213,
2/25

02-1175

LA-5833-MS
Informal Report

UC-25

Reporting Date: December 1974

Issued: January 1975

Thin Windows for Gaseous and Liquid Targets: An Optimization Procedure

by

W. V. Hassenzahl
W. H. Gray*

MASTER

*LASL summer student. Present address:
Oak Ridge National Laboratory, Oak Ridge, TN.

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

An Affirmative Action/Equal Opportunity Employer

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

In the interest of prompt distribution, this LAMS report was not edited by the Technical Information staff.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22151
Price: Printed Copy \$4.00 Microfiche \$2.25

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

THIN WINDOWS FOR GASEOUS AND LIQUID TARGETS:

AN OPTIMIZATION PROCEDURE

by

W. V. Hassenzahl and W. H. Gray

ABSTRACT

Many nuclear physics experiments require cryogenic or high-pressure gas targets with thin windows. To obtain the best data, i.e. to reduce background signals, a target window would ideally have zero thickness and zero deflection. Since this is not possible a trade off is made among target geometry, window geometry, window material, and window thickness. Measurements of deflections of a variety of window materials and geometries were performed at room temperature and in liquid nitrogen to aid in designing windows in the future. A simple graphical method for predicting the rupture pressure for several typical window materials is described. The results of the deflection measurement are compared to a finite element computer program MEMBRAN which predicts window deflection and material stresses for a uniform applied pressure.

1. INTRODUCTION

Many particle and nuclear physics experiments require cryogenic liquid, cryogenic high-pressure gas, or simply high-pressure gas targets with thin windows. Typically materials such as HAVAR,* MYLAR,[†] KAPTON,** stainless steels, and various alloys of aluminum have been used. The most frequently used materials are Mylar and Kapton which have relatively high tensile strengths, and relatively low densities and atomic numbers. An excellent review of the physical characteristics of a variety of organic materials below room temperature was compiled by Schramm, Clark, and Reed.^{1,2}

For many experiments a safe window material thickness has been determined by rule of thumb, or

back of envelope calculations. In some instances materials tests have been performed for the target geometry chosen.^{3,4} However, a comparison of the various materials on the basis of their performance as a target window has not been available.

In one particular type of experiment, the measurement of absolute cross sections, the accuracy is frequently limited by the knowledge of the average path length of the incident particles in the target material, such as liquid hydrogen. Previously the path length was determined by measuring the target length and the window deflection and shape, usually by optical methods, prior to or during the experiment.

In order that window design may be based on a complete understanding of the relationship between pressure and deflection an experimental program was carried out in two phases.

In the first, a series of measurements on various materials at room temperature and in liquid nitrogen were performed. Both the rupture pressure and the deflection of the center point of the window

*HAVAR is a nonmagnetic, corrosion resistant, cobalt-base alloy which exhibits high strength and a high fatigue endurance limit. Hamilton Technology, Inc., P.O. Box 1707, Lancaster, PA 17604

[†]Bulletin M-2C, E. I. DuPont De Nemours & Co., Film Department, Wilmington, Delaware.

**Bulletin H-1A, E. I. DuPont De Nemours & Co., Film Department, Wilmington, Delaware.

MASTER

were determined for each combination of material and window geometry as a function of pressure. These tests are described in Section II. In Section III the various materials are compared in terms of rupture pressure as a function of window thickness and diameter.

In the second phase the window shape was measured for several window materials, thicknesses, geometries, and pressures. These measurements are described in Section IV.

The general problem of target windows and their deflections is similar to the classical diaphragm problem of a thin, initially flat, elastic membrane which, when subjected to a uniform pressure, undergoes deflections which are orders of magnitude larger than the window thickness. Closed form solutions to certain classes of related problems have been obtained by using the nonlinear theory of elasticity.^{5,6,7} Complex geometries and/or inelastic material behavior, such as encountered in real membranes, require more rigorous analytical techniques. Various methods have been used to address the membrane problem.⁸⁻¹² However, only the finite element method¹³ has been successfully applied in nonaxisymmetric geometries to membrane materials having nonlinear stress-strain relationships.¹⁴⁻¹⁷ Therefore this method was chosen to solve the target window problem.

A finite element program MEMBRAN which computes membrane deflections and rupture pressure based on the window geometry and the stress-strain characteristics of the material was written. This code is briefly described in Section V. In Section VI the data from the various measurements are compared to the output of MEMBRAN. Most of the results are in good agreement and plausible explanations are given for any discrepancies. A more complete description of the measurements, the results, and the finite element program, MEMBRAN, are given by Gray.¹⁸

II. EXPERIMENTAL TECHNIQUE

The rupture pressure and gross deflection of membranes of a variety of materials were determined at room temperature and in liquid nitrogen at about 77 K. Whereas it was possible to obtain data on very large windows and all materials at room temperature, only selected measurements could be made at 77 K where the limitations of the experimental

apparatus and the greater preparation time prevented a more thorough investigation.

A. Equipment for Room Temperature Measurements

The apparatus used for the room temperature tests is shown in Figs. 1, 2, and 3. As shown in Fig. 1, the clamping mechanism was independent of the plates which defined the window geometry, thereby insuring rapid interchangeability. To closely approximate a true fixed-end boundary condition the window geometry was determined by a cutout in the top clamping plate. Since typical window materials, such as Mylar or aluminum, are not self-sealing, a separate seal was required.

Either neoprene or epoxy was used as the seal material as shown in Fig. 3. Neoprene was initially used for the room temperature measurements because of the modest preparation required. Unfortunately it does not produce a strictly fixed-end boundary condition because of the elastic behavior of the rubber. However, the reproducible results, with ruptures occurring at or near the ultimate material strength, and the minimum preparation required, led to the almost exclusive use of a neoprene seal.

Fig. 1. Cross section of apparatus for membrane deflection measurements at room temperature.

Fig. 2. Apparatus for membrane deflection measurements at room temperature. A 203-mm diameter Kapton window is being tested.

Detail of Clamp Plate Assemblies for Room Temperature Tests

Fig. 3. Detail of clamp plate assemblies for room temperature deflection measurements. Both epoxy and neoprene seals are shown.

This method of sealing is not representative of target construction as many windows will be attached to targets with epoxy.* However, there were several pairs of tests in which the only variable was the seal. In most of these tests the pressure vs deflection characteristic was identical up to the rupture pressure of one of the windows, usually the one with the epoxy seal. In some of these tests not only was the pressure vs deflection characteristic similar, but the rupture pressures were also similar.

A fixed-end circular plate free of material irregularities will fail at the center when subjected to a uniformly distributed load. Analogously a thin membrane should fail at the center due to a pressure load. Earlier measurements at the Lawrence Berkeley Laboratory¹⁹ using epoxy seals, revealed that the epoxy-window interface produces stress concentrations which result in edge failures. This topic will be discussed later; however, the effect was corroborated by our tests. Since the experimental goal was to measure material characteristics, rather than bond characteristics, neoprene was employed as the seal for many of the tests.

To measure the deflection as a function of pressure the experiment was instrumented with the following equipment. The pressure was measured by a Statham Instruments, Inc. pressure transducer, type PA 731TC-100-350/15983. This pressure transducer was calibrated with a MKS Instruments, Inc., Baratron, type 77. At the beginning of the first few experiments an H_2O pressure head was used to obtain small initial pressures. A switch to compressed nitrogen was made at about 10.6 kPa and then the membrane was pressurized to rupture. The window displacement was obtained by a Hewlett Packard displacement transducer, type 70CDT-500 (0.5-inch deflection) or 70CDT-1000 (1.0-inch deflection). A 6.00-V dc excitation was applied to the displacement and pressure transducers. The transducers' output voltage was recorded by a Hewlett Packard 2010A Data Acquisition System.

The apparatus used to measure the shape of the windows at various pressures is shown in Fig. 4.

*The epoxy employed at Los Alamos Scientific Laboratory for most cryogenic target assemblies is a 50%/50% by weight mixture of Epon 828 and Versimid 125.

Fig. 4. Apparatus for measuring membrane deflection and contour at room temperature. Two axis ways were used to provide ± 0.04 -mm positioning accuracy. A 203-mm-diameter Kapton window is shown.

This equipment was used only at room temperature where measurements of the window deflections were determined relative to a Cartesian coordinate system in the plane of the undeflected window. For each window tested the shape was determined at several pressures. For circular windows of isotropic materials the variation of one coordinate was sufficient. Deflections were measured over a full quadrant of the window for noncircular geometries and for anisotropic materials.

B. Equipment for Cryogenic Measurements

Based on the experience gained from room temperature tests, a new clamping apparatus was designed for cryogenic tests. This is shown in Figs. 5 and 6. Since epoxy is universally employed to seal cryogenic target windows, it was used exclusively during these tests.

This apparatus was placed in a small dewar and then covered with liquid nitrogen. The liquid nitrogen level was 80 to 100 mm above the membrane and did not contribute a measurable back pressure to the gauge pressure monitored during the tests. Helium, rather than nitrogen, was used to pressurize the window. To insure linear operation of the

Fig. 5. Cross section of apparatus used for membrane deflection measurements in liquid nitrogen.

Fig. 6. Apparatus used for membrane deflection measurement in liquid nitrogen. A highly deflected Kapton window is shown. (The deformation shown occurred during pressurization at room temperature.)

displacement transducer, a 3.2-mm-diameter phenolic rod served as an extension of the displacement transducers' core assembly preventing conduction of heat away from the device. Otherwise, the testing procedure was identical to that used for room temperature experiments with epoxy seals.

C. Materials and Geometries Tested

In many cases, knowing the exact deflection and shape of a window is not so important as using a window material which will have the smallest effect on a beam of particles for a given target pressure and diameter. Measurements have been made on a large number of materials of varying thicknesses, and for a variety of window geometries. A list of materials tested is given in Table I, and a list of the various window geometries is given in Table II.

Only a few of the materials were tested on all the geometries. There were several reasons for this, e.g., Havar was only available in widths up to about 127 mm and that only in limited quantities, etc.

The most extensive tests were on Mylar T and 1145-H19 aluminum. A representative set of data for these materials is given in Tables III and IV, and the values of rupture pressure are plotted as a function of window diameter in Figs. 7 and 8.

TABLE I

LIST OF MATERIALS TESTED IN THE DEFLECTION MEASURING APPARATUS. THE TEMPERATURES AT WHICH DEFLECTION VS PRESSURE AND RUPTURE DATA WERE OBTAINED ARE ALSO GIVEN

Material	Temperature
Mylar A (regular)	300 K AND 77 K
Mylar T (tensilized)	300 K AND 77 K
Aluminum 1100-0	300 K
" 1145-0	300 K AND 77 K
" 1145-H19	300 K
" 5052-0	300 K
" 5052-H38	300 K
" 6061-0	300 K
Kapton (H-Film)	300 K AND 77 K
Havar	300 K
Ti 6Al4V	300 K
Nickel	300 K
Molybdenum	300 K

TABLE II

A LIST OF THE GEOMETRIES CHOSEN FOR DEFLECTION MEASUREMENTS AT ROOM TEMPERATURE AND IN LIQUID NITROGEN

Geometry	Temperature
51 mm diameter	300 K and 77 K
102 mm diameter	300 K and 77 K
152 mm diameter	300 K
203 mm diameter	300 K
102 mm sq with 6.4 mm radius fillets	300 K
25 x 102 mm with 6.4 mm radius fillets	300 K
25 x 203 mm with 6.4 mm radius fillets	300 K
51 x 102 mm with 6.4 mm radius fillets	300 K
51 x 203 mm with 6.4 mm radius fillets	300 K
51 x 102 mm with 25 mm radius fillets	300 K and 77 K

TABLE III

RUPTURE PRESSURE AND MAXIMUM DEFLECTION OF THE MEMBRANE CENTER FOR MYLAR T AT ROOM TEMPERATURE

Window Diameter (mm)	Window Thickness (mm)	Rupture Pressure (kPa)	Maximum Deflection (mm)
50.8	0.013	172.4	16.2
101.6	0.013	85.2	32.8
101.6	0.013	77.2	30.7
152.4	0.013	39.4	32.7
50.8	0.023	299.2	14.8
101.6	0.023	153.8	30.5
101.6	0.023	141.0	25.2
101.6	0.023	172.4	29.1
152.4	0.023	111.1	42.8
152.4	0.023	103.8	35.4

TABLE IV

RUPTURE PRESSURE AND MAXIMUM DEFLECTION OF THE MEMBRANE CENTER FOR ALUMINUM 1145-H19 AT ROOM TEMPERATURE

Window Diameter (mm)	Window Thickness (mm)	Rupture Pressure (kPa)	Maximum Deflection (mm)
50.8	0.025	160.1	7.3
101.6	0.025	63.6	11.2
101.6	0.025	54.6	9.5
152.4	0.025	43.1	16.3
203.2	0.025	21.8	14.4
203.2	0.025	22.4	14.0

Fig. 7. Rupture pressure vs window diameter for two different thicknesses of tensilized Mylar, Mylar T.

Fig. 8. Rupture pressure vs window diameter for three different aluminum samples.

III. COMPARISON OF VARIOUS WINDOW MATERIALS

A graphical method for expressing rupture pressure in terms of the ratio of window thickness to window diameter was described by Doi, Morimoto, Takamatsu, and Sasaki²⁰ who tested Mylar windows by pressurizing them to rupture in an apparatus quite similar to that described here. In an earlier work by Bonyum,²¹ rupture pressures were tabulated for the purpose of designing safety pressure diaphragms. Foil thicknesses considered were only slightly greater than those presented here.

The results of the measurements of rupture pressure for several materials are plotted as a function of thickness/diameter, t/d , in Figs. 9 and 10. The curve drawn through the data for each material may be expressed in the form

$$P(\text{kPa}) = Ct/d \quad (1)$$

Fig. 9. Comparative performance of various aluminum samples and Havar as window materials at room temperature.

Fig. 10. Comparative performance of various Mylar samples as window materials at room temperature.

The value of C for the curves in Figs. 9 and 10 and for the other materials tested at room temperature and at 77 K are given in Tables V and VI respectively. Thinner windows may be used on a given target for those materials which have the larger values of C . However, since the total mass of material in the beam is also important, the density, ρ , of each material has been included, and also the ratio C/ρ which is a figure of merit of each material as a window.

Jarmie, Morrison, and Martin²² describe their experience with certain metal foils for high pressure gas targets at room temperature. They list a rupture pressure of 920 kPa for a 0.0025-mm-thick, 4.7-mm-diameter Havar target window. Using equation (1) and the data in Table V, a rupture pressure of 892 kPa can be predicted. The 3% difference is well within the experimental error associated with the

TABLE V

CHARACTERISTICS OF VARIOUS MATERIALS FOR ROOM TEMPERATURE WINDOWS, BASED ON RUPTURE PRESSURE, $P(\text{kPa}) = Ct/d$, WHERE THE THICKNESS, t , AND DIAMETER, d , ARE IN THE SAME UNITS

Window Material	C (kPa $\times 10^4$)	ρ (g/cm ³)	C/ρ (m ² /s ²) $\times 10^3$
Mylar T	67	1.4	483.
Mylar A 0.013 mm	49	1.4	350.
Mylar A 0.025 mm	40	1.4	283.
Havar	165	8.3	198.
Kapton	62	1.4	435.
Aluminum 1145-0*	12.7	2.7	47.0
A " 1145-H19*	4.17	2.7	15.4
" 5052-0	37.7	2.7	139.

*1145-0 is an inherently softer material than 1145-H19 and so undergoes larger deflections for a given pressure, but for a given geometry and thickness will rupture at a higher pressure.

TABLE VI

CHARACTERISTICS OF VARIOUS MATERIALS FOR LIQUID NITROGEN TEMPERATURE WINDOWS, BASED ON RUPTURE PRESSURE, $P(\text{kPa}) = Ct/d$ WHERE THE THICKNESS, t , AND DIAMETER, d , ARE IN THE SAME UNITS

Window Material	C (kPa $\times 10^4$)	ρ (g/cm ³)	C/ρ (m ² /s ²) $\times 10^3$
Mylar T	67	1.4	483.
Kapton	40	1.4	278.
Aluminum 1145-0	42	2.7	156.

measurement of thickness, diameter, etc., and with the variation in material characteristics from sample to sample.

Although Tables V and VI give a simple expression for the relative merits of various materials, other factors may be important in deciding the final window material and geometry. These include ease of assembly of the target, expansion and contraction of the window material relative to other target components, ease of sealing in a cryogenic environment, multiple coulomb scattering, and window deflection. The latter topic will be discussed in Section IV.

Most membranes failed catastrophically, generally with an explosive failure during the pressurization period between data points. However, a 0.051-mm-thick, 50.8-mm-diameter aluminum 1145-0 membrane did not fail catastrophically. This experiment was terminated because of the sound of gas escaping from the apparatus. Closer inspection revealed the noise was coming from a crack in the membrane surface. The crack developed near the point of maximum deflection at a pressure of 158.6 kPa.

Fig. 11 shows this membrane after its removal from the testing apparatus and a 30X magnification of the crack is shown in Fig. 12.

Fig. 11. Aluminum sample after testing at room temperature. This sample had an almost mirror-like finish up to 80% of maximum deflection at which point grain structure became apparent.

Fig. 12. Closeup view of crack in the sample shown in Fig. 11. Magnification is about 30 diameters. The crack has a linear dimension of about 0.3 mm.

These photographs demonstrate the mode of ductile fracture which characterizes many membrane failures. Since the crack is not quite at the center of the membrane where the principal strains are greatest, it shows that failure began at a microscopic instability. A discussion of failure prediction is given in Section VI where the measured and calculated rupture pressures of Mylar are compared.

IV. MEASUREMENT OF WINDOW SHAPE

The deflected shape of several windows was measured by a displacement transducer mounted on a set of lathe screws. The apparatus which is shown in Fig. 4 permitted a three-dimensional translation of the deflection measuring device. The deflections of the window surface were measured at points relative to a fixed Cartesian coordinate system in the plane of the undeflected window by translating the displacement transducer across the surface of the window. Isotropic material in circular windows required only one-dimensional radial translation of the displacement transducer; however anisotropic materials or noncircular geometries necessitated a two-dimensional translation. One quadrant of the deflected window was measured for these materials and geometries.

The surface contours of 0.025-mm-thick Kapton in a 203.2-mm-diameter window for several pressures

Fig. 13. Experimental and calculated deflections for several pressures for a circular Kapton membrane.

Fig. 14. Experimentally observed deflections of a circular Mylar T membrane both in the tensilized direction and perpendicular to the tensilized direction. Note that the deflection is smaller in the tensilized direction.

are shown in Fig. 13. Variation of one coordinate was sufficient to map the surface since Kapton, for the purpose of this experiment, was assumed isotropic. A discussion of the calculated output of MEMBRAN shown in Fig. 13 is given in Section VI.

The window surface of an anisotropic material will not be symmetrical even in a circular geometry. This is demonstrated in Fig. 14 where the contours

of a 0.023-mm-thick, 152.4-mm-diameter window made of Mylar T are given for various pressures. Mylar T is an anisotropic material which has been pre-stressed in one direction. The contours in Fig. 14 are in the directions parallel to and perpendicular to the tensilized direction.

V. MEMBRAN - A FINITE ELEMENT PROGRAM TO PREDICT WINDOW DEFLECTIONS AND RUPTURE PRESSURE

Most target windows are membrane structures, i.e., they develop load carrying capability only after deflection. These structures exhibit large deflections which may be orders of magnitude greater than the thickness, and may equal the other window dimensions. They may be subjected to pressures which cause inelastic deformation. The finite element method was chosen to model these structures because it is easily applied to complex geometries and has successfully modeled inelastic material behavior²³⁻²⁸ and large deflections.²⁹⁻³¹

Program MEMBRAN is a finite element computer program which models initially flat membrane structures. The input to MEMBRAN consists of the membrane geometry, the number and location of the finite elements, the tabulated stress-strain characteristic of the window material, and the pressures for which the deflections are to be calculated. The output of MEMBRAN consists of the membrane deflections, stresses, and strains at specified points on the surface of the membrane. Contour plots of the above quantities may be obtained directly on 35-mm film, and the points which have undergone plastic deformation may be printed out to determine the onset of inelastic material behavior. The equations that MEMBRAN solves are described briefly below.

A. Description of the Membrane Problem

A Lagrangian-Cartesian coordinate system was chosen as the reference frame for MEMBRAN. Displacement of an arbitrary point on the surface of a membrane, as shown in Fig. 15, can be represented by

$$\bar{x} = x + u \quad (2a)$$

$$\bar{y} = y + v \quad (2b)$$

$$\bar{z} = w \quad (2c)$$

where x and y are the initial, undeflected coordinates of a point, the functions u , v , and w are the displacements of a point with respect to the X , Y , and Z axes, and $\bar{x}$, $\bar{y}$, and $\bar{z}$ are the displaced

Fig. 15. Coordinate system for deflection calculations in computer program MEMBRAN.

coordinates of a point. The appropriate strain to use with this coordinate system is Green's strain tensor, which is represented by

$$\epsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right] \quad (3a)$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad (3b)$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \cdot \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} \quad (3c)$$

$$\epsilon_{yx} = \epsilon_{xy} \quad (3d)$$

Simplification of Green's strain tensor into a vector quantity was made for use in MEMBRAN; therefore, the following quantities will all be presented in vector or matrix form. Green's strain in vector form is

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} \quad (3e)$$

In more simple analyses than what is used here, e.g., the infinitesimal theory of elasticity, Green's strain is further simplified by assuming that deflections are small relative to subject dimensions. Therefore, all the derivatives in equation (3) are small and their products can be neglected. However, no assumptions about the magnitude of the derivatives are made in MEMBRAN.

Several different stress tensors can be defined for large deformation problems; however, the Piola-Kirchoff tensor is correct for the Lagrangian coordinate system used. It is obtained in vector form by post-multiplication of Green's strain with a material constitutive matrix.

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = [D]\{\epsilon\}. \quad (4)$$

For elastic isotropic materials the constitutive matrix is represented by

$$[D] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & 2 & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1/2(1-\nu) \end{bmatrix} \quad (5)$$

where E is the modulus of elasticity and ν is Poisson's ratio. Modification of equation (5) for inelastic material behavior will be discussed later.

The equation of equilibrium of an elastic membrane can be derived using the theorem of virtual work. Based on this theorem the virtual internal energy of the membrane produced by a virtual displacement is equated to the virtual work done by the pressure on the membrane. The instantaneous internal energy, U , of an elastic membrane can be expressed as

$$U = \iiint_{\text{Volume}} \{\epsilon\}^T \{\sigma\} d(\text{Volume}). \quad (6)$$

The instantaneous work, W , done on a membrane by a pressure, $\vec{p}$, can be represented by

$$W = \iint_{\text{Surface}} \vec{\delta} \cdot \vec{p} d(\text{Surface}) \quad (7)$$

where $\vec{\delta}$ is the deflected shape of the membrane.

Within the context of the finite element method, the virtual deflection, $d\vec{\delta}$, and the virtual strain, $d\{\epsilon\}$, can be related to a set of virtual nodal displacements, $d\{\delta\}$, by the following equations

$$d\vec{\delta} = [N] d\{\delta\} \quad (8a)$$

and

$$d\{\epsilon\} = [B] d\{\delta\}. \quad (8b)$$

The exact form of the matrix functions $[N]$ and $[B]$ are presented by Gray.¹⁸

Equating the virtual internal energy to the virtual work using equations (6), (7), and (8), the equation of equilibrium of an elastic membrane can be represented as

$$\iiint_{\text{Volume}} [B]^T \{\sigma\} d(\text{Volume}) = \iint_{\text{Surface}} [N]^T \vec{p} d(\text{Surface}). \quad (9)$$

Equations (5) and (9) are only valid for elastic deflections. After a material deforms inelastically, further work done by the pressure produces an irreversible deformation. The yield criterion of von Mises is used in MEMBRAN to determine if any plastic work is being done. The appropriate modifications of equation (9) to account for this were properly included in MEMBRAN. For details see Gray.¹⁸

B. Finite Element Method

The membrane structure is divided into finite elements in order that equation (9) might be solved. Three different isoparametric finite elements,^{32,33} shown in Fig. 16, were programmed into MEMBRAN.

Fig. 16. The three isoparametric elements used in MEMBRAN.

The deflection of any point within a finite element is given by the product of the shape function and the vector of its nodal displacements. Shape functions for these finite elements may be found in Ahmad, Irons, and Zienkiewicz.³⁴ The integrals located in equation (9) cannot be evaluated explicitly for isoparametric finite elements, therefore a Gaussian numerical integration was used in MEMBRAN.

Since the strains (and therefore the stresses) can be related to the deflections of the nodal points via equation (8), these deflections become the functional unknowns in equation (9). Unlike linear finite element analysis, the $[B]$ matrix is a function of deflection. This makes equation (9) nonlinear. The Newton-Raphsen simultaneous nonlinear equation solution method³⁵ was used in MEMBRAN to effect a solution for the deflections.

VI. A COMPARISON OF MEASURED PRESSURES AND DEFLECTIONS WITH THOSE CALCULATED BY MEMBRAN

The output of program MEMBRAN was compared to the data obtained via the experimental technique described in Sections II and IV. In the following comparisons all the materials were assumed to have isotropic plastic behavior in two dimensions. Since MEMBRAN cannot model anisotropic plasticity, comparisons between the deflections predicted by MEMBRAN and the observed deflections of materials tensilized in one direction such as Mylar T or aluminum 1145-H19 were not made. If it is necessary to approximate anisotropic plasticity with this program, it is suggested that directionally averaged material properties be used as input data for MEMBRAN.

MEMBRAN cannot model visco-elastic phenomenon. At room temperature if the pressure is sufficiently high, Mylar and Kapton will deform visco-elastically, i.e. they will exhibit a time-dependant deformation after a pressure has been applied. Discussion of this effect follows in Section VI. B, where MEMBRAN is compared to a room temperature Kapton experiment.

A. Measurements on an Aluminum Membrane

Program MEMBRAN was used to model a 0.76-mm-thick, 101.6-mm-square membrane made of 5052-0 (fully annealed) aluminum at room temperature. Property measurements in the plane of the sheet showed this material was nearly isotropic. By assuming symmetry only the upper right quadrant of the structure was considered for the calculation. Sixteen linear isoparametric finite elements in this quadrant, as shown in Fig. 17, were used in the analysis. The

deflections of the node points on the top and right borders of Fig. 17 were input as zero, and the deflection in the X direction along the left side and the deflection in the Y direction along the bottom were input as zero.

The stress-strain relationship for 5052-0 aluminum used for the calculation was obtained from Hecker^{36,37} and is shown in Fig. 18. Linear interpolation was used between the 14 input data points.

Fig. 17. The sixteen isoparametric elements used to model a square membrane. By symmetry only one quadrant must be modeled, and certain motions are restricted on the boundaries of the quadrant.

Fig. 18. Stress-strain curve for 5052-0 aluminum as measured by Hecker at the Los Alamos Scientific Laboratory.

The calculated output of MEMBRAN is superimposed on the experimental data for aluminum in Fig. 19. The almost exact agreement demonstrates that MEMBRAN can successfully model isotropic plasticity as this membrane deformed plastically above 30 kPa.

B. Measurements of a Kapton (H-Film) Membrane

The deflection of a 0.025-mm-thick, 203.2-mm-diameter Kapton (H-Film) membrane at room temperature was modeled with MEMBRAN using twelve linear isoparametric finite elements. By symmetry only one quadrant, as shown in Fig. 20, was used. The stress-strain relationship for Kapton used in the calculation is given in Ref. 1.

In Fig. 13 the shape of this membrane was compared to the output of MEMBRAN up to a pressure of 25.45 kPa, the highest pressure for which no significant visco-elastic effect was observed. At pressures above 25.45 kPa the output of MEMBRAN and the experimental data began to diverge, as shown in Fig. 21. A time difference of about 2 minutes was allowed between the initial and final deflection measurements at a given pressure. Visco-elasticity is characterized in this example by a slow deformation at any pressure above 25.45 kPa and results in more than one deflection at each pressure.

Fig. 19. A comparison of the measured and calculated window deflections as a function of pressure. The stress-strain characteristics of Fig. 11 were used. The agreement is excellent both in terms of calculated deflections and in the predicted rupture pressure.

Fig. 20. The twelve isoparametric elements used to model a circular membrane. By symmetry any pie-shaped section could be used, but the boundary conditions, as shown, are simpler for a quadrant.

Fig. 21. A comparison of measured and calculated values of the deflection of a Kapton membrane at room temperature. The nonlinear elasticity theory was unable to model the membrane at all and the finite element program MEMBRAN was unable to model the visco-elastic or time-dependent deformation.

Visco-elasticity also has an effect upon the values of C listed in Table V. All room temperature Mylar and Kapton experiments exhibited visco-elastic deflection. Since the time scale of these experiments was about 1 hour, the value of C should be decreased if, as is the usual case, the pressurized lifetime of the target at room temperature is greater than several hours.

C. Measurements of a Mylar Membrane at 77 K

Experimental data for a 0.025-mm-thick, 101.6-mm-diameter Mylar membrane tested at 77 K is presented in Fig. 22 along with the deflection predicted by MEMBRAN. The finite element grid used for the calculation is similar to Fig. 20 except the membrane was half as large. Stress-strain data for Mylar at 77 K are given in Ref. 1.

At 77 K Mylar and Kapton do not visco-elastically deform. Therefore, the deflection versus pressure curve for this membrane should be, and is, in good agreement until the ultimate stress is exceeded.

MEMBRAN uses the material ultimate uniaxial stress as a failure criterion. Since materials can withstand greater strains in biaxial tension than in uniaxial tension,³⁶ the calculated failure pressure in a biaxial strain condition may be low compared with experimentally achieved failure pressure. Thus MEMBRAN may be used to calculate window shapes, and deflections, but in some conditions will underestimate the ultimate failure pressure.

Fig. 22. Experimental and calculated deflections of a circular Mylar window in liquid nitrogen. The calculated rupture pressure is lower than the observed pressure due to the apparent increase in strength of materials under biaxial stress at low temperature.

Developing a biaxial failure curve for each material would be a time consuming project, not to mention the fact that the material characteristics are thickness sensitive for thin membranes, i.e., the probability of a microscopic instability, such as an inclusion or microcrack, creating a fracture in the material is greater as thickness is decreased. It is unlikely that biaxial stress data will be developed for many materials in the near future.

VII. CONCLUSIONS

Measurements of deflections and rupture pressures of a representative sample of window materials and geometries described above provide sufficient information to calculate the rupture pressure for most materials and any two-dimensional window geometries. The results of the measurements were used to formulate a graphical method for determining a safe thickness for circular windows for the commonly used window materials. For more complex geometries, or for materials not tested here, the program MEMBRAN may be used to calculate window deflection, shape, and rupture pressure given the stress-strain characteristics of the material and the window geometry.

One of the inconsistencies in the measurement of Mylar was due to failures at the window edge which may be attributed to poor epoxy joints. Another possible source of material failure is crazing of the material in the sharp bend near the metal clamp plate. The extent of this effect will vary from sample to sample. Crazing has been observed in Mylar submersed in liquid nitrogen³⁸ and in windows this would result in unpredictable premature failure.

Although a method for designing target windows has been described here, it is recommended that targets be designed in a way which will use the materials more effectively, i.e., which will minimize the window or target wall thickness. Based on the fact that a cylindrical target geometry will optimally use the tensile characteristics of the window material, a 3-cm-diameter target was fabricated using 0.023 mm-thick-Mylar T. The target, shown in Fig. 23, was tested to rupture at room temperature at 310 kPa, corresponding to 202-MPa tensile stress which is equal to the ultimate strength of the material at 293 K.

Fig. 23. Liquid hydrogen target for a $\pi \rightarrow 2\pi$ experiment at the LASL. A target having a cylindrical geometry as shown here will have a minimum window thickness.

REFERENCES

1. Schramm, R. E., Clark, A. F., and Reed, R. P., A Compilation and Evaluation of Mechanical, Thermal, and Electrical Properties of Selected Polymers, NBS Monograph 132, U.S. Government Printing Office, Sept., 1973.
2. Reed, R. P., Schramm, R. E., and Clark, A. F., "Mechanical, Thermal, and Electrical Properties of Selected Polymers," Cryogenics, Feb., 1973, p. 67.
3. Horwitz, N. and Webster, M., "Cerenkov Counter Monitor for Separated Bubble Chamber Beams," RSI, vol. 35, no. 11, 1964, p. 1525.
4. Simonds, R. and Richter, B., "Mylar Windows for High Pressure Gas Counters," RSI, vol. 34, no. 2, 1963, p. 929.
5. Friedrichs, K. O. and Stoker, J. J., "Seminar on Nonlinear Elasticity," Notes by: B. Altshuler, 1948.
6. Novozhilov, V. V., Foundations of the Nonlinear Theory of Elasticity, Graylock Press, Rochester, N.Y., 1953.
7. Green, A. E. and Adkins, J. E., Large Elastic Deformations, Clarendon Press, Oxford, 2nd ed., 1970.
8. Kao, R. and Perrone, N., "Large Deflections of Axisymmetric Circular Membranes," Int. J. Solids Structures, vol. 7, 1971, p. 1601.
9. Kao, R. and Perrone, N., "Large Deflections of Flat Arbitrary Membranes," Computers and Structures, vol. 2, 1972, p. 532.
10. Collins, D., "Characteristics of Circular Mylar Membranes in the Elastic and Plastic Stress Range," Princeton University Technical Note A-389, Sept., 1968.
11. Collins, D., "The Analysis and Design of Thin Stainless Steel Membranes Subjected to Uniform Pressure Loading," CERN-ISR-EN/70-38, Aug., 1970.
12. Glockner, P. G. and Vishwanath, T., "On the Analysis of Non-Linear Membranes," Int. J. Non-Linear Mechanics, vol. 7, 1972, p. 361-394.
13. Turner, M. J., Clough, R. W., Martin, H. C., and Topp, L. J., "Stiffness and Deflection Analysis of Complex Structures," J. Aero. Sci., vol. 23, no. 9, 1956, p. 805.
14. Oden, J. T., "Numerical Formulation of Nonlinear Elasticity Problems," ASCE J.SD., vol. 93, no. ST3, June, 1967, p. 235.
15. Oden, J. T., "Some Results of Finite Element Applications in Finite Elasticity," Computers and Structures, vol. 3, 1973, p. 175.
16. Oden, J. T. and Sato, T., "Finite Strains and Displacements of Elastic Membranes by the Finite Element Method," Int. J. Solids and Structures, vol. 3, 1967, p. 471.
17. Oden, J. T. and Kubitzka, N. K., "Numerical Analysis of Nonlinear Pneumatic Structures," Proc. 1st Int. Colloquium on Pneumatic Structures, Stuttgart, Germany, May, 1967, p. 87.
18. Gray, W. H., "An Experimental and Finite Element Investigation Into the Large Inelastic Deflection of Foils," Ph.D. Dissertation, Vanderbilt University, Nashville, Tenn., 1974.
19. Private Communication with Ed McLaughlin, LBL.
20. Doi, Y., Morimoto, T., Takamatsu, K., and Sasaki, A., "A Large Aperture Beam Window of Polyester Films for High Pressure Gas Cerenkov Counter," INS-Report-197.
21. Bonyum, M. E., "Frangible Discs as Protection For Pressure Vessels," AIChE, vol. 31, May 1935, p. 256.
22. Jarmie, N., Morrison, L. J., and Martin, J. C., "Thin Metal Windows for Gas Targets," NIM, 116, 1974, p. 451.
23. Marcal, P. V., "Finite Element Analysis with Material Nonlinearities Theory and Practice," Recent Advances in Matrix Methods of Structural Analysis and Design, ed. by Gallagher, R. H., Yamada, Y., and Oden, J. T., Univ. Alabama Press, 1969, p. 257.
24. Zienkiewicz, O. C., Valliappan, S., and King, I. P., "Elasto-Plastic Solutions of Engineering Problems 'Initial Stress', Finite Element Method," Int. J. Num. Meths. Engr., vol. 1, 1969, p. 75.

25. Newman, J. B., "Analysis of Problems Involving Nonlinear Boundary Conditions and Nonlinear Material Properties," Numerical Solution of Nonlinear Structural Problems, Papers Presented at ASME Winter Annual Meeting, Nov. 11-15, 1973, p. 51.
26. Owen, D. R. J., Nayak, G. C., Kfoury, A. P., and Griffiths, J. R., "Stresses in a Partly Yielded Notched Bar - An Assessment of Three Alternative Programs," Int. J. Num. Meths. Eng., vol. 6, 1973, p. 63.
27. Nayak, G. C. and Zienkiewicz, O. C., "Elasto-Plastic Stress Analysis. A Generalization for Various Constitutive Relations Including Strain Softening," Int. J. Num. Meths. Eng., vol. 5, 1972, p. 113.
28. Nayak, G. C., "Plasticity and Large Deformation Problems by the Finite Element Method," Ph.D. Thesis, Univ. Wales, Swansea, 1971.
29. Ebner, A. M. and Ucciferro, J. J., "A Theoretical and Numerical Comparison of Elastic Nonlinear Finite Element Methods," Computers and Structures, vol. 2, 1972, p. 1043.
30. Haisler, W. E., Stricklin, J. A., and Stebbins, F. J., "Development and Evaluation of Solution Procedures for Geometrically Nonlinear Structural Analysis," AIAA, vol. 10, no. 3, 1972, p. 264.
31. Brebbia, C. and Connor, J., "Geometrically Nonlinear Finite-Element Analysis," ASCE J. Eng. Mech., vol. 95, no. EM2, April, 1969, p. 463.
32. Zienkiewicz, O. C., "Isoparametric and Allied Numerically Integrated Elements - A Review," O.N.R. Int. Symp. Num. Comp. Methods in Structural Mechanics, Univ. Illinois, Sept. 1971.
33. Ergatondis, I., Irons, B. M., and Zienkiewicz, O. C., "Curved Isoparametric, "Quadrilateral" Elements for Finite Element Analysis," Int. J. Solids Structures, vol. 4, 1968, p. 31.
34. Ahmad, S., Irons, B. M., and Zienkiewicz, O. C., "Curved Thick Shell and Membrane Elements with Particular Reference to Axisymmetric Problems," Proc. Second Conf. Matrix Methods Structural Mech., Wright-Patterson Air Force Base, Oct. 1968, p. 557.
35. Oden, J. T., Finite Elements of Nonlinear Continuum, McGraw-Hill, 1972.
36. Hecker, S. S., "Formability of Aluminum Alloy Sheets," General Motors Corporation Research Pub. GMR-1487, Nov., 1973.
37. Private communication with S. S. Hecker, LASL.
38. Kastelic, J. R., Hiltner, A. and Baer, E., "Crazing, Yielding and Fracture in Polycarbonate and Polyethylene Terephthalate at Low Temperatures," Journal of Macromolecular Science-Physics, B7(4), 1973, p. 674-703.