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VOID FRACTION CORRELATIONS IN TWO-PHASE HORIZONTAL FLOW

*G. Papathanassiou, P. F. Maeder,
R. DiPippo and D.A. Dickinson*

*University of California
Los Alamos National Laboratory
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by

G. Papathanassiou,
P. F. Maeder, R. DiPippo*, and D. A. Dickinson

Division of Engineering
Brown University
Providence, Rhode Island 02912

May 1983

* Also, Mechanical Engineering Dept., Southeastern Massachusetts University,
N. Dartmouth, MA 02747.

ABSTRACT

This study examines some physical mechanisms which impose limits on the possible existence of two-phase flow in a horizontal pipe. With the aid of this analysis and the use of the Martinelli variable, X , a method is developed which determines the range of possible void fractions for a given two-phase flow.

This method affords a means of direct comparison among void fraction correlations, as well as between correlation predictions and experimental results. In this respect, four well-known void fraction correlations are compared against each other and with experimental results obtained in the Brown University Two-Phase Flow Research Facility.

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LIST OF SYMBOLS

Latin Letters

A	cross-sectional area
B	constant defined by eq. (57)
D	pipe diameter
F_1	function defined by eq. (A4)
F_2	function defined by eq. (A5)
F_3	function defined by eq. (A6)
F_4	function defined by eq. (A7)
f	friction factor
Fr	Froude number
h	enthalpy
k	coupling coefficient defined by eq. (25)
K	correlating function defined by eq. (59)
L	length measured along the pipe
$\dot{m}$	mass flow rate
P	pressure
Q	dynamic pressure ($= \rho w^2/2$)
Re	Reynolds number based on pipe diameter
s	shear force; entropy
T	temperature
v	specific volume
w	velocity
X	Martinelli variable
x	flowing quality
x_1	downstream coordinate
y_L	liquid volume fraction
Z	correlating function defined by eq. (60)

Greek Letters

α	void fraction
γ	density ratio ($= \rho_\ell / \rho_g$)
μ	absolute viscosity
ν	kinematic viscosity ($= \mu / \rho$)
ν_R	kinematic viscosity ratio ($= \nu_\ell / \nu_g$)
ρ	density
σ	slip ratio ($= w_g / w_\ell$)
τ	shear stress
ψ	mass flux

Subscripts

edp	refers to equal-dynamic-pressure case
ewss	refers to equal-wall-shear-stress case
g	refers to gaseous phase variable
i	refers to quantity at interface
ℓ	refers to liquid phase variable
ns	refers to no-slip case
0	refers to stagnation conditions
s	refers to superficial variable
sat	refers to saturation condition
w	refers to variable at wall

1. Introduction

From an engineering point of view, the frictional pressure drop, and the average physical and thermodynamic properties of a flowing fluid are of great interest. Such information allows us to design piping systems and other equipment to meet required specifications. When dealing with a fluid in a single phase, say, liquid or vapor, information of this nature is readily available. The situation is much less resolved for the case of two-phase, liquid-vapor flows, even for a single-component fluid. Recourse is usually made to the two-phase friction factor, often expressed as a multiplier of the single-phase, liquid-only friction factor, and to the void fraction of the flowing fluid from which the average density may be obtained. Once these two quantities are known, the average fluid properties can be found and the total pressure drop along the flow can be calculated.

The approach to providing these quantities has been that of correlations based on experiments. The first correlation in two-phase flow was developed in the late 1940's by Martinelli [1,2]. Working from a particular set of experimental observations and similarity laws, he proposed a correlation for the pressure drop and the void fraction in horizontal two-phase flow. This correlation, although widely accepted and used in many engineering applications, often deviates appreciably from experimental results.

This shortcoming of correlations, in general, gave rise to two-phase flow models which rely more on analytical considerations and less on empirical correlations. These models can be classified as: (a) "single-fluid" models which are based on the idea of replacing the two-phase mixture by an equivalent pseudo single-phase fluid; (b) "separated" models which are based on the assumption that the two phases are completely separated, each

occupying its own section of the duct; and (c) "pattern" models which are developed under the assumption of a predetermined flow-pattern structure (e.g., bubbly, slug, annular, etc.). Each of these models has its own advantages and disadvantages, but most of them still require a void fraction assumption or correlation, among others, in order for the equations to form a closed, or solvable set.

As a result, a large number of void fraction correlations have been proposed in the literature [3,6], but even in those that are most widely accepted, the standard deviation of error is about 20-30% as reported by Collier [7].

In the present study some mechanisms that impose physical limits outside which two-phase flow cannot exist will be examined. With the aid of this analysis several void fraction correlations will be tested in an attempt to shed some light on their applicability and physical phenomena that may be implied by them. A comparison between these correlations and experimental results obtained by Dickinson [8] in the Brown University Two-Phase Flow Research Facility [9] is also made.

2. Basic Relations in Two-Phase Flow

Adopting for the moment the basic notion of the separated flow model, two-phase flow can be visualized as one where the two phases, each flowing with its own velocity, are completely separated by an interface with the liquid phase occupying one portion of the duct cross-section and the gas occupying the other. The imagined phase distribution is such that the interface lies along radial lines as depicted schematically in Fig. 1. This presentation is not to be confused with the so-called "stratified"

two-phase flow pattern. It is meant to apply to any flow pattern, where, for the sake of convenience, each phase is lumped together and occupies its own segment of the pipe.

2.1 Assumptions

The underlying assumptions are:

- (a) *Equilibrium exists between the two phases.* That is, the liquid and vapor phases are in thermal, mechanical and chemical equilibrium across any section of the pipe.
- (b) *The flow is one-dimensional.* That is, the velocity vectors for each phase have only one component perpendicular to the cross-section of the pipe.
- (c) *The flow is steady.* That is, the fluid properties are averaged with respect to time.
- (d) *The system is adiabatic.* That is, there is no heat exchange between the fluid and the surroundings.
- (e) *The fluid fills the pipe completely.*

2.2 Definitions

Before any equations can be developed some definitions will be stated:

$$\text{Void Fraction: } \alpha = A_g/A \quad (1)$$

$$\text{Holdup: } 1 - \alpha = A_l/A \quad (2)$$

$$\text{Flowing Quality: } x = \dot{m}_g/\dot{m} \quad (3)$$

$$\text{Slip Ratio: } \sigma = w_g/w_l \quad (4)$$

$$\text{Density Ratio: } \gamma = \rho_l / \rho_g \quad (5)$$

$$\text{Dynamic Pressure: } Q = (1/2)\rho w^2 \quad (6)$$

$$\text{Mass Flux: } \psi = \dot{m}/A, \quad (7)$$

where $\dot{m}$ is the total mass flow rate, i.e.,

$$\dot{m} = \dot{m}_g + \dot{m}_l. \quad (8)$$

(The reader is referred to the List of Symbols at the front of the report.)

The mass flow rate of each phase can be expressed as

$$\dot{m}_g = \rho_g w_g A_g, \quad (9a)$$

and

$$\dot{m}_l = \rho_l w_l A_l. \quad (9b)$$

Using definitions (1) and (2) these become

$$\dot{m}_g = \rho_g w_g \alpha A, \quad (10a)$$

and

$$\dot{m}_l = \rho_l w_l (1-\alpha)A. \quad (10b)$$

By combining eqs. (7), (10a) and (3), we obtain

$$\psi x = \rho_g w_g \alpha. \quad (11a)$$

Similarly,

$$\psi(1-x) = \rho_l w_l (1-\alpha). \quad (11b)$$

We may derive an expression for the void fraction in terms of the quality, the slip ratio, and the density ratio by dividing eq. (10b) by (10a) and using eq. (5) to give

$$\frac{\dot{m}_l}{\dot{m}_g} = \frac{\gamma}{\sigma} \frac{1-\alpha}{\alpha} . \quad (12)$$

Combining this with eq. (3) yields

$$\frac{1-x}{x} = \frac{\gamma}{\sigma} \frac{1-\alpha}{\alpha} , \quad (13)$$

which upon rearrangement becomes

$$\sigma = \frac{x}{1-x} \frac{1-\alpha}{\alpha} \gamma . \quad (14)$$

The desired expression results from solving eq. (14) for α :

$$\alpha = \frac{1}{1 + \frac{1-x}{x} \frac{\sigma}{\gamma}} . \quad (15)$$

Therefore, the holdup may be written as

$$1 - \alpha = \frac{1 - x}{1 + x(\frac{\gamma}{\sigma} - 1)} . \quad (16)$$

Finally, we can obtain useful expressions for the vapor and liquid velocities by manipulation of eqs. (11a), (14) and (4). From eq. (11a) by rearrangement

$$w_g = \frac{\psi}{\rho_g} \frac{x}{\alpha} . \quad (17)$$

Using eq. (14) this becomes

$$w_g = \frac{\psi}{\rho_l} \sigma [1 + x(\frac{\gamma}{\sigma} - 1)], \quad (18a)$$

and using eq. (4) we obtain

$$w_l = \frac{\psi}{\rho_l} [1 + x(\frac{\gamma}{\sigma} - 1)]. \quad (18b)$$

2.3 Energy Balance

An energy balance taken between a position in the fluid where stagnation conditions prevail (denoted by subscript 0) and an arbitrary downstream station in the two-phase flow region gives

$$h_0 = h + (1-x) \frac{w_l^2}{2} + x \frac{w_g^2}{2}. \quad (19)$$

Using eq. (4) and rearranging, this becomes

$$h_0 - h = \frac{w_l^2}{2} [1 + x(\sigma^2 - 1)]. \quad (20)$$

Substitution for w_l from eq. (18b) yields

$$h_0 - h = \frac{1}{2} \left(\frac{\psi}{\rho_l}\right)^2 [1 + x(\frac{\sigma}{\gamma} - 1)]^2 [1 + x(\sigma^2 - 1)]. \quad (21)$$

2.4 Superficial Quantities

These are quantities computed on the assumption that the mass of each phase flows alone in the pipe, filling the entire cross-section.

Under this assumption, eqs. (9a) and (9b) yield the superficial velocities for the vapor and the liquid:

$$w_{gs} = \frac{\dot{m}_g}{\rho_g A}, \quad (22a)$$

and

$$w_{ls} = \frac{\dot{m}_l}{\rho_l A} \quad (22b)$$

Substitution of eqs. (10a) and (10b) in the above give

$$w_{gs} = \alpha w_g, \quad (23a)$$

and

$$w_{ls} = (1-\alpha)w_l. \quad (23b)$$

3. Physical Considerations in Two-Phase Flow

Consider the two-phase flow of a two-component mixture (e.g., air-water according to the model of the flow in Fig. 1. If we assume that each phase is incompressible, then the one-dimensional momentum equation for each phase has the form

$$\alpha A \frac{dp}{dx_1} = -\tau_{gw} \alpha D - s_i \quad (\text{vapor}) \quad (24a)$$

$$(1-\alpha) A \frac{dp}{dx_1} = -\tau_{lw} (1-\alpha) D + s_i, \quad (\text{liquid}) \quad (24b)$$

where τ_w is the wall shear stress, and s_i is the interfacial shear force per unit length acting on the liquid in the direction of motion. This has the form

$$s_i = k(w_g - w_l)|w_g - w_l|, \quad (25)$$

where k is the coupling coefficient between the two phases.

From this analysis it is evident that the two extreme values of k , namely k very large and $k = 0$, determine two limits outside which

two-phase flow cannot exist. Let us examine the consequences of these limits.

Case A: k is very large

When k is very large, it follows from eq. (25) that w_g must be nearly equal to w_l in order for the interfacial shear force to remain finite. Thus, in the limit as $k \rightarrow \infty$, we conclude that

$$w_g = w_l, \quad (26)$$

or, alternatively, the slip ratio must go to its minimum value, i.e.,

$$\sigma = 1. \quad (27)$$

Case B: $k = 0$

At the other physical extreme represented by the case when $k = 0$, eqs. (24a) and (24b) reduce to

$$\tau_{gw} = \tau_{lw}, \quad (28a)$$

or

$$Q_g f_g = Q_l f_l, \quad (28b)$$

where we have used the wall shear-stress definition, namely,

$$\tau_w = fQ. \quad (29)$$

Examination of the Moody chart [10] in the turbulent region shows that, for smooth pipes, a variation in the Reynolds number of about one order of magnitude results in only a small change in the friction factor. In a typical two-phase flow situation, the Reynolds numbers of the two phases are usually in the range of $(0.2-1.0) \times 10^6$. The ratio of the friction factors

in this case is never above 1.25. Consequently, the assumption that

$$f_g = f_l \quad (30)$$

can be justified approximately, which in combination with eq. (29) gives

$$Q_g = Q_l, \quad (31)$$

or, that the dynamic pressures of the two phases must be equal in this limiting case.

Using eq. (6), eq. (31) can be rewritten as

$$\rho_g w_g^2 = \rho_l w_l^2, \quad (32)$$

which combined with eqs. (4) and (5) yields

$$\sigma = \gamma^{0.5}. \quad (33)$$

Thus on the basis of the preceeding heuristic reasoning, it may be concluded that two-phase flow can exist only between the limits of equal phase velocities ($\sigma = 1$), and equal wall shear stresses. The latter limit is very nearly equivalent to equal dynamic pressures ($\sigma = \gamma^{0.5}$). The slip ratios corresponding to these two cases are, respectively, the minimum and maximum possible slip ratios for a given flow. Examination of eq. (14) shows that these ratios correspond, respectively, to the maximum and minimum possible void fractions.

A note of caution is appropriate at this point. Our rationale is strictly true, albeit subject to the stated approximations, for the case of two-component two-phase, horizontal flow. The momentum equations, eqs. (24a) and (24b), include only a frictional pressure drop term. For non-horizontal flows, a gravity term must be included for which the average fluid density

is needed. For one-component, two-phase flows, an acceleration term is necessary because of the volume change resulting from phase transformations, either evaporation or condensation. We will discuss the latter effect when we compare the results of our measurements on the flow of Refrigerant-114 with the void fraction correlations.

4. Void Fraction Spectrum

Typical void fraction correlations [1,5] make use of one independent variable, defined as some combination of easily measurable quantities. The correlations are simply relationships, based on experimental results and statistical data reduction, between this variable and the void fraction. These relationships are usually presented in closed analytic form, or simply as a graph.

4.1 Martinelli Variable X

In the present study the variable, X , first used by Martinelli [2] to correlate void fractions and frictional pressure drop in two-phase flow, is chosen in an effort to shed some light on the void fraction problem.

Martinelli expressed his variable as the ratio of the superficial liquid pressure drop to the superficial gas pressure drop, i.e.,

$$X^2 = \left(\frac{\Delta p}{\Delta L} \right)_{ls} / \left(\frac{\Delta p}{\Delta L} \right)_{gs} \quad (34)$$

The pressure drop in each phase alone is given by

$$\left(\frac{\Delta p}{\Delta L} \right) = (1/2) \rho w^2 f/D, \quad (35)$$

where w is the mean velocity, D is the diameter of the pipe, and f is Darcy's friction factor. Substitution of eq. (35) into eq. (34) gives

$$\chi^2 = \frac{\rho_{\ell} w_{\ell s}^2}{\rho_g w_{gs}^2} \frac{f_{\ell s}}{f_{gs}}. \quad (36)$$

Martinelli used a generalized form of the Blasius friction factor law for each phase, i.e.,

$$f_g = \frac{C_g}{Re_g^m}, \quad (37a)$$

and

$$f_{\ell} = \frac{C_{\ell}}{Re_{\ell}^n}, \quad (37b)$$

where

$$m = n = 0.2, \quad (37c)$$

and

$$C_g = C_{\ell} = 0.046, \quad (37d)$$

when both phases in the pipe are turbulent. Consequently, the friction factor for this case can be expressed as

$$f = \frac{0.046}{Re^{0.2}}. \quad (38)$$

Substitution of eq. (38) into eq. (36) gives

$$\chi^2 = \frac{\rho_{\ell} w_{\ell s}^2}{\rho_g w_{gs}^2} \left(\frac{Re_{gs}}{Re_{\ell s}} \right)^{0.2}. \quad (39)$$

Using the definition of the Reynolds number,

$$Re = \frac{\rho_w D}{\mu} \quad (40)$$

and rearranging eq. (39), we obtain

$$x^2 = \left(\frac{w_{ls}}{w_{gs}} \right)^{1.8} \left(\frac{\rho_l}{\rho_g} \right)^{0.8} \left(\frac{\mu_l}{\mu_g} \right)^{0.2} \quad (41)$$

Substitution for the superficial velocities from eqs. (22a) and (22b) and taking the square root results in

$$x = \left(\frac{\dot{m}_l}{\dot{m}_g} \right)^{0.9} \left(\frac{\rho_g}{\rho_l} \right)^{0.5} \left(\frac{\mu_l}{\mu_g} \right)^{0.1}, \quad (42)$$

which is the expression for x usually found in the literature [11,12].

Using eqs. (3) and (8), eq. (42) may be written as

$$x = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{\rho_g}{\rho_l} \right)^{0.5} \left(\frac{\mu_l}{\mu_g} \right)^{0.1} \quad (43)$$

Substitution for the term $\frac{1-x}{x}$ from eq. (13) yields

$$x = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \left(\frac{\rho_l}{\rho_g} \right)^{0.4} \left(\frac{w_l}{w_g} \right)^{0.9} \left(\frac{\mu_l}{\mu_g} \right)^{0.1} \quad (44)$$

Upon rearrangement, this becomes

$$x = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \left[\frac{\rho_l w_l^2}{\rho_g w_g^2} \left(\frac{\rho_g w_g / \mu_g}{\rho_l w_l / \mu_l} \right)^{0.2} \right]^{0.5} \quad (45)$$

From the definitions given in eqs. (6) and (40), we find

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \left[\frac{Q_l}{Q_g} \left(\frac{Re_g}{Re_l} \right)^{0.2} \right]^{0.5}, \quad (46)$$

which, using the same friction factor law, eq. (38), as before, becomes

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \left(\frac{Q_l f_l}{Q_g f_g} \right)^{0.5}, \quad (47a)$$

or

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \left(\frac{\tau_{lw}}{\tau_{gw}} \right)^{0.5}. \quad (47b)$$

This form of X is the most convenient one for the present analysis since it contains the ratio of the wall shear stresses of the two phases, which is one of the limiting constraints in two-phase flow, as we showed in Section 3. Consequently, this constraint can be easily applied to give a relationship between the minimum possible void fraction and X . Alternatively, eq. (44) rewritten in the form

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \gamma^{0.4} \sigma^{-0.9} \left(\frac{\mu_l}{\mu_g} \right)^{0.1} \quad (48)$$

can be used with $\sigma = 1$ to give a relationship between the maximum possible void fraction and X .

4.2 Special Cases

To cover the whole void fraction spectrum, the following cases can be distinguished:

Case 1: Equal wall shear stresses, i.e., $Q_g f_g = Q_l f_l$ from eq. (28b).

This is the condition for maximum slip ratio and minimum void fraction. Substitution of eq. (28b) into eq. (47) gives

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9}, \quad (49a)$$

or

$$\log X = 0.9 \log \left(\frac{1-\alpha}{\alpha} \right). \quad (49b)$$

Note that all logs are to the base 10. Comparison of eqs. (49a) and (48) gives

$$\sigma = \gamma^{4/9} \left(\frac{\mu_l}{\mu_g} \right)^{1/9}, \quad (50)$$

which is the slip ratio corresponding to this case, and hence the maximum one possible.

Case 2: Equal dynamic pressures, i.e., $Q_g = Q_l$ or $\rho_g w_g^2 = \rho_l w_l^2$ from eq. (32).

Substitution of this condition into eq. (45) yields

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \left(\frac{\rho_g w_g / \mu_g}{\rho_l w_l / \mu_l} \right)^{0.1}. \quad (51)$$

Using the definition of the kinematic viscosity,

$$\nu = \frac{\mu}{\rho} \quad (52)$$

eq. (51) becomes

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \gamma^{0.05} \left(\frac{v_l}{v_g} \right)^{0.1} \quad (53)$$

Using v_R to denote the ratio of the kinematic viscosity of the liquid phase to that of the vapor phase, eq. (53) can also be expressed as

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \gamma^{0.05} v_R^{0.1}, \quad (54a)$$

or

$$\log X = 0.9 \log \left(\frac{1-\alpha}{\alpha} \right) + 0.05 \log \gamma + 0.1 \log v_R. \quad (54b)$$

The slip ratio corresponding to this case was derived earlier and is given by

$$\sigma = \gamma^{0.5}. \quad (33)$$

Case 3: Constant Slip Ratio

In this case the general relationship between X and α is given by eq. (48), which using eq. (52) can be rewritten as

$$X = \left(\frac{1-\alpha}{\alpha} \right)^{0.9} \gamma^{0.5} \sigma^{-0.9} v_R^{0.1}, \quad (55a)$$

or

$$\log X = 0.9 \log \left(\frac{1-\alpha}{\alpha} \right) + 0.5 \log \gamma - 0.9 \log \sigma + 0.1 \log v_R. \quad (55b)$$

Substitution of the minimum value for the slip ratio, namely $\sigma = 1$, into the above equation gives the relationship between X and the maximum possible void fraction.

4.3 Computations

Using property data for Refrigerant-114 (R114) at 25°C, eqs. (49b), (54b) and (55b) have been plotted as the holdup versus X in Fig. 2, and thus the complete void fraction spectrum for a flow of R114 at 25°C has been obtained.

Examination of eq. (55b) shows that a horizontal shift in the X -direction of a constant slip-ratio curve (i.e., at a constant holdup) through a distance equal to

$$0.5 \log \frac{\gamma_1}{\gamma} + 0.1 \log \frac{v_{R1}}{v_R},$$

where γ and v_R are properties of R114 and γ_1 and v_{R1} are properties of some other fluid, will give a constant slip line of the same slip ratio corresponding to the new fluid. This means that the graph can be transformed to suit any fluid (or combination of fluids) having properties different from those of R114. To illustrate the procedure, the line corresponding to $\sigma = 1$ for an air-water flow at 25°C ($\gamma_1 = 837.8$, $v_{R1} = 0.058$) has also been included on the graph.

The calculation of the horizontal shift distance between constant slip lines can be avoided and the method simplified by the use of the property scales drawn on the graph. These scales directly give the distance that the constant slip lines should be moved, relative to the R114 lines, in order to correspond to a new fluid. The use of these scales is illustrated on the graph by the construction of the $\sigma = 1$ line for air-water flow mentioned previously.

It should be noted that the line corresponding to the equal-wall-shear-stresses case (Case 1) is universal since the fluid properties do not appear

in the correlation, eq. (49b). This line will remain at the same position on the graph for any fluid, but it will correspond to a different slip ratio according to eq. (50).

It is evident that the same method can be followed in order to transform the graph to a different temperature since the properties are assumed to depend only on temperature. The variation with temperature for R114 is also included on the graph.

Using the above method, the void fraction spectrum in two-phase flow of any fluid at any temperature can be obtained. Void fractions accurately measured experimentally or predicted by a valid correlation should fall within this spectrum.

Another important feature of this graph lies in the nature of the Martinelli variable, X , namely that it accounts for differences in stagnation conditions (see Appendix). Consequently, measured results from many experiments performed by different investigators under different stagnation conditions can be transferred onto this one graph, where they can be compared against each other, as well as against the calculated curves. In this manner, the validity of experimental results can be checked since they should fall inside the spectrum, and trends in the variation of the void fraction with some parameter (e.g., σ) can be examined.

5. Void Fraction Correlations

Dukler [13] has compared several void fraction correlations in an attempt to find one that gives the best agreement with the experimental results contained in his data bank. The correlations that he tested are the ones proposed by Martinelli [2], Hoogendoorn [4] and Hughmark [6], the latter

being a modification of a correlation originally proposed by Bankoff [3]. He came to the conclusion that, although none of these correlations was entirely satisfactory over the whole range of his data, the one by Hughmark performed the best overall, followed closely by Hoogendoorn's.

5.1 Presentation of Correlations

A. Martinelli's Correlation

This is the first correlation ever developed for two-phase flow. It is based on experimental results taken during horizontal, air-liquid*, two-phase flow. It relates the void fraction to the variable X already discussed previously. This relationship was originally presented on a graph but various closed form relationships have been proposed by later investigators [12,14] through curve fitting.

B. Hoogendoorn's Correlation

This correlation is also empirical in nature, since it has been developed out of experimental results in horizontal, air-liquid*, two-phase flow. It has the form

$$\frac{\alpha}{1-\alpha} = B \left[w_{sg} \left(1 - \frac{\alpha}{1-\alpha} \frac{w_{sl}}{w_{sg}} \right) \right]^{0.85}, \quad (56)$$

where

$$B = 0.60^{-0.85} (\text{m/s})^{-0.85}.$$

*The liquids used were water and several oils of various viscosities.

C. Bankoff's Correlation

This correlation was developed theoretically for vertical flow of a single fluid having variable density and velocity across the pipe. It has the form

$$\frac{1}{X} = 1 - \gamma \left(1 - \frac{K}{\alpha}\right), \quad (58)$$

where

$$K = 0.71 + 0.0001 P, \quad (59)$$

and P is measured in lb/in^2 .

D. Hughmark's Correlation

This is a modification of the previous correlation in which K is now related to a variable, Z , which in turn is a function of the Reynolds and Froude numbers, and the liquid volume fraction, y_ℓ , that would exist if there were no slip ($\sigma = 1$) between the phases. This relationship is given by

$$Z = \frac{Re^{1/6} Fr^{1/8}}{y_\ell^{1/4}}, \quad (60)$$

where

$$Re = \frac{\psi D}{(1-\alpha)\mu_\ell + \alpha\mu_g}, \quad (61)$$

$$Fr = \frac{w_{ns}}{gD}, \quad (62)$$

and

$$y_L = \frac{v_L}{v_L + \frac{x}{1-x} v_g} \quad (63)$$

Finally, the relationship between Z and K is presented as a graph.

Although the original Bankoff correlation was developed for vertical flow, it is claimed that this correlation predicts void fractions accurately in both vertical and horizontal two-phase flow.

5.2 Prediction of Correlations for a Given Two-Phase Flow

Using the correlations presented in Sect. 5.1, the void fraction in a horizontal flow of R114, which evaporates as it flows down a two-inch pipe, is predicted. This choice of geometry and fluid was made so that the predictions of the correlations can be directly compared with the experimental results taken at the Brown University Two-Phase Flow Research Facility (TPFRF), which has a test section of the same geometry and uses R114 as its working fluid [9].

Figure 3 shows the void fraction-temperature profile of this flow as predicted by the correlations for the following conditions (liquid R114): $T_0 = 25^\circ\text{C}$, $\dot{m} = 5 \text{ kg/s}$, $P_0 - P_{\text{sat}}(T_0) = 35 \text{ kPa}$. The temperature range of this graph is restricted to the range within which the above facility usually operates. For the sake of comparison, the profiles corresponding to equal phase velocities ($\sigma = 1$), equal dynamic pressures, and equal wall shear stresses are also included on the graph. (For the calculation methodology used to obtain this graph, see the Appendix.)

From this graph it can be seen that three of the correlations give similar predictions, diverging from each other only at very low void fractions. One of the correlations (Martinelli) crosses the equal phase

velocities line at a void fraction of about 0.4 and falls outside the physically permissible region as determined by the conditions given in Sect. 3 at lower void fractions. The fourth correlation (Bankoff) generally predicts void fractions lower than the other three. It should be noted, however, that Bankoff's correlation was developed for vertical flow, while the other three correlations mainly apply to horizontal flow.

Figure 4 is essentially the same plot as Fig. 3 but extended to cover the whole void fraction range. Again, it can be seen that the same three correlations fall close to each other over most of the graph, deviating only at the high void fraction end. Bankoff's correlation continues to predict void fractions lower than the other three and eventually crosses the lower limit line given by the equal-wall-shear-stress case, falling outside the physically permissible region at void fraction values above 0.7.

It can be concluded that the correlations give similar predictions over the greater part of the void fraction range. Disagreement mainly occurs near the two ends of the range, which is to be expected, since few experimental results, on which these correlations are based, exist at these two ends.

5.3 Comparison with Present Experimental Results

Since none of the two-phase flow measurements performed using the Brown University TPRF was made under stagnation conditions identical to those used in order to predict void fractions according to the correlations, the predicted values together with the experimental results (Table 1) are transferred to a void fraction spectrum graph (Fig. 5). On such a graph, as already mentioned, variations in stagnation conditions have been taken care

of automatically in X . Furthermore, differences that arise due to variations in downstream temperatures have been accounted for by using the temperature variation scale for R114 from Fig. 2.

On this graph (Fig. 5) the whole void fraction picture can be clearly seen. Hughmark's and Hoogendoorn's correlations appear as constant slip lines that correspond to a slip ratio of between 2 and 3, suggesting that two-phase flow proceeds with constant slip ratio. Martinelli's correlation falls between the previous two near the middle of the void fraction range and diverges at the two ends of the range. Martinelli's correlation suggests, however, that two-phase flow proceeds with an increasing slip ratio. Bankoff's correlation, although it falls between Hoogendoorn's and Hughmark's correlations at the lower half of the void fraction range, eventually diverges and levels off at a void fraction value of about 0.7. This indicates that in vertical flow there will always be droplets traveling into the vapor phase, and that complete evaporation of the liquid will never happen. While in principle such a case is not physically impossible, reliable void fraction data for vertical flow have not yet been obtained, and consequently a direct comparison with this correlation cannot be made, although experiments of such nature are currently under way at the Brown University TPRF.

Dickinson's experimental results [8] do not cover the entire void fraction range. However, within this range the void fractions observed are lower than those predicted by the correlations at a given X , although they fall close to Hoogendoorn's correlation. These results were obtained during the flow of R114 which evaporates as it flows downstream. This phase change results in an increase of the amount of vapor and consequently a decrease in the mean density of the flow. This gives rise to compressibility effects.

The pressure decreases more rapidly and the vapor is accelerated faster than the liquid. Thus, the slip ratio increases and the void fraction decreases. It can be concluded that compressibility effects reduce the void fraction in comparison to that which would exist in its absence.

None of the correlations tested takes compressibility effects into consideration, since they are developed out of experimental results obtained during two-component (e.g., air-water) two-phase flow, where the fluids can be considered incompressible. Consequently, these correlations overestimate void fractions taken during the flow of a one-component fluid. A new parameter, which takes into account compressibility effects (i.e., a Mach number), should be introduced into these correlations (in addition to the Reynolds and Froude numbers already used by Hughmark) in order to be able to predict void fractions during two-phase flow of a one-component fluid.

6. Concluding Remarks

The void fraction spectrum graph developed in this study can be a very useful tool in the future assessment of void fractions in two-phase flow. Besides showing the allowable void fraction region, it has the advantage that experimental results from many experiments obtained under any flow conditions can be indicated on it. Furthermore, curves corresponding to various void fraction correlations can easily be superimposed on it.

Consequently, this method can serve to compare experimental results against correlations and correlations against each other. It can also serve to check the range of validity of a certain correlation, or the validity of an experimental result. If an experimentally obtained result does not fall within the permissible region on this graph, then the acquisition technique should be reexamined.

This method can also help in the future to find: (a) a reliable correlation within a certain range, (b) a better overall correlation, or (c) the dependence of the void fraction on some parameter not previously considered such as the Mach number.

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TABLE 1

Experimental results taken by Dickinson [8]
during Horizontal Flow of R114 in a 2 inch diameter pipe

Experimental Run	Stagnation Temp. T_0 (°C)	Boost (kPa)	Mass Flow Rate $\dot{m}$ (kg/s)	Downstream Temp. T (°C)	α (%)	X -
9/27 B	25.75	39.24	4.99	23.66	31.35	5.64
9/27 A	27.20	29.69	4.87	22.91	58.52	2.90
9/27 C	26.94	31.81	5.12	21.08	64.62	2.13
9/28 A	25.97	37.05	5.36	19.81	66.12	2.00
9/23 A	26.39	38.92	5.45	19.54	67.64	1.81
10/05 B	27.50	28.15	4.85	22.94	61.85	2.74
9/23 B	26.30	35.72	5.38	19.19	67.90	1.74
9/28 C	25.71	41.50	5.53	19.47	70.95	1.97
9/24 A	26.42	35.58	5.39	19.31	68.09	1.74
10/04 D	35.65	13.44	4.50	30.46	55.60	2.61
10/04 C	35.67	11.59	5.21	26.05	73.87	1.39
10/04 B	34.87	18.12	5.54	23.77	73.81	1.18

FIG. 1 SCHEMATIC REPRESENTATION OF TWO-PHASE FLOW IN HORIZONTAL PIPE.

FIG. 2 HOLDUP AS A FUNCTION OF THE MARTINELLI VARIABLE FOR VARIOUS SLIP RATIOS.

FIG. 3 VOID FRACTION AS A FUNCTION OF DOWNSTREAM TEMPERATURE FOR VARIOUS CORRELATIONS, SHOWING PHYSICAL LIMITS.

FIG. 4 COMPARISON OF VOID FRACTION CORRELATIONS.

FIG. 5 HOLDUP AS A FUNCTION OF MARTINELLI VARIABLE: CORRELATIONS, SLIP RATIOS, AND PRESENT MEASUREMENTS.

APPENDIX

Calculation Methodology

During a two-phase flow experiment, one can measure the following quantities:

- stagnation temperature, T_0
- mass flow rate, $\dot{m}$
- stagnation pressure, P_0
- downstream temperature, T .

From P_0 and T_0 , the stagnation enthalpy, h_0 , can be found from property tables for the working fluid. Similarly, from T , the ratios $\gamma = \rho_\ell / \rho_g$ and $v_R = v_\ell / v_g$ can be found. Of course, one must assume thermodynamic equilibrium between the two phases at this point.

We are interested in calculating the following quantities:

- flowing quality, x
- no-slip velocity, w_{ns}
- superficial velocities, w_{gs} and $w_{\ell s}$
- Martinelli variable, X
- void fraction, α .

To obtain the quality, x , we rewrite the energy equation, eq. (21), as follows:

$$h = h_0 - 0.5(\psi/\rho_\ell)^2 \{1 + x(\frac{\sigma}{\gamma} - 1)\}^2 [1 + x(\sigma^2 - 1)]. \quad (A1)$$

From elementary considerations, we also know

$$h = h_\ell + x(h_g - h_\ell). \quad (A2)$$

These two equations combine to form an expression for x as a function of known quantities, i.e., $h_0(P_0, T_0)$, $\psi = \dot{m}/A$, $\rho_\ell(T)$, $\gamma(T)$, $h_\ell(T)$, $h_g(T)$; and the slip ratio σ which is treated as a parameter. The resulting equation is a cubic in x that may be expressed in the following form:

$$F_1 x^3 + F_2 x^2 + F_3 x + F_4 = 0, \quad (A3)$$

where

$$F_1 = \frac{\sigma^4}{\gamma^2} - \frac{2\sigma^3}{\gamma} + \sigma^2 \left(1 - \frac{1}{\gamma^2} \right) + \frac{2\sigma}{\gamma} - 1, \quad (A4)$$

$$F_2 = \frac{2\sigma^3}{\gamma} + \sigma^2 \left(\frac{1}{\gamma^2} - 2 \right) - \frac{4\sigma}{\gamma} + 3, \quad (A5)$$

$$F_3 = \sigma^2 + \frac{2\sigma}{\gamma} - 3 + 2(h_g - h_\ell)/(\psi/\rho_\ell)^2, \quad (A6)$$

and

$$F_4 = 1 - 2(h_0 - h_\ell)/(\psi/\rho_\ell)^2. \quad (A7)$$

Once a value has been selected for the parameter σ , the equation may be solved for x by an appropriate numerical method. From the value of x so obtained, one can easily calculate the corresponding value for the enthalpy h using eq. (A2). Following the custom of gas dynamics where flows are plotted using Fanno lines in enthalpy-entropy coordinates, we can compute the entropy from

$$s = s_\ell + x(s_g - s_\ell), \quad (A8)$$

and plot the state point on a Mollier diagram.

Such a procedure has been implemented in our computer program "FANNOQ" developed for the Brown University TPFRR Hewlett-Packard HP 85.

Since this method of computing x incorporates the stagnation conditions, in any subsequent expression where x is used the stagnation conditions will also be accounted for even though they may not appear explicitly in the expressions.

For the laboratory conditions achievable in the Brown University TPFRR using R114, it turns out that eq. (A3) is essentially uninfluenced by the value chosen for the slip ratio σ . The last terms in eqs. (A6) and (A7) are several orders of magnitude larger than any of the other terms in the expressions for $F_1, \dots, F_4$, because the kinetic energy, $(\psi/\rho_g)^2$, is so much smaller than the latent heat, $h_g - h_g$, or enthalpy difference, $h_0 - h_g$. Furthermore, since x is bounded by 0 and 1, eq. (A3) may be taken as approximately linear by neglecting the square and cube terms.

The following example illustrates this point.

Laboratory conditions: $\dot{m} = 2 \text{ kg/s}$, $T_0 = 25^\circ\text{C}$, $P_0 = 284 \text{ kPa}$, $T = 18^\circ\text{C}$.

Calculated functions: $\gamma = 114.19$, $\psi = 986.7 \text{ kg/m}^2 \cdot \text{s}$.

$$F_1 \approx 7.7 \times 10^{-5} \sigma^4 - 1.75 \times 10^{-2} \sigma^3 + \sigma^2 + 1.75 \times 10^{-2} \sigma - 1$$

$$F_2 \approx 1.75 \times 10^{-2} \sigma^3 - 2\sigma^3 - 3.5 \times 10^{-2} \sigma + 3$$

$$F_3 \approx 559501$$

$$F_4 \approx -30730.8$$

Since the maximum value for σ is about 10, the maximum absolute values of F_1 and F_2 are about 80 and 180, respectively. Thus, to a very good approximation, $x \approx 0.055$.

To obtain the no-slip velocity, w_{ns} , we simply use eq. (18b) with the calculated value of x and $\sigma = 1$:

$$w_{ns} = (\psi/\rho_g)\{1 + x(1-\gamma)/\gamma\}. \quad (A9)$$

To obtain the superficial velocity for each phase, w_{gs} and w_{ls} , we begin by calculating the mass flow rates of each phase:

$$\dot{m}_g = x\dot{m}, \quad (A10a)$$

and

$$\dot{m}_l = \dot{m} - \dot{m}_g. \quad (A10)$$

Then eqs. (22a) and (22b) are used:

$$w_{gs} = x\dot{m}/\rho_g A, \quad (A11a)$$

and

$$w_{ls} = (1-x)\dot{m}/\rho_l A. \quad (A11b)$$

Next, to obtain the Martinelli variable, X , we use eqs. (41), (A11a) and (A11b), and the calculated quality x . The final form is:

$$X = \left(\frac{1-x}{x}\right)^{1.8} v_R^{0.2} \gamma^{-0.8}. \quad (A12)$$

Finally, to obtain the void fraction, α , for various slip ratios, we use eq. (15):

$$\alpha = \frac{1}{1 + \frac{1-x}{x} \frac{\sigma}{\gamma}}. \quad (A13)$$

For the special cases of (1) no slip:

$$\alpha_{ns} = \frac{1}{1 + \frac{1-x}{x\gamma}} ; \quad (A14)$$

(2) equal dynamic pressures:

$$\alpha_{edp} = \frac{1}{1 + \frac{1-x}{x\gamma^{0.5}}} ; \quad (A15)$$

and (3) equal wall shear stresses:

$$\alpha_{ewss} = \frac{1}{1 + \left(\frac{1-x}{x}\right)(v_R^{1/9}/\gamma^{4/9})} . \quad (A16)$$

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