is laminar, the heat transfer coefficient in a given region is estimated by basing it on the Nusselt number

for flow with fully developed velocity and temperature profiles through a pipe with constant axial heat rate with equivalent diameter. The Nusselt number is

$$Nu = 4.364$$

so that

$$h = 4.346 \frac{k}{D_{eq}} = 4.364 \frac{(0.172)}{0.01345} = 56 \text{ BTU/hr } f^2 \text{ } ^\circ\text{F}$$

If we had based the Nusselt number on, for example, flow through a rectangular duct we find that for aspect ratios, $\frac{b}{a}$, where b is width and a is height, = 1.0, $Nu = 3.63$ and for $\frac{b}{a} = 8.0$, $Nu = 6.60$. The point being made here is that for laminar, fully developed velocity and temperature profiles, the heat transfer depends on k and D_{eq} and the constant does not change substantially with geometry. Even for a triangular duct, $Nu = 3.00$.

An estimate of the heat transfer coefficient for flow next to the insulator is found by assuming that we have flow through an annular gap. (See Figure I-9. We assume that the width of the gap, w , is the same as the equivalent diameter for the flow passages through the rod region. Since the ratio of $\frac{R^*}{R} \sim 1$, the flow is like that between parallel plates so that the Nusselt number, for laminar flow, is

$$Nu = 5.385$$

and

$$h = 5.385 \frac{k}{D_{eq}} = 5.385 \frac{(0.172)}{0.01345} = 69 \text{ BTU/hr } f^2 \text{ } ^\circ\text{F}$$

Another assumption here is that heat is given off to the stream by the rods closest to the insulator wall.

c) In this section we obtain an estimate of the maximum temperature of the graphite and "fuel" rods. For simplicity we assume a cylindrical geometry with uniform heat generation:

$$T_{\max} = \bar{T}_s + \frac{\bar{Q}r_0^2}{4k}$$

where $\bar{T}_s$ is the surface temperature, $\bar{Q}$, an average uniform heat generation and r_0 , the radius of the rod.

$\bar{T}_s$, the average surface temperature, is estimated by a knowledge of the bulk fluid temperature:

$$\bar{T}_s = \bar{T}_m + \frac{\Delta Q}{hPh}$$

In the outer graphite region it is estimated that there are 480 rods. Assuming that every fifth rod contains the mixture of LiAlO_2 plus graphite while the remaining rods are all graphite, we make the additional assumptions that all of the heat deposited in the graphite is in the solid graphite rods while the heat deposited in the LiAlO_2 is in the LiAlO_2 + graphite rods. We find that, per rod, ~ 371 BTU/hr are deposited in the graphite while ~ 1263 BTU/hr are deposited in the LiAlO_2 .

Taking the above values to be the average uniform heat generation rates in Equation II-19 (over estimations) we find that for the maximum case:

$$\frac{\bar{Q}r_0^2}{4k} = 5.76 \times 10^{-3} \text{ } ^\circ\text{F}$$

where $r_0 = 1$ cm, $k = 59$ BTU/hr $^{\circ}\text{F}$, $\bar{Q} = 1263$ BTU/hr.

Consequently $T_{\max} \approx \bar{T}_s$.

d) As a consequence of temperature difference, the heat leak approximation across the insulator and distributor plate is given by Equation II-21:

$$q = \frac{2\pi l (T_i - T_0)}{\frac{1}{R_i h_i} + \frac{1}{k_{12}} \ln \frac{R}{R_i} + \frac{1}{k_{23}} \ln \frac{R_0}{R} + \frac{1}{R_0 h_0}}$$

For the outer region we take

$R_i = 24.25$ cm, $R = 29.25$ cm, $R_0 = 29.5$ cm, $l = 50$ cm, $k_{12} = 0.289$ BTU/hrf $^{\circ}\text{F}$,
 $k_{23} = 100$ BTU/hrf $^{\circ}\text{F}$

so that

$$q_{\text{outer}} = 14138 \text{ BTU/hr}$$

For the inner region we take

$R_i = 26.75$ cm, $R = 29.25$ cm, $R_0 = 29.5$ cm, $l = 20$ cm, $k_{12} = 0.289$ BTU/hrf $^{\circ}\text{F}$,
 $k_{23} = 100$ BTU/hrf $^{\circ}\text{F}$

so that

$$q_{\text{inner}} = 7903 \text{ BTU/hr}$$

The total heat leak, $q^* =$

$$14138 + 7903 = 22041 \text{ BTU/hr} \approx 6.5 \text{ kw.}$$

This value is slightly higher than the value reported in the Ref. Design:

$$q^* = 5.5 \text{ KW}$$

The value of q^* , in turn, is used to calculate the increased temperature of He as a consequence of increased heat flow to the stream.

Table 1

Blanket and Shield Heating Per Module ~ BTU/hr

	$Q_{\text{BREM}} \times 10^{-5}$	$Q_{\text{TOTAL}} \times 10^{-5}$
First Wall		
Al Shell	3.00077	4.37762
H ₂ O		0.64723
Distributor Plate		0.17652
Be Region		
LiAl		3.53034
Be		4.22464
Inner Graphite		
Al Shell		0.42364
H ₂ O		0.18828
Al Distributor Plate		0.07061
Al ₂ O ₃		0.61193
Graphite		2.83604
LiAlO ₂		1.67103
Outer Graphite		
Al Shell		0.32950
H ₂ O		0.09414
Al Distributor Plate		0.04707
Al ₂ O ₃		0.55309
Graphite		1.42390
LiAlO ₂		1.21208

Reference

1. J. Powell, Editor: "Preliminary Reference Design of a Fusion Reactor Blanket Exhibiting Very Low Residual Radioactivity," BNL Report (in preparation).

TYPICAL BLANKET MODULE
FIG. I-1

CROSS SECTION OF
MODULE SHELL
FIG. I-2

EQUIVALENT WATER CIRCUIT

FIG. I-3

EQUIVALENT FIRST WALL

FIG. I-4

HELIUM FLOW CIRCUIT
FIG. I-5

EQUIVALENT DUCT

FIG. I-6

CROSS SECTION OF GRAPHITE ROD

FIG. I-7

COMPOSITE CYLINDER WALL
FIG. I-8

TEMPERATURE PROFILE

FIG. I-9

HELIUM BLEED OFF

FIG. I-10