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N-flation

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Abstract

The presence of many axion fields in four-dimensional string vacua can lead to a simple, radiatively stable realization of chaotic inflation.

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1 Introduction

Slow roll inflation [1, 2] is the leading candidate for early universe cosmology. However, finding a fully realistic model of inflation without fine-tuning is an ongoing endeavor [3]. In this note we present a simple module for slow roll inflation that appears to be common in known string compactifications.

The essential idea of this paper is that the inflaton is not any single field, but a collection of N fields. Any one of the fields would not slow roll for an appreciable number of e-foldings, but when taken together, these fields self-damp and can slow roll for many e-foldings[1]. The predictions of the scenario are almost identical to those of the original $m^2\phi^2$ model of chaotic inflation.

Verifying that this model of slow roll inflation is under radiative control and not tuned requires detailed knowledge of the low energy effective action of string theory, including higher order curvature terms in the action, gravitational loop corrections, and an understanding of moduli stabilization. These details are important for two reasons. First, chaotic inflation is robust from the low energy point of view, but with reasonable assumptions about the ultraviolet dynamics, a functional fine-tuning of the potential is necessary to obtain a significant number of e-foldings. We will show how this model evades these arguments. Second, we will show that while classically it appears as though inflation can last for a period of time which is parametrically long as a function of N, radiative corrections change this parametric scaling into a numerical success. This sets an upper limit to the number of e-foldings achievable without tuning. Therefore we will need to understand the sizes of radiative corrections.

The organization of the paper is as follows. In Sec. 2 we describe the general idea of N-flation. Here we make arguments about the low energy effective theory and identify what information we need from the UV theory. In Sec. 3 we discuss how N-flation appears in a wide class of string theory compactifications. In Sec. 4 we show that the inflationary predictions match those of standard $m^2\phi^2$ chaotic inflation.

[1] There are several examples of multi-field inflationary models in the literature [4, 5]. In particular, the 'Assisted Inflation' model [4] exploits a similar idea using a specific exponential potential. In each of the models of [4, 5], issues of radiative stability, N-scaling and UV sensitivity need to be addressed.

2 Pythagoras Saves Chaotic Inflation

In this section we study a field theory with a potential of the form

$$V(\phi_n) = \sum_{n=1}^{N} V_n(\phi_n). \tag{1}$$

where each V_n only depends on a single ϕ_n. Without the potential, each ϕ_n would be a Goldstone boson with independent shift symmetries $\phi_n \to \phi_n + \delta_n$. Each V_n breaks a different shift symmetry, in contrast to a general potential which would break all of the shift symmetries in one shot. We will take the potential to be periodic since the inflatons will ultimately be axions

$$V_n(\phi_n) = \Lambda_n^4 \cos\left(\frac{2\pi\phi_n}{f_n}\right) + \Lambda_n^{(2)\,4} \cos\left(\frac{4\pi\phi_n}{f_n}\right) + \cdots \tag{2}$$

where f_n is the axion decay constant and Λ_n is the dynamically generated scale of the axion potential that typically arises from an instanton expansion. This scale can be many orders of magnitude beneath the Planck scale. Higher order instanton terms will give the higher harmonics in the potential, but are of the size

$$\Lambda_n^{(2)} \simeq \frac{\Lambda_n^2}{M} \tag{3}$$

where M is a UV scale. If $\Lambda \ll M$ it is safe to drop all higher overtones of the potential. Each f_n will be less than the Planck scale, though not significantly so[2]. Multi-instanton corrections to the potential can also violate the form of the potential in Eq. 1, leading to cross couplings between the axions

$$V_{nm}^{(2)} = \frac{\Lambda_n^4 \Lambda_m^4}{M^4} \cos\left(\frac{2\pi\phi_n}{f_n}\right) \cos\left(\frac{2\pi\phi_m}{f_m}\right). \tag{4}$$

Thus, when we are in a regime where the potential in Eq. 1 is valid, we drop higher order terms in the instanton expansion. We will now show that a potential of this form can inflate.

For small field values the potential can be Taylor expanded about its minimum as

$$V_n(\phi_n) = \frac{1}{2} m_n^2 \phi_n^2 - \frac{1}{24}\lambda_n \phi_n^4 + \cdots. \tag{5}$$

where $m_n = 2\pi\Lambda_n^2/f_n$ and $\lambda_n \simeq (2\pi\Lambda_n/f_n)^4$. For simplicity, we will take all masses $m_n = m$ in this main discussion. In Sec. 2.2 we show that examples with a spectrum of masses can still inflate.

[2] In the opposite regime $f > M_P$, which may not be attainable in string theory [6], one could make a model of 'Natural Inflation' [7].

Consider an initial configuration where every axion field starts out displaced from the minimum by $\langle \phi_{n0} \rangle = \alpha_n M_P$, with the maximum displacement set by each axion decay constant

$$\alpha_n^2 \lesssim \frac{f_n^2}{M_P^2}. \tag{6}$$

Here we are tacitly assuming that we can hold each α_n fixed as we take N to be larger; we will address this issue in Sec. 3. While each field has a sub-Planckian vev, the total displacement from the origin is super-Planckian, $\sim \sqrt{N}\alpha M_P$. In polar coordinates, $\rho^2 \equiv \sum_n \phi_n^2$, the action has the form

$$\mathcal{L} \simeq (\partial\rho)^2 + \rho^2(\partial\Omega)^2 - \frac{1}{2}m^2\rho^2 + \frac{1}{24N}\lambda\rho^4 + \cdots \tag{7}$$

with $\langle \rho_0 \rangle = \sqrt{N}\alpha M_P$. The angular fields, Ω, have big kinetic terms from $\langle \rho^2 \rangle \simeq N\alpha^2$, and are easily over-damped and drop out of inflationary dynamics. The N shift symmetries force corrections to the inflaton potential to be subdominant in the large N limit so that the potential can be trusted over a distance $\sqrt{N}f > M_P$. The form of the potential in Eq. 1 is crucially important for this to work; if the potential were $SO(N)$ symmetric there would be no added control of large vevs over a one-field model with many light (and irrelevant) fields, because there would be $\mathcal{O}(N^2)$ quartic couplings. The quartic self-couplings are small and will be dropped from now on. Finally, the volume of super-Planckian field space, $\rho > M_P$ grows much larger than sub-Planckian field space as the number of fields is increased. This means that the typical initial condition in the large N limit is expected to be super-Planckian and suitable for chaotic inflation.

It is possible to use the radial variable for the inflaton. Consequently, the gross inflationary predictions of these models coincide with those of $m^2\phi^2$ chaotic inflation. Each ϕ_n field satisfies the equation of motion

$$\ddot{\phi}_n + 3H\dot{\phi}_n = -m^2\phi_n \tag{8}$$

with $3H^2 = V/M_P^2 = N\alpha^2 m^2$ and grows with the number of fields, holding the initial condition of each field fixed. Eq. 8 shows that while each scalar feels the restoring force from its own mass term, it feels the Hubble friction from the entire N-field configuration. For the initial condition $\phi_{n0} = \alpha M_P$, the theory inflates for

$$N_e = \frac{\alpha^2 N}{4} \tag{9}$$

e-foldings until $\langle \phi_n \rangle$ drops to $\sim M_P/\sqrt{N}$.

The slow roll parameters η and ϵ are diagonal matrices with each entry given by

$$\eta \equiv M_P^2 \frac{V''}{V} \sim \frac{1}{\alpha^2 N}, \quad \epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \sim \frac{1}{\alpha^2 N^2}. \tag{10}$$

Density perturbations are given by

$$\frac{\delta\rho}{\rho} \sim N\alpha^2 \frac{m}{M_P} \tag{11}$$

Since $N\alpha^2$ sets the number of e-foldings, the small parameter controlling the density perturbations $\frac{\delta\rho}{\rho} \sim 2 \times 10^{-5}$ is the inflaton mass, requiring $m \sim 10^{10}$ TeV.

2.1 Radiative Stability

In this subsection, we will study the radiative corrections to the classical action in the previous section. These corrections take several forms. In turn, we will discuss the issues of large distances in field space, renormalization of the axion potential, the renormalization of Newton's constant, and finally the breaking of global symmetries by small black holes.

There is a general worry that slow roll inflation over Planckian field distances may not make sense in a quantum theory of gravity. The rough statement is that if you go more than $\mathcal{O}(M_P)$ away from a given minimum, string scale modes can become light, and there is a different effective theory with different degrees of freedom. However, axions are periodic (with periods smaller than M_P) and we have functional control of the potential; therefore we can safely consider all field values within the effective field theory.

In order to get slow roll inflation, it was crucial that the cross couplings between different axions were small. One could worry that loop effects might destabilize this form of the potential and spoil slow roll. Each axion is endowed with its own approximate shift symmetry. In the low energy theory only the potential breaks the shift symmetry. This means that any loop induced correction to the effective potential must be proportional to the breaking and thus takes on the form

$$\delta\mathcal{L}_{\text{eff}}(\phi_n) = \sum_n b_n V_n''(\phi_n)\mathcal{R} + \sum_{mn} \frac{c_{mn}}{M_P^4} V_n(\phi_n) V_m(\phi_m) + \cdots . \tag{12}$$

The first term in Eq. 12 is the induced coupling to the Ricci scalar which arises from one-loop gravity corrections. Induced cross couplings are the same size as the multi-instanton in Eq. 4 that we safely ignored earlier. That these effects are sufficiently small not to spoil slow roll inflation can be seen from the change in the slow-roll parameter

$$\delta\eta \sim (c_{mn}\eta + b_n\varsigma) H^2/M_P^2 \tag{13}$$

where $\varsigma \equiv M_P^4 V''''/V \sim N\eta^2$. This analysis shows that there is not a "low energy" problem with chaotic inflation. In particular, corrections of the form $\phi_n^2 V/M_P^2$ are forbidden by the shift symmetries of the axions.

A serious consideration is whether super-Planckian field configurations have simply been swapped for a species problem, since a large number of fields can enhance radiative corrections (for recent discussions of similar issues see [8]). There is a quadratically divergent contribution to the effective Planck mass from each light field

$$\delta M_P^2 \simeq \pm \frac{N}{16\pi^2}\Lambda_{\rm UV}^2 \tag{14}$$

which can dilute gravity, depending on the UV-sensitive sign. Hence, the correction to the η parameter which is induced by the shift in M_P

$$\eta \simeq \frac{1}{N\alpha^2}\left(1 \pm \frac{N\Lambda_{\rm UV}^2}{16\pi^2 M_P^2}\right) \tag{15}$$

dominates at very large N. This means that one can *not* get a parametrically large number of e-foldings in a regime where the classical contribution to the gravitational coupling is dominant. There is a value of the number of axions where the suppression of η saturates

$$N \simeq 16\pi^2 \frac{M_P^2}{\Lambda_{\rm UV}^2}. \tag{16}$$

Substituting this into the expression for the number of e-foldings

$$N_e^{\rm max} \simeq 40\alpha^2 \frac{M_P^2}{\Lambda_{\rm UV}^2}. \tag{17}$$

This looks very promising, but is clearly UV sensitive. We will address this in Sec. 3.1 where we estimate $\Lambda_{\rm UV}$ for the string realizations. Whether the species problem is severe enough to spoil slow roll inflation is a detailed numerical question.

The final worry is that small black holes violate global symmetries which include the shift symmetries of the axions. These may generate unsuppressed potentials for the axions and spoil slow roll inflation. This will not be problematic in the string realization because the axions' shift symmetries will descend from short distance gauge symmetries which are not violated by black holes.

Supersymmetric Radiative Stability

Supersymmetry is not crucial for this general inflationary mechanism – the only required ingredient is that each field is endowed with its own softly broken shift symmetry. If the low energy theory is supersymmetric, then the arguments of the previous section need to be supplemented by those that we consider here. The string models we consider in the next section will have low energy supersymmetry which introduces additional issues. The axions each lie in their own chiral superfield

$$t_n = \left(\frac{\phi_n}{f_n} + iM^2\, R_n^2\right) \tag{18}$$

where R_n^2 is the modulus associated with ϕ_n, and M is a UV scale. For a supersymmetric theory, we need to use the supergravity effective potential

$$V_{\rm sugra}(\phi_n) = \exp\left(\frac{K}{M_P^2}\right)\left(|DW|^2 - 3\frac{|W|^2}{M_P^2}\right). \tag{19}$$

There are three separate quantities in the potential which might break the axion shift symmetries: $|DW|^2$, $|W|^2$ and K. The arguments of the previous section apply directly to $|DW|^2$, but we will have to consider $|W|^2$ and K separately now.

The supergravity potential contains corrections to the rigid supersymmetric potential of the form K/M_P^2 and results in the supergravity $\eta \sim \mathcal{O}(1)$ problem [9]. Because our inflatons are axions, the Kähler potential is a function of $t_n - t_n^\dagger$, which is independent of the axion, ϕ_n. Therefore the Kähler potential does not spoil slow roll inflation [10]. The Kähler potential does give a mass for the moduli R_n^2 that causes them to roll down to their respective minima quickly and decouple from inflationary dynamics.

The instanton induced superpotential is given by

$$W \simeq W_0(S) + \sum_n w_n(S)\; e^{2\pi i t_n} + \mathcal{O}(e^{4\pi i t_n}). \tag{20}$$

$W_0(S)$ parametrizes the physics which stabilizes the dilaton S; its detailed form is irrelevant for our purposes and can be approximated by

$$W_0(S) = w_0 + m^2 S + \cdots . \tag{21}$$

The leading axion potential arises from the cross-term in the F-term of S

$$|F_S|^2 = \left|\sum_n \partial_S w_n\; e^{2\pi i t_n} + m^2\right|^2 = m^2 \partial_S w_n e^{-2\pi M^2 R_n^2} \cos\left(\frac{2\pi\phi_n}{f_n}\right) + \mathcal{O}(e^{-4\pi M^2 R_n^2}). \tag{22}$$

The last term in the supergravity potential, $|W|^2$, could introduce significant cross couplings between the axions. Using the form of the superpotential in Eq. 20, we see that all cross-couplings are at least two instanton terms and therefore safe.

2.2 A Spectrum of Masses

So far we have considered the axions to have the same masses. However we expect them to have a non-trivial spectrum. In this section we briefly outline the analysis for a more general set of masses. We will show that N-flation is insensitive to this distribution.

A more realistic mass distribution is uniform on a log scale. For example, there could be hundreds of (roughly degenerate) fields in each decade of energy starting from near M_P down to 10^{10} TeV or below. This will result in sequential or multi-step inflationary periods, each one setting the stage for the next. If there are many fields at a sufficiently high density then we can approximate the field index with a continuous label $\phi_n \to \phi(n)$ with masses $m_n^2 \to m^2(n)$. For a uniform density of fields on a logarithmic energy scale, the masses take the form

$$m^2(n) = M_P^2 e^{-n/\sigma} \tag{23}$$

where σ is the density of fields per decade. If the fields start with initial conditions $\phi(n, t = 0) = \alpha M_P$, then all of the fields are over-damped if $\sigma\alpha^2 \gg 1$. At first only the heaviest fields begin sliding down the potential. After a Hubble time the heaviest fields are no longer over-damped. Instead of immediately becoming under-damped and oscillating (thereby ending inflation) they remain critically damped due to the presence of the lighter fields. Hubble changes slowly ($\dot{H}/H^2 \sim 1/\sigma\alpha^2$) so that the amount of time that a field of mass m stays critically damped is

$$\Delta t \sim \frac{H}{\dot{H}} = \frac{\sigma\alpha^2}{H} \sim \frac{\sigma\alpha^2}{m}. \tag{24}$$

During critical damping the fractional loss in amplitude is given by

$$\frac{\phi_F}{\phi_I} \sim \exp(-m\Delta t) \sim \exp(-\sigma\alpha^2). \tag{25}$$

This shows that all the potential energy of the heaviest fields is dissipated away before it can be converted into kinetic energy. Inflation proceeds until the final fields are no longer over-damped. Schematically, the first period of inflation will create a large smooth patch (solving the patch problem), the period 60 e-foldings from the end will give rise to $\delta\rho/\rho \sim 10^{-5}$, and the last period will reheat the universe.

The general lesson is that if the axion masses are densely spaced, there will be sufficient self-damping to allow inflation to proceed.

3 The Many Axions of String Theory

In oriented critical superstring theories, there is a massless two-form field, $B_{\mu\nu}$, and when the ten-dimensional theory is compactified to four dimensions there are many independent two-cycles that $B_{\mu\nu}$ can wrap. Each such cycle results in an axion at low energies. Compactification on a six-manifold M_6 leads to $N = h^{(2)}(M_6)$ such axions; N can be very large[3]. These axions have independent shift symmetries that keep them lighter than the scale of compactification even in the absence of supersymmetry [12]. Ultraviolet physics can make no contributions to the axion masses because above the scale of compactification, these global symmetries become gauge symmetries of $B_{\mu\nu}$. In particular, short distance physics (*e.g.* small black holes) cannot violate the shift symmetries of the axions. In type II theories, similar considerations apply to the higher p-form fields.

The axions are paired into chiral superfields $t_n = \phi_n/f_n + iR_n^2/\alpha'$ where R_n^2 is the modulus associated with the volume of the nth two-cycle. The volume of M_6 is related to the sizes of these cycles by

$$V_6(t) = i\frac{\alpha'^3}{6}C^{lmn}t_l t_m t_n\Big|_{\mathrm{Re}\ t=0} \tag{26}$$

where C^{lmn} are determined solely by the topology of M_6, and are integers. The form C is very sparse, having only $\mathcal{O}(N)$ nonzero entries in many models, rather than scaling as $\mathcal{O}(N^3)$. These integers may be negative; this can be interpreted as resulting from a cycle that reduces the volume of the space as it grows larger. First, notice that if all of the intersection numbers are positive, then the volume grows as N and M_P^2 falls as $1/N$, spoiling any gain in η from having many fields. However, it is generic to have negative intersection numbers, allowing string scale volumes despite the presence of many fields. The next question is whether this cancellation which makes the volume small is just the tuning to get the potential sufficiently flat. This is the question of where it is natural to stabilize moduli.

At large radius, the Kähler potential for moduli is $\alpha' K(t,\bar{t}) = -\ln\left(V_6(t-\bar{t})/\alpha'^3\right)$, with V_6 given in Eq. 26. Therefore, the axion decay constants are

$$\begin{aligned}\frac{f_{mn}^2}{M_P^2} &= \partial_m\bar{\partial}_n K = \frac{\alpha'^{\,2}C^{mnl}R_l^2}{V_6} - \frac{\alpha'^{\,2}C^{mlk}R_l^2R_k^2C^{nl'k'}R_{l'}^2R_{k'}^2}{4V_6^2} + \mathcal{O}(e^{-4\pi R^2/\alpha'}) \\ &\sim \frac{\alpha'^2R^2}{V_6} \equiv \alpha^2.\end{aligned} \tag{27}$$

where in the last approximation we have taken the volume to be slightly larger than any individual radius (which we take to be approximately the same size). Note that the complicated

[3]It follows from known examples of F-theory compactification that there exist supersymmetric string models with $\mathcal{O}(10^5)$ axions [11].

second term in the first line of Eq. 27 only affects one of the N eigenvalues of f_{mn}. Where the volume enjoys cancellations between its various terms while maintaining positivity of the metric on moduli space, one can expect larger decay constants. One important factor in determing where moduli are stabilized is the volume of moduli space. The volume of moduli space has measure factor

$$\det \frac{f_{mn}^2}{M_P^2} \sim \left(\frac{\alpha'^{\,2} R^2}{V_6} \right)^N . \tag{28}$$

Note therefore that where the measure is large, f^2/M_P^2 is also large. This indicates that on a significant portion of the volume of the moduli space, one finds string-scale volumes and $f^2 \sim M_P^2$. This is not the only factor in determining the distribution of decay constants, but others are less well-understood and are model dependent. Nevertheless, it is plausible that the moduli space volume form favors small bulk volumes [13] and hence large axion decay constants.

The axions' shift symmetry is only broken by W_n and not K, which respects $t_n \to t_n + \delta_n$ in perturbation theory. The low energy effective action for the supersymmetric theory is given by

$$\mathcal{L} = \int d^4\theta \; K\left(t - t^\dagger, S - S^\dagger\right) + \int d^2\theta \left(\sum_n^N W_n(t_n, S) + W_0(S) + \cdots \right) + \text{ h.c.} \tag{29}$$

The superpotential is

$$W_n \simeq \sum_\ell w_n^\ell(S) \; e^{2\pi i \ell t_n}; \tag{30}$$

for axions associated with the NSNS B-field, this is generated by worldsheet instantons. This is the superpotential we studied in Sec. 2.1. There are also multi-instanton terms that can wrap two different cycles and are given by

$$W_{nm}^{(2)} \simeq w_{nm}(S) \; e^{2\pi i (t_n + t_m)}. \tag{31}$$

This is the same parametric scaling as described in Eq. 4.

A similar discussion applies to the axions arising from RR p-forms in type II string theories, where the role of the worldsheet instantons is played by Euclidean D-branes.

3.1 Radiative corrections in string theory

In Sec. 2.1 we saw that we needed to estimate corrections to M_P^2 that grew with the number of axions. Any dynamics at distances shorter than the compactification scale cannot be sensitive to the number of axion fields because they all descend from a single ten-dimensional field[4]. The easiest way to proceed is to find corrections to the ten-dimensional action which after compactification become proportional to the number of fields. These operators are higher derivative corrections to the gravitational effective action that can arise both classically and at loop level. The first term that becomes sensitive to the number of axions is [15]

$$\mathcal{L}_{10D\ \mathrm{eff}} \simeq M_*^8 \left(\mathcal{R}_{10} + \zeta(3)\alpha'^3 \mathcal{R}_{10}^4 + \cdots \right) \tag{32}$$

with M_* the 10D Planck scale: $M_*^{-8} = g_s^2 (\alpha')^4 (2\pi)^7/2$. Upon compactification, $\mathcal{R}_{10}^4$ contains an $(\mathcal{R}_6 \wedge \mathcal{R}_6 \wedge \mathcal{R}_6)\mathcal{R}_4$ term and since

$$\int_{M_6} \mathcal{R}_6 \wedge \mathcal{R}_6 \wedge \mathcal{R}_6 = \frac{\chi(M_6)}{(2\pi)^3} \ , \tag{33}$$

integrating over the six internal dimensions gives a correction to Newton's constant proportional to χ. The Euler character χ is a measure of the number of light species after compactification

$$\chi(M_6) = 2(N - \tilde{N}) \tag{34}$$

where $\tilde{N}$ is the number of complex structure moduli of M.

The 4D effective Einstein-Hilbert term for string theory compactified on a Calabi-Yau six-manifold M_6 is of the form

$$\mathcal{L}_{4D\ \mathrm{eff}} = M_P^2 \left(1 + \chi(M_6) \left(\frac{\alpha'}{2\pi} \right)^3 \frac{\zeta(3)}{V_6} \right) \mathcal{R}_4 \tag{35}$$

where V_6 is the volume of the internal space. The second term, which arises from the reduction of the sigma-model four-loop $\mathcal{R}^4$ correction [15], is proportional to the "density of cycles in string units." It can be interpreted as a back reaction of the internal space to packing a huge amount of topology in a small volume.

There are higher order terms in the ten-dimensional effective action that are suppressed by more powers of $\alpha'/2\pi$. These local operators, however, can never scale more than linearly with the number of light species, because the wavefunctions of these modes are localized in the internal space.

There are also g_s corrections to the effective action. These loops can both renormalize the short distance 10D effective action and give terms that can only be written in terms

[4]For concreteness and simplicity, the discussion here is appropriate for heterotic Calabi-Yau compactification with the gauge connection set equal to the spin connection. A similar discussion would apply to examples which are known to have worldsheet instanton generated potentials, *e.g.* [14].

of operators in the 4D effective action. The renormalization of the short distance effective action (*e.g.* to $\mathcal{R}_{10}^4$) is clearly suppressed by g_s^2, and therefore is always subdominant at weak enough string coupling [16]. IR contributions to the effective action are always cut off at the KK scale. They can become sensitive to higher powers of the number of axions, but we expect that it requires n loops to become sensitive to N^n. The string loop effects are also suppressed by at least $1/6\pi$ (by standard reasoning of naive dimensional analysis [17]), so the n loop contribution to M_P^2 could be as large as

$$\left(\delta M_P^2\right)_n \sim \left(\frac{g_s^2 N}{6\pi}\right)^n M_{KK}^2. \tag{36}$$

Therefore, if $g_s^2 \lesssim 6\pi/N$, string loop corrections can be safely ignored.

The leading effect on M_P is given by

$$\frac{\delta M_P^2}{M_P^2} = \frac{\chi(M_6)}{8\pi^3}\zeta(3)\frac{\alpha'^3}{V_6} \simeq \frac{\chi(M_6)}{206}\frac{\alpha'}{R^2}\alpha^2 \tag{37}$$

where we have used Eq. 27 in the final expression. The volume of moduli space is peaked around $V_6 \sim \alpha'^3$ which coincides with the largest values of α^2. Note that this formula cannot be trusted in a regime where the correction to Newton's constant cancels the tree-level term; at small volumes, this occurs at $\chi(M_6) \lesssim -200$. Comparing our high- and low-energy estimates for the renormalization of Newton's constant, Eq. 37 and Eq. 14, we infer that $\Lambda_{\rm UV}^2 \simeq M_P^2 \frac{2\zeta(3)}{\pi}\frac{\alpha'^3}{V_6}\frac{\chi}{N}$. The number of e-foldings is set by the number of axions, rather than $\chi(M_6)$. The number of e-foldings (using Eq. 16 and setting $R^2/\alpha' = 1$) is then

$$N_e^{\rm max} \simeq \frac{2\pi^3}{\zeta(3)}\frac{N}{|\chi(M_6)|} \approx 26\frac{1}{|1-\tilde{N}/N|}. \tag{38}$$

Note that the initial value α cancels out of this expression. Thus it takes a small cancellation between the integers N and $\tilde{N}$ to get 60 e-foldings. For instance, there is a Calabi-Yau with $(N, \tilde{N}) = (251, 251)$ where the dominant correction to M_P^2 vanishes and it is possible to have $N_e \simeq N/4$.

A number of possibilities can help with the $\mathcal{O}(1)$ factor. In examples arising from type IIA string theory, the number of closed-string axions is actually $h^{(1,1)} + h^{(2,1)}$, namely $N + \tilde{N}$ in our notation. Secondly, in Eq. 38 we have conservatively taken $R^2 \sim \alpha'$; however R can be smaller, $R^2 \sim \alpha'/2\pi$, while preserving a reasonable instanton expansion. Next, there may be spaces for which the intersection form allows V_6 large preserving the fact that many decay constants satisfy $f/M_P \sim 1$. Finally, it would be interesting to study the large-N statistics of Eq. 27 on the space of CYs; any robust large-N scaling which shrinks more slowly than $1/N$ would lead to a parametric win in the number of e-foldings.

N-flation required no model building or tuning of continuous parameters to achieve slow roll inflation. The number of axions needed for the requisite number of e-foldings is (suggestively) at the high end of values available from known string compactifications.

4 Inflationary Repredictions

In this section we quickly give the standard inflationary observables. Throughout we will express our final answers in terms of N_e (which fixes $N\alpha^2$) and $\delta\rho/\rho$ (which fixes m) in order to demonstrate that the predictions are identical to those in standard chaotic inflation.

In [18], we find a general formula for the tilt which is applicable in this class of N-field models. The tilt is

$$1 - n = \frac{8}{\alpha^2 N} = \frac{2}{N_e}. \tag{39}$$

Here and below, one should set $\alpha^2 N \simeq 240$ to find the values that would be relevant for the few e-foldings visible near our present horizon.

The power in gravity waves [19] is (in the convention of [20])

$$P_g = \frac{2}{3\pi^2}\frac{V}{M_P^4} = \frac{\alpha^2 N m^2}{3\pi^2 M_P^2} = \frac{4}{3\pi^2}\frac{1}{N_e}\left(\frac{\delta\rho}{\rho}\right)^2 \tag{40}$$

at the start of inflation where $\langle\phi_n\rangle \sim \alpha M_P$.

The spectral index of the gravitational waves is

$$n_g = 2\frac{\dot{H}}{H^2} = -\frac{4}{\alpha^2 N} = -\frac{1}{N_e}. \tag{41}$$

The relative magnitude of the gravity waves to density perturbations, r, is given by

$$r \sim \frac{P_g}{P_{\mathcal{R}}} \sim \frac{32}{\alpha^2 N} = \frac{8}{N_e}. \tag{42}$$

Non-Gaussian features in the spectrum of perturbations remain a small effect. To see this, we can use the formalism in [21], where f_{NL} is given by

$$-3/5 f_{NL} = \frac{\sum_{ij} N_i N_j N_{ij}}{2(\sum_i N_i^2)^2} + \ln kL\ P/2\frac{\sum_i N_{ii}^3}{(\sum_i N_i^2)^3} = (1+6)\eta + \ln kL\ \frac{P}{2}N\eta^3 \tag{43}$$

with $N_i = \partial N_e/\partial\phi_i$, and $N_{ij} = \partial^2 N_e/\partial\phi_i\partial\phi_j$, and P is the power spectrum in the inflaton. The second term, while N enhanced, is subdominant because of the additional powers of m^2/M_P^2. This answer was to be expected from the similarity to chaotic inflation. The only difference from the calculation in *e.g.* [22] is that there are $N-1$ over-damped scalars with $m^2 \ll H^2$. However, these additional fields have very small cross couplings and are essentially free fields, and their relative contribution to the energy density (including their quantum fluctuations) is down by at least $(H/M_P)^2$ compared to the inflaton. In order for non-gaussianities to be visible, additional dynamics is needed. It would be interesting to know under what circumstances these effects would be visible.

Since the inflaton is a pseudo-Goldstone boson, the leading couplings to other fields are at least dimension 5. This lowers the reheat temperature. For instance, a typical coupling which respects the shift symmetry is $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}/M_P$. This leads to an inflaton decay width of

$$\Gamma_\phi \sim \frac{m_\phi^3}{8\pi M_P^2} \tag{44}$$

and a reheat temperature of

$$T_{\rm RH} = \sqrt{\Gamma_\phi M_P} \simeq m_\phi \sqrt{\frac{m_\phi}{8\pi M_P}}. \tag{45}$$

For typical inflaton masses, $m_\phi \sim 10^{10}$ TeV, $T_{\rm RH} = 10^7$ TeV and light gravitationally coupled particles (*i.e.* gravitinos) are not reheated, eliminating the gravitino problem.

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References

[1] A. Guth, Phys. Rev. **D23** (1981) 347; A. Linde, Phys. Lett. **B108** (1982) 389; A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48** (1982) 1220.

[2] A. Linde, Phys. Lett. **B129** (1983) 177.

[3] Recent attempts include T. Banks et al, Phys. Rev. **D52** (1995) 3548, hep-th/9503114; P. Binetruy and G. Dvali, Phys. Lett. **B388** (1996) 241; G. Dvali and S.H. Tye, Phys. Lett. **B450** (1999) 72; C. Burgess et al, JHEP 0107 (2001) 047, hep-th/0105204; C. Burgess et al, JHEP 0203 (2002) 052, hep-th/0111025; N. Jones, H. Stoica and S.H. Tye, JHEP 0207 (2002) 051, [arXiv:hep-th/0203163]; N. Arkani-Hamed, H.C. Cheng, P. Creminelli and L. Randall, JCAP **0307** (2003) 003, [arXiv:hep-th/0302034]; S. Kachru et al, JCAP **0310** (2003) 013, [arXiv:hep-th/0308055]; E. Silverstein and D. Tong, Phys. Rev. **D70** (2004) 103505, [arXiv:hep-th/0310221]; M. Alishahiha, E. Silverstein and D. Tong, Phys. Rev. **D70** (2004) 123505, [arXiv:hep-th/0404084]; J. Hsu, R. Kallosh and S. Prokushkin, JCAP **0312** (2003) 009, [arXiv:hep-th/0311077]; H. Firouzjahi and S.H. Tye, Phys. Lett. **B584** (2004) 147, [arXiv:hep-th/0312020]; J. Hsu and R. Kallosh, JHEP **0404** (2004) 042, [arXiv:hep-th/0402047]; N. Iizuka and S. P. Trivedi, Phys. Rev. D **70**, 043519 (2004) [arXiv:hep-th/0403203]; O. DeWolfe, S. Kachru and H. Verlinde, JHEP **0405** (2004) 017, [arXiv:hep-th/0403123]; S. Shandera, hep-th/0412077; G. Dvali and S. Kachru, in the Ian Kogan memorial volume, hep-th/0309095; E. Copeland and A. Rajantie, astro-ph/0501068; R. Easther, J. Khoury and K. Schalm, JCAP **0406** (2004) 006, [arXiv:hep-th/0402218]; M. Berg, M. Haack and B. Kors, Phys. Rev. D **71**, 026005 (2005), [arXiv:hep-th/0404087], arXiv:hep-th/0409282; J. Blanco-Pillado et al, JHEP **0411** 063 (2004), [arXiv:hep-th/0406230]; R. Easther, hep-th/0407042; K. Freese and D. Spolyar, hep-ph/0412145; C.P. Burgess, et al, hep-th/0501125; X. g. Chen, Phys. Rev. D **71**, 063506 (2005), [arXiv:hep-th/0408084], arXiv:hep-th/0501184; L. McAllister, arXiv:hep-th/0502001; D. Cremades, F. Quevedo and A. Sinha, hep-th/0505252. For recent reviews, see: A. Linde, hep-th/0402051; J. Cline, hep-th/0501179.

[4] A. Liddle, A. Mazumdar and F. Schunk, Phys. Rev. D **58**, 061301 (1998), [arXiv:astro-ph/9804177]; P. Kanti and K. A. Olive, Phys. Rev. D **60**, 043502 (1999), [arXiv:hep-ph/9903524], Phys. Lett. B **464**, 192 (1999), [arXiv:hep-ph/9906331]; N. Kaloper and A. R. Liddle, Phys. Rev. D **61**, 123513 (2000), [arXiv:hep-ph/9910499].

[5] Y.S.Piao et al, Phys.Rev. **D66**, 121301 (2002), [arXiv: hep-ph/0207143]; R. Brandenberger, P. Ho and H. Kao, JCAP **0411** (2004) 011, [arXiv:hep-th/0312288]; A. Jokinen and A. Mazumdar, Phys. Lett. **B597** (2004) 222, [arXiv:hep-th/0406074]; K. Becker, M. Becker and A. Krause, "M-theory inflation from multi M5-brane dynamics," hep-th/0501130.

[6] T. Banks, M. Dine, P. J. Fox and E. Gorbatov, JCAP **0306**, 001 (2003) [arXiv:hep-th/0303252].

[7] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).

[8] N. Arkani-Hamed, S. Dimopoulos and S. Kachru, arXiv:hep-th/0501082; J. Distler and U. Varadarajan, arXiv:hep-th/0507090.

[9] E. Copeland, A. Liddle, D. Lyth, E. Stewart and D. Wands, Phys. Rev. **D49** (1994) 6410.

[10] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. **85** (2000) 3572.

[11] P. Candelas, E. Perevalov and G. Rajesh, Nucl. Phys. B **507**, 445 (1997) [arXiv:hep-th/9704097].

[12] M. Green, J. Schwarz and E. Witten, *Superstring Theory: Volume II*, Cambridge University Press, 1987.

[13] J. H. Horne and G. W. Moore, Nucl. Phys. B **432**, 109 (1994), [arXiv:hep-th/9403058]; S. Ashok and M. R. Douglas, JHEP **0401**, 060 (2004), [arXiv:hep-th/0307049].

[14] R. Donagi, A. Grassi and E. Witten, Mod. Phys. Lett. A **11**, 2199 (1996) [arXiv:hep-th/9607091]; F. Denef, M. R. Douglas and B. Florea, JHEP **0406**, 034 (2004) [arXiv:hep-th/0404257]; F. Denef, M. R. Douglas, B. Florea, A. Grassi and S. Kachru, arXiv:hep-th/0503124.

[15] M. T. Grisaru, A. E. M. van de Ven and D. Zanon, Phys. Lett. B **173**, 423 (1986); D. J. Gross and E. Witten, Nucl. Phys. B **277**, 1 (1986).

[16] I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, Nucl. Phys. B **507**, 571 (1997), [arXiv:hep-th/9707013].

[17] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).

[18] M. Sasaki and E. Stewart, Prog. Theor. Phys. **95** (1996) 71, [arXiv:astro-ph/9507001].

[19] A.A. Starobinsky, Sov. Astron. Lett. **9** (1983) 302; Sov. Astron. Lett. **11** (1985) 133.

[20] H. Peiris et al, Astrophys. J. Suppl. **148** (2003) 213, [arXiv:astro-ph/0302225].

[21] D. H. Lyth and Y. Rodriguez, arXiv:astro-ph/0504045.

[22] V. Acquaviva, N. Bartolo, S. Materrese and A. Riotto, Nucl. Phys. **B667** (2003) 119, [arXiv:astro-ph/0209156]; J. Maldacena, JHEP **0305** (2003) 013, [arXiv:astro-ph/0210603].