STRINGENT LIMITS ON THE DECAYS

$K_L^0 \rightarrow \mu^+\mu^-, e^+e^-, \mu^+\mu^-$

Henry Jonathan Frisch

(Ph.D. Thesis)

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STRINGENT LIMITS ON THE DECAYS

\[ K_L^0 \rightarrow \mu^+ \mu^- \epsilon^+ \epsilon^- , \mu^+ \epsilon^- \]

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STRINGENT LIMITS ON THE DECAYS

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March 24, 1971

ABSTRACT

We have performed a search at the Bevatron for the decays \( K_L^0 \rightarrow \mu^+ \mu^- \), 
\( e^+ e^- \), \( \mu^+ e^+ \). The search was made with a double arm spectrometer 
comprised of picture frame magnets and wire spark chambers: data were 
accumulated with an on-line PDP-9 computer. Running conditions included 
both helium and vacuum in the decay volume. Approximately 1,041,000 
\( K_L^0 \rightarrow \pi^+ \pi^- \) decays were observed: these determine the normalization for 
the dilepton decay modes. No \( e^+ e^- \) or \( e^+ \mu^+ \) events were observed, 
resulting in an upper limit for the branching ratio for each of the above 
 modes of \( 1.6 \times 10^{-9} \) (90\% C.L.). One possible \( \mu^+ \mu^- \) event and at least 
four background events were detected in the data taken with helium in the 
decay volume. No \( \mu^+ \mu^- \) events were detected in the much larger data 
sample from the evacuated decay volume. If one chooses to ignore the 
hehelium data, the vacuum data alone determines a limit on the branching 
ratio for the decay \( K_L^0 \rightarrow \mu^+ \mu^- \) of \( 1.82 \times 10^{-9} \) (90\% C.L.). If one regards 
the one event in the helium as real, the branching ratio is determined to 
be \( 6.8 \times 10^{-10} \). In either case our result is in conflict with the 
theoretical lower "unitarity" limit of \( 4.8 \times 10^{-9} \) for the \( K_L^0 \rightarrow \mu^+ \mu^- \) 
branching ratio.
I. INTRODUCTION

The absence of neutral currents has long been a puzzle in the weak interactions. This thesis describes an experiment that was designed to search for weak decays involving such currents, the decays

$$K^0_L \rightarrow \mu^+ \mu^-, e^+ e^-, \mu^+ e^-.$$

The relevant theory of both the weak and electromagnetic interaction is described in Chapter II. The apparatus is described in Chapter III, and the data analysis in Chapter IV. The experimental results are presented and discussed in the next chapter. Chapter VI describes self-consistency checks on both the equipment and the analysis procedures. The final chapter discusses the significance of our new limits on the decays

$$K^0_L \rightarrow \mu^+ \mu^-, e^+ e^-, \mu^+ e^-.$$
II. THEORY

A. Introduction

Weak interactions can be described with a great deal of success by a phenomenological Hamiltonian formed by the product of two currents. The Hamiltonian that describes all presently known weak interactions can be written

\[ H_{\text{WK}} = \frac{G}{2} [ J_\mu K^\mu \dagger + J^{\dagger}_\mu K^\mu ] \] (1)

where \( J \) and \( K \) symbolize weak currents. If we make the simplest assumption that \( J \) and \( K \) are identical, we obtain

\[ H_{\text{WK}} = G J_\mu J^{\dagger}_\mu \] (2)

This is the standard hypothesis for the weak Hamiltonian.

The current was introduced into weak interactions in analogy to the current in electrodynamics. In electrodynamics the current is a four-vector constructed from the fields of the interacting particles. We thus take our general definition of a current to be a four-vector composed of the particle fields. In electrodynamics one describes interactions by the coupling of currents to the photon field. The introduction of the Intermediate Vector Boson (IVB) (or \( W \) meson) will make an analogy of the electron current interacting with the photon field and the weak current interacting with the IVB field.

What experimental facts do we have to explain this Hamiltonian? The first knowledge of the weak interaction was derived from nuclear beta-decay; as a prototype let us consider the neutron decay:

\[ n \rightarrow p e^- \bar{\nu}_e \] (3)
If we are to consider this as the product of two currents, conservation of baryon number (and of lepton number) forces us to make the neutron and proton into one current, and the electron and neutrino into the other. We will symbolize these currents by \((\bar{n}p)\) and \((\bar{e}\nu)\).

The \((\bar{e}\nu)\) current also occurs in the decay \(\Lambda \rightarrow p\bar{e}\bar{\nu}_e\), and so we are led to a strangeness changing current, symbolically \((\bar{\Lambda}p)\). This same current occurs in the decay \(\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e\), but although in our present notation one might think we would represent this current as \((\Xi^0\Sigma^-)\), it has exactly the same quantum numbers as the \((\bar{\Lambda}p)\) current, and therefore is the same current. The same quantum numbers also occur in the hadronic current involved in the semi-leptonic decays of the K meson

\[
K \rightarrow \pi e^+ \nu_e \quad (Ke3)
\]
\[
K \rightarrow \pi \mu^- \nu_\mu \quad (K\mu3)
\]

Again the current involving the hadrons has the quantum numbers \(S=1\) and \(I=1/2\). The \((\bar{\mu}\nu)\) current is clearly analogous by \(\mu-e\) universality to the \((\bar{e}\nu)\) current.

We now have three constituents for the weak current \(J\):

1. A hadronic current \((\bar{\Lambda}p)\) with the quantum numbers \(I=1/2, S=1\) which we will denote as \(J^S=1_H\).

2. A hadronic current \((\bar{n}p)\) with the quantum numbers \(I=1, S=0\) which we denote \(J^S=0_H\).

3. A leptonic current with the two terms \((\bar{e}\nu)\) and \((\bar{\mu}\nu)\). We denote this current by \(I_w = (\bar{e}\nu) + (\bar{\mu}\nu)\).

The self-product of the total current should thus give us nine terms which describe all known weak interactions. Table 1 lists the nine terms and examples of the associated reactions. It thus seems a good start to
Table 1

The 9 terms of the product $J_W^+ J_W$ and examples of the reaction they describe.

<table>
<thead>
<tr>
<th>$J_H$</th>
<th>$S = 0$</th>
<th>$J_H$</th>
<th>$S = 1$</th>
<th>$I = 1/2$</th>
<th>$L_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_H^+$</td>
<td>np → np</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_H^+$</td>
<td>$S = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I = 1/2$</td>
<td>$K \rightarrow \pi \pi \pi$</td>
<td>$\Lambda \rightarrow np$</td>
<td>$\Xi \rightarrow \Lambda \pi$</td>
<td>$\Omega \rightarrow \Xi \pi$, $\Lambda K$</td>
<td>$\Lambda p \rightarrow \Lambda p$</td>
</tr>
<tr>
<td>$I = 1$</td>
<td>$\pi \rightarrow e \nu$</td>
<td>$\pi \rightarrow \mu \nu$</td>
<td>$\pi \rightarrow \pi e \nu$</td>
<td>$p \rightarrow \pi e \nu$</td>
<td>$\sum \rightarrow \Lambda e \nu$</td>
</tr>
<tr>
<td>$I = 1$</td>
<td>$\Lambda \rightarrow p e \nu$</td>
<td>$K \rightarrow \mu \nu$</td>
<td>$K \rightarrow e \nu$</td>
<td>$K \rightarrow p e \nu$</td>
<td>$K \rightarrow \pi \mu \nu$</td>
</tr>
</tbody>
</table>
assume the weak current is of the form

\[ J_W = J_{\text{H}}^{S=0} + J_{\text{H}}^{S=1} + L_W \]

B. The Intermediate Vector Boson

Both in QED and in low energy nucleon-nucleon scattering the first order interaction can be represented as the exchange of a particle, respectively the photon and the pion. It therefore would seem reasonable to consider the weak interaction to be mediated by a particle exchange: this (as yet conceptual) particle has been named the Intermediate Vector Boson (IVB or W meson) and would have the following properties:

1. Because it couples to vector currents it has spin one.

2. It must exist in a charged state because it should mediate the reaction \( n \to p e^- \bar{\nu}_e \).

3. The IVB must be heavy as the weak interaction seems to have no momentum dependence - i.e., it appears a point interaction. (The IVB would have to have \( M > M_K \), otherwise one would see the \( K^+ \) decay into \( W^+ \gamma \).

The last point gives us an interpretation of the current-current interaction. Consider \( ve \) scattering to first order with an IVB intermediary.

\[ \begin{array}{c}
e^-
\rightarrow
\text{W}^-

\text{G}

\text{G}

\nu

\nu

\rightarrow
\text{e}^-
\end{array} \]

If we assume the lepton current \((\bar{e}v)\) is of the standard V-A form

\[ L_e = u_e \gamma \alpha (1 + \gamma_5) \bar{u}_\nu \]
and the IVB propagator is that for a spin one particle, the matrix element for the interaction is

\[ M = g^2 \bar{u}_e \gamma_\alpha (1 + \gamma_5) u_\nu \left( \frac{\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2 + M_W^2}}{2} \right) \frac{M_W}{q^2 + M_W^2} \]

where \( g \) is the weak interaction coupling constant and \( \delta_{\alpha\beta} \) is the Kronecker delta. If \( M_W^2 > q^2 \), the above expression reduces to the product of the two lepton currents - i.e., our current-current interaction.

It is clear that the current-current Hamiltonian, i.e., the point interaction, cannot be correct. For instance, in the above example of \( e\nu \) scattering, if one calculates the cross-section ignoring the IVB propagator, one finds that the cross-section is

\[ \sigma = \frac{1}{4} \frac{G^2}{\pi} s \]

where \( s \) is the square of center of momentum energy of one of the particles. The above formula cannot be correct for large \( s \), for it violates the \( s \)-wave unitarity limit \( \sigma \leq \frac{2\pi}{s} \) at \( s \approx 300 \text{ GeV} \). A \( q^2 \) dependence in the propagator for the IVB would resolve this conflict, i.e., the IVB would impose the necessary cutoff in momentum transfer.
C. Neutral Currents

The weak current we have described consists of only charged terms. It would seem reasonable on grounds of universality to include neutral terms, both hadronic and leptonic, such as $\bar{\nu}\bar{\nu}$, $\bar{p}p$, $\bar{\Lambda}\Lambda$, and $\bar{e}e$, $\bar{\nu}_\mu\nu_\mu$. There is no experimental evidence that such terms exist. There is, however, strong experimental evidence that the coupling of the hadronic neutral currents and the leptonic neutral currents is strongly suppressed relative to the coupling for the analogous charged currents. Table 2 lists interactions involving neutral currents and their present experimental upper limits.

There is no experimental evidence that the hadronic or leptonic neutral currents do not exist, only evidence that they do not mutually couple. This peculiar state of affairs comes about because in purely hadronic or purely leptonic interactions it is impossible to distinguish the neutral from the charged currents. The decay of the $K^0_S$ meson into $\pi^0\pi^0$ is an example of such a possibility. At first sight it might seem that neutral currents must be involved, as both initial and final states are neutral. One can, however, construct diagrams for the $K$ decay involving the charged currents $(\bar{\Lambda}p)$ and $(\bar{np})$.

As these charged currents are observed in $\Lambda$ and neutron $\beta$ decay, there seems to be no a priori reason to introduce the neutral $(\bar{p}p)$ and $(\bar{n}n)$ currents. In fact, if $(\bar{\Lambda}\Lambda)$ and $(\bar{n}n)$ currents existed, one could have first order $\Delta S=2$ transitions, in clear violation of experimental evidence.
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Limit (90% C.L.)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Leptonic $K^0_L$ Decays</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^0_L \rightarrow \mu^+ \mu^-$</td>
<td>$&lt; 2.6 \times 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>$K^0_L \rightarrow e^+ e^-$</td>
<td>$&lt; 1.5 \times 10^{-7}$</td>
<td></td>
</tr>
<tr>
<td>$K^0_L \rightarrow \mu^+ e^\pm$</td>
<td>$&lt; 8 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 9 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td><strong>Other $K$ Decays</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^0_L \rightarrow \mu^+ \mu^-$</td>
<td>$&lt; 7.3 \times 10^{-5}$</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ e^+ e^-$</td>
<td>$&lt; 2.5 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ \mu^+ \mu^-$</td>
<td>$&lt; 2.4 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$&lt; 3 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ \nu\bar{\nu}$</td>
<td>$&lt; 1.4 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+ \pi^0 e^+ e^-$</td>
<td>$&lt; 8 \times 10^{-6}$</td>
<td></td>
</tr>
<tr>
<td><strong>Purely Leptonic Interactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(which violate separate lepton conservation)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu^+ \rightarrow e^+ \gamma$</td>
<td>$&lt; 2.2 \times 10^{-8}$</td>
<td>Parker et al. 12</td>
</tr>
<tr>
<td>$\mu^+ \rightarrow e^+ e^- e^+$</td>
<td>$&lt; 1.8 \times 10^{-7}$</td>
<td>Alikhanov et al. 13</td>
</tr>
<tr>
<td><strong>Scattering</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_\mu + p \rightarrow \nu_\mu + p$</td>
<td>$\frac{\sigma (\nu_\mu + p \rightarrow \nu_\mu + p)}{\sigma (\nu_\mu + p \rightarrow n + \mu^+)} &lt; 0.03$</td>
<td>Block et al. 15</td>
</tr>
<tr>
<td>$\bar{\nu}_e + d \rightarrow p + n + \bar{\nu}_e$</td>
<td>$\frac{\sigma (\bar{\nu}_e + d \rightarrow p + n + \bar{\nu}_e)}{\sigma (\bar{\nu}_e + d \rightarrow n + n + e^+)} &lt; 140$</td>
<td>Block et al. 14</td>
</tr>
</tbody>
</table>
(the $\bar{\Lambda}\Lambda$ reaction corresponds to $K \rightarrow \bar{K}$ in first order).

For purely leptonic interactions it is impossible to distinguish the $(\bar{e}v)$ $(\bar{e}v)$ from the $(\bar{\nu}v)$ $(\bar{\nu}e)$ interaction due to a symmetry expressed by the Fierz relation. Thus the only place except for semi–hadronic reactions such as $K^+ \rightarrow \mu^+ \nu \bar{\nu}$, or $\nu p \rightarrow \nu p$ that one could look for neutral leptonic currents would be in interactions which either violate separate lepton number conservation, e.g. $\mu^+ \rightarrow e^+ \gamma$ or $\mu^+ \rightarrow e^+ e^+ e^-$, or in weak scattering with zero charge in both $s$ and $t$ channels, e.g., $e^+ e^- \rightarrow e^+ e^-, \nu \nu \rightarrow \bar{\nu} \bar{\nu}$. The prospects for measuring these experimentally are slim.

Hadronic neutral currents have, however, a strong theoretical appeal. This stems from the $\Delta I = 1/2$ selection rule in the strangeness changing semi–leptonic interactions. For example, consider the decay $K^0 \rightarrow \pi^0 \pi^0$. As the kaon is $I = 1/2$ and the two pions are in either an $I=0$ or an $I=2$ state ($I=1$ is forbidden by Bose statistics) we can construct such an interaction from our schematic hadronic currents with an expression of the form $(\bar{\Lambda}p)$ $(\bar{p}n)$. As the first term is an isospinor and the second has $I=1$, it is clear we can have either $\Delta I = 1/2$ or $\Delta I = 3/2$ amplitudes. To construct a pure $\Delta I = 1/2$ interaction with such a model one must resort to an interaction of the form $(\bar{p}n)$ $(\bar{\Lambda}p) + \frac{1}{2} (\bar{\Lambda}n)$ $(\bar{p}p - \bar{n}n)$. The interaction has been made pure $\Delta I = \frac{1}{2}$ by the inclusion of the neutral hadronic currents $(n\bar{n})$ and $(\bar{p}p)$.

Thus we are left with searching for a coupling between the hadronic and leptonic neutral currents as the best means of detecting neutral
currents. By far the best limits at the present on neutral current processes are the limits on the decays $K_L^0 \rightarrow \mu^+ \mu^-$, and $e^+ e^-$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ e^+ e^-$. Other modes with low limits are $K_L^0 \rightarrow e^+ e^-$, $\mu \rightarrow e \gamma$, $\mu \rightarrow e^+ e^- e^+$ (see Table 2). All of these decays depend on the existence of neutral leptonic currents, and the last three modes depend also on violation of separate lepton conservation. Thus the experimental evidence against the coupling of hadronic neutral currents to leptonic neutral currents is very good, while it is almost nonexistent for the separate existence of hadronic or leptonic neutral currents.

D. The $K_L^0 \rightarrow \ell^+ \ell^-$ Decays

The dileptonic decays $K_L^0 \rightarrow \ell^+ \ell^-$ in the current-current picture would be induced by the product of a neutral strangeness-changing hadronic current and a neutral leptonic current. If one assumes that the leptonic current has a $V-A$ structure, it is easy to derive the rate for $K_L^0 \rightarrow \ell^+ \ell^-$ in terms of some neutral weak coupling $G_N$. This calculation is performed in Appendix A. The $V-A$ coupling would predict a decay rate that is proportional to the lepton mass squared for the $e^+ e^-$ and $\mu^+ \mu^-$ decays; one would consequently expect the $e^+ e^-$ decay to be suppressed by a factor of $\frac{m_e^2}{m_\mu^2} = \frac{1}{40,000}$ for a $V-A$ interaction. One could therefore theoretically distinguish a $V-A$ coupling from a scalar or pseudoscalar coupling, which would predict equal rates for $K_L^0 \rightarrow \mu^+ \mu^-$ and $K_L^0 \rightarrow e^+ e^-$.}

For the decays $K_L^0 \rightarrow \ell^+ \ell^-$ to proceed by a first order weak process in an IVB model one would have to have neutral vector bosons (Fig. 1a). This presents a problem with isospin: vector bosons interact strongly, and presumably thus have a definite I-spin in their strong interactions.
There is no way in a standard $I$-spin or SU(3) framework to have both positive and negative mesons with strangeness less than 2 without an associated neutral meson. Thus it is unpleasant to eliminate neutral currents by eliminating the neutral IVB.

There have been several theories which have dealt with neutral currents in first order. DeRaphael, Good, and Michel have created a theory with two neutral IVB's. To eliminate the neutral leptonic currents, they have assumed the IVB's couple with such currents, but with equal couplings and opposite phases. The amplitudes for the reactions involving neutral leptonic currents thus are zero. Lee and Yang have proposed that the IVB could act both as $I=1$ and $I=1/2$ (schizons). In the strong interactions the IVB could act as $I=1$, but in the leptonic interactions it would be $I=1/2$. The contributions of the $I=-1/2 \, \overline{W}_o$ and the $I=1/2 \, \overline{W}_o$ would cancel eliminating neutral currents.

Other theories of weak interactions, such as that of Oakes, or Trieman, either simply declare that currents of the form $f^+ f^-$ do not exist, or that they do not couple to hadrons. In either case the theory would rule out the decays $K^0_L \rightarrow \mu^+ \mu^-$ or $K^0_L \rightarrow e^+ e^-$. The decays $K^0_L \rightarrow \mu^\pm e^\mp$ violate the conservation of separate lepton number. I know of no theory about such interactions.

E. Second Order Weak Interactions

The decays $K^0_L \rightarrow f^+ f^-$ could also possibly proceed via second order in the weak interaction. A typical diagram in an IVB framework of the weak interactions is given in Fig. 1b and in a 4-Fermion interaction in
Fig. 1c. The main interest in second order weak interactions stems from the fact that perhaps more can be learned about the high momentum transfer behavior of the first order interaction through them. We saw earlier that a 4-Fermion theory with V-A currents leads to divergences for c.m. energies on the order of 300 BeV. This would imply a cut-off energy of at most that value. We shall see that by considering the second order 4-Fermion interactions in the decays $K_L^0 \rightarrow \mu^+ \mu^-$ one can reduce this cut-off by approximately two orders of magnitude.

The second order calculation of the rates for $K_L^0 \rightarrow \mu^+ \mu^-$ from the diagrams 1b or 1c diverges unless one introduces a cut-off in momentum transfer. The divergence is due to the integration around the closed neutrino loop and is therefore logarithmic. Unless higher order terms tend to cancel, there must be a cut-off. Ioffe and Shabalin,\textsuperscript{21} and Mohapatra, Rao, and Marshak\textsuperscript{22} have calculated the rate for $K_L^0 \rightarrow \mu^+ \mu^-$ in terms of the cut-off $\Lambda$. The results of their current algebra calculation is that the ratio of the rates $R = \Gamma(K_L^0 \rightarrow \mu^+ \mu^-)/\Gamma(K^+ \rightarrow \mu^+ \nu)$ (neglecting terms of order $(\frac{M}{M_K})^2$) is given by

$$R = \left[ \frac{G \Lambda^2}{(4\pi)^2} \right]^2$$

If we can plug in an experimental number for $R$, we can thus solve for the cut-off parameter $\Lambda$. The result for the 4-Fermion model is

$$R = \left( \frac{G \Lambda^2}{\pi^2} \right)^2,$$

resulting in a lower value for $\Lambda$ for a given limit $R$.

The current algebra calculations of Ioffe and Shabalin, and Mohapatra, Rao, and Marshak result in a cut-off parameter $\Lambda$ independent of the mass
of the IVB. Mohapatra, et al., however, have suggested that a "strong" form factor for the IVB would lead to \( M \sim M_W \). Ioffe and Shabalin, on the other hand, have proposed that if the form factor of the IVB is instead determined by the electromagnetic interactions, then the mass of the IVB should go as

\[
M_W \sim \sqrt{\alpha} \Lambda \approx \frac{\Lambda}{12}
\]

F. Electromagnetically Induced Decays

One would expect the \( K^0_L \) to decay into lepton pairs through a first order weak, second order (in \( \alpha \)) electromagnetic interaction. A schematic diagram for such a decay is given in Fig. 2. The reasoning is as follows:

1. The \( K^0_L \) is experimentally observed to decay to two photons.\(^{23}\)
2. We believe that QED predicts the two photon–two muon coupling completely.
3. There are no states that obviously couple to two muons to interfere with the two photon intermediate state.

The calculation has been done by Beg,\(^{24}\) Sehgal,\(^{25}\) and Quigg and Jackson.\(^{26}\) Beg used a dispersion relation model to estimate an upper limit for the \( K^0 - \pi^0 \) couplings and then used a model by Drell to calculate the \( \pi^0 \to e^+ e^- \) rate. His result is an upper limit to the rate \( K^0_L \to \mu^+ \mu^- \) of

\[
\frac{\Gamma(K^0_L \to \mu^+ \mu^-)}{\Gamma(K^0_L \to \text{all})} \leq 3.8 \times 10^{-8}
\]
Sehgal assumed a vector meson dominated the form factor for one of the photons in the $K_L^0 \to \gamma \gamma$ rate to calculate the relative branching ratio

$$\frac{\Gamma (K_L^0 \to \mu^+ \mu^-)}{\Gamma (K_L^0 \to \gamma \gamma)}$$

His calculation has been repeated by Quigg and Jackson - the results are presented in Fig 3. The calculation diverges for infinite vector meson mass because of the loop integration; the result, however, is finite for a given vector meson mass. The calculation as a function of vector meson mass is represented by the line labeled KV7. Quigg and Jackson also calculated the ratio $\Gamma (K_L^0 \to \mu^+ \mu^-)/ \Gamma (K_L^0 \to \gamma \gamma)$ assuming that both photons couple to the kaon through vector mesons. The result is also shown in Fig. 3, labeled KVV. For a reasonable choice of vector meson mass, e.g., the rho mass, the KV7 coupling model predicts

$$\frac{\Gamma (K_L^0 \to \mu^+ \mu^-)}{\Gamma (K_L^0 \to \gamma \gamma)} = 1.6 \times 10^{-5}$$

and the KVV coupling model predicts

$$\frac{\Gamma (K_L^0 \to \mu^+ \mu^-)}{\Gamma (K_L^0 \to \gamma \gamma)} = 1.25 \times 10^{-5}$$

If we use the presently accepted value $^{31}$ for the experimental rate for $K_L^0 \to \gamma \gamma$ of $5.2 ^{+0.5} _{-0.4} \times 10^{-4}$ we would predict branching ratios for $K_L^0 \to \mu^+ \mu^-$ of $6.8 \times 10^{-9}$ and $8.2 \times 10^{-9}$ for the KVV and KV7 models respectively.
G. The Unitarity Bound

One can estimate a lower limit to the decay rate for $K_L^0 \to \mu^+ \mu^-$ by including only contributions from "real", i.e., on-mass-shell, photons, and ignoring contributions from virtual photons in the intermediate two-photon state. As the on-shell amplitude corresponds to the absorptive part of the amplitude, it adds incoherently with the real part of the amplitude. Thus the inclusion of only the on-shell photons should give a lower limit to the branching ratio $\Gamma(K_L^0 \to \mu^+ \mu^-)/\Gamma(K_L^0 \to \gamma \gamma)$, independent of the model of the actual $K\gamma\gamma$ coupling. This limit has been dubbed the unitarity bound, and is calculated to be $1.1 \times 10^{-5}$. If one again uses the presently accepted rate for $K_L^0 \to \gamma \gamma$, this corresponds to a branching ratio $\Gamma(K_L^0 \to \mu^+ \mu^-)/\Gamma(K_L^0 \to \text{all}) \geq 6.1 \times 10^{-9}$.

The unitarity limit calculation is very general, and as we stated earlier, rests on two assumptions and one experimentally measured quantity: The first assumption is that QED completely describes the photon-muon couplings. The second assumption is that only the two photon intermediate state contributes to the rate. The experimentally measured quantity is the rate for $K_L^0 \to \gamma \gamma$.

The second assumption above is important in that other intermediate states could destructively interfere with the two photon state, thus lowering the "unitarity limit". Pais has consequently renamed the pure two photon intermediate state limit the "primitive unitarity limit", reserving the term "unitarity limit" for the limit involving all intermediate states.
Martin, De Raphael, and Smith, have calculated possible contributions to the decay rate for \( K_L^0 \rightarrow \mu^+ \mu^- \) from 3\( \pi \) and 2\( \pi \gamma \) intermediate states. They find a lower limit for the unitarity bound of \( 4.8 \times 10^{-9} \) by assuming that these states interfere destructively. It is not clear what other states, if any, would interfere, or how one would estimate the magnitude and phase of their contributions.

We should emphasize again that a violation of the primitive unitarity limit is independent of any specific model for the \( K \rightarrow \gamma \gamma \) coupling, but would jeopardize one or both of our above assumptions, or the measured value for the \( K \rightarrow \gamma \gamma \) rate. The assumption that is clearly the easiest to discard is that the sole intermediate state is the two gamma state.

As an alternative, it is probably possible to concoct a theory in which at the weak and electromagnetic interactions conspire to eliminate or retard decays that could be induced by neutral weak currents. We already know that the charged weak currents are intimately related to the neutral electromagnetic current by CVC. Perhaps there are further such relations with the neutral weak currents.
III. DESCRIPTION OF THE APPARATUS

A. Introduction

The apparatus used for the search for the decays $K_L^0 \rightarrow \pi^+ \pi^-$ was a double armed magnetic spectrometer. (See Fig. 4.) Momentum analysis was performed with a picture frame magnet and five double gapped magnetostrictive readout wire spark chambers in each arm. Counter hodoscopes in each arm selected secondaries with high transverse momentum for the event trigger.

Two range devices each made of 17 cells of scintillation counters and steel plates provided range information to make possible the separation of muons and pions. A Freon 12 Cherenkov counter in each arm labeled electrons. Data accumulation and checking was performed with an on-line PDP-9 computer.

B. The Beam and Collimation System

A neutral secondary beam was obtained from a Cu target at the third focus of Channel I of the External Proton Beam (EPB) of the Bevatron. The production angle was centered at 3.7 degrees, and was in the vertical plane. (See Fig. 5.) The entire apparatus was therefore tilted at 3.7 degrees in the vertical plane.

Different targets were used during the data accumulation due to the requirements of other users with whom we were sharing the target. Most of the running was done with one of two copper targets: one of dimensions .090" x .200" x 5.00", and one of dimensions .200" x .300" x 4.00". The target was movable in both directions perpendicular to the EPB beam line and was periodically checked for positioning. The target was inside the Bevatron vacuum system which ended with a thin aluminum window immediately behind it. The primary proton beam intensity was monitored
by a thin foil secondary emission monitor in the EPB vacuum pipe 10 feet upstream of the target. Targeting efficiency was monitored by a three counter telescope pointing at the target at a large angle.

The target was located directly in front of the last magnet of Channel I, XM7, a 29" x 36" H magnet. This magnet served as the first sweeping magnet in our neutral beam, both steering the EPB away from the entrance to our collimation system, and sweeping charged secondary particles out of our beam.

The secondary beam was collimated and further swept by four collimators and two more sweeping magnets. The initial vertical collimation was determined by a one foot long uranium collimator (dubbed the mini-collimator) with an opening angle of 2.4 degrees. (See Fig. 5.) This was followed by an adjustable horizontal collimator with three foot uranium jaws. The jaws were remotely operated. The ends of the jaws were differentially driven so that the collimating edges of the jaws projected back to the target for all opening angles. The maximum opening angle of the collimator was 1.6 degrees; the standard data-taking position was at 1.5 degrees. Thus the maximum beam solid angle was 1.1 millisteradians; and the standard data-taking solid angle was ~ 1.0 millisteradians. The adjustable collimator gave us a great deal of flexibility, and compatibility with other users of the same target, and proved to be a powerful diagnostic tool in the early stages of the experiment.

The horizontal collimator was followed by a 13" x 36" C magnet, operated at a total line integral of 540 kgauss-inches. A re-entrant cavity in the shielding downstream of the magnet prevented deflected particles from scattering down the beam line. Downstream of the cavity
was a steel beam plug, followed by a steel rectangular "box" which served as a shadow collimator in both the horizontal and vertical planes. The last sweeping magnet was a 13" x 24" C magnet, operated at a line integral of 425-kg inches. Inside it, was a brass "box" of dimension 8" x 11" x 48" which served as the last vertical and horizontal shadow collimator. Both magnets swept in the same direction in the vertical plane preserving the left right symmetry of the apparatus.

C. Particle Fluxes in the Beam

The principal components of the beam were high and low energy neutrons, photons, and \(K_L^0\)'s. \(K_L^0\) fluxes were estimated to be \(\sim 600,000\) per pulse for \(6 \times 10^{11}\) protons on the target from \(K^+\) and \(K^-\) production curves. The data are consistent with this estimate. Neutron to \(K\) ratios were estimated as between 100:1 to 1000:1. Crude tests of interposing wood or lead in front of a thin counter held in the beam indicate that the gamma ray flux was comparable to the neutron flux.

The distribution of kaon momenta as reconstructed from the dipion decay is shown in Fig. 6. The distribution of pion momenta from the \(K_L^0 \rightarrow \pi^+ \pi^-\) decay is shown in Fig. 7. The distribution of the distance to the target of \(K_L^0 \rightarrow \pi^+ \pi^-\) decays in the vacuum decay volume is shown in Figure 8.

Runs were taken with different thickness of lead absorber from 2" to 6" inserted in the "mini-collimator", in order to attenuate the gamma-rays. We found that the lead attenuated the \(K_L^0\)'s however, and that the scattering of neutrons in the lead washed out our target as a point source, and consequently made our collimation system much less effective. This
was evidenced by the reconstruction of more neutron induced vertices from the downstream edges of the brass shadow collimator. Because of the geometry of the apparatus and because the photons and associated electrons and positrons tended to stay in the beam, the photons produced very little background in our counters or spark chambers. Our standard running condition was with no lead absorber in the beam.

Beam position and intensity were monitored for diagnostic purposes with two neutron counters hung directly in the beam in the backstop of the experiment. Each of these consisted of two inches of scintillator directly on the face of a RCA 6655 photomultiplier: The outputs were discriminated at twice the pulse height of minimizing ionizing particles.

D. The Decay Volume

The decay volume started at the end of the last collimator 300" from the target. In the early stages of the experiment (~ 13% of the data) the decay volume was filled with helium. At that time the volume started with a .005" mylar window approximately two feet inside the brass box, connected onto a thicker polyethylene boot. The boot widened out to join an aluminum sheet metal box that extended to the front spark chambers. (Figure 4.) The dimensions of the box were ~ 17 feet long, 105" wide at the widest point at the downstream end, and 14" wide at the upstream end. In the vertical direction it was 37" high at the downstream end and 18" high at the upstream.

It was immediately apparent that the neutron-helium interactions were producing large quantities of heavily ionizing background in the front spark chambers. We then replaced the helium filled decay volume with a vacuum box.
The dimensions of the vacuum box were very similar to those of the helium box except that the vertical dimension was a constant 20" over the length of the box. The sides were constructed from 2" thick aluminum plate; the top and bottom from 1" aluminum plate with 8" aluminum I beams at 20" intervals. (Aluminum was used to prevent coupling to the magnets.) The vacuum extended through a coupling upstream to the upstream edge of the brass "box" collimator. The downstream end was separated into three flanged windows by two aluminum struts, one on either side of the beam. The side windows had dimensions 18" x 40"; the central window for the neutral beam to pass through had dimensions 18" x 24".

Multiple scattering of the secondaries in the window put stringent requirements on the window thickness. The standard mylar window would have to have been ~ .030" thick, equivalent to .100 gm/cm². We used instead a 9 ounce Dacron sailcloth with .005" of mylar to render it air-tight. This corresponded to an average thickness of .043 gm/cm², less than the old window plus the helium formerly seen by the average secondary.

E. Spectrometer Magnets

The spectrometer magnets were picture frame magnets each constructed from coils from a standard LRL low-power H magnet. The yokes were flame cut from boiler plate. Each magnet had an aperture of ~ 26" x 40", and a total length of ~ 70". (See Fig. 9.) Steel pole tips sat inside flush with the coils.

The magnets were run at 1450 amps, corresponding to a line integral of 295 kgauss-inches. This line integral corresponds to a transverse momenta of 225 MeV/c, the C.M. momentum of the μ in the K→μ⁺μ⁻ decay.
The field was mapped with the LRL rapid mapper. The line integral of the field was uniform to 1% over 95% of the magnet aperture - the poor areas being close to the coils.

Magnetic shields supported from the magnet yoke reduced the stray field near the spark chambers. Additional shielding placed near the magnetostrictive wands removed polarization problems due to the magnetic field.

The magnets were supported on a three point steel stand which stood independent of the spark chamber supports. Magnet centers were 50 feet downstream from the target. The two magnets were each toed in by 6° toward the beam line to increase acceptance (see Fig. 4), and to make the average trajectory symmetric about the magnet center line.

The magnet currents were monitored with a transducer and digital volt meter (DVM) which was directly read into the DPD-9 computer after each Bevatron pulse. The computer checked after each pulse for drift. The magnet polarities were reversed after each data tape—approximately every two hours - by means of electrically driven reversing switches.

**F. The Spark Chambers**

The secondary momenta and positions were measured with 10 double-gapped magnetostrictive readout spark chambers. Two chambers of useful aperture 29" x 43" were mounted 38.5" apart directly in front of the magnet in each arm. These two chambers were toed-in toward the beam by 12° (see Fig. 4) to decrease the average deviation from normal incidence of the secondaries. Behind each magnet were three double-gapped chambers of active area 38.5" x 43", spaced 39.5" apart. The central one of these
three was rotated about an axis normal to its face by 10° to eliminate xy ambiguities caused by multiple tracks. All spark chambers were supported on a common frame. To facilitate easy and accurate replacement the chambers were mounted in a support that could slide in and out accurately on a stainless steel support bar.

There were three basic requirements the spark chambers had to meet:

1. Low mass in order to reduce multiple scattering of the secondaries.
2. Relatively high data rates, on the order of a few hundred cycles/sec.
3. Digital readout.

The low-mass requirement determined both physical and electrical characteristics of the chambers. In order to reduce the mass, the chambers did not have the conducting backing foils that are often used to reduce the inductance of the long current path due to orthogonal wires. Instead the chambers were built with two gaps, with the high voltage wires in one gap parallel to the ground wires of the neighboring gap. We thus had two coupled gaps, each with a pair of orthogonal wires. Each wire in each of the four wire planes formed half of a parallel wire transmission line with another wire in a plane of the neighboring gap. (See Fig. 10.)

The spark chamber frames were made of NEMA G-10 (fiberglass) with copper buss bars glued onto them. The spark chambers were wound with aluminum wire of .003" diameter. The wire spacing was 1mm. The gap width between wire planes was 3/8". The chambers were wound on a winding machine constructed for that purpose. The frames were wound two at a time, back to back, to minimize bowing. The wires were then glued into place and soldered to the copper busses with pure indium (no flux). The frames
were then separated, and glued together in pairs to form individual gaps. Thus each chamber was formed of two gaps of two frames each, with each frame carrying one wire plane.

The two gaps in each spark chamber were separated by two .001" sheets of mylar, so that the spark chamber was formed of two independent gas volumes. We could thus take apart the chambers and replace a defective half if the need arose.* The outer window of the chambers were of .001" mylar with 1000 A of aluminum coating. The aluminum served both to stem the diffusion of helium through the thin mylar and to allow us to apply a clearing field in the area behind the ground wires, where ions could drift out of the d.c. clearing field in the gap itself. A small positive clearing field applied to these foils greatly improved the spark chamber performance in reducing spurious sparking. The spark chamber gas was 90% neon, 10% helium: 10% of the gas was bubbled through ethyl alcohol at room temperature. The gas was recycled through a standard LRL purifying system. Destruction tests by sparking to a small area of the chamber repeatedly showed no wire failures in $10^7$ pulses on a 6" x 6" area. Two wires were broken in the approximate $5 \times 10^7$ pulses of running: the evidence is that the failures were unrelated to electrical damage.

G. Spark Chamber High Voltage

In the original design of the spark chambers the spark gap was mounted on one corner of the chamber where it drove both high voltage busses where they met at that corner. The high impedance of the buss bars, plus the inability to terminate the chamber anywhere except along the input busses, led to variations in chamber voltage across the gap.

*It did.
This was especially unpleasant in the very short space between the two high voltage planes; the very high gradients across the mylar there led to charging of the mylar and consequent sparking at high pulsing rates.

The spark chambers had been tested in a parasitic secondary beam for efficiency as a function of position prior to setting up the experiment: A small difference in efficiency was noticed but it was possible to plateau the chamber satisfactorily at a single voltage for all positions. In the high background area of our neutral beam, however, the variations in voltage led to appreciable robbing in events with multiple tracks.

This effect was corrected after taking approximately 4% of our data by adding four equally spaced tabs to each high voltage buss bar and driving each high voltage buss through 4 six foot pieces of 50 ohm cable in parallel. The cables were grounded to the ground plane of the neighboring gap. In series with each cable was a 20 ohm resistor.

The circuit diagram for the spark gap and the associated network on the spark chamber, plus a schematic of the charging system, are shown in Figure 11. The spark gap contained two 20 kV barium titanate capacitors of about 2000 picofarads (pf) each. The second of these two was separated from the first by an inductor (8 turns of hookup wire): when the two capacitors were discharged in parallel through the spark gap, the coil held off the second capacitor pulse to give us a total pulse width of approximately 100 nsec into the low impedance feed of the chamber. The spark chambers were terminated inside the spark gap box itself by a six turn coil and a 9 ohm resistance. The coil critically damped the long tail of the pulse. A MR331R diode in series with the termination permitted use of both a 50 volt DC clearing field, and a 800 volt pulsed clearing field.
The spark gap itself consisted of a Heavymet hemisphere of 1/4" radius for the high voltage electrode. The ground electrode and trigger electrode were a UL-19V Champion marine spark plug. The spark plug is cylindrically symmetric; we used the center electrode as the trigger and outer edge as the ground electrode. To maintain good electrical contact the plug threads were spring loaded with a beryllium copper washer; the gap was adjustable with the high voltage on. The plugs were cleaned about every $10^6$ pulses by wiping them across emery paper; some plugs lasted as many as $3 \times 10^7$ pulses. Rise times into a dummy load were approximately 8 nsec. Dry nitrogen was blown through the gap through a small hole at 20 psi. to clear ions. With the nitrogen the gaps performed well up to 1000 cps.

Typical operating voltages for the gaps were 13.00-14.00 kV. As the capacitance of each chamber was about 5000 pf, less than half of this appeared across the chamber gaps. The high voltage was obtained from a 4PR1000 A Eimac tube in a constant-current, voltage limited cathode follower circuit. (Fig. 11) After the spark gap had fired, the recharge system would hold the voltage to zero for approximately a millisecond to let the ions in the spark gap dissipate. The cathode follower then charged linearly to the desired (adjustable) output voltage. The charge rate was adjustable between 8mA to 800mA. The charging current also produced the pulsed clearing field on the chamber across a variable resistor. (See Fig. 11.) The pulsed and dc clearing fields entered the spark gap on a single cable which also served to return monitor signals from a voltage divider in the spark gap to further dividers in a monitor box at the experimental (people) house.
H. Spark Chamber Readout

The spark chambers had a conventional magnetostrictive readout, with standard LRL "wands" and amplifiers. Each chamber had four wire planes, and consequently had four wands, an x (horizontal) and a y (vertical) wand in each gap. Slots were milled into the frames at both ends of the wand slots so that removable fiducial tabs could be slid in and out of the slots. This feature made for easy surveying of the spark chamber positions, and was designed for easy replacement in case of failures. The fiducials were driven in series in sets of four from the spark gap through 300 Ω and were terminated in 50 Ω at the diode in the spark gap.

The wand signals were differentiated and fed into zero crossing discriminators (LRL 12X 6290 PI). The output from the discriminators then entered standard Scientific Accessories Corporation (SAC) digitizing units. The front spark chambers were allowed up to six sparks per wand; each wand fed six scalers in a Model SAC-1148 box. The rear spark chambers, shielded by the magnets, were allowed 4 scalers per wand in a Model SAC-1124 box. The SAC center finding circuit had been removed from each of the two SAC units; we found that the zero crossing discriminators did a more accurate job.

I. Helium System

Helium was used in the arms of the spectrometer to reduce multiple scattering, and in the beam line to minimize background from neutron interactions. The helium between the spark chambers was contained in 4' x 4' x 40" aluminum "boxes" with .001" aluminized (1000 A) mylar
windows. The magnets also contained helium in boxes of a similar design.

A 10 foot long aluminum box filled with helium extended between the magnets from the downstream end of the vacuum decay volume. From the end of this box a 20 foot polyethylene "wienie" bag of 3.5' diameter extended to the downstream end of the range boxes.

The aluminum boxes were all connected in parallel to manifolds of 6" aluminum pipe, which also served to support the upper end of the spark chambers. Helium was recirculated through a purification system consisting of both cold and warm molecular sieves.

J. Other Steps to Reduce Background from the Neutral Beam

On either side of the apparatus downstream of the magnets the beam was lined with 4' high steel plate: the steel was 6" thick everywhere except next to the range boxes (see section L), where space requirements limited it to 4". The 18" of steel directly behind the magnets was replaced with lead in order to reduce coupling between the magnets and the steel spark chamber support frame.

K. The Cherenkov Counters

On each side of the apparatus between the two vertical hodoscopes sat a gas Cherenkov counter. (Fig. 12) The counter was operated in a threshold mode and detected electrons (the p threshold was higher than the highest momentum produced by K decay). The counter dimensions were 48" x 48" x 100". The gas used was Freon 12 at atmospheric pressure.

The optics were quite simple - a single spherical mirror made of aluminized lucite backed with foam for rigidity was supported in the
downstream end of the counter. Five 5" RCA 4522 phototubes were placed side by side across the top of the upstream end of the counter. Each tube was equipped with a spun copper parabolic cone to increase its light gathering efficiency. Only the three central tubes were used in the data-taking. The logical or of these three tubes was constructed in Chronetics 150 series logic and was recorded with the individual tube outputs.

Each counter was tested in a parasitic beam and was better than 99% efficient over its aperture. Pion breakthrough was estimated by looking for $K_L^0 \rightarrow \pi^+ \pi^-$ events with one pion labeled as an electron and was estimated to be $< 2\%$.

L. Range Detectors

Behind the second vertical hodoscope on each side sat a range detector, consisting of one meter of graphite in two blocks, with $3/4"$ of lead on the face of each block, followed by an alternation of plastic scintillator and steel (Fig. 12). The scintillator dimensions were $3/4" \times 48" \times 48"$. The first eight scintillators were separated by a single $1" \times 48" \times 48"$ steel plate, the next four by two such plates, the following two by three plates, and the last three by four plates. The lead and graphite (plus the trigger scintillators) provided a minimum muon range of 490 MeV/c; each counter after the first corresponded approximately to a 7% increase in muon range. The average separation in range between counters corresponded to between 3 and 4 standard deviations of range straggling.

The counters were individually plateaued and checked in a parasitic beam. Efficiency was monitored in the apparatus by the PDP-9 computer.
The best check on counter efficiencies was made with identified muons from $K_{\mu 3}$ decay in the analysis on the CDC 6600 Computer. The counters were better than 99% efficient.

The first seven counters were made of Pilot F scintillator with a single Phillips 56 DVP photomultiplier mounted on one corner. The remaining ten counters were made of Nuclear Enterprise = 102 scintillator, but due to the high light absorption of that inferior scintillator, had a 56 DVP mounted on each of two corners. The two photomultiplier signals were added passively. Each phototube was mounted on a 3/4" x 2" x 6" lucite light-pipe.

The purpose of the range detectors was to discriminate between pions and muons; on the average only 11% of all pions penetrated the graphite. Figure 13a shows the range as a function of momentum for secondaries from decays consistent with being $K_{\mu 3}$ decays - i.e., events without Cherenkov counter counts, which do not satisfy the two pion hypotheses and which are kinematically compatible with being $K_{\mu 3}$. I have selected the particle with the range closest to the $\mu$ range and labeled it a muon. Fig. 13b shows in contrast the ranges of particles which accompany an electron, presumably pions from $K\pi 3$ decay. It is easily seen that muons have a well defined range while pions predominately interact in the graphite.

The number of pions that penetrate the range device is consistent with pion decay in flight.

M. Counter Hodoscopes

The first vertical hodoscope on each side, positioned directly behind the last spark chamber, consisted of twenty-eight 1.5" x .25" x 48"
vertical staves of Pilot-Y scintillator. Each counter had an RCA 6810 photomultiplier butted directly on one end. The second hodoscope on each side was positioned at the other end of the Cherenkov counter, 100" downstream of the first. It consisted of thirty 1.56" x .50" x 48" staves, also of Pilot Y, with RCA 6655 photomultipliers butted against the ends in a fashion similar to that of the first hodoscope.

Immediately behind the first vertical hodoscope on each side was a six counter horizontal hodoscope. The 6" x .250" x 48" counters were made of Pilot-Y scintillator; one RCA 6810 photomultiplier attached to an adiabatic (fluted) lightpipe sat on the outboard end.

N. Chronotrons

The last set of counters in the trigger was a pair of 4' x 4' x 3/4" single counters, dubbed chronotrons, one mounted directly behind the second vertical hodoscope on each side. The purpose of these counters was to discriminate against neutron interactions in the helium of the original design of the experiment, and to reduce accidental triggers. This was done by time of flight, i.e., the two counters were put in fast coincidence with each other before being put in the trigger. Events with one pion and one proton of energy less than approximately 2.2 BeV were completely eliminated due to the difference in arrival times over the approximately thirty feet from the average vertex in the decay volume.

The chronotron coincidence had a time resolution of approximately six nsec, although the maximum transit time across a four foot counter is greater than six nsec. Each counter had four RCA 8575 photomultipliers, two on the upper edge, two on the lower. (See Fig. 14,) The two upper
photomultipliers and independently, the two lower, were added passively. The sum of each pair was fed into a Chronetics model 154 discriminator with the output pulse length set to 3 nsec. These two output pulses, one from the upper pair and one from the lower pair, were then fed into a chronotron circuit (Fig. 14). The chronotron works as follows; the pulses enter opposite ends of a tightly wound coil of 50 ohm cable on which are strung 1N914 diodes at 6" (1/2 nsec) intervals. A d.c. bias was put on the diode such that a single pulse from one of the discriminators would not pass the diode, but the sum of two such pulses would. Thus when the two pulses cross, one sees an output pulse from one of the diodes. All of the diodes were tied by very short leads to a common point, from which the output pulse was fed into another Chronetics model 154 discriminator. If one regards the counter and the cables as being a large circle which is traversed by light or a pulse at a constant speed, then it is clear that if two pulses start in opposite directions from any point of the circle that they cross at the antipodal point at a time that is a constant of the circle. As the output pulse occurs when the two discriminator pulses cross in the cable, the delay time for this chronotron output pulse was therefore independent of where in the counter a particle had passed. Actual slewing and jitter in the chronotrons was measured to be approximately 1/2 nanosecond.

O. The Event Trigger Requirements and Logic

The event trigger was designed to provide high acceptance of two body decays relative to the much more copious three body decays of the kaon. An additional constraint was to minimize mass in the region of the
momentum measurement of the secondaries to reduce multiple scattering. The rejection of three body decays was accomplished by selecting high transverse momentum secondaries by the two vertical hodoscopes on each side of the apparatus after the magnets. The magnets were set at a field value whose line integral corresponded to the transverse momentum of the two muon decay of the neutral kaon, $P_T = 225$ MeV/c. A particle entering the magnet with this transverse momentum (defined relative to the beam direction) would exit parallel to the beam line. If one considers two body decays in the kaon center of momentum frame (c.m.) with the secondaries emerging normal to the beam direction, the transverse momentum requirement is clearly equivalent to a mass requirement. The condition that secondaries emerge from the magnets parallel to the beam line is dependent only on transverse momentum (the magnets disperse only in transverse momentum, thanks to Lorentz). The trigger requirement is therefore independent of the kaon momentum.

The requirement for high transverse momentum was enforced by putting in coincidence each of the twenty-eight counters of the front hodoscopes on a side of the spectrometer with any of six associated rear counters, thus defining six angular bands of approximately $15$ milliradians each. Thus, for example, the fourth most inboard counter in the right front hodoscope, #4, would be in coincidence separately with any of the six most inboard of the right rear hodoscope, 1-6; #5 would be in coincidence with counters 2-7 in the rear, etc. This massive set of coincidences can be expressed as a 6 x 28 network (see Fig. 15), with each intersection being the coincidence circuit for one of the front counters and one of the six associated rear counters. The six columns of such a matrix would correspond to the six angular bins.
The matrices and the rest of the trigger logic, with the exception of the above mentioned chronotron logic, were composed entirely of MECL II Flogic.\(^{(30)}\) The hodoscope counter outputs were fed into LRL Flogic dual discriminators with thresholds of 185 mv, and output pulse widths of 28 nsec. One of two independent output pulses from these discriminators was fed directly into the hodoscope coincidence matrix, the other into a 140 nsec delay line. A coincidence between one front counter and any of its associated six rear ones would result in a column output from the matrix. To suppress triggers from Ke3 decays, the first and second columns of each side (corresponding to lower transverse momenta) were put in anticoincidence with the Cherenkov counter on that side. This anticoincidence suppressed the Ke3 rate by 60%.

The column outputs of the left and right coincidence matrices were fed by cables to a separate box, the trigger logic box (TLB), in which they were required in coincidence together with the or of the front horizontal hodoscope on each side, and with the and of the two chronotron outputs.

The coincidence generated a fast strobe and an event trigger. The event trigger closed a fast off gate (FOG), set an event trigger flag for the computer, fired the spark chambers, and initialized the readout system. The strobe returned to the hodoscope matrices where it met in coincidence with the second output from each of the hodoscope discriminators emerging from the delay line. The output of this coincidence was fed to a register; one bit per counter. All of the hodoscopes, both vertical and horizontal had such a system.

The advantage of this delay line system was that a strobe and trigger
were generated by the column outputs - in a general or of the many specific configurations which could trigger the system. Each counter, however, was recorded as to whether or not it fired by being later strobed into a register. The system was also flexible for adding external components, e.g., the Cherenkov anticoincidence on the inner columns. Dead time was reduced to a minimum as only event triggers closed the fast off gate.

P. Event Readout System

The event trigger started the readout cycle. The SAC boxes for the spark chambers and the hodoscope and counter systems were read out 18 bits at a time by a standard LRL NIDBUS(29) system. The event trigger started an approximate 40 μsec. gate which clamped off all the SAC logic to gate out the spark chamber electrical noise. After this the SAC box digitized incoming sparks for ~ 200 μsec. Readout then started sequentially, first with the 12 words of the hodoscope matrices and the range counter, Cherenkov counter and horizontal hodoscope registers, then the 192-18-bit words of the spark chamber scalers. These passed through a standard LRL PDP-9 interface and into the PDP-9 memory.

Q. The PDP-9 Computer

The PDP-9 stored 16 events of 205-18-bit words each in 2 buffers. When a buffer was full it started writing the buffer onto magnetic tape on one of two Ampex TM9 tape devices. It also monitored counter and chamber performance by means of fourteen displays which could be selected by sense switches to be displayed on a Tektronix 611 display.
scope. Possible displays were of on-line events, hodoscope counting rates, the Cherenkov counter and range counter rates and accidentals, range counter efficiencies, spark chamber multiplicity and pairs, spark chamber errors, spark chamber resolutions, and counting rates at the hodoscope matrix points. The PDP-9 also checked events in the buffer for missing fiducials, spark chamber (SAC) sequence errors, and zero scalers in the SAC box. Upon detection of an error it would grunt audibly and type a message on the teletype. Total dead time due to the computer was ~ 8 msec/event. Other utility programs could be loaded from a system tape to check the SAC boxes, discriminator gates, etc.
IV. ANALYSIS

A. Introduction

The crux of the analysis lies in the fact that the apparatus has almost the same geometric acceptance for the CP violating two pion decay of the \( K^0_L \) as for the dileptonic decays \( e^+e^-, \mu^+\mu^- \). The dipion decay thus gives us an easy normalization for the dilepton decays. The number of dipion events detected is given by

\[
N_{\pi\pi} = N_K \epsilon_{\pi\pi} B_{\pi\pi}
\]

where \( N_K \) is the number of kaons decaying in our decay volume, \( \epsilon_{\pi\pi} \) is the geometric efficiency of the apparatus for the dipion decay, and \( B_{\pi\pi} \) is the branching ratio

\[
B_{\pi\pi} = \frac{\Gamma (K^0_L \rightarrow \pi\pi)}{\Gamma (K^0_L \rightarrow \text{all})}.
\]

Similarly the number of \( \mu^+\mu^- \) pairs detected is given by

\[
N_{\mu\mu} = N_K \epsilon_{\mu\mu} B_{\mu\mu}
\]

where \( N_K \) is identical to above, \( \epsilon_{\mu\mu} \) is the geometric efficiency of the apparatus for dimuon events, and \( B_{\mu\mu} \) is the branching ratio

\[
B_{\mu\mu} = \frac{\Gamma (K^0_L \rightarrow \mu\mu)}{\Gamma (K^0_L \rightarrow \text{all})}.
\]

One can therefore eliminate \( N_K \), resulting in the branching ratio for \( \mu^+\mu^- \) pairs

\[
B_{\mu\mu} = B_{\pi\pi} \frac{\epsilon_{\pi\pi}}{\epsilon_{\mu\mu}} \frac{N_{\mu\mu}}{N_{\pi\pi}}.
\]
Thus if one knows the relative efficiency of the apparatus for \( \pi \) pairs and \( \mu \) pairs from \( K \) decay, all one has to do is count the separate kinds of events to establish the branching ratio for the dilepton decays.

Because both the dipion events and the dilepton events are two body decays, they satisfy the same constraints. The dipion events thus also provide calibration standards for the different constraints — i.e., one can judge whether or not cuts are reasonable by how the sample of two pion events acts under the cuts. The geometric differences between the two kinds of events are very small, and as long as one does not make a discrimination between muons and pions one would expect the two classes of events to behave similarly.

Because both classes of events are two body decays, one has the following constraints:

1. **Invariant mass**: the invariant mass of the two secondaries (properly identified) should be the kaon mass. The two pion events thus conveniently calibrate our mass scale. The invariant mass distribution of all events with non-electron secondaries with invariant masses greater than 476 MeV is shown in Fig. 16 (with a loose cut on point 3 below).

2. **Vertex**: the two secondary tracks should meet at a point in the decay volume. The distribution of the amount the two tracks miss is shown for the two pion decays in Fig. 17.

3. **Apparent kaon origin**: in the two body kaon decays the components of the secondary momenta transverse to the kaon direction should be equal by momentum conservation. This is equivalent to the statement that having measured the secondary momenta, the reconstructed kaon momentum should
pass through the target origin. This is a two-dimensional constraint.

The two requirements that both tracks meet in a vertex and that the vertical components of the transverse momentum be equal are equivalent to a coplanarity constraint among the two secondaries and the primary kaon. The distance between the kaon origin and the target center for all events is shown in Fig. 18. The corresponding density of events (after dividing out the area factor $2\pi r^2$) is shown in Fig. 19. The effect of the target cut on the invariant mass distribution for the dipion events can be seen by comparing Fig. 16a to Fig. 16b. The effect is to reduce the many-body (three particles or greater) background without changing the two-body peak.

The following points are also important to the analysis and should be made clear:

1. As long as no distinction is made between pions and muons, any programming errors should affect dipion events and dilepton events equally: i.e., the final answer should be invariant to such errors if they exist.

2. Because most of the momentum of the secondaries is due to the laboratory momentum of the kaon, the differences in transverse momentum do not greatly change the momentum distributions of the secondaries for different types of decays (assuming they are all almost transverse).

3. There is a good source of muons with which to calibrate the range devices from $K^\mu_3$ decay: by point 2 above these cover approximately the same momentum spectrum as one would expect from $K^0_L \rightarrow \mu^+ \mu^-$ decay.

4. There is a good source of electrons with which to check the Cherenkov counters from $K^e_3$ decay.
5. By reducing the magnet line integral to a value corresponding to the transverse momentum of the $K \to \pi^+\pi^-$ decay one can measure to good accuracy the relative $\mu^+\mu^-$ and $\pi^+\pi^-$ geometric acceptances. The only kinematic difference between a $K \to \pi^+\pi^-$ decay with the magnets at the corresponding transverse momentum and a $K \to \mu^+\mu^-$ decay with the magnets set at the higher corresponding momentum is a small difference in opening angle. The effect of this difference is identical to moving forward or backward slightly in the decay volume, and is easily shown to be very small. Thus the ratio $\varepsilon_{\pi\pi}/\varepsilon_{\mu\mu}$ in the expression for the dilepton branching ratio above can be experimentally measured with our apparatus.

B. The Analysis Computer Programs

Due to the large amount of data, 45 million total events and approximately 1 million good two body events, we have not done a constrained fit to the two-body hypothesis. The procedure was rather to consider an event as a two body candidate if it could possibly satisfy a two body hypothesis. This was done by reconstructing the vector momenta and positions, assuming Cherenkov counter tagged secondaries were electrons, and asking if under the assumptions that other secondaries were $\pi$'s or $\mu$'s one could satisfy very crude two body criteria. These criteria were wide cuts in the invariant mass, kaon origin in the target plane, vertex miss (i.e., how far apart the two secondary tracks were at point of closest approach), and decay volume. At this stage there was no criterion on range to label muons. All non Cherenkov-tagged particles were first assumed to be muons — if under such an assumption the kinematics was approximately
two body (the actual cuts are listed below), the event was labeled a di-
muon candidate.

This philosophy was workable because of the extremely good momentum
and spatial resolution of the apparatus, the reassuring presence of the
$K^0_L \to \pi^+\pi^-$ decays, and because there is very little background to the dilepton
decays. The major background for $\mu^+\mu^-$ decays is due to $K\mu3$ decays in which
a $\pi$ decays in flight in the magnet, leading to a mismeasured momentum.
As there are relatively few such events that satisfy even crude two–body
criteria, we could afford to be generous in our initial selection of
dilepton candidates.

C. First Pass Analysis

The following pages are a detailed description of the analysis
programs; all but the most masochistic readers are implored to skip to
page 44.

The data were analyzed in a multiple pass fashion. On the first
pass, after being reconstructed and identified, the two–body data were
condensed onto a separate file in their original data format. At the same
time all events that were reconstructed, whether or not they satisfied
two–body criteria, were written out in reconstructed format (e.g., masses,
momenta, angles, etc.) onto a separate file. The reasons for this proce-
dure were:

1. One should be able to easily re–analyze any part of the two–body
data in case of errors or changes in programming, surveying, etc.
2. The two-body events were less than 10% of the total data and could therefore be reconstructed twice at a negligible increase in computer time.

3. It would be prohibitively expensive, and not necessarily useful, to reconstruct all of the three body data twice.

The data tapes from the PDP-9 were taken to the CDC 6600 where they were written in full on the IBM 1360 Photo Digital Storage System (Chipstore). The tape was then read from the Chipstore and processed event by event, all events which reconstructed being written out on a file which was then copied back onto the Chipstore.

The first step was to unpack the data from the 18-bit word PDP-9 format into a 60 bit CDC 6600 word format. The spark chamber coordinates were then transformed into laboratory coordinates and the number of sparks per chamber counted. If any pair of gaps failed to have any sparks the event was immediately rejected at this stage. The hodoscopes were interrogated as to which counters were lit - if more than 4 were lit in any given bank, or if any were missing (an infrequent error) the event was rejected.

The lit counters in the front and rear vertical hodoscopes which satisfied the trigger criterion defined a band in the rear spark chambers in which the track must lie. The X' and Y' sparks in the rotated chamber were unrotated in all combinations - those X (horizontal) combinations that fell in the horizontal band were flagged and the associated (vertical) Y value calculated. The program then selected sparks in the end gaps of
the rear gaps and cycled through the sparks in the middle gap looking for those that fell within .4" of the line between the end sparks. When any set of 3 sparks or more was found which satisfied the requirement that at least one spark be in each of the 3 pairs of gaps, the sparks were fitted to a straight line. If the fit was good the sparks were flagged. When all sparks in the outer gaps had been cycled through, the program repeated the process with the unflagged sparks in the inner gaps of the end chambers. The program stored only 3 tracks per side — if more than these were found, it replaced the last one found with any later ones which had better $\chi^2$ fits.

For each track found in the X (horizontal) plane there is an associated Y (vertical) value in the rotated chambers. That vertical point and the horizontal hodoscopes then define bands in the other two rear spark chambers in which the other Y sparks should lie. These were found by permuting and constraining the sparks to a straight line. The best fit found was used. If two possible tracks were found in Y for a single X track, and 3 X tracks had not yet been found, the program duplicated the X track associated with the sparks in the rotated chamber and associated it with both Y tracks.

The rear track was then extended into the magnet and its intercept with the magnet midplane calculated. The requirement that the particle had come from the decay volume determined limits on the area of the front chambers that had to be searched by a search procedure much the same as for the rear chambers, except with the requirement that the front track came within a specified distance of the rear track at the magnet midplane.
The Y front track was searched for in a narrow band defined by shooting the rear track through the magnet with a first order correction due to edge focusing.

Again if more than one X or Y front track were found and the track limit of 3 had not yet been reached the rear tracks were updated accordingly.

1. **Momentum Analysis**

The momenta of the secondaries was calculated by using an effective length approximation for the magnets, i.e.

\[ P = \frac{P_T (\text{MeV}/c)}{\sin \beta_{\text{out}} - \sin \beta_{\text{in}}} \]

where \( \beta_{\text{out}} \) and \( \beta_{\text{in}} \) are the angles of the secondaries in the horizontal plane going into and out of the magnet, respectively. \( P_T \) for the magnets was tabulated in a 12 x 5 XY grid; the correct value was interpolated from the position of traversal of the magnet midplane. The grid was for only one half of the magnet – the magnet was assumed symmetric around its horizontal midplane. Such an effective length parameterization leads to a mass resolution of \( \sim 1.7 \text{ MeV} \) (see Fig. 16): it however is poor for any trajectory that has a steep dip angle in the magnet. (Most of our events are approximately flat.) Better resolution can be obtained (see Fig. 20) by integrating the secondary trajectory through the measured field; this was impossible to do for all events because of the expense in computer time.

2. **Cuts**

a. **Vertex Positions**
The tracks on each side were required to come within a certain distance of each other — this distance was .5" at the downstream end of the decay value and increased linearly to .8" at the upstream end. As the plane of the secondaries in most events was flat in the laboratory frame this distance has been approximated by the vertical distance between the two tracks when they crossed in the plan view.

b. Decay Volume

The vertex position was known in the upstream-downstream direction to approximately a few inches. Cuts were made on the vertex position to require that the vertex be downstream of the collimator end and that it be in the neutral beam as defined by the collimators.

c. Target Requirements

A two-body decay should allow the vector kaon momentum to be reconstructed back to its origin in the target. Thus all two body candidates (both n+π- and p+π-) were required to have the reconstructed kaon origin to be within a radius of 3" of the target center. This cut is sublimely generous as can be seen in Fig. 19.

d. Invariant Mass

The last requirement was that the invariant mass of the two secondaries, considered first as two leptons, then as two pions should fall within the range 476 MeV to 520 MeV. The Cherenkov counter identification of electrons was assumed correct, but range was not used in distinguishing pions from muons. If there were multiple tracks the requirement was that if any of the combinations satisfied the above cuts the event was considered two body.
The two body events were stored in a separate buffer from the other reconstructed events. When either buffer was full it was written out into an output file — the two body events in packed PDP-9 data format, the three body events in a reconstructed format with kinematic and spatial parameters. The two different kinds of records were distinguished by their different lengths.

D. Second Pass Analysis

The second pass program was similar to the first except that it did not have to deal with the three body events, and consequently was slightly simpler. The track recognizing apparatus was identical, as was the kinematic reconstruction and cuts. This time, however, the data was written out, 11 events to the 506 word record, in a reconstructed format that no longer contained spark positions, but contained target, vertex, momentum, mass, range, and Cherenkov counter information, and positions and angles in the magnets.

E. Further Analysis

A third pass was then made with a fast histogramming program. Events were histogrammed according to their invariant mass identification by the second pass program. Dilepton candidate events were printed out for personal attention. To facilitate further analysis of the dilepton events, all of the data were searched and all dilepton candidates with invariant mass greater than 480 MeV and with muon ranges (where applicable) within two counters of 3 standard deviations below the average muon range
were written out onto a separate file. The range of a particle was defined as the furthest counter of the furthest pair of consecutive struck counters. The reader should note that this is the first time the muon range was used in identification.

Most of these events are either K\(_{\mu 3}\) events in which the pion has decayed in flight inside the magnet, or events with multiple tracks in the front spark chambers which were mismatched with rear tracks by the reconstruction program. In either of these cases the effective length approximation will give an incorrect momentum.

It was possible to discriminate against such cases by integrating (in the jargon of the trade, orbitting or ray-tracing) the secondary momenta through the magnetic field using the measured three-dimensional field map. For each secondary particle the track emerging from the magnet was linearly extrapolated back to the magnet midplane. The difference between this intersection and the intersection point measured with the spark chamber information was computed. This difference, for each secondary in every event, was compared to those for identified muons from K\(_{\mu 3}\) decays which passed through the same area of the magnet with similar momentum. Any event in which a secondary fell outside of 4\(\text{rms}\) deviations from the center of the muon distribution was discarded.

The momenta of the secondaries in the remaining events were then corrected by an amount necessary to make the orbit emerge from the magnet at the same angle as the spark chamber track. To check the procedure of correcting the momentum a small sample of the K\(_L^0\) → π\(^+\)π\(^-\)
events were also orbitted. The invariant mass distribution of these events is shown in Fig. 20; this is to be compared to the $\pi^+\pi^-$ events in Fig. 16. It is clear that such a procedure improves the mass resolution from 1.7 MeV to ~1.1 MeV.

The invariant mass of a number of the dilepton events changed by more than our mass resolution. The events that changed, however, have secondaries that enter the magnet with large dip angles. These are cases for which the effective length approximation is not good for the magnetic field (though we should note that a 2 MeV shift – a rather large shift – is still less than 0.5%). It is reasonable that the dilepton candidates should have a larger spread of tilt angles than the dipion events, for the tilt lowers the effective transverse momentum. As the dipion events are already at a lower (206 MeV) transverse momentum, it is reasonable that we accept fewer tilted dipion events.
A. Introduction

For reasons which will become clear below, I have treated the two samples of data from the two different running conditions of helium or vacuum in the decay volume as separate experiments. The justification for this is that the dilepton data from the helium clearly has a background due to the interaction of neutrons in the helium. The actual analysis programs for the two sets of data were identical except for occasional minor changes necessitated by changes in the Berkeley computer center system.

B. The Dipion Events

The total number of kaons examined by our apparatus was derived from the decay $K_L^0 \rightarrow \pi^+\pi^-$. The invariant mass distribution of all events without electrons, with an apparent kaon origin within 3" of the target, and with a dipion invariant mass within 22 MeV of the kaon mass is shown in Fig. 16 for the vacuum data, and in Fig. 21 for the helium data. The number of dipion events has been estimated by subtracting a linear background from the data. The total number of dipion events in the vacuum and the helium data after the subtraction is 903,000 and 138,000, respectively.

The background under these dipion events is due almost entirely to the decay in flight of pions from the decay $K_L^0 \rightarrow \pi^+\pi^-$. We have estimated the number of such dipion events rejected in either our angular requirement in the trigger or in the background subtraction to be 28%. The effective number of dipion events is therefore 1,254,000 and 192,000, respectively.

The relative geometrical efficiencies of the apparatus for the decays $K_L^0 \rightarrow \mu^+\mu^-$ and $K_L^0 \rightarrow \pi^+\pi^-$ were measured by comparing the ratio (number of
dipion events)/(incident proton on the target) at the two magnet settings corresponding to the transverse momentum of the appropriate decay. This was done four times during the data taking. Of these the first time was with the magnets slightly miscalibrated so that they were actually running at a field 1.3% lower than the correct settings. Such an error would have the tendency to make the relative efficiency $\varepsilon_{\pi\pi}/\varepsilon_{\mu\mu}$ closer to one, as the dipion events are not bent inward as close to the edge of our angular acceptance as they should be. The value for the ratio $\varepsilon_{\pi\pi}/\varepsilon_{\mu\mu}$ measured for these data was $69 \pm 5\%$.

The ratio $\varepsilon_{\pi\pi}/\varepsilon_{\mu\mu}$ was measured the second time with the magnets at the correct settings. The measurement was with helium in the decay volume and a very high ($10^{12}$ protons) beam on our target. The result was $\varepsilon_{\pi\pi}/\varepsilon_{\mu\mu} = 78 \pm 4\%$. I have discarded this measurement entirely because the many extra tracks from the high beam intensity and the helium affects the data at the lower magnet setting more heavily than at the higher setting. This is due to the difference in trigger rate at the two settings.

The ratio $\varepsilon_{\pi\pi}/\varepsilon_{\mu\mu}$ was measured twice with the vacuum decay volume and with the magnets set at the correct settings. Each result gives $62 \pm 3\%$. This result is entirely consistent with the $69\%$ measured at the lower magnet setting, and is the value used in the analysis below.

C. The Decay $K_L^0 \rightarrow e^+e^-$

No dielectron events with secondary momenta less than 2.1 BeV/c, target positions less than 1.6", and invariant mass within 20 MeV of the kaon mass were detected in the vacuum data. In the helium data three events which would have passed the above cuts except they had invariant masses of 490 MeV, 504 MeV, and 508 MeV were detected. Since all three of these
events are more than 6 MeV (approximately 5 sigma) from the kaon mass, I consider them background due to neutron interactions in the helium. The combined helium and vacuum data determine a 90% confidence level limit on the branching ratio for $K_L^0 \rightarrow e^+e^-$ of

$$\frac{\Gamma(K_L^0 \rightarrow e^+e^-)}{\Gamma(K_L^0 \rightarrow \text{all})} \leq 1.6 \times 10^{-9}$$

D. The Decays $K_L^0 \rightarrow e^+\mu^-$

No events with invariant masses within 7 MeV of the kaon mass, with target positions less than 1.6", and with ranges within two counters and three standard deviations of the muon range were detected in the vacuum data. Two such events were seen in the helium data. One candidate at mass 491 MeV was missing the first fourteen counts of its sixteen counter range (range was defined as the last two consecutive counters lit; no other requirement was imposed). This event clearly had an accidental range count. The other candidate, at mass 496 MeV had a target radius of 1.3" (approximately 1% probable) and had a 1.125 BeV electron which penetrated five counters into the range device. Examination of electron ranges for electrons from $K_{e3}^-$ decays at this momentum showed that only 2% of the electrons penetrate this far or beyond. These two independent criteria disqualify this event as a candidate. We thus conclude that there are no $K_L^0 \rightarrow e^+e^-$ events in our data, implying a 90% confidence level limit of

$$\frac{\Gamma(K_L^0 \rightarrow \mu^+\mu^-)}{\Gamma(K_L^0 \rightarrow \text{all})} \leq 1.6 \times 10^{-9}$$
E. The Decay $K_L^0 \rightarrow \mu^+ \mu^-$

The invariant mass spectrum of all events without an electron on either side of the apparatus, target position less than 1.6", and with a dimuon invariant mass greater than 476 MeV is shown in Fig. 22 and Fig. 23 for evacuated and helium-filled decay volumes, respectively. In these figures both secondaries have been assumed to be muons; no range information has been used. The sharply falling spectrum on the low mass side of the plots is due to the $K_L^0$ decay. The striking background across the whole mass region in Fig. 23 is presumably due to the interaction of neutrons in the beam with helium. The much smaller flat background in the vacuum plot (Fig. 22) is due to the looseness of the reconstruction program cuts: multiple track events, if they can be reconstructed as two body events, are always so reconstructed. While this philosophy tends to miss no candidates, it sometimes accumulates spurious identifications which can be discarded by more sophisticated fitting.

The invariant mass distributions corresponding to those of Figures 22 and 23 are shown in Figures 24 and 25 after a cut in muon range. The range cut was determined as follows: for each of the 17 counters the momentum corresponding to a muon stopping 3σ (straggling) short of its mean range was computed. Thus a secondary with a measured momentum greater than that momentum would be expected to reach that counter. Any secondary stopping two counters short of that counter or beyond would be considered a muon. This cut thus corresponds approximately to between a
7 and 10 standard deviation cut in range. (One has to be cautious on quoting standard deviations for ranges in the range device as the measurement of the range is quantized by the fact that there are only seventeen counters, each corresponding to a given range measurement.)

The momentum of each secondary from all the events shown in Figures 24 and 25 was then integrated through the magnetic field, using a momentum derived from the effective length approximation as an initial guess. The dimuon invariant mass plot corresponding to these events is shown in Figures 26 and 27, corresponding to vacuum and helium data, respectively. After the range cut only nine events remain on the mass plot for the vacuum data above 490 MeV; a great many more (nineteen) remain on the plot for the helium data. I make the following observations about these plots and their difference: (1) The ratio of the low mass end of the spectrum for the helium and vacuum data is approximately the same as the ratio of dipion events, as one would expect from \( \pi \mu \) decay with the pion decaying into a muon in flight. (2) Some of the events at high dimuon mass have shifted in mass radically, many moving completely off of the plot. This is entirely reasonable for events which result from bad mismatches between front and rear tracks in the magnet; the effective length approximation has no requirement of continuity for a track, it merely takes the difference of the sines of the incoming and outgoing angles. The integrating of orbits, however, requires some continuity both spatially and in the angles, and consequentially drastically changes the momenta of tracks to achieve this spatial continuity. Because matching errors by the program will tend to give almost arbitrary invariant masses, the program errors are most visible in the region of invariant mass that is not already heavily
The ratios of events remaining in the helium data to those remaining in the vacuum data is much higher than the ratio of amounts of data. The mass distribution of events in the helium is approximately flat, like the background seen before the range cut (Fig. 23). Because the data taken with helium in the decay volume clearly has a background problem for the $K_L^0 \rightarrow \mu^+\mu^-$ decay that is not present in the vacuum data, I feel justified in treating the two running conditions as physically different, separate experiments.

F. The Result for the Evacuated Decay Volume

The major sources of background for the $K_L^0 \rightarrow \mu^+\mu^-$ decay are either $K\mu\nu$ decays with a pion decaying in flight in the magnet or misidentifications of tracks by the analysis programs. One can discriminate against these backgrounds by comparing the trajectory of the secondaries in the dilepton events with those of bona fide muons from $K\mu\nu$ decay. For each trajectory from the dimuon candidates, muons with topologically similar trajectories were selected from the $K\mu\nu$ data sample. These muons were integrated through the magnetic fields. The figure of merit used for the continuity of a trajectory was the difference between the intersection of the track in the rear spark chambers and the magnet midplane and the intersection of the linear extrapolation of the integrated trajectory and the magnet midplane. The distribution of this figure of merit for the muons from $K\mu\nu$ decay was formed and an rms deviation was calculated. The cross-hatched events in Fig. 26 are those that survive a 4σ cut on this figure of merit for continuity of trajectories. All events with mass above 490 MeV except one are eliminated. The one event remaining is missing the first nine counters out of an eleven counter range and is
clearly an accidental range count. Figure 28 shows the vacuum data dimuon invariant mass plot between 476 MeV and 512 MeV with the same cuts as the unhatched events in Fig. 26 except that a 3σ range cut has been applied. There are no events remaining above 490 MeV.

Table 3 gives the masses, target positions, ranges and figures of merit of the continuity of the trajectories through the magnets for events with invariant mass greater than 490 MeV and range within 2 counters and 3σ of the expected range. The standard deviations quoted for the trajectories are derived from comparison with topologically similar orbits of muons from K → πμν. No estimates were made on the left trajectory if the right trajectory failed a four-sigma cut.

It is clear that every event has at least two four-sigma problems. Because the large number of standard deviations tell us little about the probability for distributions which are surely not Gaussian, I have not made any estimate on the probabilities of these events: A generous upper limit would be 1%. We consequently can quote a 90% level on the rate $K_L^0 \rightarrow \mu^+\mu^-$ from the vacuum data alone:

$$\frac{\Gamma(K_L^0 \rightarrow \mu^+\mu^-)}{\Gamma(K_L^0 \rightarrow \text{all})} \leq 1.82 \times 10^{-9}$$

G. The Result for the Helium-Filled Decay Volume

There are nineteen events with mass above 490 MeV/c² which have muon ranges within 2 counters and 3σ of the correct range and which have a target radius of less than 1.6 inches. (See Fig. 25.) When this is compared to the nine events in the six times larger data sample from the
Table III

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<th>Event</th>
<th>Mass (MeV)</th>
<th>Target radius (inches)</th>
<th>Range Probability left</th>
<th>Range Probability right</th>
<th>Orbit Continuity left</th>
<th>Orbit Continuity right</th>
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<td>8σ</td>
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<td>503.7</td>
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<td>5.3σ</td>
<td>-</td>
<td>12.5σ</td>
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<tr>
<td>3</td>
<td>504.2</td>
<td>1.54</td>
<td>5.9σ</td>
<td>8 counters missing</td>
<td>1.2σ</td>
<td>1.2σ</td>
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<tr>
<td>4</td>
<td>490.7</td>
<td>1.57</td>
<td>2.0σ</td>
<td>1.0σ</td>
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<td>4.8σ</td>
<td>9.3σ</td>
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<td>8.6σ</td>
<td>5.0σ</td>
<td>-</td>
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<td>7</td>
<td>493.3</td>
<td>1.53</td>
<td>5.6σ</td>
<td>3.5σ</td>
<td>-</td>
<td>8.0σ</td>
</tr>
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<td>1.8σ</td>
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<td>-</td>
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</table>
vacuum data; it is clear that there is a background problem in the helium data.

Figure 29 shows the $\mu^+\mu^-$ invariant mass plot after integrating the trajectories through the magnetic field (but without any cut on continuity) and after requiring a 3$\sigma$ cut on the range. Seven events remain with invariant masses above 490 MeV. This plot should be compared to the analogous plot for the vacuum data, Fig. 28. A four rms deviation cut on trajectory continuity with a two counter and 3$\sigma$ cut on range leaves eight events as shown by the cross-hatched events in Fig. 27. Either of the above two more-or-less independent cuts eliminates all possible candidates in the vacuum data. Table IV contains the masses, target positions, ranges and a crude $\chi^2$ probability estimate for each of the events with mass $> 490$ MeV that passes the $4\sigma$ orbit cut and the 2 counter + 3$\sigma$ range cut.

This crude estimated $\chi^2$ has been made by treating the invariant mass, target distributions, ranges and orbit continuity as independent variables. The problems with this are: (1) The range is quantized so that one can have a range that seems short by about 1.5$\sigma$ that has hit the correct counter. This has been dealt with by comparing the momentum of the secondary with the distribution of momenta for muons from $K\mu3$ decays which have a range given by the same counter of the range device. A standard deviation was estimated by calculating $1/3$ width at $1/3$ height for the momentum distribution of the muons from $K\mu3$. This estimate is crude but not unreasonable. (2) The six variables are not independent. Cross terms should be small, however. The correct (and much more complicated) way of handling this is a fully constrained fit to the $\mu\mu$ hypothesis.
<table>
<thead>
<tr>
<th>Event</th>
<th>Mass (MeV)</th>
<th>Target radius (inches)</th>
<th>Range left right</th>
<th>Orbit continuity left right</th>
<th>$\chi^2$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>496.85</td>
<td>0.71</td>
<td>1.2σ 2.4σ</td>
<td>1.0σ 2.6σ</td>
<td>18.5</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>2</td>
<td>498.7</td>
<td>0.94</td>
<td>1.85σ 0.05σ</td>
<td>0.04σ 1.6σ</td>
<td>12.3</td>
<td>4.5%</td>
</tr>
<tr>
<td>3</td>
<td>493.4</td>
<td>1.1</td>
<td>5.0σ .96σ</td>
<td>1.35σ .8σ</td>
<td>51.8</td>
<td>$&lt;10^{-4}$</td>
</tr>
<tr>
<td>4</td>
<td>500.9</td>
<td>1.49</td>
<td>1.2σ 4.4σ</td>
<td>0.2σ 4.6σ</td>
<td>42.9</td>
<td>$&lt;10^{-4}$</td>
</tr>
<tr>
<td>5</td>
<td>493.2</td>
<td>0.67</td>
<td>7.5σ 7σ</td>
<td>1σ 1.1σ</td>
<td>115.2</td>
<td>$&lt;10^{-4}$</td>
</tr>
<tr>
<td>6</td>
<td>494.4</td>
<td>1.0</td>
<td>1.3σ 0.3σ</td>
<td>1.7σ 1.2σ</td>
<td>21.8</td>
<td>1%</td>
</tr>
<tr>
<td>7</td>
<td>496.8</td>
<td>0.4</td>
<td>0.28σ 1.0σ</td>
<td>1.07σ .6σ</td>
<td>4.25</td>
<td>65%</td>
</tr>
<tr>
<td>8</td>
<td>501.0</td>
<td>0.81</td>
<td>0.8σ 5.9σ</td>
<td>0.8σ 8.5σ</td>
<td>49.9</td>
<td>$&lt;10^{-4}$</td>
</tr>
</tbody>
</table>
One can see from Table IV that one event satisfies our criteria for a $K_L^0 \rightarrow \mu^+\mu^-$ event with high probability. One other event has approximately 5% probability, and one has .5% probability. The other events have a negligible probability.

We are therefore left with two alternative explanations of the good event: (1) It is a $K_L^0 \rightarrow \mu^+\mu^-$ decay. (2) It is a background event that has happened to fall squarely in the middle of our criteria for the $K_L^0 \rightarrow \mu^+\mu^-$ decay. To estimate the relative probability of alternatives (1) and (2), one can look at the $\chi^2$ distribution of the events in Table IV: if the reader considers it approximately flat he should choose alternative (2); if he would be happy putting a cut at, e.g., $\chi^2 = 10$, he chooses alternative (1). If he chooses alternative (1), the sum of the vacuum and helium data determines the branching ratio to be

$$\frac{\Gamma(K_L^0 \rightarrow \mu^+\mu^-)}{\Gamma(K_L^0 \rightarrow \text{all})} = 6.8 \times 10^{-10}$$

based on one event.

If he chooses alternative (2), it is perhaps most reasonable to ignore the helium data and quote only the vacuum data limit of

$$\frac{\Gamma(K_L^0 \rightarrow \mu^+\mu^-)}{\Gamma(K_L^0 \rightarrow \text{all})} \leq 1.82 \times 10^{-9} \ (90\% \ C.L.)$$

A more sophisticated full constrained fit will do a much more accurate job of computing $\chi^2$'s, and hopefully will decide between the two alternatives. Until then there is no clear evidence as to whether or not our one event is a true $K_L^0 \rightarrow \mu^+\mu^-$ decay.
VI. SELF-CONSISTENCY CHECKS ON
THE APPARATUS AND ANALYSIS PROCEDURES

It is philosophically impossible to prove that one could observe an event which has never before been detected. One can, however, show that it would take unknown forces or very high order correlated mistakes to prevent one from observing such events. This experiment happily lends itself to many such self-consistency checks. In too many experiments one has to rely not only on the veracity but on the competence of the experimenter. I would like to show that in this experiment that almost any error in either the equipment or the analysis would show up in the data as described below. (This implies that the reader must rely on my veracity but not on my competence.) One way of describing this situation is to say that I challenge the reader to imagine that he is faced with the task of fooling us into missing the decay $K_L^O \rightarrow \pi^+\pi^-$ while retaining all of the rest of our data. He is allowed to modify our equipment or programs in any way, at any time. A clever reader will find this task possible, but very complicated.

I list below the self-consistency checks in the form of a catechism (with truly deserved apologies to James Joyce and Leopold Bloom):

(Question 1): How do you know the apparatus detects events with the geometry of the $K\mu\mu$ decay?

Answer: The decay $K_L^O \rightarrow \pi\pi$ is geometrically almost identical to the decay $K_L^O \rightarrow \mu\mu$. The only differences are small differences in opening angle and transverse momentum. The different opening angle is easily approximated by a similar decay of a kaon of slightly different momentum.
(approximately 10%) at a slightly different distance from the target. If one looks at the distribution in these variables (Figs. 6 and 8) it is clear that the geometrical acceptance is not a strong function of these variables and is therefore essentially unchanged by these shifts. The difference in transverse momentum is easily checked by lowering the magnet settings to correspond to the transverse momentum of the \( K_{\pi 3} \) decay. As this is how we measured the normalization, and because the relative acceptances at the two magnet settings agree with our Monte Carlo calculations, this cannot be a problem.

(Question 2): How do you know that you are not rejecting muons in the trigger logic?

Answer: 1) There is no range or muon requirement in the trigger logic. Muons are thus indistinguishable to the logic from pions. 2) As all pions, muons, and electrons are relativistic, there can be no dependence on the timing of the counters.

(Question 3): How do you know that your muon identification criteria (e.g., within two counters and 3 rms deviations) is reasonable?

Answer: The most stringent test of this (although it is a stringent test of everything else we have done) is that the relative branching ratio for the \( K_{\mu 3} \) and \( K_{e 3} \) decays, after unfolding the detection efficiency, is in very good agreement with the correct value. Any mistake in muon identification would change this ratio drastically. 2) Fig. 13a shows the ranges of particles consistent with being muons in that they are accompanied by a stopping particle that does not count in the Cherenkov counter (i.e., a pion) and are consistent with \( K_{\mu 3} \) decay. The arrows
indicate the expected muon range. In contrast the ranges of particles consistent with being pions in that they are accompanied by an electron and are consistent with Ke3 decay are shown in Fig. 13b. The two counter plus three sigma cut on the muon range looks very generous. The approximately 10% of the pions that penetrate the range box are consistent with pion decay in flight.

(question 4): From the analysis of Ke3 decays it is clear that you are sensitive to single muon events. How do you know that your software does not reject dimuon events?

Answer: 1) One thousand fake $\mu\mu$ events were constructed by taking real $\pi\pi$ events and adding bits to the raw data buffer to fake a muon range of within ± 3 counters of the correct range. All of these events were then analyzed with the actual reconstruction programs exactly as for any of the data. The dimuon invariant mass was made to be close to the kaon mass by changing the magnet normalization – an input parameter always read from cards in the reconstruction programs. All 1000 of these events survived reconstruction and were labeled dimuon events by the programs. 2) The better dimuon events in the helium data were again reconstructed by the actual copies of the programs used predominantly on the vacuum data (the programs were identical for the two data sets but had to be periodically reloaded onto the data cell to accommodate computer system changes – this check was to detect cards reversed by the card reader, parity errors, computer hardware errors, etc.). All events reconstructed the second time.
(Question 5): Although there is nothing in the apparatus that depends on the muon ranges, suppose the apparatus in some way rejects events with two particles each of which counts in the range devices.

Answer: A secondary pion from the $K_L^0 \rightarrow \pi^+ \pi^-$ decay satisfies our muon criterion a certain fraction $F$ of the time (it is momentum dependent). To detect such a bias one therefore has to ask if the fraction of events in which both pions satisfy the muon criterion is approximately $F^2$. To within the uncertainty caused by differences in the momentum distribution this condition is satisfied. This estimate would have to be wrong by a factor of 10 to account for our conflict with the electromagnetically predicted rate for the decay $K_L^0 \rightarrow \mu^+ \mu^-$. 
VII. CONCLUSIONS

A. The Electromagnetic Interaction

It is clear that our measurement of the branching ratio is in conflict with the calculations of Jackson and Quigg, Sehgal, etc., for the electromagnetically induced decay. More important is that it is almost an order of magnitude smaller than the lower "unitarity" bound. To repeat part of the theoretical introduction, let me list again the assumptions that go into this lower bound:

1. The branching ratio \( \frac{\Gamma(K_L^0 \rightarrow \gamma\gamma)}{\Gamma(K_L^0 \rightarrow \text{all})} \) is \( 5.2 \pm 0.5 \times 10^{-4} \). \(^{31}\)
2. QED describes the muon-photon couplings completely.
3. No other intermediate state except the two photon state contributes to the decay amplitude.

Because of the 10% error on the two gamma branching ratio it is very hard to change that number enough to account for our discrepancy. As for assumption number two, one does not tamper with a theory that works well everywhere because of a discrepancy in a calculation that contains many unknown factors. Assumption number three is clearly open to question: the calculation of Martin, et al., however, including the \( \pi\pi\gamma \) and \( \pi\pi\pi \) intermediate states, is still in conflict with our number.

B. Neutral Currents?

Because of the above conflict with the expected rates from first order weak, second order (in alpha) electromagnetic decay, it is not clear that any of the predications from first order weak neutral current or second order charged current calculations are valid. I will discuss them
as if they are: one should keep in mind, however, that "something funny is going on". The expression derived in Appendix A for the ratio of the neutral to charged coupling constants, assuming a V–A interaction was

\[ \left| \frac{G_N}{G} \right| = \sqrt{\frac{1/49 \cdot \Gamma(K^0_L \rightarrow \mu^+ \mu^-)}{\Gamma(K^+ \rightarrow \mu \nu)}} \]

If we consider only the vacuum data upper limit (90% C.L.) to be valid, we derive a value for the ratio \( G_N/G \) of \( |G_N/G| \leq 6.25 \times 10^{-5} \) (90% C.L.). If instead we consider the one event in the helium to be a valid event, and consider that it was created by a weak neutral current, then we derive

\[ \left| \frac{G_N}{G} \right| \approx 4 \times 10^{-5} \]

We see that in either case the neutral weak current is coupled much less strongly to the strangeness changing hadronic current than its charged counterpart.

C. Second Order Weak Interactions

The 90% C.L. limit on the ratio \( \frac{\Gamma(K^0_L \rightarrow \mu^+ \mu^-)}{\Gamma(K^0_L \rightarrow \text{all})} \) derived from the vacuum data alone gives us a limit on the weak interaction cutoff parameter of \( \Lambda \leq 19 \text{ BeV} \).

in the IVB picture of Joffe and Shabalin, or Mohapatra. This is much smaller than the approximate 300 BeV estimate made considering \( \nu-e \) scattering and unitarity. The 4–Fermion picture gives an even smaller limit \( \Lambda \leq 7 \text{ BeV} \).
This limit, like the limit derived from the $K_L - K_S$ mass difference, is already low enough so that experiments at NAL should see observable effects. If we use the one event branching ratio derived from considering the one event in the helium data to be real and to be the result of a second-order weak decay, then we find a value for the weak interaction cutoff parameter of

$$\Lambda \approx 15 \text{ BeV}$$

Joffe and Shabalin's estimate of the IVB mass, for the cases of the vacuum limit alone, or the helium branching ratio, are respectively 1.6 BeV and 1.25 BeV. In both cases the mass is on the order of, or less than, the current experimental limits derived from neutrino experiments. This would probably mean that Joffe and Shabalin's conjecture that the form factor of the $W$ is determined by its electromagnetic interactions is not correct. The conjecture of Mohapatra, et al. that instead $M_W \sim \Lambda$ is not jeopardized by our results.
ACKNOWLEDGEMENTS

This experiment could not have been completed without the hard work of many people. I would especially like to thank William A. Wenzel, Leroy T. Kerth, and Alan R. Clark for the patience and wisdom they have shown toward me over the last four years. Special thanks are also due to Tom Elioff and R. Clive Field for the large contributions they have also made to this experiment.

Thanks are due to the other graduate student who received his degree from this experiment, Rolland P. Johnson, who contributed much hard work and many critical decisions to the success of the experiment.

The vast bulk of the data-taking of our year-long run was done by the Lofgren Physics Group technicians: William Baldock, Fred Goozen, John Gustinis, Elaine Lucas, Timothy Nuzum, Gerald Schnurmacher, Thomas Weber, and John Wilson. Finally, none of this would have been completed (and even less of it would be legible) without the Lofgren Group secretaries, Anna Mae Morrish and Peggy Fox.

Many thanks are due to the very competent Bevatron crews and staff.

This work was done under the auspices of the Atomic Energy Commission and of my wife, Priscilla.
Calculation of the rate $K_L^0 \to \mu^+ \mu^-$

Assume a V-A matrix element of the form

$$M = \frac{G}{\sqrt{2}} f_K^* P_K \bar{u}_1 \gamma^\alpha (1 + \gamma_5) u_2$$

where $G_N$ is a "neutral weak coupling constant"

- $f_K$ is $K_L^0$ form factor
- $P_K$ is the kaon 4-momentum
- $P_1, P_2$ are the 4-momenta of the two muons
- $m$ is the muon mass, $M_K$ the kaon mass

$$M = \frac{G_N}{\sqrt{2}} f_K^* \bar{u}_1 (\not{P}_1 + \not{P}_2) (1 + \gamma_5) u_2$$

$$|M|^2 = 2 G_N^2 f_K^2 m^2 \text{trace} \left( (\not{P}_1 + m)(\not{P}_2 + m) \right)$$

$$= G_N^2 f_K^2 m^2 (P_1 \cdot P_2 + m^2)$$

The decay rate $W$ is given by

$$W = \frac{(2\pi)^4}{(2\pi)^6} \frac{1}{(2m_K^2)} \int \frac{|M|^2}{2E_1 2E_2} \ 2^3P_1 \ 2^3P_2 \ 2^4(P_K - P_1 - P_2)$$

$$W = \frac{G_N^2 f_K^2 m^2}{(2\pi)^2 m_K} \int \frac{(P_1 \cdot P_2 + m^2)}{E_1 E_2} \ 2^3(P_1 + P_2) \ 2^3(m_K - E_1 - E_2)$$

Let $A = \frac{G_N^2 f_K^2 m^2}{(2\pi)^2 m_K}$
Integrate over $d^3p_1$

$$W = A \int \frac{P_1 \cdot P_2 + m^2}{E_1 E_2} \, d^3p_2 \, \delta(m_K - E_1 - E_2)$$

where now $P_1 = -P_2 = P$, $E_1 = \sqrt{P^2 + m^2} = E_2$

$$W = 2A \int d^3p_2 \, \delta(m_K - E_1 - E_2)$$

$$= 8\pi A \int \frac{p^2 dp}{P^2} \, \delta(m_K - 2\sqrt{P^2 + m^2})$$

$$= 4\pi A \, P \sqrt{P^2 + m^2} \left| P = \frac{m_K^2 - 4m^2}{2} \right.$$}

$$W = \frac{G^2 f^2_{K} m^2 m_K}{4\pi} \left( 1 - \frac{4m^2}{m_K^2} \right)^{1/2}$$

Calculation of the rate $K^+ \rightarrow \mu^+ \nu$

Again the V-A matrix element is of the form

$$M = \frac{G}{\sqrt{2}} \, f_{K}^{\mathcal{P} \mu} \bar{\mu} Y_{\alpha} (1 + Y_5) \, u_{\nu}$$

$$= \frac{G}{\sqrt{2}} \, f_{K}^{\mathcal{P} \mu} (\mathcal{P}_{\mu} + \mathcal{P}_{\nu})(1 + Y_5) \, u_{\nu}$$

$$= \frac{Gf_{K}}{\sqrt{2}} \, m \, \bar{\mu} \, (1 + Y_5) \, u_{\nu}$$

-- continued
\[ |M|^2 = \frac{G^2 r^2}{2} \, \text{m}^2 \text{ trace } \left[ (\not{p}_\mu + m)(1 + r_5) \, \not{p}_\nu (1-r_5) \right] \]

\[ = \frac{G^2 r^2}{2} \, \text{m}^2 \text{ trace } \left[ (\not{p}_\mu + m)(2 + r_5) \, \not{p}_\nu \right] \]

\[ = 4 G^2 r^2 m^2 \, P_\mu \cdot P_\nu \]

Again the rate \( W \) is given by

\[ W = \frac{(2\pi)^4}{(2\pi)^6 \, 2m_K} \int \frac{d^3 p_\mu \, d^3 p_\nu}{2E_\mu \, 2E_\nu} |M|^2 \, \delta(m_K - E_\nu - E_\mu) \, \delta^3(\not{p}_\mu + \not{p}_\nu) \]

\[ = \frac{G^2 r^2 m^2}{8\pi^2 m_K} \int \frac{d^3 p_\mu \, d^3 p_\nu}{E_\mu \, E_\nu} \, P_\mu \cdot P_\nu \, \delta(m_K - E_\nu - E_\mu) \, \delta^3(\not{p}_\mu + \not{p}_\nu) \]

Let \( B = \frac{G^2 r^2 m^2}{8\pi^2 m_K} \), Let \( P \equiv P_\nu \)

\[ = B \int \frac{d^3 p}{\sqrt{p^2 + m^2}} \left( \sqrt{p^2 + m^2} \, P + P^2 \right) \delta(m_K - \sqrt{p^2 + m^2} - P) \]

\[ = B \int \frac{p^2 \, dp \, d\Omega}{\sqrt{p^2 + m^2}} \, \sqrt{p^2 + m^2} \, P \delta(m_K - \sqrt{p^2 + m^2} - P) \]

\[ = 4\pi B \, \sqrt{p^2 + P} \cdot \frac{m_K^2 - m^2}{2m_K} \]

\[ = 4\pi B \left( \frac{m_K^2 - m^2}{2m_K} \right)^2 \]

-- continued
We can now use the calculated rates for \( K_L^0 \rightarrow \mu^+ \mu^- \) and \( K^+ \rightarrow \mu^+ \nu \) and our experimental limit on the rate for \( K_L^0 \rightarrow \mu^+ \mu^- \) to estimate limit the ratio \( (G_N/G) \), i.e., to calculate by how much the neutral couplings are suppressed relative to the charged couplings.

The experimental limit on the rates is

\[
\frac{W(K_L^0 \rightarrow \mu^+ \mu^-)}{W(K^+ \rightarrow \mu^+ \nu)} = \frac{\text{0.07/sec}}{5.18 \times 10^7/\text{sec}} = 1.35 \times 10^{-9}
\]

The calculated ratio from above is

\[
\frac{W(K_L^0 \rightarrow \mu^+ \mu^-)}{W(K^+ \rightarrow \mu^+ \nu)} = \frac{G_N^2 f_K^2 m^2_{K^0} m_{K^+}}{4\pi} \left( 1 - \frac{4m^2_{\mu \mu}}{m_{K^0}^2} \right)^{1/2}
\]

\[
= \frac{G^2 f_K^2 m^2_{K^+} m_{K^0}}{8\pi} \left( 1 - \frac{m^2_{\mu \mu}}{m_{K^+}^2} \right)^2
\]
For simplicity equating $f_K^+$ and $f_K^0$, and $m_K^+$ and $m_K^0$, we have

\[
R \equiv \frac{W_{K_L}^0 \rightarrow \mu^+ \nu}{W_{K^+} \rightarrow \mu^+ \nu} = \left[ \frac{G_N^2}{G^2} \right] \left[ \frac{\left( 1 - \frac{4m^2}{m_K^2} \right)^{1/2}}{\left( 1 - \frac{m^2}{m_K^2} \right)^2} \right]
\]

Substituting $m_K \approx 495$, $m = 105$

\[
R = 0.49 \left[ \frac{G_N^2}{G^2} \right]
\]

Now experimentally $R \leq 1.9 \times 10^{-9}$

\[
\Rightarrow \left| \frac{G_N^2}{G^2} \right| \leq \left( \frac{1.9}{0.49} \right) \times 10^{-9} = 3.90 \times 10^{-9}
\]

or \[
\left| \frac{G_N}{G} \right| \leq 6.25 \times 10^{-5}
\]

We can thus see that the coupling of neutral leptonic currents to neutral hadronic currents is suppressed by over 4 orders of magnitude.
FOOTNOTES AND REFERENCES

1. The interaction must be $S$-wave if it occurs at a point; the unitarity limit is thus $\sigma_T \leq \frac{2\pi}{8}$ as $\lambda = 0$.

2. I am indebted to R. H. Hildebrand and R. F. Stiening for some of these references.


15. The Fierz relation is

\[ [\bar{u}_a \gamma^\alpha (1 + \gamma_5) u_b] [\bar{u}_c \gamma^\alpha (1 + \gamma_5) u_d] = - [\bar{u}_a \gamma^\alpha (1 + \gamma_5) u_d] [\bar{u}_c \gamma^\alpha (1 + \gamma_5) u_b] \]

Thus if we start with the matrix element from \( (\bar{e}_e v_e) \) \( (e_e) \) currents in \( \nu_e \to \nu_e \) scattering we can easily transform to \( (\bar{e}_e v_e) \) \( (e_e v_e) \) currents with only a change of phase that is indetectable if there are no interfering terms. We should note here that if \( \nu\nu \) currents coupled as strongly to \( (\bar{e}_e e) \) currents as \( (\bar{e}_v v) \) currents couple to \( (\bar{e}_v v) \) currents the interaction \( \nu_e \to \nu_e \) should have zero amplitude in the simple 4-Fermion V-A theory because of this cancellation due to the Fierz relation. This clearly would prevent the violation of the unitarity limit predicted at c.m. energies \( \approx 300 \) BeV.

16. See Appendix A for the calculation. Physically this is easily understood because the average helicity \( 2 < -V/C > \) of the leptons is larger for electrons than for muons, and consequently has less overlap with the spin zero kaon.


FIGURE CAPTIONS

Fig. 1. (a) The diagram involving a neutral IVB (W) that would induce first order neutral currents.
        (b) The second order weak diagram involving only charged W's.
        (c) The second order 4-Fermion interaction.

Fig. 2. The diagram for the $K_L^0 \rightarrow \mu^+\mu^-$ decay through the two photon intermediate state.

Fig. 3. The branching ratio $\frac{K_L^0 \rightarrow \mu^+\mu^-}{K_L^0 \rightarrow \text{all}}$, assuming the two photon intermediate state. The rising curves result from assuming either that one vector meson (KVγ) or two vector mesons (KVV) dominate the kaon form factor. The line labeled unitarity limit corresponds to assuming the photons are on-shell in the two photon intermediate state.

Fig. 4. The plan view of the double-arm spectrometer.

Fig. 5. The elevation view of the collimation system and target for the neutral beam.

Fig. 6. The distribution of kaon momenta reconstructed from the two pion decay.

Fig. 7. The distribution of secondary pion momentum from $K_L^0 \rightarrow \pi^+\pi^-$ decays as detected by our apparatus for decays in the vacuum decay volume.

Fig. 8. The distribution of distance from the target for $K_L^0 \rightarrow \pi^+\pi^-$ decays in the vacuum decay volume.

Fig. 9. The picture frame magnet used in one arm of the double-arm spectrometer.

Fig. 10. Schematic of the double-gapped magnetostrictive readout spark chambers.
Fig. 11. Schematic of the spark gap high voltage recharging system, and clearing field supply.

Fig. 12. The Freon 12 threshold Cherenkov detector used for labeling electrons, and the range detector used to separate muons and pions.

Fig. 13. (a) The range as a function of momentum for the secondary with range closest to the expected muon range from decays without electrons.

(b) The range as a function of momentum for secondaries which accompany an electron, i.e., presumably pions.

Fig. 14. Schematic of the chromotron and its circuitry.

Fig. 15. The coincidence matrix for the front and rear hodoscopes of one arm of the spectrometer by which one can select high transverse momentum events.

Fig. 16a, b The dipion invariant mass for all events in the vacuum decay volume without electrons and with dipion invariant masses between 476 MeV and 516 MeV.

The two graphs 16a and 16b require that the kaon appear to originate within a distance from the target of 3" and 1.6" respectively. Momenta are calculated with an effective length approximation; no range information was used.

Fig. 17. The miss in the vertical plane at the point of closest approach of the two tracks in the arms of the spectrometer for all events without electrons in the vacuum data.

Fig. 18. The distance between the apparent kaon origin and the target center for all events without electrons in the vacuum data.
Fig. 19. The density of events in the target plane corresponding to the distribution of events in Fig. 18. The long tail is due to three body events.

Fig. 20. The dipion invariant mass spectrum for a small sample of events for which the momentum has been calculated by integrating the trajectory through the measured magnetic field (ray tracing). The target cut is 1.6".

Fig. 21. The dipion invariant mass for all events in the helium decay volume without electrons and with dipion invariant masses between 476 MeV and 516 MeV.

Fig. 22. The dimuon invariant mass spectrum between 476 and 512 MeV, for all events without an electron, for the data taken with the vacuum decay volume. No range information has been used - all particles are assumed to be muons. The two distributions 22a and 22b correspond to target cuts of 3" and 1.6" respectively. All momenta are calculated with the effective length approximation.

Fig. 23. The dimuon invariant mass spectrum between 476 and 512 MeV, for all events without an electron, for the data taken with the helium decay volume. No range information has been used - all particles are assumed to be muons. The two distributions 22a and 22b correspond to target cuts of 3" and 1.6" respectively. All momenta are calculated with the effective length approximation. The striking background at high invariant masses is due to neutron - helium interactions.
The dimuon invariant mass spectrum between 476 and 512 MeV, for all events without an electron, for the data taken with the vacuum decay volume. The muon range is required to be within two counters plus 3σ of the expected muon range. The target cut is 1.6". All momenta are calculated with the effective length approximation.

The dimuon invariant mass spectrum between 476 and 512 MeV, for all events without an electron, for the data taken with the helium decay volume. The muon range is required to be within two counters plus 3σ of the expected muon range. The target cut is 1.6". All momenta are calculated with the effective length approximation.

The dimuon invariant mass spectrum between 476 and 512 MeV, for all events without an electron, and with apparent kaon origin within 1.6" of the target center from the data taken in vacuum. The muon range is required to be less than 2 counters plus 3 standard deviations short of the expected muon range. All momenta are calculated by integrating the trajectories through the magnetic field. The cross-hatched events are those that survive a 4σ cut on trajectory continuity.

The dimuon invariant mass spectrum between 476 and 512 MeV, for all events without an electron, and with apparent kaon origin within 1.6" of the target center from the data taken in helium. The muon range is required to be less than 2 counters plus 3 standard deviations short of the expected muon range. All momenta are calculated by integrating the trajectories through the magnetic field. The cross-hatched
events are those that survive a 4σ cut on trajectory continuity.

Fig. 28. The dimuon invariant mass spectrum between 476 and 512 MeV for all events without an electron and with apparent kaon origin within 1.6" of the target center from the data taken in vacuum. The muon range is required to be less than 3 standard deviations short of the expected muon range. All momenta are calculated by integrating the trajectories through the magnetic field.

Fig. 29. The dimuon invariant mass spectrum between 476 and 512 MeV for all events without an electron and with apparent kaon origin within 1.6" of the target center from the data taken in helium. The muon range is required to be less than 3 standard deviations short of the expected muon range. All momenta are calculated by integrating the trajectories through the magnetic field.
Fig. 1
ELECTROMAGNETICALLY INDUCED $K_L^0 \rightarrow \mu^- \mu^+$

1.) VECTOR DOMINANCE ($\rho$) (JACKSON & QUIGG)

\[
\frac{\Gamma (K_L^0 \rightarrow \mu^- \mu^+)}{\Gamma (K_L^0 \rightarrow \gamma \gamma)} \approx 1.3 - 1.7 \times 10^{-5}
\]

\[
\frac{\Gamma (K_L^0 \rightarrow \mu^- \mu^+)}{\Gamma (K_L^0 \rightarrow \gamma \gamma)} \cdot \frac{\Gamma (K_L^0 \rightarrow \mu^- \mu^+)}{\Gamma (K_L^0 \rightarrow \gamma \gamma)} = 6.7 - 8.8 \times 10^{-9}
\]

2.) "PRIMITIVE" UNITARITY LIMIT

\[
\frac{\Gamma (K_L^0 \rightarrow \mu^- \mu^+)}{\Gamma (K_L^0 \rightarrow \gamma \gamma)} \geq 1.17 \times 10^{-5}
\]

\[
\frac{\Gamma (K_L^0 \rightarrow \mu^- \mu^+)}{\Gamma (K_L^0 \rightarrow \gamma \gamma)} \geq 6.1 \times 10^{-9}
\]

3.) "LESS PRIMITIVE" UNITARITY LIMIT (MARTIN et al.)

INCLUDE $3\pi$ AND $2\pi\gamma$ INTERMEDIATE STATES

\[
\frac{\Gamma (K_L^0 \rightarrow \mu^- \mu^+)}{\Gamma (K_L^0 \rightarrow \gamma \gamma)} \geq 4.8 \times 10^{-9}
\]

Fig. 2
Fig. 6

KAON MOMENTUM (MeV)

EVENTS / 150 MeV/c

900 1800 2700 3600

900 1800 2700 3600

30,000

25,000

20,000

15,000

10,000

5,000

0
Fig. 7
DECAY DISTANCE FROM TARGET (inches)

XBL 711-84

Fig. 8
Fig. 10
Fig. 11
Fig. 13
Fig. 14
Fig. 15
Fig. 16
VERTEX MISS AT POINT OF CLOSEST APPROACH (inches)

Fig. 17
DISTANCE FROM TARGET CENTER TO THE
K^L APPARENT ORIGIN IN THE TARGET PLANE

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Fig. 18
DISTANCE FROM TARGET CENTER TO THE
\( K^0 \) APPARENT ORIGIN IN THE TARGET PLANE

Fig. 19
Fig. 20

$\pi^-\pi^+$ INVARIANT MASS (MeV)
WITH FIELD INTEGRATION

XBL 711-94
Fig. 22
Fig. 23
Fig. 25
Fig. 26

\( \mu^- \mu^+ \) IN Variant Mass (MeV)

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Fig. 26
Fig. 27

\( \mu^- \mu^+ \) INVARIANT MASS (MeV)

XBL 712-158
Fig. 28
Fig. 29
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