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III. Asymmetric Confocal Resonator**

by

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OPTICAL DESIGN OF A REACTION CHAMBER FOR WEAKLY ABSORBED LIGHT

III. ASYMMETRIC CONFOCAL RESONATOR

by

J. J. Devaney

ABSTRACT

NOTICE
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An asymmetric confocal resonator is studied for the purpose of maximizing useful absorption in the enclosed medium. Ray matrices are developed, the product of which may be rendered in a very simple form when the number of passes is an integral power of 2. Simple approximate ray matrix products are also presented for any pass number. Ray positions are computed for a beam of width $2C$ and divergence δ . The value of δ for maximum absorption is determined. A simple optical system for optimum beam insertion is examined. An elementary characterization of lasers is developed consisting of a disk source plus a lens or aperture. Optimum chamber parameters are given. Beam and image sizes as a function of pass number are given with consequent beam energy densities. Integrated energy densities on the mirrors and at focus are given. Numerical examples are given for a near diffraction-limited beam laser and a X50 diffraction-limited beam laser. Closed form expressions for the useful absorption of the medium are derived when absorber depletion is negligible. The pass-numbers for arrival of the beam at the center, for the minimum beam size, and for commencement of beam leakage out of the chamber are determined. We present an order of magnitude correction for beam depletion and evaluate the (depleted) beam absorption by HP-9820A computer. We tabulate a number of representative numerical results and give comparisons with other chambers.

I. INTRODUCTION

This is the third in a series of reports^{1,2} dealing with the illumination of a reaction chamber by weakly absorbed light which, as a specific example, we will take here to be of $0.39\text{-}\mu\text{m}$ wavelength and having a cavity medium linear absorption of $6.11 \times 10^{-7} \text{ cm}^{-1}$. However, our results are general and not limited to the examples given in this report, nor does this work specifically depend on the earlier reports.

The region to be illuminated is of width W , of length L , and of height h . In this report we will illuminate the region or chamber by means of two concave spherical or paraboloid of revolution mirrors* having a common focal point with unequal focal lengths, so forming an asymmetric confocal resonator, which is an unstable system. This resonator has an interesting and useful property in that one family of rays introduced externally can

*The mathematical analysis of this study is restricted to paraxial rays.

be trapped indefinitely in the resonator, in seeming contradiction to its description as an unstable resonator. However, as we will show in detail, this family of rays is trapped indefinitely, because at precisely one angle of entrance, or, alternatively, focused at precisely one point, the ray position has a fortuitous cancellation of terms that otherwise would cause ray divergence. Because rays from all other angles or positions diverge, only a beam fraction of measure zero is indefinitely trapped. However, real beams can be trapped long enough for useful fractional absorption. We expect the asymmetric confocal resonator to be especially useful for the efficient illumination of very wide (in W) but narrow chambers (in L and h). That indeed proves to be true, as we will show, but under the corresponding restricted circumstances that mirror tolerances or media adverse intensity limits be not exceeded by the beam concentrations. Surprisingly, even for very narrow L and h , the canted-mirror design¹ does respectably as well in useful reaction absorption as the asymmetric confocal resonator (like within a factor of 2 for our examples) when the beam quality is more typical of the highly radiant lasers. Moreover, the canted mirror uses plane or slightly cylindrical mirrors and has lower medium and mirror intensities. We will present comparative calculations in order to display the special merits of the two designs.

II. RAY MATRICES

Refer to Fig. 1. The origin of coordinates is the center of mirror M_1 with the z -axis along the center line and positive toward mirror M_2 . The positive x -axis extends toward the center of the insertion lens of focal length f . The positive y -axis is up out of the paper and perpendicular to it. Mirror M_1 has a focal length f_1 ; M_2 has f_2 and they are of size $L = 2l + 4c$, except M_1 has a $2C$ diameter

Fig. 1. Asymmetric confocal resonator reaction chamber.

hole for the insertion lens of focal length f , and diameter $2C$. M_1 and M_2 are spaced a distance W apart and have a common focal point so that

$$f_1 + f_2 = W. \quad (1)$$

Defining the ratio $f_1/f_2 \equiv S$ with S always greater than 1, we can solve for

$$f_1 = WS/(1 + S) \text{ and } f_2 = W/(1 + S). \quad (2)$$

Consider first a ray at coordinates $z, y = 0$ with position $x = x_0$ and slope $x' = x'_0$ and use the matrix methods of Kogelnik and Li³ to transverse the chamber plus be reflected at M_2 , then having position x_1 and slope x'_1 , recrossing the chamber and reflecting at M_1 , then having position x_2 and slope x'_2 . The subscripts thus indicate the number of passes of the ray across the chamber plus one reflection. We write

$$\begin{bmatrix} x_2 \\ x'_2 \end{bmatrix} \equiv M \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}, \quad (3)$$

and after a little algebra the effect of two passes on the ray, including two reflections, one at M_2 , the other at M_1 , gives:

$$M = \begin{bmatrix} -S & W(1 - S) \\ 0 & -(1/S) \end{bmatrix} \quad (4a)$$

as the ray transfer matrix. For any even number of passes we need only reapply the matrix repeatedly so that

$$\begin{bmatrix} x_{2n} \\ x'_{2n} \end{bmatrix} = M^n \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}, \quad n \text{ integral.} \quad (4b)$$

Thus,

$$M^2 = \begin{bmatrix} S^2 & \frac{W}{S} (S - 1)(S^2 + 1) \\ 0 & S^{-2} \end{bmatrix} \quad (5)$$

transforms a ray through 4 passes and reflections;

$$M^4 = \begin{bmatrix} S^4 & \frac{W}{S^3} (S - 1)(S^2 + 1)(S^4 + 1) \\ 0 & S^{-4} \end{bmatrix} \quad (6)$$

transforms a ray through 8 passes and reflections;

$$M^6 = \begin{bmatrix} S^6 & \frac{W}{S^5} (S - 1)(S^2 + 1)(S^8 + S^4 + 1) \\ 0 & S^{-6} \end{bmatrix} \quad (7)$$

transforms a ray through 12 passes and reflections; and so forth.

Evidently regularities exist so that, after some analysis, we find that we can write

$$M^n = \begin{bmatrix} S^n & \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} \\ 0 & S^{-n} \end{bmatrix} \quad (8)$$

for $n = 2^0, 2^1, 2^2, 2^3, 2^4, \dots$

To interpolate between these values of n , we introduce (after some algebra) the matrix

$$M_{n,m} = \begin{bmatrix} S^m & \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} & S^{n+m}(S^m - 1) \\ 0 & S^{-m} & \end{bmatrix} \quad (9)$$

with the property that

$$M^{n+m} \approx M_{n,m} M^n = \begin{bmatrix} S^{n+m} & \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} \cdot S^{2m} \\ 0 & S^{-(n+m)} \end{bmatrix} \quad (10)$$

for m any integer between n and $2n$ inclusive, and $n = 2^0, 2^1, 2^2, 2^3, 2^4, \dots$. The approximation is in the (1,2) element of the matrix and is good for $S^{2n} \gg 1$.

For exact matrices one should combine the exact expressions (4) and (8). These matrices are exceedingly useful in displaying at a glance the ultimate fate of various beams and chamber configurations. However, before exhibiting such interesting cases, we shall complete our matrix repertoire by formulating matrices for other than two passes at a time which the M and N matrices are restricted to do. We need such a "by pass" description to make sure the ray does not exceed the mirror width or re-enter the exit, especially in the first few passes. From Kogelnik and Li³ the ray matrix for moving a ray across a space W and then through a lens of focal length f_1 is:

$$P_1 = \begin{bmatrix} 1 & W \\ -f_1^{-1} & 1 - Wf_1^{-1} \end{bmatrix} \quad (11)$$

so that, using (2):

$$P_2 = \begin{bmatrix} 1 & W \\ -\frac{1+S}{W} & -S \end{bmatrix} \quad (12)$$

takes a ray across W and reflected through mirror M_2 , and

$$P_1 = \begin{bmatrix} 1 & W \\ -\frac{(1+S)}{WS} & -\frac{1}{S} \end{bmatrix} \quad (13)$$

takes a ray across W and reflected through mirror M_1 . As a check we note that, from (4),

$$P_1 P_2 = M. \quad (14)$$

Our matrix analysis for the resonator proper is now complete and we are ready to study actual beam behavior.

III. RAY POSITION

Refer again to Fig. 1. We introduce a beam of radius C with its center at $x_{oc} = l + C$, $y = 0$, $z = 0$, and with slope such that the center of the beam passes through the confocal point for then such a central ray will be trapped permanently as will be shown. The slope of the beam center is then:

$$x'_{oc} = -\frac{l+C}{f_1} = -\frac{(l+C)(1+S)}{WS}. \quad (15)$$

We suppose further that the beam at $z = 0$ has an effective divergent slope of δ (convergence of $-\delta$). Labeling at $z = 0$ the upper edge of the beam 1, the center C , and the lower edge 2, we have the following initial ray vectors in x -space (y will be treated later) upon which our ray matrices operate:

$$x_{01} = l + 2C, \quad x'_{01} = -\frac{(l+C)(1+S)}{WS} + \delta \quad (16)$$

$$x_{0C} = l + C, \quad x'_{0C} = -\frac{(l+C)(1+S)}{WS} \quad (17)$$

$$x_{02} = l, \quad x'_{02} = -\frac{(l+C)(1+S)}{WS} - \delta. \quad (18)$$

Observe that we may calculate any of the three rays by determining only ray 1 and then performing the transformation $l + C \rightarrow l + C$, $C \rightarrow 0$, $\delta \rightarrow 0$ for the center and $l + C \rightarrow l + C$, $C \rightarrow -C$, $\delta \rightarrow -\delta$ for ray 2.

Accordingly we calculate:

$$x_{2n,1} \equiv \begin{bmatrix} x_{2n,1} \\ x'_{2n,1} \end{bmatrix} = M^n \begin{bmatrix} x_{0,1} \\ x'_{0,1} \end{bmatrix} \equiv M^n x_{0,1} \quad (19a)$$

From (16) and (8);

$$\begin{bmatrix} x_{2n,1} \\ x'_{2n,1} \end{bmatrix} = \begin{bmatrix} C S^n + (\ell + C) S^{-n} + \delta \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} \\ -\frac{(\ell + C)(1 + S)}{W S^{n+1}} + \delta S^{-n} \end{bmatrix} \quad (19b)$$

whence, by our prescription:

$$\begin{bmatrix} x_{2n,C} \\ x'_{2n,C} \end{bmatrix} = \begin{bmatrix} (\ell + C) S^{-n} \\ -\frac{(\ell + C)(1 + S)}{W S^{n+1}} \end{bmatrix} \quad (20)$$

similarly for $x_{2n,2}$.

Note the position of the center after $2n$ passes, $x_{2n,C} = (\ell + C) S^{-n}$, $S = f_1/f_2$ being greater than 1 implies that the center of the beam approaches the center of the chamber monotonically and is therefore indefinitely trapped! This is the source of the permanently trapped external ray in the asymmetric confocal resonator. Now refer to $x_{2n,1}$ of (19b), compare this expression with $x_{2n,C}$, and note that if the initial position of the beam is not precisely at the center, i.e., if not at $C \equiv 0$, or if the angle is not precisely that of center ray, i.e., $\delta = 0$, then the dominant behavior of the ray position $x_{2n,1}$ is not S^{-n} but S^{+n} ; and so in place of being trapped (that is, going to the center as S^{-n}), the ray eventually diverges from the center at a rate proportional to S^n . Thus, for real beams only a set of measure zero of the beam is indefinitely trapped.

The foregoing remarks are strictly correct and properly explain both the advantage and limitation of the asymmetric confocal resonator as a beam "trapping" device; but we will now make use of results (19) and (20) to retain bundles of rays - all at appropriate angles. One bundle will be permanently trapped, but it too is a set of measure zero in a real beam, so that the following analysis is not a contradiction of the foregoing analysis of single rays, but rather an expansion thereof.

Observe in $x_{2n,1}$ of (19b) that the divergent beams have the form

$$[C + \delta \frac{WS}{S+1}] S^n, \quad (21)$$

so diverging rapidly for large n since $S > 1$, with the remaining terms of the form

$$[(\ell + C) - \frac{\delta WS}{S+1}] S^{-n}, \quad (22)$$

thus converging rapidly. Now in (21) or (19b), if we set

$$\delta = -\frac{C(S+1)}{WS} \approx -\frac{C}{f_1}, \quad (23)$$

as we are permitted to do by the freedom of choice of the focal length of the insertion lens f , the diverging terms then drop out and we are left with the bundle of rays for which δ is precisely proportional to C or, alternatively, that all pass through the confocal point. Again we caution that rays having δ proportional to C is a set of measure zero in a real beam, which we discuss after characterizing lasers and describing an optical insertion system.

Before continuing let us briefly mention the rays whose y -coordinate is not zero. Crucial to our description is the usual assumption that the action of optical elements in one dimension is independent of that in another. That is to say, we assume that our system aberrations are negligible to that extent. Under the independence hypothesis then, in (19) and (20) we replace x by y and set $\ell \equiv 0$ and otherwise draw the same conclusions, to wit, the bundle of rays, whatever their x or y initial coordinate, that precisely pass through the confocal point are permanently trapped. All other rays eventually diverge. How fast we determine below.

IV. BEAM INSERTION OPTICS

Let hats (^) denote parameters along the insertion-beam axis (see Figs. 1, 2) so that $\hat{z}$ is measured along the beam centerline from the center of the insertion lens as origin; and $\hat{x}$ and $\hat{y}$ are perpendicular to $\hat{z}$ at the insertion lens and are in the plane of the paper and perpendicular to it,

Fig. 2. Insertion optics.

respectively. Let the laser beam at the laser be of radius b and divergence half-slope Δ . Our purpose is to focus the laser beam to a converging half-slope of $\delta = -C(S+1)/(WS)$ from Eq. (23), thus giving optimum trapping of the beam. The beam position at the laser $\hat{z} = -d$ is $\hat{x}_{L,1} = b$, $\hat{x}'_{L,1} = +\Delta$. The ray transfer matrix from the laser to just beyond the lens, f , $\hat{z} = 0^+$, is then³

$$M_I = \begin{bmatrix} 1 & d \\ -1/f & 1 - (d/f) \end{bmatrix} \quad (24)$$

so that

$$\hat{x}_{01} = M_I \hat{x}_{L1}, \quad (25)$$

or

$$\begin{bmatrix} \hat{x}_{01} \\ \hat{x}'_{01} \end{bmatrix} = \begin{bmatrix} C \\ \delta \end{bmatrix} = \begin{bmatrix} 1 & d \\ -1/f & 1 - (d/f) \end{bmatrix} \begin{bmatrix} b \\ \Delta \end{bmatrix}. \quad (26)$$

Similarly for ray C and ray 2, note that δ , the slope, is here a negative quantity (see Fig. 2).

Equation (26) leads to two expressions:

$$b + d\Delta = C, \quad (27)$$

which simply describes the spread of the beam from the laser to the insertion lens, and

$$-\frac{b}{f} + \Delta - \frac{d\Delta}{f} = \delta, \quad (28)$$

which fixes f in order to achieve the half-slope, δ . With (27) and (23), Eq. (28) becomes:

$$f = C/(\Delta - \delta) = \left[\frac{\Delta}{C} + \frac{(S+1)}{WS} \right]^{-1}. \quad (29)$$

This identical equation would follow from ray 2 for then C , Δ , and δ would all change sign. Ray C is the vector $[0,0]$ and so transforms into itself.

The description of a laser beam in terms of a width plus a half-angle divergence is adequate for Gaussian coherent beams or for other beams in simple optical geometries. For photometry in more complicated optics such as our conical system, we need a better characterization of the beam, to which we now turn.

V. LASER CHARACTERIZATION

We need a description of a laser adequate to give the gross features of the device in sufficient detail to enable us to compute the photometry of complex optical systems in both the near and far fields of the various image spaces, and to at least a useful approximation. The purpose here is to characterize a partially incoherent laser beam well enough to determine reaction chamber properties, leaving the fine details for more sophisticated (and time-consuming) theory such as that of Fox and Li,⁴ of Terrell,⁵ or of Suydam,⁶ all based on Huygen's principle. Laser beams that are nearly diffraction limited should be treated by the well-known coherent Gaussian-mode beam optics.

We suggest then that a laser be characterized by a convex lens of aperture $2C$ equal to the effective exit window of the laser, together with a flat-plate disk light source of diameter $2b$ located at the focal point, f , of lens C (see Fig. 3). The brightness of source b is a function of position on the disk b , and for that matter the transmission of the lens C can also be made a function of position. Such a laser model, or its image space equivalent below, is easily propagated through an optical system by successive imaging. Moreover, the characterization fits the gross features of lasers, the limiting features of lasers, and varies smoothly and plausibly between limiting features. For example, a diffraction-limited coherent beam is described by setting b equal to zero. The true diffraction, as in the real laser, being given by the size of C . At the other extreme, when $b/f \gg 0.61 \lambda/C$ (or $\lambda/\pi C$), the model describes the divergence of a super-radiant or near superradiant laser, with the intensity on the disk b , giving the variation of the intensity with angle.

Actually we may transform the disk b of Fig. 3 into the image space of the lens C to obtain the

Fig. 3. Laser characterization.

Fig. 4. Equivalent laser characterization.

fully equivalent characterization given in Fig. 4. Thus e is the image space transform of the disk b . C is the exit pupil of the system and determines the diffractive spread of the laser beam. The transmission may as before be varied across C . e is a flat plate light source with the ratio e/d_0 giving the intrinsic nondiffractive spread of the laser beam. As before (over b), the intensity of the source as a function of angle is given by the (photometric) brightness function over the disk whose radius is e .

As an example, we work a first approximation that certainly does not exhaust the details of the model, much less the complexity of a real beam. We thus describe a beam for which we have only two experimental numbers, namely, the laser exit beam diameter of 0.5 cm and a distant angular spread of 1.3 mrad. These values determine $C = 0.25$ cm, and $e/d_0 = 1.3 \times 10^{-3}$ rad. By setting d_0 at the appropriate distance to give the proper diffraction, to wit, $C/d_0 = \lambda/\pi C$ for a coherent Gaussian beam whence for, say $\lambda = 0.39 \mu\text{m}$, a distance $d_0 = 5020$ cm and thus $e = 6.5$ cm are obtained. We now can image e through our system by geometrical optics and get thereby a first and, for most photometric purposes, a more than adequate analysis.

Again we caution that our characterization and certainly the above over-simplified model is not intended to be all things to all people. Rather we submit the model as a way to use a paucity of experimental data to make first-order calculations that must be carefully examined at each succeeding step to make sure that the model does not over-simplify physical reality. With these caveats, the model proves highly useful.

VI. RAY POSITION AND OPTIMIZATION FOR A DISK (i.e., DIVERGENT LASER) SOURCE

Refer to Fig. 4. Let now the exit pupil C also contain the insertion lens C , where the focal length is f . We desire to duplicate Section III, that is, to insert the beam from the disk source e in the optimum way given to us from Section II. It will be no surprise to find that that optimization images the source e on the confocal point of the mirrors.

We must propagate our beam "edges" and center to just beyond the lens C where they will then be inserts for the theory of Section III [Eqs. (16), (17), and (18)] with δ optimized as in (23). The ray matrix, H , which takes a ray from d_0 to just beyond a lens of focal length f , is:

$$H = \begin{bmatrix} 1 & d_0 \\ -\frac{1}{f} & 1 - \frac{d_0}{f} \end{bmatrix}. \quad (30)$$

Our disk source rays from $\hat{x}_S = +e$ are (Fig. 4):

$$\hat{x}_{Su} = \begin{bmatrix} +e \\ (C - e)/d_0 \end{bmatrix} \text{ (upper),} \quad (31)$$

$$\hat{x}_{Sm} = \begin{bmatrix} +e \\ -e/d_0 \end{bmatrix} \text{ (middle),} \quad (32)$$

and

$$\hat{x}_{Sl} = \begin{bmatrix} +e \\ -(C + e)/d_0 \end{bmatrix} \text{ (lower).} \quad (33)$$

The rays from the center are obtained from (31), (32), and (33) by setting $e = 0$, and from $-e$ by changing the sign of e in these three equations.

Applying H to (31), (32), and (33) gives the rays $\hat{x}_{Ou}$, $\hat{x}_{Om}$, and $\hat{x}_{Ol}$ in the insertion system.

The insertion center line is given by (17) so that conversion from the insertion system to the confocal mirror system of coordinates is simply obtained by adding (17) to $\hat{x}_0$. Thus,

$$\hat{x}_0 = \begin{bmatrix} l + C \\ -A \end{bmatrix} + \hat{x}_0 \quad (34)$$

where $A \equiv (\ell + C)(1 + S)/(WS)$.

Putting this sequence together we have, originating at $+e$:

$$X_{0u} = \begin{bmatrix} \ell + 2C \\ -A + \frac{C - e}{d_0} - \frac{C}{f} \end{bmatrix}, \quad (35)$$

$$X_{0m} = \begin{bmatrix} \ell + C \\ -A - \frac{e}{d_0} \end{bmatrix}, \quad (36)$$

and

$$X_{0s} = \begin{bmatrix} \ell \\ -A - \frac{(C + e)}{d_0} + \frac{C}{f} \end{bmatrix}, \quad (37)$$

to which we may apply our confocal mirror ray tracing matrices M^n , Eq. (9); M^{n+m} , Eq. (10); P_1 , Eq. (13); and P_2 , Eq. (12).

However, first we should optimize our insertion angles per Eq. (23) and Section V. Refer to Figs. 2 and 4. The insertion lens allows us to match only one set of edge rays of the beam to the convergent angle δ . The optimum is clearly to use the center ray of such edge rays, namely the rays from $e = 0$. Thus in (29) we put $\Delta = C/d_0$, and it becomes:

$$\frac{1}{f} = \frac{1}{d_0} + \frac{(S + 1)}{WS} = \frac{1}{d_0} + \frac{1}{f_1}, \quad (38)$$

thus, as asserted, imaging the disk e at the mirror confocal point, which is precisely at f_1 . Combining Eqs. (30)-(38) with (8) we obtain the ray positions:

$$X_{2n,u} = \begin{bmatrix} (\ell + 2C)S^{-n} - \frac{e}{d_0} \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} \\ -S^{-n} \left\{ \frac{(\ell + C)(1 + S)}{WS} + \frac{e}{d_0} \right\} \end{bmatrix}, \quad (39)$$

$$X_{2n,m} = \begin{bmatrix} (\ell + C)S^{-n} - \frac{e}{d_0} \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} \\ -S^{-n} \left\{ \frac{(\ell + C)(1 + S)}{WS} + \frac{e}{d_0} \right\} \end{bmatrix}, \quad (40)$$

and

$$X_{2n,l} = \begin{bmatrix} \ell S^{-n} - \frac{e}{d_0} \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)} \\ -S^{-n} \left\{ \frac{\ell(1 + S)}{WS} - \frac{e}{d_0} \right\} \end{bmatrix}; \quad (41)$$

all precise for n a positive integer power of 2.

Note that in (39)-(41), e and d_0 appear only in the ratio e/d_0 .

In Section III we learned that the rate at which the untrapped rays of the beam expanded or left the cavity was proportional to S^n . Consequently, to optimize beam retention we want to make S as small as possible (by definition, $S \geq 1$). However $S = 1$ is a symmetric cavity and returns the beam to the entrance after four passes. See Fig. 1. The best we can do is to make S just large enough so that the second beam return just misses the entrance. We will adjust S here so that the whole beam just misses the entrance, realizing that for some physical beams we might want to sacrifice a portion of the beam edge so as to obtain greater entrapment of the remainder and consequently greater total useful absorption. From Figs. 1 and 4 or from Eqs. (39)-(41) our critical ray is then $X_{4,u}$ for $x_S = -e$, because after four passes (the first return on the entrance side) it is the highest. Our limiting condition, which we use to determine S , has this highest ray just hit the lowest entrance point ℓ . From (39) that gives:

$$(\ell + 2C)S^{-2} + \frac{e}{d_0} \frac{W(S^4 - 1)}{S(S + 1)} = \ell, \quad (42)$$

or

$$\frac{e}{d_0} WS(S^3 + S^2 + S + 1) - \ell S^2 + (\ell + 2C) = 0,$$

a quartic equation for S . Except for the entrance window at $\ell + C$ with radius C , the x, y plane radius of all mirrors has been taken to be $\ell + 2C$, so there is no leakout here beyond the mirror edges. Leakout is only possible through the entrance window for near optimum geometries. Also, further diffraction losses are negligible for most applications because the beam concentration relative to mirror size leaves little in the wings to "spill over" (see also Ref. 7).

VII. BEAM SIZE, INTENSITIES, BREAKDOWN

Since for n, m each being either large or small the matrices (8) and (10) are very close to each other, we can use (8) alone with the pair (39) and (41) to give us the beam spot size on both mirrors - accurately on one (M_1) and roughly on the other (M_2) - and as a rough upper bound on beam size in between (Fig. 1) for all n -values, not just $n = 2^k$ for which the relation is precise on M_1 .

Accordingly we form the beam diameter:

$$D_{2n} \equiv \overline{\Delta x_{2n}} = x_{2n,u}(-e) - x_{2n,l}(+e). \quad (43)$$

Substituting (39) and (41) and consolidating,

$$D_{2n} = \frac{2C}{S^n} + \frac{2e}{d_0} \frac{W(S^{2n} - 1)}{S^{n-1}(S + 1)}. \quad (44)$$

Equation (44) again illustrates beam convergence and subsequent divergence which we demonstrated in Section III from ray behavior. The beam begins ($n = 0$) at a diameter of $2C$ and is steadily compressed (all this except for the periodic refocusing, see Fig. 1), and were it not for a nonzero divergence, e/d_0 , the beam would be indefinitely compressed by a factor of S per two passes and trapped.

But real beams do have a nonzero divergence, e/d_0 , however small, and since the second term of (44) is proportional to S^n whereas the first and third are proportional to S^{-n} , the second term eventually dominates and the beam diverges. It is clear from (44) that a diffraction-limited (i.e., minimum-divergence) beam remains in the chamber longest. Clearly then, the beam shrinks to a minimum size (apart from the periodic refocused part discussed below) which is also the minimum mirror spot size. That size occurs near:

$$S^{2n} \sim \frac{C d_0 (1 + S^{-1})}{eW}, \text{ for } S^{2n} \gg 1, \quad (45)$$

or

$$2n \sim \ln \left\{ \frac{C d_0 (1 + S^{-1})}{eW} \right\} / \ln S, \text{ for } S^{2n} \gg 1,$$

and is:

$$D \sim 4 \sqrt{\frac{C e W}{d_0 (1 + S^{-1})}} \sim 4 \frac{C}{S^n} \text{ for } S^{2n} \gg 1. \quad (46)$$

Let us give two examples. (1) Suppose we have a 30-mJ beam of diameter $2C = 0.56$ cm and divergence $e/d_0 = 1.3$ mrad in a confocal chamber of size $W = 20$ cm by $L = 1$ cm, with useful beam absorption $(1 - T) = 1.222 \times 10^{-5}$ per pass. Then the criterion (42) leads to $S \sim 1.26$ so that $D \sim 0.25$ cm at $2n = 12.8$, or, say 12 with a single reflection mirror intensity of 0.6 J/cm^2 for perfect mirror reflectivity, otherwise less. (2) Suppose now we have a near-diffraction-limited coherent beam of 130 mJ, diameter $2C = 0.2$ cm, $e/d_0 = 1.6 \times 10^{-4}$ in the same chamber with the same absorption, then (42) gives $S = 1.0958$ and we find that $2n \sim 44.7$ or, say, 44 from Eq. (45), giving $D \sim 0.052$ from Eq. (46) giving a single-reflection mirror intensity of 61 J/cm^2 for perfect mirror reflectivity, otherwise less.

Let us now determine the size of the focused spot, which for the first focus is just a disk of radius:

$$e_0 = \frac{e}{d_0} f_1 = \frac{e}{d_0} \cdot \frac{WS}{1 + S}, \quad (47)$$

using (2). The focused image (47) occurs precisely at the confocal point [per (38)] where the image is refocused every second pass by the optical properties of the confocal system.

Optical imaging shows that the image size increases in the ratio $f_1/f_2 = S$ per round trip (2 passes) so that the image radius is

$$e_{2n} = \frac{e}{d_0} \frac{W S^{n+1}}{(1 + S)}. \quad (48)$$

If the beam fluence (energy density) at the entrance window is $I_0 = E_0/\pi C^2$ where E_0 is the initial beam energy, then the fluence at any focus before leakout is $I_{2n} = E_0/\pi e_{2n}^2 = I_0 (C^2/e_{2n}^2)$ and the integrated central fluence at focus is:

$$I \equiv \sum_{n=0} I_{2n} = I_0 \frac{C^2 (1 + S)^2}{W^2 (e/d_0)^2 S^2} \sum_{n=0} S^{-2n}, \quad (49)$$

$$I = I_0 \left[\frac{C}{W(e/d_0)} \cdot \frac{1 + S}{S} \right]^2 \frac{S^2 - S^{-2n}}{S^2 - 1},$$

$$I \sim I_0 \left[\frac{C(1 + S)}{W(e/d_0)} \right]^2 \cdot \frac{1}{S^2 - 1},$$

where $\bar{n}$ is some upper limit and where equality in the last formula is true for $S^{2\bar{n}+2} \gg 1$.

Equation (49) has been computed without regard for losses. Suppose the loss per pass (e.g., due to mirror reflectivity, gas absorption, etc.) is $1 - \Lambda$, then the central fluence will also decrease by Λ^2 per two passes so that in (49) the last factor of the second line need only be transformed by $S \rightarrow S/\Lambda$ or:

$$I = I_0 \left[\frac{C}{W(e/d_0)} \left(\frac{1+S}{S} \right) \right]^2 \frac{S^2 - \Lambda^2 (S/\Lambda)^{-2\bar{n}}}{S^2 - \Lambda^2} \quad (50)$$

$$I \lesssim I_0 \left[\frac{C(1+S)}{W(e/d_0)} \right]^2 \frac{1}{S^2 - \Lambda^2}$$

$$= \left[\frac{(1+S)}{W(e/d_0)} \right]^2 \frac{E_0}{\pi(S^2 - \Lambda^2)}$$

where equality is good for $(S/\Lambda)^{2\bar{n}+2} \gg 1$.

For our above example 1 and $\Lambda = 1$ ($\bar{n} = 23$ wholly within - see later discussion) $I \lesssim 123 \text{ J/cm}^2$ -pulse integrated (0 losses), for $\Lambda = 0.99$, $I \lesssim 119 \text{ J/cm}^2$ -pulse; for $\Lambda = 0.95$, $I \lesssim 105 \text{ J/cm}^2$ -pulse. These are the summed intensities per pulse over an effective period of 30.7 ns (46 passes).

In Example 2 for no losses the intensity mounts to an incredible $88\,400 \text{ J/cm}^2$ -pulse and for $\Lambda = 0.99$ we have $80\,400 \text{ J/cm}^2$ -pulse ($\bar{n} \sim 100$ wholly within) within about 130 ns at the confocus. The effect of these high integrated fluxes upon the gas must be carefully checked. Breakdown and unwanted reactions are possible; depletion of the absorber near the confocal point is likely. We propose crude depletion corrections in Section IX.

As we have noted earlier, the beam walks toward the optical axis of the cavity. It is important to know how soon it gets there and what the maximum summed energy load on the mirrors is. The former can be calculated from (41) by determining that n' which makes $x_{2n,e} = 0$.

Since usually $S^{2n'} \gg 1$, we can use that approximation to find:

$$S^{2n'} \approx \frac{\ell(1+S^{-1})}{(e/d_0)W} \quad (51)$$

or

$$2n' \approx 2n \left[\frac{\ell(1+S^{-1})}{(e/d_0)W} \right] / \frac{1}{2} S,$$

good for $S^{2n'} \gg 1$, giving the pass number $2n'$ at which the beam just reaches the center. Taking the first-order approximation that the beam (a central core at least) is approximately uniform and using (44) the beam intensity is:

$$I_{2n} = \frac{4E_0}{\pi D_{2n}^2} \Lambda^{2n} \quad (52)$$

where $1 - \Lambda$ gives the intrinsic or cavity loss per pass defined above.

For $\Lambda < 1$ and n' large, this intensity should be examined for n small (zero), otherwise near $n = \bar{n}$ or (45) for the maximum mirror intensity per single reflection. For $\Lambda \sim 1$ we should also check the summed maximum total intensity per pulse which occurs at the center and is obtained by summing (52) above $n = n'$,

$$I_c = \frac{4E_0}{\pi} \sum_{2n=2n'}^{\infty} \frac{\Lambda^{2n}}{D_{2n}^2}$$

$$= \frac{4E_0}{\pi} \sum_{2n=2n'}^{\infty} \frac{\Lambda^{2n}}{\left(\frac{2C}{S^n} + \frac{2e}{d_0} \frac{W(S^{2n} - 1)}{S^{n-1}(S+1)} \right)^2} \quad (53)$$

Note that the summation is made over $2n$, not n .

This is a rather formidable expression, however we can easily obtain an accurate simple approximation. Use first the approximation that $S^{2n} \gg 1$, then observe from comparing (51) to (45) that since almost always $\ell > C$ (the chamber is bigger than the beam radius), in fact usually $\ell \gg C$; then in the sum (53) $n \geq n' > \bar{n}$. Now $\bar{n}$ corresponds to the (half) pass number at which the two terms of D_{2n} are equal; above that number the second term dominates. So in the sum we make but a small error, compensating to boot, by omitting the first term and third term of D_{2n} , obtaining thereby:

$$I_c \approx \frac{E_0}{\pi \left(\frac{e}{d_0} W \right)^2} \left(\frac{S+1}{S} \right)^2 \sum_{2n=2n'}^{\infty} (\Lambda/S)^{2n},$$

or

$$I_c \sim \frac{E_0}{\pi W^2 \left(\frac{e}{d_0}\right)^2} \left(\frac{S+1}{S}\right)^2 \frac{(\Lambda/S)^{2n'}}{1 - (\Lambda/S)^2}, \quad (54)$$

which is the total central fluence incident on the mirrors. The total axial fluence is greater than or equal to $(1+R)I_c$, where equality occurs right at the mirrors.

Using (51) and (54) divided by 2, the central point of the mirrors is reached at $2n' \sim 18$ for Example 1 and at $2n' \sim 70$ for Example 2, with summed mirror energy densities per pulse of:

Example 1:

1.6 J/cm ² -pulse	no losses
1.3 J/cm ² -pulse	1% loss per pass
0.5 J/cm ² -pulse	5% loss per pass

Example 2:

141 J/cm ² -pulse	no losses
64 J/cm ² -pulse	1% loss per pass
2.5 J/cm ² -pulse	5% loss per pass

The reader is reminded that these central-mirror fluences are achieved from an infinite number of passes, nonetheless it is possible that in Example 2 any mirrors limited by heat damage (i.e., impurity inclusions) as against dielectric breakdown (i.e., intrinsic) would be damaged at the higher fluences.

VIII. USEFUL ABSORPTION OR REACTION - NO DEPLETION

In this section we calculate the fraction of light that is usefully absorbed in the reaction chamber medium. Losses occur in the mirrors, in useless medium absorption, and by escape of the beam edges. For calculational simplicity, we lump the former two mechanisms of loss into one, $1-R$, where R is the mirror reflectivity, $\Lambda \equiv RT$, and T is medium transmission. We divide our absorption algorithm into two parts, that for the beam wholly within the chamber (save for the negligible diffraction losses post entry), and that when edge leakage exists. Consequently we need to determine the pass number, $2\bar{n}$, at which the beam begins to leak out of the chamber. To within four passes (because the mirrors have a size radius $\ell + 2C$ except at the entrance, where the radius is ℓ) the beam will begin to leak

out when its outer rays reach the radius ℓ . Using (44), the number of passes, $2\bar{n}$, is determined by the equation:

$$CS^{-\bar{n}} + \frac{e}{d_0} \frac{WS}{S+1} (S^{\bar{n}} - S^{-\bar{n}}) = \ell. \quad (55)$$

Again, if $S^{2n'}$ and $S^{2\bar{n}}$ are much larger than one, as in most situations, then, because $\bar{n}$ is considerably larger than n' or $\underline{n}$, $S^{2\bar{n}} \gg 1$ is better satisfied so that

$$\frac{e}{d_0} \frac{WS}{S+1} S^{\bar{n}} \sim \ell$$

or

$$\bar{n} \sim \ell n \left| \frac{\ell(S+1)}{W(e/d_0)S} \right| / \ell n S \quad (56)$$

for $S^{2\bar{n}} \gg 1$ is a very good approximation indeed. Note that $2n' = \bar{n}$ from (51), that is, the beam begins to walk out of the chamber in precisely double the number of passes that it took to reach the center.

Now then, when the beam is wholly within the chamber its useful energy absorption, U_{2n} , per pass is clearly:

$$U_{2n} = (1-T)E_{2n}, \quad (57)$$

where E_{2n} is the total beam energy,

$$E_{2n} = E_0 \Lambda^{2n}. \quad (58)$$

The total energy usefully absorbed up to and including $2n = 2\bar{n}$, U_I , is then:

$$U_I = \sum_{2n=0}^{2\bar{n}} U_{2n} = (1-T)E_0 + (1-T)\Lambda E_0 + (1-T)\Lambda^2 E_0 + \dots + (1-T)\Lambda^{2\bar{n}} E_0$$

$$U_I = (1-T)E_0 \frac{1 - \Lambda^{2\bar{n}+1}}{1 - \Lambda}. \quad (59)$$

Beyond $2\bar{n}$ we suffer beam-edge escape. Using (44) in the now even better approximation $S^{2\bar{n}} \gg 1$, the beam area is:

$$Q_{2n} \equiv (\text{Area})_{2n} \sim \pi \left[\frac{e}{d_0} \frac{WS}{S+1} S^{\bar{n}} \right]^2, \quad S^{2\bar{n}} \gg 1, \quad (60)$$

so that to our internal beam loss, Λ , we must include the factor:

$$\pi \ell^2 / Q_{2n} \sim \left[\frac{\ell(S+1)}{W(e/d_0)S} s^{-n} \right]^2 = s^{2\bar{n}-2n} \quad (61)$$

using (56).

Our useful absorption per pass beyond $2\bar{n}$ is:

$$U'_{2n} = (1-T)E_{2n} \frac{\ell^2 \pi}{Q_{2n}} \sim (1-T)E_0 \Lambda^{2n} s^{2\bar{n}-2n},$$

$$s^{2\bar{n}} \gg 1, n \geq \bar{n}. \quad (62)$$

The contribution from the leaking passes is:

$$U_{II} = \sum_{2n=2\bar{n}+1}^{\infty} U'_{2n} = (1-T)E_0 \Lambda^{2\bar{n}+1} s^{-1}$$

$$\times \left[1 + \frac{\Lambda}{S} + \left(\frac{\Lambda}{S}\right)^2 + \left(\frac{\Lambda}{S}\right)^3 + \dots \right],$$

$$U_{II} = (1-T)E_0 \frac{\Lambda^{2\bar{n}} (\Lambda/S)}{1 - (\Lambda/S)}. \quad (63)$$

The total useful absorption is:

$$U = U_I + U_{II}$$

$$= (1-T)E_0 \left[\frac{1 - \Lambda^{2\bar{n}+1}}{1 - \Lambda} + \frac{\Lambda^{2\bar{n}} (\Lambda/S)}{1 - (\Lambda/S)} \right]. \quad (64)$$

This result, or the ratio U/E_0 , is the chief figure of merit of the cavity. It can be made large for:

1. a high density of useful absorber,
2. a high cross section for useful absorption, both of which make $(1-T)$ large.
3. a large initial beam energy, E_0 ,
4. a small (useless) loss, which makes Λ large,
5. a small, well collimated beam which makes $\bar{n}$ large, so approaching the theoretical limit of $U = [(1-T)/(1-\Lambda)]E_0$ for the cavity parameters of absorption and reflectivity (infinite trapping). However, the beam or its divergence cannot be decreased indefinitely without exceeding mirror tolerances or unwanted reaction limits, even for low power beams (see the discussion above, Example 2),

6. a small beam, allowing S to approach 1.

In our examples,

1. For $R = 1.0^*$	$U/E_0 = 5.19 \times 10^{-4}$
$R = 0.99$	$U/E_0 = 4.24 \times 10^{-4}$
$R = 0.95$	$U/E_0 = 2.16 \times 10^{-4}$
2. For $R = 1.0^*$	$U/E_0 = 18.58 \times 10^{-4}$
$R = 0.99$	$U/E_0 = 9.55 \times 10^{-4}$
$R = 0.95$	$U/E_0 = 2.44 \times 10^{-4}$

Thus, the higher passes possible with a well-collimated beam are useless for poor reflectivity, as expected; compare $R = 0.95$ above.

IX. DEPLETION ESTIMATES - MACHINE CALCULATION

As our examples in Section VII have shown, we get high integrated intensities for even weak laser beams both at the cavity center line and also at the confocus. Pending studies of other chamber designs, it is this writer's view that a detailed study of depletion for this chamber is premature. However, some indication of depletion is required in order to fairly compare this design to other designs, especially that of straight-mirror designs^{1,2} which, with their nearly uniform irradiation, are much less sensitive to localized depletion.

Accordingly we make only a crude calculation and claim it useful for an order of magnitude estimate of depletion only. Moreover, we deplete only the focused part of the beam, there is also a center-line depletion of the parallel beam which, in our approximation, it is appropriate to neglect. Thus a large depletion correction must also imply that our correction is inadequate.

Our depletion correction, then, is to keep the absorber density constant until the absorbed photon density precisely equals the absorber density and then deplete the absorber to zero density by simply excluding that volume from absorption. For example, for a one-level ground state and a three-level excited state activated by light, we deplete to zero density when the absorber number density ratio reaches $\Delta N/N = 3/4$. For this calculation we smoothly interpolate the beam uniformly between each mirror spot and the images. If the radius, x , of the conical volumes so formed reaches a critical radius, x_c , that is, when

* $R = 1$ means $\Lambda = T = 0.999987779$ in this example.

$$\frac{\Delta N}{N} = \frac{\sigma}{h\nu} \frac{E}{\pi x_c^2} \quad (65)$$

then the computer (an HP-9820A) excludes the absorption volume for all further values of $x \leq x_c$. Equation (65) is derived from the fact that absorption is proportional to the absorber particle density N , to the absorbing cross section σ , and to the photon intensity. The transmission, T , is then increased by the ratio of total path to unexcluded path, and the useful absorption is reduced inversely by that ratio. Thus, for each pass, an average correction term, $0 \leq K_{2n} < 1$ is calculated and the useful absorption of (57) and (62) is reduced by the factor $(1 - K_{2n})$. The loss factor is corrected from $A = RT$ to $A = R[T(1 - K_{2n}) + K_{2n}]$. The computer solves for S , calculates the correction terms, corrects each pass accordingly, and sums the useful absorptions keeping track of the energy remaining in the beam. The results are presented in Section X.

X. RESULTS

Selected numerical results, optimized for maximum absorption, are presented in Table I. Of course, depletion and energy densities are dependent on the assumed beam energy; here 30 mJ. The linear absorption, σN , is $6.11 \times 10^{-7} \text{ cm}^{-1}$, and the wavelength, λ , is $0.39 \text{ } \mu\text{m}$. The number in parenthesis should be read as the exponent of ten in a multiplicative factor, thus $1.3(-3)$ means 1.3×10^{-3} , etc. The maximum spot energy density on the mirrors is the maximum intensity per reflection (Column 10). The summed energy densities (Columns 11 and 12) are summed over repeated passes during the whole pulse and are therefore appropriate to heat loads. Impurity heat loads as well as total heat loads are important in practical designs ("dirty" mirror breakdown). The summed focused maximum medium energy density is the energy dumped per pulse in the medium at the confocal point; it is included as a check on overloading a portion of the medium itself. Note also that a 0.01-cm-diameter 30-mJ beam itself has a fluence of 382 J/cm^2 ; we hardly propose that our insertion lens and mirrors can handle those fluences at this time. We have included these smaller beams to give a wide range of numerical parameters for cavity behavior itself. Our choice of a 30-mJ energy beam is only intended to be one

practical choice; the reader may scale our results for others. Last, we have used but one insertion element, namely the converging lens, diameter 2C, of Fig. 1. The use of more elements, a Galilean telescope, for instance, permits some further optimization not investigated here.

In Table II we present a limited comparison of three cavity designs, namely our asymmetric confocal chamber, a chamber using canted mirrors to walk a beam down the chamber by repeated reflections to an eventual standstill and then return,¹ and a folded optical delay line which walks a beam along Lissajous figures on symmetric stable spherical mirror resonators.^{9,10} Table II shows the asymmetric confocal chamber to be far and away the best chamber for useful absorption of light in a weakly absorbing medium. That is true, but the sample is biased and moreover useful absorption is not the only criterion for comparison. The confocal chamber is best for light absorption exclusively. It's worst for uniformity of illumination. The confocal design is only grossly superior when very small mirror losses and other losses make many passes meaningful. Even at a reflectivity of 0.99 in a 20-cm chamber and our very low absorptivity of $6 \times 10^{-7} \text{ cm}^{-1}$, other chambers can be made to closely approach confocal performance (Table II, numbers 1 to 4). At lower reflectivities or much greater absorptivities there is just no point in going to sophisticated designs such as the asymmetric confocal; they do only slightly better than plane-mirror chambers. Surprisingly, mirror spot intensities in the confocal are low, often lower than the folded design for instance, because the focused spot occurs nearly midway between the mirrors and the beam diverges to the mirrors.

In summary, the asymmetric confocal mirror reaction chamber should be considered when one or more of the following factors apply: for a high aspect ratio cavity (W/L is large); when highest beam useful absorption is desired, with consequent high reflectivity mirrors needed, together with a low-loss medium so as to allow many passes; when the time of reaction is not limiting (many passes); when beam power is not excessively high; for high beam concentration; and for well-collimated light beams.

Contra-indication for use of the asymmetric cavity are: complete lack of uniform illumination

TABLE I
SELECTED OPTIMIZED NUMERICAL RESULTS, ASYMMETRIC CONFOCAL CHAMBER ($\sigma_N = 6.11 \times 10^{-7} \text{ cm}^{-1}$, $\lambda = 0.39 \text{ } \mu\text{m}$, LASER BEAM ENERGY = 0.03 J)

#	Chamber Size WxLxh (cm ³)	Effective Laser Beam Diameter, 2C (cm)	Laser Beam Divergence (DL is Dif- fraction Limited) e/d ₀ **	Mirror Reflec- tivity, R	Fractional Useful Ab- sorption, U/E ₀ (no depletion), Eq. (64)	Theoretical Max. Absorp- tion Fraction	Passes Wholly Within Cham- ber, 2π, Eq. (56)	Fraction- al Use- ful Ab- sorption With Depletion U/E ₀ (ma- chine)	Maximum Beam Spot En- ergy Density on Mir- rors Eq. (45) (46), J/ cm ² -refl.	Summed Central Mirror Energy Density, Eq. (54) J/cm ² - pulse	Summed Focused Energy Density Eq. (50) J/cm ² - pulse	Focal Length Ratio S = f ₁ /f ₂
1	20x3.5x1	0.5	1.3 (-3)	0.999	6.94 (-4)	120.7 (-4)	52.	6.94(-4)	0.665	1.80	163.6	1.18794
2	20x3.5x1	0.5	1.3 (-3)	0.995	6.20 (-4)	61.1 (-4)	52.	6.20(-4)	0.622	1.59	160.5	1.18794
3	20x3.5x1	0.5	1.3 (-3)	0.99	5.42 (-4)	12.2 (-4)	52.	5.42(-4)	0.572	1.36	156.8	1.18794
4	20x3.5x1	0.5	1.3 (-3)	0.98	4.22 (-4)	6.11 (-4)	52.	4.22(-4)	0.483	1.00	150.0	1.18794
5	20x3.5x1	0.5	1.3 (-3)	0.96	2.76 (-4)	3.05 (-4)	52.	2.76(-4)	0.342	0.54	138.1	1.18794
6	20x3.5x1	0.5	1.3 (-3)	0.90	1.22 (-4)	0.61 (-4)	52.	1.22(-4)	0.117	0.082	112.5	1.18794
7	32.5x3.5x1x20*	0.5	1.3 (-3)	0.98	3.96 (-4)	6.11 (-4)	45.7	3.95(-4)	0.485	1.03	148.0	1.19105
8	32.5x3.5x1x20*	0.5	1.3 (-3)	0.96	2.68 (-4)	3.05 (-4)	45.7	2.68(-4)	0.345	0.56	136.4	1.19105
9	32.5x3.5x1x20*	0.5	1.3 (-3)	0.90	1.22 (-4)	1.22 (-4)	45.7	1.22(-4)	0.120	0.08	111.4	1.19105
10	20x3.5x1	0.5	5(-5)DL	0.99	7.67 (-4)	12.2 (-4)	92	7.07(-4)	12.21	29.6	1.08(5)	1.18339
11	20x3.5x1	0.2	2.39(-4)DL	0.99	10.97 (-4)	12.2 (-4)	212	10.30(-4)	5.26	6.52	4740.	1.062765
12	20x3.5x1	0.1	4.77(-4)DL	0.99	12.03 (-4)	12.2 (-4)	394	11.06(-4)	4.46	2.17	5345.	1.030036
13	20x3.5x1	0.1	9.54(-4)	0.99	11.93 (-4)	12.206(-4)	345	11.50(-4)	2.83	1.38	1330.	1.030218
14	20x3.5x1	0.05	9.54(-4)DL	0.99	12.20 (-4)	12.206(-4)	706	10.85(-4)	2.5	0.35	2143.	1.014768
15	20x3.5x1	0.05	19.08(-4)	0.99	12.19 (-4)	12.206(-4)	604	12.13(-4)	↑	0.29	532.	1.01494
16	20x3.5x1	0.025	19.08(-4)DL	0.99	12.206(-4)	12.206(-4)	1201	code limits	↑	0.0191	759.	1.007496
17	20x3.5x1	0.01	47.7 (-4)DL	0.99	12.206(-4)	12.206(-4)	2371	code limits	↑	2.91(-5)	161.9	1.003037
18	20x3.5x1	0.05	19.08(-4)	0.999	59.5 (-4)	120.74 (-4)	604	59.4 (-4)	↑	6.90	829.	1.01494
19	20x3.5x1	0.05	19.08(-4)	0.9999	79.1 (-4)	1089.0 (-4)	604	79.0 (-4)	↑	9.55	878.	1.01494
20	20x3.5x1	0.025	19.08(-4)DL	0.999	89.3 (-4)	120.74 (-4)	1201	code limits	↑	9.33	1548.	1.007496
21	20x3.5x1	0.025	19.08(-4)DL	0.9999	151.51 (-4)	1089.0 (-4)	1201	code limits	↑	17.96	1730.	1.007496
22	20x3.5x1	0.01	47.7 (-4)DL	0.999	112.54 (-4)	120.74 (-4)	2371	code limits	↑	4.29	519.	1.003037
23	20x3.5x1	0.01	47.7 (-4)DL	0.9999	284.3 (-4)	1089.0 (-4)	2371	code limits	↑	16.04	667.	1.003037

*Note active volume only 20 cm of W = 32.5 cm.

**Our near-field (Fresnel Number > 1) diffraction limited is $\lambda/\pi C$, far field (Fresnel Number < 1) is $0.61 \lambda/C$.

TABLE I (Continued)

SELECTED OPTIMIZED NUMERICAL RESULTS, ASYMMETRIC CONFOCAL CHAMBER ($\sigma_N = 6.11 \times 10^{-7} \text{ cm}^{-1}$, $\lambda = 0.39 \text{ }\mu\text{m}$, LASER BEAM ENERGY = 0.03 J)

#	Chamber Size WxLxh (cm ³)	Effective Laser Beam Diameter, 2C (cm)	Laser Beam Divergence (DL is Dif- fraction Limited) e/d ₀ **	Mirror Reflec- tivity, R	Fractional Useful Ab- sorption, U/E ₀ (no depletion), Eq. (64)	Theoretical Max. Absorp- tion Fraction	Passes Wholly Within Cham- ber, 2n, Eq. (56)	Fraction- al Use- ful Ab- sorption With Depletion U/E ₀ (ma- chine)	Maximum Beam Spot En- ergy Density on Mir- rors Eq. (45) (46), J/ cm ² -refl.	Summed Central Mirror Energy Density, Eq. (54) J/cm ² - pulse	Summed Focused Energy Density Eq. (50) J/cm ² - pulse	Focal Length Ratio S = f ₁ /f ₂
24	100x10x1	0.25	3.136(-4)	0.995	120.2(-4)	122.2(-4)	1012.	120.0(-4)	~0.53	0.37	1098.	1.01278
25	100x10x1	0.25	1.5 (-4)	0.998	271.7(-4)	305.5(-4)	1130.	268.5(-4)	1.6	3.2	5770.	1.01276
26	100x10x1	0.1	4.77 (-4)DL	0.999	538.2(-4)	611.1(-4)	2387.	538.2(-4)	~2.	2.2	1375	1.005062
27	100x10x1	0.1	4.77 (-4)DL	0.9999	1290.9(-4)	6110.5(-4)	2387	1290.8(-4)	~2.	6.6	1611.	1.005062
28	100x10x1	0.05	9.54 (-4)DL	0.999	571.3(-4)	611.1(-4)	4219.	code limits exceeded	Approx. fails	8.0	585.	1.002534
29	200x10x1	0.25	3.136(-4)	0.995	237.0(-4)	244.4(-4)	900.	237.0(-4)	~0.35	0.24	273.	1.01282
30	200x10x1	0.25	1.5 (-4)	0.998	519.4(-4)	611.1(-4)	1019.	518.6(-4)	~0.89	1.74	1435.	1.01278
31	700x10x1	0.5	1.568(-4)	0.995	713.1(-4)	788.4(-4)	396.	713.0(-4)	~0.13	0.25	50.8	1.02629
32	700x10x1	0.5	6.9 (-5)	0.998	1239.6(-4)	1762.5(-4)	463.	1239.5(-4)	0.31	0.97	291.	1.02612
33	700x10x1	0.2	2.385(-4)DL	0.999	2296.7(-4)	2996.8(-4)	926.	2296.6(-4)	Approx. fails	0.58	58.6	1.01033
34	700x10x1	0.2	2.385(-4)DL	0.9999	3379.0(-4)	8105.8(-4)	926.	3379.0(-4)	must use	0.90	63.5	1.01033
35	700x10x1	0.1	4.77 (-4)DL	0.999	2746.9(-4)	2996.8(-4)	1569.	code limits exceeded	exact form from	0.35	25.9	1.005215
36	700x10x1	0.1	4.77 (-4)DL	0.9999	4566.5(-4)	8105.8(-4)	1569.	code limits exceeded	Eq. (44)	0.72	29.9	1.005215

TABLE II

SELECTED COMPARATIVE RESULTS BETWEEN THE CONFOCAL, CANTED, AND DELAY CHAMBERS (EACH OPTIMIZED FOR MAXIMUM ABSORPTION WITH A SINGLE OPTIMIZED INSERTION ELEMENT). NO DEPLETION. CONFOCAL REFERS TO THE ASYMMETRIC CONFOCAL CHAMBER OF THIS REPORT, CANTED TO THE CYLINDRICAL OR PLANE CANTED MIRRORS OF I¹, AND DELAY TO A FOLDED OPTICAL DELAY CAVITY IN THE FORM OF SYMMETRIC AFOCAL MIRRORS WITH OFF-AXIS BEAM PROPAGATION.^{9,10}

#	Chamber Size WxLxh (cm ³)	Diameter 2C (cm)	Mirror Reflec- tivity R	Theoreti- cal Maxi- mum Use- ful Beam Absorp- tion	Confocal Fractional Useful Beam Absorption	Wholly Within Chamber (Total Passes =)	Canted Fractional Useful Beam Absorption	Passes (Mostly Within Chamber)	Delay Fractional Useful Beam Absorption	Passes
1	20x3.5x1	0.5	0.99	12.206(-4)	7.67(-4)	92.	4.35(-4)	46.	4.09(-4)	42
2	20x3.5x1	0.2	0.99	12.206(-4)	10.97(-4)	212.	8.26(-4)	119.		
3	20x3.5x1	0.1	0.99	12.206(-4)	12.03(-4)	394.	9.83(-4)	200.		
4	20x3.5x1	0.05	0.99	12.206(-4)	12.20(-4)	706.	>10. (-4)	>215.		
5	100x10x1	0.25	0.998	305.5 (-4)	271.7 (-4)	1130.	113. (-4)	226		
6	200x10x1	0.25	0.998	611.1 (-4)	519.4 (-4)	1019.	117. (-4)	181.		
7	700x10x1	0.5	0.998	1762.5 (-4)	1239.6 (-4)	463.	344. (-4)	95.		

with consequent possible high heat loads on mirrors and in the medium for the higher energy beams, expense of curved mirrors, relative inflexibility of design compared to the canted mirror design which can optimize multiple beam sizes and divergences by simply changing the cant angle.

In short, and as Table II shows, in its special realm of operation the asymmetric confocal mirrors can do exceptionally well in usefully reacting weakly absorbed light.

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