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The Depletion of Burnable Poison in Endurance Calculations

***R. C. Dahlberg
F. D. Judge***

March 1, 1960

Operated for the
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UC-34 Physics and Mathematics
(TID-4500, 15th Edition)

THE DEPLETION OF BURNABLE POISON
IN ENDURANCE CALCULATIONS

R. C. Dahlberg
F. D. Judge

March 1, 1960

Richard C. Dahlberg
Authorized Classifier

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Date

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KNOLLS ATOMIC POWER LABORATORY
Schenectady, New York
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ABSTRACT

Three methods of relating the burnable poison concentration in a reactor to the fuel concentration during the life of the core are presented. These methods correspond to the following ways of using burnable poison: (1) in discrete lumps, (2) in a homogeneous mixture with the fuel, and (3) a combination of (1) and (2). Nuclear data relevant to the $U^{235}\text{-}B^{10}$ system are presented.

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THE DEPLETION OF BURNABLE POISON IN ENDURANCE CALCULATIONS

R. C. Dahlberg, F. D. Judge

INTRODUCTION

In most long-endurance cores, burnable poison is a necessary ingredient. As the poison is depleted, reactivity is added to the core; this approximately compensates for the reactivity loss resulting from fuel depletion. Obviously, accurate endurance studies require an accurate treatment of the poison depletion. In particular, the ratio of the poison load to the fuel load must be determined correctly as a function of the core life. Since only the fuel depletion rate is determined by the power level, the problem is to relate the poison depletion to the fuel depletion.

The poison load is related to the fuel load for the following three cases of current interest:

1. Discrete Poison Depletion: The poison is in the core in the form of discrete lumps and its self-shielding factor is independent of the fuel load.
2. Homogeneous Poison Depletion: The poison in the core is intimately mixed with the fuel in the fuel elements and, hence, must always have the same self-shielding factor as the fuel.
3. A combination of the above two methods.

In case 1, either two-group or one-group depletion is examined; that is, the epithermal poison self-shielding factor may be different from the thermal factor (two-group) or it may be the same as the thermal factor (one-group). In cases 2 and 3, only one-group depletion is examined.

NOTATION

f	Self-shielding factor.
$\bar{f}$	Average self-shielding factor.
K	Self-shielding parameter (constant).
γ	Self-shielding parameter (constant).
$C(E)$	Self-shielding parameter, a function of the neutron energy.
t	Time.
$P(t)$	Fraction of poison remaining at time t .
$P_d(t)$	Fraction of discrete poison remaining at time t .
$P_h(t)$	Fraction of homogeneous poison remaining at time t .
$U(t)$	Fraction of uranium (fuel) remaining at time t .
ϕ_i	Total neutron flux in energy group i .
σ	Average microscopic absorption cross section in energy group i .

Subscripts

1	Epithermal energy group.
2	Thermal energy group.
u	Uranium (fuel).
p	Poison.
P_h	Homogeneous poison
P_d	Discrete poison.

DEPLETION SCHEMES

In developing the equations that relate the fuel loads to the poison loads, the following conditions were assumed:

1. Fuel and burnable poison are only consumed during reactor operation; they are not produced.
2. There is only one species of fuel and one species of poison.
3. The power is constant for a given interval of time. This assumption does not limit the usefulness of the schemes, since any type of reactor operation can be expressed in terms of equivalent full power hours of operation.

It should be noted that the self-shielding factor is defined here as the ratio of the average flux in the absorber to the average flux in the moderator.

The self-shielding factor of a discrete poison lump or plate is assumed to be of the form:

$$f_{pd}(E, t) = \frac{K_{pd}}{1 + C_{pd}(E)P_d(t)} \quad (1a)$$

With both poison and fuel in the plate,

$$f_{ph}(E, t) = f_u(E, t) = \frac{K_u}{1 + C_u(E)U(t) + C_{ph}(E)P_h(t)} \quad (1b)$$

and for a plate containing no poison,

$$f_u(E, t) = \frac{K_u}{1 + C_u(E)U(t)} \quad (1c)$$

Initially, $U(0) = P(0) = 1.0$. C_u and C_p are functions of the energy and, in general, proportional to the optical thickness of the fuel and poison plates, respectively. For B^{10} , a $1/v$ absorber, C_p is inversely proportional to E ; for U^{235} , C_u is a more complicated function of the energy.

When the self-shielding factor is averaged over a specified energy interval, one can express the factors in the following form:

$$\text{discrete poison,} \quad \overline{f_{pd}}(t) = \frac{K_p}{1 + \gamma_{pd}P_d(t)} \quad (2a)$$

poison and fuel in the plate,

$$\overline{f_{pf}}(t) = \overline{f_u}(t) = \frac{K_u}{1 + \gamma_u U(t) + \gamma_{ph} P_h(t)} \quad (2b)$$

and no poison in a fuel plate,

$$\bar{f}_u(t) = \frac{K_u}{1 + \gamma_u U(t)} \quad (2c)$$

The error involved in assuming that the self-shielding has the time dependence shown in Equations (2) is difficult to determine. Presumably, a reasonably accurate calculation of $\bar{f}_p$ and $\bar{f}_u$ can be made for the undepleted core. Also, at the end of life, in the heavily depleted parts of the core, the asymptotic or position self-shielding factors K_u and K_p may be determined. However, the shape of the curve between these two end points may be a complicated function of the detailed core geometry and composition and is only approximated by Equations (2).

Discrete Poison Depletion - Two-Group*

The time dependence of the fuel and poison concentrations is given by the following expressions:

$$\frac{dP}{dt} = -\bar{f}_{p1} P(t) \phi_1 \sigma_{p1} - \bar{f}_{p2} P(t) \phi_2 \sigma_{p2} \quad (3a)$$

$$\frac{dU}{dt} = -\bar{f}_{u1} U(t) \phi_1 \sigma_{u1} - \bar{f}_{u2} U(t) \phi_2 \sigma_{u2} \quad (3b)$$

It is convenient to introduce two quantities, w and α , which are defined as follows:

$$w \equiv \phi_1 / \phi_2 \quad (4a)$$

$$\alpha \equiv \sigma_1 / \sigma_2 \quad (4b)$$

The product $w\alpha$ is the ratio of epithermal to thermal absorptions; $1 + w\alpha$ is then the ratio of total absorption to thermal absorptions. With the use of these quantities, Equations (3) become:

$$\frac{dP}{dt} = -\phi_2 P \sigma_{p2} (w\alpha \bar{f}_{p1} + \bar{f}_{p2}) \quad (5a)$$

$$\frac{dU}{dt} = -\phi_2 U \sigma_{u2} (w\alpha \bar{f}_{u1} + \bar{f}_{u2}) \quad (5b)$$

By dividing (5a) by (5b) and using the self-shielding factors as defined in Equation (2a), the following expression is readily obtainable:

*The subscript d in P_d is omitted in the discussion, since the reference is clearly to discrete poison.

$$\frac{dU}{U \left(w\alpha_u \frac{K_{u1}}{1 + \gamma_{u1}U} + \frac{K_{u2}}{1 + \gamma_{u2}U} \right)} = \frac{\sigma_{u2}}{\sigma_{p2}} \frac{dP}{P \left(w\alpha_p \frac{K_{p1}}{1 + \gamma_{p1}P} + \frac{K_{p2}}{1 + \gamma_{p2}P} \right)} \quad (6)$$

The integration of Equation (6) between times t and t' , corresponding to U and U' and P and P' , is straightforward, yielding:

$$\begin{aligned} & \left[\ln \frac{U' (w\alpha_u \gamma_{u2} K_{u1} + \gamma_{u1} K_{u2}) + (w\alpha_u K_{u1} + K_{u2})}{U (w\alpha_u \gamma_{u2} K_{u1} + \gamma_{u1} K_{u2}) + (w\alpha_u K_{u1} + K_{u2})} \right] \times \left[\frac{\gamma_{u1} + \gamma_{u2}}{w\alpha_u \gamma_{u2} K_{u1} + \gamma_{u1} K_{u2}} \right. \\ & \quad \left. - \frac{1}{w\alpha_u K_{u1} + K_{u2}} - \frac{(w\alpha_u K_{u1} + K_{u2}) \gamma_{u1} \gamma_{u2}}{(w\alpha_u \gamma_{u2} K_{u1} + \gamma_{u1} K_{u2})^2} \right] + \frac{\gamma_{u1} \gamma_{u2} (U' - U)}{(w\alpha_u \gamma_{u2} K_{u1} + \gamma_{u1} K_{u2})} \\ & \quad + \frac{1}{w\alpha_u K_{u1} + K_{u2}} \ln \frac{U'}{U} \\ & = \frac{\sigma_{u2}}{\sigma_{p2}} \times \left\{ \left[\ln \frac{P' (w\alpha_p \gamma_{p2} K_{p1} + \gamma_{p1} K_{p2}) + (w\alpha_p K_{p1} + K_{p2})}{P (w\alpha_p \gamma_{p2} K_{p1} + \gamma_{p1} K_{p2}) + (w\alpha_p K_{p1} + K_{p2})} \right] \right. \\ & \quad \times \left[\frac{\gamma_{p1} + \gamma_{p2}}{w\alpha_p \gamma_{p2} K_{p1} + \gamma_{p1} K_{p2}} - \frac{1}{w\alpha_p K_{p1} + K_{p2}} - \frac{(w\alpha_p K_{p1} + K_{p2}) \gamma_{p1} \gamma_{p2}}{(w\alpha_p \gamma_{p2} K_{p1} + \gamma_{p1} K_{p2})^2} \right] \\ & \quad \left. + \frac{\gamma_{p1} \gamma_{p2} (P' - P)}{(w\alpha_p \gamma_{p2} K_{p1} + \gamma_{p1} K_{p2})} + \frac{1}{w\alpha_p K_{p1} + K_{p2}} \ln \frac{P'}{P} \right\} \quad (7) \end{aligned}$$

It should be noted that in Equation (7) the right-hand side is identical to the left-hand side, except for the factor σ_{u2}/σ_{p2} and the obvious fact that the left-hand side refers to the fuel and the right-hand side refers to the poison.

In practice, U and P are usually known, since these are the relative loads at the beginning of a time step. Each can be taken as equal to 1.0. U' is determined by the energy requirements of the core and P' is then given by the above transcendental relationship which may be solved by standard techniques.

Discrete Poison Depletion - One-Group

Often, it is convenient to use only one-group depletion; that is, assign the same self-shielding factor to all energy groups. This is true, for example, if the self-shielding factor for the discrete poison is

obtained by a solution experiment where the amount of homogeneous poison equivalent to the discrete poison is determined outside of any group or energy structure.

Equation (7) is reduced to the following simple expression when the thermal and epithermal self-shielding factors for the fuel and the poison are made equal:

$$\ln \frac{U'}{U} + \gamma (U' - U) = \frac{w\alpha_u + 1}{w\alpha_p + 1} \frac{K_u}{K_p} \frac{\sigma_{u2}}{\sigma_{p2}} \left\{ \ln \frac{P'}{P} + \gamma_p (P' - P) \right\} . \quad (8)$$

This can be further simplified by noting that

$$\frac{w\alpha_u + 1}{w\alpha_p + 1} \cdot \frac{\sigma_{u2}}{\sigma_{p2}} = \frac{\sigma_u}{\sigma_p} , \quad (9)$$

where the cross sections on the right-hand side are appropriate averages over both energy groups.

Homogeneous Poison Depletion - One-Group*

When the poison is mixed homogeneously with the fuel in the fuel element, the self-shielding factor of Equation (2b) is applicable. The equations governing the fuel and poison depletion are:

$$\frac{dP}{dt} = -\phi P(t) \sigma_p \bar{f}_p \quad (10a)$$

$$\frac{dU}{dt} = -\phi U(t) \sigma_u \bar{f}_u , \quad (10b)$$

and where the cross sections are averages over all energy and ϕ is, as before, the appropriate total flux. Proceeding exactly as in the case of discrete poison, one obtains:

$$\frac{P'}{P} = \left(\frac{U'}{U} \right)^{\sigma_p / \sigma_u} . \quad (11)$$

Combination of Discrete and Homogeneous Poison Depletion - One-Group

We start, as before, by writing the equations governing the depletion:

$$\frac{dU}{dt} = \phi U(t) \bar{f}_u \sigma_u , \quad (12a)$$

where $\bar{f}_u = \bar{f}_p$

*The subscript h is omitted in P_h since the reference is clearly to homogeneous poison.

$$\frac{dP}{dt} = \frac{dP_d}{dt} + \frac{dP_h}{dt} = -\phi P_h \bar{f}_{ph} \sigma_{pd} - \phi P_d \bar{f}_{pd} \sigma_p \quad (12b)$$

or, rearranging the latter,

$$\frac{dP_h}{dt} + \phi P_h \bar{f}_{ph} \sigma_p = - \left[\frac{dP_d}{dt} + \phi P_d \bar{f}_{pd} \sigma_p \right] \quad (12c)$$

If P_h and P_d are assumed to be independent, both sides must equal a constant. This constant must be zero since at high burnups, dP_h/dt and $P_h \rightarrow 0$. Hence,

$$\frac{dP_h}{dt} = -\phi P_h \bar{f}_{ph} \sigma_p \quad (13)$$

$$\frac{dP_d}{dt} = -\phi P_d \bar{f}_{pd} \sigma_p \quad (14)$$

Since $\bar{f}_u = \bar{f}_{ph}$, the relationship between the homogeneous poison load and the fuel load is given by Equation (11); that is,

$$\frac{P'_h}{P_h} = \left(\frac{U'}{U} \right)^{\sigma_p/\sigma_u} \quad (15)$$

In Equation (15) P_h is displayed as a function of U ; hence, the self-shielding factor of the fuel and homogeneous poison can be written as:

$$\bar{f}_u = \bar{f}_{ph} = \frac{K_u}{1 + \gamma_u U + \gamma_{ph} P_{h_0} (U/U_0)^{\sigma_p/\sigma_u}} \quad (16)$$

where P_{h_0} and U_0 are the initial values of P_h and U . The self-shielding factor is now a function of the fuel load only. Equations (12a), (14), and (16) can now be combined to yield a relationship between the discrete poison and the fuel load:

$$\frac{P'_d}{P_d} + \gamma_{pd} (P'_d - P_d) = \frac{K_{pd}}{K_u} \frac{\sigma_p}{\sigma_u} \left[\ln \frac{U'}{U} + \gamma_u (U' - U) + \gamma_{ph} \frac{\sigma_u}{\sigma_p} \left(U'^{\sigma_p/\sigma_u} - U^{\sigma_p/\sigma_u} \right) \right] \quad (17)$$

Here, as before, the integration has been between t and t' (i.e., U and U' , P and P') and use has been made of the fact that $P_{h_0} = U_0 = 1$.

Equations (15) and (17) are the basic equations in the third depletion scheme.

APPLICATION OF THE DEPLETION SCHEMES

All of the depletion schemes can be applied to parts of the core as well as to the whole core. For example, in calculational models, the fuel region can be divided into several core subregions. In each of the subregions, any one of the depletion schemes can be used. If a discrete poison element is included explicitly in the model, this region may be "doped" with a small amount of fuel and treated as a regular fuel region.

DATA FOR THE U^{235} - B^{10} SYSTEM*

The U^{235} - B^{10} system represents an important type of core in current reactor technology. Many cores of interest fulfill all of the assumptions listed under Depletion Schemes. Nuclear data that are required for the application of the formulas given for U^{235} and B^{10} under Depletion Schemes are presented below.

In all cases, average nuclear parameters of some sort must be used. Accurate averages are difficult to obtain since a full description of the flux (weighting parameter) in space and energy is impossible to obtain. Still more fundamentally, it is not clear just how to compute the average parameters; i.e., just what weighting parameters should be used. Recognition of these difficulties, however, does not eliminate the need for calculations. To facilitate the calculations, a relatively simple neutron-energy spectrum has been used which has been found extremely useful.** The spatial problem has not been studied.

In the thermal group, a hardened Maxwellian spectrum has been assumed. The effective temperature of the hardened spectrum can be approximately related to the moderator temperature as follows:

$$E_{\text{eff}}(\text{ev}) = \left(1 + a \frac{\Sigma_a(E_0)}{\xi \Sigma_s} \right)^b E_0(\text{ev}) \quad (18)$$

The values of a and b in current use at KAPL are $3/4$ and 2 , respectively; E_0 is the moderator temperature in electronvolts. The thermal group is assumed to extend up to $4 E_{\text{eff}}$. It can be shown from neutron conservation arguments that for moderately black cores, the $1/E$ epithermal spectrum would intersect the thermal Maxwellian spectrum at about this energy. In the epithermal group, a $1/4$ spectrum is assumed. This group extends from $4 E_{\text{eff}}(\text{ev})$ to 184 kev which is at about the bottom of the fission source spectrum.

*Cross section data were obtained from BNL-325.

**R. W. Deutsch, "Computing 3-Group Constants for Neutron Diffusion," Nucleonics, January 1957, 47-51.

In the two-group discrete poison depletion scheme, a value for $w\alpha$ is required. As mentioned before, this is simply the ratio of the epithermal to the thermal absorptions, and for B^{10} , a $1/v$ absorber, the ratio is independent of E_{eff} , viz, 0.355, under the above spectrum assumption. For U^{235} , $w\alpha$ is a slowly varying function of energy and is plotted in Figure 1 as a function of the hardened temperature, in electron-volts. The curve is reasonably well-fitted by the polynomial:

$$w\alpha_u = 0.252 + 4.77 E_{eff} - 34.67 E_{eff}^2,$$

where E_{eff} is the hardened temperature in electron volts.

FIGURE 1 RATIO OF EPITHERMAL ABSORPTIONS TO THERMAL ABSORPTIONS IN U^{235} vs EFFECTIVE ENERGY

Also needed in the two-group depletion is the ratio of the average microscopic thermal U^{235} absorption cross section to that for B^{10} . This ratio is also a slowly varying function of E_{eff} , as shown in Figure 2.

In one-group depletion, the ratio of average absorption cross sections also enters but the ratio is over-all energy. This ratio is shown in Figure 3.

FIGURE 2 RATIO OF AVERAGE THERMAL U^{235} ABSORPTION CROSS SECTION TO AVERAGE THERMAL B^{10} ABSORPTION CROSS SECTION vs EFFECTIVE ENERGY

FIGURE 3 RATIO OF AVERAGE B^{10} ABSORPTION CROSS SECTION TO AVERAGE U^{235} ABSORPTION CROSS SECTION AVERAGING OVER ALL ENERGY vs EFFECTIVE ENERGY