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Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
The column buckling device fuel element vibration suppressor has been analyzed and re-designed. Pre-kinking of the column shows resultant lowering of the strain hardening effects of a straight column and the axial deformation required to sustain the total column load.

Design of the new CBD is presented and further recommendations are included.
I. INTRODUCTION

Development of a launch vibration suppression device has involved the investigation of several possible designs (Ref. 1). The most promising device, which best conformed to the design criteria (Ref. 1), was the column buckling device (CBD). Analysis and testing of a suitable CBD to be compatible with the current reference ZrH design has continued through the present (Ref. 2-9). However, recent investigation into fabrication processes has revealed the necessity of placing the CBD in the cup plug end (inlet) of the fuel element rather than the hotter, blind end (outlet). The collapse load during various phases of operation can effect stressing of the clad and hydrogen barrier. Thus, further analysis and experimental verification must be performed to stabilize the vibration suppressor design point. Design and Analysis of the CBD is reported in sections II - IV assuming that the column could collapse during at-power operation (long-term creep collapse) or during a transient (short-term collapse at scram). However, the collapse mode discussion is dependent upon the generated loads in the CBD and thus, this section (V) is presented after the loading analysis.

II. CURRENT CBD DESIGN - ANALYSIS & EXPERIMENT

Collapse loads for various design CBD's have been experimentally determined at different temperatures. The pertinent data from these tests appears in Table I. Load versus deflection curves for certain of these tests appear in Figures 1 - 10. The data from this table has been used to study various analytical approaches to the CBD design problem. Calculations using the standard Euler equation for long columns (Ref. 10, 11, 12) and the Reduced Euler Equation (for buckling beyond the proportional limit) (Ref. 12-16) have been compared with the experimental data. These methods predict total column load capability two orders of magnitude above the measured loads, and are therefore not applicable to the problem at hand. However, the secant equation (Ref. 7, 10, 11) predicts column loads which are only somewhat lower than the reported loads of Table I. A short discussion of the secant
TABLE 1: CBD DATA

Experimental Data for CBD development (Ref 24)

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<th>TEST NUMBER</th>
<th>DESIGN MATERIAL</th>
<th>END DESIGN TYPE</th>
<th>COLUMN LENGTH (in)</th>
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Design: 502 ss wire; annealed 100 hrs @ 1400°F; furnace cooled in vacuum; 0.045" diameter; 2 flat ends (H) 0.45" long

Data: 2 samples at R.T. (70°F)

Collapse rate is 0.002 "/min/min
FIGURE 2

CBD Collapse Curves

Test#:

Design:

Data:

Collapse Rate is 0.002 in/in/min.
**FIGURE 3**

**CBD Collapse Curves**

**Desig**n: 502 SS wire; 0.052" dia.; annealed 100 hr @ 1400°F — furnace cooled in vacuum;
2 flat ends (H); 0.45" long

**Data**: P-E curve — 4 samples at R.T. (70°F)

**Collapse rate** = 0.002 m/m/mm
Design: 502 stainless steel wire; 0.049" dia; annealed 100 hr @ 1400°F — furnace cooled in vacuum; reference ends — 1 round/1 flat (H); 0.45" long

Data: P-6 curve; 3 samples at R.T. (70°F)

Collapse rate = 0.002 w/in/in
Test#: 402 401

Design: 502 ss wire; annealed 100 hr. @ 1400°F; furnace cooled in vacuum; 0.045" diameter; reference ends - 1 flat/1 round (T); 0.45" long

Date: 2 samples at R.T. (70°F)

Collapse rate: 0.002 "/m/min
FIGURE 6

CBD Collapse Curve

Test #: 501

Design: 502 SS wire; 0.052" dia; annealed 100hr
@ 1400°F - furnace cooled in vacuum;
2 flat ends; 0.45" long

Data: P-6 curve; 1 sample at ~1100°F

Collapse rate = 0.002 \text{in/mm/mm}
FIGURE 7

CBD Collapse Curves

Test #: 601  602  603

Design: 502 88 wire; 0.052" dia.; annealed 100 hr.
@ 1400°F - furnace cooled in vacuum;
2 flat ends; 0.45" long

Data: P-6 curve - 3 samples at ~ 1350°F

Collapse rate = 0.002 m/in/mm
Design: 405 ss wire; annealed 100 hrs @ 1400°F; furnace cooled in vacuum; 0.053" diameter; reference design - 1 ft/round end (H); 0.45" long

Data: 2 samples at R.T. (70°F)

Collapse Rate is 0.002 w/m/min
Design: 405 ss, annealed 100 hrs @ 1400°F; furnace cooled in vacuum;

0.053" dia; reference design - 1 flat/round end (H); 0.45" long

Data: 3 samples at 1350°F.

Collapse Rate is 0.002"/in./min.
Test #: 901

Design: 405 as unwr. annealed 100 hr @ 1400°F - furnace cooled in vacuum; 0.052" dia.; reference ends - 1 flat/1 round (19); 0.45" long

Data: 2 samples at R.T. (70°F)

Collapse rate = 0.002 in/in/min
equation follows. The formulation is based on a statistical analysis of real column behavior (since no column is perfectly straight or the applied loads perfectly concentric). Thus, for the design of a real column, a probable crookedness translated into an effective load eccentricity may be assigned. We therefore have

\[ \sigma_{\text{max}} = \frac{P}{A} \left[ 1 + \left( \frac{ec}{r^2} \right) \sec \frac{L}{r} \sqrt{\frac{P}{4EA}} \right] \]  

(1)

where

- \( \sigma_{\text{max}} \) = maximum fiber stress at yield - yield point (for alloyed steels) (Ref. 11)
- \( P \) = column load at failure
- \( A \) = column cross-sectional area
- \( e \) = equivalent eccentricity
- \( c \) = distance from central bending axis to extreme fiber on compression side of member = column radius for "straight" column = \( R \)
- \( r \) = radius of gyration for column = \( 1/2R \) for circular column
- \( \frac{ec}{r^2} \) = Ecc = eccentric ratio for column \( \approx 0.25 \) for "straight", short column (Ref. 11)
- \( L \) = effective column length = distance between points of inflection
- \( E \) = elastic modulus

Referring to Figure 11, we note that the effective length of the CBD column is approximately \( L_{\text{total}}/2 \). Figure 12 illustrates the effects of column offset on the failure mode. It should be noted that the misnomer eccentricity has been used to label the offset effect. Due to the fixed end design of the CBD, and the parallel alignment of the CBD end caps (resulting from
FIGURE 11 - Effective Length of CBD

FIGURE 12 - Offset Column Effects
the parallel loading of the fuel), any statistical analysis of the column can only establish that concentric loading exists with any amount of offset. Thus, offset has no effect on collapse load.

Data correlation is presented in Table II. The values in column 4 have been calculated using Equation (1) with \( \sigma_{\text{max}} = 0.2\% \) offset yield strength at given temperature, and Ecc = 0.25. Note that these values are only 70-85\% of the reported failure loads in column 7. The secant collapse loads have been plotted on Figures 1-10, and a review of the curves reveals that the secant equation is predicting the load at the initiation of plastic deformation. This is especially demonstrated in Figures 1-4. Thus, the elastic behavior of the CBD is entirely predictable analytically. This loading will be further denoted as the maximum elastic loading.

The maximum elastic load is the design load for the CBD during launch vibration. This becomes obvious by noting that any plastic (non-recoverable) axial deformation of the column results in increased loading (due to greater axial fuel freedom of motion) of the CBD. Increased loading promotes further deformation, thus the column must be designed to a given safety factor within the elastic design. The design point of the CBD is therefore analytically predictable.

The reported loads are such that further collapse requires diminished loading and will be denoted as the maximum plastic collapse loads. That these loads exist above the maximum elastic loading is a strain-hardening phenomenon. The plastic strain-hardening effect is demonstrated in Fig. 13. Note that the compressive stress which exists above the proportional limit of the material exists only on the concave side of the column, and exists only during a state of initial buckling. Thus, the additional loading from the maximum elastic load to the maximum plastic collapse load is a function of the rate of strain hardening (and thus the rate of loading or deformation) and the stability of the state of initial buckling.
### TABLE 2: SECANT CALCULATIONS

**SECANT CALCULATIONS in use:** $G_{\text{map}} = 0.290 \text{ offset yield stress (col. 3)}$

Column 3 data from USS steel and Carpenter Steel handbooks.

Extrapolated to 1350°F.

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<th>TEST NUMBER</th>
<th>MATH/TEMP (°F)</th>
<th>-2% off yield (ksi)</th>
<th>MAX LOAD FROM SEC. EQR. WITH ECC = 2.5 (lbs)</th>
<th>OFFSET LOAD FROM COLLAPSE CURVES (lbs)</th>
<th>COLLAPSE LOAD P (from Table 1) (lbs)</th>
<th>$\Delta P_{\text{calc}}$ (lbs)</th>
<th>$\Delta P_{\text{exp}}$ (lbs)</th>
<th>% Corr.</th>
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Strain Hardening of CBD due to Initial Buckling

Central Bending Axis is at average stress

Increase in avg. stress above proportional limit due to additional compression from bending

Average stress in column = 6.2% yield

Decrease in average stress due to tension from bending

This section of column is strain-hardening due to plastic deformation. This offers a resistance to further buckling.
Again, referring to the load-deflection curves, the axial CBD deformation during the strain hardening phenomenon can be obtained. Measurement of these deformations are illustrated in Figure 14. The data is presented in Table III. The method of predicting the bowing due to axial deformation (as listed in Table III) is shown in Figure 15. Note that this method overpredicts the bowing but qualitatively demonstrates the effects of kinking versus strain-hardening.

The last column in Table III demonstrates the correlation between column diameter and bowing to realize the maximum plastic collapse load. When this condition exists, it can be seen that the position of the central bending axis (see Figure 13) is such that the buckling moment generated by the load at the offset center of the column exceeds the opposite moment generated by the strain hardening phenomenon. This is in direct proportion to the column diameter. The larger the column diameter, the greater axial deformation is required to reach the maximum plastic collapse load.

Note also that the difference between the maximum plastic collapse load and the maximum elastic load (see Table II: columns 8 and 9) also correlates (within large limits) to column diameter. Thus, diameter determines the duration of the deformation required for maximum plastic collapse and also the magnitude of the plastic strain-hardening for a "straight" column. However, were it possible to decrease the axial deformation necessary to cause full plastic collapse, and also the maximum plastic collapse load, the total maximum load generated by the CBD in the element would decrease. In addition, the possibility of realizing this load during a transient (shutdown) would diminish.

Pre-kinking of the CBD can provide the needed load and deformation decrease. A discussion of analytical methods for pre-kinked column load capability prediction follows.
Sample Collapse Curve (Load-Deflection)
for "straight" CBD

Maximum Plastic Collapse Load

Secondary Predicted Load
(Maximum Elastic Load)

Duration of Strain Hardening
**TABLE III**

**STRAIN HARDENING: AXIAL DEFORMATION AND COLUMN BOWING EFFECTS**

<table>
<thead>
<tr>
<th>Test No.*</th>
<th>( \Delta L_1^{**} ) (mils)</th>
<th>( \Delta L_2 ) (mils)</th>
<th>( B,^{***} ) bowing resulting from ( \Delta L_2 )</th>
<th>( B/Col ) Dia x100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>6.0</td>
<td>13.0</td>
<td>27.0</td>
<td>60.0</td>
</tr>
<tr>
<td>102</td>
<td>5.4</td>
<td>12.0</td>
<td>25.0</td>
<td>55.5</td>
</tr>
<tr>
<td>103</td>
<td>6.0</td>
<td>13.0</td>
<td>27.0</td>
<td>60.0</td>
</tr>
<tr>
<td>105</td>
<td>12.0</td>
<td>12.0</td>
<td>25.0</td>
<td>55.5</td>
</tr>
<tr>
<td>201</td>
<td>6.0</td>
<td>15.0</td>
<td>29.0</td>
<td>55.8</td>
</tr>
<tr>
<td>202</td>
<td>5.0</td>
<td>17.0</td>
<td>31.0</td>
<td>59.5</td>
</tr>
<tr>
<td>203</td>
<td>6.0</td>
<td>17.0</td>
<td>31.0</td>
<td>59.5</td>
</tr>
<tr>
<td>204</td>
<td>7.0</td>
<td>17.0</td>
<td>31.0</td>
<td>59.5</td>
</tr>
<tr>
<td>302</td>
<td>8.0</td>
<td>12.0</td>
<td>25.0</td>
<td>51.0</td>
</tr>
</tbody>
</table>

* Only measurable tests have been included.

** The assumption has been used that all bowing results from \( \Delta L_2 \) only.

*** \( B \) is assumed to be \( 1/2 \) of the bowing predicted by the model in Figure 15.
Prediction Method For Bowing

\[
\left( \frac{L}{4} \right)^2 = \left( \frac{L}{4} - \frac{AL^2}{2} \right)^2 + B^2
\]

\[
B^2 = \frac{L \cdot AL^2}{4} - \frac{AL^2}{2}
\]

\[
B \approx \frac{\sqrt{L^2}}{2}
\]

for \( L = .45 \)

\[
B \approx \frac{\sqrt{L^2}}{3}
\]
III. PRE-KINKED CBD DESIGN - ANALYSIS

The applicability of the secant formulation to the present design led to investigation of a similarly derived equation for pre-bent short columns (Ref. 11).

\[ \sigma_{\text{max}} = \frac{P}{A} \left[ 1 + \frac{Bc}{r^2} \frac{8EA}{P(L/r)^2} \left[ \sec \left( \frac{P}{4EA} \left( \frac{L}{r} \right)^2 \right) \frac{1}{2} - 1 \right] \right] \]

The nomenclature is similar to that for Equation (1) and in addition:

- \( L \) = effective column length = distance between points of inflection
  
  - \( L_{\text{total}}/2 \)

- \( B \) = effective maximum initial deflection = 1/2 of maximum distance from centerline of unbent section to centerline of bent section (a result of the effect of fixed ends versus pinned ends)

For comparison, a plot of calculated collapse load (maximum elastic load) for a "straight" and pre-kinked column is presented in Figure 16. Note that the "straight" column is designed for maximum elastic collapse (with \( E_c = 0.25 \)) and the maximum plastic load is assigned by adding an estimated strain-hardening load differential. In comparison, the pre-kinked column has a smaller plastic deformation effect because it is designed with an initial deflection. Thus, for pre-kinking, the elastic and plastic loadings are very close. The design diameter of the kinked column therefore must be greater in that (1) the design load is to be taken as the elastic load, and (2) the large initial deflection reduces the loading capabilities of the column.

IV. EXAMPLE DESIGN OF "STRAIGHT" AND PRE-KINKED CBD's

At the present time, no analysis has been performed to establish the total effective static load exerted by the segmented fuel during launch vibration. Thus, design of the CBD can only proceed to establish load/column diameter relationships. However, it is interesting and informative to assume a design load and establish design parameters for the two CBD's.
FIGURE 16
Collapse Load
for
STRAIGHT column with
carrying eccentric load
and
KINKED column with
varying eccentricity

L = 0.48 m
D = 0.05 m
G = 4.100E6 psi
E = 30,000,000 psi

P, Calculated Collapse Load, lbs.

ECC 0

BOW 0

(ratio)

(in mil)

0.25

0.50

0.75

1.0

5

10

15

20
We assume a total effective static load at $70^\circ F$ of 90 pounds. The CBD design material is chosen as 405 SS for its yield strength versus temperature properties and its tendency not to air-harden (aluminum additive). 

$\sigma_{\text{max}} = 41000$ psi at $70^\circ F$ (Ref. 19). The elastic modulus is $30,000,000$ psi (Ref. 24). The remaining design parameters appear in Table IV below:

**TABLE IV**

"STRAIGHT" COLUMN

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. plastic collapse load</td>
<td>90. pounds at $70^\circ F$</td>
</tr>
<tr>
<td>Max. elastic load (0.8 x plastic)</td>
<td>72. pounds at $70^\circ F$ (Design Load)</td>
</tr>
<tr>
<td>Column length</td>
<td>0.45 inches</td>
</tr>
<tr>
<td>Effective eccentricity (nominal)</td>
<td>0.25</td>
</tr>
<tr>
<td>Column diameter</td>
<td>0.053 inches</td>
</tr>
<tr>
<td>Axial deformation required to sustain max. plastic load</td>
<td>0.015 inches at $70^\circ F$</td>
</tr>
</tbody>
</table>

PRE-KINKED COLUMN

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. plastic collapse load</td>
<td>76. pounds at $70^\circ F$</td>
</tr>
<tr>
<td>Max. elastic load (.95 x plastic)</td>
<td>72. pounds at $70^\circ F$ (Design Load)</td>
</tr>
<tr>
<td>Column length</td>
<td>0.45 inch</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Deflection (mils)</th>
<th>Column Diameter (inch)</th>
<th>Axial deformation to sustain P_{\text{max}} (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.061</td>
<td>0.013</td>
</tr>
<tr>
<td>12</td>
<td>0.063</td>
<td>0.012</td>
</tr>
<tr>
<td>15</td>
<td>0.066</td>
<td>0.010</td>
</tr>
</tbody>
</table>
V. CBD COLLAPSE MODES

There are two possible collapse modes for the CBD:

1. Long-term collapse mode:

The normal, at-power mode of collapse for the CBD is initiated by axial fuel swelling. A plot of axial fuel growth versus time for a ring one element in the core test reactor is reported separately (as a classified document) in Ref. 17. It can be inferred from this plot and the fuel swelling correlation that the maximum creep rate applied to the CBD column is 0.01%/hr (a conservative estimate which decreases rapidly with time). From Fig. 17 (Ref. 19) the maximum stress generated in the column due to this creep rate is estimated to be 16,000 psi (assuming CBD operated at approximately 1100°F). Assuming a maximum column diameter of 0.070", the stress corresponds to a cladding load of 62 pounds, or a cladding stress of approximately 1000 psi. Fig. 18 (Ref. 18) demonstrates that this stress generates a negligible creep rate in the Incolloy 800 cladding. Thus, the long term mode of collapse would seem to have no adverse effects on the fuel element.

2. Short-term collapse mode:

There exists the possibility of a short-term collapse in which the entire column load is generated in the cladding. The collapse load of the CBD is dependent upon the operating temperature and the point in lifetime at which the short-term deformation occurs. Fig. 19 illustrates a short-term collapse during an early lifetime scram to room temperature (Refs. 22 and 23). Fig. 20 shows the effect of a shutdown axial fuel CBD gap for a ring one element undergoing a change from an operational temperature profile to a given isothermal core temperature.

Returning to the nomenclature of Fig. 14, we define \( \Delta L_{\text{collapse}} = \Delta L_1 + \Delta L_2 \) to be the total axial deformation required to attain the maximum plastic collapse load. Minimizing \( \Delta L_{\text{collapse}} \) is preferable. The effects of
**Figure 2.18** Secondary Creep Rate

- **Uniaxial -**

Incoloy 800

\[ \dot{\varepsilon}_2 = 4.3369 \times 10^{-11} \cdot 7.5/322 \cdot e^{-93.7133/T} \]

- **Primary Creep Rate -** \(2.3 \dot{\varepsilon}_2\)

Strain at primary secondary intersection is defined by

\[ \varepsilon_{int} = 0.0528 \dot{\varepsilon}_2 \]

- **Strain Rate** in/in/hour

- **10000**

- **1000**

- **100**

- **10**

- **10^3**

- **10^5**

- **10^6**
Short Term Collar Button Collapse Mode

**#1**
Room Temp.
B.O.L., $t=0$
5 mil Axial Gap (Assembly Tolerance)

**#2**
Operating Temp
B.O.L., $t=0$
~50 mil Axial Gap

**#3**
Operating Temp.
$t = t_o$
0 mil Axial Gap
~50 mil Axial Swelling

**#4**
Room Temp., isothermal
$t = t_o$
After Scram,
~4.5 mil Collapse

No Scale
FIGURE 2.0

Δ Fuel-CBD-Axial Gap
From Operating Temp Profile

Core Test
Ring One Element
Incalloy 800 cladding

Example:
1 ft Fuel-CBD gap during operation 3.10 mils

Then: Fuel-CBD gap at
Isothermal core temp of $T = 400^\circ F$ is

$\Delta T \approx 20 \times 10^{-3} \text{ mils}$

$T$, Isothermal Core Temp, $^\circ F$
the large $\Delta L_{\text{collapse}}$ which exists with the "straight" column design is well demonstrated in Fig. 21, in which the "straight" column of Table IV has been analyzed using the CBALLS (Column Behavior At Low Level Swelling) code on the Honeywell 430 system. A listing of CBALLS appears in Appendix A.

CBALLS correlates the effects of (1) Secant calculations of the maximum elastic load using temperature dependent yield strength date for 405 stainless steel (Fig. 22), (2) strain hardening on the maximum plastic collapse load using an experimentally determined temperature correlation, (3) differential expansion between the fuel and cladding as shown in Fig. 20, (4) variation of $\Delta L_{\text{collapse}}$ with temperature as determined from the experimental data, (5) variation of the generated column load with axial deformation of the CBD as determined from the experimental data, and (6) decrease in the generated column load with increased axial deformation after reaching the maximum plastic collapse load (as extrapolated from the experimental data).

Assuming that the reactor has been shut down from an operating temperature profile to an isothermal inlet temperature condition, followed by further reduction in isothermal core temperature to $T_o$, the total column load generated at $T_o$ can be found from Fig. 21 as a function of axial fuel swelling (assuming a 0.005" axial assembly gap at room temperature). This can then be correlated to a reactor operating lifetime through Ref. 17. Note that the analytical methods in CBALLS are valid only for axial fuel swelling less than that amount, $\Delta L_{\text{crit}}$, necessary for fuel CBD contact during operation. No CBD creep evaluation curves are in existence at the present time and the effects of high temperature buckling through creep cannot be evaluated. Thus, no curves exist in Fig. 21 for $\Delta L_{\text{fuel swell}} > \text{Axial assembly gap} + 0.044$" (where 0.044" is the differential expansion gap in going from an isothermal room temperature core to an operating temperature profile).
FIGURE 2-1
Total Load Generated in CRD
Under Following Conditions:
1) Core at operating temp.
   profile.
2) Core shunt for rated temp. (approximately)
3) Core temp (corrected)
   restored to 115.
4) Small and steady gap
5) Incl. of B00 Girding

Load Generated in Column, lbs.

$T_0$, Isothermal Core Temp, °F

10 15 20 25 30 35 40 45

0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900 950 1000
Figure 22

0.2% Offset Yield Strength for H05 Steel (Fig. 19)

Fully Annealed
0 → 1500°F
VI. DISPLACEMENT OF THE COLLAPSED COLUMN

The CBD must be capable of sustaining an 0.250 inch axial deformation. The possibility of the outer column fiber contacting the ceramic hydrogen barrier upon column failure has been raised. Fig. 23 illustrates a "Worst-Case" collapse mode in which (1) the entire deformation is taken in a short section at the center of the column, (2) there is no length change in fibers on the compression side of the member, and (3) the deformed column section is perpendicular to the load centerline. Application to the pre-kinked column of Fig. 26 reveals that 0.25 inch collapse is possible and displacement is within the radius of the cup plug (with a maximum 0.060 inch cup plug wall). However, a length increase of the CBD column could be very beneficial in that (1) the total lateral displacement of a collapsed longer column will be less than that for a shorter column, (2) the length increase will cause only a second-order change in the design load, and (3) a 0.010 inch length increase allows design of an upper CBD loading plate which does not enter into the recessed cup plug (Fig. 24). Thus, the CBD design which follows is based upon an 0.55 inch column.

VII. CBD DESIGN: "STRAIGHT" VERSUS PRE-KINKED

Pre-kinking of the CBD results in a reduction of the strain-hardening effects: (1) the additional loading due to hardening is decreased, (2) the axial column deformation required to sustain the total load is decreased (Fig. 25), I, therefore, submit the preferred CBD design as outlined in Fig. 26.

VIII. CBD ELASTIC EFFECTS

The effective spring constant (K) for a "straight" column can be equated directly to the elastic modulus of the column material. This is well demonstrated in the initial (elastic) slope of the load-deflection curves (Fig. 1-10). This slope can be equated directly to the material modulus of elasticity (E). The correlation is strikingly close. Thus,

\[ K = \frac{EA}{L} = \frac{ETR^2}{L} \]
Displacement of Collapsed Column

SLOPE INFECTION POINT

d = diameter of column
D = I.D. of cup plug

B = Bowing (initial)

#1 BEFORE COLLAPSE

0.25 collapse possible if \( \left( \frac{3d}{4} - d \right) > 0.25 \)

Collapse Assumptions:
1. No Column Length Change on Compression Side of Member (Worst Case Bend)
2. 0.25 Inch Collapse Minimum

#2 AFTER 0.25 INCH COLLAPSE

\[ X = \text{radius of bent column section} \]
\[ X = 0.125 + (1.5)d \]

AND \( 2x < D \) for no damage to cup plug
Schematic of Lengthened CBD Collapse

FUEL

CLAD

COLLAPSED

CUP PLUG

All dimensions in inches

NO SCALE
Load - Deflection Curves for
Pre-kinked versus "Straight" CBD

Assume:
1) Pre-kinked column diameter increased to sustain same design load as "straight" column
2) ~ 15 mils of prekinking
Prekinked CBD Design

O.D. OF UPPER PLATE LESS THAN CLAD I.D.

0.066" COL. DIAM.

0.015" ± 0.003" DEFLECTION OF BOWED SECTION

O.D. OF LOWER PLATE LESS THAN CUP PLUG I.D.

DESIGN MATERIAL: 405 SS FOR COLUMN

FABRICATION: PREKINK MAY BE DONE AT ANY POINT IN ASSEMBLY, BUT CBD MUST BE FULLY ANNEALED AFTER ASSEMBLY.

NOTE A* - SLOPE INFLECTION POINT OF BEND SHOULD OCCUR AS CLOSE AS POSSIBLE TO ONE-FORTH OF THE TOTAL COLUMN LENGTH FROM THE END PLATES. THIS MOST CLOSELY APPROXIMATES THE NATURAL BENDING MODE.

SCALE: 5" = 1"
For 405 stainless steel, we assume that $E = 30,000,000$ psi, $R = 0.0265$ inch, and $L = 0.45$ inch. Then $K = 14,700$ lbs/in. This corresponds to a natural resonance frequency (assuming direct attachment to a one pound mass fuel element) of 1200 hertz.

Unfortunately, a similar analysis is not applicable to a pre-kinked column in that lateral response to axial deformation could lower the effective spring constant. This has been illustrated qualitatively in Fig. 25. Furthermore, any analysis of elastic effects in a pre-kinked column would tend to be overly complex (i.e., finite element analysis). Experimental determination of the spring constant would, however, be inexpensive, simple, and accurate (in that the elastic modulus was accurately determined in the "Straight" column testing).

IX. SUMMARY OF RECOMMENDATIONS

Most of the following changes have been recommended or inferred in the previous sections, as noted. The remaining recommendations are substantiated below:

1. Pre-kinking of the column buckling device to minimize strain hardening effects during the short-term collapse mode. (Sec. II, III, IV, V, and VII).

2. Experimental testing of the pre-kinked CBD to determine the effective spring constant for the device. (Sec. VIII).

3. A 0.10 inch increase in the column length (from 0.45 to 0.55 inch) to eliminate upper CBD plate interference with the cup plug (and thus eliminate the necessity for the smaller diameter section of the fuel rod).

4. Reactor transient analysis to determine the possibility of other modes of collapse for the CBD. (To supplement the material of Section V).

5. Elimination of the ceramic coating on the upper plate of the CBD.
In that the primary function of the coating was removal of the possibility of fuel-metal interaction during high temperature operation (1350°F) and in that the CBD will now operate at lower temperatures (1100°F), I recommend that the coating be eliminated.

6. Investigation of local stressing of the ceramic barrier during short-term loading of the CBD at low temperatures.

The effects of a line contact in the cup plug could set up large stresses in the ceramic barrier. Increased hydrogen permeation rates could result from such stressing.

REFERENCES


19. United States Steel Handbook, Type 405, USS12AL


24. Personal Communication, K. E. Moore, Jan. 1971
CBALLS:

COLUMN BEHAVIOR AT LOW LEVEL SWELLING

This code uses secant formulation and experimentally determined parameters to calculate the CBD load generated by shutting down the reactor from an operating temperature profile to an isothermal core temperature as a function of axial fuel growth.

10:00
110 REALT(21),SW(26),YS(21),FRACP(21),P MAX,P,L,LMAX
120 COMM0NR,EFFL,ICNTL
130 DATAT/60.100,150,200,250,300,350,400,450,500,
140 & 550,600,650,700,750,800,900,950,1000,1035/
150 DATASW/0,0.005,0.005,0.02,0.025,0.03,0.035,0.04,0.045,0.05,
160 & 0.05,0.06,0.065,0.07,0.075,0.08,0.085,0.09,0.095,1.
170 & 105,11,115,12,125/
180 C YS ARRAY IS YIELD STRENGTHS OF 405 SS
190 DATAS/41000,40500,39900,39400,38500,38000,
200 & 37400,36800,36400,36000,35500,35000,34200,33500,
210 & 32600,31600,30900,29500,28300,26400,19500/
220 C FRAC ARRAY DESCRIBES THE LOADING CURVE
230 C FOR THE CBD IN FRACTIONAL INCREMENTS
240 C OF PMAX AS A FUNCTION OF FRACTIONAL
250 C INCREMENTS OF LMAX.
260 DATAFRACP/0.,.5,1,26
270 & .545,.68,.782,.815,.87,.92,.945,.977,
280 & .99,.991,.992,.993,.995,.997,.999,1,1,1,1,1,995,.98/
280 C 1000FORMAT(//
290 & INIT ASSEMBLY GAP = "3PF8.1" MILS//
300 & FUEL SWELLING = "3PF6.1" MILS//
310 1001FORMAT(OPF9.5X,OPF9.2)
320 3PF9.5X,
330 & FUEL-CBD CONTACT AT "OPF7.0" DEGREES//
340 & TEMP LOAD")
350 6969FORMAT(/////)
360 C READ THE COLUMN DIAMETER, LENGTH, AND
370 C THE INITIAL (REGM TEMP) AXIAL GAP
380 PRINT,"DIA, L, ASSEMBLY GAP*",**
390 READ(50),DIA,L,GAP
400 R=DIA/2
410 EFFL=L/2
420 C DO-LOOP FOR VARIOUS SWELLINGS
430 DC500ISH=1.26
440 C CALCULATE GAPX: THE OPERATIONAL GAP FOR
450 C A GIVEN FUEL SWELLING
460 GAPX=GAPI+0.0438461-SW(ISW)
470 C CALCULATE TCCN: THE CONTACT TEMPERATURE
480 C FOR THIS SWELLING
490 TCCN=650-13000*GAPX
500 PRINT 1000,GAPI,SW(ISW)
510 C CHECK GAPX FOR OPERATIONAL CONTACT
CBALLS CONTINUED

520 IF(GAPX.LT.0)G0T069
530 PRINT 1002,TC0N
540C DO-LOOP FOR TEMPERATURE ARRAY
550 DC400IT=1,21
560 DELLSW=(TI(IT)-650)/13000
570C CALCULATE GAP: THE GAP AT A GIVEN TEMP
580 GAP=SW(ISW)-GAPI-0.0436461-DELLSW
590 IF(GAP.LE.0)G0T0499
600 SMAX=YS(IT)
610 XI=SMAX*3.14*R*R
620C USE NEWTON-RAPHSON ITERATION TO CALCULATE
630C PMAX FROM SECANT EQUATION
640 CALLNEWT(SMAX,XI,0.01,0.5,0.002,500.0F,YS,XF)
650C CALCULATE PMAX INCLUDING STRAIN HARDENING
660 PMAX=XF*YS(IT)**0.0643)/1.55
670C CALCULATE LMAX: THE DELTA LENGTH OF CBD AT
680C WHICH PMAX REDUCES BACK TO .98*PMAX
690 LMAX=0.03-(PMAX-10)/8200
700C IF GAP EXCEEDS LMAX, USE DIFFERENT CALCULATION FOR LOAD
710 IF(GAP.GT.LMAX)G0Te299
720C USE FRAC ARRAY TO CALCULATE POSITION OF COLLAPSE CURVE
730 FRACI=GAP/LMAX
740 IGAP1=IFIX(FRACI/0.05)+1
750 IGAP2=IGAP1+1
760 FRAC1=FRACI-(IGAP1-1)*0.05
770 FRACP1=FRACP(IGAP2)-FRACP(IGAP1)
780 FRACP=FRACP(IGAP1)+FRACP1*FRAC1/0.05
790 P=PMAX*FRACP
800 G0T0300
810 299CONTINUE
820 GAPr=GAP-LMAX
830C CALCULATE LOAD IS DELTA LENGTH GREATER THAN LMAX
840 P=0.95*PMAX-25*GAPr/0.016
850 IF(P.LE.0)G0T0499
860 G0T0300
870 499?0=0
880 G0T0300
890 300PRINT 1001,T(IT),P
900 400CONTINUE
910 500CONTINUE
920 PRINT 6969
930 G0T01
940 69FINT"," FUEL HAS CONTACTED CBD"
950 P=IT 6969
960 G0T01
970 END
100C SUBROUTINE YS,XI,DX,DXM,EPS,N,F,F,Y,F,XF
1010C NEWTON-RAPHSON ITERATION METHOD
1020 REALP,L,PI,ECC
1030 1X2=AX
CBALLS CONTINUED

1040 DEL=DX*X2
1050 D0300=D1*N
1060 Y2=FNEWT(X2)
1070 IF(I.EQ.1)G0T020
1080 100IF(ABS((Y2-YS)/YS).LE.EPS)G0T0200
1090 DEL=-(Y2-YS)/(Y2-Y1)*(X2-X1)
1100 DEL=SIGN(MIN1(ABS(DEL),ABS(DXM*X2)),DEL)
1110 20X1=X2
1120 X2=X2+DEL
1130 30Y1=Y2
1140 900PRINT 902
1150 902FORMAT(//' WARNING---MAXIMUM NUMBER OF ITERATIONS'//)
1160 I=N
1170 200XF=X2
1180 YF=Y2
1190 NF=I
1200 RETURN
1210 END
2000 FUNCTION FNEWT(X)
2010 C0MM0N:EFFL,ICNTL
2020 F=SGRT(X/9.42E7)
2030 2040 F=EFFL*X/(R*R)
2050 2060 F=1+0.25/C0S(F)
2070 FNEWT=F
2080 RETURN
2090 END