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INTERNAL TEMPERATURES
OF FUEL ELEMENTS DURING ENVIRONMENTAL TESTING
(Title Unclassified)

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ABSTRACT

This report presents an analytical study on the environmental tests performed on nuclear rocket fuel elements. The significance and interpretation of measured fuel element surface temperatures are discussed.

An electric analog and mathematical models of the fuel elements were studied to determine the effect of radiation losses on the temperature gradients in the fuel elements during environmental testing. The results permit the determination of maximum internal temperatures from surface temperature measurements.

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INTRODUCTION

To date, all nuclear fuel elements proposed for a nuclear rocket have been tested environmentally in a high pressure vessel using an external helium atmosphere. During such tests, the heat generated in the element by resistance heating is removed by forced convection heat transfer to the hydrogen coolant flowing in parallel channels within the element, by surface radiation to the shielding, and by convective heat transfer to the helium atmosphere. The mechanisms of heat generation and heat transfer encountered in the environmental tests are different from those expected to occur in the nuclear reactor. The major differences are:

1. The element is tested in the absence of any nuclear radiation.
2. The exterior of the element is bathed in helium, not stagnant hydrogen.
3. The power generation results in a temperature distribution unlike that expected in a nuclear reactor; and
4. The test elements have additional heat losses as a result of surface radiation to the surroundings and convective heat transfer to the helium atmosphere.

In view of the differences between environmental test conditions and those occurring under actual reactor operating conditions, the data obtained from these tests must be interpreted with great care. These conditions must be accepted in the present design of environmental tests, but the effect of surface radiation on the temperature gradients in the fuel elements and the effect of resistance heating on the axial temperature distribution can be taken into consideration. In these environmental tests, the fuel element temperatures were measured by optical pyrometers at four axial positions, along the surface but not interior to the element. Therefore, an analytic approach is used to predict the maximum fuel element temperatures on the basis of measured surface temperatures. The techniques used for the various types of thermal calculations are discussed in order to provide a sound basis for extracting reliable information from the environmental tests.

AXIAL TEMPERATURE DISTRIBUTION

In the environmental tests, the fuel elements were heated by electrical resistance. Since the local power generation is the product of the local resistance times the current squared, the only variation in power generation must be due to the variation of resistivity along the axis of the element. For carbon or graphite compositions, the variation of resistivity is slight and it can be considered as a second order effect. The elements are coated with niobium carbide on the inside of the heat transfer surfaces; the coating constitutes a possible parallel path to the graphite. However, if the coating cracks, discreet and discontinuous current paths are created, thus contributing little if anything to the overall resistivity of the element. Assuming uniform power generation, then the maximum temperature must exist at the downstream or hot end of the element, and a linear axial temperature distribution would be expected. Typical axial element surface temperatures, observed with an optical pyrometer, are shown in Figure 1. The surface temperatures were measured at four axial locations and it was evident from the data obtained that the temperature can be approximated by a linear function. Early tests using large chucks, resulted in deviations from this linear relation. The relatively large mass of the chuck provided a good conduction path and materially modified the axial temperature distributions. Early estimates for the maximum axial temperature based on the four measured surface temperatures used second order interpolation, presented in Appendix A. Later this interpolation was modified to the method discussed in the second part of the same appendix.

The linear behavior of the temperature is predicted from the development of the temperature profile developed in Appendix B.

ESTIMATES OF MAXIMUM INTERNAL TEMPERATURES

The basic problem is one of the correct interpretation of the data obtained from electrical resistance heating tests in terms of anticipated behavior for nuclear reactor applications. Since the rate of corrosion of hydrogen on the graphite-uranium carbide matrix is essentially a function of temperature, the maximum temperature in the interior of the element is of prime importance. To accomplish this estimate, two approaches were used: (1) an electrical analog model, and (2) simplified mathematical model yielding a closed form solution.

1. Analog Study of Radial Temperature Gradients

It can be shown that any radiation loss from a fuel element would result in temperature gradients within the element, the maximum temperature being somewhere in the interior. The complexity of the fuel element geometry precludes an exact mathematical analysis on the heat transfer involved, but the system can be readily studied by an electric analog.

The electric simulation technique used is discussed in Appendix C. The results of the study are presented in Figure 2 as a plot of the maximum interior to radiating surface temperature difference versus the percent heat loss by radiation. It can be seen that the temperature difference increases steadily with increasing heat loss. From the analog study, it was observed that the point of maximum temperature within the element moved towards the center with increasing radiation loss. The same figure also presents the results of the calculations of the cylindrical model as a comparison.

2. Cylindrical Analytic Model

The cylindrical model consisted of a fuel element segment divided into three regions, incorporating the three coolant channels in the form of three ring-interface heat sinks, one bordering each region, representing convective heat loss

to the hydrogen. The derivation and solution of the equations for this model is presented in Appendix D. The results of this model are compared with those of the analog model in Figure 2 for the cited conditions. The figure presents maximum temperature differences, internal to surface, versus the percent radiation loss. It is apparent from the comparison that the agreement between the electric analog and the cylindrical mathematical model is good in the region of $0 < \eta < 0.8$. Near the limit of $\eta = 1$ the large discrepancies between the electric analog and the cylindrical model can be attributed to the fact that the electric analog has a reduced area available for conduction due to the hydrogen channels thus requiring a higher temperature difference than the solid cylindrical models. Figure 3 presents both temperature curves on a dimensionless basis, that is, referred to twice the cylindrical maximum temperature difference defined as $Q/2\pi K$ as used in Appendix D.

Sample calculation:

In Table 1 a summary of the surface temperatures is presented together with shield temperatures taken at a location virtually the same as T_4 . As an example of the numerical calculations (discussed in detail below), Run 96 was chosen. Table 2 compares the maximum observed surface temperatures with the maximum calculated temperatures and the maximum estimated internal temperature for the same series of tests.

Numerics for Run 96

1.0 Estimated maximum surface temperature.

Using the estimated maximum surface temperature
equation from Appendix A, Section 2.0

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$$\begin{aligned}T_{sm} &= T_4 + 0.006 (T_4 - T_2) \\&= 4510 + 0.006 (1690) = 4520^{\circ}\text{R}\end{aligned}$$

2.0 Estimate of radiation from element to shield.

$$\frac{q_r}{A} \text{ (BTU/sec. in}^2\text{)} = 3.3 \cdot 10^{-7} \left[\left(\frac{T_m}{100} \right)^4 - \left(\frac{T_{sh}}{100} \right)^4 \right]$$

For a maximum temperature of $T_m = 4520^{\circ}\text{R}$ and a shield temperature of 2390°R (T_{sh})

$$\begin{aligned}\frac{q_r}{A} &= 3.3 \cdot 10^{-7} (4.174 \cdot 10^6 - .326 \cdot 10^6) \\&= 1.27 \text{ BTU/sec. in}^2\end{aligned}$$

Surface area of 0.75 inch hex

$$A = \frac{0.75}{2} \cdot \frac{2}{\sqrt{3}} \cdot 6 \cdot 1 = 2.60 \text{ in}^2/\text{inch length}$$

$$q_r = 1.27 \times 2.60 = 3.30 \text{ BTU/inch length-sec.}$$

3.0 Power generated.

$$Q = 578 \text{ KW} \times \frac{3413 \text{ BTU/KW-hr}}{3600 \text{ sec/hr}} \times \frac{1}{52 \text{ inches}}$$

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$$Q = 10.54 \text{ BTU/inch length-sec}$$

$$\eta = \frac{q_r}{Q} = .313$$

4.0 Estimation of maximum temperature.

$$T_{im} - T_{sm} = B \frac{Q}{2\pi K}$$

where $B = .112$ for cylindrical model and $B = .072$ for the electrical analog. These data are taken from Figure 3.

$$\frac{Q}{2\pi K} = \frac{578 \text{ KW} \times 3413 \text{ BTU/KW-hr}}{4.33 \text{ ft}} \cdot \frac{1}{2\pi \times 18.5 \text{ BTU/hr-ft-}^\circ\text{R}} = 3919^\circ\text{R}$$

Therefore,

$$\begin{aligned} T_{im} &= .112 \times 3919 + 4520 \\ &= 439 + 4520 = 4959^\circ\text{R for the cylindrical model} \\ &= .072 \times 3919 + 4520 \\ &= 282 + 4520 = 4802^\circ\text{R from the electric analog} \end{aligned}$$

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TABLE 1

Test Location (x)	Power, KW	-----Observed Temperature - °R -----				
		T ₁ (4")	T ₂ (15")	T ₃ (37")	T ₄ (48")	T _{sh} (48")
96	578	2420	2820	3960	4510	2390
97	641	-----	-----	-----	4510	2480
98	720	-----	-----	-----	4820	2550
99	562	2540	3000	4060	4460	2345
100	576	2580	3030	4130	4490	2390

TABLE 2

Test	Maximum Observed Temperature - °R	Maximum Calculated Surface Temperature - °R	Maximum Calculated Internal Temperature - °R
	T ₄	T _{sm}	T _{im}
96	4510	4520	4959
97	4510	(4510)*	4940
98	4820	(4820)*	5396
99	4460	4469	4892
100	4490	4499	4928

*No estimate possible other than T₄.

CONCLUSIONS

With the aid of electric analog, the temperature measurements available at the present are adequate for the determination of local fuel element temperature gradients and local gaseous temperatures. The analog study was made with equal convective heat transfer to all the coolant channels. Non-uniform convective heat transfer can be readily incorporated into the electric model.

The accuracy of the electric analog is expected to be good where potential differences are relatively large but poor where potential differences become small. For this reason the confidence level of the analog model would improve as the ratio of the radiating energy loss to the power generated, η , approaches unity. At low values of this parameter, the analytic solution should be more accurate. For the tests reported, values of η fall in the range from .275 to .313. Accordingly the analytic solution is used, as being more accurate, to calculate the maximum internal temperatures which are tabulated in Table 2.

The analytic model checks the electric analog within the degree expected. Consideration must be given to the degree with which each approach simulates the real case. The electric analog introduces the current to the conducting paper model in definite and discreet number of points. While the number of points used is large, a difference does exist between the finite and differential approach. Obviously the accuracy of the measurements are also reflected in the accuracy of the solution. The analytic approach, too, can be improved especially in the areas of internal heat removal. At present either approach can be considered to be no better than the value of the thermal conductivity K for the beaded fuel element presently determined to be 20 BTU/hr-ft-°R at 3000°R and 17 BTU/hr-ft-°R at 4500°R. For the calculation in this report a mean value of 18.5 BTU/hr-ft-°R was used. Considering all the factors, it is recommended that use of the analytic solution be restricted to values of η less than 0.5.

FUTURE WORK

Future work in the area of analysis to augment and interpret high temperature fuel element testing will consist of the following steps.

1. Larger and smaller scale models of conducting paper to verify the absence of scale effects.
2. Improvement of the analytic model to incorporate reduced conduction areas corresponding to the voids resulting from the channels.
3. Analytic and analog studies necessary to redistribute the heat removal at the channels in order to reduce the differences between surface and peak internal temperatures. Once this information is available elements will be orificed to reflect this heat removal redistribution.

Figure 1

Figure 2

Figure 3

APPENDIX "A"

ESTIMATION OF MAXIMUM SURFACE TEMPERATURE

1.0 Complete Parabolic Assumption

Since surface temperature measurements of fuel elements are limited to a finite number of points in the corrosion furnace, the probability of the maximum temperature occurring at a point other than the four directly observable points is high. An interpolation of the maximum temperature can be made by assuming that the three highest observed temperatures determine a second order equation which can then be differentiated for the maximum temperature value.

Let the following nomenclature be used:

Length from cold end of fuel element = x

Temperature at location x = T

Temperatures T_2 , T_3 and T_4 are measured at positions $x_2 = 15$ in. ;

$x_3 = 37$ in. ; $x_4 = 48$ in.

Assuming a parabolic temperature profile in the form

$$T - T_4 = a (x_4 - x) + b (x_4 - x)^2$$

and evaluating "a" and "b" in terms of x_2 , T_2 , x_3 and T_3 yield

$$a = \frac{1}{x_3 - x_2} \left[(T_2 - T_4) \frac{x_3 - x_4}{x_2 - x_4} - \frac{x_2 - x_4}{x_3 - x_4} (T_3 - T_4) \right]$$

and

$$b = \frac{1}{x_3 - x_2} \left(\frac{T_3 - T_4}{x_3 - x_4} - \frac{T_2 - T_4}{x_2 - x_4} \right)$$

Maximum temperature occurs when

$$\frac{d(T - T_4)}{d(x - x_4)} = 0$$

at a value of

$$x - x_4 = -\frac{a}{2b}$$

which when substituted in the parabolic temperature profile yields a value of

$$T_{\max} = T_4 + \frac{1}{4} \frac{1}{x_3 - x_2} \frac{\left[\frac{x_4 - x_2}{x_4 - x_3} (T_3 - T_4) + \frac{x_4 - x_3}{x_4 - x_2} (T_4 - T_2) \right]^2}{\left(\frac{T_3 - T_4}{x_4 - x_3} + \frac{T_4 - T_2}{x_4 - x_2} \right)}$$

which with the numerics of the system simplifies to

$$T_{\max} = T_4 + \frac{1}{24} \frac{(9T_3 - 8T_4 - T_2)^2}{3T_3 - 2T_4 - T_2}$$

If the surface temperature is essentially linear as shown on Figure 1, the previously derived parabolic temperature corrections are invalid because the denominator $3T_3 - 2T_4 - T_2$ approaches zero.

2.0 End Limited Parabolic Assumption

An alternate approach for estimating the correction is based on the assumption that a parabola exists in the range between the last measured temperature T_4 at x_4 and its end x_5 . Assuming that the temperature in this region is given by

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$$T = T_4 + a(x - x_4) + b(x - x_4)^2,$$

the slope is

$$\frac{dT}{dx} = a + 2b(x - x_4)$$

The slope at $x = x_5$ must be assumed. For this development it is considered to be m times the value at x_4 . Hence we have the conditions that at

$$x = x_4 \quad \frac{dT}{dx} = \frac{T_4 - T_2}{x_4 - x_2} = a + 2b(x_4 - x_4) = a$$

$$x = x_5 \quad \frac{dT}{dx} = -m \frac{T_4 - T_2}{x_4 - x_2} = a + 2b(x_5 - x_4)$$

Then

$$a = \frac{T_4 - T_2}{x_4 - x_2}$$

$$b = -\frac{1}{2} \frac{1+m}{x_5 - x_4} \frac{T_4 - T_2}{x_4 - x_2} = -\frac{1}{2} \frac{1+m}{x_5 - x_4} a$$

So that the maximum temperature occurs when

$$x = x_4 + \frac{x_5 - x_4}{1+m}$$

and has a value of

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$$T_{\max} = T_4 + \frac{1}{2} \frac{x_5 - x_4}{x_4 - x_2} \frac{T_4 - T_2}{1 + m}$$

which for the numerics used previously becomes

$$T_{\max} = T_4 + .0606 \frac{T_4 - T_2}{1 + m}$$

The temperature drops rapidly on the last part of the element between x_4 and x_5 and must be comparable in magnitude to $T_4 - T_2$. Since the distance x_4 to x_5 (4 inches) is less than 1/8 that between x_2 and x_4 (33 inches), the absolute value of the temperature slope from x_4 to x_5 must be more than 8 times that between x_2 and x_4 , that is, $m > 8$. Using a value of approximately 9, the equation becomes

$$T_{\max} = T_4 + 0.006 (T_4 - T_2)$$

While other curve shapes could be assumed, the temperature maximum cannot be materially different than the value predicted by the above equation, since the linearity of the temperature precludes a temperature of $T > T_4$ at a location of $x < x_4$, and the requirement of a rapid drop in temperature in the region $x_4 < x < x_5$ limits the temperature rise. The maximum temperature assuming a continued linear increase in the temperature would be

$$T_5 = T_4 + \frac{x_5 - x_4}{x_4 - x_2} (T_4 - T_2)$$

which numerically becomes

$$T_5 = T_4 + .1212 (T_4 - T_2)$$

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APPENDIX "B"

Axial Temperature Distributions

In order to obtain one dimensional axial temperature distributions in closed form solutions a simple analysis was made. The differential energy balance equating the increase in gas enthalpy to the power dissipation is

$$\dot{w} C_p dT = dP = \left(\frac{dP}{dx} \right) dx$$

or in integrated for

$$\dot{w} C_p (T - T_o) = \dot{w} (H - H_o) = \int_o^x \frac{dP}{dx} dx.$$

The differential heat transfer is

$$h (T_w - T) dA = dP$$

and if the differential area is the product of the circumference and the differential length ($dA = S dx$)

$$Sh (T_w - T) = \frac{dP}{dx} \quad \text{so that}$$

$$T_w = T + \frac{\frac{dP}{dx}}{hS}$$

If electrical heating is considered with constant resistance $\frac{dP}{dx} = \frac{P_o}{x_o}$ and on paper substitution yields

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$$T = T_o + \frac{P_o}{\dot{w} C_p} \frac{x}{x_o}$$

and

$$T_w = T + \frac{P_o}{h S x_o} = T_o + \frac{P_o x}{\dot{w} C_p x_o} + \frac{P_o}{h S x_o}$$

The gas temperature would be linear if C_p were constant and departure from linearity would be measured by the deviation of C_p . The wall temperature would be influenced by the specific heat as well as the heat transfer coefficient. Linearity would be determined if both were constant.

In the event that a sinusoidal power distribution such as expected in a nuclear device were considered such as

$$P = \frac{P_o}{2} \left(1 - \cos \frac{\pi x}{x_o} \right)$$

so that

$$\frac{dP}{dx} = \frac{P_o \pi}{2 x_o} \sin \pi \frac{x}{x_o}$$

for such a case

$$T = T_o + \frac{P_o}{2 \dot{w} C_p} \left(1 - \cos \frac{\pi x}{x_o} \right)$$

and

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$$T_w = T_o + \frac{P_o}{2\dot{w}C_p} \left(1 - \cos \frac{\pi x}{x_o} \right) + \frac{P_o \pi}{2hSx_o} \sin \pi \frac{x}{x_o}$$

Curves on Figure B-1 show the axial temperature distributions for the two cases for both gas and wall temperatures. The constant heating curves correspond to electrical heating, and the cosine heating curves approximate nuclear heating.

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Figure B-1

APPENDIX "C"

1.0 Electric Analog of Heat Conduction in the Fuel Element

The fuel element in question is 52 inches in length and hexagonal in shape, .75 inches across flats. The element contains nineteen 0.1 inch holes arranged as shown in Figure C-1. In the experimental tests, the fuel element was placed inside a furnace. The gas was then heated to desired temperature levels by resistance heating of the element. As power increased, element and gas temperatures as well as radiation losses increased correspondingly. If radiation losses from the fuel element occur, heat must be transferred from the element to the surface by conduction, resulting in a temperature gradient between the element interior and the radiating surface. In order to ascertain the effect of hydrogen corrosion on the fuel element, the magnitude of the temperature gradients must be known.

Let Q_T denote the total electrical energy input into the element. Then, assuming steady-state operation

$$Q_T = Q_C + Q_R \quad (1)$$

where Q_C is the total heat transferred to the gas by convection and Q_R is the total radiation loss. The radiation loss can be given by the equation

$$Q_R = \frac{K_{eff} A_{eff}}{\delta_{eff}} (T_m - T_s) \quad (2)$$

where A_{eff} is the effective heat transfer area and δ_{eff} is the effective distance between the point of maximum temperature and the radiating surface. Under actual conditions, the temperature of the element varies radially as well as along the entire length of the element; consequently, the rate of heat transfer

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FUEL ELEMENT SECTION

SCALE: 1 Inch = 0.141 Inch

Figure C-1

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to the gas in the nineteen holes is expected to vary. But, for the purpose of this study, it is assumed that

1. There is uniform heat generation within the element.
2. The inside walls of all the flow passages at any location are at the same temperature.
3. The rate of heat transfer to all the flow passages is the same.
4. The outside surface temperature at any location along the element is constant.
5. Radiation heat flux and heat flux to hydrogen is uniform along the element.

2.0 Electrical Simulation Procedure

It is apparent from Figure C-1 that a one-twelfth segment of the fuel element (as shown by the cross-hatched area) provides adequate geometrical simulation of the entire element. Accordingly, an electric circuit for Type II PAL analog* was set up as shown schematically in Figure C-2 and the attached figure underneath.

A number of identical resistors with one end shaped in the form of pins were connected to the battery as shown in the underlay on Figure C-2. The resistors are represented by arrow tails. The pins were allowed to penetrate the conducting paper, and silver paint was used to provide complete contact between the pins and the conducting paper. The use of a large number of resistors (with resistances much greater than the conducting paper) distributed uniformly over the segment provided excellent simulation of uniform heat generation.

The values of the resistors are shown in Figure C-2. From similarity,

*"Poissonian and Laplacian Field Analog for the Solution of Complex Heat Conduction Problems", WANL-TNR-059, June, 1962.

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ELECTRIC ANALOG FOR FUEL ELEMENT HEAT TRANSFER

- == Adiabatic Surface
- Equipotential Surface (Silvered)
- /// Conducting Paper

Each Grid Point Resistance 20×10^6 ohms

DISTRIBUTION NETWORK

C-2 Underlay

$\frac{Q_R}{Q_T} = \frac{I_R}{I_T}$. The electric analogs to heat transfer are as follows:

<u>Heat Transfer</u>	<u>Electric Analog</u>
$\frac{Q_T}{52}$	I_T
$\frac{Q_R}{52} = \frac{K_{eff} W_{eff} (T_m - T_s)}{\mathcal{J}_{eff}}$	$I_R = \frac{W_{eff} \Delta E_m}{\rho \mathcal{J}_{eff}}$
K_{eff}	$\frac{1}{\rho}$
$T_m - T_s$	ΔE_m

where ρ is the electrical resistivity of the conducting paper.

Radiation loss per inch of element length per hour is given by the equation:

$$\frac{Q_R}{52} = \frac{I_R}{I_T} \frac{Q_T}{52} \quad (3)$$

But

$$\frac{Q_R}{52} = \frac{K_{eff} W_{eff} (T_m - T_s)}{\mathcal{J}_{eff}}$$

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Eliminating $\frac{Q_R}{52}$,

$$\frac{I_R}{I_T} \frac{Q_T}{52} = \frac{K_{eff} W_{eff} (T_m - T_s)}{\rho \delta_{eff}} \quad (4)$$

where W_{eff} is the effective width across which heat is conducted.

But

$$I_R = \frac{W_{eff} \Delta E_m}{\rho \delta_{eff}} \quad (5)$$

or

$$\frac{W_{eff}}{\delta_{eff}} = \frac{I_R}{\Delta E_m} \quad (6)$$

Substituting into 4 ,

$$T_m - T_s = \frac{I_R}{I_T} \frac{Q_T}{52} \frac{\Delta E_m}{K_{eff} \rho I_R} \quad (7)$$

Since the standard consists of two one-inch square pieces of conductivity paper,

$$\Delta E_{st'd} = \frac{I_T \rho}{2} \quad (8)$$

Hence

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$$T_m - T_s = \frac{I_R Q_T}{I_T 104 K_{eff}} \frac{\Delta E_m}{\Delta E_{st'd}} \quad (9)$$

Assuming $Q_T = 700 \text{ KW or } 2.43 \times 10^6 \text{ BTU/hr}$

and

$$K_{eff} = 18.5 \text{ BTU/hr-ft-}^\circ\text{R or } 1.54 \text{ BTU/hr-in-}^\circ\text{R} \quad (10)$$

Then $T_m - T_s = 2.34 \times 10^4 \Delta E_m \frac{I_R}{I_T}$, since $\Delta E_{st'd} = .65 \text{ volts}$

Referring to the circuit diagram shown in Figure C-2, let

$$E = E_{4-7} = E_{4-9} = E_{4-11} = E_{4-13}$$

From the parallel circuit relationship,

$$E + 1000 I_G = E_{4-5} + 1000 I_R \quad (11)$$

But

$$I_T = I_R + 4 I_G$$

or

$$I_G = \frac{I_T - I_R}{4} \quad (12)$$

Substituting into 11 and simplifying,

$$4 (E - E_{4-5}) + 1000 I_T = 5000 I_R \quad (13)$$

Also

$$E_{3-4} = 1000 I_T \quad (14)$$

From these relationships, $\frac{I_R}{I_T}$ can be readily calculated from the electrical measurements. In accordance with the assumptions made for the problem, the variable resistors R_1 , R_2 , R_3 , and R_4 in Figure C-2 were adjusted to give the same value of E . R_5 was then varied so as to give different ratios of radiative heat flux to total heat generated. For each ratio, the maximum voltage drop between a point in the paper and the radiating surface was determined by trial and error with a moving point contactor. The results of the measurements and the calculations are summarized below:

$$\Delta E_{st'd} = E_{2-3} = .65 \text{ V}$$

$$E_{4-7} = E_{4-9} = E_{4-11} = E_{4-13} = .6 \text{ V} = \text{constant, fixed}$$

$$E_{3-4} = .6 \text{ V, a constant}$$

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$E_{4-5}, \text{ Volts}$	$\Delta E_m, \text{ Volts}$	$\Delta T, ^\circ\text{R}$	Q_R/Q_T	$\frac{\Delta T}{Q/2\pi K}$
0	.24	5620	1	1.18
.05	.15	3280	.933	.690
.15	.12	2250	.80	.474
.30	.09	1265	.60	.266
.40	.05	545	.467	.115
.55	.044	275	.267	.056
.625	.0297	118	.167	.025
.75	.0104	0	0	0

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APPENDIX D

INTERNAL TEMPERATURE DIFFERENCES IN FUEL ELEMENTS

1.0 Model

The analysis to determine the temperature distribution in the cross section of a model of the fuel element is on a simplified version of the nineteen hole hexagonal element. The mathematical model consists of three concentric annular rings with heat abstraction on all ring interfaces. The model is depicted below

The total heat removed in a 360° (2π) sector at locations $\rho = a_1, a_2, a_3$ and a_4 would be respectively q_1, q_2, q_3 and q_r . These values could be considered as the heat removed from the center hole, the six hole ring, the twelve hole ring, and the surface of the hexagonal fuel elements, respectively.

2.0 Equation of Heat Transfer

The differential equation for the case of a conducting solid with homogeneous heat generation is

$$K \nabla^2 T = -\dot{q}_v \quad \text{where } \dot{q}_v = \text{volumetric heat generation.}$$

and in radial coordinates

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r} \frac{\partial}{\partial r}$$

which for radial symmetry reduces to

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}$$

or

$$r \frac{d^2 T}{dr^2} + \frac{dT}{dr} + \frac{\dot{q}_v}{K} = 0$$

but if $p = \frac{dT}{dr}$, $\frac{dp}{dr} = \frac{d^2 T}{dr^2}$

$$r \frac{dp}{dr} + p + \frac{\dot{q}_v}{K} r = 0.$$

Since $d(pr) = rdp + pdr$

$$d(pr) + \frac{\dot{q}_v}{K} r dr = 0$$

$$pr + \frac{\dot{q}_v}{K} \frac{r^2}{2} = C_1$$

or

$$\frac{dT}{dr} = \frac{C_1}{r} - \frac{\dot{q}_v}{K} \frac{r}{2}$$

which on final integration yield

$$T = C_1 \text{Log } r + C_2 - \frac{\dot{q}_v}{K} \frac{r^2}{4}$$

For a cylinder of radius R ($\rho = 1$) the heat generated per unit length is $\dot{q}_v \pi R^2 = Q$
so that

$$\frac{\dot{q}_v r^2}{4K} = \frac{\dot{q}_v \pi R^2}{4K \pi} \frac{r^2}{R^2} = \frac{Q}{4 \pi K} \rho^2$$

and the solution can be rewritten as

$$T = C + C_1 \text{Log } \rho - \frac{Q}{4 \pi K} \rho^2$$

where

$$C = C_2 - C_1 \text{Log } R$$

3.0 Boundary Value Determinations

3.1 Nomenclature

3.1.1 Definitions

Region 1 is domain bounded by $0 < \theta < 2\pi$, $a_1 < \rho < a_2$

Region 2 is domain bounded by $0 < \theta < 2\pi$, $a_2 < \rho < a_3$.

Region 3 is domain bounded by $0 < \theta < 2\pi$, $a_3 < \rho < a_4 = 1$.

Temperature in any region 1, 2, 3 is T_1, T_2, T_3 .

Surface temperature at $\rho = 1$ is T_s .

3.2 Energy Conservation Conditions

3.2.1 $\sum q_i = 0$

General case of interface conditions yields

Heat conducted to interface = Heat dissipated from interface
 + Heat conducted from interface

3.2.2 Interface boundary conditions for conduction and dissipation.

3.2.2.1 $\rho = a_1$

$$0 = q_1 - 2\pi K \rho \left(\frac{dT_1}{d\rho} \right)_{\rho=a_1}$$

3.2.2.2 $\rho = a_2$

$$-2\pi K \rho \left(\frac{dT_1}{d\rho} \right)_{\rho=a_2} = q_2 - 2\pi K \rho \left(\frac{dT_2}{d\rho} \right)_{\rho=a_2}$$

3.2.2.3 $\rho = a_3$

$$-2\pi K \rho \left(\frac{dT_2}{d\rho} \right)_{\rho=a_2} = q_3 - 2\pi K \rho \left(\frac{dT_3}{d\rho} \right)_{\rho=a_3}$$

$$3.2.2.4 \quad \rho = a_4 = 1$$

$$-2\pi K \rho \left(\frac{dT_3}{d\rho} \right)_{\rho=a_4} = q_r \text{ (radiated from surface)}$$

3.2.3 Interface conditions for temperature.

The assumption is made that the temperature is a continuous function of ρ , so that

$$T_i \text{ at } \rho = a_i = T_{i+1} \text{ at } \rho = a_i$$

which when referred to the mathematical model becomes

$$3.2.3.1 \quad \rho = a_2$$

$$T_1(a_2) = T_2(a_2)$$

$$3.2.3.2 \quad \rho = a_3$$

$$T_1(a_3) = T_3(a_3)$$

$$3.2.3.3 \quad \rho = a_4 = 1$$

$$T_3(1) = T_s$$

3.2.4 Energy considerations.

3.2.4.1 Conservation

$$Q = q_r + \sum_i q_i$$

3.2.4.2 Dimensionless radiation loss

$$\eta = q_r / Q = q_4 / Q$$

4.0 Solution of Equations

Using the general solution

$$T = C + C_1 \text{Log } \rho - \frac{Q}{4\pi K} \rho^2 \text{ at } \rho = 1 \text{ with the conditions 3.2.2.4}$$

and 3.2.3.3 yields

$$\left. \frac{dT_3}{d\rho} \right|_{\rho=1} + \frac{q_r}{2\pi K} = \left. \frac{C_1}{\rho} - \frac{Q}{2\pi K} \right|_{\rho=1} \quad \text{from 3.2.2.4}$$

$$C_1 = \frac{Q}{2\pi K} - \frac{q_r}{2\pi K} = \frac{Q}{2\pi K} (1 - \eta)$$

so that

$$T_3(\rho) = C + \frac{Q}{2\pi K} (1 - \eta) \text{Log } \rho - \frac{Q}{4\pi K} \rho^2$$

When $\rho = 1$

$$T_3(1) = T_s = C - \frac{Q}{4\pi K} \rho^2 \quad \text{from 3.2.3.3}$$

and

$$T_3(\rho) = T_s + \frac{Q}{2\pi K} (1 - \eta) \text{Log } \rho + \frac{Q}{4\pi K} (1 - \rho^2)$$

Using conditions 3.2.2.3 and 3.2.3.2 yields

$$T_2(\rho) = T_s + \frac{Q}{2\pi K} (1 - \eta) \log \rho - \frac{q_3}{2\pi K} \log \rho/a_3 + \frac{Q}{4\pi K} (1 - \rho^2).$$

Repeating the process for region 1 using conditions 3.2.2.1 and 3.2.3.1 yields

$$T_1(\rho) = T_s + \frac{Q}{2\pi K} (1 - \eta) \log \rho - \frac{q_3 + q_2}{2\pi K} \log \rho/a_2 - \frac{q_3}{2\pi K} \log a_2/a_3 \\ + \frac{Q}{4\pi K} (1 - \rho^2)$$

q_1 can be determined from conditions 3.2.2.1 where $q_1 = 2\pi K \rho \frac{dT}{d\rho} \Big|_{\rho=a_1}$

where

$$\frac{dT_1(\rho)}{d\rho} = \left(\frac{Q}{2\pi K} (1 - \eta) - \frac{q_3 + q_2}{2\pi K} \right) \frac{1}{\rho} - \frac{Q}{4\pi K} (2\rho)$$

and

$$\frac{dT_1(a_1)}{d\rho} = \frac{Q}{2\pi K} \left\{ \left(1 - \eta - \frac{q_3 + q_2}{Q} \right) \frac{1}{a_1} - a_1 \right\}$$

hence

$$q_1 = Q \left\{ (1 - \eta) - \frac{q_3 + q_2}{Q} - a_1^2 \right\}$$

4.1 Summary of Equations

Region 1. $a_1 < \rho < a_2$

$$\frac{T_1 - T_s}{Q/2\pi K} = \frac{1}{2} (1 - \rho^2) - (1 - \eta) \left\{ \text{Log } \frac{1}{\rho} - \frac{q_2 + q_3}{Q} \text{Log } \frac{a_2}{\rho} - \frac{q_3}{Q} \text{Log } \frac{a_3}{a_2} \right\}$$

Region 2. $a_2 < \rho < a_3$

$$\frac{T_2 - T_s}{Q/2\pi K} = \frac{1}{2} (1 - \rho^2) - (1 - \eta) \left\{ \text{Log } \frac{1}{\rho} - \frac{q_3}{Q} \text{Log } \frac{a_3}{\rho} \right\}$$

Region 3. $a_3 < \rho < 1$

$$\frac{T_3 - T_s}{Q/2\pi K} = \frac{1}{2} (1 - \rho^2) - (1 - \eta) (\text{Log } \frac{1}{\rho})$$

4.2 Reduction of Hexagonal Fuel Element to Circular Mathematical Model

$$\text{Effective Radius } \pi r^2 = 6 \frac{h^2}{4} \tan 30^\circ$$

where h = distance across flats of hexagon or $r = 0.525 h$

4.2.1 Equivalent radius of outer surface

$$R = 0.525 h, h = 0.753, R = 0.396$$

4.2.2 Equivalent radius of 12 holes

$$r_3 = 0.525 h, h_3 = .5968, r_3 = .3133$$

therefore

$$\frac{r_3}{R} = .7912$$

4.2.3 Equivalent radius of 6 holes

$$r_2 = .525 (.301) = .1582$$

$$a_2 = \frac{r_2}{R} = .3996$$

4.2.4 Equivalent radius of center hole

$$r_1 = .094 \text{ diameter hole}$$

$$a_1 = \frac{.047}{.396} = .1187$$

4.2.5 Equivalent gap at interfaces representing "m" finite holes

$$\pi R^2 (\overline{a_i + \delta_i^2} - \overline{a_i - \delta_i^2}) = m \frac{\pi d^2}{4}$$

or

$$\delta_i = \left(\frac{d}{R} \right)^2 \frac{m}{16 a_i}$$

$$\text{for } d = .094" \quad R = .396"$$

$$\delta_i = .00417 \frac{m}{a_i}$$

4.2.6 For region 2 to 3 interface

$$a_3 = .7912 \quad m = 12$$

$$\delta_3 = .0632$$

4.2.7 For region 1 to 2 interface

$$a_2 = .3996 \quad m = 6$$

$$\delta_2 = .0626$$

- 4.3 Using the assumption that each of the nineteen holes has the same heat loss yields

$$\sum q_i = 19 q_i$$

$$q_1 = q_i$$

$$q_2 = 6 q_i$$

$$q_3 = 12 q_i$$

Hence, from 2.2.1

$$\frac{q_1}{Q} = (1 - \eta) - \frac{q_2 + q_3}{Q} - a_i^2 = \frac{(1 - \eta - a_i^2)}{19}$$

$$q_2 = 6 q_i \text{ or } \frac{q_2}{Q} = \frac{6 (1 - \eta - a_i^2)}{19}$$

$$q_3 = 12 q_i \text{ or } \frac{q_3}{Q} = \frac{12 (1 - \eta - a_i^2)}{19}$$

which now yields the following equations containing the appropriate numerics based on the following rounded out values.

$$a_1 = 0.12$$

$$a_2 = 0.40$$

$$a_3 = 0.80$$

$$a_4 = 1.00$$

$$\sigma = 0.06$$

$$\text{Region 1.} \quad 0.12 < \rho < 0.40$$

$$\frac{T - T_s}{Q/2\pi K} = \frac{1}{2} (1 - \rho^2) - (1 - \eta) \log \frac{1}{\rho} + \frac{18}{19} (0.9856 - \eta) \log \frac{.4}{\rho} + \frac{12}{19} (0.9856 - \eta) \log 2$$

$$\text{Region 2.} \quad 0.40 < \rho < 0.80$$

$$\frac{T - T_s}{Q/2\pi K} = \frac{1}{2} (1 - \rho^2) - (1 - \eta) \log \frac{1}{\rho} + \frac{12}{19} (0.9856 - \eta) \log \frac{.8}{\rho}$$

$$\text{Region 3.} \quad 0.80 < \rho < 1.00$$

$$\frac{T - T_s}{Q/2\pi K} = \frac{1}{2} (1 - \rho^2) + (1 - \eta) \log \rho$$

The previous equations were used to calculate the axial temperature distributions shown in Figure D-1 and the data was used in the preparation of Figures 2 and 3.

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CYLINDRICAL MODEL TEMPERATURE DIFFERENCE DISTRIBUTION

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D-1