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H. D. Ferguson

J. E. Spencer

Klaus Halbach*

*Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720.

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87545

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A GENERAL ION-OPTICAL CORRECTION ELEMENT

by

H. D. Ferguson, J. E. Spencer, and Klaus Imbach

ABSTRACT

A general purpose type of multipole magnet is described which provides some unique advantages. It produces a very uniform dipole field which can be rotated about the longitudinal axis of the magnet. Higher order multipoles can also be rotated and can be excited simultaneously without the use of independent coils. A magnet having octupole geometry was built and shown to verify the basic ideas.

I. INTRODUCTION

Most beam line systems consist of combinations of dipoles, quadrupoles, and occasionally higher order multipoles, arranged to provide a desired optical performance. Regardless of the specific application, it is often useful to make small changes in the system either to correct small errors and misalignments, to tune it or make measurements on it that reveal its optical properties, or possibly to match its optics to a particular experiment or other comparable system. The multipurpose magnets described here fill these roles. Two such magnets have been in operation in the Los Alamos Meson Physics Facility (LAMPE) since the spring of 1973 and can be used, depending on the bussing arrangement, to produce either a dipole field of arbitrary orientation, a quadrupole field of arbitrary orientation, or an octupole field with an orientation determined by the physical orientation of the magnet.

Benaroya and Ramler¹ developed a magnet capable of producing a dipole field with arbitrary orientation by using two sets of coils arranged in a 24-pole circular steel structure. The coils were arranged to provide $\cos\theta$ - and $\sin\theta$ -like current distributions to give the two orthogonal field components. Qualitatively, the same result can be achieved with a design that represents another extreme--namely, two sets of coils of the

window-frame type in a square steel frame. When one attempts to design a general-purpose magnet, it is clear that a large range and variety of characteristics have to be considered, such as the type, strength, and quality of fields to be produced; the size and complexity of the steel and coil configurations; and the bussing, power supplies, and control arrangements. We have concluded that an octupole configuration is a good compromise solution whenever it is necessary to produce both dipole and quadrupole fields. Depending on the intended use, a 12-pole magnet might be a better choice, if one also wants to produce sextupole fields with arbitrary orientation.

Because the rule "to produce a $2N$ -pole field, excite the poles like $\sin N\theta$ or $\cos N\theta$ " would appear to be too crude in the case of an octupole, we discuss in detail the proper excitation and expected field quality in the next section. The specific design details of the iron/coil configuration and the bussing and power supply setup are discussed in Sec. III; results for the prototype octupole system are discussed and compared with the theoretical expectations in Sec. IV.

II. THEORY

Since we are primarily concerned here with the fields in the region around the optical axis of the magnet, or with the line integrals of these fields

taken through the magnet (including the fringe fields), we need to consider only two-dimensional fields. The discussion of harmonics in multipole magnets is given in Ref. 2, and we use the results of that treatment, with modifications in notation only.

Magnetic fields in a vacuum that have components only in the x-y plane and do not depend on the third space coordinate can conveniently be derived from either a scalar potential V or a vector potential that needs to have only a component A in the direction perpendicular to the x-y plane. Inspection of the magnetostatic equations shows that A and V are harmonically conjugate, or more specifically, that the function

$$F(z) = A(x,y) + iV(x,y) \quad (1)$$

is an analytic function of the complex variable $z = x + iy = re^{i\theta}$. The magnetic field components H_x and H_y are obtained from F through

$$H^* = H_x - iH_y = i dF/dz, \quad (2)$$

where H^* is the complex conjugate of H .

It is convenient to expand F in a Taylor series

$$F(z) = \sum_{n=1}^{\infty} C_n (z/r_0)^n, \quad (3)$$

where r_0 is a normalization length to be specified below and the C_n are the multipole coefficients. In a perfect multipole magnet, the sum in Eq. (3) has only one term.

A scalar potential surface, which can represent a pole contour, would then be obtained by setting the imaginary part of F equal to a constant. If we talk specifically about the pole of an ideal octupole that intersects the x-axis at $x = r_0$ (Fig. 1), its contour will be given by

$$\text{Im } i(z/r_0)^4 = (r/r_0)^4 \cos 4\theta = 1. \quad (4)$$

If only this one pole is excited by V_0 ampere turns, the coefficients C_n are purely imaginary and proportional to V_0 , that is, C_n has the form

$$C_n = iV_0 j_n. \quad (5)$$

Fig. 1. Reference geometry, pole designation, and excitation pattern for Table I.

with j_n being real and independent of V_0 . The expression for the fields then takes the form

$$H^* = -\frac{V_0}{r_0} \sum h_n (z/r_0)^{n-1}, \quad (6)$$

with

$$h_n = nj_n.$$

In Table III of Ref. 2, the values of $nj_n/4$ are given for an ideal octupole for $n = 1 - 24$ and are reproduced here in Table I up to $n = 16$ after multiplication by 4 to give h_n . We can say the following about the numerical values tabulated for the h_n .

1. The fact that $h_n = 0$ when n is an even multiple of four is a consequence of symmetry and not pole contour.
2. $h_n = 0$ for odd multiples of four is true only for ideal pole shapes and will not hold if one deviates from the ideal contour [Eq. (4)] in a significant way.
3. The values of h_n for all other n do not depend very strongly on the pole shape for small values of n ($n \leq 10$).

Depending on the excitation pattern one uses, harmonics which would be present for single-pole excitation (Table I) may not be present for reasons of symmetry regardless of coefficient magnitude. This will become clearer in the succeeding discussion.

TABLE I
CALCULATED MAGNITUDES OF HARMONICS FOR OCTUPOLE
AND DUODECAPOLE MAGNETS FROM REF. 2

n	$h_n (N = 4)$	$h_n (N = 6)$
1	0.229	0.158
2	0.399	0.293
3	0.492	0.399
4	0.500	0.469
5	0.436	0.503
6	0.313	0.500
7	0.160	0.464
8	0	0.400
9	-3.54 E-2	0.313
10	-4.12 E-2	0.213
11	-2.52 E-2	0.106
12	0	0
13	2.24 E-2	-2.71 E-2
14	3.22 E-2	-4.03 E-2
15	2.46 E-2	-4.19 E-2
16	0	-3.22 E-2

Another result from Ref. 2 is the expression for the field when a pole is at an angular location $\alpha_n = m(n/4)$ and is excited by V_m .

$$H^* = -\frac{V_m}{r_0} \sum h_n \cdot e^{-inm(n/4)} \cdot (z/r_0)^{n-1} \quad (7)$$

When the octupole shown in Fig. 1 is excited like a conventional octupole, i.e. poles -2,0,2,4 are excited with $+V_0$ ampere turns, and poles -3,-1,1,3 are excited with $-V_0$, application of Eq. (7) shows that the only harmonics that can be nonzero are $n = 4(2u+1)$, $u = 0,1,\dots$. Furthermore, if the poles have the ideal shapes, $h_{4(2u+1)} = 0$ for $u \geq 1$, the resulting field is that of an ideal octupole,

$$H^* = -\frac{V_0}{r_0} 8 \cdot h_4 (z/r_0)^3 = -\frac{V_0}{r_0} 4 \cdot (z/r_0)^3 \quad (8)$$

If the pole contour deviates significantly from the one given by Eq. (4), the odd multiples of four may appear with substantial strength. However, if one intends to use only a fraction of the aperture, the lowest allowed harmonic, $n = 12$, is cut down in amplitude relative to $n = 4$ by $(r/r_0)^8$ from what it would be at full aperture, making it generally acceptable for use as a corrective element.

The excitation of the octupole shown in Fig. 1 required to produce a quadrupole field oriented such that $H_y = 0$ on the x-axis is obtained by exciting poles 0 and 4 with V_0 , and poles 2 and -2 with $-V_0$. Application of Eq. (7) shows that the only allowed harmonics are $n = 2(2u+1)$, so that one gets

$$H^* = -\frac{V_0}{r_0} 4 \cdot \sum_{u=0}^{\infty} h_{2(2u+1)} (z/r_0)^{4u+1} \quad (9)$$

The coefficients in Table I are, of course, only a rough approximation if the pole shape is as different from the ideal one as is the case for the prototype pole contour shown in Fig. 2. In cases where one cannot allow strong higher harmonics, such as the duodecapole, one has no other choice but to increase the magnet aperture. As outlined above, changing the pole contour would have little effect on h_6 .

If one wants to produce a quadrupole field oriented so that $H_x = 0$ on the x-axis, one excites poles -1 and 3 with V_0 , and -3 and 1 with $-V_0$, and the field is described by the right side of Eq. (9) multiplied by i . Arbitrary orientation of the quadrupole field axes is then achieved by mixing these two modes of excitation. If the first mode of excitation has a strength, $V_0 \cos \alpha$, and the second, $V_0 \sin \alpha$, then the effective angular rotation (6) of the axes of the resulting quadrupole relative to those induced by the first mode of excitation will be $\theta = \alpha/2$.

To produce a dipole field oriented such that $H_y = 0$ on the x-axis, poles 2 and -2 (Fig. 1) should not be excited. Furthermore, it is clear that if

Fig. 2. Comparison of pole contour used for the magnet shown in Figs. 3 and 4 with the ideal octupole contour derived in the text.

one excites pole 0 with V_0 , pole 4 should be excited with $-V_0$. Similarly, the excitation of pole -1 should be the same as the excitation of pole 1, whereas poles 3 and -3 must have the same magnitude of excitation as poles 1 and -1, but opposite sign. Finally, to make the field as uniform as possible, the magnitude of excitation of poles 1 and -1 relative to V_0 can be used to cancel the contributions from the next-higher allowed harmonic, $n = 3$. It is easy to see that this can be accomplished if poles 1 and -1 are excited by $V_0/\sqrt{2}$. Using Eq. (7), to superimpose the contributions from the individual poles excited in this pattern, shows that the contributions to $n = 5$ also cancel, so that one obtains

$$H^* = -\frac{V_0}{r_0} 4 \sum h_n (z/r_0)^{n-1}; \quad n = 1, 7, 9, 15, 17, \dots \quad (10)$$

Again, the coefficients h_n are only a rough guide to the strength of the multipoles which actually occur. Furthermore, the cancellation of multipoles has nothing to do with the specific magnitudes of these coefficients, but depends only on symmetry and excitation pattern.

To produce a dipole field oriented with $H_x = 0$ on the x-axis, one uses the same excitation pattern as above, but rotated by 90° . Superposition of these two excitations can then give a dipole field at an arbitrary orientation.

If one wants a field pattern not oriented along the x-axis (the $\phi = 22.5^\circ$ line going symmetrically between two adjacent poles, one simply superimposes the excitation pattern that gives Eq. (10) with the same pattern rotated by 45° . This means that poles 0 and 1 should be excited with $V_0 (1 + \sqrt{0.5})$, poles -1 and 2 should be excited with $V_0 \sqrt{0.5}$, and the remaining poles with the corresponding negative values. The field is obtained by multiplying the various terms in the sum in Eq. (10) by

$$1 + e^{-i\pi/4} = 2 \cos \frac{\pi}{8} \cdot e^{-i(n-1)\frac{\pi}{8}} e^{-i\frac{\pi}{8}},$$

yielding

$$H^* = -\frac{V_0}{r_0} 4 \sqrt{2 + \sqrt{2}} \cdot e^{-i\frac{\pi}{8}} \sum h_n \cdot \frac{2 \cos(n\pi/8)}{\sqrt{2 + \sqrt{2}}} \cdot (ze^{-i\pi/8}/r_0)^{n-1},$$

$$n = 1, 7, 9, 15, 17, \dots \quad (11)$$

For $n = 1, 7, 9, 15, 17, \dots$, $2 \cos(n/8)/\sqrt{2 + \sqrt{2}} = 1, -1, -1, 1, \dots$, that is, the magnitudes of the harmonics, relative to $n = 1$, are unchanged, only the signs are reversed for $n = 7, 9, 23, 25, \dots$. To produce the same dipole field strength, the second pattern requires 1.082 times the ampere turns of the first pattern.

We point out that such a magnet can be used to produce all of the above fields simultaneously, i.e., dipole, quadrupole, and octupole, provided one arranges the coils and power supplies appropriately. This feature, together with the ability to rotate the dipole and quadrupole fields, makes the magnet quite versatile. The actual field uniformity achievable for the dipole field using the technique described above should be quite good, especially compared to more conventional magnets, as we show in a later section. If for some reason one needs to use a larger fraction of the aperture than is compatible with a required field uniformity, it might be preferable to adopt a 12-pole design. We leave it as an exercise to the readers to convince themselves that the following characteristics are achievable with this kind of magnet.

1. A rotatable dipole field which has only the harmonics $n = 1, 11, 13, 23, \dots$,
2. A rotatable quadrupole field which has only the harmonics $n = 2, 10, 14, 22, \dots$,
3. A rotatable sextupole field which has only the harmonics $n = 3, 9, 15, \dots$, and
4. Any linear combination of the above.

III. OCTUPOLE DESCRIPTION

The system which was built has an octupole geometry as shown in Figs. 3 and 4. For reasonably small currents ($i \leq 200$ A), we have found it best to excite the different poles using a single, unregulated bipolar power supply connected to a bank of independent regulators shown schematically in Fig. 5. Then, depending on the number of regulators as well as the bussing arrangement adopted, it is possible to produce dipole, quadrupole, and octupole fields. For instance, if only four regulators are used they should drive the four pairs of opposing poles with a bussing arrangement which allows reversing the relative polarity between individual pole pairs. Depending on the buss connection, one can then obtain

Fig. 3. One of the octupole steering magnets designed for tuning a beam at LAMPF. The gap opening is 5.28 cm to accommodate a 5.08-cm beam pipe. Magnet fits into a <30-cm space along the beam line and delivers a dipole field integral of >25 kG-cm with a deviation from uniformity $<\pm 10^{-4}$ over the central quarter of its aperture.

either a dipole or a quadrupole and octupole field. However, if one requires only a rotatable dipole field, then just two independent regulators are required; and if all multipole fields are required simultaneously, a total of eight independent regulators is necessary.

In the magnet shown in Figs. 3 and 4, opposing pairs of poles were excited together so that only four regulators per magnet were used. Buss bars were provided between each pair of opposing poles so that the relative magnetic polarity could be easily reversed for each pair by shifting the buss bar on its switching assembly (Fig. 3). The buss bars were fabricated from commercial quality copper, and the insulation components were fabricated from paper base phenolic and permalloy rods and nuts.

Magnet pole pieces were fabricated from Armco ingot iron, rough machined to size, and subsequently stress relieved before final machining. It was

Fig. 4. View of magnet along the beam direction. Potting epoxy for the coils was loaded with alumina powder to improve its properties in high-radiation fields. Aluminum ring near the beam pipe periphery maintains fixed coil positions.

hoped that the Armco poles would reduce the effects of residual fields at zero excitation. The pole contour used is shown in Fig. 2, where it is compared to that for an ideal octupole. For the 5.28-cm bore shown in the figures, the contour is a 0.25-cm by 45° chamfer on a 1.27-cm pole width. With these dimensions, we attempted to approximate the ideal contour in an integral sense in the region nearest the center of the magnet. This might have been done better by using a wider pole, but the iron for this was not readily available. Geometrical constraints from coil allocation often outweigh the importance of matching the theoretical pole contour as indicated in the previous section. We should point out, however, that considerable attention was given to what the highest fields in the iron were likely to be using these dimensions. The ends of the poles, at entrance and exit, were machined to a radius to simplify coil winding and their axial lengths were made as long as possible compared to the bore radius to minimize end effects.

Fig. 5. Schematic layout of the power supply, regulators, and controls' interface. Not all of the coil circuits are shown in the figure.

Each pair of opposing poles was provided with its own series water cooling circuit - yielding a total of four parallel circuits originating from the magnet-mounted supply and return manifolds. The coils were kept as distant from the aperture of the magnet as practical as shown in Figs. 4 and 6. Each coil consists of 10 turns of 0.555-cm-o.d., 0.318-cm-i.d. OFHC copper tube. This tubing was wound to approximate form and then sheathed with fiberglass tubing after winding and prior to potting. Shell Epon 815 epoxy filled with 900 mesh aluminum oxide (using Shell Diethanolamine hardener for radiation resistance) was used to pot the coils. Each coil was then vacuum filled and cured to 70°C overnight.

The circular yoke was fabricated from commercially available SAE C1015 steel tubing, stress relieved before machining. It has a bore of 16.6 cm

with 26.6-cm o.d. and a 22-cm length. To allow accurate positioning of the poles, slots were cut in the yoke with a vertical boring mill. Pairs of opposing poles were individually installed with an allowable separation error of $\sqrt{2}r_0 \times 10^{-3}$, or 1e.s.

The magnet built to these specifications is rated for 2500 At/pole with a voltage drop of 1.75 V/pole. Each pole has a 10 full turns with a mean length per turn of ~48 cm. Coil power is calculated to be 430 W/pole at 250 A. Water flow per circuit is 2.2 l/min with a 14-kg/cm² pressure drop through the coils. With a total flow of 9 l/min through four parallel circuits, coil water velocity is calculated at 30 cm/s and ΔT is calculated to be 5°C.

It should be emphasized that this system was designed with certain specific needs in mind, mainly to provide a reasonably large dipole field integral

Fig. 6. Basic geometry of half of magnet used in POISSON calculations, with predicted field distribution when the coils containing circles are excited to give a dipole distribution in the horizontal plane.

with good uniformity in a location with significant restrictions on the physical length. The versatility of the magnet is clearly enhanced by its small overall size (it occupies only a 40-cm-o.d. by 29-cm-long volume), its light weight (≈ 70 kg), its unique bussing arrangement, and its comparatively high fields. Furthermore, the use of commonly stocked items resulted in the use of less than 100 man-h of machine shop time for this design. One obvious modification in the present design would be the use of tape-wound coils, which would simplify the cooling and thereby reduce both size and cost as well as considerably simplify its use in some applications.

Before discussing the magnetic measurements which verify the conclusions of Sec. II, a word is in order about the practicality of such a system. To some extent, the verdict depends on the application and the control instrumentation available. For those cases where one requires either a dipole or quadrupole configuration with an arbitrary field orientation, the controls can be arranged exactly the same as for conventional magnets of these types, i.e., one- or two-knob systems. However, for multipurpose operation, it is simpler and more efficient to interface the current controls through a computer which "interprets" knobs assignable as dipole, quadrupole, and octupole. In any system which already uses computers to set magnets, this should entail nothing more than some additional software and the control hardware.

In all of the measurements discussed in the next section, the currents were set remotely and

monitored with the LAMPF control computer. For instance, besides automatically setting various stored dipole, quadrupole, and octupole solutions with different cycling procedures, the three currents — $I_{xy} = I(1, -3)$, $I_x = I(2, -2)$ (or I_y), and $I_{yx} = I(3, -1)$ — required to provide a dipole field in the y (or x) direction, were set simultaneously with the computer by assigning a single "tweak" knob at the computer console in very much the same way one would manually vary the current of a conventional dipole magnet. This particular mode has been used for more than two years for the two steering magnets used in the LAMPF switchyard.

IV. OCTUPOLE PERFORMANCE AND CAPABILITIES

In searching for a high-field and high-field-uniformity steering magnet for experimental ray tracing,³ a number of different geometries and types of magnets were considered. As discussed above, the most versatile system has a 12-pole geometry, whereas the most practical one is the 8-pole magnet. For our purposes, both of these systems have the attractive feature of providing corrections for rotational misalignments of quadrupole twister units. They both have many other uses, and in this section we discuss a number of measurements that were made to verify that such magnets can actually be made to work essentially as predicted in Sec. II.

The overall design of the octupole described in Sec. III was based on a rather complete computer simulation of the system which assumed finite permeability and two-dimensional fields. These calculations were done with the code POISSON* for the geometry shown in Fig. 6. The only symmetry assumed in the calculations was midplane symmetry, so that we were able to make certain checks on the calculations. In addition, a number of other geometries were studied with this code and some of the results, which are representative of each basic type of magnet, have been included as a comparison to the octupole performance. In the discussion below, we consider each excitation mode of the octupole separately. We emphasize that all of the comparisons to theory which are given in the figures below come from the two-dimensional POISSON calculations.

* POISSON is an improved version of TRIM and was developed by J. R. Spoerl, R. F. Holsinger, and Klaus Halbach.

A. Dipole Excitation

Some of the basic dipole types studied during this investigation are shown in Figs. 7 and 8. Figure 7a shows the conventional window-frame geometry which was evaluated, and Fig. 7b shows a variant of that design which converts easily into the basic H-type geometry of Fig. 8. A variant of the H-type geometry which has some similarity with the final octupole design is shown in Fig. 8b. Figures 6-8 were all produced with POISSON and show the magnetic field lines throughout the magnet in each case.

Figure 9 shows the predicted distributions in the median plane, plotted as percentage deviation versus the distance from the center, in units of the available half-aperture. The required excitation (ampere-turns) to produce the same field B_0 at the center of the magnet was used in each case. The solid line in the upper quadrants gives the results for the window-frame geometry. Although this design provides a reasonable field uniformity, its major shortcoming is the magnitude of the field which is obtainable—particularly for a double-field system, i.e., when it is required to supply both B_x and B_y with the same magnet. If one tries to improve on this particular aspect by going to a double H-type

design, one gets results that are generally bracketed by the solid and dashed curves shown in the lower quadrants. While the absolute field level increases

Fig. 8. Basic geometry of half the H-type geometry calculated with POISSON. Figure 8a shows what could be called a conventional geometry and Fig. 8b is an attempted design improvement.

Fig. 7. Basic geometry of half the window-frame geometry calculated with POISSON. Figure 7a shows the conventional coil configuration and Fig. 7b, a variant, converts easily into H-type.

Fig. 9. Results of POISSON calculations for different types and geometries of dipole magnets. Upper curve shows percentage deviation from uniformity across the gap of conventional window-frame design; lower curves show results for H-types and octupole design described in text.

for a given number of ampere-turns, the field uniformity decreases drastically. The solid line here represents a conventional square pole whereas the dashed line is intended to show the futility of varying this basic pole contour to improve field uniformity. This can be understood by noting that the pole width must generally be less than the gap height in the symmetric double H-design.

Finally, the dot-dashed line shows the predicted results for the 8-pole system. The field uniformity calculated for poles with square contours (Fig. 6) is $<10^{-4}$ over the central quarter-aperture. Furthermore, in contrast to the window-frame design, the induction field obtainable for a given number of ampere-turns per pole is comparable to the H-type design.

Figure 10 gives the results of the integral field measurements on the 8-pole magnet when excited according to the prescriptions of Sec. II. The upper and lower curves compare the results of a $\pm 5\%$ excitation error from the prescribed setting in the secondary currents I_{xy} and I_{yx} . In the case

Fig. 10. Results of integral field measurements of magnet shown in Figs. 3 and 4 when operated in the dipole mode compared to central field predictions of POISSON.

of the positive current overvalues for I_{xy} and I_{yx} , the field uniformity was $\pm 0.25\%$ over half the aperture, which is still better than is obtained with most conventional dipole design geometries used for spectrometers. The dots in the figure are the predicted values from Fig. 9. The measured curve is somewhat better than predicted, certainly better than one part in 10^4 over the central quarter of the magnet.

To see how the previous excitation history of the magnet affects the field distribution, the measurements of Table II were made. This table gives the normalized, fractional gap width ($\delta x/G$) over which the field integral is constant to the specified percentage. The first row (Cols. 1-3) shows the width over which the total field variation is $<0.01\%$ due to bringing the currents directly from zero to their final values (1) with no overshoot (0.0), (2) with a 50% overshoot (0.5), and (3) a 100% overshoot (1.0). The last two columns of the first row give the results for a 100% overshoot in the cases when there is 5% excess current in the two secondary coils (I_{xy} and I_{yx}) and when the measurements are made above the median plane ($y = 0$) by an amount $y/G = 0.12$. From these data, it is clearly better to set the dipole fields by approaching the final value from above in applications where extended field uniformity is important, even though at no time during the cycling should the fields be >10 kG anywhere in the iron of the poles or yoke.

A related effect is the degree to which linear superposition applies in this kind of magnet because

TABLE II
FRACTIONAL CURRENT OVERSHOOT
($I_{\max} - I_{\text{final}}$)/ I_{final}

$\int_0^{100} \frac{(B_y - B_0) dx}{B_0}$ (percent)	0.0	0.5	1.0	1.0 ^a	1.0 ^b
0.01	0.11	0.21	0.26	0.08	0.16
0.05	0.23	0.29	0.31	0.11	0.19
0.10	0.29	0.33	0.34	0.16	0.22
0.50	0.41	0.43	0.44	0.48	0.35
1.0	0.47	0.49	0.49	0.52	0.43
5.0	0.64	0.65	0.65	0.66	>0.75

^aCurrent in secondary coils (I_{xy} and I_{yx}) at 5% excess.

^bMeasurements above median plane ($y = 0$) by an amount $y/G = 0.12$.

this essentially determines how versatile and how easy it will be to operate. Figure 11 shows the results of measuring magnetization curves for three different spatial orientations of the resultant dipole field vector. The line in the figure is a straight line drawn through the boxes representing the data for a field at 45° to the other two. The magnet is linear over the full range from 0 to 25 kG-cm.

B. Quadrupole Excitation

Figure 12 shows the results of exciting the magnet as a conventional quadrupole, i.e., of exciting every other pole centered at 45° to the horizontal and vertical axes. The plot gives the actual quadrupole field across the x-axis using the same integral coil as used for the dipole measurements of Figs. 10 and 11. The straight line with constant slope was drawn to show the degree of deviation from a constant gradient field and the dots are again the POISSON predictions for the square pole contour. The gradient is constant to better than one part in 10^3 over the central quarter-aperture. Although the system does not provide a good quadrupole field in the vicinity of the bore

Fig. 11. Results of integral field measurements showing superposition of dipole fields. Solid line is a straight line through measured values when field is at 45° to vertical and horizontal fields.

Fig. 12. Results of integral field measurements of magnet shown in Figs. 3 and 4 when operated in quadrupole mode with central field predictions of POISSON.

radius, it is significant how close the measurements come to the predictions.

The integral harmonic content of this excitation pattern was also measured and is given in Table III in percent of the $n = 2$ component extrapolated to the bore radius. The range of variation of the different multipoles with excitation current was negligible, varying generally by less than $\pm 0.02\%$ of the quadrupole component over the full range of current variation which was measured (50-200 A). As expected from the discussion in Sec. II, the largest contributions come from the $n = 6$, duodecapole term. Although this term dominates, others which might have been expected to be zero because of symmetry are still surprisingly large, possibly owing to symmetry violations in the assembly even though assembly was rather carefully controlled.

TABLE III

INTEGRAL HARMONIC CONTENT OF QUADRUPOLE
AND OCTUPOLE EXCITATION PATTERNS FOR 5.28-cm BORE

n	Quadrupole (Percent of n=2)	Octupole (Percent of n=4)
2	100.00	0.31
3	0.18	0.34
4	0.31	100.00
5	0.60	0.32
6	105.90	0.23
7	1.37	0.46
8	1.00	0.62
9	1.24	0.86
10	11.20	1.00
12	2.11	8.36
14	29.30	1.55
16	3.20	3.16

Figure 13 shows the results of exciting the magnet in different patterns to check whether linear superposition applies, i.e., whether one can easily rotate the quadrupole field distribution as in Fig. 11 for the dipole excitation. The plot was obtained by setting the various currents according to the proper prescription and then measuring the integral. This was done for a range of currents. Again, the solid line is a straight line through the 22.5° rotated field.

C. Octupole Excitation

When all currents were excited with equal magnitudes and alternating signs to produce an octupole distribution, we obtained the results given in the third column of Table III. The numbers are the integral field harmonic contents, expressed in percent of $n = 4$ at the bore radius. The first allowed harmonic above $n = 4$ is $n = 12$, which is the largest contribution even though this term would be zero for the ideal contour. From the numbers in Table III, this system would appear to be an acceptably good octupole over the central half of the aperture although no attempt was made to improve it either before or after fabrication except as discussed in Sec. III.

V. DISCUSSION

We have shown that general-purpose, multipole magnets are possible that have very specific

advantages and which are inexpensive and easy to use. In the case of the octupole geometry, it is easy to build magnets which provide multipoles or linear combinations of multipoles, oriented at any angle about the beam, that are accurate to better than $\pm 0.5\%$ over half the aperture.

There are many possible applications where such magnets would be useful, and they have been used at LAMPF in three quite different ways. Singly, they have been used as high-field steering magnets and as general-purpose quadrupoles to correct angular misalignments of a beam rotator system. In pairs, they have been used in conjunction with a zero-phase space beam to create any combination of x , θ , y , ϕ (and δ) phase space points for experimental ray tracing.³

A single octupole, serving as a quadrupole (Q) in a quadrupole-dipole (QD) or QDD spectrometer system could be used in the conventional way or to correct effects such as kinematic line-broadening⁴ associated with the finite acceptance angles in the scattering plane. In multiples, they could be used in periodic accelerating structures or high-resolution

Fig. 13. Results of integral field measurements showing superposition of quadrupole fields. Solid line is a straight line through measured values when the quadrupole field is rotated at 22.5°, intermediate between measurements for field orientations of 0° and 45°.

beam lines. These applications are all valid, but we believe these systems could prove useful for any application requiring one or more of the following characteristics.

- Unique, high-field uniformity of the dipole excitation.
- Small size of the magnets, for the field integrals which are provided.
- Ability to superimpose independently variable, multipole fields within the same magnetic volume rather than disperse them serially along a system.
- The ability to orient these fields at any angle about the axis of the magnet.

A comment is necessary concerning the overall cost of such a system - including its power supply and controls. As already discussed, it is possible to run several octupoles on a single unregulated power supply (bipolar). This results in a compact system which is easy and inexpensive to control. Furthermore, and perhaps of more importance, it is simple to use. Even if one does not have a computer available, it is possible to have a system which functions essentially the same way as the one described, using microprocessors which can easily and inexpensively handle all of the logical operations needed to run the octupole or duodecapole.

Finally, this prototype correction element has functioned well enough that two of the authors (KH and JES) have applied for a patent on these types of systems.

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