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REPORT

**RELATIVISTIC ION
IN ELECTROMAGNETIC WAVE**

(BATTELLE-NORTHWEST PROGRAM ORBIT)

**BERNARD H. DUANE
REACTOR PHYSICS DEPARTMENT**

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By

Bernard H. Duane

Theoretical Physics Unit
Reactor Physics Department

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RELATIVISTIC ION IN ELECTROMAGNETIC WAVE
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Bernard H. Duane

ABSTRACT

Battelle-Northwest program Orbit constructs and plots eight-dimensional phase-space orbits for a relativistic charged particle driven by an electromagnetic plane wave.

MATHEMATICAL PHYSICS

EQUATIONS OF MOTION

Consider the relativistic motion of a charged particle driven by an electromagnetic plane wave, as illustrated in Figure 1. In the four-dimensional space-time of special relativity⁽¹⁾ spanned by the line element

$$ds^2 = dx^2 + dy^2 + dz^2 + (ict)^2 = -c^2 d\tau^2, \quad (1)$$

a charge q having rest energy mc^2 is at four-position $\underline{X} = (x \ y \ z \ ict)$ with four-momentum $\underline{P} = (p_x \ p_y \ p_z \ iE/c) = m \ d\underline{X}/d\tau$ at proper time τ . The electromagnetic potential-momentum field

$$\underline{A} \cos(\underline{K} \cdot \underline{X}) \quad (2)$$

has propagation vector $\underline{K} = (0 \ 0 \ k \ i\omega/c)$, amplitude $\underline{A} = (A \ 0 \ 0 \ 0)$, and phase $\underline{K} \cdot \underline{X} = kz - \omega t$. The propagation vector has null magnitude and is perpendicular to the wave amplitude.

$$\underline{K} \cdot \underline{K} = 0 = k^2 - \omega^2/c^2, \text{ and } \underline{K} \cdot \underline{A} = 0. \quad (3)$$

The magnetic force field $[B_y = kA \sin(\omega t - kz)]$ and the electric force field $[E_x = \omega A \sin(\omega t - kz)]$ appear respectively as the space-like ($\underline{B}$) and time-like ($\underline{E}/ic$) components of the potential-momentum curl,

$$(\underline{A} \underline{K} - \underline{K} \underline{A}) \sin(\underline{K} \cdot \underline{X}). \quad (4)$$

In this representation, Einstein's equations of motion take the form

$$d\underline{P}/d\tau = (q/m)(\underline{A} \underline{K} - \underline{K} \underline{A}) \cdot \underline{P} \sin(\underline{K} \cdot \underline{X}). \quad (5)$$

FIRST INTEGRALS

Contracting this equation of motion upon the propagation vector $\underline{K}$ with the help of Equation (3) yields the phase integral

$$\begin{aligned} \underline{K} \cdot d\underline{P}/d\tau &= 0, \text{ or } \underline{K} \cdot \underline{P} = \underline{K} \cdot \underline{P}_0, \text{ or} \\ \underline{K} \cdot \underline{X} &= \underline{K} \cdot \underline{X}_0 + \underline{K} \cdot \underline{P}_0 \tau/m. \end{aligned} \quad (6)$$

$$\underline{X} = (x \ y \ z \ ict), \ \underline{P} = (p_x \ p_y \ p_z \ iE/c), \ \underline{A} = (A \ 0 \ 0 \ 0), \ \underline{K} = (0 \ 0 \ k \ i\omega/c)$$

$$ds^2 = dx^2 + dy^2 + dz^2 + (dict)^2 = -c^2 d\tau^2$$

$$d\underline{P}/d\tau = (q/m)(\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot \underline{P} \ f'(\underline{K} \cdot \underline{X})$$

Matrix-Exponential Solution

$$\underline{K} \cdot \underline{X} = \underline{K} \cdot \underline{X}_0 + \underline{K} \cdot \underline{P}_0 \ \tau/m$$

$$\underline{P} = \{ \exp[(q/\underline{K} \cdot \underline{P}_0)(\underline{K} \underline{A} - \underline{A} \underline{K}) \ f(\underline{K} \cdot \underline{X}) \big|_0^{\tau}] \} \cdot \underline{P}_0$$

$$\underline{X} = \underline{X}_0 + \int_0^{\tau} \underline{P}(\tau') \ d\tau'/m$$

FIGURE 1

ION IN ELECTROMAGNETIC WAVE

With the help of this phase integral, the equations of motion (5) now integrate by inspection to the matrix-exponential solution

$$\underline{P} = \{ \exp[(q/K \cdot \underline{P}_0)(\underline{K} \underline{A} - \underline{A} \underline{K}) \cos(\underline{K} \cdot \underline{X}) |_0^\tau] \} \cdot \underline{P}_0. \quad (7)$$

The matrix power series defining the matrix exponential function terminates here with its second-order term, since from Equation (3)

$$(\underline{K} \underline{A} - \underline{A} \underline{K})^2 = (\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot (\underline{K} \underline{A} - \underline{A} \underline{K}) = -\underline{K} \underline{A}^2 \underline{K},$$

so

$$(\underline{K} \underline{A} - \underline{A} \underline{K})^3 = -(\underline{K} \underline{A}^2 \underline{K}) \cdot (\underline{K} \underline{A} - \underline{A} \underline{K}) = 0. \quad (8)$$

The complete first integral, Equation (7), thus simplifies to its low order terms,

$$\begin{aligned} \underline{P} = \underline{P}_0 + (q/K \cdot \underline{P}_0)(\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot \underline{P}_0 \cos(\underline{K} \cdot \underline{X}) |_0^\tau \\ - (1/2)[(qA/K \cdot \underline{P}_0) \cos(\underline{K} \cdot \underline{X}) |_0^\tau]^2 \underline{K} \underline{K} \cdot \underline{P}_0. \end{aligned} \quad (9)$$

Translated back into three-space language and arranged as a numeric computation sequence, Equations (6) and (9) become

$$\begin{aligned} p_x/mc = p_x^0/mc - (qA/mc) \{ \cos[(E^0/mc^2 - p_z^0/mc)\omega\tau + \omega t_0 \\ - kz_0] \} |_0^\tau, \end{aligned} \quad (10)$$

$$p_y/mc = p_y^0/mc, \quad (11)$$

$$\begin{aligned} p_z/mc = p_z^0/mc + (1/2)(p_x/mc - p_x^0/mc)(p_x/mc + p_x^0/mc)/(E^0/mc^2 \\ - p_z^0/mc), \end{aligned} \quad (12)$$

$$E/mc^2 = E^0/mc^2 + (p_z/mc - p_z^0/mc). \quad (13)$$

Program Orbit constructs the first integrals as a function of proper time τ by direct evaluation of the four boxed equations.

SECOND INTEGRALS

Multiplying the momentum integral, Equation (9), by $d\tau/m$ and integrating with the help of the phase integral, Equation (6), now gives for the orbit equations,

$$\begin{aligned}\underline{X} = \underline{X}_0 + \underline{P}_0 \tau / m + [q / (\underline{K} \cdot \underline{P}_0)^2] (\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot \underline{P}_0 [\sin(\underline{K} \cdot \underline{X}) \\ - \underline{K} \cdot \underline{X} \cos(\underline{K} \cdot \underline{X}_0)]_0^\tau - (1/2)(qA/\underline{K} \cdot \underline{P}_0)^2 \{ \underline{K} \cdot \underline{X} [1/2 + \cos^2(\underline{K} \cdot \underline{X}_0)] \\ - 2 \sin(\underline{K} \cdot \underline{X}) \cos(\underline{K} \cdot \underline{X}_0) + (1/4) \sin(2\underline{K} \cdot \underline{X}) \}_0^\tau \underline{K}.\end{aligned}\quad (14)$$

Translated back into three-space language and arranged as a numeric computation sequence, these orbit equations become

$$\omega_0 = (E^0/mc^2 - p_z^0/mc) \omega, \quad (15)$$

$$\begin{aligned}kx = kx_0 + (p_x^0/mc) \omega \tau + (qA/mc) (\omega/\omega_0) [\omega_0 \tau \cos(\omega t_0 - kz_0) \\ - \sin(\omega_0 \tau + \omega t_0 - kz_0)]_0^\tau,\end{aligned}\quad (16)$$

$$ky = ky_0 + (p_y^0/mc) \omega \tau, \quad (17)$$

$$\begin{aligned}kz = kz_0 + (p_z^0/mc) \omega \tau \\ + (qA/mc) (\omega/\omega_0)^2 (p_x^0/mc) [\omega_0 \tau \cos(\omega t_0 - kz_0) \\ - \sin(\omega_0 \tau + \omega t_0 - kz_0)]_0^\tau \\ + (1/2)(qA/mc)^2 (\omega/\omega_0)^2 \{ \omega_0 \tau [1/2 + \cos^2(\omega t_0 - kz_0)] \\ + (1/4) \sin 2(\omega_0 \tau + \omega t_0 - kz_0) \\ - 2 \sin(\omega_0 \tau + \omega t_0 - kz_0) \cos(\omega t_0 - kz_0) \}_0^\tau,\end{aligned}\quad (18)$$

$$\omega t = \omega t_0 + \omega_0 \tau + kz - kz_0. \quad (19)$$

Program Orbit constructs the second integrals as a function of proper time τ by direct evaluation of these boxed equations (divided by 2π to simplify phase recognition).

BREMSSTRAHLUNG INTEGRAL

An accelerated charge radiates energy from its rest frame at the proper time rate⁽²⁾

$$dE_Y/d\tau = (\mu_0 c/6\pi)(q d\underline{P}/mc d\tau)^2, \quad (20)$$

where $\mu_0 c$ is the impedance of free space (377 ohms). Making use of Equations (5), (6), and (8) reduces this expression to

$$\begin{aligned} dE_Y/mc^2 &= (\mu_0 q^2/6\pi m)(q/mc)^2 (\underline{P}/mc) \cdot (\underline{K} \underline{A} - \underline{A} \underline{K}) \cdot (\underline{A} \underline{K} \\ &\quad - \underline{K} \underline{A}) \cdot (\underline{P}/m) \sin^2(\underline{K} \cdot \underline{X}) d\tau, \\ &= (\mu_0 q^2/6\pi m)(qA/mc)^2 (\underline{P}/mc) \cdot \underline{K} \underline{K} \cdot (\underline{P}/m) \sin^2(\underline{K} \cdot \underline{X}) d\tau, \\ &= (\mu_0 q^2/6\pi m)(qA/mc)^2 (\underline{K} \cdot \underline{P}_0/mc) \sin^2(\underline{K} \cdot \underline{X}) d(\underline{K} \cdot \underline{X}). \end{aligned} \quad (21)$$

The radiative energy E_Y thus integrates to

$$\begin{aligned} E_Y/mc^2 &= (\mu_0 q^2/12\pi m)(qA/mc)^2 (\underline{K} \cdot \underline{P}_0/mc) [\underline{K} \cdot \underline{X} \\ &\quad - (1/2) \sin(2 \underline{K} \cdot \underline{X})]_0^\tau. \end{aligned} \quad (22)$$

Translated back into three-space language and arranged for numeric evaluation as a function of proper time τ , this integrated "Beschleunigungsstrahlung" energy becomes

$$\begin{aligned} E_Y/mc^2 &= (kr_0/3)(qA/mc)^2 (\omega_0/\omega) [\omega_0 \tau - (1/2) \sin 2(\omega_0 \tau \\ &\quad + \omega t_0 - kz_0)]_0^\tau, \end{aligned} \quad (23)$$

where $r_0 = \mu_0 q^2/4\pi m$ is the "charge radius" defined classically* by equating its rest energy mc^2 to its electrostatic energy $q^2/4\pi\epsilon_0 r_0$.

* For an electron, $r_0 = (2.81785 \pm 0.00004) \times 10^{-15}$ m.

PULSED WAVE TRAINS

The preceding matrix-exponential analytic solution for the relativistic motion of an ion driven by an electromagnetic wave evolved as a recent spin-off of some 25 years of mathematical physics work at Hanford on a far more intricate problem having somewhat similar mathematical structure (nuclear transmutation theory⁽³⁾). Conversely, the domain of applicability of these matrix-exponential ion orbits appears to extend considerably beyond this first application, and certainly includes ions driven by pulsed plane-wave trains of quite arbitrary pulse shape. The legend of Figure 1 displays this pulsed wave solution explicitly, and it can be expected to become available in program Orbit as the need arises.

MAXIMUM ENERGY

Analyzing the momentum integrals for maximum attainable energy leads to combining Equations (12) and (13) in the form

$$E/mc^2 = E^0/mc^2 + (1/2)[(p_x/mc)^2 - (p_x^0/mc)^2]/(E^0/mc^2 - p_z^0/mc). \quad (24)$$

Along any orbit, maximum energy E/mc^2 thus corresponds to maximum x-momentum magnitude $|p_x/mc|$. From Equation (10) it can be seen that p_x/mc oscillates between the limits

$$p_x^\pm/mc = p_x^0/mc - (qA/mc)[\pm 1 - \cos(\omega t_0 - kz_0)],$$

and thus reaches a maximum magnitude of

$$|p_x/mc|_{\max} = |p_x^0/mc + (qA/mc) \cos(\omega t_0 - kz_0)| + |qA/mc|. \quad (25)$$

With respect to variation of initial time-phase ωt_0 , this expression maximizes at

$$|p_x/mc|_{\max} = |p_x^0/mc| + |2qA/mc|, \quad (26)$$

where the progressive capitalization (from max to Max) refers to further maximization. The corresponding maximum energy, Equation (24), becomes

$$E_{\text{Max}}/mc^2 = E^0/mc^2 + |2qA/mc|(|p_x^0/mc| + |qA/mc|)/(E^0/mc^2 - p_z^0/mc). \quad (27)$$

Further maximization with respect to variation of the initial momentum direction is considerably simplified by shifting the representation to four-momentum-space polar coordinates, as defined by

$$E/mc^2 = \cosh\theta, \quad 0 \leq \theta \leq \infty, \quad (28)$$

$$p_z/mc = \sinh\theta \cos\psi, \quad 0 \leq \psi \leq \pi, \quad (29)$$

$$p_y/mc = \sinh\theta \sin\psi \sin\phi, \quad -\pi \leq \phi \leq \pi, \quad (30)$$

$$p_x/mc = \sinh\theta \sin\psi \cos\phi. \quad (31)$$

The motivation for this shift, of course, is to impose the radial constraint

$$(E/mc^2)^2 - (p_x/mc)^2 - (p_y/mc)^2 - (p_z/mc)^2 = 1 \quad (32)$$

upon the momentum variations. In this representation, the maximum energy, Equation (27), takes the form

$$\begin{aligned} \cosh\theta_{\text{Max}} = & \cosh\theta_0 + |2qA/mc|(\sinh\theta_0 \sin\psi_0 |\cos\phi_0| \\ & + |qA/mc|)/(\cosh\theta_0 - \sinh\theta_0 \cos\psi_0). \end{aligned} \quad (33)$$

With respect to variation of the azimuthal momentum angle ϕ_0 , this expression maximizes at

$$\begin{aligned} \cosh\theta_{\text{MAX}} = & \cosh\theta_0 + |2qA/mc|(\sinh\theta_0 \sin\psi_0 + |qA/mc|)/(\cosh\theta_0 \\ & - \sinh\theta_0 \cos\psi_0), \end{aligned} \quad (34)$$

where further capitalization (Max to MAX) refers to further maximization.

Differentiating this expression to locate its maximum with respect to variation of the initial-momentum polar angle ψ_0 , and then maneuvering to solve for ψ_0 , yields sequentially,

$$\begin{aligned} (\cos\psi_0)/(\sinh\theta_0 \sin\psi_0 + |qA/mc|) &= (\sin\psi_0)/(\cosh\theta_0 \\ &\quad - \sinh\theta_0 \cos\psi_0), \end{aligned} \quad (35)$$

$$\cosh\theta_0 \cos\psi_0 - |qA/mc| \sin\psi_0 = \sinh\theta_0,$$

$$\begin{aligned} \cos[\psi_0 + \tan^{-1}(|qA/mc| \operatorname{sech}\theta_0)] &= (\sinh\theta_0)/[\cosh^2\theta_0 \\ &\quad + (qA/mc)^2]^{1/2}, \end{aligned}$$

$$\psi_0 + \tan^{-1}(|qA/mc| \operatorname{sech}\theta_0) = \tan^{-1}([1 + (qA/mc)^2]^{1/2} \operatorname{csch}\theta_0),$$

$$\begin{aligned} \tan\psi_0 &= \{[1 + (qA/mc)^2]^{1/2} \cosh\theta_0 \\ &\quad - |qA/mc| \sinh\theta_0\} / \{\cosh\theta_0 \sinh\theta_0 + |qA/mc|[1 \\ &\quad + (qA/mc)^2]^{1/2}\}. \end{aligned} \quad (36)$$

The structure here suggests setting

$$|qA/mc| = \sinh\alpha, [1 + (qA/mc)^2]^{1/2} = \cosh\alpha, \quad (37)$$

which leads to the further simplification,

$$\begin{aligned} \tan\psi_0 &= (\cosh\theta_0 \cosh\alpha - \sinh\theta_0 \sinh\alpha)/(\cosh\theta_0 \sinh\theta_0 \\ &\quad + \cosh\alpha \sinh\alpha), \\ &= \cosh(\theta_0 - \alpha)/(1/2)(\sinh 2\theta_0 + \sinh 2\alpha), \\ &= \cosh(\theta_0 - \alpha)/[\sinh(\theta_0 + \alpha) \cosh(\theta_0 - \alpha)], \end{aligned}$$

or finally,

$$\begin{aligned} \cot\psi_0 &= \sinh(\theta_0 + \alpha) = \sinh[\cosh^{-1}(E^0/mc^2) \\ &\quad + \sinh^{-1}(|qA/mc|)]. \end{aligned}$$

(38)

Second differentiation of Equation (34) via similar maneuvers yields a negative second derivative,

$$\partial^2 \cosh \theta_{\text{MAX}} / \partial \psi_0 \partial \psi_0 = -2(\sinh \theta_0 \sinh \alpha \csc \psi_0) / (\cosh \theta_0 - \sinh \theta_0 \cos \psi_0), \quad (39)$$

which establishes the extremal Equation (38) as a maximum.

The corresponding maximum energy, obtainable now by mentally slipping the polar angle Equation (38) into Equation (34) with the help of Equation (35), becomes

$$\begin{aligned} \cosh \theta_{\text{MAX}} &= \cosh \theta_0 + 2 \sinh \alpha \sinh(\theta_0 + \alpha), \\ &= \cosh[(\theta_0 + \alpha) - \alpha] + 2 \sinh \alpha \sinh(\theta_0 + \alpha), \\ &= \cosh(\theta_0 + \alpha) \cosh \alpha + \sinh(\theta_0 + \alpha) \sinh \alpha, \\ &= \cosh(\theta_0 + 2\alpha), \text{ or} \end{aligned}$$

$$\boxed{\theta_{\text{MAX}} = \theta_0 + 2\alpha,} \quad \text{or} \quad (40)$$

$$\boxed{E_{\text{MAX}}/mc^2 = \cosh[\cosh^{-1}(E^0/mc^2) + 2 \sinh^{-1}(|qA/mc|)],} \quad (41)$$

where again further capitalization (MAX to MAX) refers to further maximization.

This simple and useful expression, in resumé, gives the maximum attainable relativistic energy E_{MAX} for an ion of rest energy mc^2 and charge q driven by an electromagnetic plane wave having momentum-potential amplitude A . To attain this theoretical maximum, the particle must begin its orbit with the local electric field passing through zero, must have its initial momentum (if any) in the plane swept out by the traveling electric field and oriented at an angle ψ_0 , as given by Equation (38), from the propagation vector toward the rising electric force, and must ride the wave until it has seen half a wave go by.

The ultra-relativistic limit for the optimal injection angle, Equation (38), is displayed more clearly by the alternative form

$$\csc\psi_0 = \cosh(\theta_0 + \alpha) = \cosh[\cosh^{-1}(E^0/mc^2) + \sinh^{-1}(|qA/mc|)]. \quad (42)$$

The maximum-energy function, Equation (40), clearly displays the orbit physics as a Lorentz rotation, and the form of Equation (41) is convenient for slide-rule work, but electronic computers prefer the equivalent algebraic mess

$$E_{\text{MAX}}/mc^2 = (E^0/mc^2)(1 + 2|qA/mc|^2) + 2|qA/mc|[(1 + |qA/mc|^2)(E^0/mc^2 + 1)(E^0/mc^2 - 1)]^{1/2}, \quad (43)$$

for both accuracy and speed.

MEAN ENERGY

For some applications, mean energy may provide a more informative measure of performance than the preceding maximum energy. Mentally averaging the energy integral, Equation (13), over its proper time period along an orbit, with the help of the momentum integrals, Equations (12) and (10), yields

$$\begin{aligned} \langle E/mc^2 \rangle &= \int_0^{2\pi} (E/mc^2) d\omega_0 \tau / \int_0^{2\pi} d\omega_0 \tau, \\ &= E^0/mc^2 + (1/2) [\langle (p_x/mc)^2 \rangle - (p_x^0/mc)^2] / (E^0/mc^2 - p_z^0/mc), \text{ or} \\ \langle E/mc^2 \rangle &= E^0/mc^2 + (qA/mc) \{ (p_x^0/mc) \cos(\omega t_0 - kz_0) \\ &\quad + (1/2)(qA/mc) [(1/2) + \cos^2(\omega t_0 - kz_0)] \} / (E^0/mc^2 - p_z^0/mc). \end{aligned} \quad (44)$$

This expression gives the average relativistic energy along an orbit.

For a random distribution of initial time-phase ωt_0 , further phase averaging now provides

$$\begin{aligned} \langle\langle E/mc^2 \rangle\rangle &= \int_0^{2\pi} \langle E/mc^2 \rangle d\omega t_0 / \int_0^{2\pi} d\omega t_0, \text{ or} \\ \langle\langle E/mc^2 \rangle\rangle &= E^0/mc^2 + (1/2)(qA/mc)^2 / (E^0/mc^2 - p_z^0/mc). \end{aligned} \quad (45)$$

Shifting to four-momentum spherical coordinates to facilitate further averaging over a random distribution of initial three-momentum directions yields, finally,

$$\begin{aligned} \langle\langle\langle \cosh\theta \rangle\rangle\rangle &= \int_0^\pi \langle\langle \cosh\theta \rangle\rangle \sin\psi_0 d\psi_0 / \int_0^\pi \sin\psi_0 d\psi_0, \\ &= \cosh\theta_0 + (1/4)(qA/mc)^2 \int_0^\pi (\cosh\theta_0 \\ &\quad - \sinh\theta_0 \cos\psi_0)^{-1} \sin\psi_0 d\psi_0, \\ &= \cosh\theta_0 + (1/4)(qA/mc)^2 \operatorname{csch}\theta_0 \ln[(\cosh\theta_0 \\ &\quad + \sinh\theta_0)/(\cosh\theta_0 - \sinh\theta_0)], \\ &= \cosh\theta_0 + (1/4)(qA/mc)^2 \operatorname{csch}\theta_0 \ln(e^{\theta_0}/e^{-\theta_0}), \text{ or} \end{aligned}$$

$$\langle\langle\langle \cosh\theta \rangle\rangle\rangle = \cosh\theta_0 + (1/2)(qA/mc)^2 (\theta_0/\sinh\theta_0), \quad \text{or} \quad (46)$$

$$\begin{aligned} \langle\langle\langle E/mc^2 \rangle\rangle\rangle &= E^0/mc^2 + (1/2)(qA/mc)^2 [(\cosh^{-1}E^0/mc^2) \\ &\quad / \sinh(\cosh^{-1}E^0/mc^2)]. \end{aligned} \quad (47)$$

This expression, in resumé, gives the mean relativistic energy $\langle\langle\langle E \rangle\rangle\rangle$ for ions of rest energy mc^2 and charge q driven by an electromagnetic plane wave having momentum-potential amplitude A , proper-time averaged along orbits, and averaged over a random distribution of initial time phase and initial momentum direction.

NUMERICSCOMPUTATION LOGIC

The source deck consists of about 250 cards (Appendix), formulated in the machine-independent dialectal intersection of Univac-1107 Fortran-IV-CSC and IBM-7090 Fortran-IV-13, and designed for both modular growth versatility and freedom from obsolescence.

Complete input instructions appear at the beginning of the main program. Input consists of the electromagnetic potential momentum qA/mc , the charge-radius phase $kr_0/2\pi$, the initial position-phase and time-phase of the particle $(kx_0/2\pi, ky_0/2\pi, kz_0/2\pi, \omega t_0/2\pi)$, its initial momentum $(p_x^0/mc, p_y^0/mc, p_z^0/mc)$, the proper-time output phase interval $\omega_0 \Delta\tau/2\pi$, the terminal proper-time phase $\omega_0 \tau/2\pi$, and the range and resolution of the graphical output.

Numeric rounding is used throughout the calculations to minimize accrual of truncation bias. As can be gathered by scanning subroutine Famdr (Floating add-multiply-divide round), rounding is performed simply by adding twice the low order part of each arithmetic operation answer to its high order part prior to final truncation. Subroutine Spill writes a line of warning and diagnostic information in the output upon the occurrence of any spill or division by zero.

Subroutine Albert (Einstein) evaluates the momentum integrals, Equations (10), (11), (12), (13); the position integrals, Equations (16), (17), (18), (19); and the radiation integral, Equation (23), all as a function of proper time. This key subroutine has been quite fully optimized with emphasis upon numeric reliability, unrestricted numeric range, and unrestricted problem size.

PLOT LOGIC

All numeric output is written in duplicate on two separate drum (or disc) files. In a fully developed remote transmission environment, the first drum feeds decimal output to either a desk typewriter or line printer under real-time control of the using analyst. The second drum periodically feeds accrued binary output to the plotter logic under automatic time-lag control to provide cross-plot and plot-overlay versatility.

Subroutine Plot scans the plot drum for all relevant data, assembles eight-dimensional phase-space cross plots, calls upon subroutine Map to transform each data point to Cartesian coordinates via flexible curvilinear coordinate transformation logic, translates and scales the plots to fit the display domain, and feeds fully processed cross plots and plot overlays to the plotter interface. Subroutine Map is preset to an identity transformation but can be overloaded freely by the using analyst to produce polar plots, logarithmic scales, conformal mapping transformations, confocal quadric coordinate plots, or other exotic patterns.

ION ORBITS IN LASER BEAMS

PROPOSED EXPERIMENT

To investigate these effects experimentally, consider focusing a laser beam upon a cloud of electrons emitted by a hot tungsten ring encircling the focal domain, as sketched in Figure 2. Peak laser beam intensity appears to be limited principally by the dielectric strength of the lasing medium. Focusing the laser beam down to within the coherent diffraction limit, of the order of half a wave length in diameter, provides an enormous controllable increase in beam intensity, rising as the inverse square of the beam diameter.

Based upon his accrued measurements of laser pulse energies and pulse structures, W. Vali estimates⁽⁴⁾ that low power lasers presently available within Battelle-Northwest Laboratories, operated in the giant pulse mode, should generate pulse energies of about 0.01 J in 6×10^{-13} sec, or $10^{11}/6$ W of beam power, at the neodymium wave length of 1.06μ . Focused to within the first zero of the Bessel function ($kr_0 = 2.4048$), this beam power corresponds to a beam intensity of

$$\begin{aligned} \langle \text{ExH} \rangle &= (10^{11}/6) / \pi (2.4048 \times 1.06 \times 10^{-6} / 2\pi)^2 \\ &= 3.2232 \times 10^{22} \text{ W/m}^2. \end{aligned} \quad (48)$$

The corresponding electromagnetic field amplitudes become

$$\begin{aligned} E &= (\mu_0 c 2 \langle \text{ExH} \rangle)^{1/2} = (376.73 \times 2 \times 3.2232 \times 10^{22})^{1/2} \\ &= 4.9281 \times 10^{12} \text{ v/m}, \end{aligned} \quad (49)$$

$$H = E / \mu_0 c = 4.9281 \times 10^{12} / 376.73 = 1.3081 \times 10^{10} \text{ a/m}, \quad (50)$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 1.3081 \times 10^{10} = 1.6438 \times 10^4 \text{ v}\cdot\text{s/m}^2, \quad (51)$$

$$\begin{aligned} A &= B/k = 1.6438 \times 10^4 / (2\pi / 1.06 \times 10^{-6}) \\ &= 2.7732 \times 10^{-3} \text{ v}\cdot\text{s/m}. \end{aligned} \quad (52)$$

FIGURE 2
ELECTRONS IN LASER BEAM

For an electron, the potential momentum ratio qA/mc becomes

$$\begin{aligned} qA/mc &= -2.7732 \times 10^{-3} \times 2.99793 \times 10^8 / 0.510976 \times 10^6 \\ &= -1.6270. \end{aligned} \quad (53)$$

The physical constants used here are $\mu_o c = 4\pi \times 10^{-7} \times 2.997930 \times 10^8$ ohms for the impedance of free space, $mc^2 = 0.510976$ MeV for the rest energy of the electron, and $c = 2.997930 \times 10^8$ m/sec for the velocity of light.

An informative plausibility check on these fantastic magnitudes is obtainable quite simply by noting, with reference to Figure 2, that scaling the focused electric field back to the one-centimeter diameter of our neodymium laser rod yields for the electric field within the laser,

$$\begin{aligned} E_o &= E r_o / R_o = 4.9281 \times 10^{12} (2.4048 \times 1.06 \times 10^{-6} / \pi) / 0.01 \\ &= 3.9986 \times 10^8 \text{ v/m}, \end{aligned}$$

just below the dielectric strength of quartz glass (6.7×10^8 v/m).⁽⁵⁾

THEORETICAL PREDICTIONS

For a low density cloud of electrons which start from rest ($E^o/mc^2 = 1$), as in the experiment proposed in Figure 2, the maximum energy, Equation (41), and mean energy, Equation (47), respectively, simplify to

$$E_{MAX}/mc^2 = \cosh[2 \sinh^{-1}(|qA/mc|)] = 1 + 2 (qA/mc)^2, \quad (54)$$

$$\langle\langle E/mc^2 \rangle\rangle = 1 + (1/2) (qA/mc)^2. \quad (55)$$

Subtracting off the electron rest energy ($mc^2 = 0.510976$ MeV),⁽⁶⁾ and then making use of the datum, Equation (53), now gives for the maximum and mean kinetic energies:

KINETIC ENERGY PREDICTIONS

$$\text{Maximum: } T_{\text{MAX}}/mc^2 = 2(qA/mc)^2, \text{ or} \quad (56)$$

$$\begin{aligned} T_{\text{MAX}} &= 2(-1.6270)^2 0.510976 \\ &= 2.705 \text{ MeV.} \end{aligned} \quad (57)$$

$$\text{Mean: } \langle\langle T/mc^2 \rangle\rangle = (1/2)(qA/mc)^2, \text{ or} \quad (58)$$

$$\begin{aligned} \langle\langle T \rangle\rangle &= (1/2)(-1.6270)^2 0.510976 \\ &= 0.6764 \text{ MeV.} \end{aligned} \quad (59)$$

These electron kinetic energy predictions rise linearly with laser beam power, according to Equations (56) and (58).

Evaluating the orbit integrals, Equations (16), (18), and (19), for an electron which starts from rest with the electric field passing through zero ($x_o = y_o = z_o = t_o = 0 = p_x^o = p_y^o = p_z^o$, $E^o = mc^2$) and which rides the wave until it has seen half a wave go by ($\omega_o \tau = \pi$) yields as the laboratory position/wavelength and time/period ratios for the maximum energy state,

$$kx/2\pi = (1/2)(qA/mc) = -0.8135, \quad (60)$$

$$kz/2\pi = (3/8)(qA/mc)^2 = 0.9927, \quad (61)$$

$$\omega t/2\pi = (1/2) + (3/8)(qA/mc)^2 = 1.493. \quad (62)$$

The corresponding momentum ratios and momentum direction can be read off the momentum integrals, Equations (10) and (12), as

$$p_x/mc = 2(qA/mc) = -3.254, \quad (63)$$

$$p_z/mc = 2(qA/mc)^2 = 5.295, \quad (64)$$

$$\tan\psi = p_x/p_z = mc/qA = -0.6146. \quad (65)$$

Two such maximum energy states appear as the prominently encircled points on the orbits shown in Figure 3. The second arises for the mirror-image initial time-phase $\omega t_0 = \pi$.

For electrons which start from rest ($x_0 = y_0 = z_0 = 0 = p_x^0 = p_y^0 = p_z^0$), the orbit integrals, Equations (16) and (18), simplify somewhat to the form

$$kx = (qA/mc) [\omega\tau \cos(\omega t_0) - \sin(\omega\tau + \omega t_0)]_0^\tau, \quad (66)$$

$$\begin{aligned} kz = & (1/2)(qA/mc)^2 \{ \omega\tau [1/2 + \cos^2(\omega t_0)] \\ & + (1/4) \sin 2(\omega\tau + \omega t_0) \\ & - 2 \sin(\omega\tau + \omega t_0) \cos(\omega t_0) \}_0^\tau. \end{aligned} \quad (67)$$

Despite their deceptive algebraic simplicity, these orbit equations display phenomenally intricate functional behavior, as can be gathered by a glance at Figure 3, which plots position phase ($kx/2\pi$, $kz/2\pi$) as a function of both proper-time phase $\omega\tau/2\pi$ and initial time-phase $\omega t_0/2\pi$. Program Orbit constructed this machine plot as a mosaic-superposition cross-plot of the output of 24 successive change cases which swept both proper phase $\omega\tau/2\pi$ and initial phase $\omega t_0/2\pi$ through two complete cycles at intervals of $1/24$ cycle (15°), to generate the entire family of orbits, Equations (66) and (67), in one run. This plot has been arranged to display the actual visual appearance of the electron orbits in the laser beam, if mercury vapor is permitted to leak into the vacuum in sufficient concentration to make the electron ionization tracks visible. Each penned line traces out an electron orbit. Each point symbol marks a point tabulated in the numerical output (Appendix). The point symbols cycle through 12 shapes on successive orbits to aid the eye in tracking an electron through its orbit. The laser beam is incident from the left of Figure 3, with its focal domain centered at the origin (0, 0).

The electromagnetic wave hurls the electrons first transversely in the direction of the electric force field and then downstream along intricately cusped orbits dependent upon initial phase.

Combined scrutiny of this highly informative plot and of these highly deceptive orbit equations reveals that along each orbit the electron reaches maximum relativistic energy at the points of inflection and returns to rest energy at the cusps! With the isolated exception of two degenerate orbits which zigzag asymptotically downstream (the two centermost orbits in Figure 3), the orbits tend generally to zigzag about the maximum energy orbits which appear in Figure 3 as the dense black lines of orbit coalescence. The two orbits along which the maximum attainable energy, Equation (56), is reached are tilted snakes which have no cusps. Their angle of tilt can be inferred from Equations (60) and (61) as

$$\tan^{-1}(x/z) = \pm \tan^{-1}(4mc/3qA) = \pm \tan^{-1}0.8195 = \pm 39^{\circ}20.1'. \quad (68)$$

The upper and right scales in Figure 3 give the laboratory distances in wavelengths for the conditions of the proposed experiment, and the structure of the orbit Equations (66) and (67) makes it clear how to rescale the same plot to fit other experimental conditions. Variation of the driving field qA/mc scales the transverse x-axis linearly and the axial z-axis quadratically. The left and bottom scales of Figure 3 increase the driving field qA/mc by a factor of 100, to illustrate the rescaling procedure.

The preceding plot reappears in Figure 4, prepared at twice the level of detail, with both proper time phase $\omega\tau/2\pi$ and initial time phase $\omega t_0/2\pi$ swept through two cycles at intervals of 1/48 cycle (7.5°), to display more vividly the statistical distribution of orbits arising from random initial phase conditions. The dense black lines of high-energy-orbit

coalescence show up quite sharply here as the most prominent visible characteristic of the electron beam. To complete the picture, visualize the three-dimensional orbit plot generated by permitting the origin of Figure 4 to flutter about the three-dimensional focal domain of the laser beam. This final aspect of the statistical averaging logic leads to the prediction that the electron beam will appear as twin round high-energy prongs of the order of half a wavelength in diameter, thrust out of the focal domain of the laser beam in the plane of its electric field, and enveloped in some low energy fuzz.

PROPOSED MEASUREMENTS

A decisive (two significant figure) measurement of the maximum electron energy, for comparison with the prediction, Equation (57), and a photograph of the electron beam, for comparison with Figure 4, are key initial objectives.

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APPENDIX

BITS

APPENDIX

BITS

The source deck for program Orbit is followed by numeric input for the orbits displayed in Figure 3 and numeric output for the maximum energy orbit (initial time phase $\omega t_0/2\pi = 0$) and for the minimum energy orbit (initial time phase $\omega t_0/2\pi = 1/4$).

000001	C	CHARGED PARTICLE IN ELECTROMAGNETIC WAVE	
000002	C	BATTELLE-NORTHWEST PROGRAM ORBIT	
000003	C#ORBIT	INPUT DEFINITION	NEE BIT
000004	C QA	POTENTIAL MOMENTUM RATIO QA/MC (X POTENTIAL $A \cdot \cos(2 \cdot \pi \cdot (T/TT - Z/L))$)	1 EXP
000005	C (X	ELECTRIC FIELD $E = -DA/DT$, Y MAGNETIC FIELD $B = \text{CURL}(A)$, Z PROPAGATION)	
000006	C X0	INITIAL X-POSITION PHASE X0/L (L IS LABORATORY PHOTON WAVE LENGTH)	0 EXP
000007	C Y0	INITIAL Y-POSITION PHASE Y0/L (L IS LABORATORY PHOTON WAVE LENGTH)	0 EXP
000008	C Z0	INITIAL Z-POSITION PHASE Z0/L (L IS LABORATORY PHOTON WAVE LENGTH)	0 EXP
000009	C T0	INITIAL TIME PHASE T0/TT (TT IS LABORATORY PHOTON PERIOD)	0 EXP
000010	C PX0	INITIAL X-MOMENTUM RATIO PX0/MC (M=REST MASS, C=PHOTON SPEED)	0 EXP
000011	C PY0	INITIAL Y-MOMENTUM RATIO PY0/MC (M=REST MASS, C=PHOTON SPEED)	0 EXP
000012	C PZ0	INITIAL Z-MOMENTUM RATIO PZ0/MC (M=REST MASS, C=PHOTON SPEED)	0 EXP
000013	C DTP	OUTPUT TICK-TOCK TIME PHASE DTP/TTP (TTP IS PROPER PHOTON PERIOD)	1/8 EXP
000014	C TP	TERMINAL PROPER TIME PHASE TP/TTP (TTP IS PROPER PHOTON PERIOD)	1 EXP
000015	C R	CHARGE RADIUS RATIO R/L (L IS LABORATORY PHOTON WAVE LENGTH)	0 EXP
000016			
000017	C MAPS	NUMBER OF PHASE-SPACE CROSS PLOTS (NONE 0, MAXIMUM 100)	0 INT
000018	C	MINUS MAPS MEANS MORE MAPS LATER (PLOT-TAPE END-OF-FILE BYPASS)	
000019	C	PLUS MAPS MEANS LAST MAPS (PLOT-TAPE END-OF-FILE, REWIND, UNLOAD)	
000020			
000021	C ((ZZNSS(K,L,M),K=1,5),L=1,2),M=1,MAPS	SCALES (L=1=XMILS, L=2=YMILS)	EXP
000022	C END PHASE	PHASE END PHASE N(L,M) END PLOT (MILS) PLOT END	
000023	C ZZNSS(1,L,M) ZZNSS(2,L,M) ZZNSS(3,L,M) ZZNSS(4,L,M) ZZNSS(5,L,M)		
000024	C (N=1=X N=2=Y N=3=Z N=4=T N=5=PX N=6=PY N=7=PZ N=8=E N=9=ER N=10=TP)		
000025			
000026	C	DATA TRANSFORM FLEXIBLY VIA SUBROUTINE MAP(XY)	
000027	C	THEN TRANSLATE AND SCALE LINEARLY TO MATCH ENDS	
000028	C	PLOTTER GRID, RANGE, SYMBOL, AND CURVE ALTERATION OPTIONS	
000029	C MESHX	(10 9 8 7 6 5 4 3 2 1)=(1 2 3 4 5 6 7 8 9 10) MIL X-GRID	10 INT
000030	C MESHY	(10 9 8 7 6 5 4 3 2 1)=(1 2 3 4 5 6 7 8 9 10) MIL Y-GRID	10 INT
000031	C	1 MIL GRID SPANS 20000 MILS 2-10 MIL GRIDS SPAN 30000 MILS	
000032	C MARK	(0 1 2 3 4 5 6 7 8 9 10 11)=(*) SYMBOL	1 INT
000033	C MARK	(0 1 2 3 4 5 6 7 8 9 10 11)=(. 1 2 3 4 5 6 7 8 9 0 -) NUMBER	1 INT
000034	C	MINUS MARK SUPERPOSES PLOTS, WITH CYCLIC MARKS	
000035	C LINE	(2 4 8)=(LINES POINTS ARCS) CURVE-SEGMENTATION MODE	2 INT
000036			
000037	C	PHOTON SPEED C = 2.99793+8 METER/SECOND	
000038	C	ELECTRON REST ENERGY = M*C*C = 0.510976+6 ELECTRON VOLTS	
000039	C	ELECTRON CHARGE Q = -1.60206-19 COULOMBS	
000040	C	ELECTRON RADIUS R = Q*Q/(4*PI*EPSILON*M*C*C) = 2.81785-15 METER	
000041	C	ELECTRON QA/MC=-398.54244 FOR 1.+15 JOUL/SEC WITHIN FIRST BESSEL ZERO	
000042	C	PROTON REST ENERGY = M*C*C = 938.211+6 ELECTRON VOLTS	
000043			
000044	C MATERIAL	LASER WAVELENGTH L RADIUS/WAVE R/L	

000045	C	AL203	CR+++	6943.-10 M	4.0585482-9
000046	C	GLASS	ND+++	1.06-6 M	2.6583490-9
000047	C	CAF2	U+++	2.511-6 M	1.1222023-9
000048	C	HE	NE	1.160-6 M	2.4291810-9
000049					
000050		DIMENSION DRUM(7)			UNIVAC
000051		CALL ETIME			UNIVAC
000052		CALL SETDR(4,524288,262144,DRUM)			UNIVAC
000053		CALL SETIO(5,1)			UNIVAC
000054		CALL SETIO(6,2)			UNIVAC
000055		CALL SETEOF(\$32766)			UNIVAC
000056	C	CALL MODIFY(6)			IBM
000057					
000058	C	ENTER ORBIT LOOP			
000059		100	REWIND 4		
000060	C	LOAD INPUT LIST INPUT COMPUTE AND LIST ORBIT PLOT ORBIT			
000061		CALL LOAD			
000062		CALL LIST			
000063		CALL ALBERT			
000064		CALL PLOT			
000065	C	NEXT ORBIT			
000066		GO TO 100			
000067		32766	STOP		UNIVAC
000068		END			

5. LIST PAGE

A-3

BNWL-452

@ ELT PAGE,1,670227, 50374

000001	C	48-LINE PAGE, 2-LINE MARGINS TOP/BOTTOM, TIME/PAGE TOP LEFT/RIGHT	
000002		SUBROUTINE PAGE(LINES)	
000003		DATA LEAF/0/,LINE/0/	
000004		IF(LINES) 30,20,10	
000005	10	LINE=LINE-LINES	
000006		IF(LINE) 30,30,50	
000007	C	LINES=0 BEGIN PAGE 1	
000008	20	LEAF=0	
000009	C	LINES=- TURN PAGE	
000010	30	LEAF=LEAF+1	
000011	C	WRITE (6,40) LEAF	IBM
000012	C	40 FORMAT(11H1 00H00MOUS98XI11)	IBM
000013		CALL ETIMEF(TIME)	UNIVAC
000014		TIME=TIME/60.	UNIVAC
000015		WRITE (6,40) TIME,LEAF	UNIVAC
000016	40	FORMAT(5H1TIME1PE15.7,1HM88XI11)	UNIVAC
000017		LINE=48-IABS(LINES)	
000018	50	RETURN	
000019		END	

6. LIST SPILL

A-4

BNWL-452

@ ELT SPILL,1,670324, 26203

```
000001      C      IF SPILL OR DIVISION BY ZERO, SAY WHAT, WHERE, NOW WHAT
000002      SUBROUTINE SPILL(WHERE)
000003      DIMENSION YESNO(2)
000004      DATA YESNO(1)/6H      /,YESNO(2)/6HNO      /
000005      CALL OVERFL(I)
000006      CALL DVCHK(J)
000007      IF((I+J).EQ.4) GO TO 200
000008      CALL PAGE(1)
000009      WRITE (6,100) YESNO(I),YESNO(J),WHERE
000010      100 FORMAT(15H BAD POINT NEXT,2XA3,5HSPILL,2XA3,21HDIVISION BY ZERO I
000011      1N A6,2X14HNO PROPAGATION)
000012      200 RETURN
000013      END
```

7. LIST FAMDR

A-5

BNWL-452

Q ELT FAMDR,1,670220, 14279

000001	.	FUNCTION ZERO(A) RETURNS DEAD ZERO IF A IS MULTIPLE ZERO	FAMDR*00
000002	.	FUNCTION FADR(A,B) RETURNS A+B ROUNDED	FAMDR 01
000003	.	FUNCTION FMPR(A,B) RETURNS A*B ROUNDED	FAMDR 02
000004	.	FUNCTION F DPR(A,B) RETURNS A/B ROUNDED	FAMDR 03
000005	.	METHOD ADD TWICE LOW ORDER PART TO HIGH ORDER PART BH DUANE BNW	FAMDR 04
000006	A0	EQU 12	FAMDR 05
000007	A1	EQU 13	FAMDR 06
000008	B11	EQU 11	FAMDR 07
000009	\$(2)	LIT	FAMDR 08
000010	\$(1)		FAMDR 09
000011	FADR*	NOP	FAMDR 10
000012	LA	A0,*0,B11	FAMDR 11
000013	FA	A0,*1,B11	FAMDR 12
000014	J	FRN	FAMDR 13
000015	FMPR*	NOP	FAMDR 14
000016	LA	A0,*0,B11	FAMDR 15
000017	FM	A0,*1,B11	FAMDR 16
000018	FRN	FA A1,A1	FAMDR 17
000019	FA	A0,A1	FAMDR 18
000020	TOP	A0,(0400400000000)	FAMDR 19
000021	SZ	A0	FAMDR 20
000022	J	3,B11	FAMDR 21
000023	F DPR*	NOP	FAMDR 22
000024	LA	A0,*0,B11	FAMDR 23
000025	FD	A0,*1,B11	FAMDR 24
000026	FA	A1,A1	FAMDR 25
000027	FD	A1,*1,B11	FAMDR 26
000028	J	FRN+1	FAMDR 27
000029	ZERO*	NOP	FAMDR 28
000030	LA	A0,*0,B11	FAMDR 29
000031	JGD	B11,FRN+2	FAMDR 30
000032	END		FAMDR*31

8. LIST LOAD

A-6

BNWL-452

@ ELT LOAD,1,670402, 5862

```
000001      SUBROUTINE LOAD
000002      COMMON/GO/R,QA,DTP,TP,X0,Y0,Z0,T0,PX0,PY0,PZ0,E0,X,Y,Z,T,PX,PY,PZ
000003      1,E,ER,PT,MAPS,ZZNSS(5,2,100),MESHX,MESHY,MARK,LINE
000004      NAMELIST/ORBIT/R,QA,DTP,TP,X0,Y0,Z0,T0,PX0,PY0,PZ0
000005      1,MAPS,ZZNSS,MESHX,MESHY,MARK,LINE
000006      DATA R/0./,QA/1./,DTP/.125/,TP/1./,X0/0./,Y0/0./,Z0/0./,T0/0./
000007      1,PX0/0./,PY0/0./,PZ0/0./,MAPS/0/,MESHX/10/,MESHY/10/,MARK/01/
000008      2,LINE/2/
000009      READ (5,ORBIT)
000010      RETURN
000011      END
```

9. LIST LIST

@ ELT LIST,1,670402, 5866

```

000001      SUBROUTINE LIST
000002      COMMON/GO/R,QA,DTP,TP,X0,Y0,Z0,T0,PX0,PY0,PZ0,E0,X,Y,Z,T,PX,PY,PZ
000003      1,E,ER,PT,MAPS,ZZNSS(5,2,100),MESHX,MESHY,MARK,LINE
000004      CALL PAGE(-6)
000005      WRITE (6,100) R,QA,DTP,MAPS,MESHX,MESHY,MARK,LINE
000006      100 FORMAT(41X40HCHARGED PARTICLE IN ELECTROMAGNETIC WAVE/45X32HRAITEL
000007      1LE-NORTHWEST PROGRAM ORBIT//3X11HBODY RADIUS5X9HPOTENTIAL6X9HTICK-
000008      2TOCK/4X9HR/L WAVES5X11HQA/MC RATIO4X12HDTP/TTP CYCS4X11HPHASE-SPAC
000009      3E4X11HPLOT X-GRID4X11HPLOT Y-GRID4X11HPLOT SYMROL3X12HPLOT SEGMENT
000010      4/1P3E15.7,4X4HMAPSI7,4X5HMESHXI6,4X5HMESHYI6,4X4HMARKI7,3X4HLINEI8
000011      5)
000012      INKS=IABS(MAPS)
000013      IF(INKS.LT.1) GO TO 500
000014      CALL PAGE(3)
000015      WRITE (6,200) INKS
000016      200 FORMAT(1H /31X50HPHASE-SPACE MAPS ((ZZNSS(K,L,M),K=1,5),L=1,2),M=1
000017      1,I6/25X9HEND PHASE6X9HPHASE END8X5HPHASE8X24HEND PLOT (MILS) PLOT
000018      2END)
000019      DO 300 M=1,INKS
000020      CALL PAGE(3)
000021      300 WRITE (6,400) ((ZZNSS(K,L,M),K=1,5),L=1,2)
000022      400 FORMAT(1H /(21X1P5E15.7,2H X/21X1P5E15.7,2H Y))
000023      500 CALL PAGE(4)
000024      WRITE (6,600)
000025      600 FORMAT(1H /120H LAB ENERGY      MOMENTUM      MOMENTUM      MOMENTUM      P
000026      10SITION      POSITION      POSITION      LAB TIME      BREMSSTRAHL PROPER T
000027      2IME/120H E/MCC RATIO PX/MC RATIO PY/MC RATIO PZ/MC RATIO X/L      WAV
000028      3ES Y/L      WAVES Z/L      WAVES T/TT CYCLES E/MCC RATIO TP/TTP CYCS/1H
000029      4)
000030      RETURN
000031      END

```

10. LIST ALBERT

A-8

BNWL-452

@ ELT ALBERT,1,670402, 5872

```

000001      C      INTEGRATE EINSTEIN HAMILTONIAN H=(P-Q*A(X))**2/2*M=-M*C**2/2
000002      SUBROUTINE ALBERT
000003      COMMON/GO/R,QA,UTP,TP,X0,Y0,Z0,T0,PX0,PY0,PZ0,E0,X,Y,Z,T,PX,PY,PZ
000004      1,E,ER,PT,MAPS,ZZNSS(5,2,100),MESHX,MESHY,MARK,LINE
000005      DIMENSION QPT(10)
000006      EQUIVALENCE (QPT,X)
000007      DATA PI2/6.28318531/
000008
000009      E0=SQRT(FADR(1.,FADR(FMPR(PZ0,PZ0),FADR(FMPR(PX0,PX0),FMPR(PY0,PY0
000010      1))))))
000011      C0=FADR(E0,ABS(PZ0))
000012      IF (PZ0.GT.0.) C0=FDPR(FADR(1.,FADR(FMPR(PX0,PX0),FMPR(PY0,PY0))),C
000013      10)
000014      C1=FDPR(QA,C0)
000015      C2=FMPR(C0,PI2)
000016      C3=FMPR(FDPR(FMPR(R,PI2),3.),FMPR(FMPR(QA,QA),C0))
000017      CYCLE0=FADR(T0,-Z0)
000018      PHASE=FMPR(PI2,CYCLE0)
000019      COS0=COS(PHASE)
000020      SIN0=SIN(PHASE)
000021      SIN20=FMPR(2.*SIN0,COS0)
000022      COS20=FADR(.5,FMPR(COS0,COS0))
000023      PT=-DTP
000024      C      ENTER PROPER TIME STEP LOOP
000025      100 PT=FADR(DTP,PT)
000026      PHASE=FMPR(FADR(CYCLE0,PT),PI2)
000027      DPX=FMPR(QA,FADR(COS0,-COS(PHASE)))
000028      PX=FADR(PX0,DPX)
000029      PY=PY0
000030      PZ=FADR(PZ0,FDPR(FMPR(DPX,FADR(DPX/2.,PX0)),C0))
000031      E=FADR(C0,PZ)
000032      CYCLE=FDPR(PT,C0)
000033      DSIN=FADR(SIN(PHASE),-SIN0)
000034      WAVE=FMPR(PT,PI2)
000035      DX=FMPR(C1,FADR(FMPR(WAVE,COS0),-DSIN))
000036      X=FADR(X0,FADR(FMPR(PX0,CYCLE),FDPR(DX,PI2)))
000037      Y=FADR(Y0,FMPR(PY0,CYCLE))
000038      DSIN2=FADR(SIN(2.*PHASE),-SIN20)/2.
000039      DZ=FDPR(FADR(FMPR(PX0,DX),FMPR(FMPR(C1/2.,QA),FADR(FMPR(WAVE,COS20
000040      1),FADR(DSIN2/2.,-FMPR(2.*COS0,DSIN))))),C2)
000041      Z=FADR(Z0,FADR(FMPR(PZ0,CYCLE),DZ))
000042      T=FADR(FMPR(E0,CYCLE),FADR(T0,DZ))
000043      ER=FMPR(FADR(WAVE,-DSIN2),C3)
000044      CALL SPILL(6HALBERT)

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BNWL-452

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000045      CALL PAGE(1)
000046      WRITE (6,200) E,PX,PY,PZ,X,Y,Z,T,ER,PT
000047      200 FORMAT((1P10E12.5))
000048      WRITE (4) (QPT(N),N=1,10)
000049      IF (ABS(PT).LT.ABS(TP)) GO TO 100
000050      C   CLOSE PROPER TIME LOOP (PLOT LAST TP/DTP LOGICAL RECORDS ON UNIT 4)
000051      END FILE 4
000052      REWIND 4
000053      RETURN
000054      END

```

```

11.      LIST PLOT

```

```

000001      C      TEN-DIMENSIONAL PHASE SPACE (X Y Z T PX PY PZ E ER TP) CROSS PLOTS
000002      SUBROUTINE PLOT
000003      COMMON/GO/R,QA,DTP,TP,X0,Y0,Z0,T0,FX0,FY0,FZ0,E0,X,Y,Z,T,FX,FY,FZ
000004      1,E,ER,PT,MAPS,ZZNSS(5,2,100),MESHX,MESHY,MARK,LINE
000005      DIMENSION QPT(10),DS(2),DSDZ(2),XY(100,2),QP(2)
000006      EQUIVALENCE (QPT,X)
000007      DATA NXY/100/
000008
000009      INKS=IABS(MAPS)
000010      IF(INKS.LT.1) GO TO 1900
000011      C      SET SCALES TO 1000 COUNTS/INCH (TO ALTER, OVERLOAD MESHX MESHY)
000012      CALL SCALE(MESHX,MESHY)
000013
000014      DO 1800 INK=1,INKS
000015      CALL ETIMEF(TIME)
000016      TIME=TIME/60.
000017      CALL PAGE(2)
000018      WRITE (6,1000) INK,TIME
000019      1000 FORMAT(1H /44X7HDO PLOTI4,6H TIME1PE15.7,1HM)
000020      C      WRITE (6,1000) INK
000021      C1000 FORMAT(1H /47X7HDO PLOTI4,16H TIME 00H00M00S)
000022      C      SET UP PLOT SCALER AND SHIFTER
000023      DO 1100 L=1,2
000024      DS(L)=FADR(ZZNSS(4,L,INK),ZZNSS(5,L,INK))/2.
000025      1100 DSDZ(L)=FDR(FADR(ZZNSS(5,L,INK),-ZZNSS(4,L,INK)),FADR(ZZNSS(2,L,INK),-ZZNSS(1,L,INK)))
000026      C      SET ORIGIN AT PLOT CENTER (FOR MAXIMUM RANGE -9999 TO 9999 COUNTS)
000027      CALL ORIGIN(INT(DS(1))/10,INT(DS(2))/10)
000028      MARKS=IABS(MARK)
000029      NX=ZZNSS(3,1,INK)+.25
000030      NY=ZZNSS(3,2,INK)+.25
000031
000032
000033      C      SCAN OUTPUT DRUM FOR DATA IN PLOT DOMAIN INERTIAL PLOTTER LOGIC
000034      NP=0
000035      TT=-DTP
000036      1200 TT=FADR(DTP,TT)
000037      READ (4) (QPT(N),N=1,10)
000038      QP(1)=QPT(NX)
000039      QP(2)=QPT(NY)
000040      C      DATA TRANSFORM FLEXIBLY VIA SUBROUTINE MAP(XY)
000041      CALL MAP(QP)
000042      C      THEN TRANSLATE AND SCALE LINEARLY TO MATCH ENDS
000043      DO 1300 L=1,2
000044      XY(NP+1,L)=-DS(L)+(QP(L)-ZZNSS(1,L,INK))*DSDZ(L)

```

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000045      IF (ABS(XY(NP+1,L)).GT.ABS(DS(L))) GO TO 1400
000046      1300 CONTINUE
000047      NP=NP+1
000048      1400 IF ((NP.LT.NXY).AND.(ABS(TT).LT.ABS(TP))) GO TO 1200
000049
000050      C      IF SPILL OR DIVISION BY ZERO, WRITE WHAT, WHERE, NOW WHAT
000051      CALL SPILL(6H, PLOT)
000052      IF (NP.LT.1) GO TO 1600
000053      C      SPEW BLOCKS OF UNIFORM LOGIC TO UTILIZE PLOTTER INERTIAL MOMENTUM
000054      CALL MODE(4,MARKS)
000055      CALL TOLP(XY(1,1),XY(1,2),NP)
000056      CALL MODE(LINE,MARKS)
000057      CALL TOLP(XY(1,1),XY(1,2),NP)
000058      IF (NP.LT.NXY) GO TO 1600
000059      DO 1500 L=1,2
000060      1500 XY(1,L)=XY(NP,L)
000061      NP=1
000062      GO TO 1200
000063
000064      C      MINUS MARK SUPERPOSES PLOTS, WITH CYCLIC MARKS
000065      1600 IF (SIGN(1,MARK).GT.0) GO TO 1700
000066      MARK=-MOD(MARKS+1,12)
000067      GO TO 1800
000068      1700 IF (INK.NE.MAPS) CALL WAIT
000069      1800 REWIND 4
000070
000071      IF (MAPS.GT.0) CALL HALT
000072      1900 RETURN
000073      END

```

12. LIST MAP

Q ELT MAP,1,670223, 67119

000001		SUBROUTINE MAP(XY)
000002		DIMENSION XY(2)
000003	C	DATA TRANSFORM FLEXIBLY VIA SUBROUTINE MAP(XY)
000004	C	THEN TRANSLATE AND SCALE LINEARLY TO MATCH ENDS
000005		RETURN
000006		END

END CUR

```

- RUN          5,CUB068,10,100          BH DUANE..703 BLDG..EXT 6-4448.. ORBIT 0
- TEL BNW      JOB B      TAPES U0356,PLOT          ORBIT 1
-RX ASG A=U0356,H=PLOT          ORBIT 2
-N XQT CUR          ORBIT 3
  IN A          ORBIT 4
  TRI A          ORBIT 5
-N XQT ORBIT          ORBIT 6
$ORBIT X0=0., YU=0., ZU=0., QA=-1.6270427, DTP=4.1666667-2, TP=2., DOCUMENT
ZZNSS(1,1,1)= -.2, 2.3, 3., 0., 9842.5, X=Z
ZZNSS(1,2,1)= -1.8, 1.8, 1., 0., 7086.6, Y=X
MAPS=-1, MARK=-1, T0=-.45833333, T0=11/24
R=2.6583490-9, PX0=0.0000000+0, PY0=0.0000000+0, PZ0=0.0000000+0, $ ND+++
$ORBIT T0=-.41666667, $ T0=10/24
$ORBIT T0=-.375, $ T0=09/24
$ORBIT T0=-.33333333, $ T0=08/24
$ORBIT T0=-.29166667, $ T0=07/24
$ORBIT T0=-.25, $ T0=06/24
$ORBIT T0=-.20833333, $ T0=05/24
$ORBIT T0=-.16666667, $ T0=04/24
$ORBIT T0=-.125, $ T0=03/24
$ORBIT T0=-8.3333333-2, $ T0=02/24
$ORBIT T0=-4.1666667-2, $ T0=01/24
$ORBIT T0=0., $ T0=00/24
$ORBIT T0=4.1666667-2, $ T0=01/24
$ORBIT T0=8.3333333-2, $ T0=02/24
$ORBIT T0=.125, $ T0=03/24
$ORBIT T0=.16666667, $ T0=04/24
$ORBIT T0=.20833333, $ T0=05/24
$ORBIT T0=.25, $ T0=06/24
$ORBIT T0=.29166667, $ T0=07/24
$ORBIT T0=.33333333, $ T0=08/24
$ORBIT T0=.375, $ T0=09/24
$ORBIT T0=.41666667, $ T0=10/24
$ORBIT T0=.45833333, $ T0=11/24
$ORBIT T0=.5, MAPS=1, $ T0=12/24
- MSG LABEL TAPE H PLOT TAPE H ON BENSON-LEHNER PLOTTER PLOT 1
- MSG BH DUANE(6-4448) CUB068 703 BLDG 700 AREA 31MAR67 PLOT 2
- MSG 1 PLOT KE-46-1510 (10X10/CM 18X25 CM) HOLES AT TOP PLOT 3
- MSG SYMBOL HEAD SAVE TAPE PLOT 4
- FIN X

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BNWL-452

TIME 9.3608398-01M

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CHARGED PARTICLE IN ELECTROMAGNETIC WAVE
BATTELLE-NORTHWEST PROGRAM ORBIT

BODY RADIUS	POTENTIAL	TICK-TOCK												
R/L WAVES	QA/MC RATIO	DTP/TTP CYCS	PHASE-SPACE	PLOT X-GRID	PLOT Y-GRID	PLOT SYMBOL	PLOT SEGMENT							
2.6583489-09	-1.6270427+00	4.1666666-02	MAPS	-1	MESHX	10	MESHY	10	MARK			-0	LINE	2
PHASE-SPACE MAPS ((ZZNSS(K,L,M),K=1,5),L=1,2),M=1, 1														
END PHASE		PHASE END	PHASE	END PLOT (MILS)		PLOT END								
-2.0000000-01		2.3000000+00	3.0000000+00	0.0000000		9.8425000+03 X								
-1.8000000+00		1.8000000+00	1.0000000+00	0.0000000		7.0865999+03 Y								
LAB ENERGY	MOMENTUM	MOMENTUM	MOMENTUM	POSITION	POSITION	POSITION	LAB TIME	BREMSSTRAHL	PROPER TIME					
E/MCC RATIO	PX/MC RATIO	PY/MC RATIO	PZ/MC RATIO	X/L	WAVES	Y/L	WAVES	Z/L	WAVES	T/TT CYCLES	E/MCC RATIO	TP/TTP CYCS		
1.00000+00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000		
1.00154+00	-5.54401-02	0.00000	1.53680-03	-7.71766-04	0.00000	1.28500-05	4.16795-02	1.73911-10	4.16667-02	1.73911-10	4.16667-02	4.16667-02		
1.02376+00	-2.17982-01	0.00000	2.37582-02	-6.11095-03	0.00000	4.01213-04	8.37345-02	1.33515-09	8.33333-02	1.33515-09	8.33333-02	8.33333-02		
1.11355+00	-4.76550-01	0.00000	1.13550-01	-2.02737-02	0.00000	2.92478-03	1.27925-01	4.20649-09	1.25000-01	4.20649-09	1.25000-01	1.25000-01		
1.33091+00	-8.13521-01	0.00000	3.30908-01	-4.69149-02	0.00000	1.16395-02	1.78306-01	9.05248-09	1.66667-01	9.05248-09	1.66667-01	1.66667-01		
1.72714+00	-1.20593+00	0.00000	7.27137-01	-8.88389-02	0.00000	3.29990-02	2.41332-01	1.56086-08	2.08333-01	2.41332-01	2.08333-01	2.08333-01		
2.32363+00	-1.62704+00	0.00000	1.32363+00	-1.47809-01	0.00000	7.50370-02	3.25037-01	2.31520-08	2.50000-01	3.25037-01	2.50000-01	2.50000-01		
3.09746+00	-2.04815+00	0.00000	2.09746+00	-2.24426-01	0.00000	1.45788-01	4.37454-01	3.06954-08	2.91667-01	4.37454-01	2.91667-01	2.91667-01		
3.97818+00	-2.44056+00	0.00000	2.97818+00	-3.18089-01	0.00000	2.51328-01	5.84662-01	3.72515-08	3.33333-01	5.84662-01	3.33333-01	3.33333-01		
4.85735+00	-2.77754+00	0.00000	3.85735+00	-4.27034-01	0.00000	3.93956-01	7.68956-01	4.20975-08	3.75000-01	7.68956-01	3.75000-01	3.75000-01		
5.60896+00	-3.03610+00	0.00000	4.60896+00	-5.48459-01	0.00000	5.70998-01	9.87665-01	4.49688-08	4.16667-01	9.87665-01	4.16667-01	4.16667-01		
6.11567+00	-3.19865+00	0.00000	5.11567+00	-6.78706-01	0.00000	7.74618-01	1.23295+00	4.61301-08	4.58333-01	1.23295+00	4.58333-01	4.58333-01		
6.29454+00	-3.25409+00	0.00000	5.29454+00	-8.13521-01	0.00000	9.92725-01	1.49273+00	4.63040-08	5.00000-01	1.49273+00	5.00000-01	5.00000-01		
6.11567+00	-3.19865+00	0.00000	5.11567+00	-9.48336-01	0.00000	1.21083+00	1.75250+00	4.64779-08	5.41667-01	1.75250+00	5.41667-01	5.41667-01		
5.60896+00	-3.03610+00	0.00000	4.60896+00	-1.07858+00	0.00000	1.41445+00	1.99779+00	4.76391-08	5.83333-01	1.99779+00	5.83333-01	5.83333-01		
4.85735+00	-2.77754+00	0.00000	3.85735+00	-1.20001+00	0.00000	1.59149+00	2.21649+00	5.05105-08	6.25000-01	2.21649+00	6.25000-01	6.25000-01		
3.97818+00	-2.44056+00	0.00000	2.97818+00	-1.30895+00	0.00000	1.73412+00	2.40079+00	5.53565-08	6.66667-01	2.40079+00	6.66667-01	6.66667-01		
3.09746+00	-2.04815+00	0.00000	2.09746+00	-1.40262+00	0.00000	1.83966+00	2.54800+00	6.19126-08	7.08333-01	2.54800+00	7.08333-01	7.08333-01		
2.32363+00	-1.62704+00	0.00000	1.32363+00	-1.47923+00	0.00000	1.91041+00	2.66041+00	6.94560-08	7.50000-01	2.66041+00	7.50000-01	7.50000-01		
1.72714+00	-1.20593+00	0.00000	7.27137-01	-1.53820+00	0.00000	1.95245+00	2.74412+00	7.69994-08	7.91667-01	2.74412+00	7.91667-01	7.91667-01		
1.33091+00	-8.13522-01	0.00000	3.30909-01	-1.58013+00	0.00000	1.97381+00	2.80714+00	8.35555-08	8.33333-01	2.80714+00	8.33333-01	8.33333-01		
1.11355+00	-4.76550-01	0.00000	1.13550-01	-1.60677+00	0.00000	1.98253+00	2.85753+00	8.84015-08	8.75000-01	2.85753+00	8.75000-01	8.75000-01		
1.02376+00	-2.17983-01	0.00000	2.37582-02	-1.62093+00	0.00000	1.98505+00	2.90172+00	9.12728-08	9.16667-01	2.90172+00	9.16667-01	9.16667-01		
1.00154+00	-5.54402-02	0.00000	1.53681-03	-1.62627+00	0.00000	1.98544+00	2.94377+00	9.24341-08	9.58333-01	2.94377+00	9.58333-01	9.58333-01		
1.00000+00	0.00000	0.00000	0.00000	-1.62704+00	0.00000	1.98545+00	2.98545+00	9.26080-08	1.00000+00	2.98545+00	1.00000+00	1.00000+00		
1.00154+00	-5.54401-02	0.00000	1.53680-03	-1.62781+00	0.00000	1.98546+00	3.02713+00	9.27819-08	1.04167+00	3.02713+00	1.04167+00	1.04167+00		
1.02376+00	-2.17982-01	0.00000	2.37581-02	-1.63315+00	0.00000	1.98585+00	3.06919+00	9.39431-08	1.08333+00	3.06919+00	1.08333+00	1.08333+00		
1.11355+00	-4.76550-01	0.00000	1.13550-01	-1.64732+00	0.00000	1.98838+00	3.11338+00	9.68145-08	1.12500+00	3.11338+00	1.12500+00	1.12500+00		
1.33091+00	-8.13521-01	0.00000	3.30908-01	-1.67396+00	0.00000	1.99709+00	3.16376+00	1.01660-07	1.16667+00	3.16376+00	1.16667+00	1.16667+00		
1.72714+00	-1.20593+00	0.00000	7.27137-01	-1.71588+00	0.00000	2.01845+00	3.22678+00	1.08217-07	1.20833+00	3.22678+00	1.20833+00	1.20833+00		
2.32363+00	-1.62704+00	0.00000	1.32363+00	-1.77485+00	0.00000	2.06049+00	3.31049+00	1.15760-07	1.25000+00	3.31049+00	1.25000+00	1.25000+00		

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TIME	9.5747069-01M								24
3.09746+00	-2.04815+00	0.00000	2.09746+00	-1.85147+00	0.00000	2.13124+00	3.42291+00	1.23303-07	1.29167+00
3.97818+00	-2.44056+00	0.00000	2.97818+00	-1.94513+00	0.00000	2.23678+00	3.57011+00	1.29859-07	1.33333+00
4.85735+00	-2.77754+00	0.00000	3.85735+00	-2.05408+00	0.00000	2.37941+00	3.75441+00	1.34705-07	1.37500+00
5.60896+00	-3.03610+00	0.00000	4.60896+00	-2.17550+00	0.00000	2.55645+00	3.97312+00	1.37577-07	1.41667+00
6.11567+00	-3.19865+00	0.00000	5.11567+00	-2.30575+00	0.00000	2.76007+00	4.21840+00	1.38738-07	1.45833+00
6.29454+00	-3.25409+00	0.00000	5.29454+00	-2.44056+00	0.00000	2.97818+00	4.47818+00	1.38912-07	1.50000+00
6.11567+00	-3.19865+00	0.00000	5.11567+00	-2.57538+00	0.00000	3.19628+00	4.73795+00	1.39086-07	1.54167+00
5.60896+00	-3.03610+00	0.00000	4.60896+00	-2.70563+00	0.00000	3.39990+00	4.98324+00	1.40247-07	1.58333+00
4.85735+00	-2.77754+00	0.00000	3.85735+00	-2.82705+00	0.00000	3.57695+00	5.20195+00	1.43118-07	1.62500+00
3.97818+00	-2.44056+00	0.00000	2.97818+00	-2.93600+00	0.00000	3.71957+00	5.38624+00	1.47964-07	1.66667+00
3.09746+00	-2.04815+00	0.00000	2.09746+00	-3.02966+00	0.00000	3.82511+00	5.53345+00	1.54521-07	1.70833+00
2.32363+00	-1.62704+00	0.00000	1.32363+00	-3.10628+00	0.00000	3.89587+00	5.64587+00	1.62064-07	1.75000+00
1.72714+00	-1.20593+00	0.00000	7.27137-01	-3.16525+00	0.00000	3.93790+00	5.72957+00	1.69607-07	1.79167+00
1.33091+00	-8.13521-01	0.00000	3.30908-01	-3.20717+00	0.00000	3.95926+00	5.79260+00	1.76163-07	1.83333+00
1.11355+00	-4.76549-01	0.00000	1.13550-01	-3.23381+00	0.00000	3.96798+00	5.84298+00	1.81009-07	1.87500+00
1.02376+00	-2.17982-01	0.00000	2.37581-02	-3.24797+00	0.00000	3.97050+00	5.88717+00	1.83881-07	1.91667+00
1.00154+00	-5.54399-02	0.00000	1.53679-03	-3.25331+00	0.00000	3.97089+00	5.92922+00	1.85042-07	1.95833+00
1.00000+00	0.00000	0.00000	0.00000	-3.25409+00	0.00000	3.97090+00	5.97090+00	1.85216-07	2.00000+00

DO PLOT 1 TIME 9.6847330-01M

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BNWL-452

TIME 1.4315918+00M

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CHARGED PARTICLE IN ELECTROMAGNETIC WAVE
BATTELLE-NORTHWEST PROGRAM ORBIT

BODY RADIUS	POTENTIAL	TICK-TOCK	PHASE-SPACE	PLOT X-GRID	PLOT Y-GRID	PLOT SYMBOL	PLOT SEGMENT
R/L WAVES	QA/MC RATIO	DTP/TTP CYCS	MAPS	MESHX	MESHY	MARK	LINE
2.6583489-09	-1.6270427+00	4.1666666-02	MAPS -1	10	10	MARK -6	2

PHASE-SPACE MAPS ((ZZNSS(K,L,M),K=1,5),L=1,2),M=1, 1
END PHASE PHASE END PHASE END PLOT (MILS) PLOT END

-2.0000000-01 2.3000000+00 3.0000000+00 0.0000000 9.8425000+03 X
-1.8000000+00 1.8000000+00 1.0000000+00 0.0000000 7.0865999+03 Y

LAB ENERGY	MOMENTUM	MOMENTUM	MOMENTUM	POSITION	POSITION	POSITION	LAB TIME	BREMSSTRAHL	PROPER TIME
E/MCC RATIO	PX/MC RATIO	PY/MC RATIO	PZ/MC RATIO	X/L WAVES	Y/L WAVES	Z/L WAVES	T/TT CYCLES	E/MCC RATIO	TP/TTP CYCS
1.00000+00	0.00000	0.00000	0.00000	0.00000	0.00000	1.53277-12	2.50000-01	-2.14481-19	0.00000
1.08867+00	-4.21110-n1	0.00000	8.86667-02	-8.82357-03	0.00000	1.24285-03	2.92910-01	7.54342-09	4.16667-02
1.33091+00	-8.13521-n1	0.00000	3.30908-01	-3.46930-02	0.00000	9.54156-03	3.42875-01	1.40995-08	8.33333-02
1.66182+00	-1.15049+00	0.00000	6.61817-01	-7.58453-02	0.00000	3.00614-02	4.05061-01	1.89455-08	1.25000-01
1.99273+00	-1.40906+00	0.00000	9.92725-01	-1.29476-01	0.00000	6.46930-02	4.81360-01	2.18168-08	1.66667-01
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