NONLINEAR DYNAMICAL SYSTEMS

DoE DE-FG02-95ER25238: Final report

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Introduction

This award is approaching the end of a second and final year of no-cost extension, and the following is the final report on the work completed since the start of the second renewal period: Jan 1, 2001. It supplements the annual reports submitted in 2001 and 2002. In the renewal proposal I envisaged work in three main areas:

- Analytical and topological tools for studying flows and maps
- Low dimensional models of fluid flow
- Models of animal locomotion

Since the last full report and proposal for 2003, submitted in Fall 2002, I have requested and received two no-cost extensions, prompted by my students' receipt of fellowship support from Princeton, thereby substantially reducing their support costs. These extensions enabled me to continue the work through the calendar years 2004 and the present year 2005. I provided brief reports and statements of work in those requests, but here I give a more complete summary of achievements since Fall 2002, and list all relevant publications. I maintain the categories of the last full report.

The nominal final year 2003 and no-cost extensions of 2004 and 2005 provided partial summer salary for me, partially supported my PhD students T.R. Smith and Raffaele Ghigliazza, who graduated in Fall 2003 and Fall 2004 respectively, and provided partial support for my current graduate students J. Seipel and R. Kukillaya and postdoctoral fellow K. Rogale-Plazonic. Smith worked on fluid flow, and the others on animal locomotion and dynamical systems problems arising therefrom.

This DoE award has supported my core research in dynamical systems methods, but since the basic work and applications are inextricably linked, much of the work described below is also related to three further research groups supported by NIMH, NSF and NIH, and my contributions are partially supported by those agencies. I am a member of the NIMH Silvio M. Conte Research Center for 'Cognitive and Neural Mechanisms of Conflict and Control' at Princeton, in which I direct a mathematical and computational modelling group including two postdoctoral fellows and two graduate students. The overall project leader is J.D. Cohen (Psychology, Princeton University).

I am also co-PI on an NSF Frontiers in Biological Research award on Neuromechanical Systems Biology, collaborating with R.J. Full (Integrative Biology, University of California, Berkeley, PI), D. Koditschek (Electrical Systems Engineering, University of Pennsylvania, co-PI), J. Guckenheimer (Mathematics,

Cornell University, co-PI) and (J. Miller, Computational Biology, Montana State University, co-PI).

I am likewise a co-PI on a recent NIH award ‘An Integrated Locomotion Model for Lamprey Swimming’ (although I take no salary, and will derive only partial support for a postdoctoral fellow in 2006). The PI is Avis Cohen (Zoology, University of Maryland) and Alexander Smits (Mechanical and Aerospace Engineering, Princeton) and Thelma Williams (University College London) are co-PIs.

1. Analytical and topological tools for studying flows and maps

1.1 Nonlinear boundary value problems and discretization The paper with Gabor Domokos of the Technical University of Budapest on nonlinear boundary value problems has now appeared [1].

1.2 PDE problems The paper with Georgi Medvedev, a former Princeton University/Institute for Advanced Study Veblen Fellow has appeared [2]. We have continued research discussions since his move to Drexel University in 2002, but have not collaborated closely.

My PhD student Jaime Cisternas, who was supported on an NSF grant held jointly with Y. Kevrekidis (now expired), has graduated and accepted a faculty position at the University of the Andes, Chile. Our joint work on chemico-thermo-mechanical instabilities is complete and has appeared [3-6].

1.3 Hamiltonian dynamics My work with NSF Postdoctoral Fellow with Industry Roy Goodman and Michael Weinstein of the Fundamental Math Center at Lucent Technologies/Bell Laboratories has also concluded. Goodman has moved to a faculty position at the New Jersey Institute of Technology: we remain in close touch but are not actively collaborating. Our joint work on nonlinear optics and soliton trapping has appeared [7,8]. At the time of the last full report, I had also hoped to start a collaboration with S.A. Cohen of the Princeton Plasma Physics Laboratory, but this did not develop, although I did serve on the thesis committee of his student A. Landsman, who has since graduated.

1.4 Interacting systems and group behavior I serve on thesis committees of several of Naomi Leonard’s students, who work on control methods for arrays of autonomous underwater vehicles (flocks and flotillas). C. Woolsey and E. Fiorelli have graduated (the former has moved to a faculty position at VPI), but my projected collaboration with Fiorelli did not develop into a paper. However, my interest in collective behaviors of (semi-) autonomous agents is beginning to inform some of the neurobiological work described below.

1.5 Neurobiology and oscillator models My PhD student Eric Brown has graduated and is currently an NSF Mathematical Sciences Postdoctoral Fellow at New York University. Our joint work on oscillator networks modeling brain areas involved in neurotransmitter release, and models for decision-making, has appeared in [9-14], and two substantial further papers are in review.

sion and in press [15a,b]. Brown's thesis won a 2004 CGS/UMI Distinguished Dissertation Award and he has just won a 5 year Burroughs-Wellcome Faculty Fellowship.

This work is primarily supported by the NIMH Conte Center noted above, and also benefitted substantially from the presence of J. Moehlis, who was an NSF Mathematical Sciences Postdoctoral Fellow at Princeton from 2000-2003, and is now a faculty member at the University of California at Santa Barbara, and a current NSF Mathematical Sciences Postdoctoral Fellow, Tyler McMillen.

2. Low-dimensional models of fluid flow

My PhD student Troy Smith graduated in Fall 2003 and has held a postdoctoral fellowship in Control and Dynamical Systems at CalTech since then. He has recently accepted a staff position at United Technologies Research Center, which he will join early in 2006. Our joint work is described in [16-19]. I believe that these collectively constitute an unusually thorough study of low-dimensional models of (weakly) turbulent flows in constrained geometries. We use proper orthogonal decomposition (POD) to derive empirical subspaces, on which the Navier-Stokes equations are projected, giving (very) low dimensional ODE systems. These are then studied by dynamical systems methods, illuminating the physical mechanisms involved in sustaining turbulence. This work is the centerpiece of the DoE award, which provided its sole support, excepting the involvement of J. Moehlis (see above).

3. Models of animal locomotion

I continue working on models of insect locomotion, with the ultimate goal of developing integrated neuromechanical models, and I have recently begun a new project on swimming. We hope to both illuminate key biological questions of stability and control of locomotion, and to provide insights and analytical tools for the creation and study of 'bio-inspired' robots. The major support for this project comes from NSF and NIH, but my own involvement, and that of several students, has been partially supported by the DoE award. In insect locomotion I am building on the work of John Schmitt (PhD 2001, now a faculty member at Oregon State University). Schmitt used (relatively) simple bipedal models of horizontal plane rigid body dynamics to demonstrate passive stability, supporting a conjecture by our experimental collaborator R.J. Full of the University of California at Berkeley. The parts of this work that appeared since my last full report are in [20,21].

My PhD student Raffaele Ghigliazza graduated in Fall 2004, and is currently in a Mathematical Finance program. He performed detailed analyses of spring-loaded inverted pendulum (SLIP) models of running [22,23], proving that they are passively stable in some parameter ranges; he developed models of neural central pattern generators [24,25], and a preliminary model of an hexapedal insect with biologically realistic muscles actuating simplified legs [26]. Ghigliazza also collaborated with D. Koditshek and his postdoctoral fellow R. Altendorfer in studies of leg-placement control strategies for legged robots [27,28]. His work has laid a sound foundation for our current move into integrated neuromechan-

ical models.

My current students Justin Seipel (5th year, expected to graduate in 2006) and Raghavendra Kukillaya (2nd year) continue this work. Seipel is focussing on mechanical models with ‘simple’ actuated legs more characteristic of machines than animals, and he is considering both horizontal plane and full spatial dynamics [29-31]. He is currently collaborating with D. Koditshek’s laboratory to improve his preliminary model of the hexapedal robot RHex. Kukillaya is continuing Ghigliazza’s neuromechanical work, beginning to build a hexapedal horizontal plane model with jointed legs actuated by Hill-type muscle models, so that we may link the CPG-motoneuron models to the mechanical system.

A postdoctoral fellow, Kristina Rogale-Plazonic, also worked on this project during 2004 and 2005. We are preparing a paper on the stability of SLIP models and the parameteric dependencies of their characteristic bouncing frequencies, in response to questions and experimental data from Full’s lab. Ms Rogale-Plazonic is currently on leave, and I expect that a new postdoctoral fellow, Manoj Sreenivasan, will join the project after graduating late this year.

A substantial review article on legged locomotion models is currently in press [31], and in the new project on swimming, Tyler McMillen and I have recently completed a paper on actuated elastic rods as models of eel-like propulsion [32].

In addition to our primary method for exposition of research: publications in archival journals as referenced above and listed below, the members of my group and I make numerous contributed and invited conference presentations. Highlights include a plenary lecture at the International Symposium on Nonlinear Theory and its Applications (NOLTA2004), in Fukuoka, Japan, December 2004, and the opening plenary lecture at the Fifth EUROMECH Nonlinear Dynamics Conference (ENOC-2005), Eindhoven University of Technology, the Netherlands, August 2005. I gave a public lecture in the Math Matters series at the Institute of Mathematics and its Applications in Minnesota on December 8th, 2005 and I shall be a lecturer at the 19th International Physics Summer School: Turbulence and Coherent Structures in Fluids, Plasmas and Granular Flows, to be held January 16-20, 2006, in Canberra, Australia. See <http://www.ima.umn.edu/2005-2006/PUB12.8.05/>

<http://www.rphysse.anu.edu.au/ccs/SUMMERSCHOOLS/SS19/index.php>

Following the list of archival publications, I append a list of recent conference presentations, several of which involve full papers or extended abstracts.

Archival publications

[1] G. Domokos and P. Holmes (2003) *Proc. Roy. Soc. Lond. A* 459, 1535-1561. On nonlinear boundary value problems: Ghosts, parasites and discretizations.

[2] G. Medvedev, K. Ono and P. Holmes (2003) *European J. of Appl. Math.* 14, 343-367. Traveling wave solutions of the degenerate Kolmogorov-Petrovski-Piscunov equation.

- [3] J. Cisternas and P. Holmes (2002) *Math. and Computer Modelling* 36 (3), 233-243. Buckling of extensible thermoelastic rods.
- [4] J. Cisternas, P. Holmes and I.G. Kevrekidis (2003) *Physica D* 177 (1-4), 71-100. Buckling in response to applied heat sources.
- [5] J. Cisternas, P. Holmes, I.G. Kevrekidis and X. Li (2003) *J. Chem. Phys.* 118 (7), 3312-3328. CO oxidation on thin Pt crystals: Temperature slaving and the derivation of lumped models.
- [6] F. Cirak, J. Cisternas, A.M. Cuitiño, G. Ertl, P. Holmes, I.G. Kevrekidis, M. Ortiz, H.H. Rotermund, M. Schunack and J. Wolff (2003) *Science* 300, 1932-1936. Oscillatory thermo-mechanical instability of an ultrathin catalyst.
- [7] R. Goodman, P. Holmes and M. Weinstein (2002) *Physica D* 161, 21-44. Interaction of sine-Gordon kinks with defects: Phase space transport in a two-dimensional model.
- [8] R. Goodman, P. Holmes, and M.I. Weinstein (2004) *Physica D* 192 (3-4), 215-248. Strong NLS soliton-defect interactions.
- [9] R.Y. Cho, L.E. Nystrom, E. Brown, A.D. Jones, T.S. Braver, P. Holmes and J.D. Cohen (2002) *Cognitive, Affective and Behavioral Neuroscience* 2 (4), 283-299. Mechanisms underlying performance dependencies on stimulus histories in a two-alternative forced choice task.
- [10] E. Brown, P. Holmes and J. Moehlis (2003) Globally coupled oscillator networks. In “Perspectives and Problems in Nonlinear Science: A celebratory volume in honor of Lawrence Sirovich”, pp 183-215, Ed. E. Kaplan, J.E. Marsden, and K.R. Sreenivasan, Springer Verlag, New York.
- [11] E. Brown, J. Moehlis, P. Holmes, E. Clayton, J. Rajkowski and G. Aston-Jones (2004) *J. Comp. Neurosci.* 17 (1), 13-29. The influence of spike rate and stimulus duration on brainstem noradrenergic neurons.
- [12] E. Brown, J. Moehlis and P. Holmes (2004) *Neural Computation* 16 (4), 673-715. On the phase reduction and response dynamics of neural oscillator populations.
- [13] E. Brown, J. Gao, P. Holmes, R. Bogacz, M. Gilzenrat and J.D. Cohen (2005) *Int. J. Bifurcation and Chaos* 15 (3), 803-826. Simple neural networks that optimize decisions.
- [14] P. Holmes, E. Brown, J. Moehlis, R. Bogacz, J. Gao, G. Aston-Jones, E. Clayton, J. Rajkowski and J.D. Cohen (2005) *IEICE Transactions on Fundamentals on Electronics, Communications and Computer Science* 88 (10), 2496-2503. Optimal decisions: From neural spikes, through stochastic differential equations, to behavior.
- [15a] R. Bogacz, E. Brown, J. Moehlis, P. Holmes and J.D. Cohen (2005) *Psych. Rev. (in revision)*. The physics of optimal decision making: A formal analysis of performance in two-alternative forced choice tasks.

- [15b] T. McMillen and P. Holmes (2005) *J. Math. Psych.* (*in press*). The dynamics of choice among multiple alternatives.
- [16] J. Moehlis, T. Smith, P. Holmes and H. Faisst (2002) *Physics of Fluids* 14 (7), 2493-2507. Models for turbulent plane Couette flow using the proper orthogonal decomposition.
- [17] T.R. Smith, J. Moehlis and P. Holmes (2005) *J. Fluid Mech.* 538, 71-110. Low-dimensional models for turbulent plane Couette flow in a minimal flow unit.
- [18] T.R. Smith, J. Moehlis and P. Holmes (2005) *Nonlinear Dynamics* 41, 275-307. Low-dimensional modelling of turbulence using the proper orthogonal decomposition: A tutorial.
- [19] T.R. Smith, J. Moehlis and P. Holmes (2005) *Physica D* 211, 347-376. Dynamics of an 0:1:2 $O(2)$ -equivariant system: Heteroclinic cycles and periodic orbits.
- [20] R.J. Full, T. Kubow, J. Schmitt, P. Holmes and D. Koditschek (2002) *Integrative and Comparative Biology* 42, 149-157. Quantifying dynamic stability and maneuverability in legged locomotion.
- [21] J. Schmitt and P. Holmes (2003) *Biological Cybernetics* 89 (1), 43-55. Mechanical models for insect locomotion: Active muscles and energy losses.
- [22] R.M. Ghigliazza, R. Altendorfer, P. Holmes and D. Koditschek (2003) *SIAM J. on Applied Dynamical Systems* 2 (2), 187-218. A simply stabilized running model.
- [23] R.M. Ghigliazza, R. Altendorfer, P. Holmes and D. Koditschek (2005) *SIAM Review* 47 (3) 519-549. A simply stabilized running model. (Updated version of [22] invited for republication in SIGEST.)
- [24] R.M. Ghigliazza and P. Holmes (2004) *SIAM J. on Applied Dynamical Systems* 3 (4), 636-670. Minimal models of bursting neurons: The effects of multiple currents and timescales.
- [25] R.M. Ghigliazza and P. Holmes (2004) *SIAM J. on Applied Dynamical Systems* 3 (4), 671-700. A minimal model of a central pattern generator and motoneurons for insect locomotion.
- [26] R.M. Ghigliazza and P. Holmes (2005) *Regular and Chaotic Dynamics* 10 (2), 193-225. Towards a neuromechanical model for insect locomotion: Hybrid dynamical systems.
- [27] R. Altendorfer, D. Koditschek and P. Holmes (2004) *Int. J. Robotics Research* 23 (10-11), 979-999. Stability analysis of legged locomotion models by symmetry-factored return maps.
- [28] R. Altendorfer, D. Koditschek and P. Holmes (2004) *Int. J. Robotics Research* 23 (10-11), 1001-1012. Stability analysis of a clock-driven rigid-body

SLIP model of RHex.

[29] J.E. Seipel, P. Holmes and R.J. Full (2004) *Biological Cybernetics* 91, 76-90. Dynamics and stability of insect locomotion: A hexapedal model for horizontal plane motions.

[30] J.E. Seipel and P. Holmes (2005) *Int. J. Robotics Research* 24 (8) 657-674. Running in three dimensions: Analysis of a point-mass sprung-leg model.

[31] J.E. Seipel and P. Holmes (2005) *Int. J. Robotics Research* (in review). Three-dimensional dynamics and stability of multi-legged runners.

[32] P. Holmes, R.J. Full, D. Koditschek and J. Guckenheimer (2006) *SIAM Review* (in press). Dynamics of legged locomotion: Models, analyses, and challenges.

[33] T. McMillen and P. Holmes (2005) *J. Math. Biology* (in review). An elastic rod model for anguilliform swimming.

Conference papers (*) and abstracts (2003-5)

[c1]* R. Altendorfer, R. Ghigliazza, P. Holmes and D. Koditschek (2002) Exploiting passive stability for heirarchical control. Clawar, Paris, France, Sept 25-27, 2002.

[c2] E. Brown, J. Moehlis, P. Holmes, E. Clayton, J. Rajkowski and G. Aston-Jones (2002) The influence of spike rate and stimulus duration on response in *locus coeruleus*. Systems Level Neuroscience Workshop, Mathematical Biosciences Institute, Ohio State University, Columbus. OH, Nov 18-22, 2002.

[c3]* R. Altendorfer, D. Koditschek and P. Holmes (2003) Towards a factored analysis of legged locomotion models. IEEE International Conference on Robotics and Automation (ICRA'03), Grand Hotel in Taipei, Taiwan, May 12-17, 2003.

[c4] P. Holmes (2003) Optimal decisions: From neural oscillations to stochastic differential equations. IAS Program for Women in Mathematics, 10th anniversary reunion, May 16-18, 2003, Princeton, NJ.

[c5]* T.R. Smith, J. Moehlis and P. Holmes (2003) Minimal plane Couette flow turbulence: a low-dimensional, uncoupled model. 13th International Couette Taylor Workshop, July 3-5, 2003, Universitat Politècnica de Catalunya, Barcelona, Spain.

[c6] P. Holmes (2003) Piecewise-holonomic mechanics, hybrid dynamical systems and escaping cockroaches, and Optimal decisions: from neural spikes, through stochastic differential equations, to behavior. London Mathematical Society regional meeting on Nonlinear Dynamics, Southampton University, UK, Oct 24-26, 2003.

[c7] R. Bogacz, J.M. Moehlis, E.T. Brown, P. Holmes, J.D. Cohen (2003)

Neural mechanisms for decision optimization, 2003 Abstract Viewer/Itinerary Planner. Society for Neuroscience, Washington DC, CD-ROM, Program No. 197.6.

[c8]* T.R. Smith, J. Moehlis and P. Holmes (2003) Modeling and control of minimal flow unit turbulence in plane Couette flow. 42nd IEEE CDC, Hyatt Regency Maui, Hawaii, Dec 9-12, 2003.

[c9] P. Holmes, T.R. Smith and J. Moehlis (2003) Dynamics of an 0-1-2 $O(2)$ -equivariant system: Heteroclinic cycles and periodic orbits. Workshop on Bifurcation Theory and Spatio-Temporal Pattern Formation in PDE, Fields Institute for Research in Mathematical Sciences, Toronto, Dec 11-13, 2003.

[c10] R.J. Full, J. Seipel and P. Holmes (2004). Dynamic stability model predicts constraints in sprawled posture running. Final Program and Abstracts, p. 286, Society of Integrative and Comparative Biology, Annual Meeting and Exhibition, New Orleans, Jan 4-9, 2004.

[c11] R. Bogacz, E. Brown, J. Moehlis, P. Holmes and J.D. Cohen (2004). Reward optimization in decision making. Workshop on The Neurobiology of Decision-making: Theory and Experiment, Cold Spring Harbor Laboratory, NY, Mar 24, 2004.

[c12] J. Gao and P. Holmes (2004) Model reduction and optimization in a decision-making task (poster). Computational and Systems Neuroscience 2004, Cold Spring Harbor Laboratory, NY, Mar 24-28, 2004.

[c13] E. Brown, J. Moehlis, P. Holmes, E. Clayton, J. Rajkowski and G. Aston-Jones (2004). From spikes to speed-accuracy via the locus coeruleus (poster). Computational and Systems Neuroscience 2004, Cold Spring Harbor Laboratory, NY, Mar 24-28, 2004.

[c14] P. Holmes (2004). Low-dimensional models of plane Couette flow. Invited lecture for International Workshop on Bifurcation Theory and Applications, Shanghai Jiaotong University, PRC, May 23-26, 2004.

[c15] P. Holmes (2004). Applied dynamical systems: From low to high dimensions and back again. International Workshop on Nonlinear Dynamics and Stochastic Partial Differential Equations, Morningside Center of Mathematics, Chinese Academy of Sciences, Beijing, May 27-31, 2004.

[c16] R. Bogacz, E. Brown, P. Hu, P. Holmes and J.D. Cohen (2004). Optimization of decision making: speed-accuracy trade-off maximizing reward rate. 4th Forum of European Neuroscience, Lisbon, Spain, July 10-14, 2004.

[c17]* J. Seipel and P. Holmes (2004) Three-dimensional running is unstable but easily stabilized. 7th International Conference on Climbing and Walking Robots, CLAWAR 2004, Madrid, Spain, Sept 22-24, 2004.

[c18]* P. Holmes, E. Brown, J. Moehlis, R. Bogacz, J. Gao, P. Hu, G. Aston-Jones, E. Clayton, J. Rajkowski and J.D. Cohen (2004) Optimal decisions: From

neural spikes, through stochastic differential equations, to behavior. Invited plenary lecture, International Symposium on Nonlinear Theory and its Applications. NOLTA2004, Fukuoka, Japan, Nov 29 - Dec 3, 2004.

[c19] P. Holmes (2004). Low-dimensional models of plane Couette flow. Invited lecture at International Symposium on Complexity Modelling and its Applications, Faculty of Engineering, University of Tokyo, Japan, Dec 5-8, 2004.

[c20] P. Holmes and R. Ghigliazza (2005). A central pattern generator for insect locomotion. Invited lecture at Coupled 60: A Focussed Research Group Workshop on the Dynamics, Classification and Applications of Coupled Systems, University of Houston, TX, Feb 3-6, 2005.

[c21] P. Holmes (2005). Poincaré's mistake and the origins of 'Chaos Theory.' Invited talk at the Third Annual Workshop on Applied Mathematics and Computational Physics, Budapest University of Technology and Economics, March 18th, 2005.

[c22] P. Holmes (2005). Piecewise-holonomic mechanics, hybrid dynamical systems, and escaping cockroaches. Invited lecture at Mathematical Association of America New Jersey Section Meeting, Middlesex Community College, Edison, NJ, April 2, 2005.

[c23] P. Holmes (2005). A central pattern generator for insect locomotion. Minisymposium presentation at the Conference on Frontiers in Applied and Computational Mathematics, New Jersey Institute of Technology, May 13-15, 2005.

[c24] P. Simen, P. Holmes and J.D. Cohen (2005). Performance adaptation by a drift-diffusion-based decision making circuit. Lecture at the 9th International Conference on Cognitive and Neural Systems, Center for Adaptive Systems and Department of Cognitive and Neural Systems, Boston University, May 18-21, 2005.

[c25] P. Holmes and T. McMillen (2005). What's optimal about decision-making for two and more choices? Invited presentation at Neurobiology of Decision-Making, Banbury Center, Cold Spring Harbor Laboratory, Long Island, NY, May 22-25, 2005.

[c26]* R. Bogacz, E. Brown, J. Moehlis, P. Holmes and J.D. Cohen (2005) How a biological decision network can implement a statistical test. Presentation at the workshop on Modelling Natural Action Selection, Edinburgh University, UK, July 30-31, 2005.

[c27]* P. Holmes (2005). Ninety plus thirty years of nonlinear dynamics: More is different and less is more. Opening plenary lecture at the Fifth EU-ROMECH Nonlinear Dynamics Conference, Eindhoven University of Technology, The Netherlands, Aug 7-12, 2005. Invited paper in special ENOC issue of *Int. J. Bifurcation and Chaos* 15(9), 2703-2716.

[c28] P. Holmes (2005). A swimming rod with a mind of its own (or at least a muscle). Plenary lecture at IMA Conference on Recent Advances in Nonlinear Mechanics (RANM-05), University of Aberdeen, Scotland, Aug 29-Sept 1, 2005.