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MASTER

MICROINSTABILITIES IN INHOMOGENEOUS PLASMA

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INTRODUCTION

In common with our previous investigations our effort for the past year has been aimed at investigating microinstabilities that may occur in the three major approaches to controlled fusion, namely open-ended and toroidal devices, and Astron. For mirror geometries we have been studying the stability of ion cyclotron electrostatic waves in finite geometry where confinement is achieved by the introduction of an axially varying electrostatic potential and/or axially varying magnetic field. Information gained from this analysis is expected to shed some light on the feasibility of using end plates to recover through direct conversion some of the energy loss through the mirrors as was recently proposed. We have also continued our numerical investigation of convective vs absolute instabilities in open-end systems, and progress was made in writing and procuring Algorithms which would solve the problem in two dimensions. Of relevance to toroidal systems we have extensively studied shear stabilization of drift cyclotron waves. It is found that, at low densities, these waves are sufficiently stabilized by the shear necessary to stabilize low frequency drift waves, but more stringent shear is required at high densities. Our analysis was concerned mainly with shear stabilization of highly localized wave packet type of disturbance, but in lieu of some recent work on shear stabilization of low frequency normal modes we have initiated similar investigation in connection with the drift cyclotron modes which in turn has resulted in a calculation based on the energetics of the system to demonstrate the equivalence of the two methods. In Astron.-related problems our efforts have been focused on investigating the stability of longitudinal modes in a system consisting of a cold background plasma and a cylindrical relativistic electron beam in a uniform magnetic field. We have shown that instabilities occur at multiples of the gyrofrequency of the relativistic electrons, and that at low beam densities the instability occurs at the hybrid

frequency of the background electrons. These results are in good agreement with recently reported results on instabilities which occur during plasma buildup in Astron. Along with these specific investigations we have also continued our study of nonlinear aspects of plasma instability and turbulence. Having rederived from basic principles all the necessary equations we have begun to effectively assess the various approximations employed by others in the study of plasma turbulence and their true applicability to physically meaningful systems. The highlights of these investigations are briefly discussed in the following pages.

I.° Shear Stabilization of Drift Waves

In a recent paper dealing with Shear stabilization of low frequency drift waves, Pearlstein and Berk¹ gave a stability condition which is essentially identical to one previously reported by Rutherford and Frieman². This was interesting in that the two papers used two separate methods and seemingly dealt with two different problems.

In the earlier work, reference 2, the authors concerned themselves with the effect of shear on a highly localized wave packet type of disturbance in recognition of the fact that spatially localized normal mode disturbances could not exist in the field geometries studied. That is, the second order differential equation for the wave potential did not have solutions which were evanescent at large distances from the region of interest. Therefore, a partial differential equation in space and time was derived for a superposition of oscillatory solutions of the original equation, certain expansions were performed, and a condition was derived for the Shear necessary to limit the growth of these convective wave packets.

The Pearlstein and Berk paper dealt with the stabilization of an absolute eigenmode instability by the use of a method first reported by Berk et. al.³. In this analysis, the boundary condition is not the requirement that the solution be evanescent, but rather that the energy flow in the eigenmode be away from the region of interest.

We have shown that the identity of the low frequency stability conditions in the two cases is not fortuitous, by demonstrating that the correspondence holds equally well for the drift cyclotron instabilities. In fact, if both methods start from the same general differential equation of the form

$$\delta^2 \frac{d^2 \phi(x)}{dx^2} + [A(w) + B(w)x^2 + i C(w, x)] \phi(x) = 0 \quad (1)$$

the general stability conditions obtained are identical. Currently work is underway to determine just why this correspondence between the stability of two seemingly different instabilities, one absolute and the other convective, obtains. It can be shown, by rather crude methods, that intermediate equations in both methods can be identified with an independently derived energy flow relation for electrostatic waves in a nearly transparent dispersive medium. Qualitatively it appears that under the "worst case" approximation, the wave packet corresponds to a small portion of the eigenmode since the wave packet velocity is identical to the group velocity of the eigenmode.

There are two major questions that remain to be completely answered. First can a complete correspondence be shown even when the stability condition in the eigenmode case is for onset of the instability, and the one in the wave packet case is a rather arbitrary limit on the growth of the disturbance? The answer is perhaps not completely, but it should be possible to make the identification of the two methods more quantitative than it is at present.

Second, is the eigenmode case truly just a special case of the apparently more general wave packet approach, or does the eigenmode approach correspond to the only valid use of the wave packet approach? The question is now being considered by re-examining the latter method with the emphasis on physical interpretation of the quantities and steps involved, and consistency of approximations.

II. Astron Related Instabilities

The problem considered here is the stability of fluti-like modes in a system consisting of a cold background plasma and a cylindrical relativistic electron beam situated in a uniform magnetic field. Using a distribution function for the relativistic electrons of the form

$$f_{OE} = N \delta(u_{\perp} - u_{\perp 0}) \delta(L_{\theta} - L_{\theta 0}) \delta(u_z) \quad (2)$$

where $u_{\perp}$ and u_z are the reduced velocities perpendicular and parallel to the magnetic field respectively, and L_{θ} is the cononical angular momentum, we find that long wave length oscillations with frequency near multiples of relativistic electron gyrofrequency are most unstable for a dilute beam i. e. a beam density much smaller than the background density, the instability occurs at the hybrid frequency of the background electrons i. e. at

$$l = \gamma_{\perp 0} (1 + w_{pe}^2 / \Omega_c^2)^{1/2} \quad (3)$$

where l is the harmonic number, $\gamma_{\perp 0}$ is the measure of relativity of the beam, and w_{pe} and Ω_c are the plasma frequency and cyclotron frequency of the background electrons respectively. The maximum growth rate in this case is given by

$$\text{Im } w = w_{E0} \frac{\beta_0}{2} (\ell \bar{\Delta})^{1/2} \quad (4)$$

where w_{E0} is the cyclotron frequency of the relativistic specie, β_0 is their relativistic parameter and $\bar{\Delta}$ is effectively the beam thickness in units of the relativistic electrons Larmor radius, a_E .

The above instability is quenched as the background plasma density increases. However, short wave length oscillations become unstable with growth rates

given by

$$\text{Im } w = \frac{w_{PE}}{2} \left(\frac{l}{k_{\perp} a_E} \right) \frac{\beta_0}{|\epsilon|} \sqrt{l \bar{\Delta}} \quad (5)$$

where w_{PE} is the plasma frequency of the background whose dielectric is ϵ .

The interesting feature about this instability is that it maximizes at a specific harmonic number given by

$$\bar{l} \approx \sqrt{3} \gamma_{\perp 0} (w_{Pe} / \Omega_e) \quad (6)$$

These results are in general agreement with recently reported experiments on Astron⁴. It was shown that during plasma build up an instability at the hybrid frequency occurs which disappears as the density increases. However, another instability occurs at harmonics of the gyrofrequency which maximizes at a specific value of l .

This analysis has been submitted to the physics of fluids for publication.

III. Instabilities in Finite Mirror Geometry

Velocity space instabilities which have been observed in open-ended systems, have been extensively studied using linear, infinite plasma theory⁵. It is clear, however, that true assessment of the effects of these instabilities on actual devices requires re-examination of these problems by incorporating the effects of finite geometry. In line with this we have started to examine the familiar ion cyclotron instabilities in mirror geometry where confinement is provided for by the introduction of an axially varying electrostatic potential and/or an axially varying magnetic field. The inclusion of the electrostatic potential is particularly relevant at this time in view of recently reported

effort⁶ of attempting, through direct conversion, to recover some of the energy loss through the mirrors by placing end plate collectors.

The procedure we have followed thus far closely resembles the method used by Berkand Book⁷ in a similar one-dimensional problem. Additional complexities have, however, resulted in our attempt to solve the three dimensional problem where we are interested in the stability of waves propagating at an arbitrary angle to the magnetic field.

Assuming an electrostatic perturbation of the form

$$\phi(\underline{r}, t) = \phi_1(z) \exp i \left\{ k_{\perp} x + \int_0^z k_z(z') dz' - \omega t \right\} \quad (7)$$

we have reduced to the problem to solving an integro-differential equation of the form

$$L\phi_1(z) \exp i \int_0^z k_z(z') dz' = \int_{-\infty}^{+\infty} D(z' - z, \frac{z' + z}{z}) \phi_1(z') \exp i \cdot \int_0^{z'} k_z(z'') dz'' dz' \quad (8)$$

where $L(z)$ is a differential operator, and D is another operator that contains a double integration over the energy E and magnetic moment μ of the particles i.e.

$$E = \frac{1}{2} m v_z^2 + \mu B(z) + e \phi(z) \quad (9)$$

$$\mu = \frac{m v_{\perp}^2}{2B(z)} \quad (10)$$

At the present we are attempting to solve eq (8) for an arbitrary applied electrostatic potential from which we hope to deduce the dispersion equation which we will analyze for stability. A quadratic potential along with quadratic axial variation in the magnetic field seem to indicate that stability

results may be readily obtained. Work is progressing in this direction.

IV. Convective and Absolute instabilities in Two-dimensional Geometry

Also of relevance to mirror machines is the ascertaining of whether certain dangerous ion cyclotron instabilities propagating at an arbitrary angle to the magnetic field are convective or absolute. By contrast to the already known results in one-dimensional propagation - along the magnetic field - represented by the first two of the following equations.

$$D(\omega, \underline{k}) = 0$$

$$\partial D / \partial k_{11} = 0$$

$$\partial D / \partial k_{\perp} = 0$$

where D is the dispersion relation, the two-dimensional problem requires solving the three equations simultaneously to obtain the saddle point in the complex frequency plane. Using loss cone distributions we have written-guided by a program obtained from ORNL-a computer program which would solve these equations. It has been tested on the one-dimensional problem and has reproduced results obtained by others.

V. Non-linear Theory and Plasma Turbulence

Due to the avalanche of papers on non-linear phenomena in plasma in recent years, and the multitude of approximations and assumptions contained there in, we have found it necessary to rederive the basic equations from first principles so that we can apply them to some physical problems of interest with the added assurance of truly assessing the physical implications of these approximations.

For a plasma in externally applied magnetic field, and using the Klimontovich formalism for the basic equations we have used the projection operator technique to obtain equations for the first order distribution function

and the fluctuations spectrum. We are attempting to find the conditions under which it is meaningful to describe the system with the first order distribution function and the fluctuations spectrum. An immediate problem arises: If we can excite a normal mode of the system, when can we think of this as a new degree of freedom?

We have shown that if we replace the perturbed orbit propagators by the unperturbed orbit propagators and ignore time dependence of the first order distribution functions we obtain Kadomtsev's turbulent theory⁸. If we expand the perturbed orbit propagators in powers of the fluctuating electric field we obtain Manheimer's results⁹. This latter author has applied his equations to the study of ion cyclotron modes in a loss-cone system but with the drastic simplification of assuming zero Larmor radius for the ions.

Presently we are investigating the non-linear aspects of the ion cyclotron instability (for mirror confinement), and the drift cyclotron instability (for torodial confinement) in detail. Specifically we are studying the non-linear mode coupling of two cyclotron modes to give a third, and presently we are in the process of modeling the plasma so that we can obtain meaningful approximations of the matrix elements in the equations.

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