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THE SNAP 2
RADIATIVE CONDENSER ANALYSIS

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THE SNAP 2
RADIATIVE CONDENSER ANALYSIS

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CONTENTS

	Page
I. Introduction	1
II. Two Phase Flow.	2
A. Pressure Drop	2
B. Mercury Hold-up.	7
III. Heat Transfer	11
A. Condensing Heat Transfer.	11
B. Tube to Fin Heat Transfer.	11
C. Radiant Heat Transfer	12
IV. Meteoroid Protection	25
A. Meteoroid Flux	25
B. Meteoroid Penetration	25
C. Armor Requirements	28
D. Erosion	33
V. Reactor-Radiator Configuration	36
A. Scattered Radiation	36
B. Activation	36
VI. Minimum Weight Radiator-Condenser Design	39
References	47
Appendices	
I. Fluid Flow Equations	49
II. Radiator Equations	55
Nomenclature	59

TABLES

I. Mercury Condenser Fluid Hold-up.	9
II. Meteoroid Penetration.	29

FIGURES

	Page
1. Correlation of Lockhart and Martinelli for Two-Phase Flow . . .	4
2. Pressure Drop Parameter (ϕ_1) <u>vs</u> Temperature	6
3. Pressure Drop Parameter (ϕ_2) <u>vs</u> Temperature	8
4. Martinelli Correlation of Liquid Volume Fraction	10
5. Error Introduced in Equation (10) by Neglecting Sink Temperature .	13
6. Radiative Heat Flux <u>vs</u> Temperature	14
7. Radiator Geometry	16
8. Fin Effectiveness - η <u>vs</u> Radiation Modulus - M_r	17
9. Radiator Area Requirements for Subcooler Section	19
10. Emissivity <u>vs</u> Temperature for Various Surfaces.	21
11. A Comparison of the Emissive Characteristics of Anodized Aluminum with the Distribution for Thermal Radiation at 1000°R .	22
12. Emissivity <u>vs</u> Temperature (Data from NACA-TN-4206)	24
13. Meteoroid Flux <u>vs</u> Mass Near Earth (Data from Reference 10) . .	26
14. Material Thickness <u>vs</u> Radiator Area (Aluminum)	31
15. Material Thickness <u>vs</u> Radiator Area (Steel)	32
16. Time for 10% Loss of Area <u>vs</u> Material Thickness	35
17. Several Radiator-Shield Configurations	37
18. Radiator Geometry	40
19. Detail of Tube and Fin Arrangement	41
20. Radiator Weight <u>vs</u> Number of Tubes	43
21. Radiator Weight <u>vs</u> Fin Effectiveness.	44
22. Radiator Weight <u>vs</u> Radiator Area	46

I. INTRODUCTION

The heat rejection system for space power plants is a major weight item, and for high power systems may be the predominant one. For this reason considerable effort can profitably be expended to insure the design of minimum weight systems.

The 3 kw SNAP 2 power system, currently under development by Atomic International for the Missile Projects Branch of the Atomic Energy Commission Aircraft Reactors Office, uses a mercury Rankine cycle to convert heat, generated in a nuclear reactor, to electrical energy. The Rankine cycle requires the rejection of heat by condensation of the working fluid, and because of the space environment, the heat rejection must ultimately be accomplished by thermal radiation to space.

The SNAP 2 design employs a direct-condensing radiator. In this design, the heat released in the condensation process is radiated to space from a large extended surface, consisting of fins which are attached to tubes containing the condensing fluid. This is the type of system considered in this report.

The large number of variables involved in the design of a radiator condenser makes it impossible to prepare a single design curve. The different aspects of the radiator-condenser design problem are considered in Sections II, III, IV, and V. The data developed in these sections are then combined in Section VI where a procedure leading to the determination of a minimum weight heat rejection system is developed. An example of the procedure is presented for the SNAP 2 radiator design point which requires the mercury working fluid to be condensed at 600°F. The example illustrates the major point that minimum weight heat rejection systems can only be determined when all the design parameters are simultaneously considered.

II. TWO PHASE FLOW

A. PRESSURE DROP

Viscous drag is the mechanism by which condensed mercury is removed from the radiative condenser. Mercury enters the radiator-condenser as saturated vapor, and is condensed at substantially constant temperature by being subjected to a constant heat flux throughout the length of the tube. After the vapor has been completely condensed the liquid is subcooled by about 200°F before leaving the radiator. Subcooling of the condensate is required to reduce the possibility of pump cavitation, and to provide a low-temperature bearing and alternator coolant.

The design of a minimum weight radiator requires an accurate prediction of the pressure drop associated with the condensing mercury as it flows through the radiator tubes. This must be done with some precision since each 1 psi of static pressure loss in the condenser tubes lowers the saturation temperature of the condensing mercury by about 14°F, which results in a 5% drop in radiating power at this temperature.

Many investigators have correlated pressure drop data for two-phase flow in tubes. Of these, the correlation of Lockhart and Martinelli¹ is probably the best and most widely used. They were able to correlate pressure drop for two-phase, two-component flow in the nondimensional form,

$$\left(\frac{dP}{dl}\right)_{\text{TPF}} = \phi_g^2 \left(\frac{dP}{dl}\right)_g, \quad \dots (1)$$

where

$$\left(\frac{dP}{dl}\right)_{\text{TPF}} = \text{two phase frictional pressure gradient,}$$

$$\left(\frac{dP}{dl}\right)_g = \text{pressure gradient per unit tube length of the gas (vapor) phase alone,}$$

$$\phi_g = \text{dimensionless parameter which is a function only of } \frac{\left(\frac{dP}{dl}\right)_l}{\left(\frac{dP}{dl}\right)_g};$$

where $\left(\frac{dP}{dl}\right)_l$ = pressure gradient per unit tube length of the liquid phase alone.

The experimental data of Lockhart and Martinelli were plotted in terms of ϕ_g and $\sqrt{\frac{\left(\frac{dP}{dl}\right)_l}{\left(\frac{dP}{dl}\right)_g}}$ for different combinations of flow regimes, e.g., turbulent gas-laminar liquid, turbulent gas-turbulent liquid. While these correlations were for two-component flow, extension of these correlations to boiling or condensing (i.e., two-phase one-component flow) has been suggested by Martinelli and Nelson² and has been tested with some success. In support of this extrapolation, McAdams³ has found that friction arising from transfer of momentum between phases of a one-component system was of little importance under his experimental conditions. Furthermore, Lockhart and Martinelli stated that their correlation was independent of flow mechanism, whether mist, annular or stratified flow existed. Unpublished experimental work being conducted by AI and its subcontractors indicates that these correlations predict reasonable values for condensing pressure drop in tubes.

The Lockhart and Martinelli correlation corresponding to the SNAP 2 condenser conditions is reproduced in Figure 1. In the range of interest this can be represented by

$$\phi_g = 1.76 X^{0.0825}, \quad \dots (2)$$

where

$$X = \sqrt{\frac{\left(\frac{dP}{dl}\right)_l}{\left(\frac{dP}{dl}\right)_g}}. \quad \dots (3)$$

By combining Equations (1), (2), and (3) with the appropriate values of $(dP/dl)_g$ and integrating the resulting expression to account for the changing flow conditions throughout the length of the tube, an expression for the frictional pressure drop can be derived (see Appendix I). The result is given by:

Figure 1. Correlation of Lockhart and Martinelli for Two-Phase Flow

$$\left(\frac{\Delta P}{L}\right)_{\text{TPF}} = \frac{0.402 W_T^{1.684} \mu_g^{0.316}}{N^{1.684} D^{4.684} \rho_g g_c} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.0825} \quad \dots (4)$$

where

- W_T = weight flow
- μ_g = viscosity of vapor
- μ_l = viscosity of liquid
- N = number of tubes
- D = inside tube diameter
- ρ_g = density of vapor
- ρ_l = density of liquid
- g_c = gravitational constant.

Any consistent system of units may be used.

Combining constants and fluid properties, Equation (4) can be reduced to:

$$(\Delta P)_{\text{TPF}} = \left(\frac{L W_T^{1.684}}{N^{1.684} D^{4.684}}\right) \phi_1 \quad \dots (5)$$

The parameter, ϕ_1 , is plotted in Figure 2 for mercury for the range of temperatures of interest where:

$$\phi_1 = \frac{0.402 \mu_g^{0.316}}{\rho_g g_c} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.0825} \quad \dots (6)$$

In addition to the frictional pressure drop given by Equation (5) there is a pressure rise due to the momentum loss of the high velocity mercury vapor as it traverses the condenser tube. Since the fluid velocity is essentially zero at the exit of the condenser this is given by:

Figure 2. Pressure Drop Parameter (ϕ_1) vs Temperature

(A)

$$(\Delta P)_M = - \left(\frac{1.62 W_T^2}{\rho_g g_c D^4 N^2} \right), \quad \dots (7)$$

where the negative sign indicates a pressure rise. Combining constants and fluid properties, Equation (7) is reduced to

$$(\Delta P)_M = - \phi_2 \left(\frac{W_T^2}{D^4 N^2} \right) \quad \dots (8)$$

where

$$\phi_2 = \frac{1.62}{\rho_g g_c} \quad \dots (9)$$

The parameter, ϕ_2 , is plotted in Figure 3 for mercury for a range of temperatures. Equations (5) and (8) were used to calculate the pressure drop for the condensers which are considered in this report. It must be emphasized that this analysis is valid only if the static pressure drop is a small fraction of the inlet pressure. It may be safely used here if the static pressure drop is 2.5 psi or less.

B. MERCURY HOLD-UP

Hold-up, or mercury inventory, in the condensing section of the radiator is difficult to predict with any degree of confidence. It is of interest, however, to investigate the upper and lower limits of inventory to determine the importance it has on system weight.

The lower limit is predicted by assuming all vapor in the condenser. The inventory is then equal to the vapor density times the condenser volume, or about 0.05 lb for a typical SNAP 2 condenser.

The upper limit on mercury inventory is given by the Lockhart and Martinelli correlation. The reason for taking this as the upper limit is as follows: Lockhart and Martinelli correlated volume fraction for two-phase two-component flow, in which the slip velocity is constant. In a condenser, the slip near the inlet can

Figure 3. Pressure Drop Parameter (ϕ_2) vs Temperature

be predicted from the Lockhart and Martinelli correlation; however for the rest of the condenser, the slip will be less than the predicted value since the vapor is slowed down by reduction of its mass flow rate, while the liquid continues to move rapidly. The SNAP 0-gravity mercury condenser test data taken in parabolic-flight aircraft at WADD show that while the vapor velocity at the interface is zero, the small liquid droplets strike the interface at velocities about 50 ft/sec for some cases. Thus for a significant portion of the condenser the liquid moves faster than the vapor (negative slip velocity). Since higher velocity of liquid relative to vapor means lower liquid residence time, and thus lower liquid inventory, we may safely state the Lockhart and Martinelli correlation predicts too high a value of mercury hold-up. (By entirely analogous reasoning, one would predict that Lockhart and Martinelli predict too high a value of hold-up in boiling. Data of Dengler⁴ confirm this.)

The Martinelli parameter, X , is given in Equation A-11 of Appendix I as a function of l/L , and Figure 4 plots the Martinelli correlation of R_l vs X . Then $\bar{R}_L$ is obtained graphically. At 600°F, $\bar{R}_l = 0.04$, and the mercury inventory is about 10 lb, where R_l is the liquid phase volume fraction.

Table I shows the comparison between the lower limit (all-gas model) and the upper limit (Martinelli model) of mercury hold-up in the SNAP 2 condenser. An intermediate value of inventory, derived by assuming that the average liquid velocity is equal to the average vapor velocity (no-slip model), is also shown.

TABLE I
MERCURY CONDENSER FLUID HOLD-UP

	R_l	Mercury Inventory for Typical SNAP 2 Condenser
All-gas model	0.0	0.05 lb
No-slip model	0.0012	0.5 lb
Martinelli model	0.04	10 lb

Only experimental data can supply the answer to the problem of predicting hold-up. It will be what it will be, and the designer can do little to affect it except in the subcooler beyond the condenser section. Since the condenser holdup is less than 10 lb in any case, it does not affect system weight to any appreciable degree.

Figure 4. Martinelli Correlation of Liquid Volume Fraction

III. HEAT TRANSFER

The heat transfer problem associated with the design of a radiative condenser is complicated in that it includes: condensing heat transfer under a complex and changing flow regime, heat conduction through the tube walls to the fins, and thermal radiation from the surface of the fins to the space environment. In addition, for the design under consideration in this report, there is a requirement that subcooling of the condensate be achieved. The need to design the lightest-weight system to do the job requires that all the above phenomena be thoroughly analyzed and experimentally verified.

A. CONDENSING HEAT TRANSFER

Values recorded in the literature^{5,6} for heat fluxes obtained with condensing mercury vapor are of the order of 500,000 Btu/hr-ft². The radiant heat flux from the surface of the fins (at 600°F) is something less than 2000 Btu/hr-ft² and the ratio of tube area to radiating surface area is on the order of ten to one. From the above criteria it is seen that the temperature drop from the condensate to the tube walls will be less than 1 degree. The effect of the "0-gravity" environment on the heat transfer coefficients and/or the heat transfer mechanism still remains to be evaluated. An experimental program has been initiated, as part of the SNAP 2 development program, to investigate this problem. This program, called OFF-BACE (Orbital Force Field Boiling and Condensing Experiments), is directed toward determining the effects of "0-gravity" conditions on the boiling and condensing of mercury. For the purposes of this study, the limiting thermal resistance was assumed to be that of thermal radiation from the surface of the radiator. Preliminary data from the "OFF-BACE" aircraft experiments indicate that this is the case.

B. TUBE TO FIN HEAT TRANSFER

The temperature drop between the tube and the fin will depend on the nature of the bond between them. Work has been undertaken at Atomics International to determine suitable ways of bonding steel tubes to aluminum. Preliminary analysis has indicated that with any reasonable bond, the thermal resistance is quite small, and hence, was not included in this analysis.

C. RADIANT HEAT TRANSFER

1. Temperature

Radiant heat transfer is described by the following equation:

$$Q = \epsilon \sigma A (T_o^4 - T_{\text{sink}}^4), \quad \dots (10)$$

where

Q = radiative power (Btu/hr)

ϵ = surface emissivity

A = surface area (ft^2)

σ = Stephen Boltzman constant = 0.171×10^{-8} Btu/hr- ft^2 - $^{\circ}\text{R}^4$

T_o = radiating surface temperature ($^{\circ}\text{R}$)

T_{sink} = radiative sink temperature ($^{\circ}\text{R}$).

The exact determination of T_{sink} is a complicated calculation requiring a detailed knowledge of the specific mission and conditions involved. A detailed study⁷ for a satellite orbiting the earth at a 300 mile altitude, reveals that the effective radiative sink temperature is about 0°F . This value is sufficiently low in comparison with the range of T_o under consideration in this report (600°F) as to permit the neglect of this term. The error introduced by neglecting T_{sink} in Equation (10) is shown in Figure 5 for various values of T_{sink} and T_o . With this assumption, Equation (10) is greatly simplified and reduces to:

$$Q = \epsilon \sigma A T_o^4. \quad \dots (11)$$

Equation (11) is plotted in Figure 6 for values of ϵ from 0.2 to 1.0.

2. Fin Effectiveness

A quantity termed the fin effectiveness is introduced to assist in the evaluation of the performance of the tube-and-fin radiator. It is defined as the ratio of the heat rejected by the fin to that which would be rejected if the entire fin were maintained at the base temperature. The model used to calculate the

Figure 5. Error Introduced in Equation (10) by Neglecting Sink Temperature

Figure 6. Radiative Heat Flux vs Temperature

fin effectiveness is shown in Figure 7A. Expressed mathematically:

$$\eta \equiv \frac{\int_0^{B/2} T_x^4 dx}{B/2 T_o^4} \quad \dots (12)$$

where

η = fin effectiveness

B = tube spacing (ft)

T_x = temperature at a point on the fin ($^{\circ}\text{R}$)

x = distance along fin (ft).

This equation is derived in Appendix II and was solved numerically on an IBM-704. The results are given in Figure 8 as a function of the dimensionless parameter M_r , defined as:

$$M_r = \frac{B^2 \epsilon \sigma T_o^3}{kt} \quad \dots (13)$$

where

k = conductivity of fin material (Btu/hr-ft- $^{\circ}\text{R}$)

t = fin thickness (ft).

By using the curve in Figure 8, the fin effectiveness may be evaluated for a given material, geometry, and base temperature (T_o).

3. Radiator Area Requirements

a. Condensing

For the condensing portion of the radiator, the tube temperature remains constant until the fluid is completely condensed. This will be true only if the static pressure drop is kept small since the condensate and condensing vapor are always in thermal equilibrium. If this condition is met, the area requirements for the condensing portion of the radiator can be obtained by combining

Figure 7. Radiator Geometry

Figure 8. Fin Effectiveness - η vs Radiation Modulus - M_r

Equation (11) with the definition of fin effectiveness.

$$Q = \epsilon \sigma \eta A T_o^4 \quad \dots (14)$$

Figure 6 can be used directly to obtain the required area if the ϵ as shown is considered to be equal to the product, $\epsilon \times \eta$.

b. Subcooler

In the subcooler section the radiant heat rejection is accompanied by a sensible heat loss of the fluid. The temperature decrease of the fluid results in temperature gradients both perpendicular and parallel to the direction of fluid flow. This complicates the analysis, but by combining the model of the condensing (constant temperature) fin with that of a radiator which experiences a coolant temperature drop, an expression can be derived to give the area requirements for the tube-fin subcooler configuration (see Appendix II). The result is given by:

$$Q = 3 \epsilon \sigma \eta A \Delta T \left\{ \frac{1}{(T_{in} - \Delta T)^3} - \frac{1}{T_{in}^3} \right\}^{-1} \quad \dots (15)$$

where

Q = radiative power (Btu/hr)

A = area required (ft²)

T_{in} = fluid temperature into subcooler (°R)

T_{out} = fluid temperature out of subcooler (°R)

ΔT = temperature drop of fluid passing through the
subcooler (°R) = $(T_{in} - T_{out})$

η = fin effectiveness (evaluated at some intermediate
temperature).

Equation (15) is plotted in Figure 9 for various values of T_{in} and ΔT . This figure can be used to determine the area requirements for the subcooler section.

Figure 9. Radiator Area Requirements for Subcooler Section

4. Emissivity

Since metallic surfaces in general have low emissivities, they are not good radiators. To reduce radiator weight, it is necessary to apply a surface coating of a high-emissivity material. A survey of the literature indicates that certain ceramic coatings, such as Al_2O_3 , ZrO_2 , and ZrSiO_4 , are attractive from the standpoint of high-emissivity. The emissivity for these materials is shown in Figure 10.

Absorptivity may be defined as the ratio of the energy absorbed by a surface to the total energy incident upon the surface. This ratio is strongly dependent upon the wave length of the incident radiation, since incident photons of different energies are absorbed in different processes. Incident radiation in the visible spectrum raises the energy levels of the orbital electrons. Infrared radiation raises the vibrational and kinetic energy levels of the molecules. In addition, the quantization of these energy levels gives a plot of the absorptivity (or emissivity, since $\epsilon_\lambda = a_\lambda$) vs wave length which has a series of strong absorption (or emission) peaks.

In Figure 11, the spectral emissivity of anodized aluminum is plotted vs radiation wave length. Superimposed on this is the power emission spectrum of a black body at 1000°R . The total power emitted (or absorbed) by the anodized aluminum is the integral of the product of the emissivity at each wave length times the power at that wave length.

In the case of anodized aluminum the theoretical emissivity at 1000°R calculated from the ratio of the areas in Figure 11, is 0.697 (Reference 7). In particular, anodized aluminum is a poor radiator at this temperature since it has low emissivity at wave lengths corresponding to the maximum power emission at 1000°R . Yet at certain higher and lower temperatures, coinciding with strong absorption wave lengths, the emissivity is much higher. Thus, it is important to realize that the smooth curves shown in Figure 10, based on only a few points, do not demonstrate the true character of the emissivity. Most important, the sharp variation of emissivity with wave length make it impossible to extrapolate, and the values of the emissivity at the design point temperature must be measured at that temperature.

Figure 10. Emissivity vs Temperature for Various Surfaces

Figure 11. A Comparison of the Emissive Characteristics of Anodized Aluminum with the Distribution for Thermal Radiation at 1000°R

*DATA ON ANODIZED ALUMINUM FROM TRANS. ASME, 76, MAY 1954, p. 551-2

In addition to the foregoing considerations, a ceramic coating applied to metal will have some insulating effects, with the result that the surface of the coating may be as much as 15°F cooler than the metal to which it is attached. The thickness of the coating seems to make a minor contribution to this effect and most of the temperature drop may be attributed to the ceramic-metal interface. This effect will reduce the radiating temperature and, hence, the effective emissivity, since the emissivity for design calculations is based on the metal temperature. However, it is this effective emissivity that should be used as a criteria for the selection of a coating, since it includes both of the variables to be considered, and it should also be used in design calculations, rather than the strict physical emissivity.

Figure 12 presents additional emissivity data for various oxidized metals as a function of temperature.⁸ The curve for mild steel shows this material to be attractive but the high weight associated with steel fins precludes them from consideration in this temperature range.

Figure 12. Emissivity vs Temperature
(Data from NACA-TN-4206)

IV. METEOROID PROTECTION

At the present time there is considerable doubt associated with the methods used to calculate the amount of armor required to provide adequate meteoroid protection. The best effort to date to correlate the small amount of experimental data with a reasonable analytical model has been by Bjork.^{9,10} While admittedly his results are probably conservative, his approach appears justifiable in view of the meager experimental data available. It is essentially his procedure which has been followed to determine the armor requirements for meteoroid protection of the radiator.

There are two methods by which meteoroids may damage the radiator. The first is puncture of fluid lines, causing loss of working fluid. The second is erosion of the radiator surface, or surface coatings, causing a change in surface emissivity. Both of these effects must be considered in the design of a space radiator, and both are quite sensitive to the assumed flux density of meteoroids.

A. METEOROID FLUX

The flux of meteoroids given in Reference 10 is:

$$\phi = 10^{-12} M^{-10/9}, \quad \dots (16)$$

where

ϕ = number of particles of mass greater than M per square meter per second

M = mass of particle in grams.

This expression is plotted in Figure 13, along with experimental points derived from satellite data. This curve is in agreement with that predicted by Whipple from visual observations if one accepts a mass of 30 grams as the mass of a zero magnitude meteor.¹⁰

B. METEOROID PENETRATION

The theoretical model postulated by Bjork⁹ predicts the depth of penetration of projectiles impinging upon thick targets at hypersonic velocities. The

Figure 13. Meteoroid Flux vs Mass Near Earth (Data from Reference 10)

calculations based on this model agree in both size and shape (hemispherical) with experimental craters produced in aluminum at 6.3 km/sec and in iron at 6.8 km/sec. The results of the theoretical analysis are summarized by the equation:

$$p = K'(Mv)^{1/3} \quad \dots (17)$$

where

p = penetration depth (cm)

M = mass of projectile (grams)

v = projectile velocity in (km/sec)

K' = constant depending upon materials;

for aluminum projectiles impinging on aluminum targets -

$$K' = 1.09$$

for iron projectiles impinging on iron targets - $K' = 0.606$.

The modification of Equation (17) to account for projectiles which have densities different than the target material is uncertain,¹¹ as is the value of the meteoroid density itself. Some evidence exists which indicate that the meteoroid density may be that of stone, ~ 2.3 g/cc. Hence, Equation (17) will be a good approximation for stone meteoroids impinging upon aluminum, but will predict too high a value for the depth of penetration in materials which have a density significantly greater than that of aluminum.

While Equation (17) was derived for thick targets, enough information was obtained from Bjork's¹² analysis to deduce that if a projectile penetrates a depth, p , in a thick target it will just penetrate a sheet of the same target material which is $1.5 p$ thick, thus:

$$t_1 = 1.5 p \quad \dots (18)$$

where

t_1 = required thickness of material (cm).

Equations (16) and (17) may be combined to obtain a penetrating flux which is given as:

$$\psi = 10^{-12} t_i^{-10/3} K^{10/3} v^{10/9} \quad \dots (19)$$

where

ψ = the number of penetrations of target thickness
 t_i per square meter per second

$$K = 1.5K'.$$

This relationship is shown in Table II for the velocity distribution stated by Whipple.¹⁰

C. ARMOR REQUIREMENTS

The meteoroid punctures may be expected to follow a Poisson distribution which is given as:

$$P(n) = \frac{(\psi A' \tau')^n \exp^{-(\psi A' \tau')}}{n!} \quad \dots (20)$$

where

$P(n)$ = is the probability that n punctures will occur in time
 τ' of a sensitive area A' .

For the case of the fluid type radiator under consideration, the allowable number of punctures of the tube walls is zero, hence Equation (20) reduces to:

$$P(0) = \exp^{-\psi A' \tau'} \quad \dots (21)$$

where

$P(0)$ = the probability that no punctures will occur in time τ'
of a sensitive area A' .

Combining Equations (19) and (21) yields:

TABLE II
METEOROID PENETRATION

Visual Magnitude	Mass (gm)	Velocity (km/sec)	Aluminum Thickness (cm)	Steel Thickness (cm)	Penetrating Flux (Penetrations/m ² -sec)
0	25.0	28.0	14.5	8.06	2.80×10^{-14}
1	9.95	28.0	10.7	5.93	7.79×10^{-14}
2	3.96	28.0	7.85	4.36	2.17×10^{-13}
3	1.58	28.0	5.78	3.21	6.02×10^{-13}
4	0.628	28.0	4.25	2.36	1.68×10^{-12}
5	0.250	28.0	3.13	1.74	4.67×10^{-12}
6	9.95×10^{-2}	28.0	2.30	1.28	1.30×10^{-11}
7	3.96×10^{-2}	28.0	1.69	0.940	3.62×10^{-11}
8	1.58×10^{-2}	27.0	1.23	0.684	1.00×10^{-10}
9	6.28×10^{-3}	26.0	0.894	0.496	2.80×10^{-10}
10	2.50×10^{-3}	25.0	0.649	0.360	7.78×10^{-10}
11	9.95×10^{-4}	24.0	0.471	0.261	2.17×10^{-9}
12	3.96×10^{-4}	23.0	0.341	0.190	6.03×10^{-9}
13	1.58×10^{-4}	22.0	0.248	0.138	1.67×10^{-8}
14	6.28×10^{-5}	21.0	0.179	9.96×10^{-2}	4.67×10^{-8}
15	2.50×10^{-5}	20.0	0.130	7.21×10^{-2}	1.30×10^{-7}
16	9.95×10^{-6}	19.0	9.38×10^{-2}	5.21×10^{-2}	3.61×10^{-7}
17	3.96×10^{-6}	18.0	6.78×10^{-2}	3.76×10^{-2}	1.01×10^{-6}
18	1.58×10^{-6}	17.0	4.90×10^{-2}	2.72×10^{-2}	2.79×10^{-6}
19	6.28×10^{-7}	16.0	3.53×10^{-2}	1.96×10^{-2}	7.78×10^{-6}
20	2.50×10^{-7}	15.0	2.54×10^{-2}	1.41×10^{-2}	2.17×10^{-5}
21	9.95×10^{-8}	15.0	1.87×10^{-2}	1.04×10^{-2}	6.03×10^{-5}
22	3.96×10^{-8}	15.0	1.37×10^{-2}	7.63×10^{-3}	1.68×10^{-4}
23	1.58×10^{-8}	15.0	1.01×10^{-2}	5.62×10^{-3}	4.66×10^{-4}
24	6.28×10^{-9}	15.0	7.44×10^{-3}	4.13×10^{-3}	1.30×10^{-3}
25	2.50×10^{-9}	15.0	5.47×10^{-3}	3.04×10^{-3}	3.61×10^{-3}
26	9.95×10^{-10}	15.0	4.03×10^{-3}	2.24×10^{-3}	1.01×10^{-2}
27	3.96×10^{-10}	15.0	2.96×10^{-3}	1.64×10^{-3}	2.80×10^{-2}
28	1.58×10^{-10}	15.0	2.18×10^{-3}	1.21×10^{-3}	7.77×10^{-2}
29	6.28×10^{-11}	15.0	1.60×10^{-3}	8.90×10^{-4}	0.217
30	2.50×10^{-11}	15.0	1.18×10^{-3}	6.55×10^{-4}	0.603
31	9.95×10^{-12}	15.0	8.67×10^{-4}	4.82×10^{-4}	1.68

$$t_1 = \frac{2.5 \times 10^{-4} K_v^{1/3} (A' \tau')^{3/10}}{\left[\ln \frac{1}{P(o)} \right]^{3/10}} \quad \dots (22)$$

For values of $P(o) >$ than 0.90 there is little error incurred by making the following approximation:

$$t_1 = 2.5 \times 10^{-4} K_v^{1/3} \left[\frac{A' \tau'}{1 - P(o)} \right]^{3/10} \quad \dots (23)$$

Assuming an average meteoroid velocity of 30 km per second and converting to engineering units, Equation (23) reduces to:

for aluminum

$$t' = 43.6 \left[\frac{A \tau}{1 - P(o)} \right]^{0.3} \quad \dots (24)$$

for steel

$$t' = 24.2 \left[\frac{A \tau}{1 - P(o)} \right]^{0.3} \quad \dots (25)$$

where

t' = required thickness in mils

A = area in ft^2

τ = time of exposure in years

$P(o)$ = probability that no punctures will occur in time τ of area A .

These equations are plotted in Figures 14 and 15 for aluminum and steel respectively for various values of $P(o)$ from 0.90 to 0.99 for τ equal to one year. These curves were then used to determine the tube armor thickness required for radiator designs considered in another section of this report.

41

Figure 14. Material Thickness vs Radiator Area (Aluminum)

Figure 15. Material Thickness vs Radiator Area (Steel)

Another interesting approach to the meteoroid protection problem is that of the meteoroid "bumper." The usual concept has a piece of material placed around, and at some distance from the point to be protected. The meteoroid, by passing through this light piece of material, shatters and its mass is dispersed over a larger area. Preliminary calculations indicate that this is indeed the case but insufficient analytical techniques or experimental evidence exists at the present time to base radiator designs on this approach.

D. EROSION

The surface of the radiator will likely be subjected to an erosion process from the meteoroids. If high emissivity surface coatings are employed they may be gradually worn away by the eroding action of the meteoroids. An approximate measure of the magnitude of the erosion problem may be deduced from Equation (19). Since the collisions produce a hemispherical crater of radius p , the amount of area removed by such punctures is:

$$da = 10^{-4} \pi p^2 dn \quad \dots (26)$$

where

da = area removed in square meters

p = penetration depth in cm

dn = number of hits with penetration between p and $p + dp$.

The number of hits with penetration greater than p is given by multiplying Equation (19) by the area and the exposure time which yields:

$$n = 10^{-12} p^{-10/3} v^{10/9} A' \tau' K'^{10/3} \quad \dots (27)$$

so that

$$dn = -1/3 \cdot 10^{-11} p^{-13/3} v^{10/9} A' \tau' K'^{10/3} dp . \quad \dots (28)$$

The area removed by hits of penetration exceeding p is:

$$a = \int_p^{\infty} \pi 10^{-4} p^2 dn = \pi/4 \cdot 10^{-15} v^{10/9} A' \tau' K'^{10/3} p^{-4/3} . \quad \dots (29)$$

For small values of a/A' , multiple hits on the same area may be neglected. The ratio, a/A' , which is the fraction of the original surface coating of thickness p that has been removed after a time τ' , may be calculated from Equation (29). $T_{1/10}$, the time required for removal of one-tenth of the original surface, is shown in Figure 16 as a function of surface-coating thickness.

This calculation shows that there should be a negligible effect on the radiator surface. If high emissivity coatings are employed, it is possible that they may be eroded slightly. However, a hole which is just the depth of the coating thickness should not result in any loss in emissivity since the product of the view factor and the increased area (due to the formation of the hole) should be approximately unity. Other effects such as spallation of the coatings could seriously effect the thermal characteristics of the radiator surface. (It should be noted that this could have a severe effect on optical reflecting surfaces such as mirror collectors.)

Figure 16. Time for 10% Loss of Area vs Material Thickness

V. REACTOR-RADIATOR CONFIGURATION

A. SCATTERED RADIATION

The large area of the radiator serves as a scattering medium for the radiation emanating from the reactor, and deflects a portion of this radiation into the payload section. This requires additional shielding over and above that required if the radiator were absent. Since the shield weights represent a sizeable fraction of the total system weight, in the low power range (< 30 kwe unmanned and < 1000 kwe manned), it is important to select configurations where this scattering problem is minimized. This is usually done by selecting a radiator configuration so as to have it lie within the shadow of the reactor shield. The reactor shield shadow is determined by the separation distance between the reactor and payload, the reactor size and shape, and the projected area of the payload section. Several radiator-shield configurations, designed with the scattered radiation problem in mind, are shown in Figure 17. What has been said concerning the radiator is applicable for the vehicle structure since this also will serve as radiation scattering or activation centers.

It is impossible at this time to do more than to indicate configurations where the scattering problem is at least minimized. This is because the shielding requirements can only be established in terms of a specific payload and mission. Payloads may range from hard tubes, which will probably requires no shielding, to manned missions which will require a large amount of shielding. In general, the scattered radiation problem increases with increasing payload radiation sensitivity. The particular shape, configuration, or geometry of the radiator will be further influenced by the vehicle employed and the installation, structural, and aerodynamic requirements imposed by the vehicle or mission.

B. ACTIVATION

For systems which employ boiling directing in the reactor core and condensing in the radiator, neutron activation of the working fluid could be a problem since it introduces an extended diffuse source of radiation which could require significant payload shielding depending on the allowable dose rates, fluid activation

Figure 17. Several Radiator-Shield Configurations

cross section and type of radiation emitted. In general, this problem only arises when manned systems are considered since the allowable radiation levels are lowest for manned missions. It is possible that the payload shielding required for Van Allen or cosmic radiation may mitigate this particular consideration.

VI. MINIMUM WEIGHT RADIATOR-CONDENSER DESIGN

To illustrate how the various design variables are combined to determine a minimum weight heat rejection system, a specific example is presented. The conditions selected corresponds to the design point of the SNAP 2 power system and are listed below:

- 1) Power - 3 kw electrical
- 2) Working fluid - mercury
- 3) Cycle overall efficiency - 6.0%
- 4) Condensing temperature - 600°F
- 5) Vehicle skin geometry - 5 ft diameter cylinder
- 6) Required subcooling $\Delta T = 200^\circ\text{F}$
- 7) Surface emissivity - 0.90
- 8) Condenser static pressure drop - 1.0 psi
- 9) Meteoritic protection - 90% probability of no puncture for 1 year life
- 10) Materials - fins - aluminum
armor - aluminum
tubes - steel (30 mil tube wall).

The radiator geometry is shown in Figure 18. The condensing tubes are located inside a 5 ft diameter cylinder. The outside surface of the cylinder forms the fins for the tubes and skin of the vehicle. The selection of this particular geometry is arbitrary, since the design of a power system for a space vehicle will most certainly be modified by the mission and payload requirements, and the specific vehicle limitations and/or requirements. This particular geometry does, however, offer several advantages. It can be made integral with the vehicle skin and does not require extension or orientation; thus the packaging and remote operation requirements are minimized. The detail of the tube and fin are shown in Figure 19. The meteoroid armor is made up of the fin material and the tube walls. If additional armor material was found to be required, a piece of aluminum of appropriate thickness was inserted between the fin and the

Figure 18. Radiator Geometry

$r_o = 0.149$ in
 $B = 4.32$ in
 $\Delta t = 0.030$ in
 $t = 0.025$ in
 $d = 0.157$ in

Figure 19. Detail of Tube and Fin Arrangement

tube. This armor was shaped to present a constant thickness of material irrespective of the direction of arrival of the meteoroid. Meteoroids which penetrate the opposite side of cylinder are not considered, since they are shattered and dispersed. The fins act as a meteoroid "bumper" for the tubes which are located directly opposite the point of penetration.

Figure 20 shows the variation in weight for the conditions listed above, and for a constant value of fin effectiveness = 0.80. The value selected for the fin effectiveness sets the radiator area, since $\eta = A_o/A$, where A is the actual area and A_o is the area required if the radiator surface were all at the condensing temperature. As the number of tubes is increased, the thickness of fins decrease in order to maintain a constant effectiveness. The tube weight increase is due primarily to the increase in the number of tubes since there is only a slight decrease in the tube radius as the number of tubes is made larger. The radius of the tubes is determined by the flow model postulated and the assigned pressure drop. A constant value of 30 mils for the tube wall thickness was selected as being a minimum practical value. The required weight of armor increases with the number of tubes due to the additional tubes and to the increase in the amount of armor material per tube required, as the vulnerable area (projected tube area only) increases. The mercury holdup is also shown to increase with increasing tube number. This is due primarily to the weight of mercury associated with the subcooling section. Appropriate allowances were made for inlet and exit headers and for brazing or welding materials. These weight penalties are included in the tube weight curve shown in Figure 20; no structural weight penalty has been assumed. As can be seen from Figure 20, the total weight curve passes through a broad minimum at about 45 tubes for the assumed value of fin effectiveness of 0.80.

In order to show that the selected fin effectiveness of 0.80 results in a minimum weight system, the procedure outlined above was repeated for several different values of η . The results are plotted in Figure 21, where the total radiator weight is plotted vs η . This shows that the optimum value of η is 0.75. Figure 22 shows the variation in area requirements as a function of weight. Since there is only a small difference in weight between $\eta = 0.70$ and $\eta = 0.80$, one would be inclined to select the value of $\eta = 0.70$ since, as shown in Figure 21, 25% fewer tubes are required.

Figure 20. Radiator Weight vs Number of Tubes

Figure 21. Radiator Weight vs Fin Effectiveness

If there is an additional constraint imposed upon the radiator in terms of a maximum area limitation, the weight penalty resulting from this particular constraint can be assessed from Figure 22. For the SNAP 2 system an area limitation of 110 ft^2 is imposed. From Figure 22 it is seen that this is very close to the minimum weight value and hence only a small weight penalty is incurred in satisfying this requirement.

There may be additional constraints imposed by practical considerations such as limits in tube radius, structural considerations, or minimum fin thickness requirements; however, the optimization procedure can still be followed and an optimum system determined within a given set of constraints.

The design procedure presented here points out that the heat rejection system must be optimized on a system basis, in which many factors must be considered. One cannot separate the flow problems from the heat transfer or meteoroid puncture problems, since they are all closely interrelated. To achieve minimum weight systems, all must be considered simultaneously.

Figure 22. Radiator Weight vs Radiator Area

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APPENDIX I

FLUID FLOW EQUATIONS

Lockhart and Martinelli correlated pressure drop data for two-phase flow in the nondimensional form:

$$\left(\frac{dP}{d\ell}\right)_{\text{TPF}} = \phi_g^2 \left(\frac{dP}{d\ell}\right)_g, \quad \dots (A-1)$$

where ϕ_g is a function only of $\frac{(dP/d\ell)_\ell}{(dP/d\ell)_g}$.

Another parameter, X , is defined as the square root of the ratio of the liquid phase pressure gradient to the gas phase gradient:

$$X = \sqrt{\frac{(dP/d\ell)_\ell}{(dP/d\ell)_g}}. \quad \dots (A-2)$$

The gradients for the two phases are calculated as if each phase flowed alone. When the liquid phase is in laminar flow ($Re_\ell < 1000$) and the vapor flow is turbulent ($Re_g > 2000$),

$$\left(\frac{dP}{d\ell}\right)_\ell = \frac{32V\mu_\ell}{g_c D^2} \quad \dots (A-3)$$

and

$$\left(\frac{dP}{d\ell}\right)_g = \frac{2fu^2 \rho_g}{g_c D} \quad \dots (A-4)$$

$$= \frac{2(0.046)(Re_g)^{-0.2} u^2 \rho_g}{g_c D}.$$

The velocities used in Equations (A-3) and (A-4) are the superficial values which are defined as follows:

$$V = \frac{W_l}{\rho_l A} = \frac{W_T(1-x)}{\rho_l A} \quad \dots (A-5)$$

$$u = \frac{W_g}{\rho_g A} = \frac{W_T x}{\rho_g A} \quad \dots (A-6)$$

Then

$$X = \sqrt{\frac{16}{0.046} \frac{W_l}{W_g} \frac{\rho_g}{\rho_l} \frac{\mu_l}{\mu_g} Re_g^{-0.8}} \quad \dots (A-7)$$

It should be noted that values of Reynolds' numbers, velocities, and flow rates change continuously along the length of the condenser. Therefore, local values must be used. If q/A is constant, as it will be for condensing vapors,

$$(Re)_g = (Re)_{g0} \left(1 - \frac{l}{L}\right), \quad \dots (A-8)$$

$$u = u_0 \left(1 - \frac{l}{L}\right), \quad \dots (A-9)$$

and

$$\frac{W_l}{W_g} = \frac{l/L}{1 - l/L} \quad \dots (A-10)$$

Combining Equations (A-7), (A-8), and A-10),

$$X = 18.65 (Re)_{g0}^{-0.4} \left(1 - \frac{l}{L}\right)^{-0.4} \left(\frac{l/L}{1 - l/L}\right)^{0.5} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.5} \quad \dots (A-11)$$

The Lockhart and Martinelli correlation is shown in Figure 1. The curve is well approximated over the SNAP 2 condenser conditions by:

$$\phi_g = 1.76 X^{0.0825} \quad \dots (A-12)$$

Combining Equations (A-11) and (A-12) we obtain:

$$\left(\frac{\Delta P}{L}\right)_{\text{TPF}} = 0.804 \frac{\rho_g u_o^2}{g_c D} (\text{Re}_{go})^{-0.316} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.0825} \int_0^1 \left(1 - \frac{l}{L}\right)^{1.6} \left(\frac{l}{L}\right)^{0.825} d\left(\frac{l}{L}\right) \quad \dots (A-13)$$

The integral may be evaluated by the use of a binomial expansion:

$$\int_0^1 \left(1 - \frac{l}{L}\right)^{1.6} \left(\frac{l}{L}\right)^{0.825} d\left(\frac{l}{L}\right) = 0.333 \quad \dots (A-14)$$

Substituting into Equation (A-13), we obtain:

$$\left(\frac{\Delta P}{L}\right)_{\text{TPF}} = 0.268 \frac{\rho_g u_o^2}{g_c D} (\text{Re}_{go})^{-0.316} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.0825} \quad \dots (A-15)$$

Remembering that $f_o = 0.079 (\text{Re}_{go})^{-0.25}$, we may write Equation (A-15) as

$$\left(\frac{\Delta P}{L}\right)_{\text{TPF}} = 1.67 \left(\frac{2f_o \rho_g u_o^2}{g_c D}\right) (\text{Re}_{go})^{-0.066} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.0825} \quad \dots (A-16)$$

or

$$\left(\frac{\Delta P}{L}\right)_{\text{TPF}} = 1.67 \left(\frac{dP}{dl}\right)_o (\text{Re}_{go})^{-0.066} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g}\right)^{0.0825} \quad \dots (A-17)$$

Equation (A-17) is the generalized form of the Martinelli correlation for frictional pressure drop. The pressure rise due to momentum is given by

$$\int_{P_o}^{P_E} dP_m = \int_{u_o}^{u_e} \frac{\rho u du}{g_c} . \quad \dots (A-18)$$

Since (ρu) is constant throughout the condenser, and since the exit liquid velocity is very small,

$$P_o - P_E = - \frac{\rho_g u_o^2}{g_c} . \quad \dots (A-19)$$

Adding the friction and momentum terms:

$$P_o - P_E = 1.67L \left(\frac{dP}{dl} \right)_o (Re)_{go}^{-0.066} \left(\frac{\rho_g \mu_l}{\rho_l \mu_g} \right)^{0.0825} - \frac{\rho_g u_o^2}{g_c} . \quad \dots (A-20)$$

Equation (A-20) gives the net static pressure drop in a total condenser.

For a condenser consisting of n tubes in parallel:

$$(Re)_{go} = \frac{Du \rho_g}{\mu_g} = \frac{4W_T}{\pi Dn \mu_g} \quad \dots (A-21)$$

and

$$u_o = \frac{4W_T}{n\pi D^2 \rho_g} . \quad \dots (A-22)$$

Equation (A-20) may be written:

$$P_o - P_E = \left(\frac{L W_T^{1.684}}{N^{1.684} D^{4.684}} \right) \phi_1 - \left(\frac{W_T^2}{N^2 D^4} \right) \phi_2, \quad \dots (A-23)$$

where the first term on the right side of the equation represents the frictional pressure drop, and the second term, the momentum pressure rise. The parameters, ϕ_1 and ϕ_2 , contain the fluid properties, and for mercury, are shown as a function of temperature in Figures 2 and 3.

Finally, it is of interest to compare the two-phase friction pressure drop with the pressure drop that would exist if the liquid had no effect on the fluid friction (the "pure-gas" pressure drop); that is, if the liquid were removed from the system the instant it condensed, and the gas flow rate decreased linearly along the condensing length.

The inlet Reynold's number for SNAP 2 condenser will be about 10,000; the dynamic viscosity ratio, $(\rho_g \mu_l / \rho_l \mu_g)$, is 0.0024 at 600°F. Equation (A-17) becomes:

$$\left(\frac{\Delta P}{L} \right)_{TPF} = 0.545 \left(\frac{dP}{dl} \right)_o. \quad \dots (A-23)$$

To find the "pure-gas" pressure drop, the Fanning friction equation may be integrated over the condenser length.

$$dP = \frac{2f u^2 \rho_g dl}{g_c D} \quad \dots (A-24)$$

and

$$f = 0.079 \text{ Re}^{-0.25}. \quad \dots (A-25)$$

Substituting local values of Re and u from Equations (A-21) and A-22):

$$dP = 0.16 \frac{\rho_o^2 u_o^2}{g_c D} (Re_o)^{-0.25} \left(1 - \frac{\ell}{L}\right)^{1.75} d\ell. \quad \dots (A-26)$$

Integrating from $\ell = 0$ to $\ell = L$, we obtain:

$$\left(\frac{\Delta P}{L}\right)_{gF} = 0.364 \left(\frac{dP}{d\ell}\right)_o. \quad \dots (A-27)$$

Comparing Equations (A-23) and (A-27), it is seen that

$$\frac{(\Delta P/L)_{TPF}}{(\Delta P/L)_{gF}} = 1.5. \quad \dots (A-28)$$

Thus, the effect of the liquid's presence is to raise the pressure drop by about 50% over the value for the gas flowing alone.

APPENDIX II

RADIATOR EQUATIONS

The following analysis considers the combination of two specific types of radiators. One is a tube and fin design in which the tube temperature remains constant (as is typical of a condensing radiator), the second is a flat plate radiator in which there is a temperature gradient in the direction of fluid flow.

A. CONSTANT TEMPERATURE TUBE

The heat conducted along the fins in the x direction is given by:

$$q_x = -kA_n \frac{dT_x}{dx}, \quad \dots (A-29)$$

where A_n is the area normal to the direction of heat flow. The change in heat flow per unit length under steady state conditions must be equal to the heat radiated by the elemental area ydx (see Figure 7A). That is,

$$dq_x = -kA_n \frac{d^2T_x}{dx^2} dx = -\epsilon\sigma T_x^4 ydx \quad \dots (A-30)$$

which reduces to:

$$\frac{d^2T_x}{dx^2} - \frac{\epsilon\sigma}{kt} T_x^4 = 0. \quad \dots (A-31)$$

Equation (A-31) is nondimensionalized by making the substitutions: $\beta = x/B$ and $\theta = \frac{T_x}{T_o}$

$$\frac{d^2\theta}{d\beta^2} - \frac{T_o^3 \epsilon \sigma B^2}{kt} \theta^4 = 0 \quad \text{or} \quad \frac{d^2\theta}{d\beta^2} - M_R \theta^4 = 0. \quad \dots (A-32)$$

M_R is a nondimensional parameter referred to as the radiation modulus, and is equal to $T_o^3 \epsilon \sigma B^2 / kt$.

The boundary conditions for Equation (A-32) are:

$$\frac{d\theta}{d\beta} = 0 \quad \text{at} \quad \beta = \frac{1}{2} \quad \dots (A-33)$$

and

$$\theta = 1 \quad \text{at} \quad \beta = 0 \quad \dots (A-34)$$

Of primary importance to the designer is the fin effectiveness, defined as the ratio of the heat rejected by the fin to that which would be rejected if it were all at the base temperature, T_o .

Expressed mathematically:

$$\eta = \frac{2\epsilon\sigma y}{2\epsilon\sigma y} \frac{\int_0^{B/2} T_x^4 dx}{\int_0^{B/2} T_o^4 dx} \quad \dots (A-35)$$

or in terms of the previous substitutions

$$\eta = 2 \int_0^{1/2} \theta^4 d\beta \quad \dots (A-36)$$

Equations (A-32) and (A-36) have been solved numerically. The resulting plot of η vs M_R is shown in Figure 8.

B. FLAT PLATE

By taking a heat balance about a flat plate radiator in which there is a temperature gradient in the direction of fluid flow (still assuming radiation from

only one side) the following equation results (Figure 7B):

$$-c_p W dT = \epsilon \sigma T^4 y dx, \quad \dots (A-37)$$

which reduces to:

$$A = \frac{c_p W}{3 \epsilon \sigma} \left\{ \frac{1}{T_{out}^3} - \frac{1}{T_{in}^3} \right\} . \quad \dots (A-38)$$

Equation (A-38) can be modified by substituting the following identities:

$$Q_R = c_p W \Delta T \quad \dots (A-39)$$

$$\Delta T = T_{in} - T_{out} \quad \dots (A-40)$$

to give

$$A = \frac{Q_R}{3 \epsilon \sigma \Delta T} \left\{ \frac{1}{(T_{in} - \Delta T)^3} - \frac{1}{T_{in}^3} \right\} . \quad \dots (A-41)$$

This is the radiator area required to radiate the power, Q_R .

C. TUBE FIN (TUBE TEMPERATURE NOT CONSTANT)

The case of the finned tube design in which the temperature does not remain a constant (where heat is lost by a sensible temperature drop of the fluid in traversing the radiator), the radiator can be analyzed by combining the two approaches previously considered to give:

$$A = \frac{Q_R}{(3 \epsilon \sigma \Delta T) \eta} \left\{ \frac{1}{(T_{in} - \Delta T)^3} - \frac{1}{T_{in}^3} \right\} \quad \dots (A-42)$$

where η is evaluated at same intermediate temperature.

The radiator area requirements for the subcooler section in this report were determined by Equation (A-42). The assumption implicit in Equation (A-42) is that the heat conducted by the fin in the direction of the fluid flow is small.

NOMENCLATURE

A = area (ft^2)
 A' = area (cm^2)
 a = absorptivity
 B = tube spacing (ft)
 c_P = heat capacity (Btu/lb-°F)
 D = inside tube diameter (ft)
 f = Fanning friction factor
 g_c = gravitational constant $\left(32.2 \frac{\text{lb-mass-ft}}{\text{lb-force-sec}}\right)$
 K = $1.5K'$
 K' = constant, dependent on material properties
 k = thermal conductivity of fin (Btu/hr-ft-°R)
 L = condenser length (ft)
 l = distance along condenser (ft)
 M = mass (grams)
 M_R = radiation modulus
 N = number of tubes
 n = number of meteoroid hits
 P = pressure (lb/ft^2)
 $P(n)$ = probability that n punctures will occur in time τ' of a sensitive area A'
 $P(o)$ = Probability that no punctures will occur in time τ' of a sensitive area A'
 p = penetration depth (cm)
 Q = radiative power (Btu/hr)
 Re = Reynold's number
 T_o = radiating surface temperature (°R) (also tube temperature)
 T_{sink} = radiative sink temperature (°R)
 T_{in} = fluid temperature into subcooler (°R)
 T_{out} = fluid temperature out of subcooler (°R)
 T_x = temperature at a point on the fin (°R)
 R_l = liquid phase volume fraction

$T_{1/10}$ = time for 10% loss of area
 t = fin thickness (ft)
 t_i = required thickness for meteorite protection (cm)
 t' = required thickness for meteorite protection (mils)
 u = gas velocity (ft/sec)
 V = liquid velocity (ft/sec)
 v = projectile, or meteoroid, velocity (km/sec)
 W = flow rate (lb/sec)
 X = dimensionless parameter of Lockhart and Martinelli
 x = distance along fin perpendicular to direction of fluid flow (ft) (also vapor quality)
 y = distance along flat plate radiator, parallel to direction of fluid flow (ft)

GREEK

β = dimensionless length
 θ = dimensionless temperature
 ϵ = surface emissivity
 η = fin effectiveness
 λ = wave length
 ϕ_g = dimensionless parameter of Lockhart and Martinelli
 ϕ = number of meteorite particles of mass greater than M per square meter per second
 ϕ_1, ϕ_2 = fluid property parameters; defined by Equations 6 and 9 respectively.
 μ = viscosity (lb-mass/ft-sec)
 σ = Stephen Boltzman constant ($0.171 \times 10^{-8} \text{ Btu/hr-ft}^2\text{-}^\circ\text{R}^4$)
 ρ = density (lb-mass/ft³)
 ψ = number of penetrations of target thickness t_i (cm) per square meter per second
 τ = time of exposure in years
 τ' = time of exposure in seconds

SUBSCRIPTS

T = total
 l = liquid phase
 g = gas phase (gF refers to gas friction)
 TPF = two phase friction
 M = momentum
 o = entrance
 E = exit